EMMI Rapid Reaction Task Force
Fluctuations and Correlations of conserved charges in nuclear collisions – Challenges and Future Prospects,
GSI, Darmstadt, Germany, 7 Nov. 2023

Some Topics on 2nd Order Fluctuations

Masakiyo Kitazawa (YITP, Kyoto)

Contents

A Coin Game

① Bet 25 Euro

② You get head coins of

A. 50 x 1 Euro

B. 25 x 2 Euro

Same expectation value.

A Coin Game

① Bet 25 Euro
② You get head coins of

A. 50 x 1 Euro

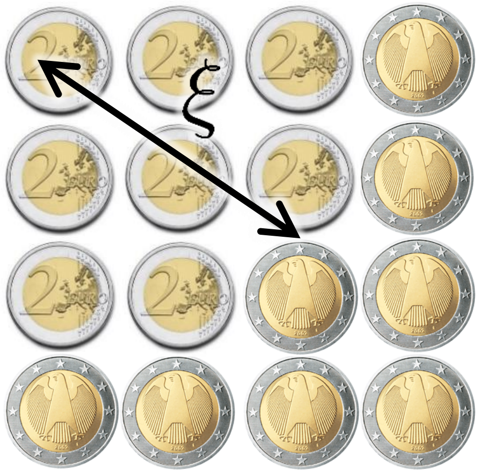
B. 25 x 2 Euro

C. 1 x 50 Euro

Same expectation value.
But, different fluctuation.

Fluctuations in HIC @ 2nd Order

Search for QCD CP



Fluctuation
increases

Stephanov, Rajagopal, Shuryak, 1998; 1999

Onset of QGP

Fluctuation
decreases

Asakawa, Heinz, Muller, 2000;
Jeon, Koch, 2000

Higher Order Cumulants

A. 50 x 1 Euro

B. 25 x 2 Euro

$$2\langle \delta€^2 \rangle = \langle \delta€^2 \rangle$$

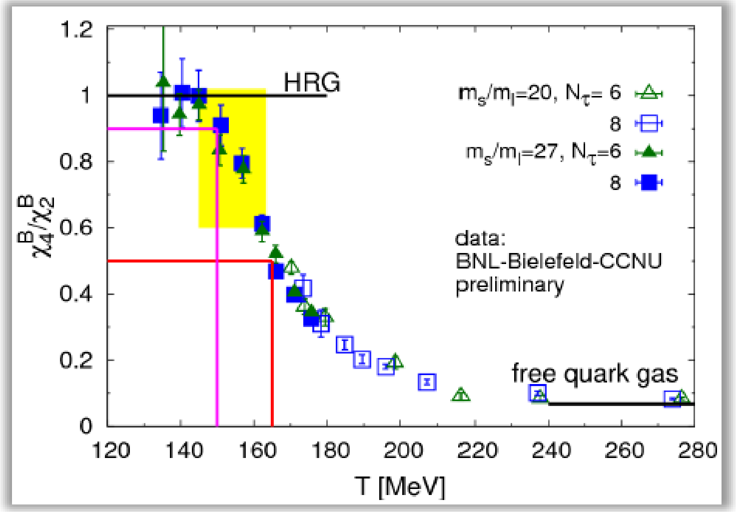
$$4\langle \delta€^3 \rangle = \langle \delta€^3 \rangle$$

$$8\langle €^4 \rangle_c = \langle €^4 \rangle_c$$

Asakawa, MK,
PPNP 90, 299
(2016)

Non-Gaussian Fluctuations

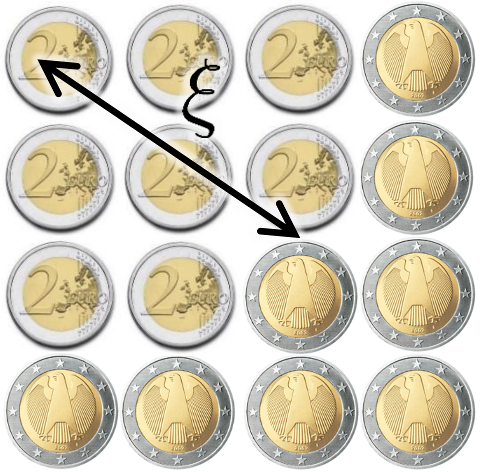
Onset of QGP



Fluctuation **decreases**

Ejiri, Karsch, Redlich, 2006

Search for QCD CP



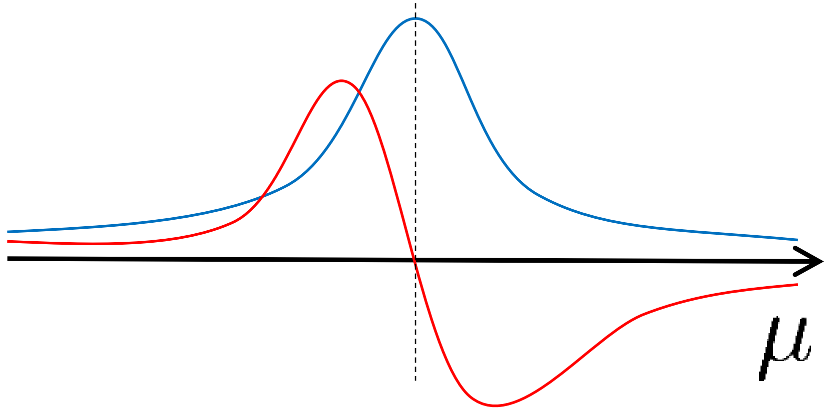
Fluctuation **increases**

Stephanov, 2009

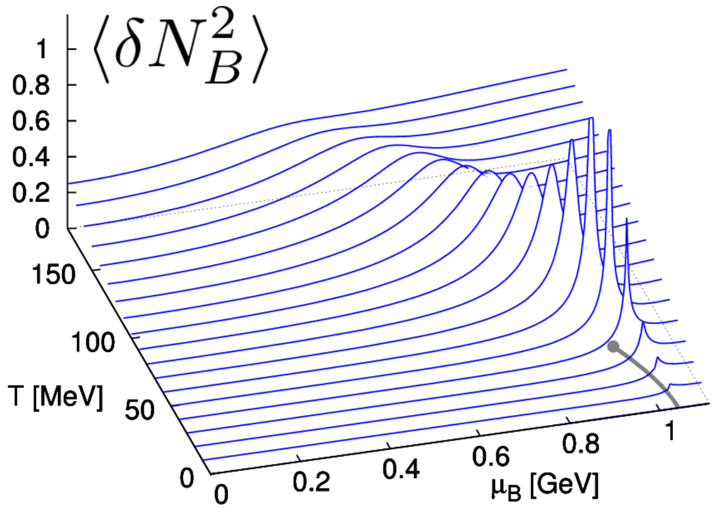
Sign of Higher Order Cumulants

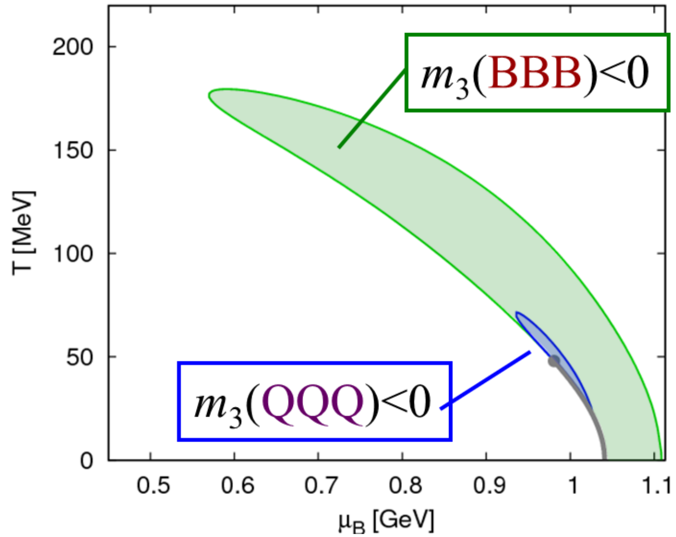
Geometric Interpretation

$$\langle \delta N^3 \rangle = T \frac{\partial \langle \delta N^2 \rangle}{\partial \mu}$$

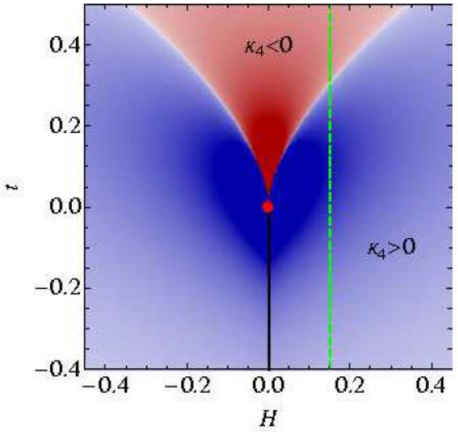


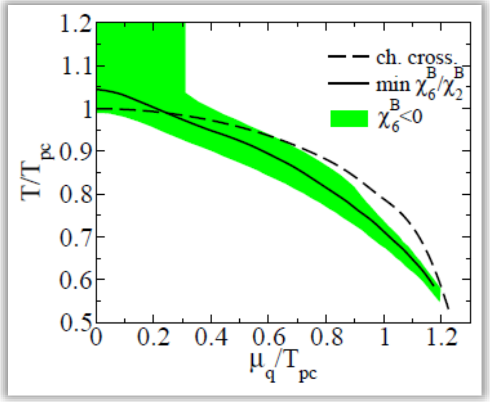
Asakawa, Ejiri, MK, 2009





Asakawa, Ejiri, MK, 2009





Stephanov, 2011;
Friman, Karsch, Redlich, Skokov, 2011; …

Fragile Higher Order Cumulants

Ex.: Relation b/w baryon & proton # cumulants

(with approximations)

MK, Asakawa, 2012

$$\begin{cases} 2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle_{\mathrm{free}} \\ 2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle_{\mathrm{free}} \\ 2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \cdots \end{cases}$$

genuine info. Poisson noise

Higher orders are more seriously affected by efficiency loss.
Volume fluctuations are another source of the modification.

Contents

D-Measure = Net-charge Fluctuations

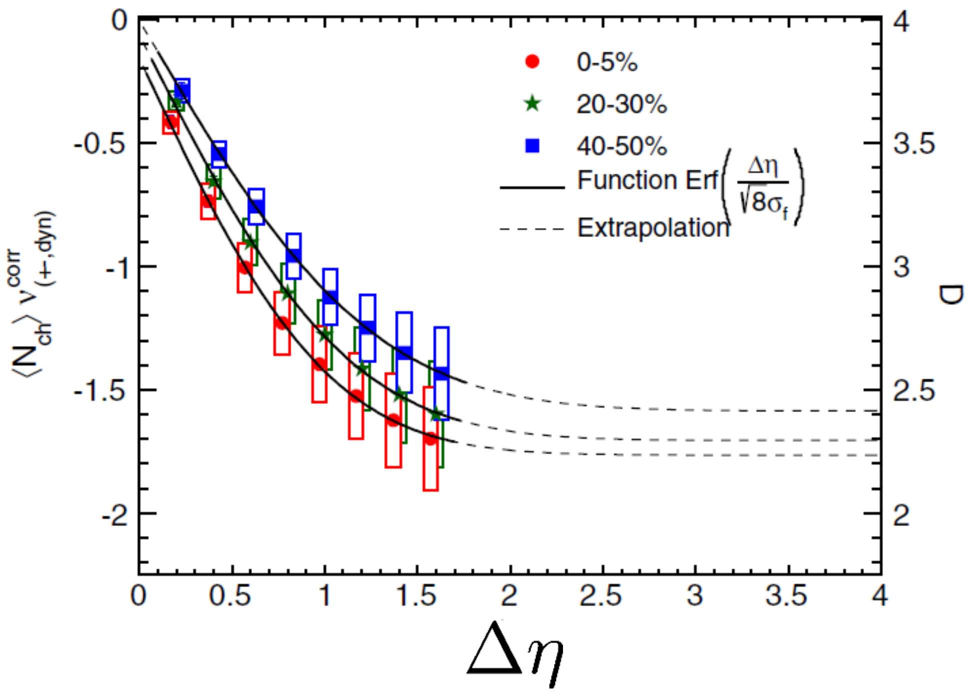
D-measure

$$D = 4\frac{\langle N_Q^2 \rangle_c}{\langle N_Q^{tot} \rangle}$$

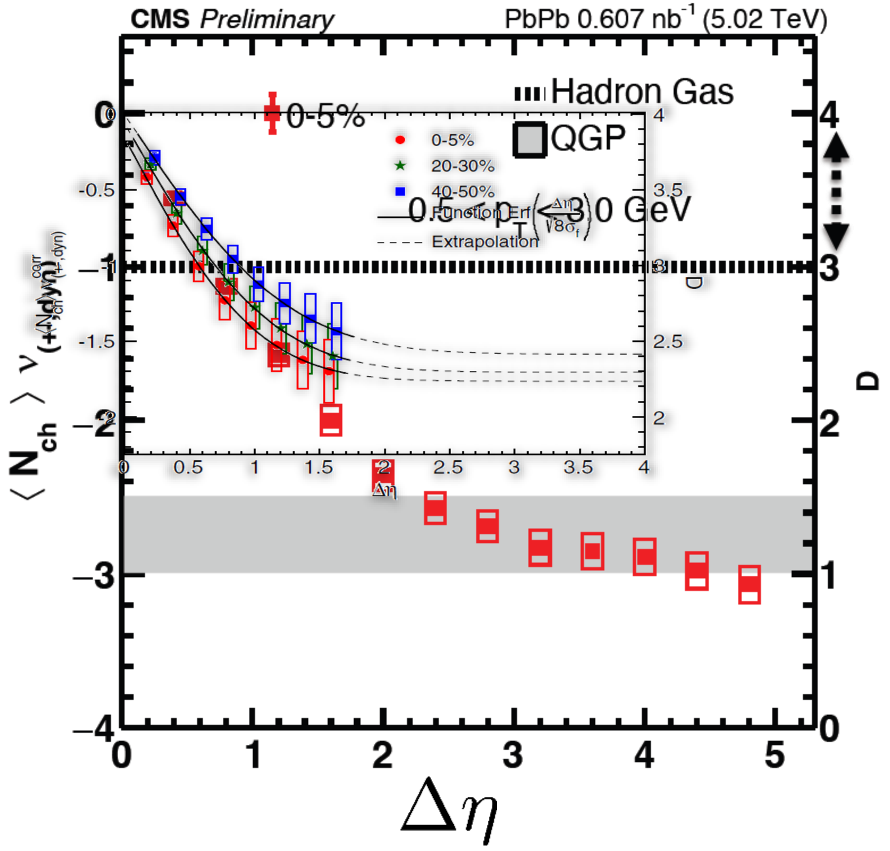
- Hadron gas $D \simeq 3$
- QGP $D \simeq 1 - 1.5$

