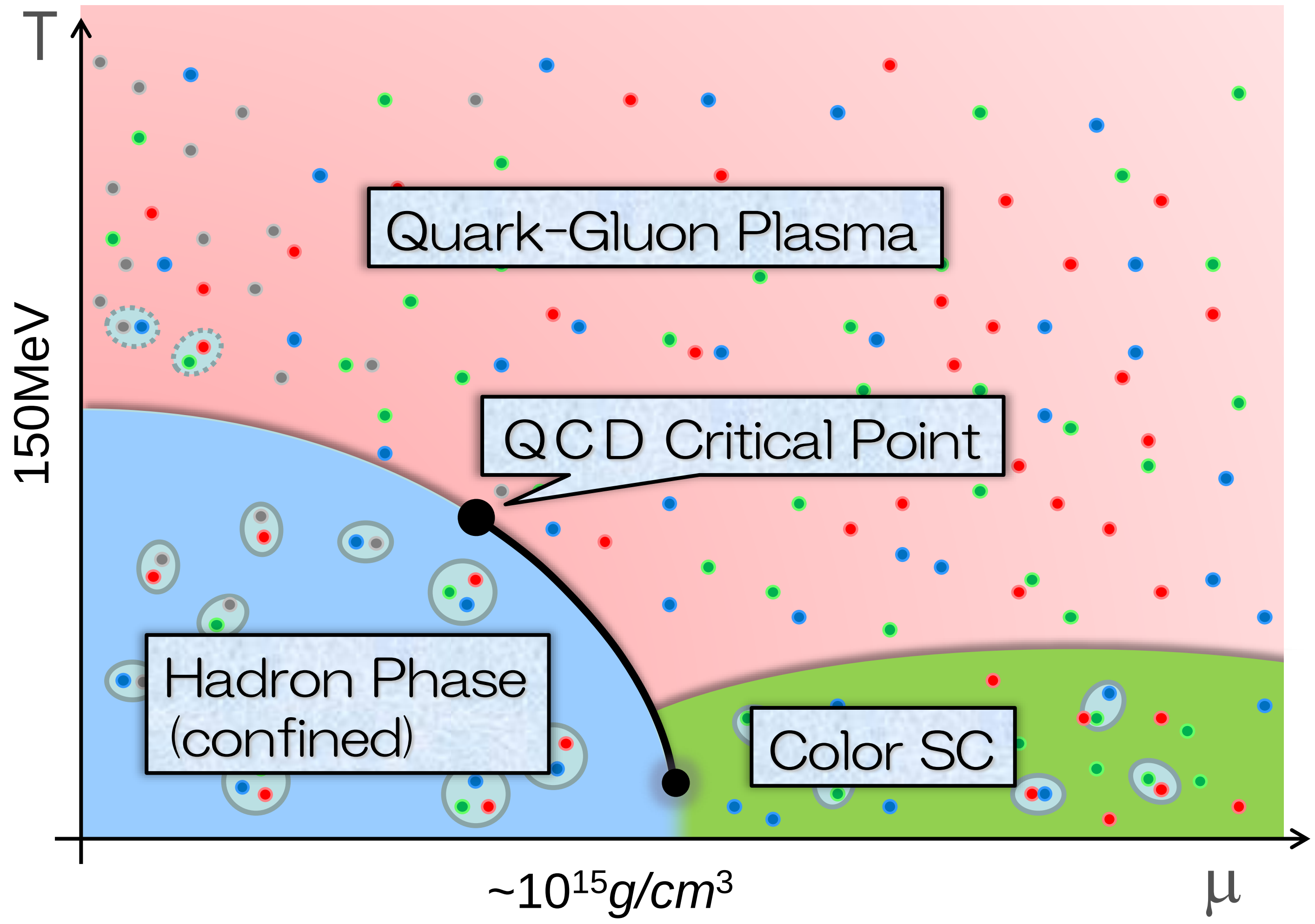
Theoretical Study of QCD Phase Diagram at High Baryon Density

Masakiyo Kitazawa
(YITP, Kyoto)

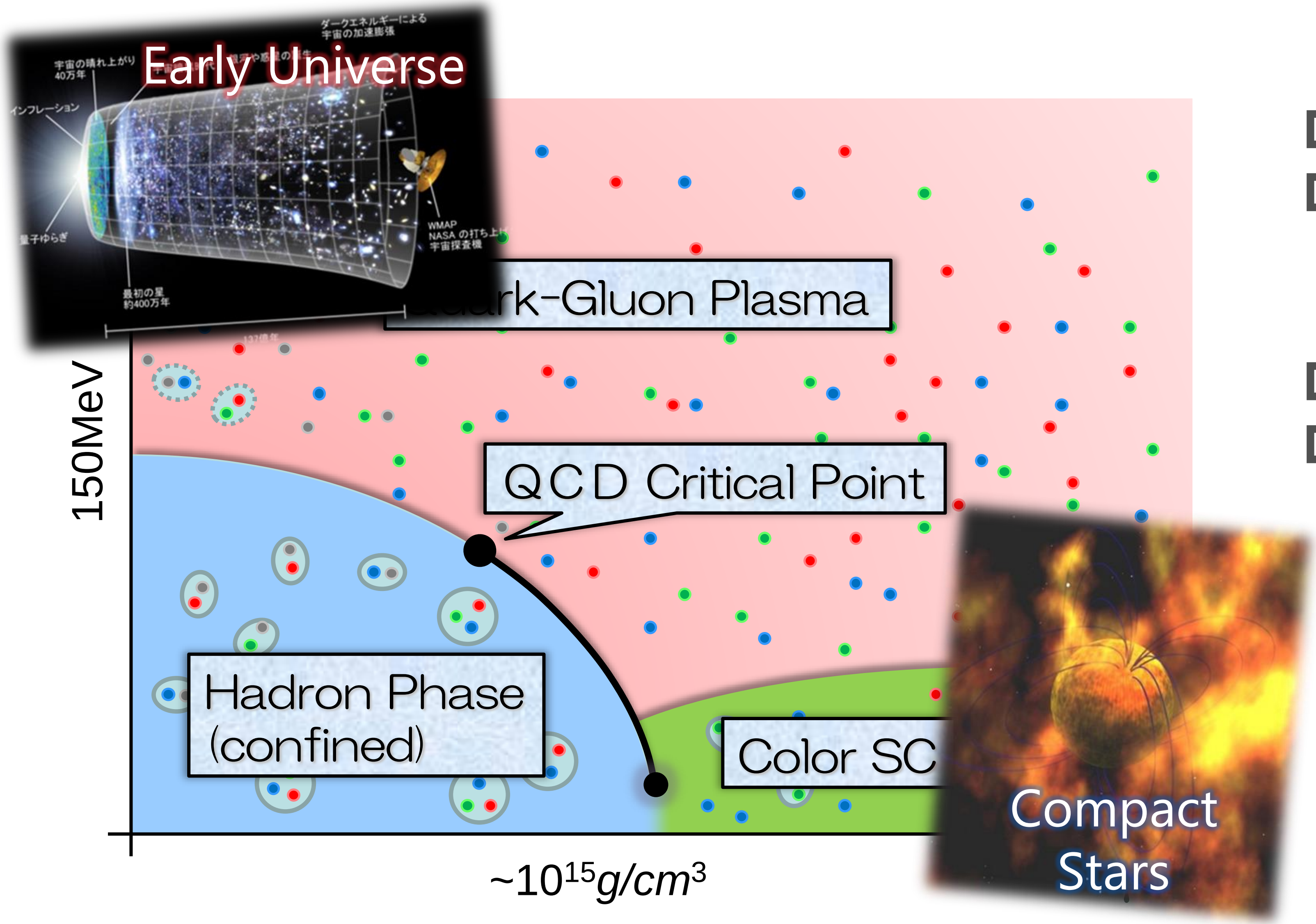
6th Joint Meeting of the APS Division of Nuclear Physics and the Physical Society of Japan,
Nov. 27, 2023, Waikoloa, Hawaii, USA

QCD Phase Diagram



- Crossover at $\mu = 0$
- Possible first-order transition and QCD critical point in dense region
- Multiple QCD-CP? MK+ ('02)
- Color superconducting phases in dense and cold quark matter

QCD Phase Diagram



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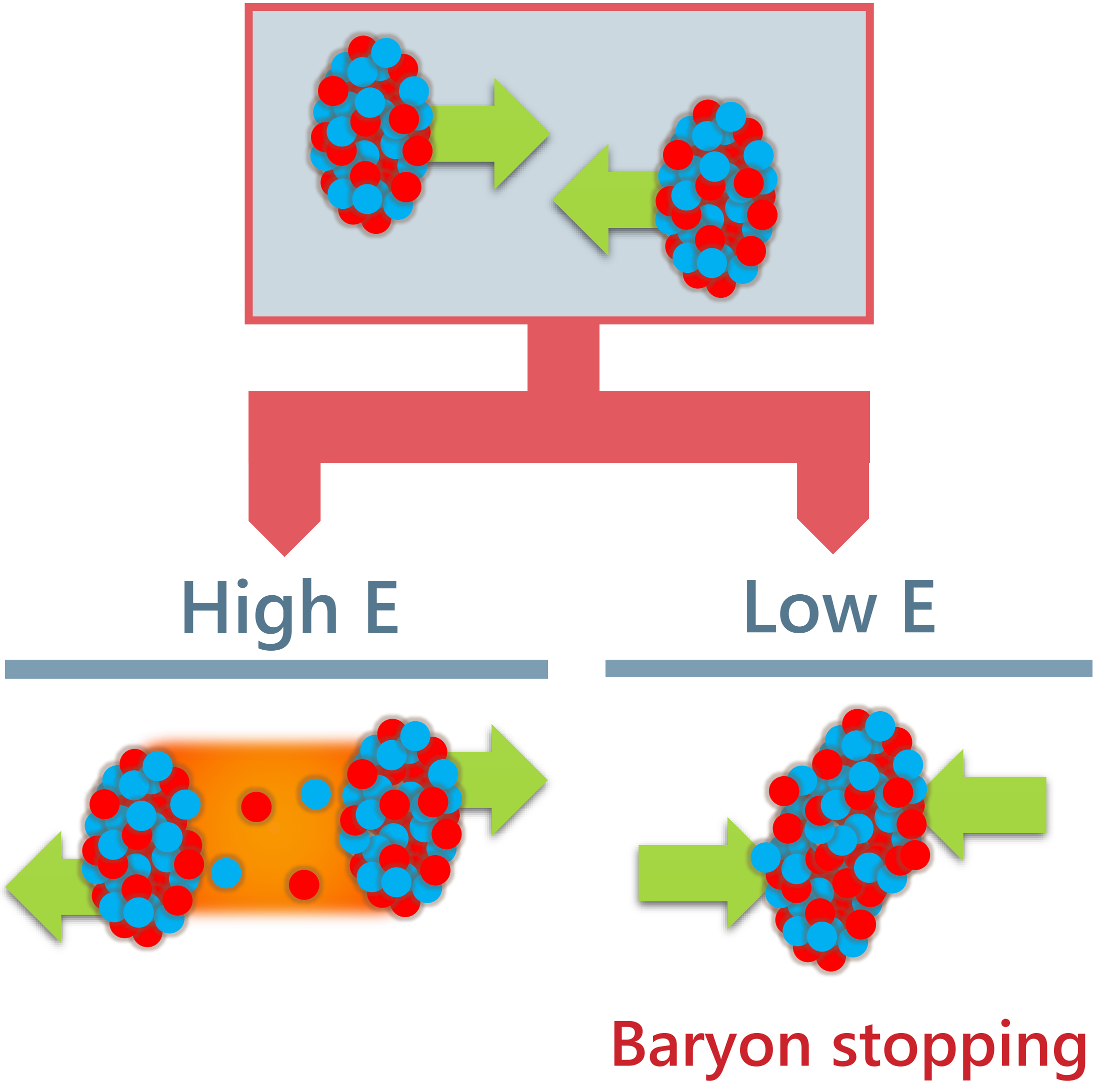
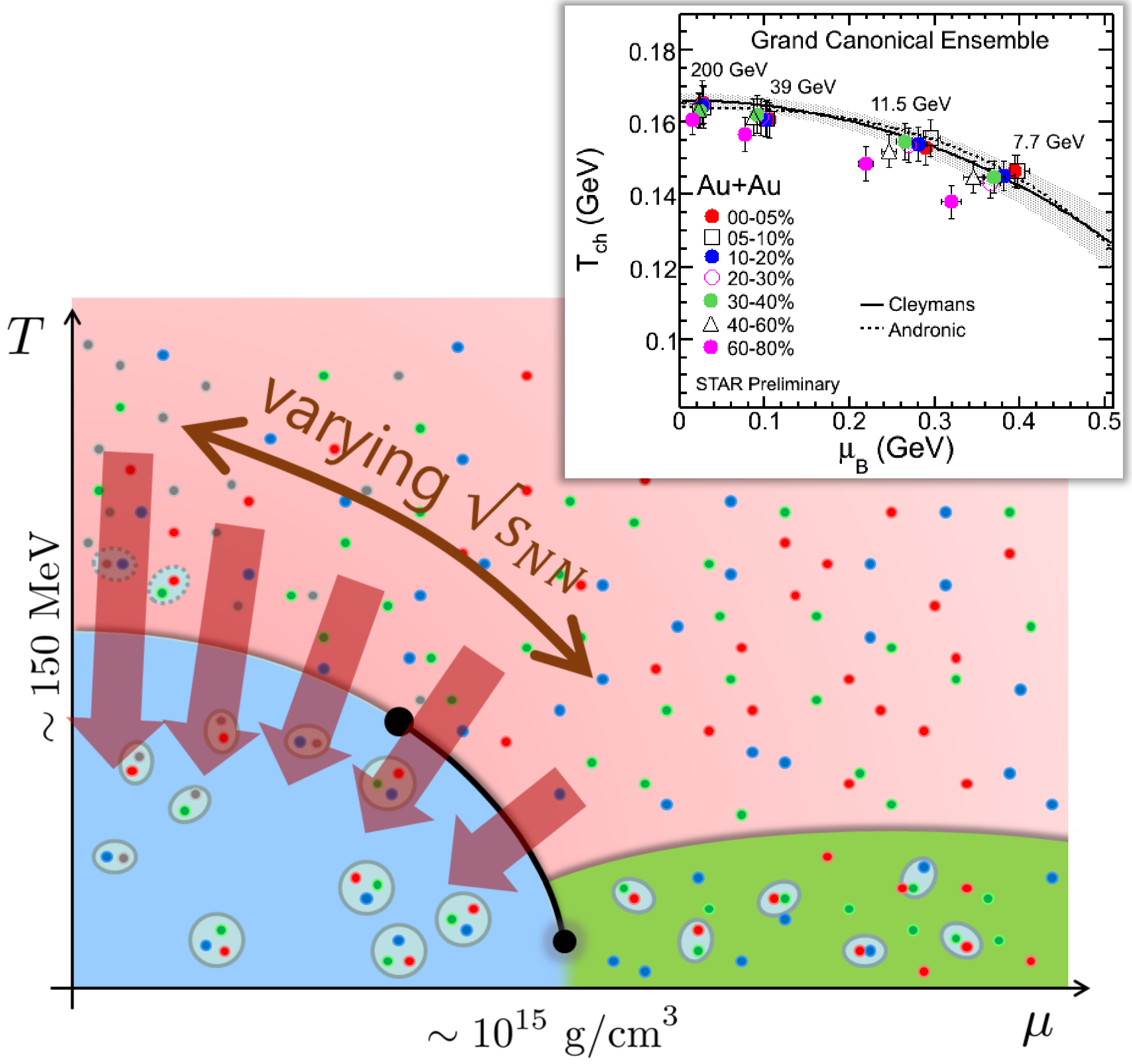
Beam-Energy Scan



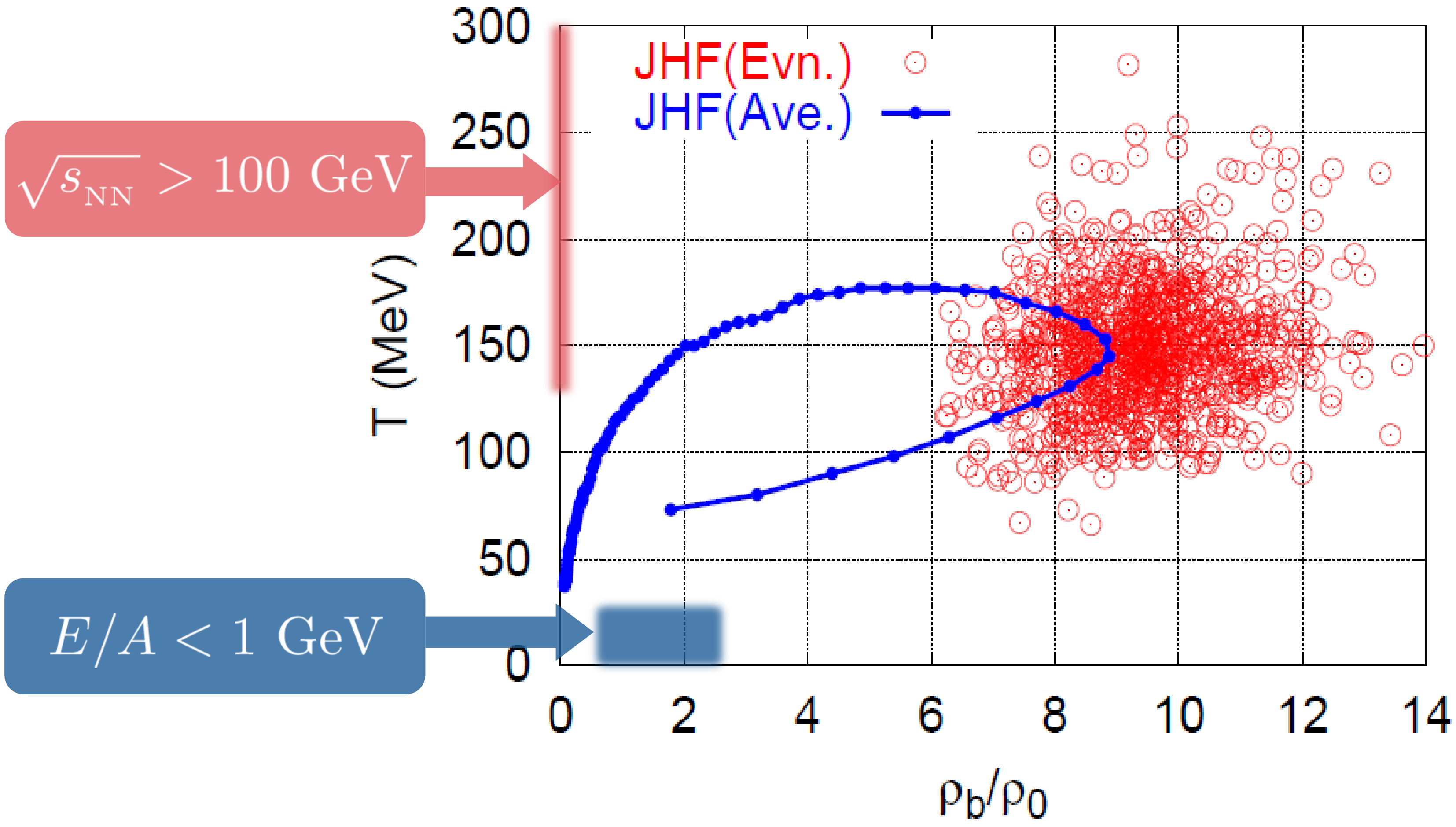
STAR, 2012



Highest Baryon Density

$$E/A = 20\text{GeV}$$

$$\sqrt{s_{NN}} \simeq 6\text{GeV}$$

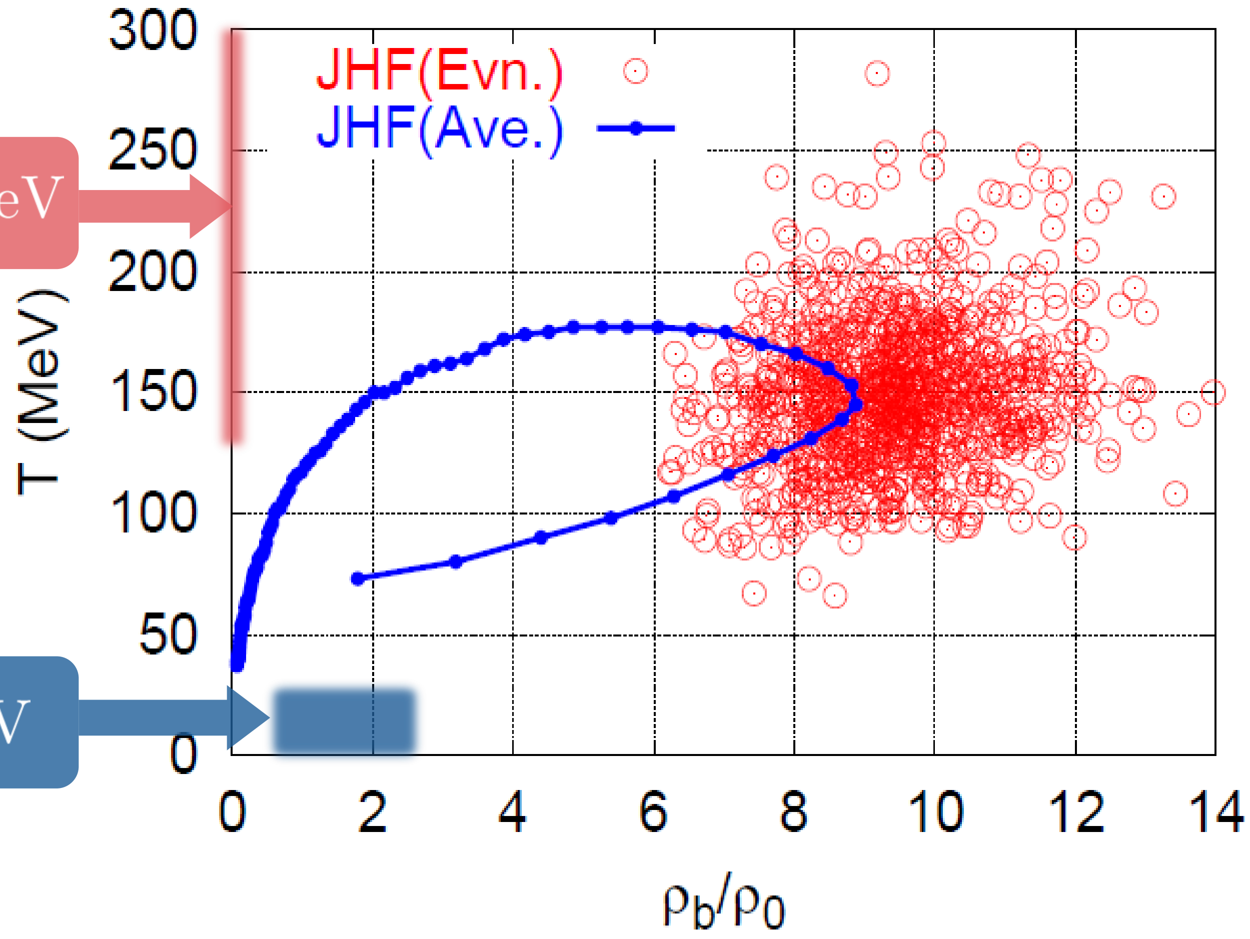


Ohnishi, 2002

Highest Baryon Density

$\sqrt{s_{\rm NN}} > 100$ GeV

$E/A < 1$ GeV



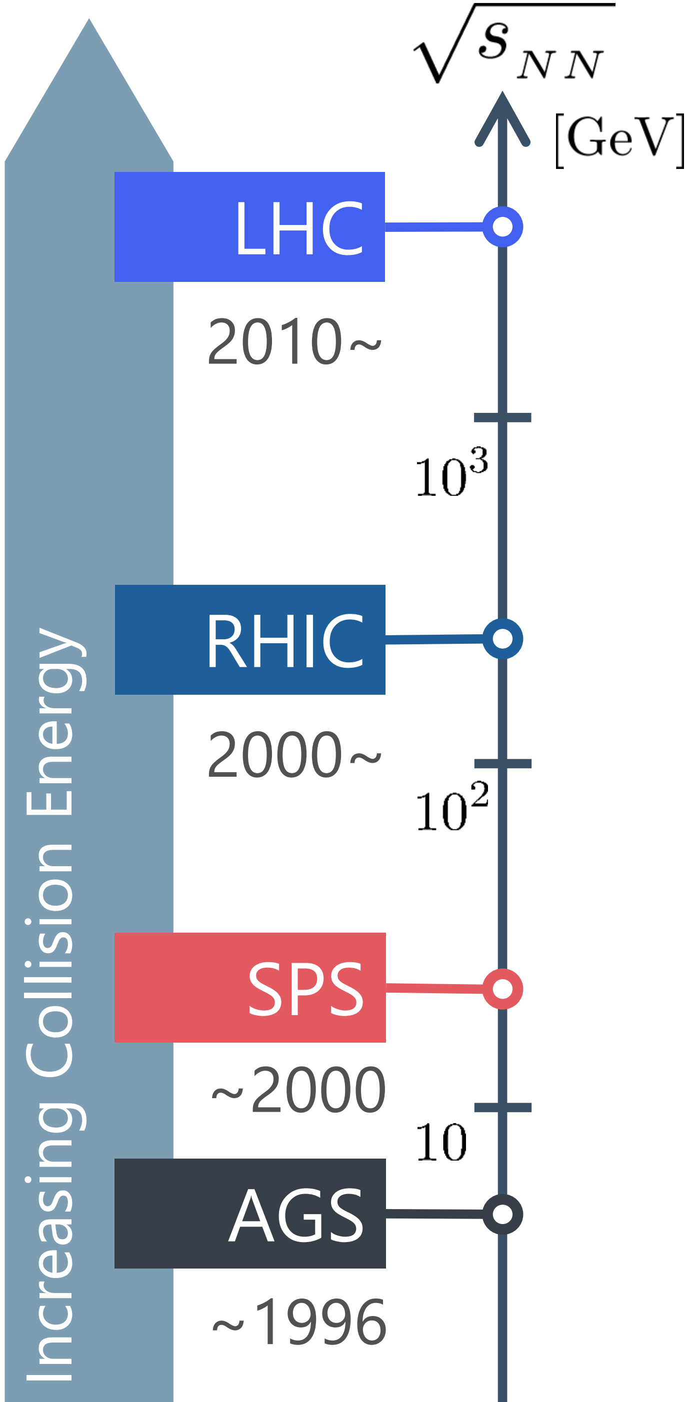
Ohnishi, 2002

$$E/A = 20\text{GeV}$$

$$\sqrt{s_{NN}} \simeq 6\text{GeV}$$

Akira Ohnishi
1964-2023
passed away silently
on May. 16, 2023

History / Current Status of HIC



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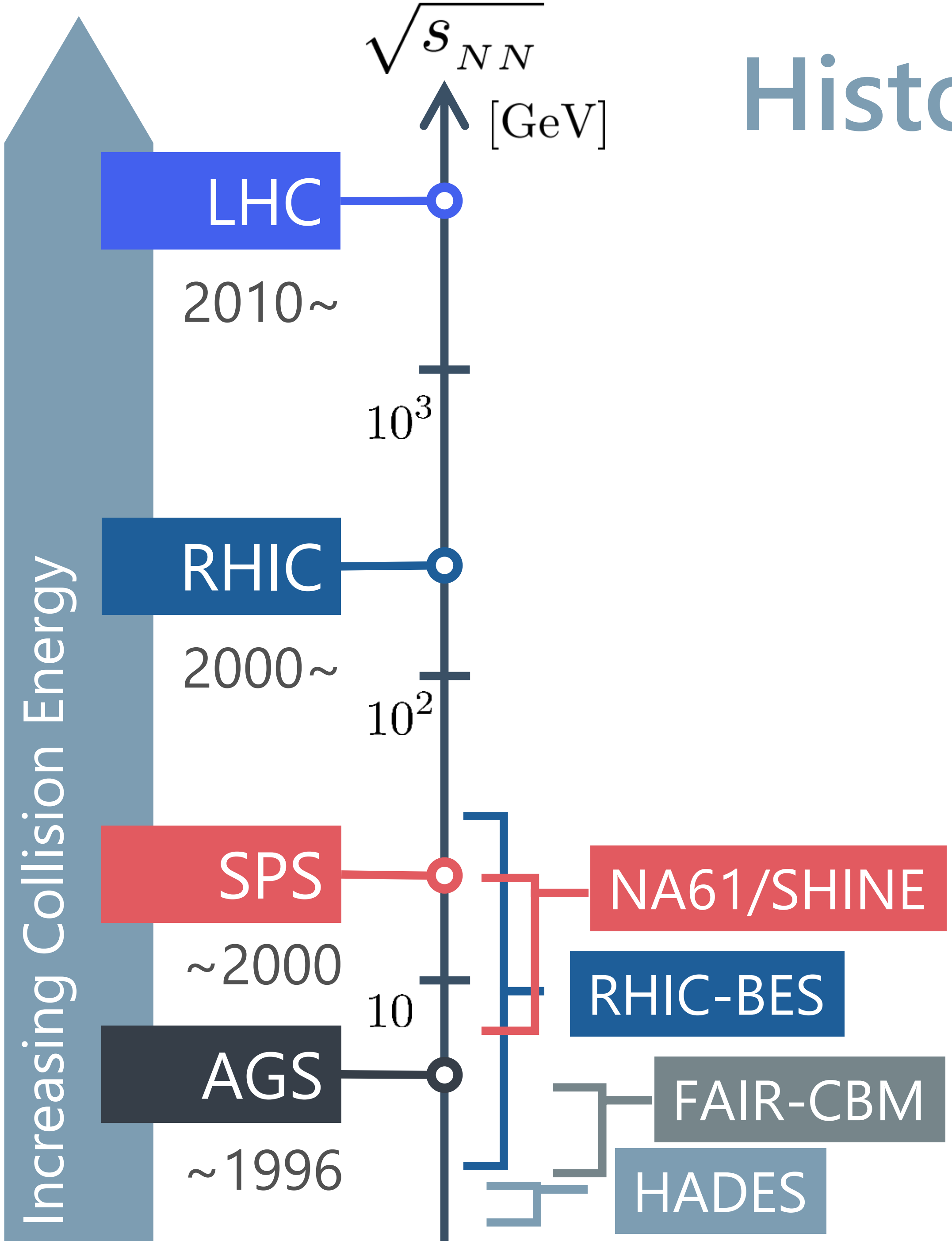

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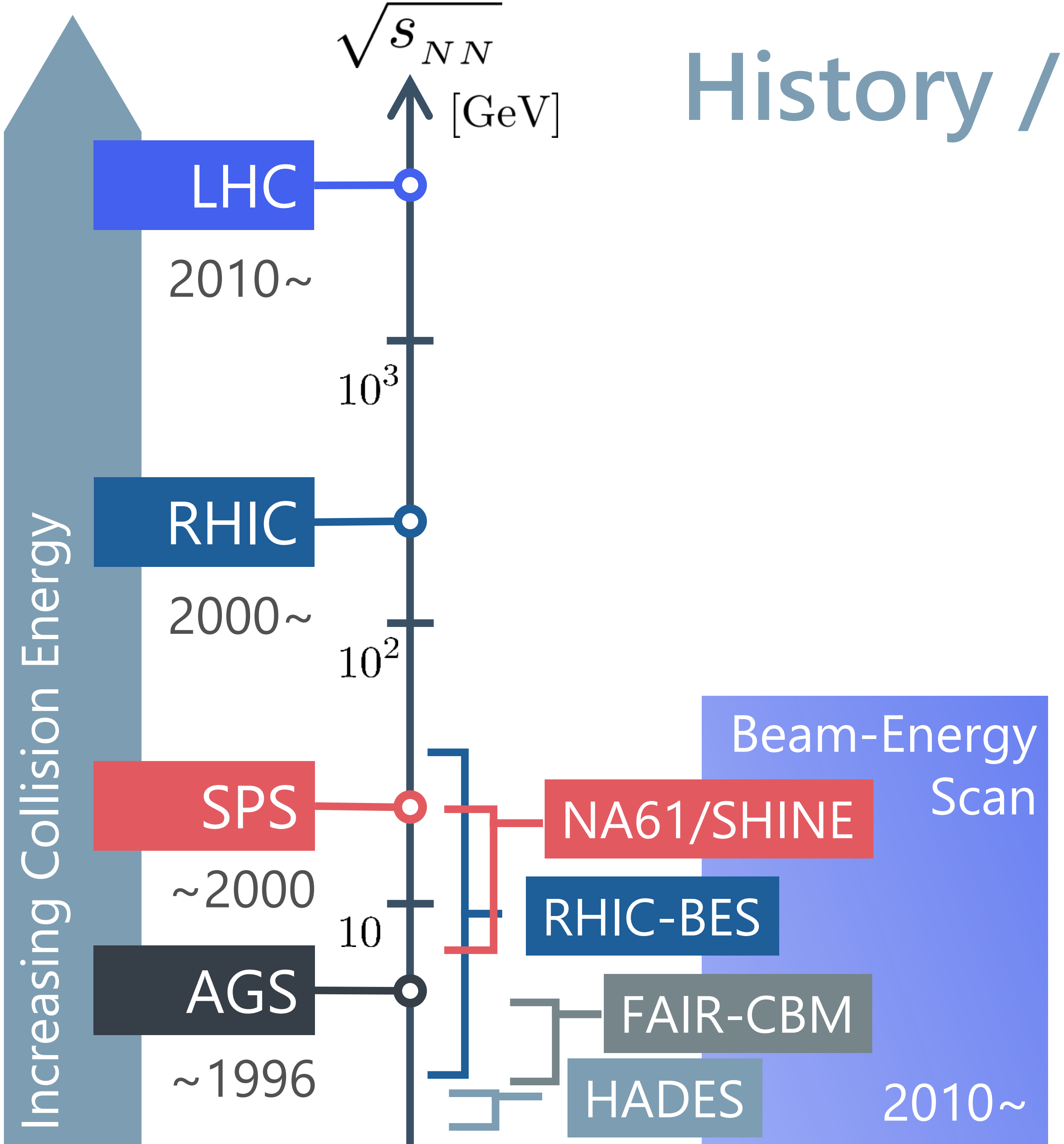