Jeon, Koch, 2000

ALICE, PRL2013

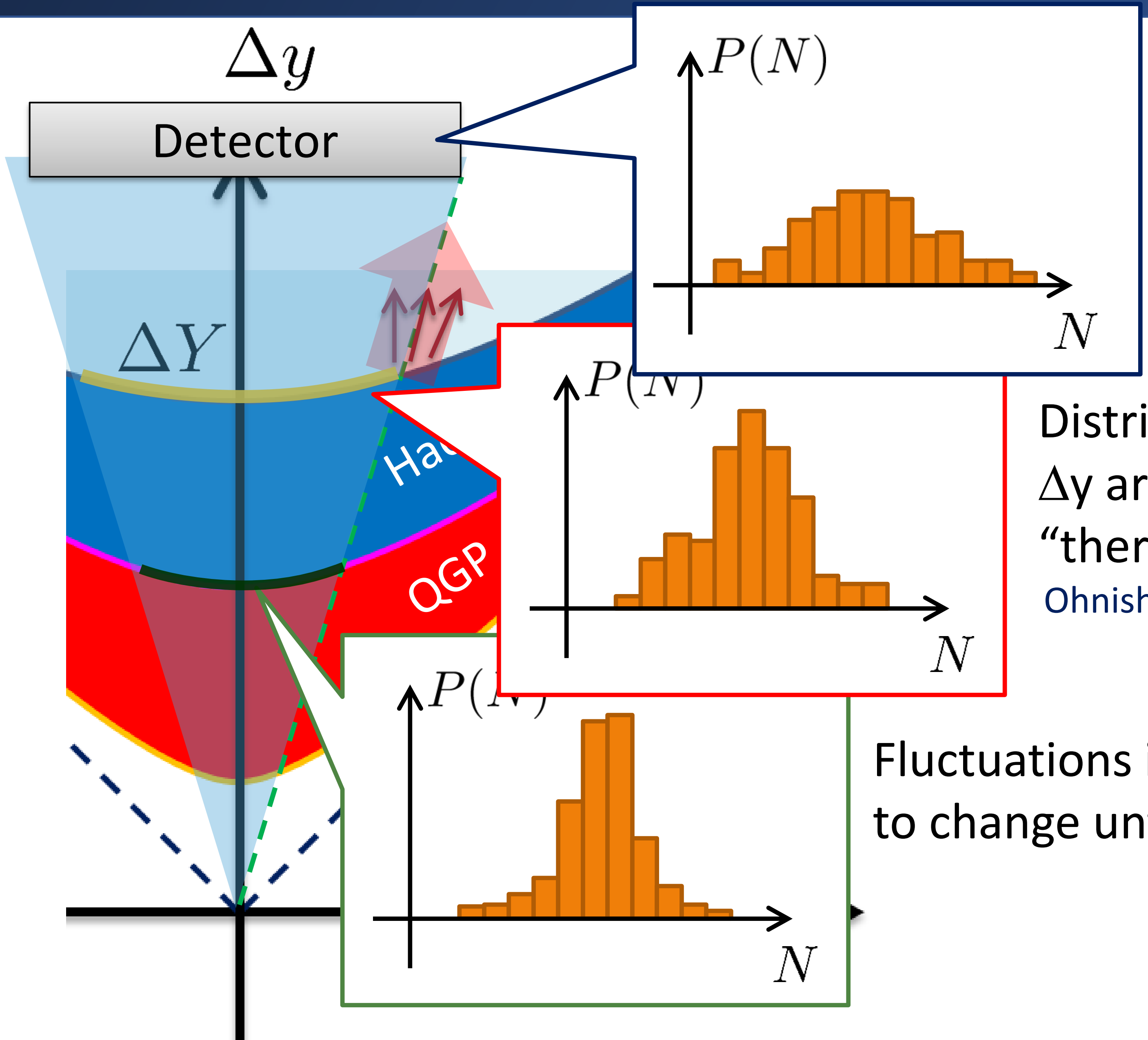


CMS (Tuo, QM2023)



$0.5 < p_T < 3.0$ GeV

Diffusion of Fluctuations

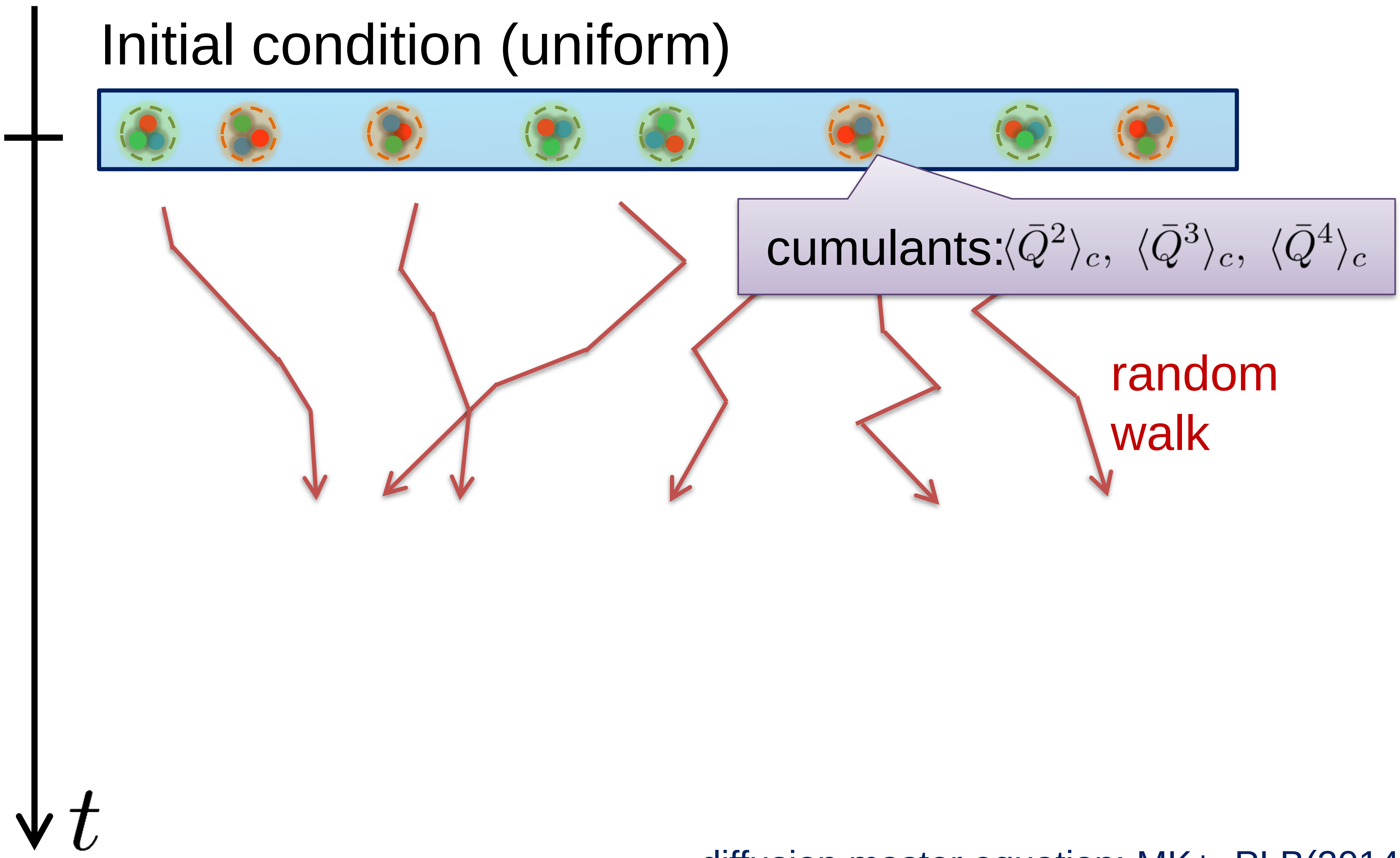


Distributions in ΔY and Δy are different due to "thermal blurring".

Ohnishi, MK, Asakawa, PRC(2016)

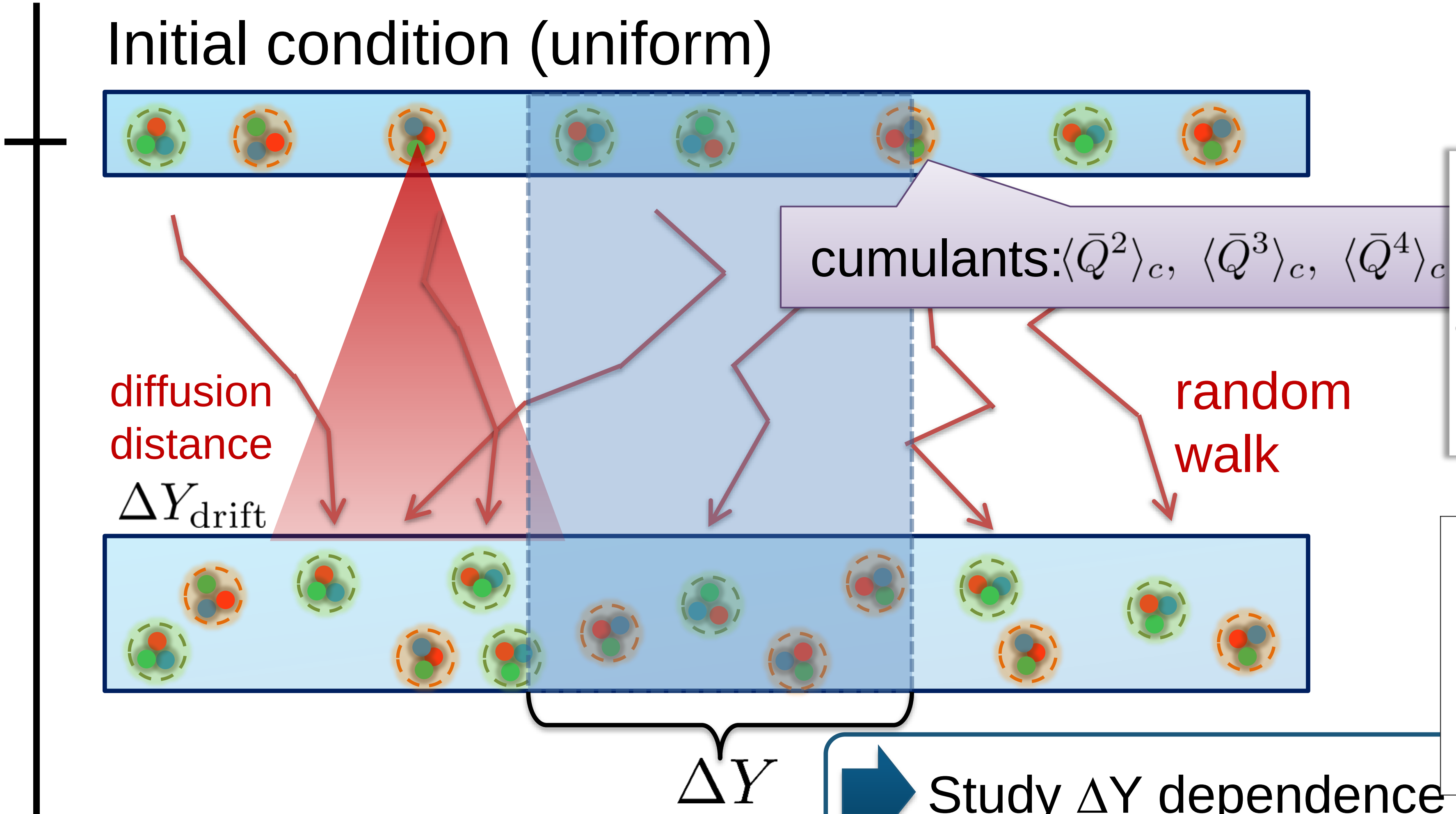
Fluctuations in ΔY continue to change until kinetic f.o.

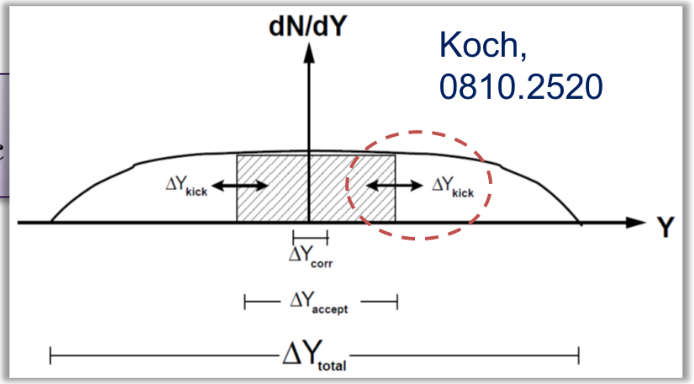
(non-interacting) Brownian Particle Model