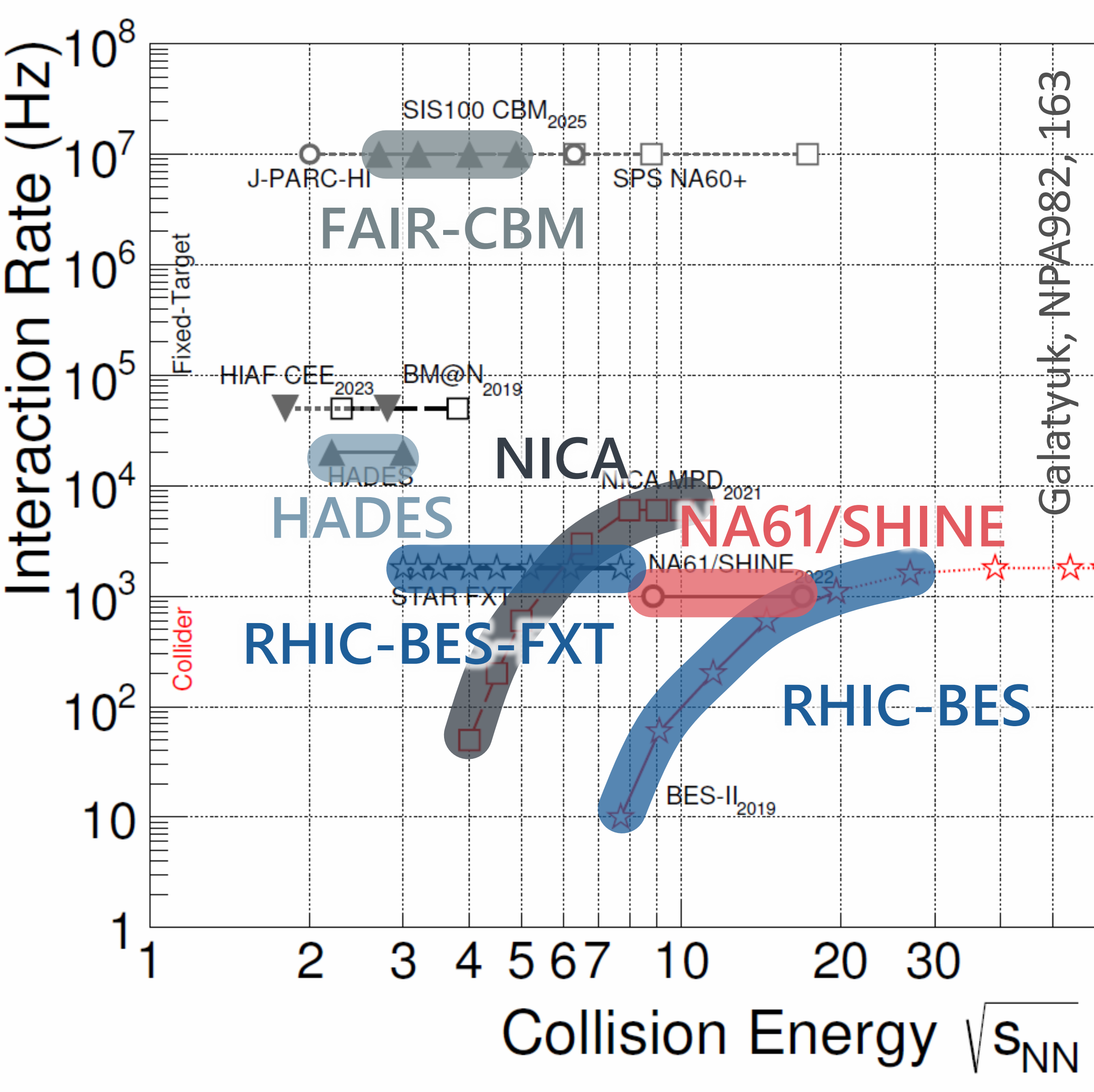

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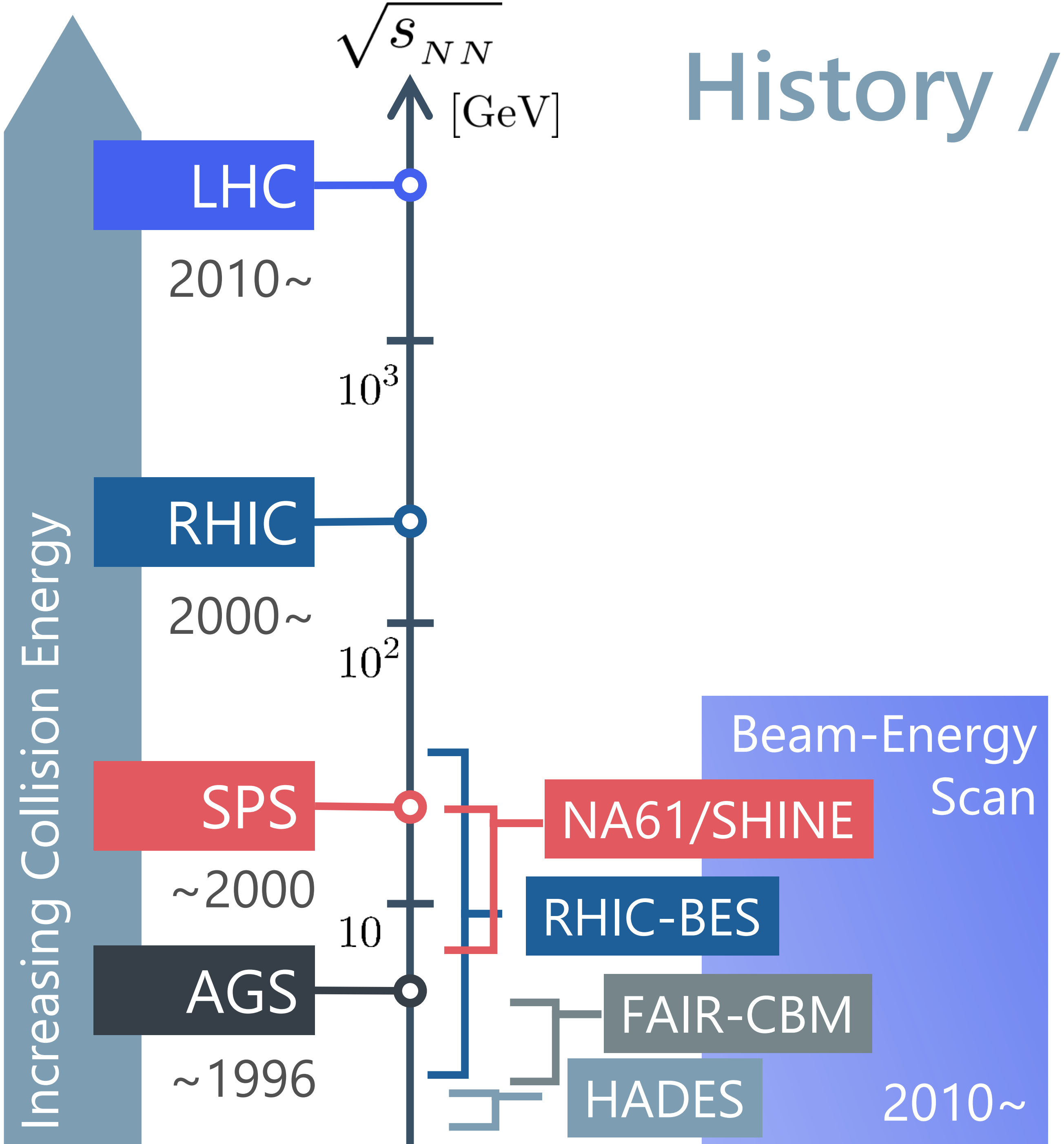
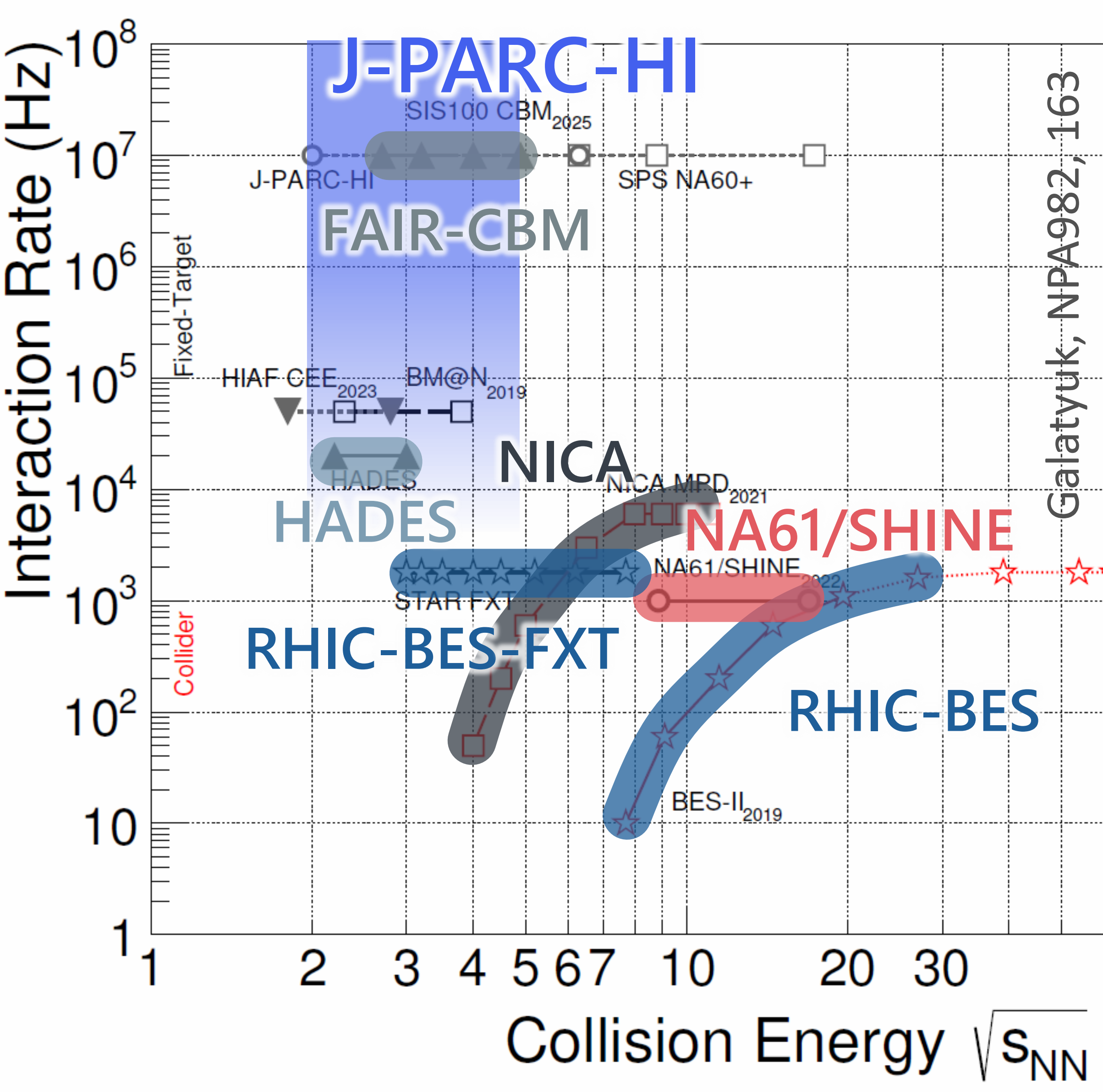
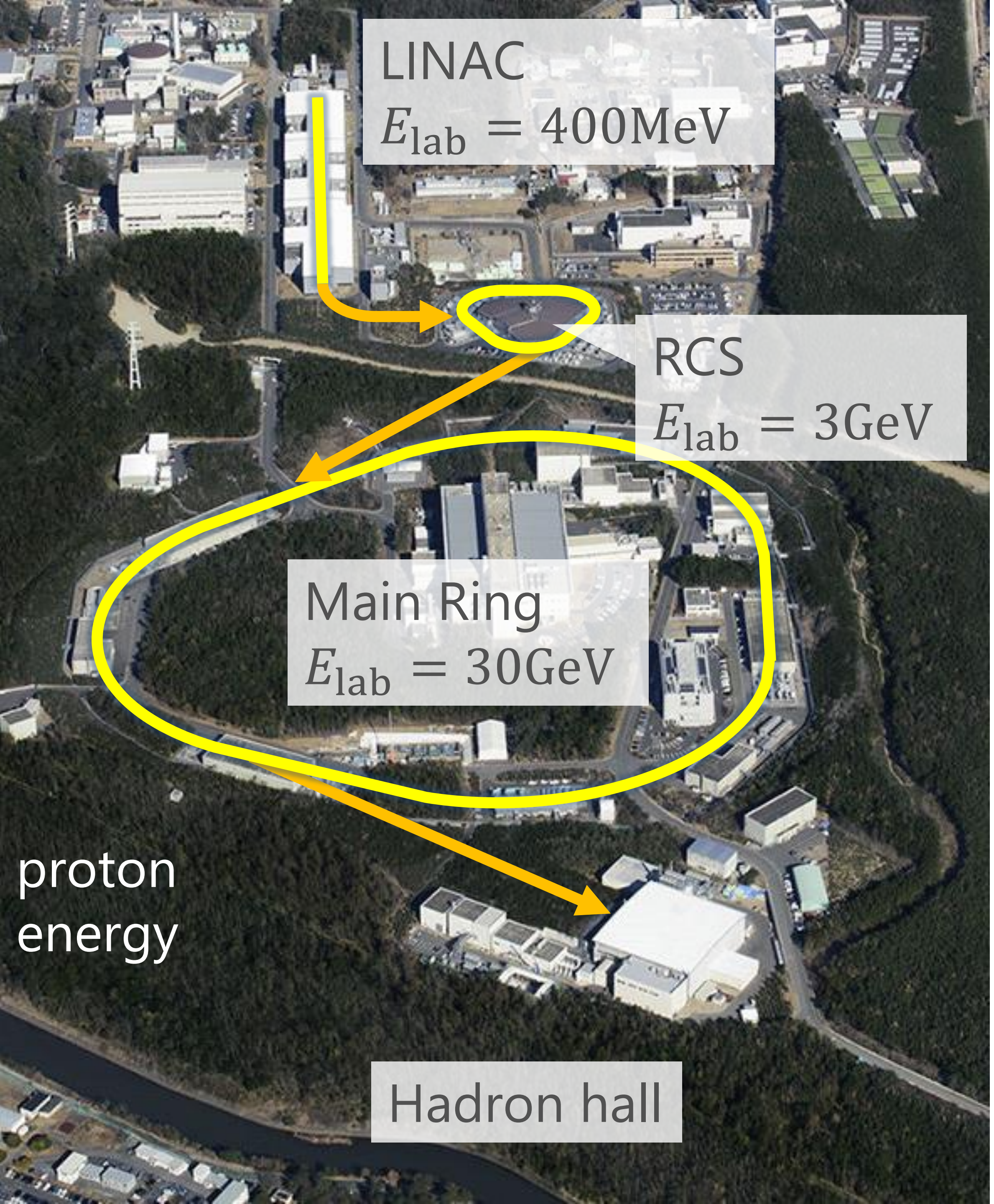