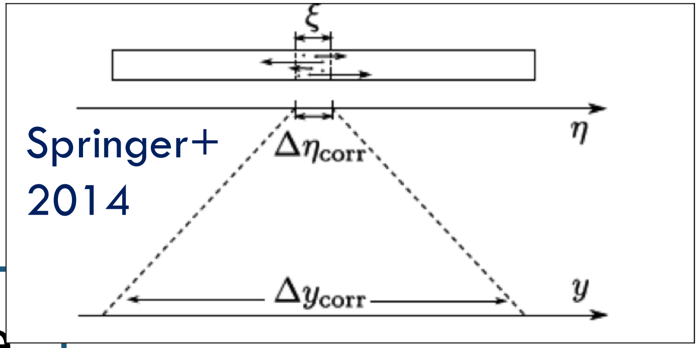


diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

(non-interacting) Brownian Particle Model







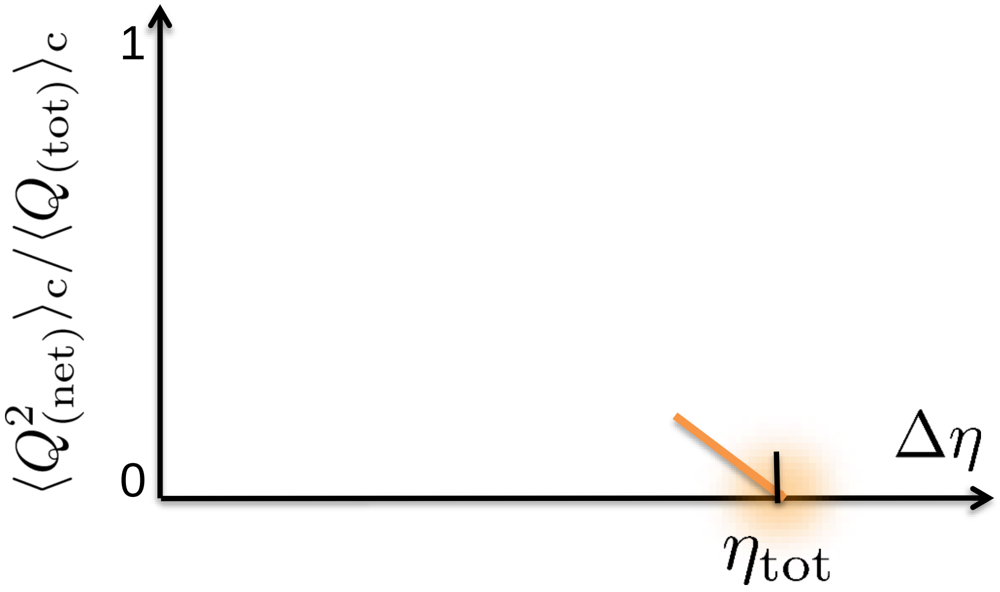
$t \to \infty$
Poisson distribution

Study ΔY dependence

diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

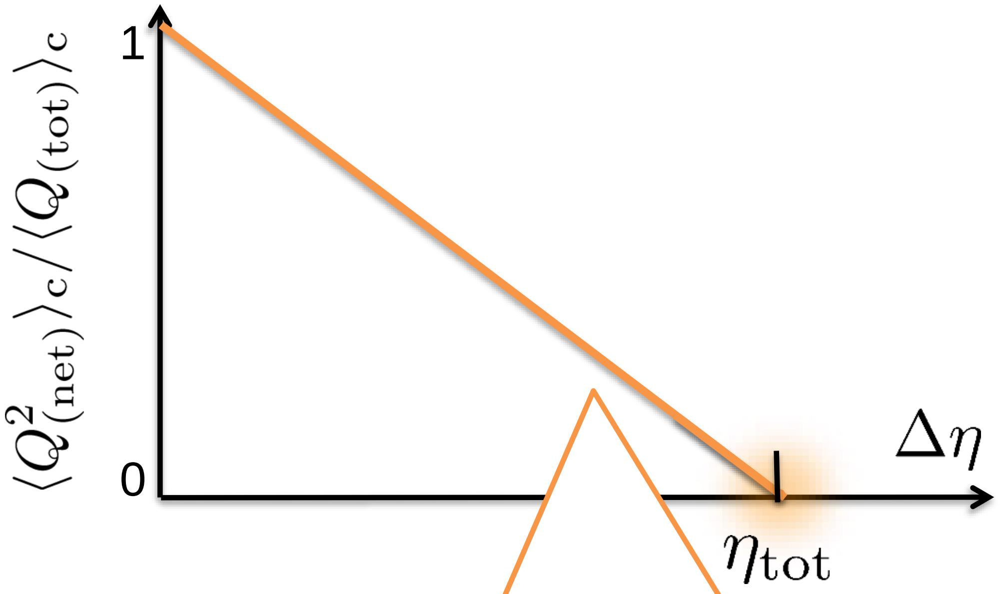
Global Charge Conservation

Conserved charges in the total system do no fluctuate!



Global Charge Conservation

Conserved charges in the total system do no fluctuate!



GCC Effect without local correlations

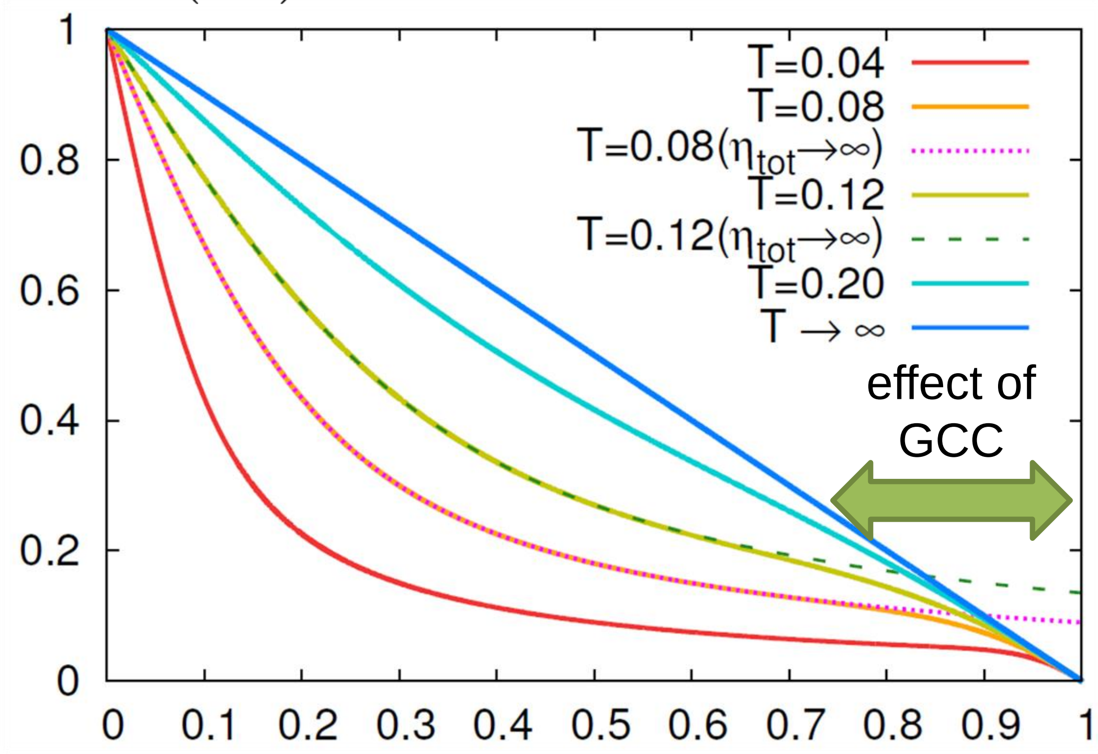
$$\langle \delta N^2 \rangle_{\mathrm{GCC}} = \langle \delta N^2 \rangle_{\mathrm{inf}} \times \left(1 - \frac{\Delta\eta}{\eta_{\mathrm{tot}}}\right)$$

Diffusion in Finite Volume

Sakaida, Asakawa, MK, PRC, 2014

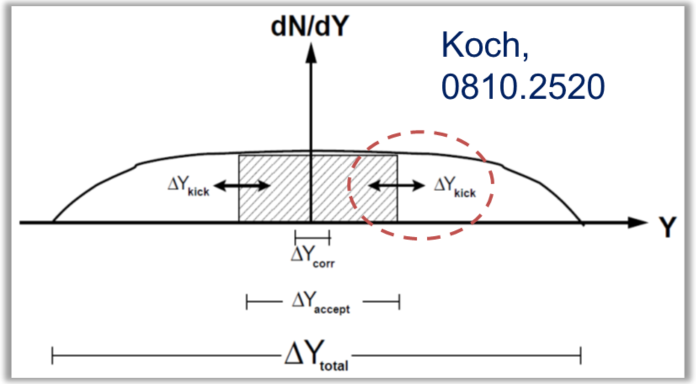
Result of the diffusion model in finite volume

$\langle Q^2_{(\mathrm{net})}\rangle_c / \langle Q_{(\mathrm{tot})}\rangle_c$

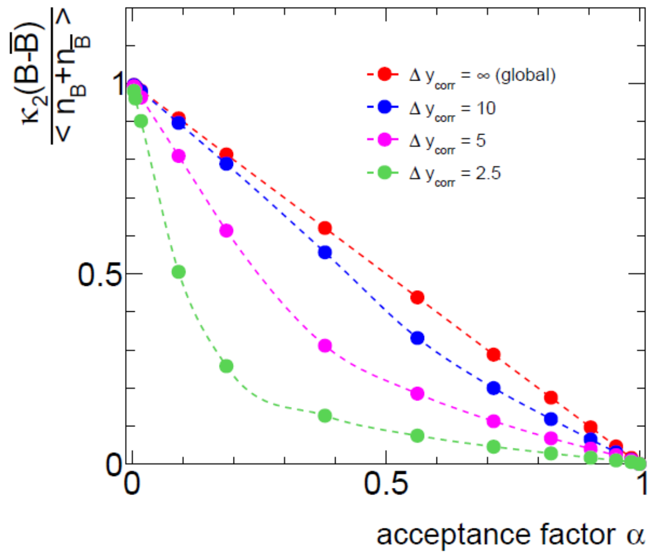


$\Delta\eta/\eta_{\mathrm{tot}}$

- $d(\tau)$: diffusion distance
- η_{tot}: total rapidity



$$T = \frac{d(\tau)}{\eta_{\mathrm{tot}}}$$

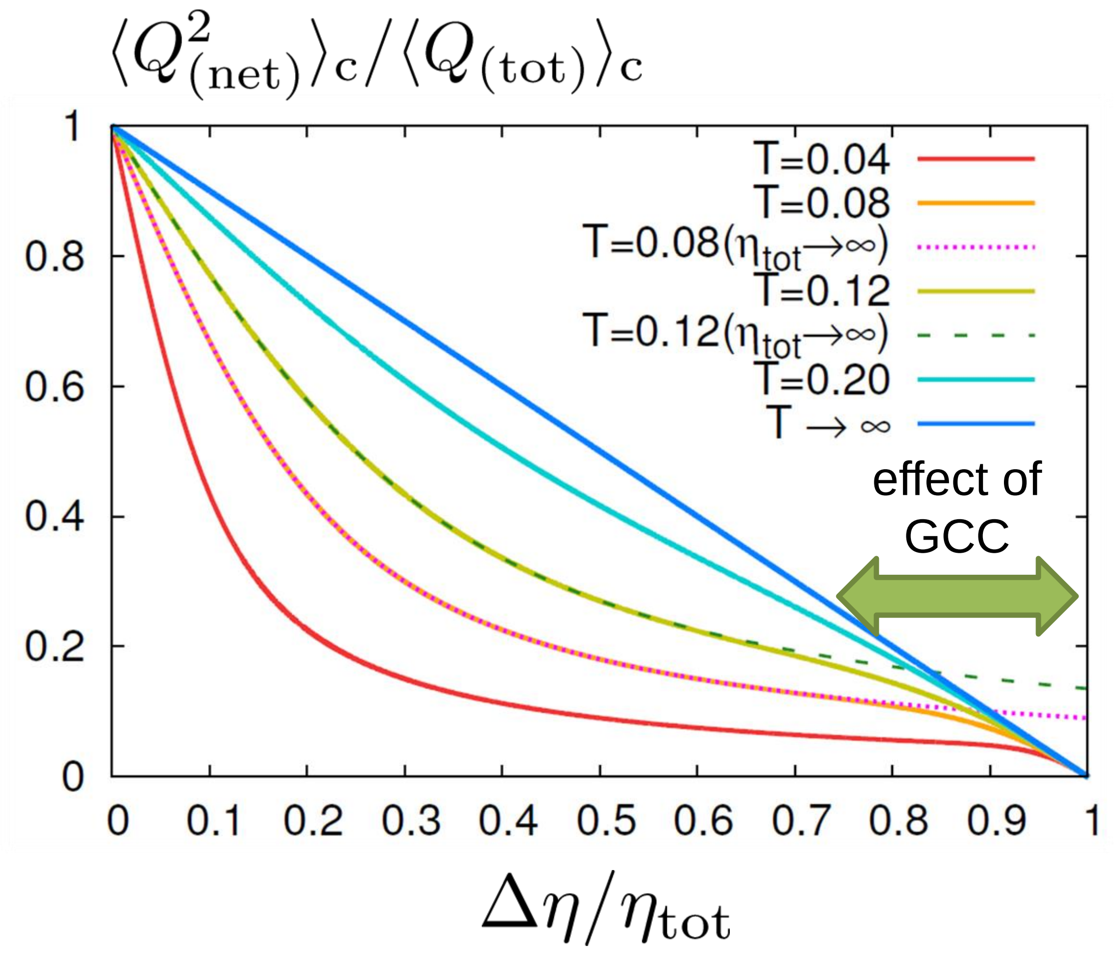


PBM, Rustamov, Stachel
1907.03032

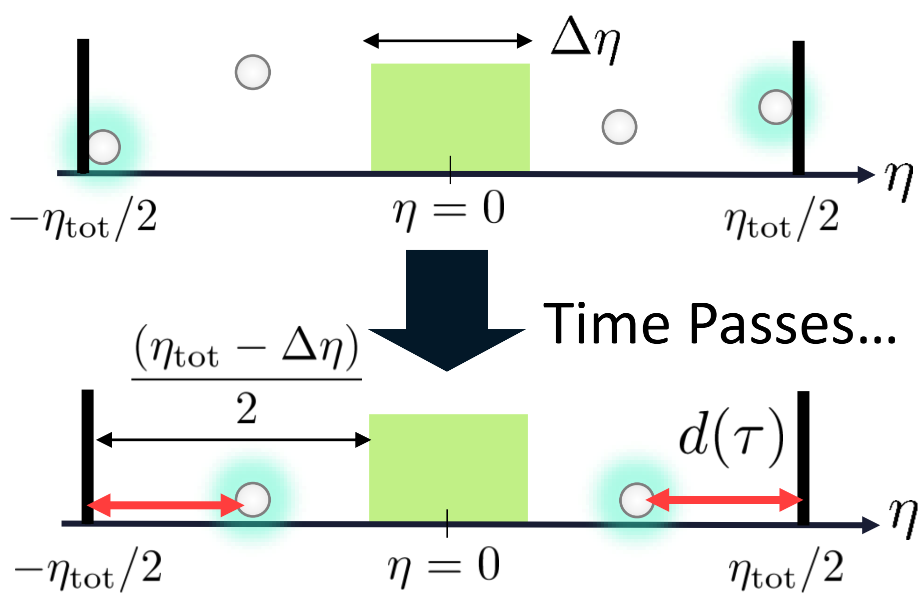
Diffusion in Finite Volume

Sakaida, Asakawa, MK, PRC, 2014

Result of the diffusion model in finite volume



Physical Interpretation

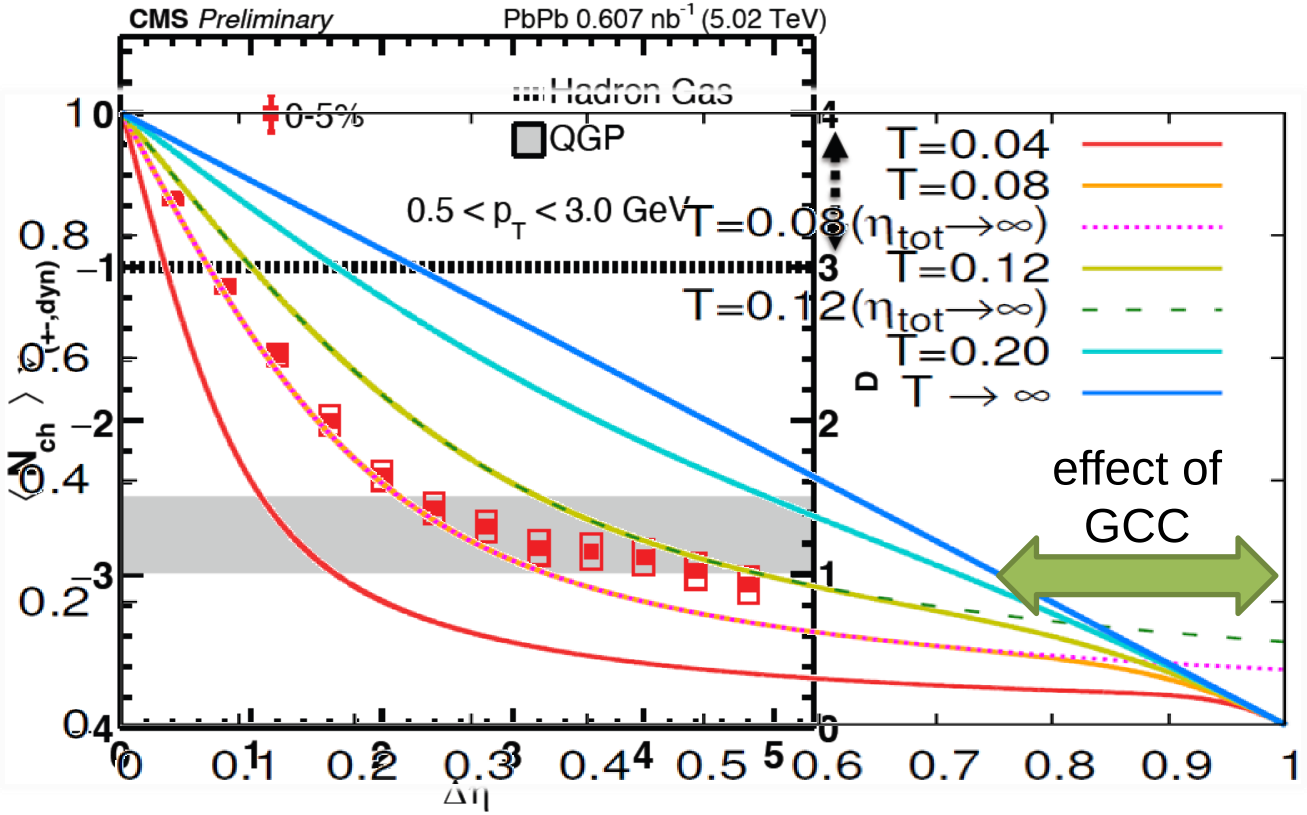


$d(\tau)$: Averaged Diffusion Distance

η_{tot} : Total Length of Matter

Comparison with CMS Result

MK, in progress for preparation

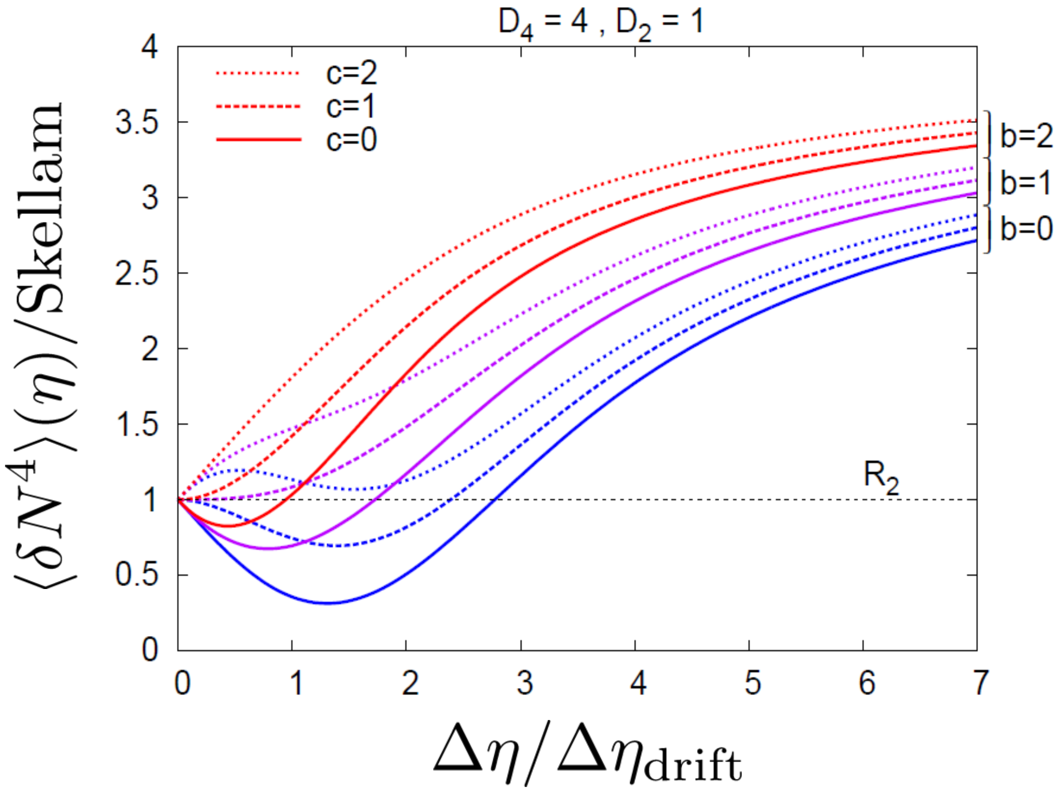


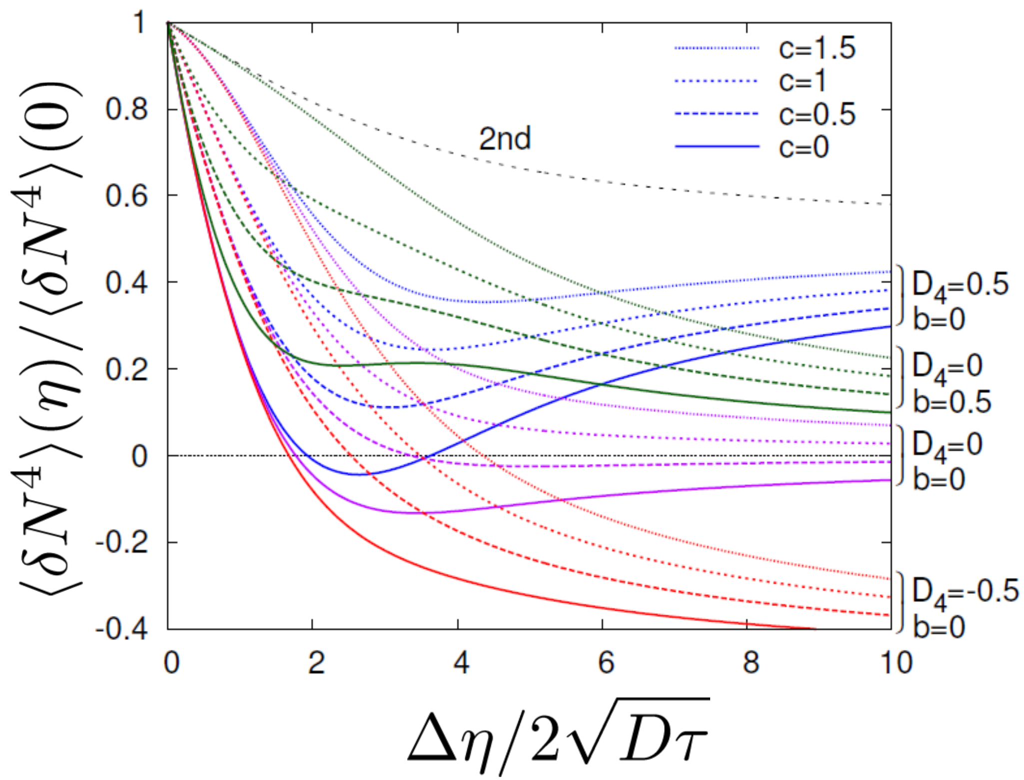
- CMS result is well described by a simple diffusion model in finite volume.
- Effects of the global charge conservation do not affect the results for $\Delta\eta < 5$.

Higher Order Cumulants in the Diffusion Model

MK+ (2014); MK (2015)

Predictions for higher order cumulants





- Higher order cumulants can have various non-monotonic $\Delta\eta$ dependence reflecting the initial condition.
- **Their experimental measurement gives us insights into it!**

Revisiting Estimates of D-measure

D-measure

$$D = 4\frac{\langle N_\mathrm{Q}^2\rangle_\mathrm{c}}{\langle N_\mathrm{Q}^\mathrm{tot}\rangle}$$

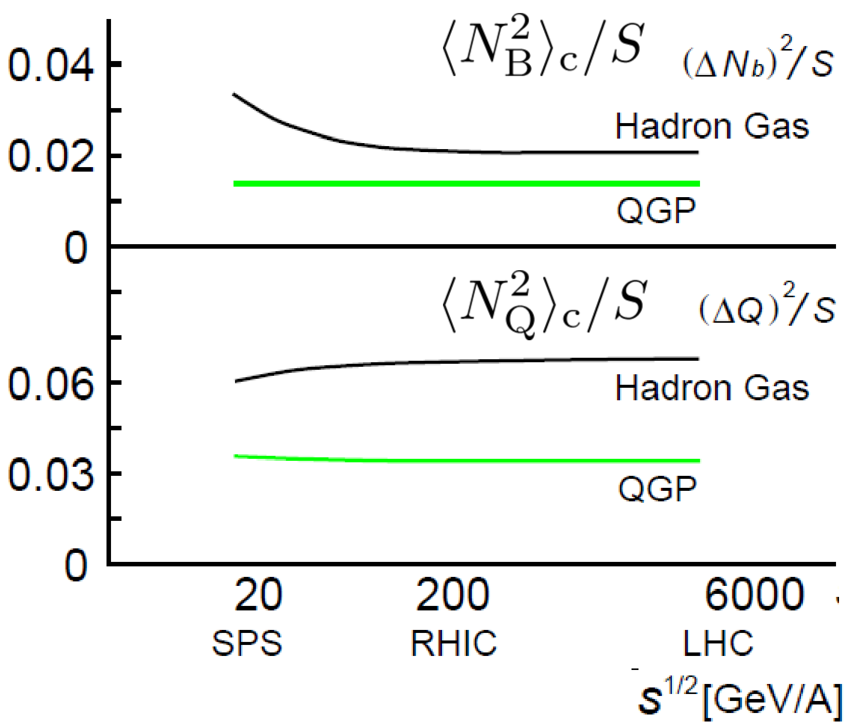
- Hadron gas
  $D \simeq 3$
- QGP
  $D \simeq 1 - 1.5$

Jeon, Koch, 2000

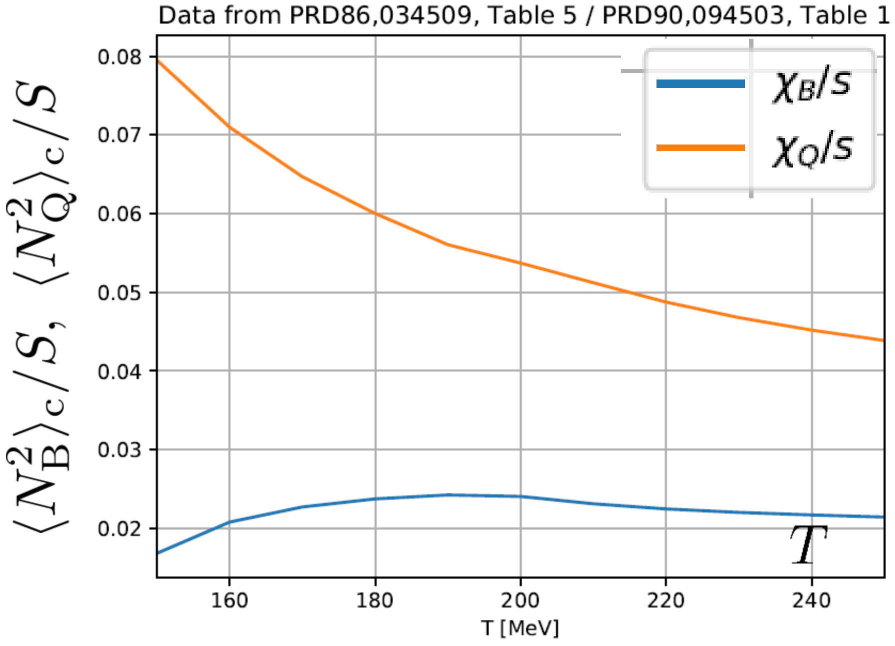
$$D \sim \frac{\langle N_\mathrm{Q}^2\rangle_\mathrm{c}}{S}$$

Prediction in 2000

Asakawa, Heinz, Mueller ('00)



From Recent Lattice Data

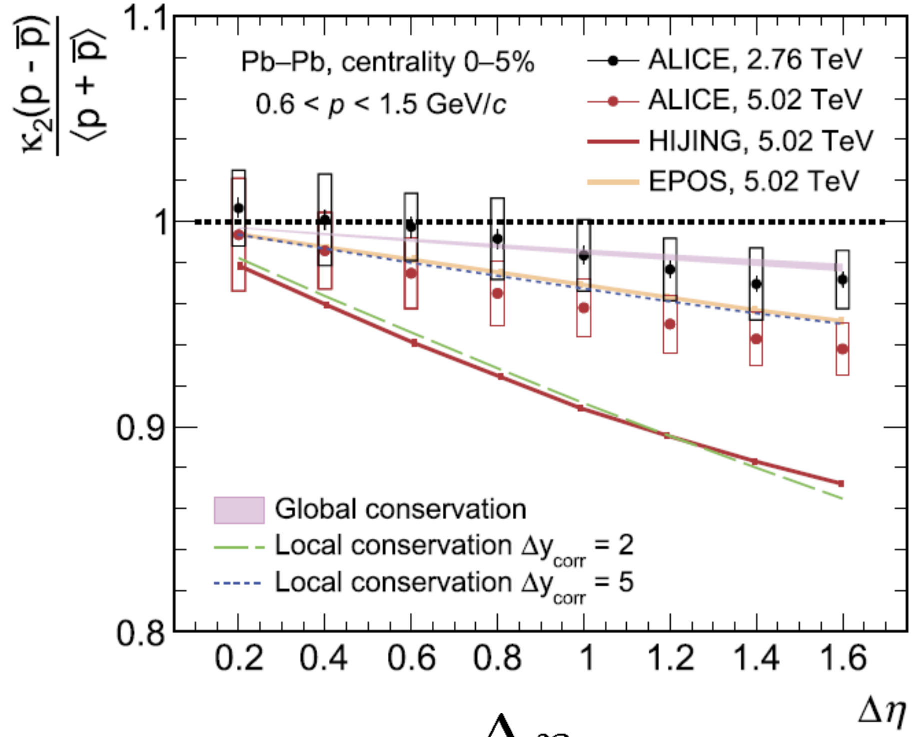


Bazavov+ ('12); Bazavov+ ('14)

- Estimate on $\langle N_B^2\rangle, \langle N_Q^2\rangle$ should be made using latest lattice results.
- The value of $\langle N_B^2\rangle/S$ are insensitive to T.