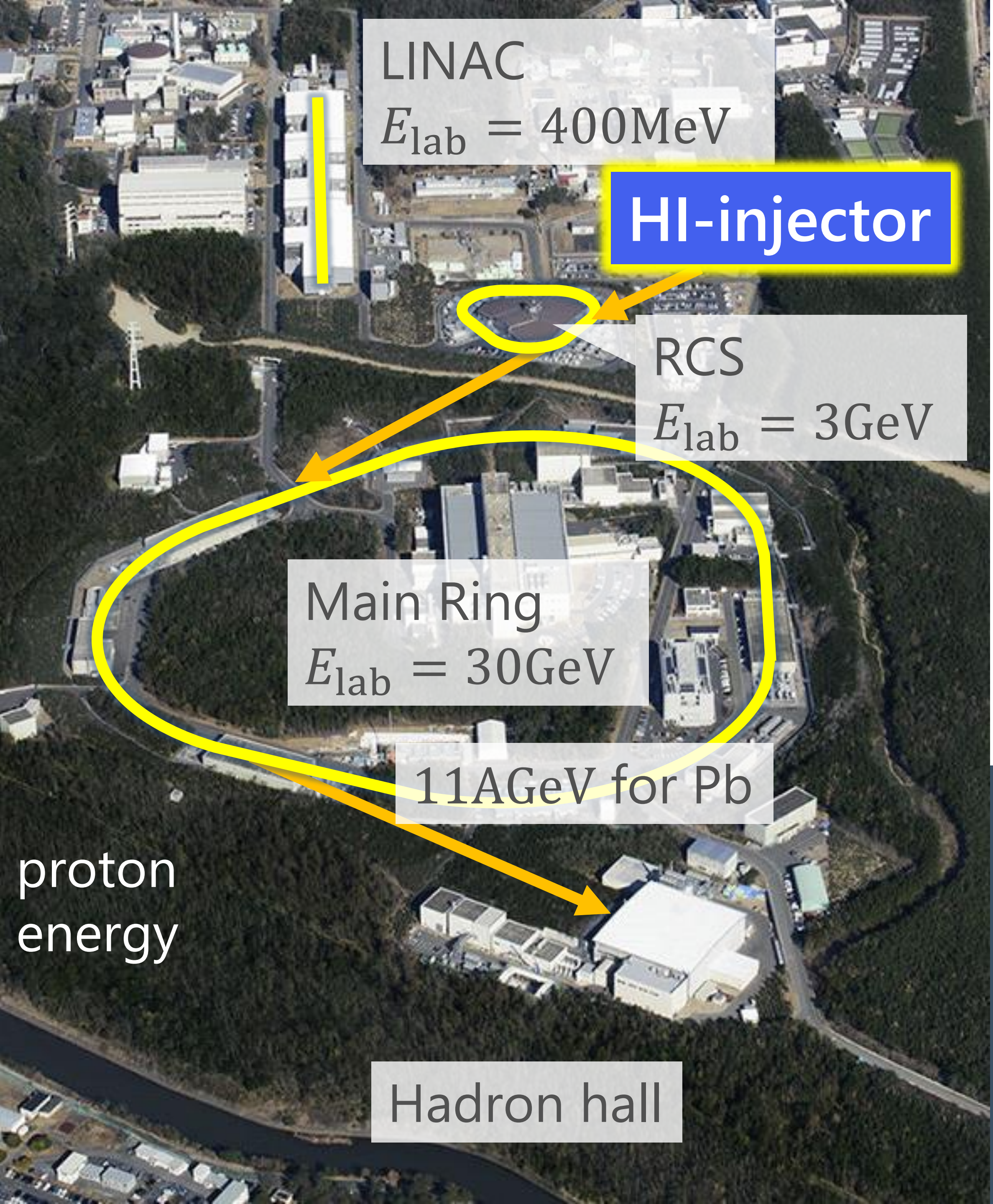
J-PARC

Accelerators

- LINAC
- RCS
- Main Ring(MR)
- High intensity $I = 1\text{MW}$

Purposes

- Hadron/Nuclear physics
- Neutrino physics
- Material/Life science



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J-PARC-HI

J-PARC Heavy Ion Program

High intensity

×

Intermediate energy

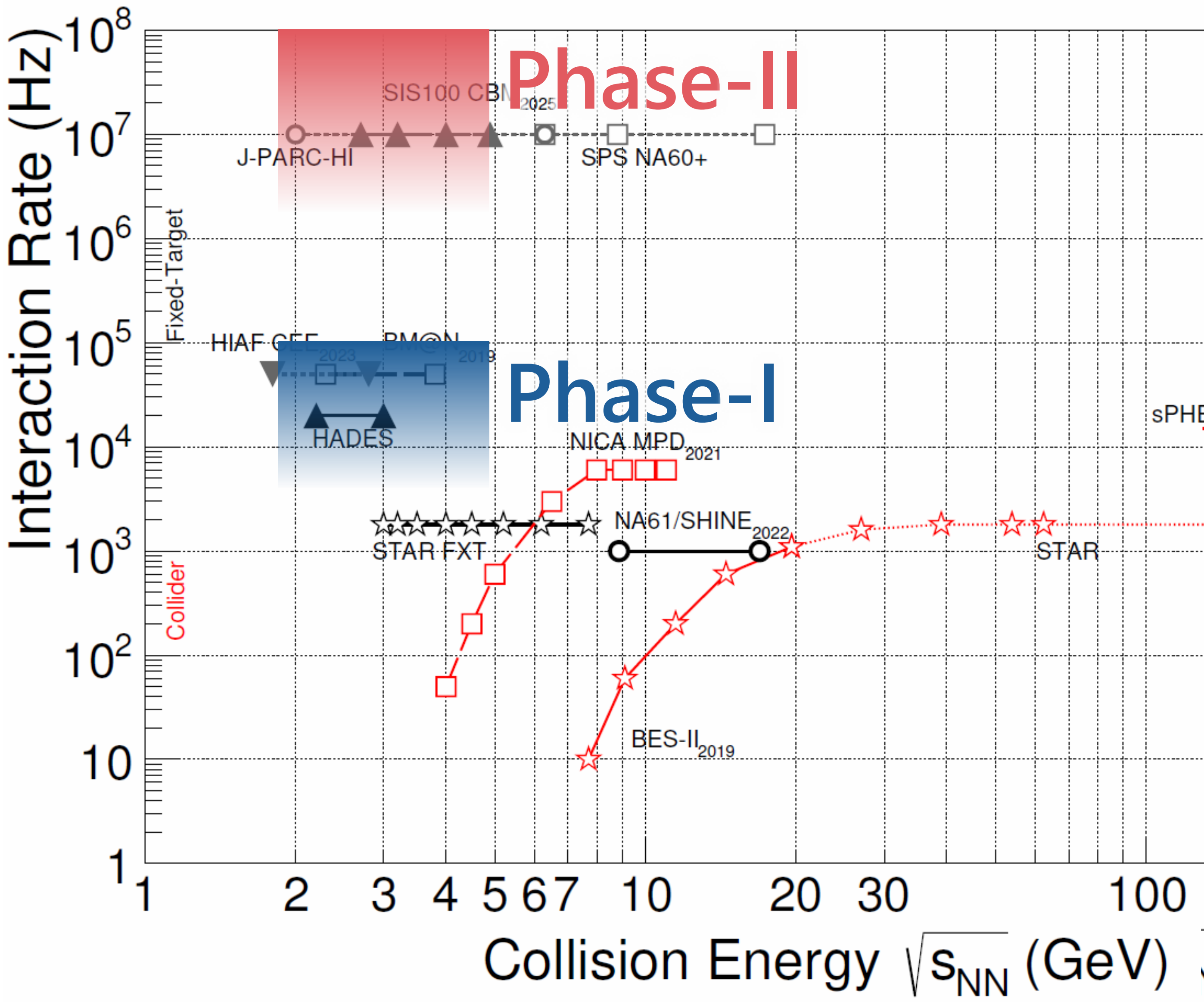
J-PARC-HI Staging Plan

Phase-I

- KEK-BS booster
- E16+α spectrometer

Phase-II

- **NEW** HI booster
- **NEW** spectrometer



Event-by-event Fluctuations of Conserved Charges

A Coin Game

① Bet 25 Euro
② You get head coins of

A. 50 x 1 Euro	B. 25 x 2 Euro
	

Same expectation value.

A Coin Game

① Bet 25 Euro
② You get head coins of

A. 50 x 1 Euro

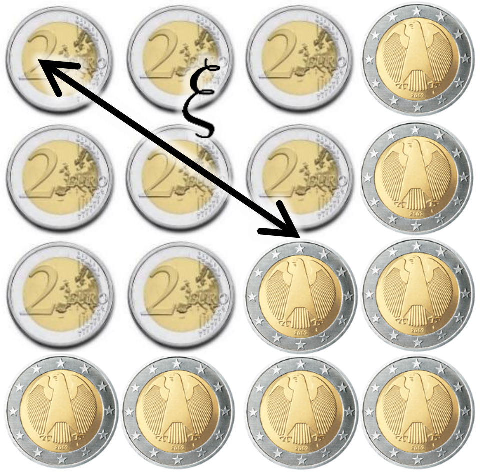
B. 25 x 2 Euro

C. 1 x 50 Euro

Same expectation value.
But, different **fluctuations**.

Fluctuations in HIC @ 2nd Order

Search for QCD CP



Fluctuation
increases

Stephanov, Rajagopal, Shuryak, 1998; 1999

Onset of QGP

Fluctuation
decreases

Asakawa, Heinz, Muller, 2000;
Jeon, Koch, 2000

Higher Order Cumulants

A. 50 x 1 Euro

B. 25 x 2 Euro

$$\langle \delta€^2 \rangle = \frac{1}{2} \langle \delta€^2 \rangle$$

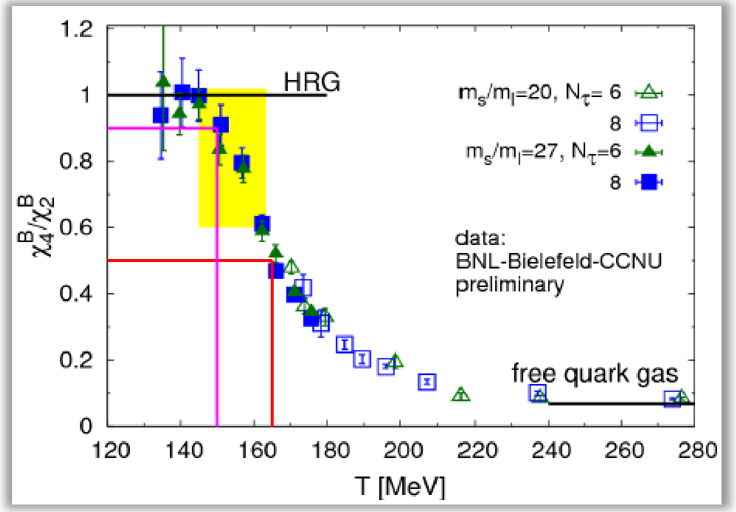
$$\langle \delta€^3 \rangle = \frac{1}{4} \langle \delta€^3 \rangle$$

$$\langle €^4 \rangle_c = \frac{1}{8} \langle €^4 \rangle_c$$

Asakawa, MK, PPNP 90, 299 (2016)

Non-Gaussian Fluctuations in HIC

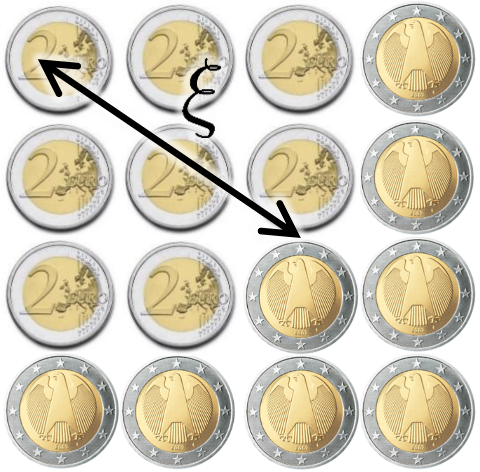
Onset of QGP



Fluctuation
decreases

Ejiri, Karsch, Redlich, 2006

Search for QCD CP



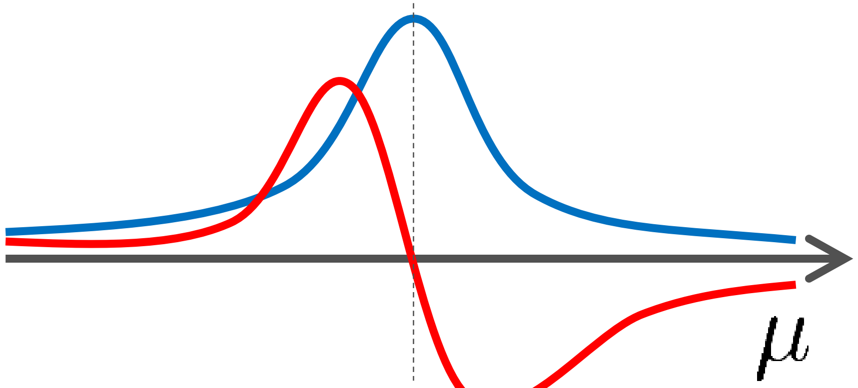
Fluctuation
increases

Stephanov, 2009

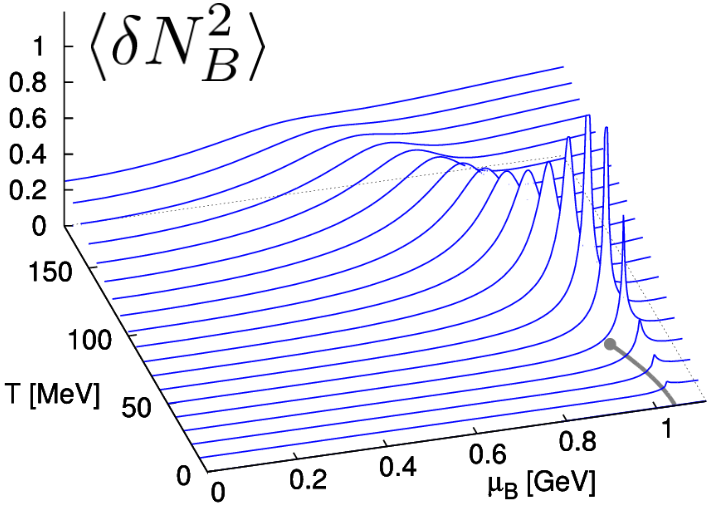
Sign of Higher Order Cumulants

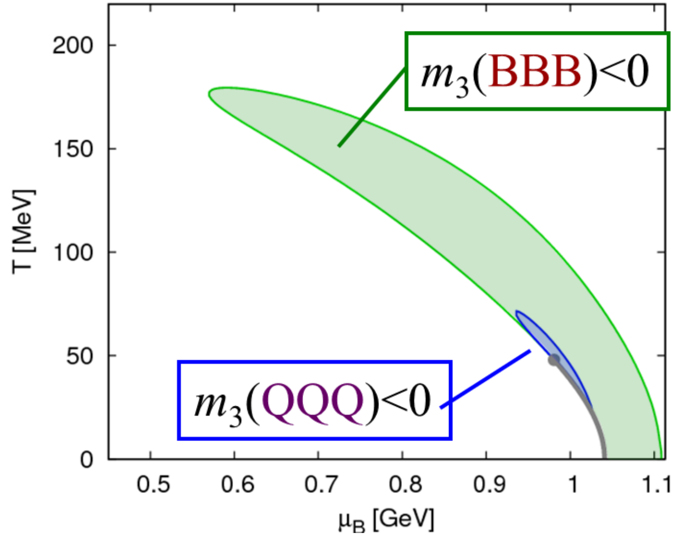
Geometric Interpretation

$$\langle \delta N^3 \rangle = T \frac{\partial \langle \delta N^2 \rangle}{\partial \mu}$$

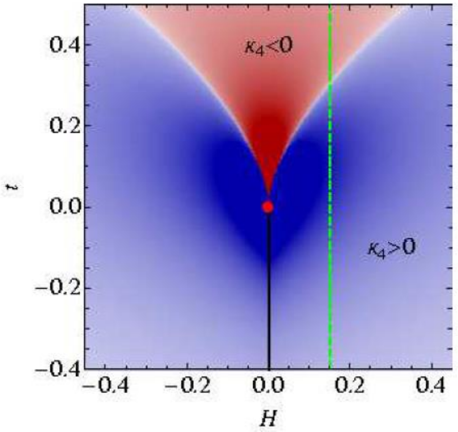


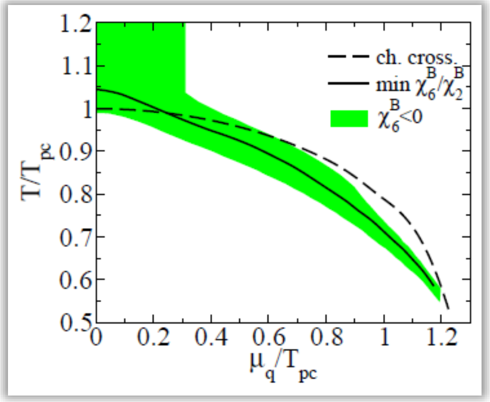
Asakawa, Ejiri, MK, 2009





Asakawa, Ejiri, MK, 2009





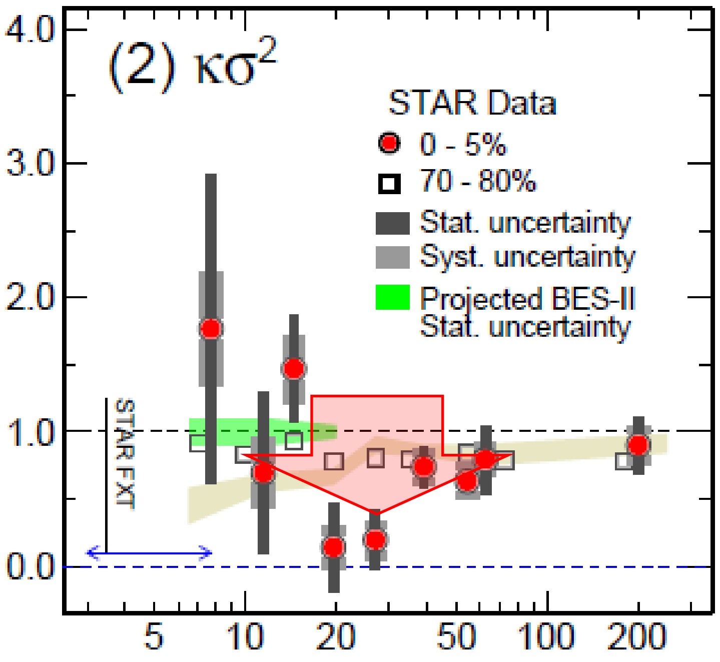
Stephanov, 2011;
Friman, Karsch, Redlich, Skokov, 2011; …

Experimental Results

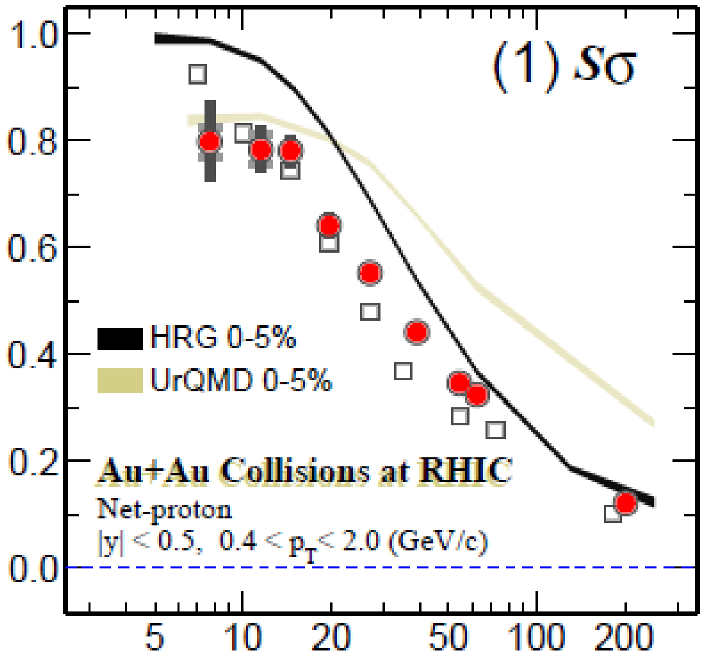
Net-proton number cumulants

STAR, 2020 (2001.02852)

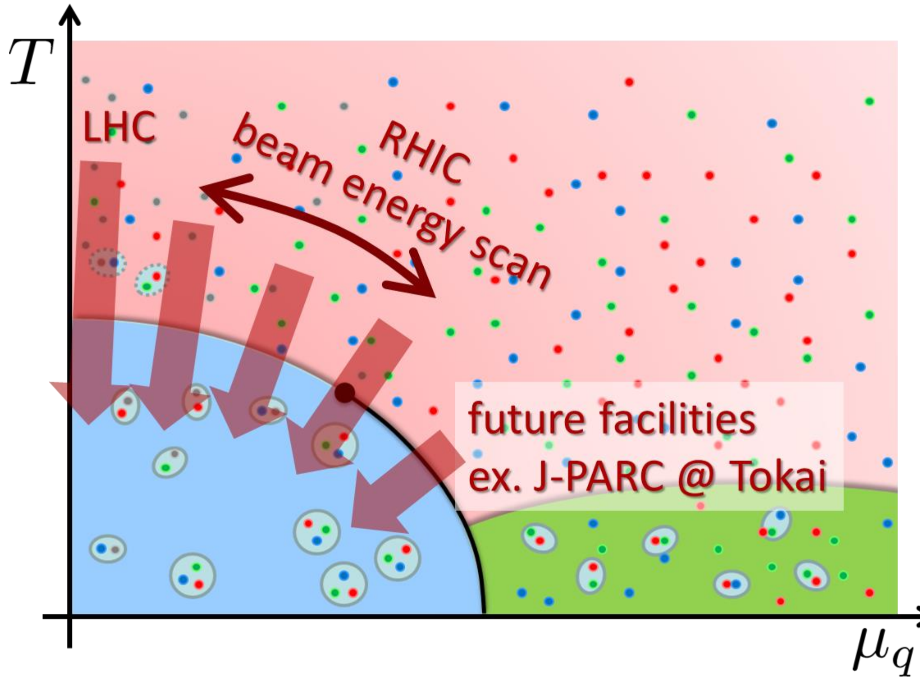
$\langle N_p^4 \rangle_c / \langle N_p^2 \rangle_c$



$\langle N_p^3 \rangle_c / \langle N_p^2 \rangle_c$



Non-Poisson and non-monotonic behaviors of the higher order cumulants.



Questions

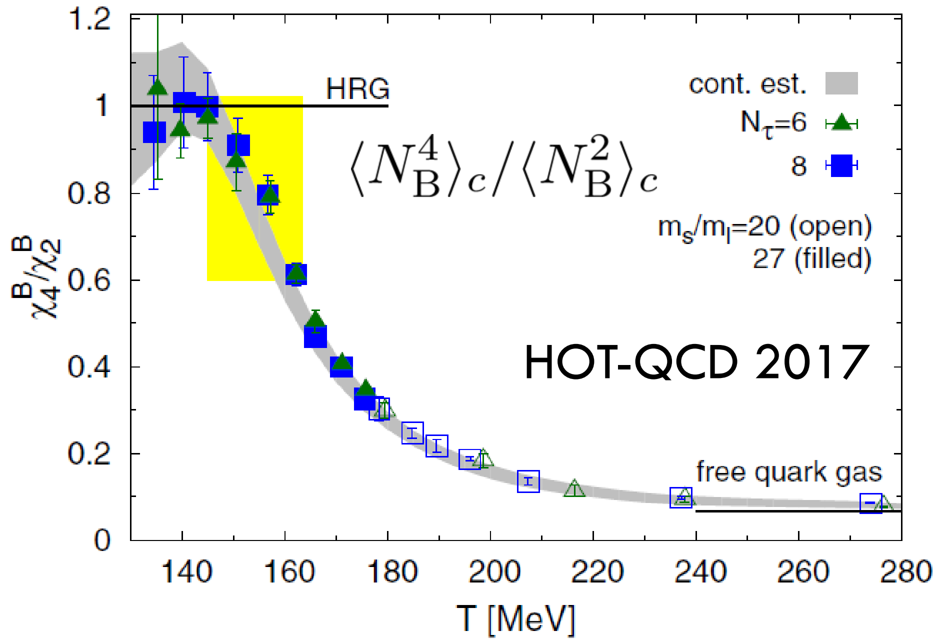
- ☐ When are these fluctuations generated?
- ☐ Is the use of proton # cumulants as a proxy of baryon's justified?

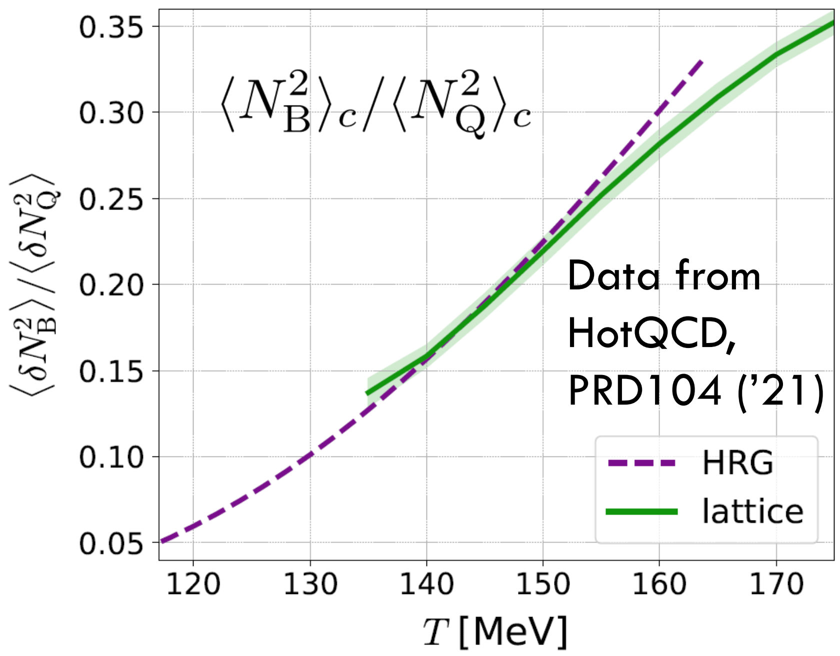
$\langle N_B^2\rangle_c/\langle N_Q^2\rangle_c$

MK, Esumi, Nonaka, 2205.10030

$\langle N_{\rm B}^2\rangle_c/\langle N_{\rm Q}^2\rangle_c$

- Ratio of 2nd order: Suppress uncertainties from various experimental effects compared with higher orders.
- Almost linear T dependence around T_c^*.





Experimental Data

MK, Esumi, Nonaka, 2205.10030

$\langle N_p^2 \rangle_c$

STAR, PRC104,024902 (2021)

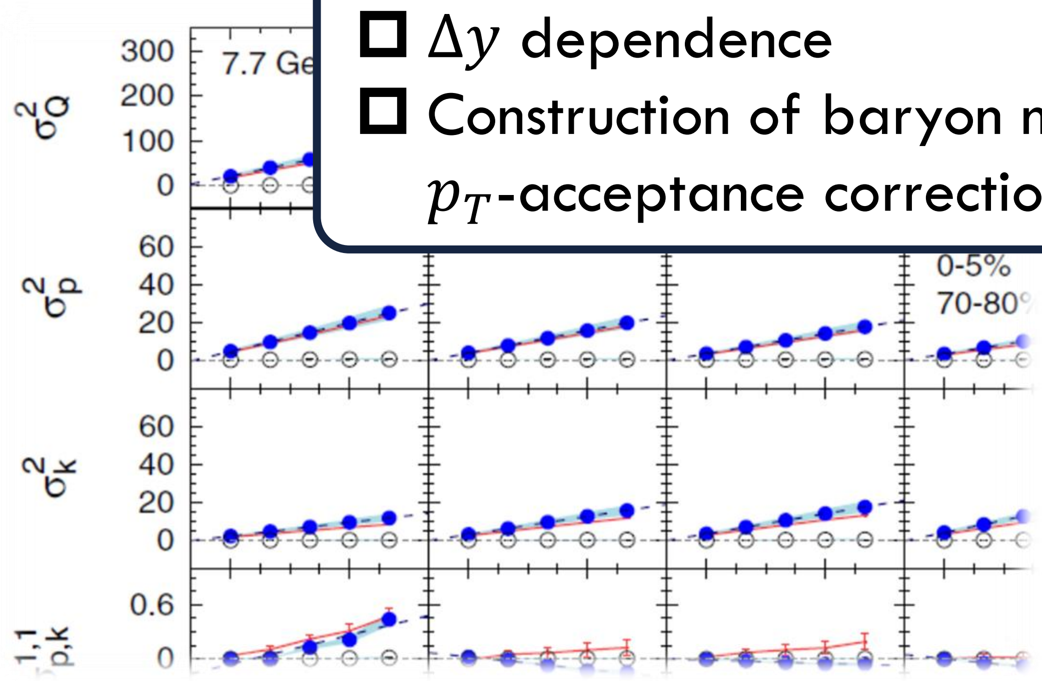
- proton cumulants up to 4th order
- **rapidity window** Δy
- $0.4 < p_T < 2.0$GeV/c

$\langle N_Q^2 \rangle_c$

STAR, PRC100,014902 (2019)

- 2nd mixed cumulants of p, ,pi, K, Q
- **pseudo-rapidity window** $\Delta\eta$
- $0.4 < p_T < 1.6$GeV/c
- Total charge: private comm. A. Chattergee

- $\sqrt{s_{NN}} = 200$GeV
- 0-5% centrality
- Δy dependence
- Construction of baryon number, p_T-acceptance correction



- proton → baryon cumulants MK, Asakawa,'12;'12
- Rapidity is better than pseudo-rapidity Ohnishi, MK, Asakawa, '16
- Wider acceptance is more desirable.

p_T-Acceptance Correction

MK, Esumi, Nonaka, 2205.10030

p_T Acceptance

$0.4<p_T<1.6$ [GeV/c]

PRC100,014902('19)

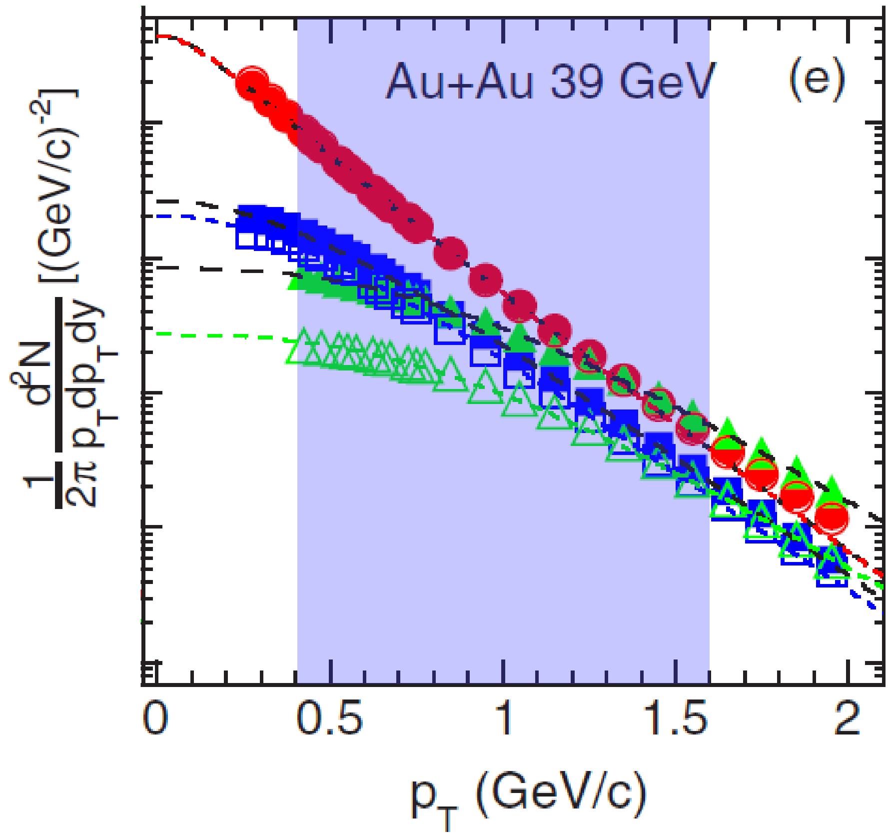
$0.4<p_T<2.0$ [GeV/c]

PRC104,024902('21)

Particles in p_T space

- Electric charge: **49%**
- Protons: **82%**

blast wave model @ $\sqrt{s_{NN}}$=200 GeV



Modification by p_T-cut should be corrected.
This study: Binomial distribution model.