Net-Proton Fluctuations @ ALICE

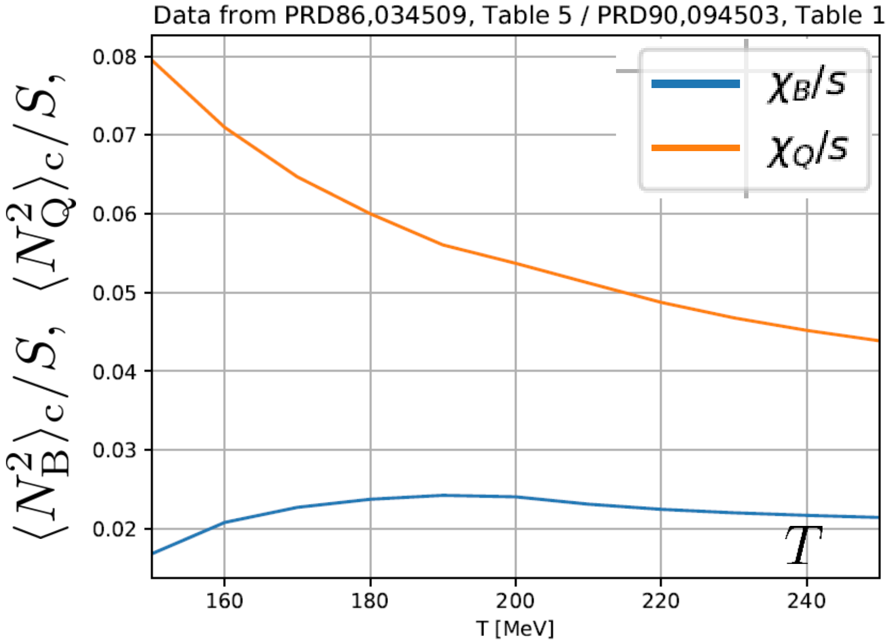
Net proton fluctuation



$\Delta\eta$

ALICE, PLB (2023)

From Recent Lattice Data



Bazavov+ ('12); Bazavov+ ('14)

- ☐ Weak T dependence of $\langle N_B^2 \rangle / S$ may explain the ALICE data on the proton fluctuation.

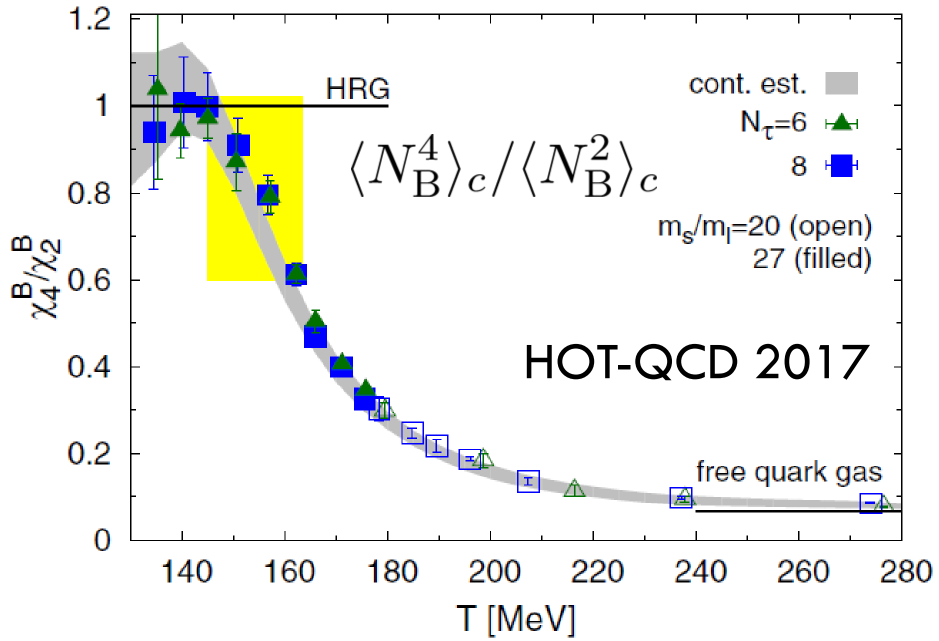
Contents

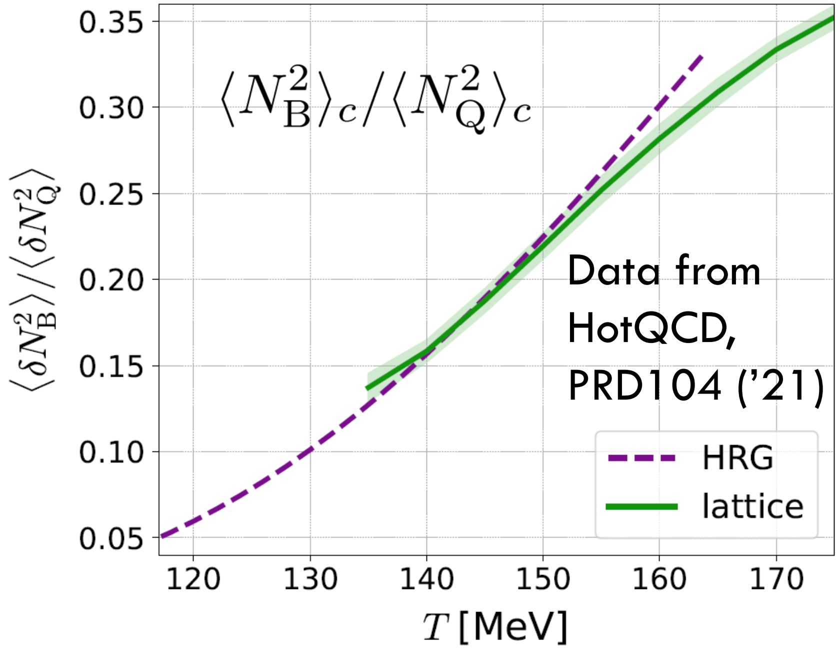
$\langle N_B^2\rangle_c/\langle N_Q^2\rangle_c$

MK, Esumi, Nonaka, 2205.10030

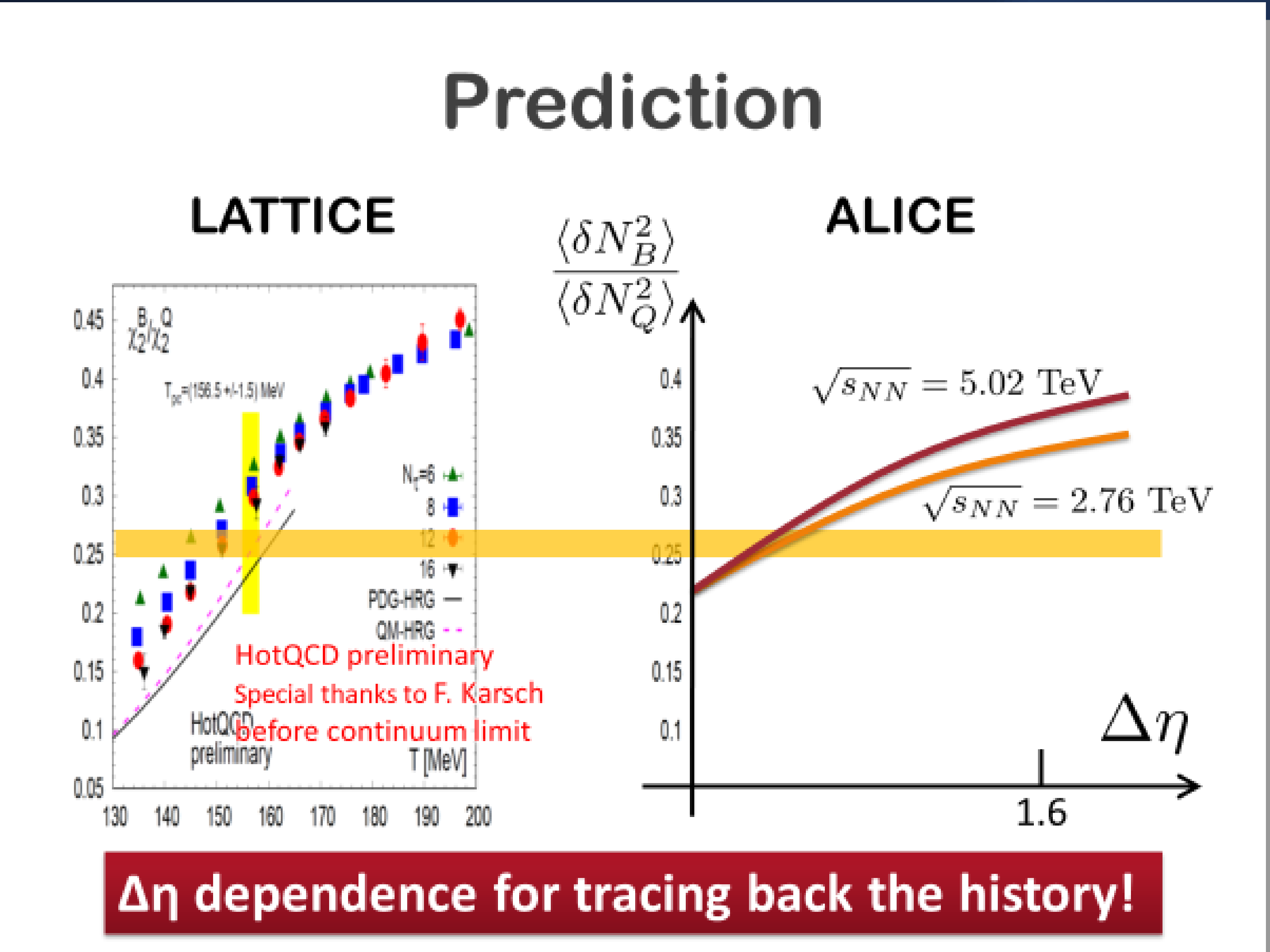
$\langle N_{\rm B}^2\rangle_c/\langle N_{\rm Q}^2\rangle_c$

- Ratio of 2nd order: Suppress uncertainties from various experimental effects compared with higher orders.
- Almost linear T dependence around T_c^*.





From my Slide for RRTF2019



Experimental Data

$\langle N_p^2 \rangle_c$

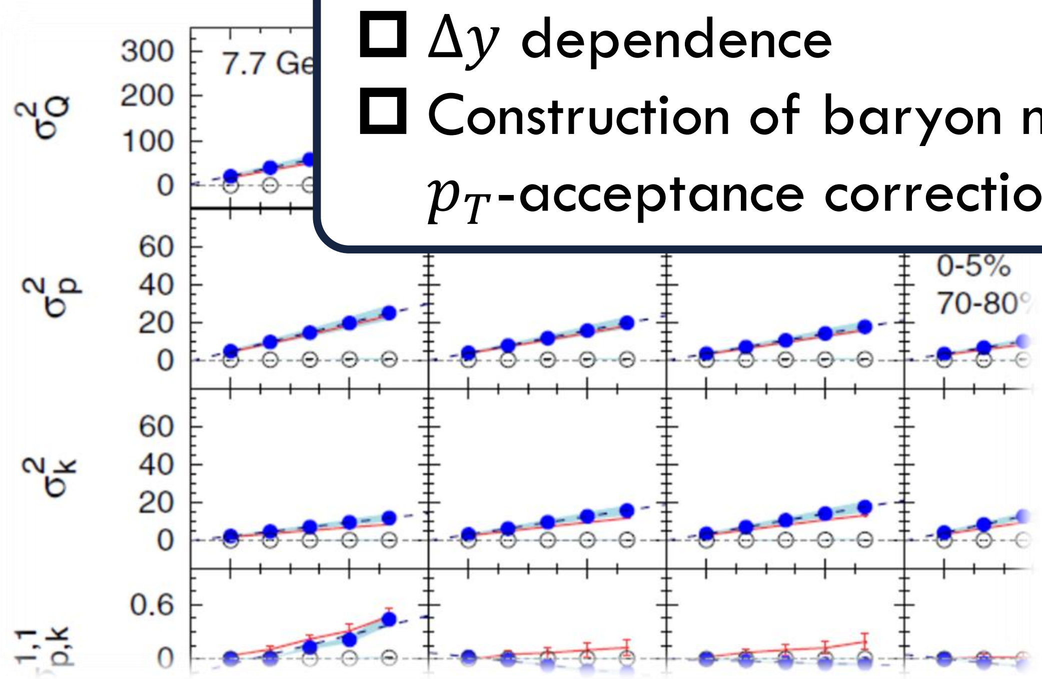
STAR, PRC104,024902 (2021)

- proton cumulants up to 4th order
- **rapidity window** Δy
- $0.4 < p_T < 2.0$GeV/c

$\langle N_Q^2 \rangle_c$

STAR, PRC100,014902 (2019)

- 2nd mixed cumulants of p, ,pi, K, Q
- **pseudo-rapidity window** $\Delta\eta$
- $0.4 < p_T < 1.6$GeV/c
- Total charge: private comm. A. Chattergee



- ☐ $\sqrt{s_{NN}} = 200$GeV
- ☐ 0-5% centrality
- ☐ Δy dependence
- ☐ Construction of baryon number, p_T-acceptance correction

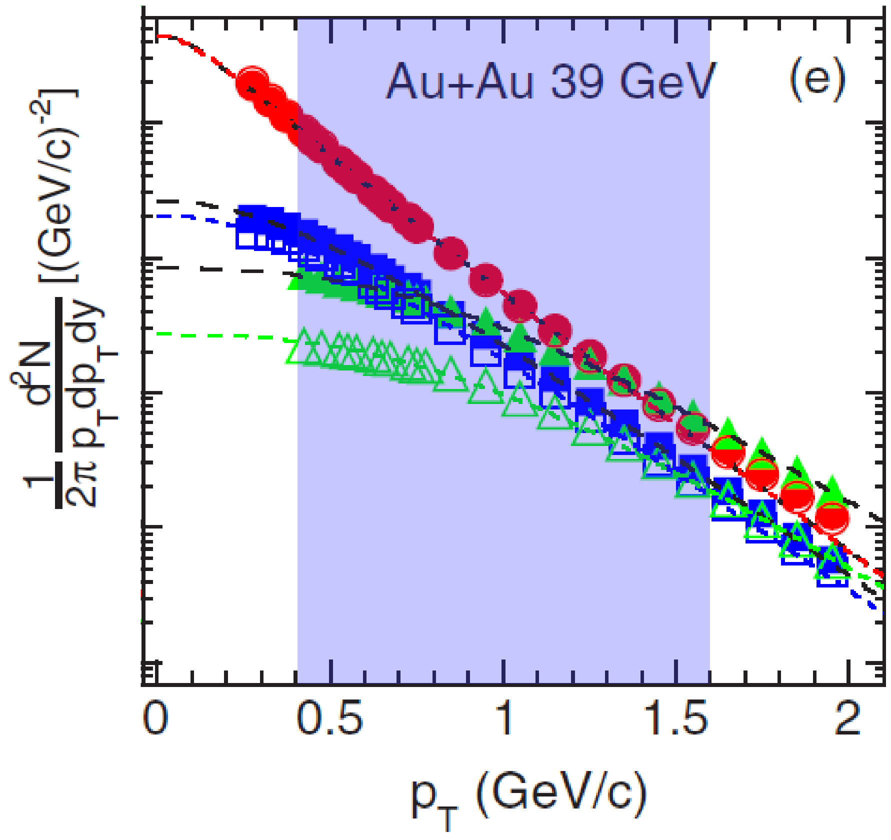
- ☐ proton → baryon cumulants MK, Asakawa,'12;'12
- ☐ Rapidity is better than pseudo-rapidity Ohnishi, MK, Asakawa, '16
- ☐ Wider acceptance is more desirable.

p_T-Acceptance Correction

p_T Acceptance

0.4<p_T<1.6 [GeV/c]
PRC100,014902('19)

0.4<p_T<2.0 [GeV/c]
PRC104,024902('21)



Particles in p_T space

- Electric charge: **49%**
- Protons: **82%**

blast wave model @ $\sqrt{s_{NN}}$=200 GeV

Modification by p_T-cut should be corrected.
This study: Binomial distribution model.