$$\langle N_{\rm net}^2\rangle_c^{\rm corrected} = \frac{1}{p^2}\Big(\langle n_{\rm net}^2\rangle_c - (1-p)\langle n_{\rm tot}\rangle_c\Big)$$

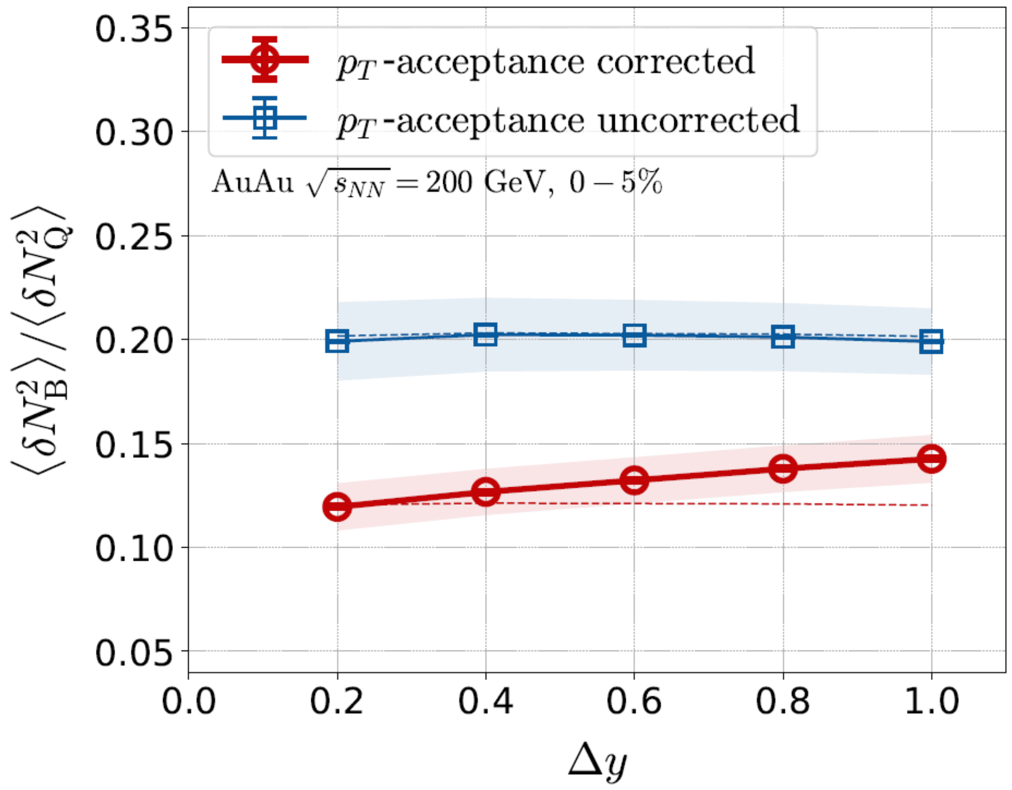
MK, Asakawa, '12, '12

- correction in HRG: Alba+'14; '15; …

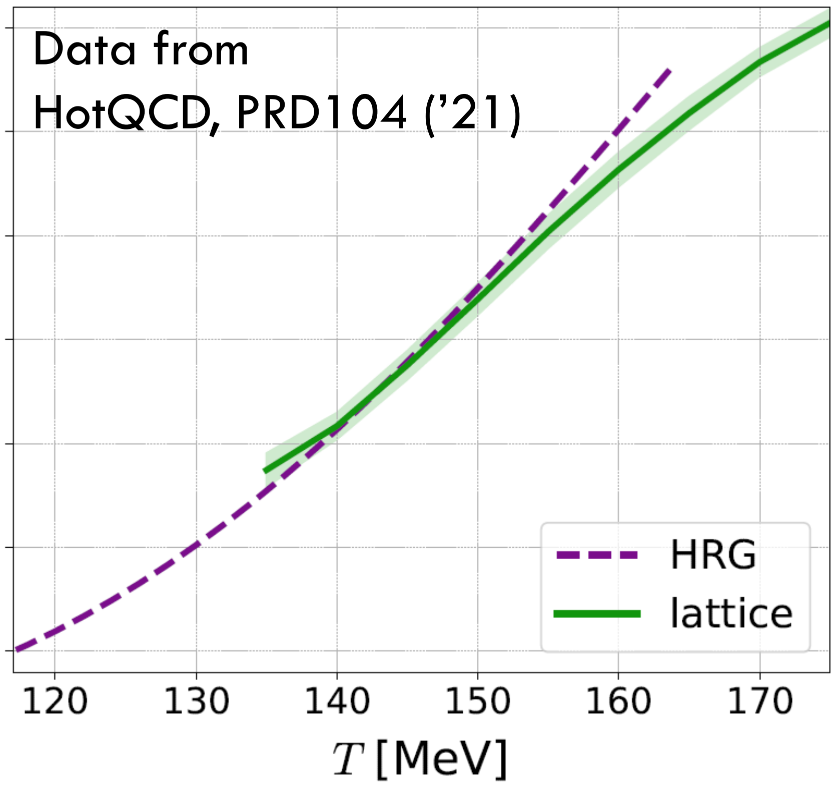
HIC vs HRG&LAT

MK, Esumi, Nonaka, 2205.10030

From data @ STAR



HRG+Lattice



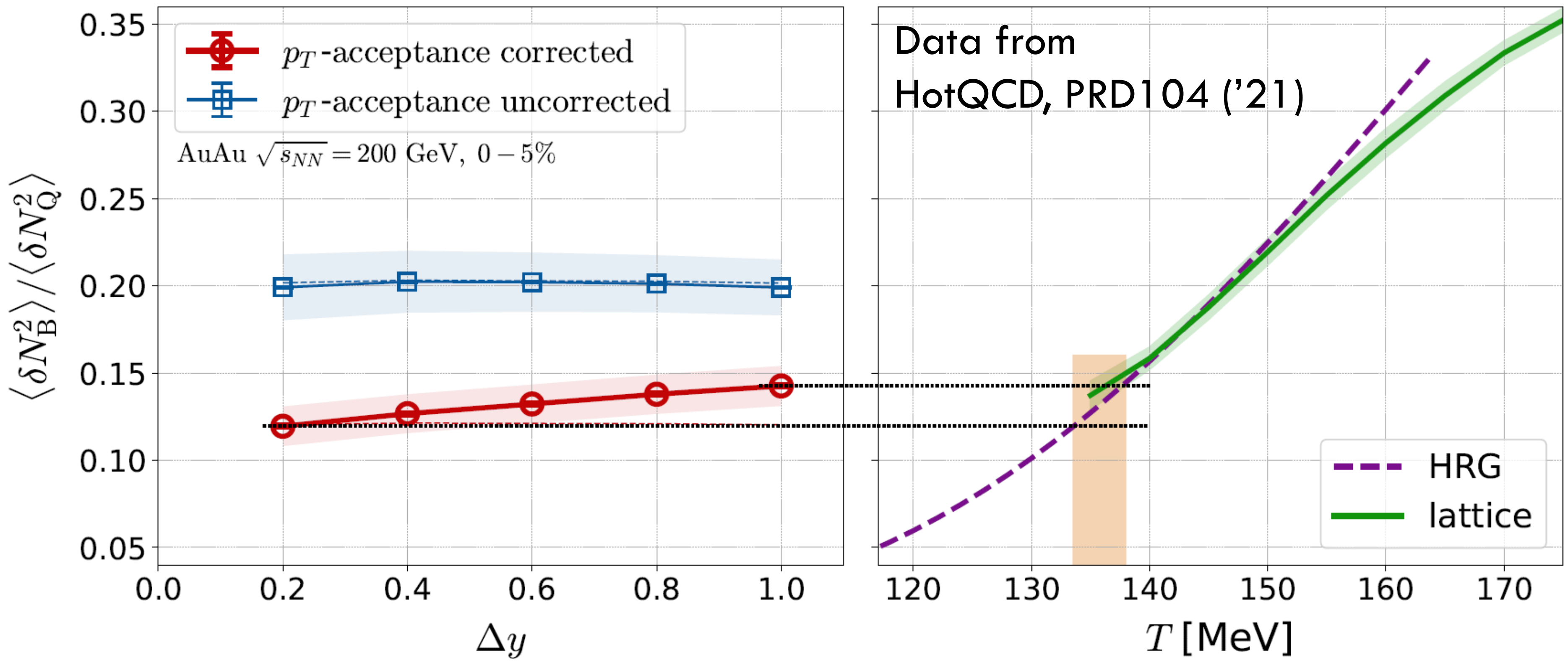
- $T = 134 \sim 138$ MeV
- Significantly lower than $T_{\rm chem}$

HRG: QMHRG2020
Bollweg+, PRD104, 7 ('21)
Volume dep. corrected
plot by MK

HIC vs HRG&LAT

MK, Esumi, Nonaka, 2205.10030

From data @ STAR HRG+Lattice



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Bollweg+, PRD104, 7 ('21)
Volume dep. corrected
plot by MK

D-Measure = Net-charge Fluctuations

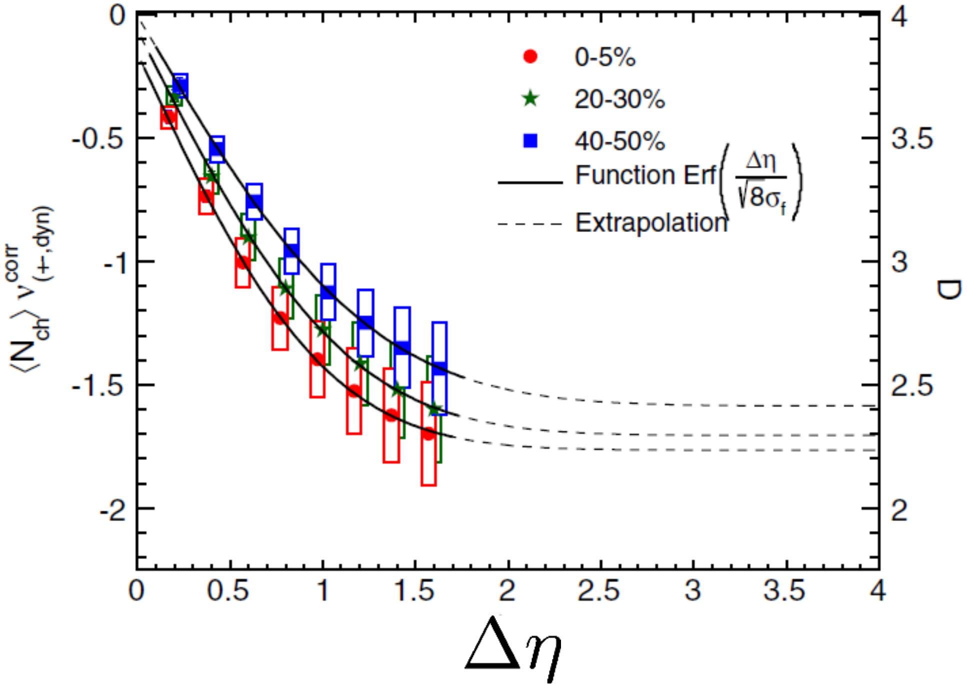
D-measure

$$D = 4\frac{\langle N_Q^2 \rangle_c}{\langle N_Q^{tot} \rangle}$$

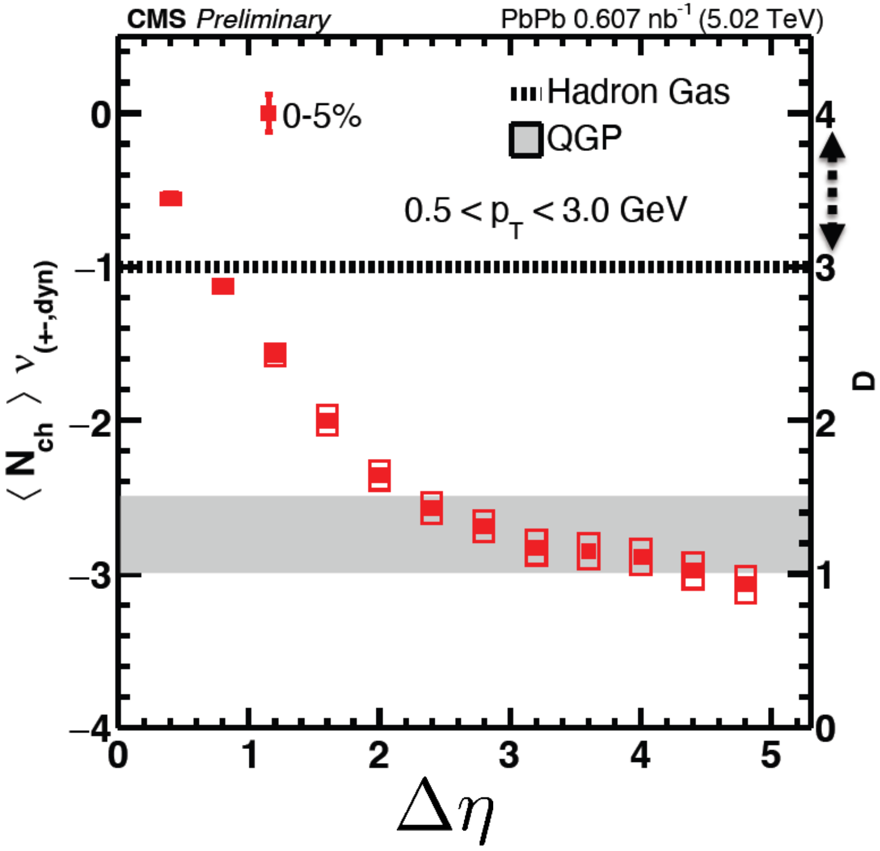
- Hadron gas
  $D \simeq 3$
- QGP
  $D \simeq 1 - 1.5$

Jeon, Koch, 2000

ALICE, PRL2013



CMS (Tuo, QM2023)



$$0.5 < p_T < 3.0\ \text{GeV}$$

D-Measure = Net-charge Fluctuations

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$$D = 4\frac{\langle N_Q^2 \rangle_c}{\langle N_Q^{tot} \rangle}$$

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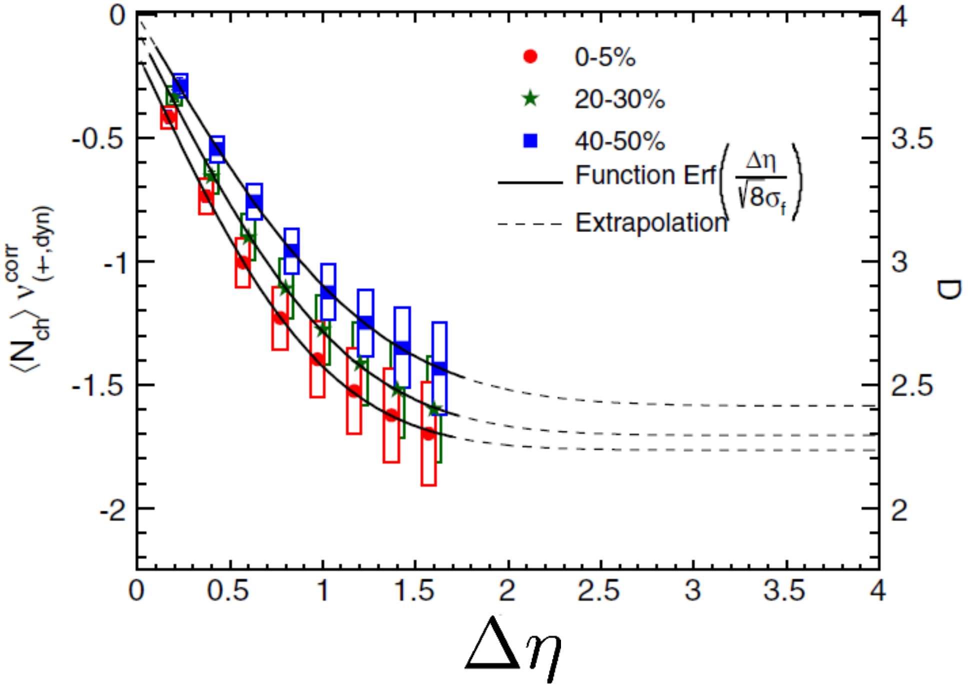
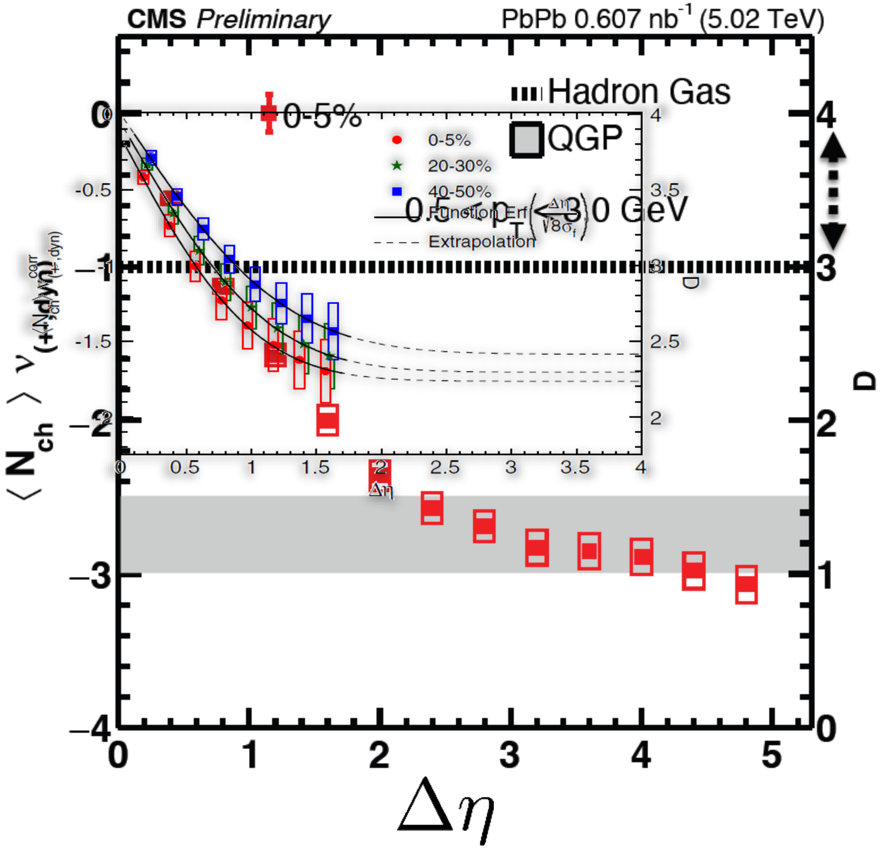
Jeon, Koch, 2000