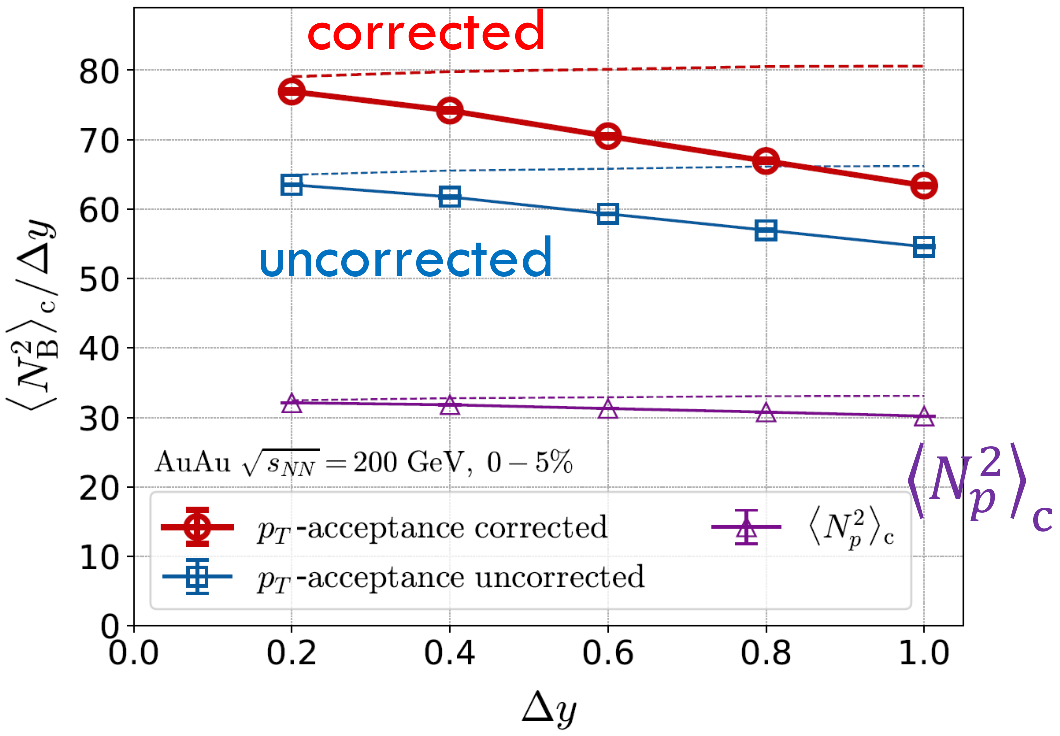
$$\langle N_{\rm net}^2\rangle_c^{\rm corrected} = \frac{1}{p^2}\Big(\langle n_{\rm net}^2\rangle_c - (1-p)\langle n_{\rm tot}\rangle_c\Big)$$

MK, Asakawa, '12, '12

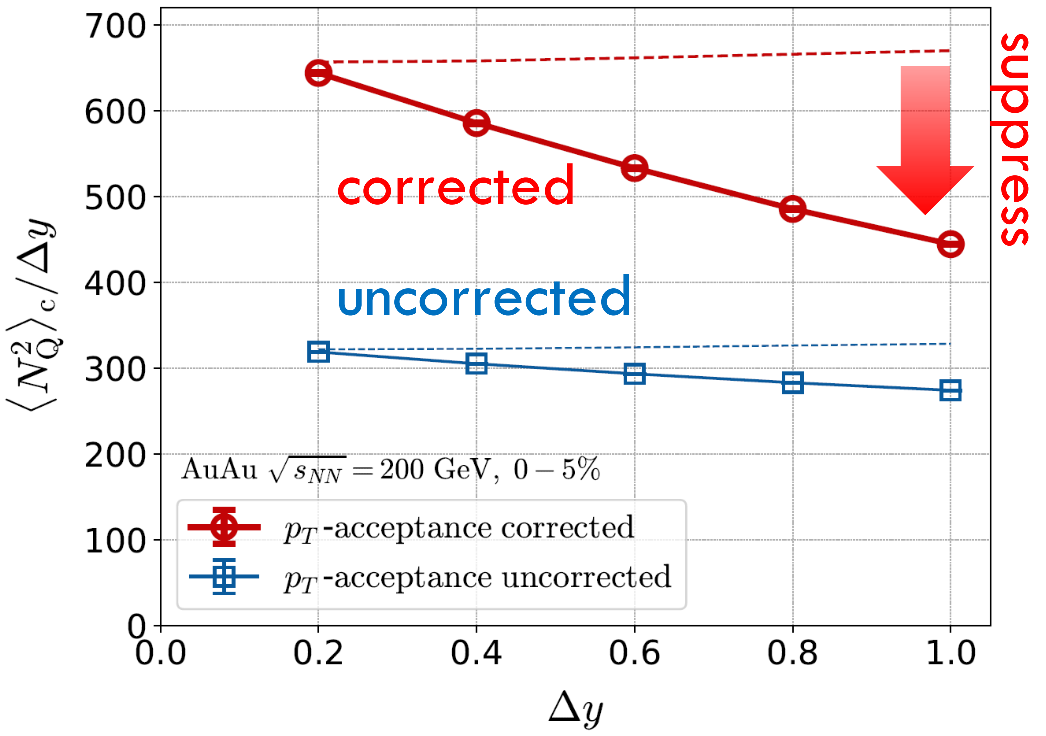
Cumulants: Proton→Baryon & Acceptance Correction

Data from STAR, '19, '21

$\langle N_B^2 \rangle_c / \Delta y$



$\langle N_Q^2 \rangle_c / \Delta y$



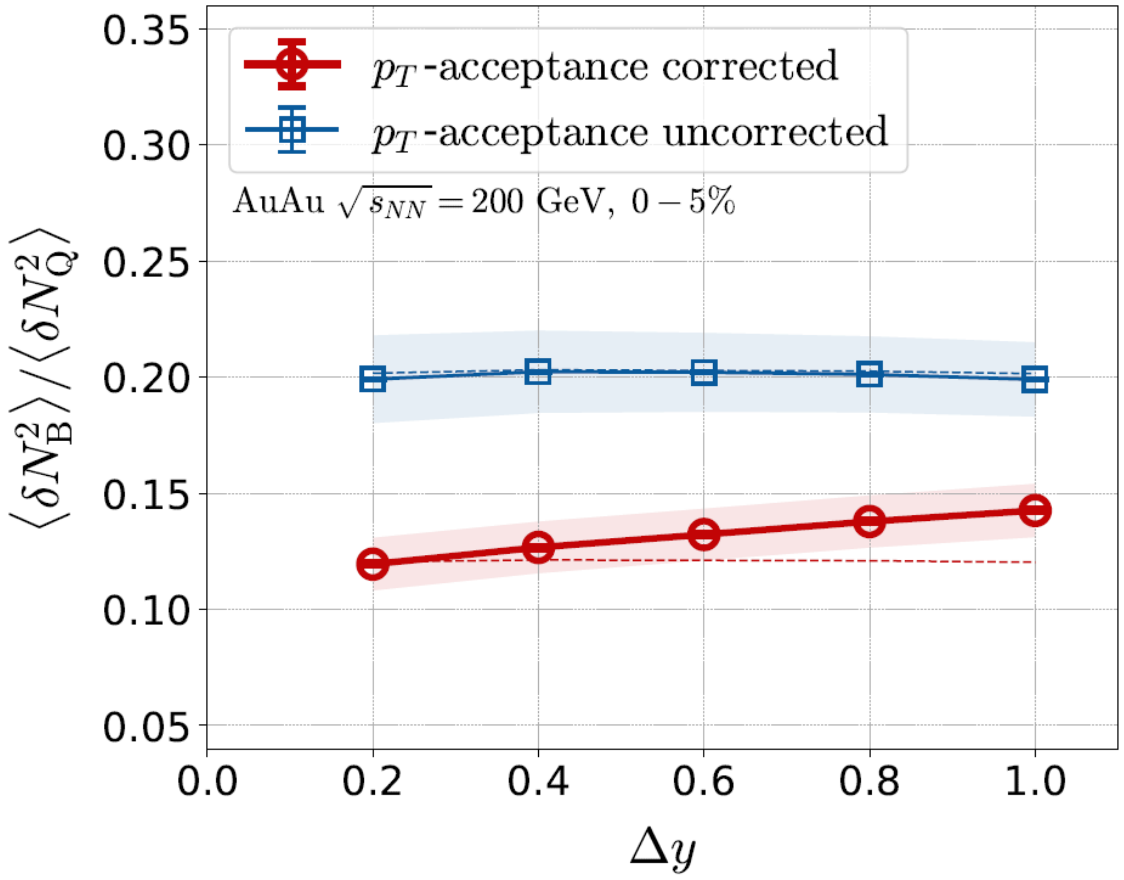
dashed:
$\langle N^{tot} \rangle$
Poisson

- Deviation from Poissonian is clarified by the acceptance correction.

$$\langle N_{\rm net}^2 \rangle_c^{\rm corrected} = \frac{1}{p^2}\Big(\langle n_{\rm net}^2 \rangle_c - (1-p) \langle n_{\rm tot} \rangle_c \Big)$$

MK, Asakawa, '12, '12

$\langle N_B^2\rangle_c/\langle N_Q^2\rangle_c$



p_T-acceptance **un**corrected

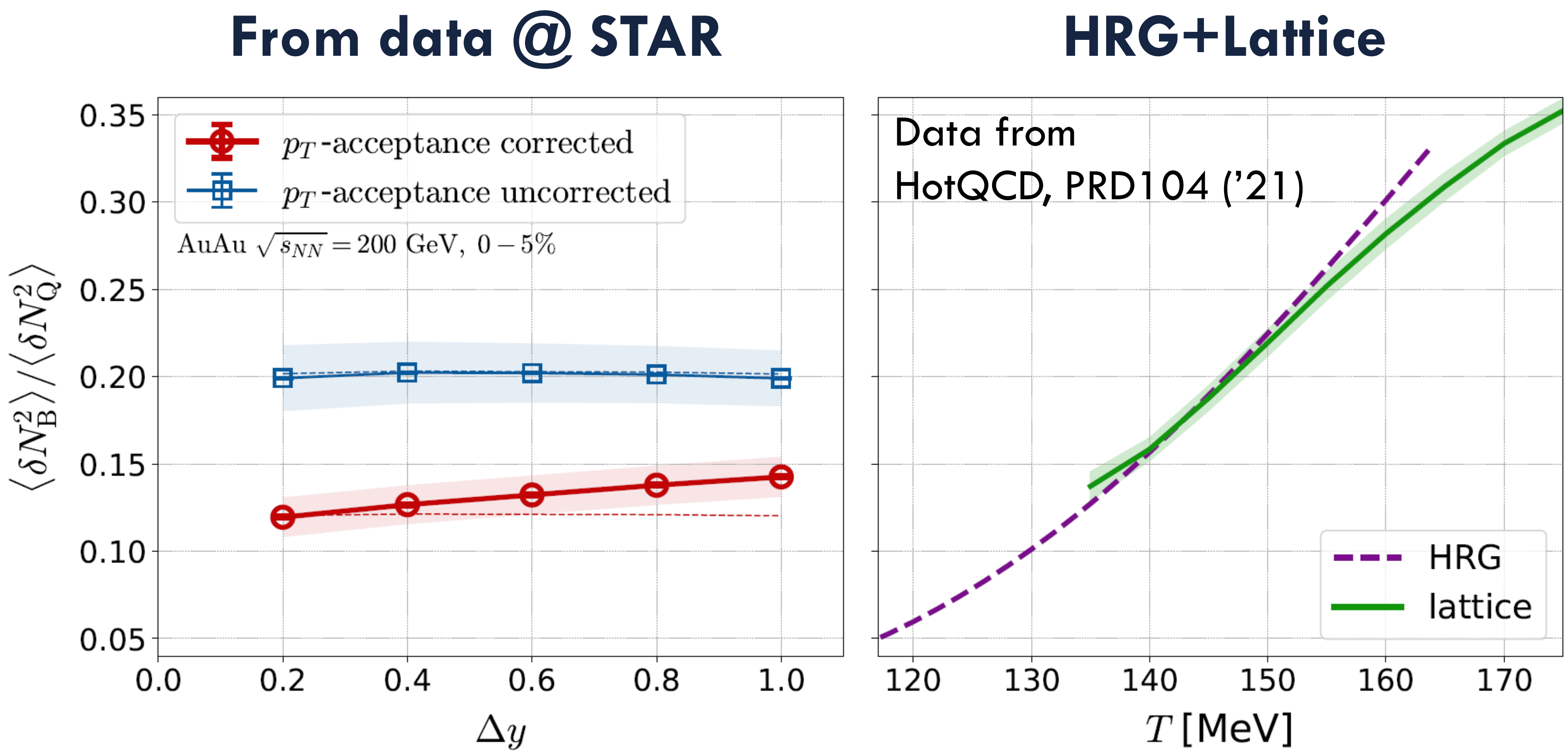
p_T-acceptance corrected

dashed: Poisson

- ☐ $\langle N_B^2\rangle_c/\langle N_Q^2\rangle_c$ becomes smaller due to the p_T-acceptance correction.
- ☐ Clear Δy dependence ➡ non-thermal effects behind fluctuations