ALICE, PRL2013



CMS (Tuo, QM2023)

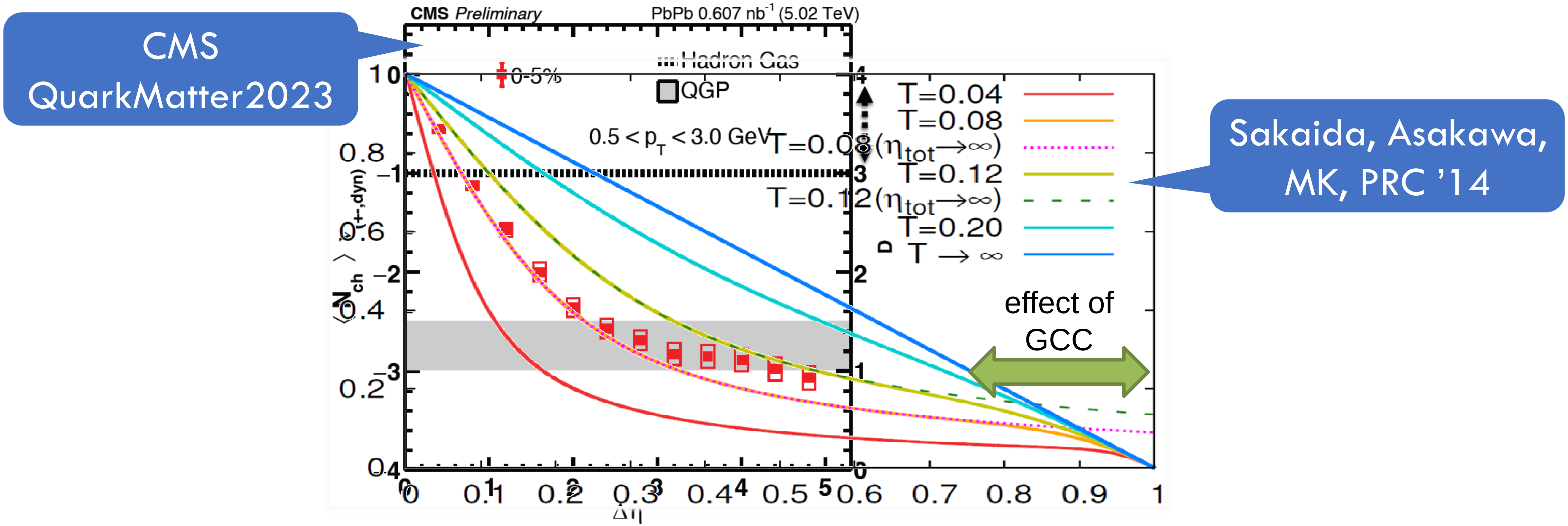


$0.5 < p_T < 3.0$ GeV

Comparison with a Diffusion Model in Finite Volume

MK, in progress

- Diffusion model in finite volume Sakaida, Asakawa, MK, PRC, 2014



- CMS result is well described by a simple diffusion model in finite volume.
- Effects of the global charge conservation do not affect the results for $\Delta\eta < 5$.

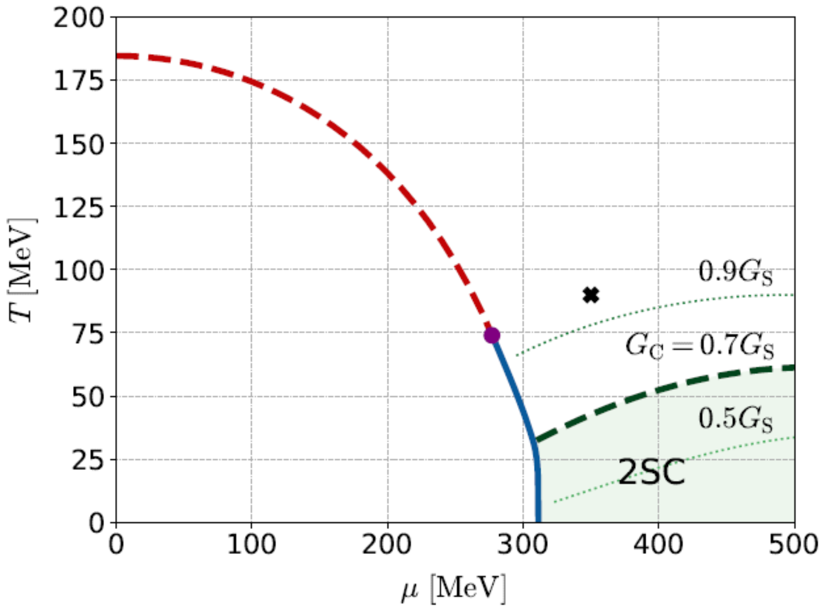
Dilepton Production
as experimental observables of
Color Superconductivity & QCD-CP

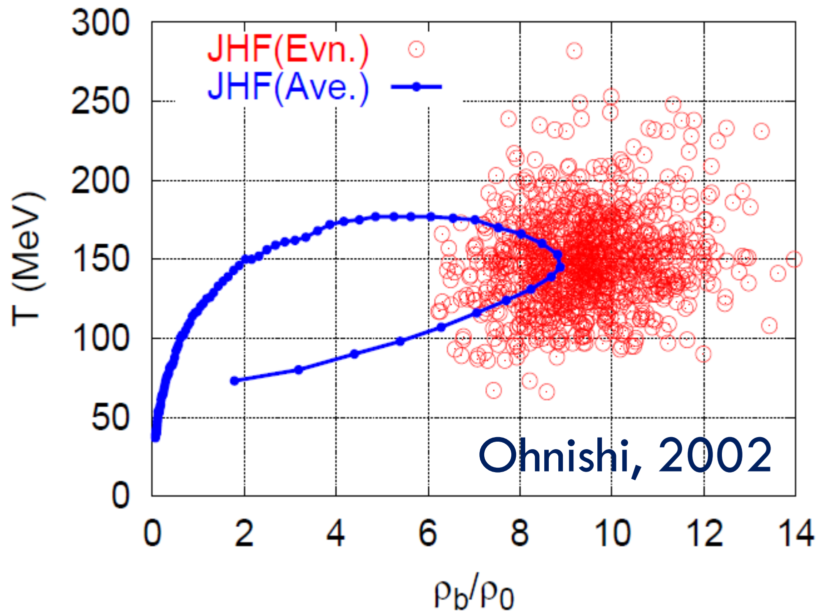
Nishimura, MK, Kunihiro, PTEP2022, 093D02; PTEP2023, 053D01; in prep

Observing CSC in HIC

- **Difficulties**
  - CSC would not be created if Tc is not high enough.
  - Even if created, its lifetime would be short.
  - Since CSC is created in the early stage, its signal would be blurred during the evolution in later stage.

- **Strategy** in our study:
  - Focus on precursory phenomena of CSC
  - Use dilepton production as an observable

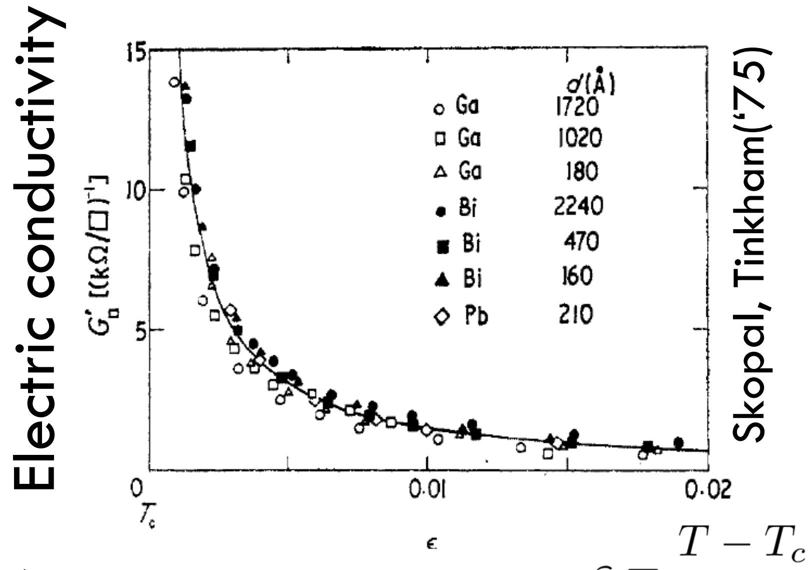




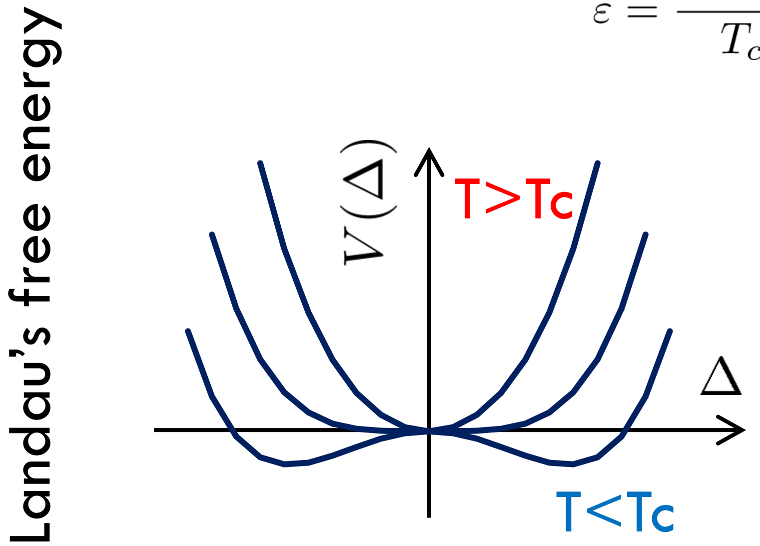
Precursor of CSC

- ☐ **Anomalous behavior of observables near but above Tc of SC**
  - ☐ electric conductivity
  - ☐ magnetic susceptibility
  - ☐ pseudogap

- ☐ Enhanced pair fluctuations is one of the origins of precursory phenomena.
- ☐ More significant phenomena in strongly-coupled systems.



$$\varepsilon = \frac{T - T_c}{T_c}$$

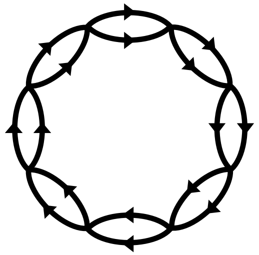


Precursor of Color Superconductivity

MK, Koide, Kunihiro, Nemoto, '03, '05

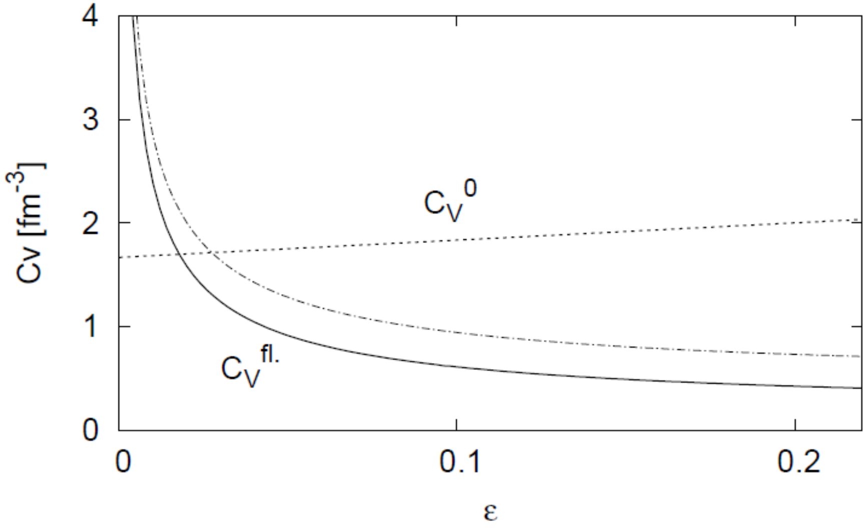
□Thermodynamic Potential

$\Omega =$



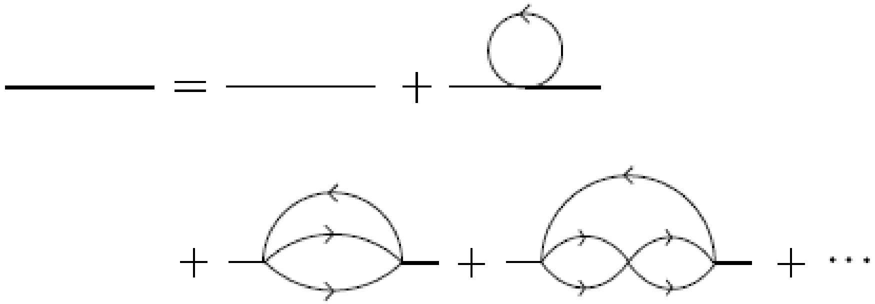
Specific heat

$$c = -T\frac{\partial^2\Omega}{\partial T^2}$$

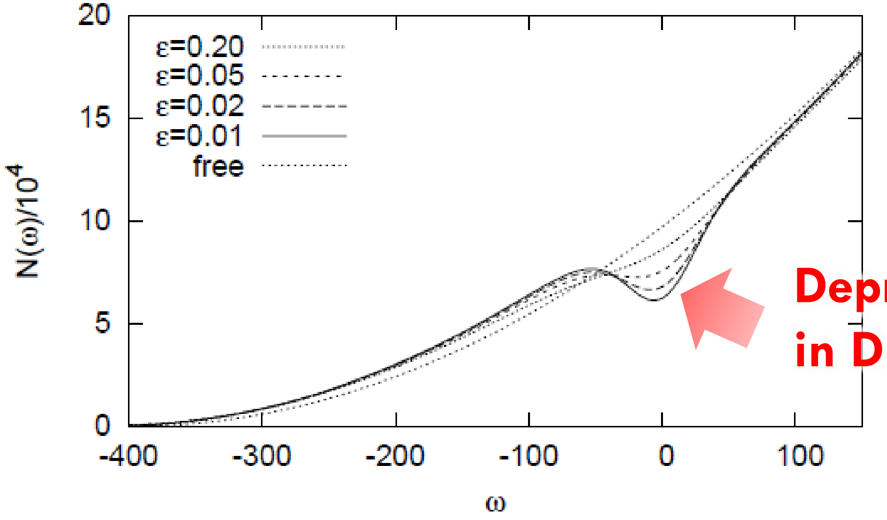


$$\varepsilon = \frac{T - T_c}{T_c}$$

□Pseudogap



density of states



Depression in DoS above Tc

Model

NJL model (2-flavor)