HIC vs HRG&LAT

From data @ STAR / HRG+Lattice

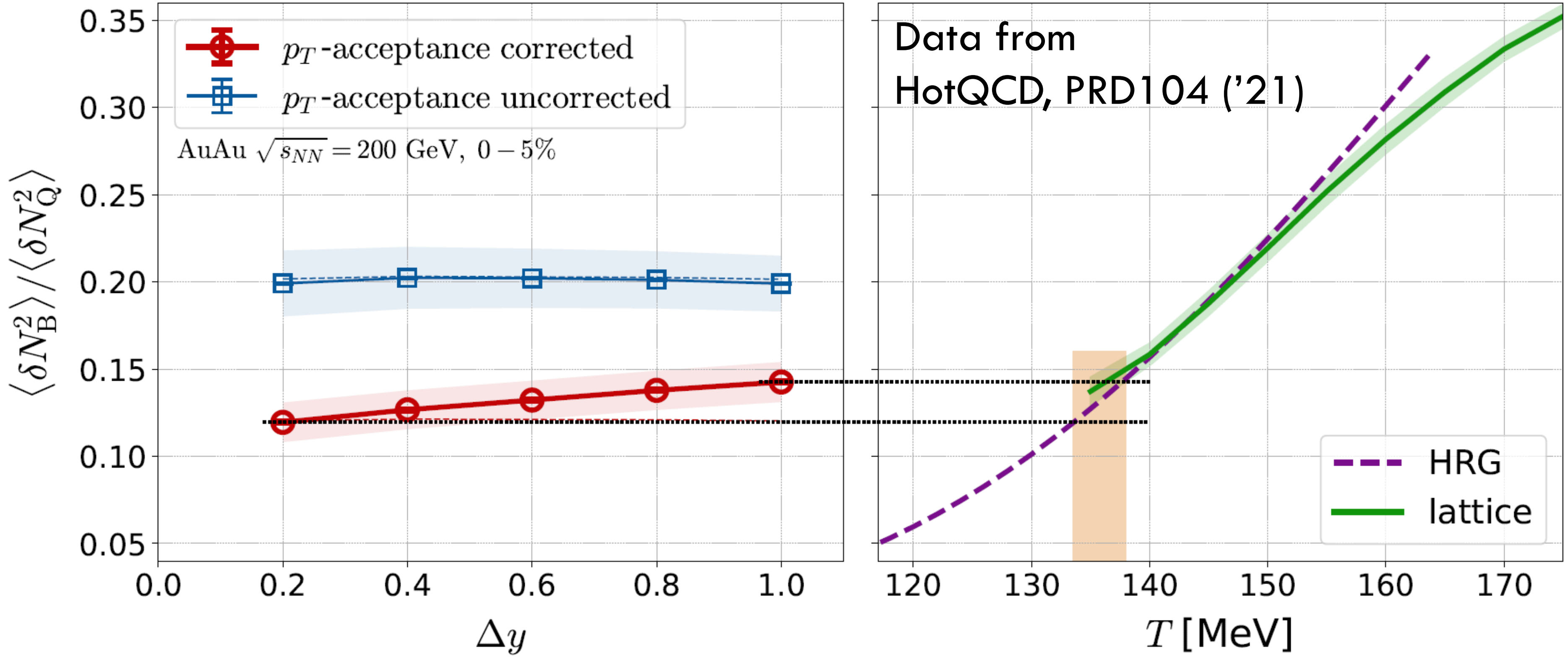


- $T = 134 \sim 138$ MeV
- Significantly lower than $T_{\rm chem}$

HRG: QMHRG2020
Bollweg+, PRD104, 7 ('21)
Volume dep. corrected
plot by MK

HIC vs HRG&LAT

From data @ STAR　　HRG+Lattice



- $T = 134 \sim 138$ MeV
- Significantly lower than $T_{\rm chem}$

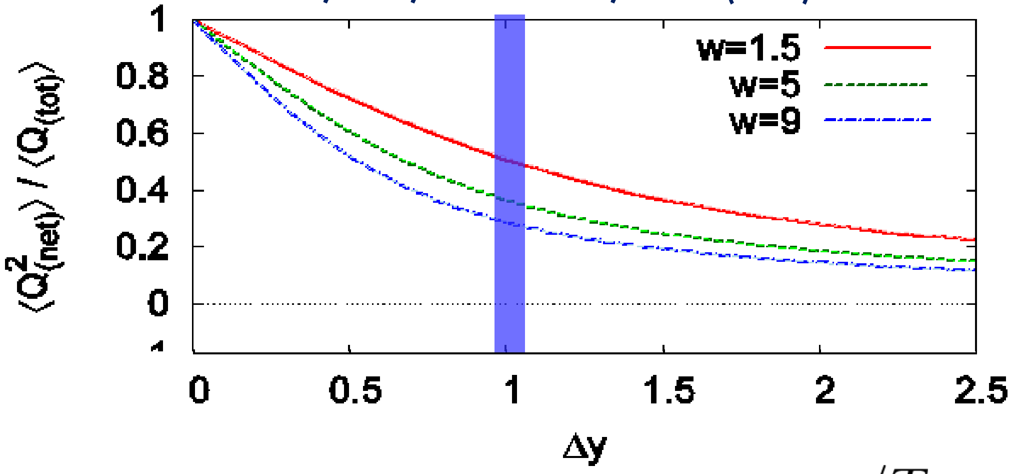
HRG: QMHRG2020
Bollweg+, PRD104, 7 ('21)
Volume dep. corrected
plot by MK

Effect of Diffusion and Rapidity Conversion

- Blurring due to diffusion & rapidity conversion ($Y \to y$)
  - Stronger modification in Q than B
- Resonance Decays
  - About 30% charged particles come from RD
  - Enhancement of charged particles

$\Rightarrow$
- Increase $\langle N_Q^2 \rangle$
- Reduce $\langle N_B^2 \rangle_c / \langle N_Q^2 \rangle_c$

Ohnishi, MK, Asakawa, PRC ('16)

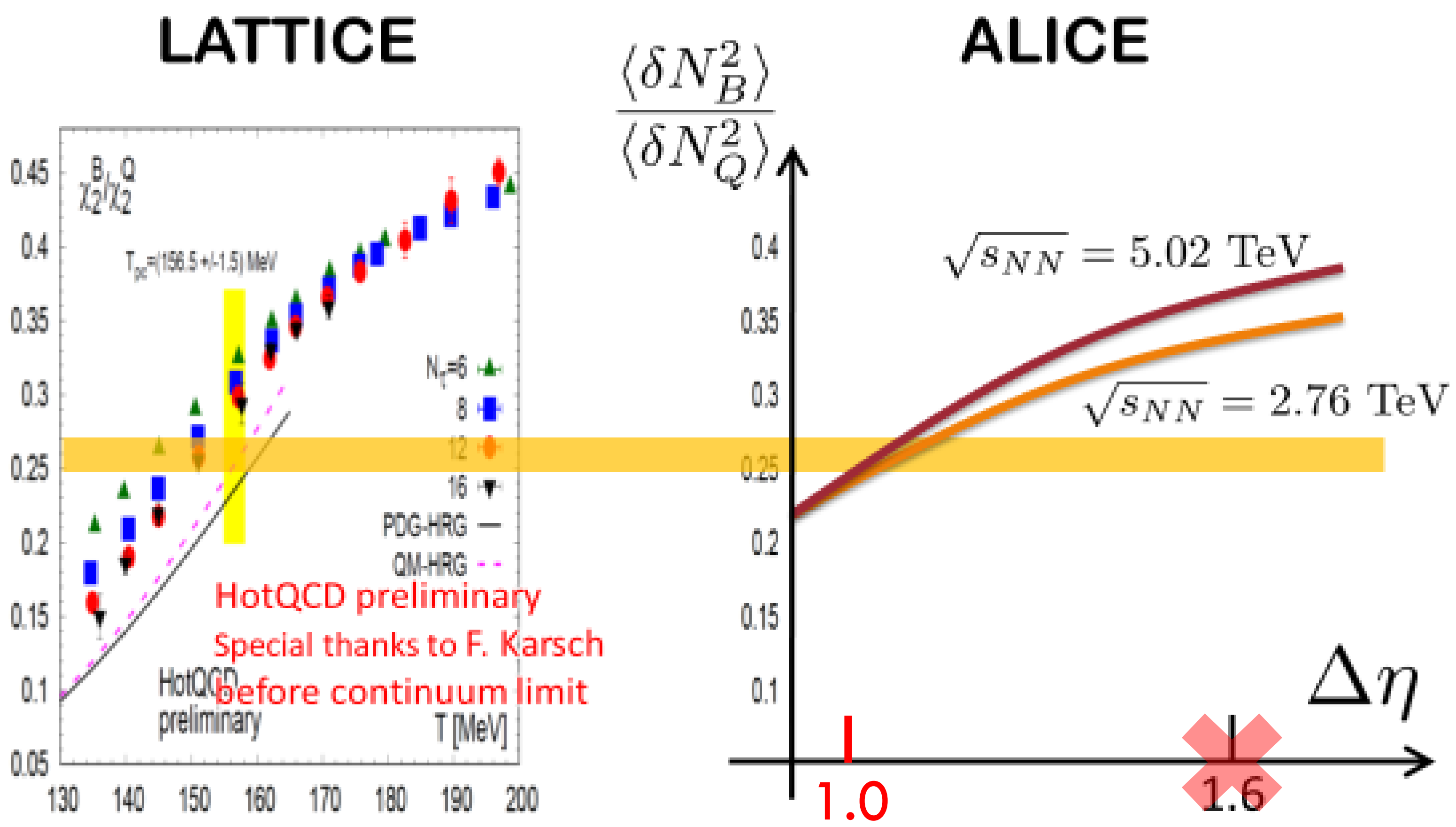


$w = m/T$

These effects will be more important for higher order cumulants!

From my Slide for RRTF2019

Prediction



Δη dependence for tracing back the history!

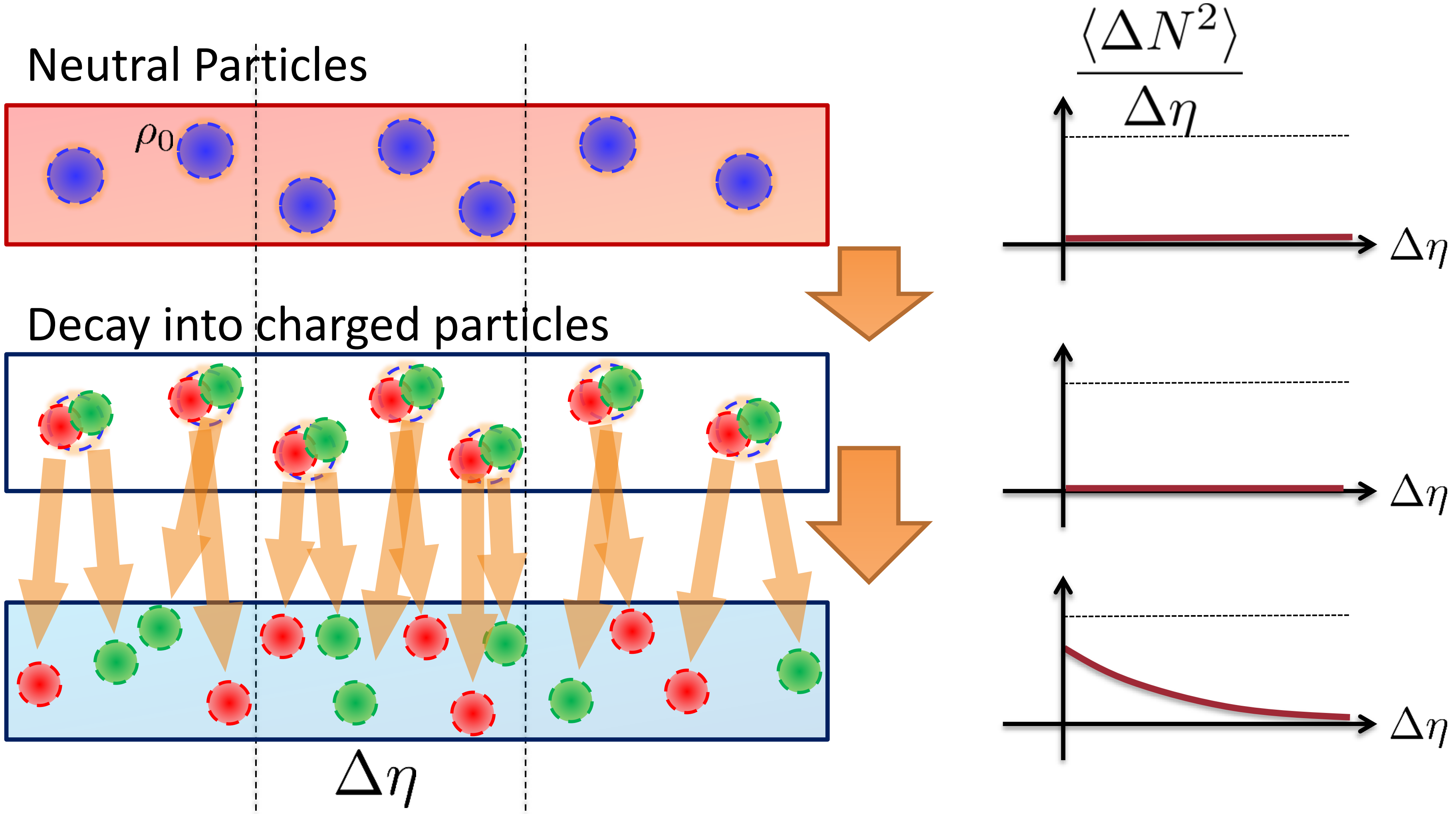
Summary

- ☐ $\Delta\eta$ dependence of fluctuations encodes the information of its evolution.
- ☐ New CMS result looks consistent with a simple diffusion picture.

- ☐ Temperature extracted from baryon/charge ratio is significantly lower than the one at chemical freezeout.
- ☐ p_T acceptance correction is essential to extract correct cumulants.

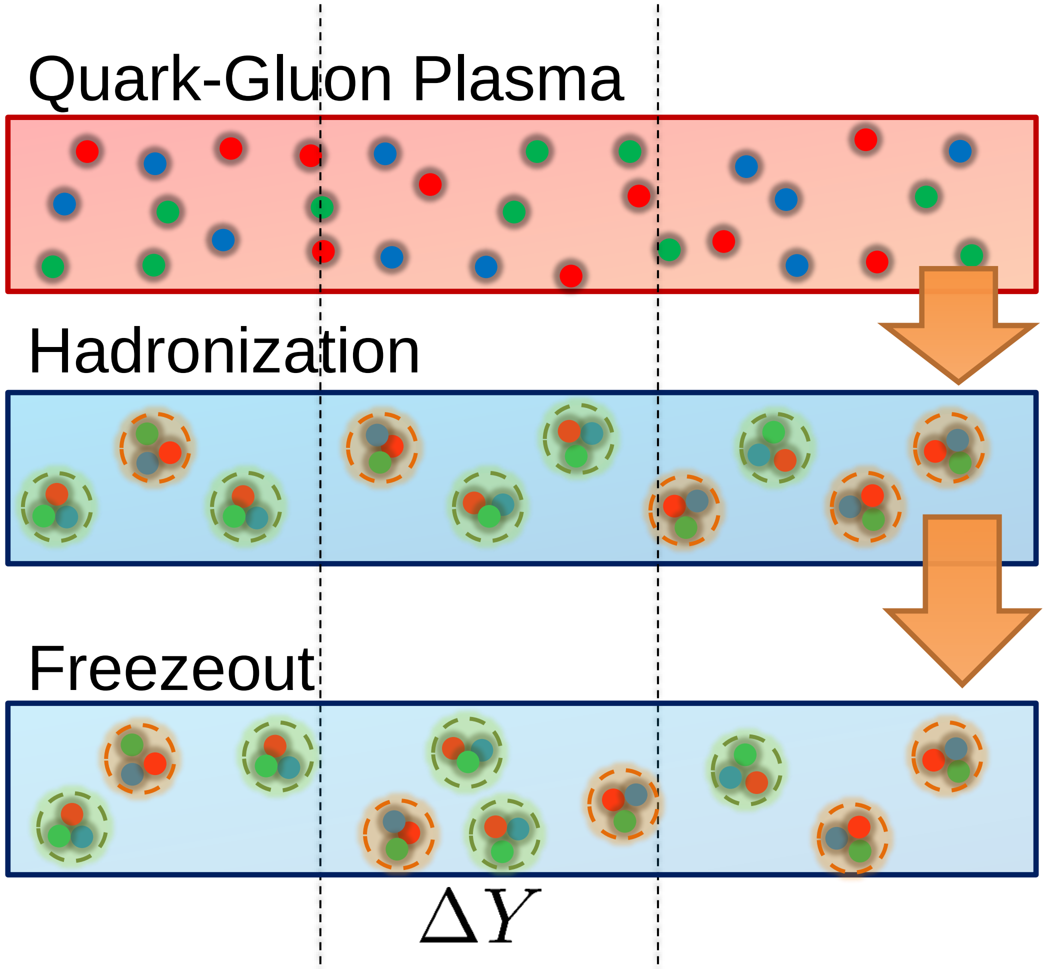
- ☐ **Further experimental analysis on various cumulants with wider Δy and p_T are awaited!**

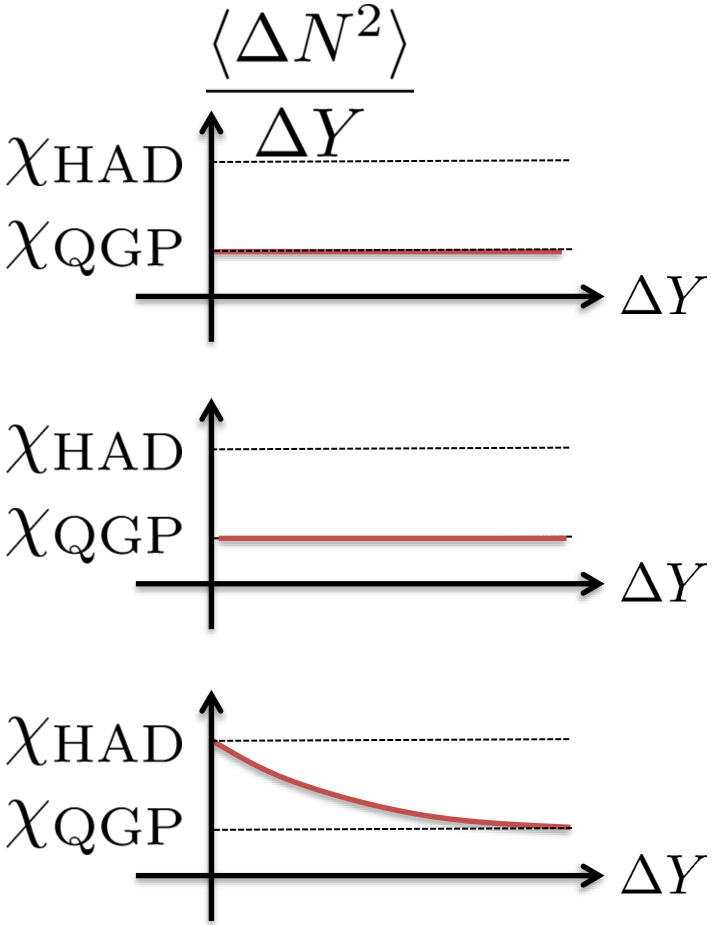
Resonance Decays



The larger Δη, the slower diffusion.

Evolution of Conserved-charge Fluctuations





Fluctuations of CC are modified by the diffusion.

Relaxation time becomes longer as ΔY →large.

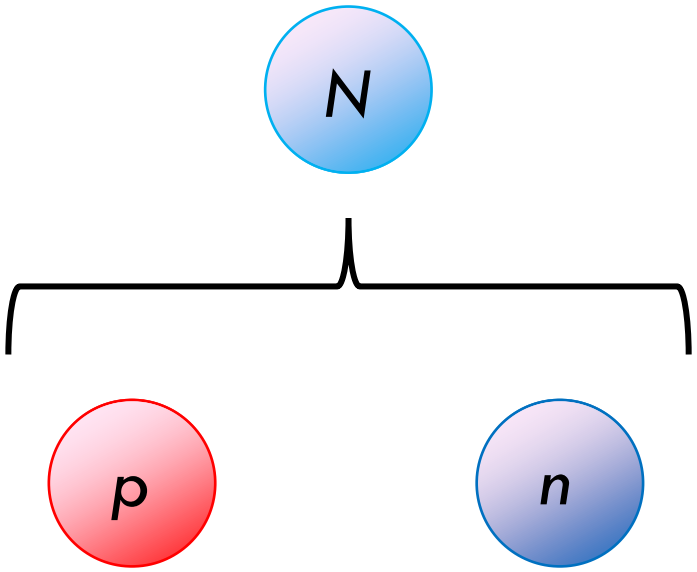
Experiments on $\langle N_Q^2 \rangle$

- ☐ No QGP signal @ RHIC ('02, '03)
- ☐ QGP signal? @ ALICE ('12)

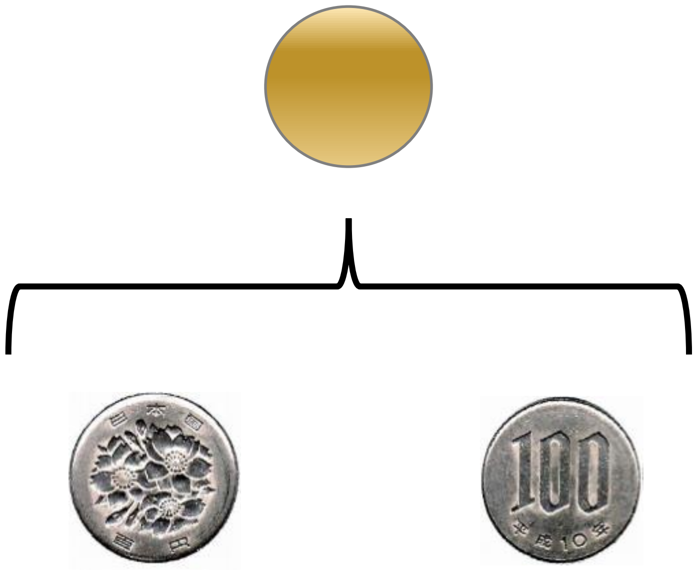
Asakawa, Heinz, Muller, 2000

Jeon, Koch, 2000; Shuryak, Stephanov, 2001

Nucleon's Isospin as Two Sides of a Coin



a coin



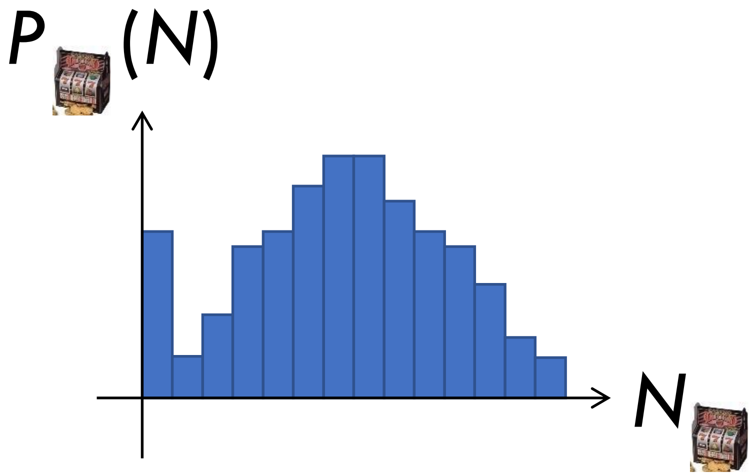
Nucleons have
two isospin states.

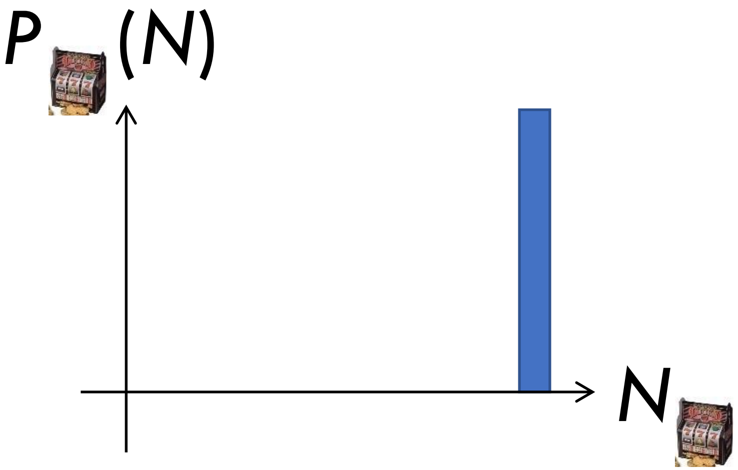
Coins have two sides.

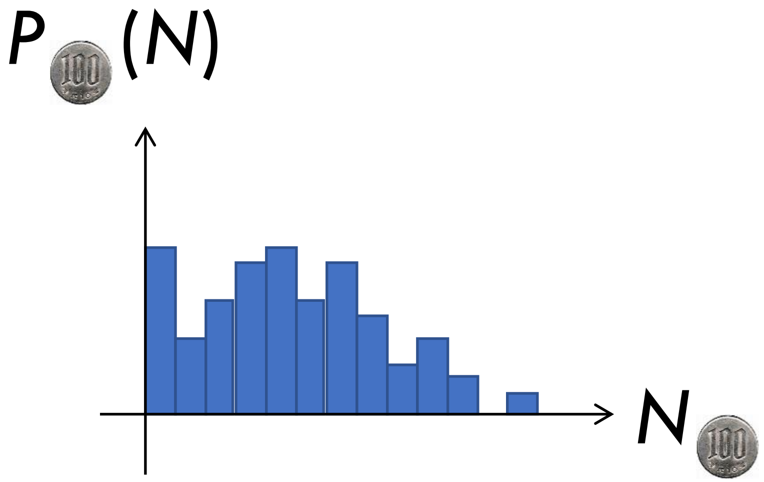
MK, Asakawa, 2012;2012

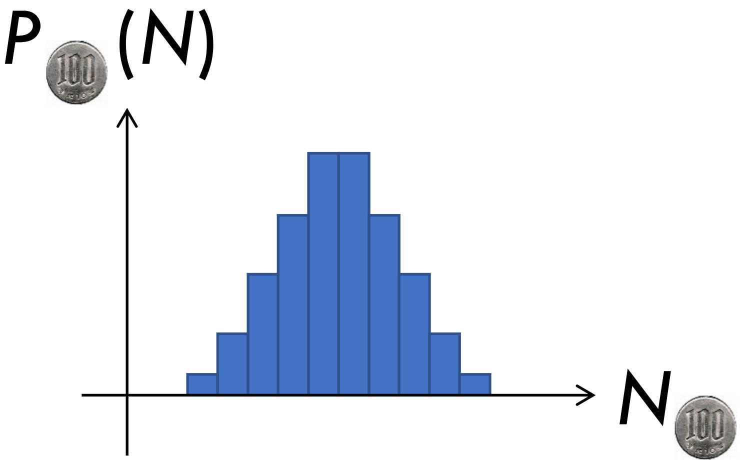
Slot Machine Analogy

Fixed # of coins

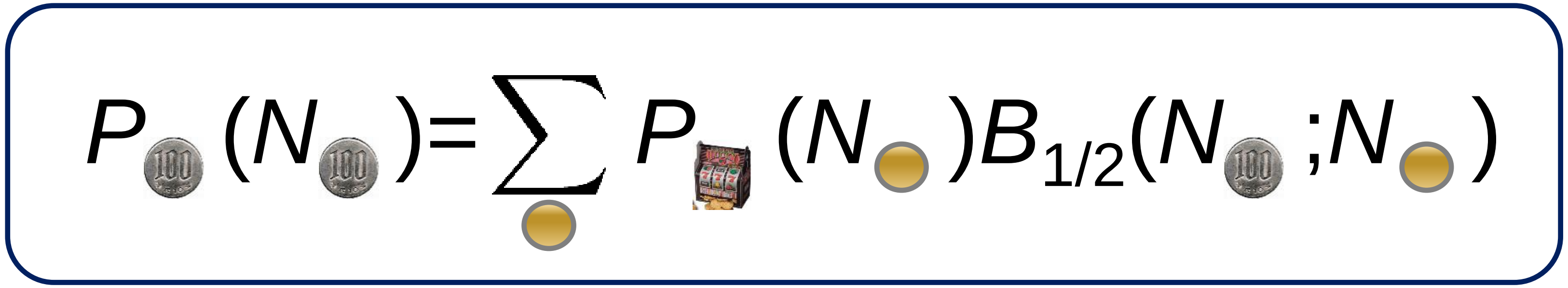








Reconstruction of Total Coin



□Example

$$
\begin{cases}
2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \cdots
\end{cases}
$$

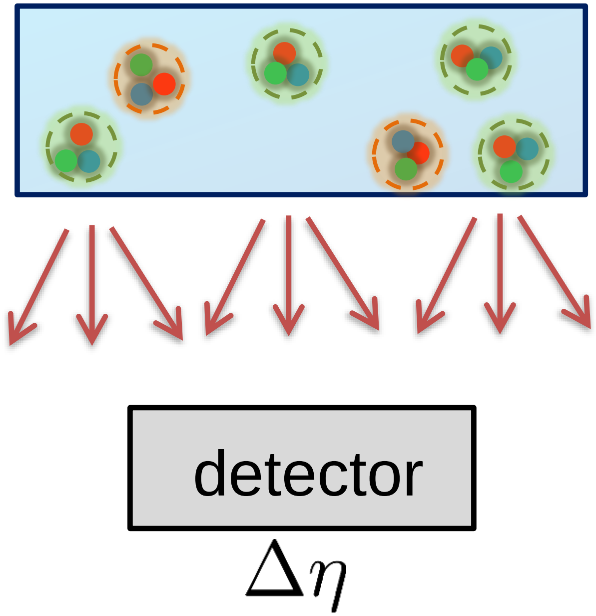
genuine info. Poisson noise

Note: Higher order cumulants are more fragile.

MK, Asakawa, 2012;2012

Difficulties of Low-Energy Collisions

- Large contribution from global charge conservation
- Violation of Bjorken picture



Careful treatment is required to interpret fluctuations at low beam energies!