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \mathcal{L}_S + \mathcal{L}_C$$

$$\mathcal{L}_S = G_S\left((\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \tau \psi)^2\right)$$

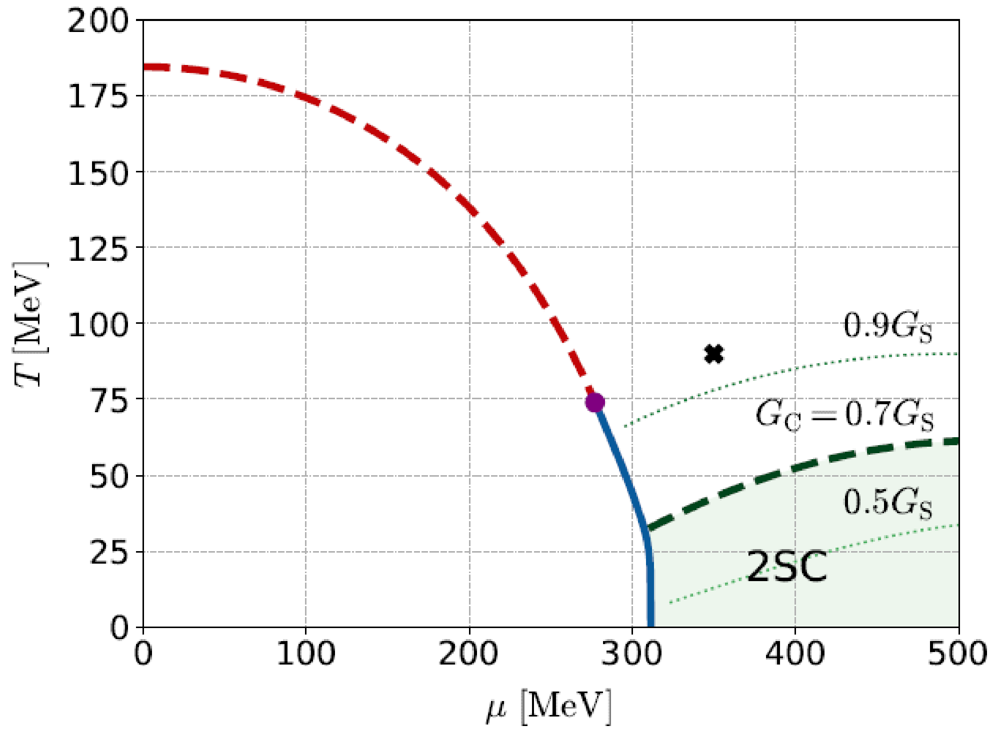
$$\mathcal{L}_C = G_C\left((\bar{\psi} i\gamma_5 \tau_A \lambda_A \psi^C)(\text{h.c.})\right)$$

diquark interaction

Parameters

$$G_S = 5.01\ \text{GeV}^{-2}, \quad \Lambda = 650\text{MeV}, \quad m_q = 0$$

Phase Diagram in MFA



- Order of phase transition
  - 2nd in the MFA
  - can be 1st due to gauge fluctuation

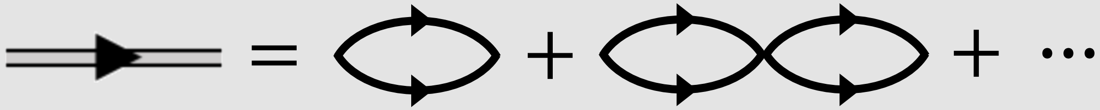
Matsuura+('04), Giannakis+('04)
Noronha+('06), Fejos, Yamamoto('19)

Di-quark Fluctuations

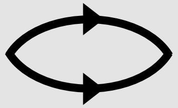
☐ Diquark Propagator

$$D^R(x) = \langle [\Delta^\dagger(x), \Delta(0)] \rangle \theta(t) =$$ 

☐ Random Phase Approximaion



$$= \frac{Q^R(\boldsymbol{k},\omega)}{1 + G_C Q^R(\boldsymbol{k},\omega)}$$

$$Q^R(\boldsymbol{k},\omega) =$$ 

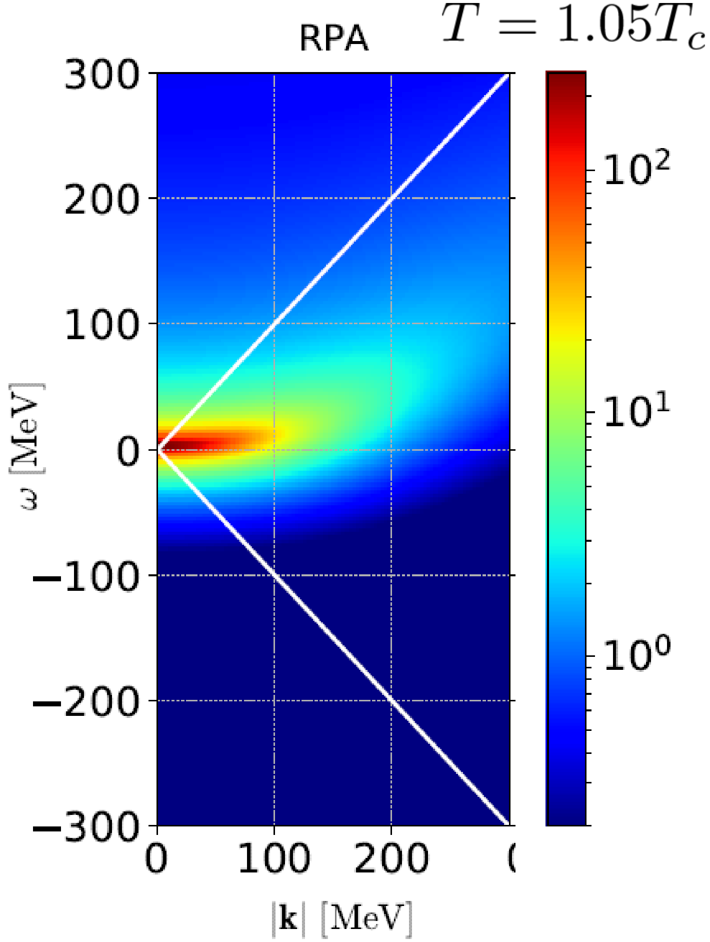
- ☐ Diquark field becomes massless at T=Tc
- ☐ Soft mode of CSC transition
- ☐ Strength in the space-like region

MK, Koide, Kunihiro, Nemoto, '01,'05

Dynamical Structure Factor

$$S(\boldsymbol{k},\omega) = -\frac{1}{\pi}\frac{1}{1-e^{-\beta\omega}}\mathrm{Im}\mathrm{D}^{\mathrm{R}}(\mathrm{k},\omega)$$

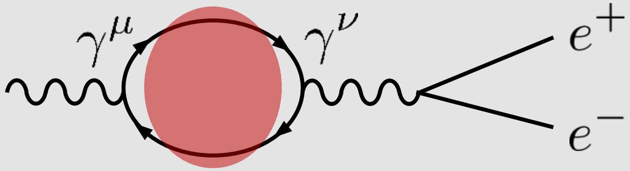
$T = 1.05 T_c$



Photon Self-Energy: Precursor of CSC

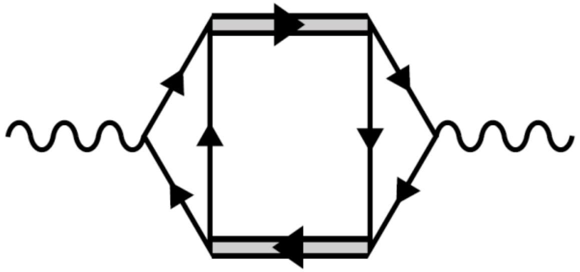
□ Dilepton Production Rate

$$\frac{d^4\Gamma}{dk^4} = \frac{\alpha}{12\pi^4}\frac{1}{k^2}\frac{1}{e^{\beta\omega-1}}\mathrm{Im}\Pi^{R\mu}_{\mu}(k)$$

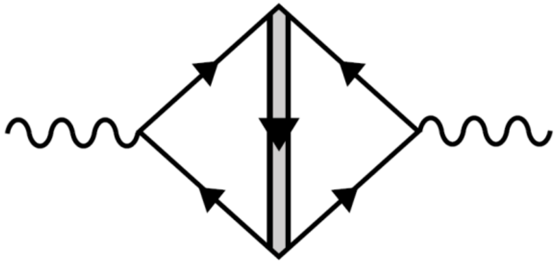


□ Effect of Di-quarks on $\Pi^{\mu\nu}(k)$

Aslamasov-Larkin term



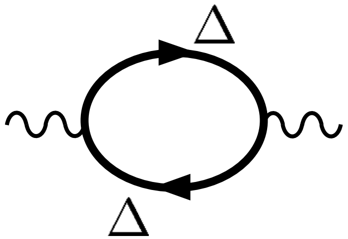
Maki-Thompson term

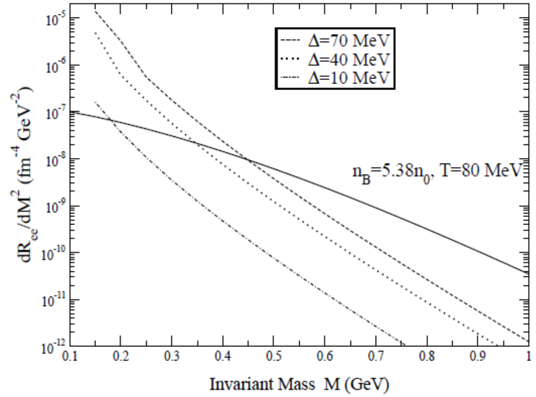


Well-known diagrams in metallic SC
for describing paraconductivity

□ DPR from CFL phase

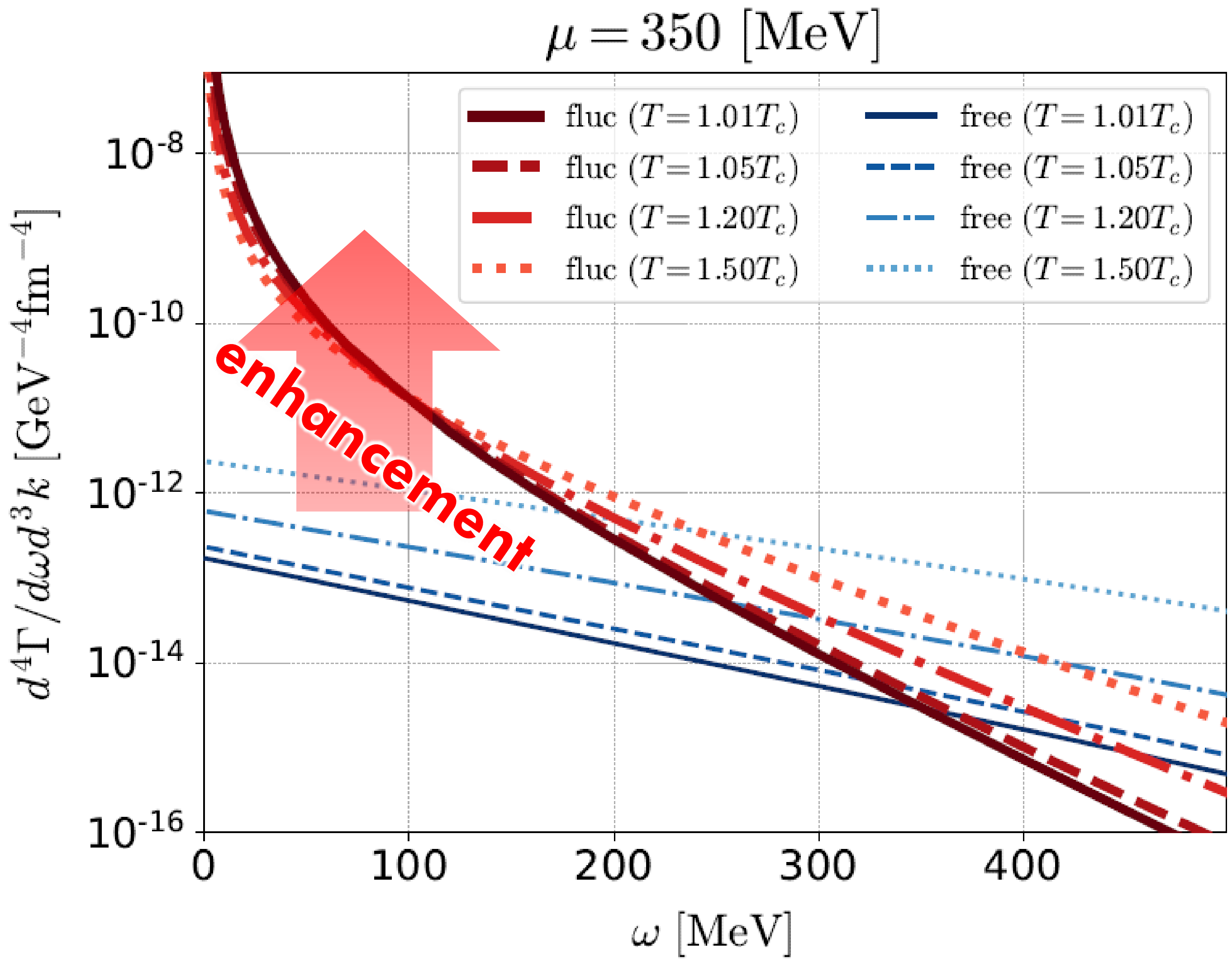
Jaikumar, Rapp, Zahed ('02)





Production Rate at $\boldsymbol{k} = 0$

Nishimura, MK, Kunihiro ('22)



Red: fluctuation contribution

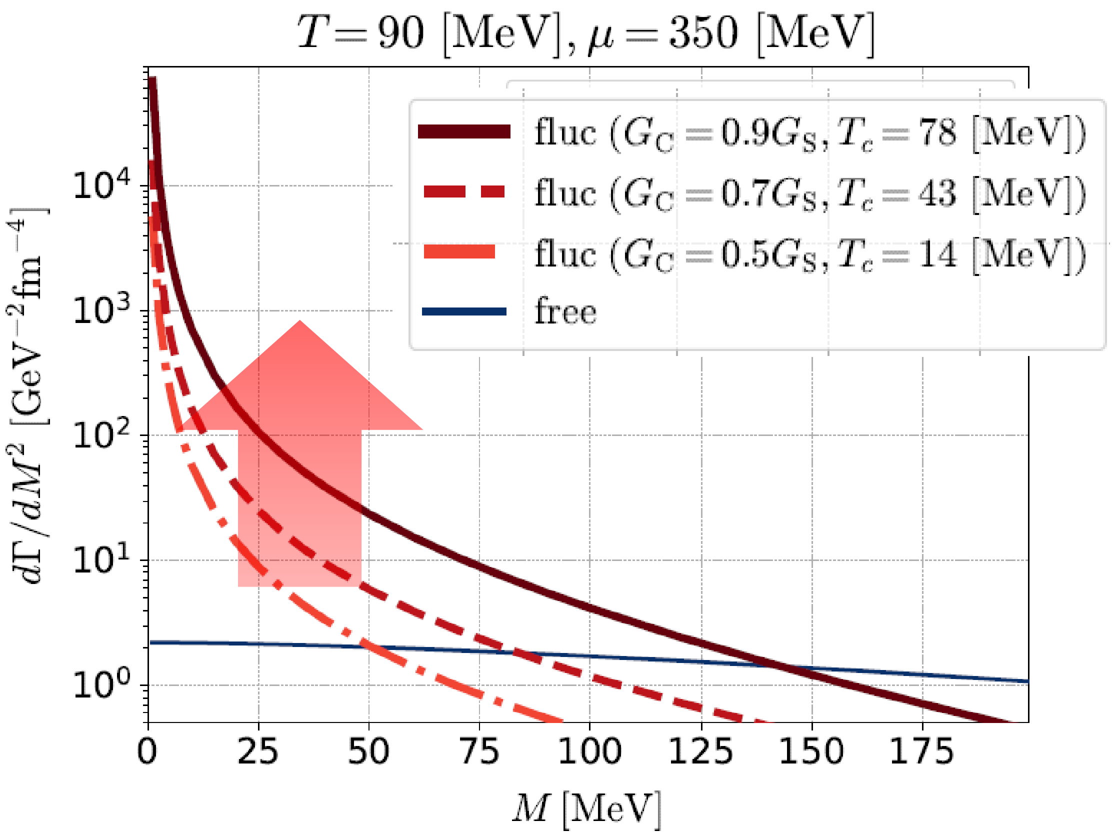
Blue: free quarks

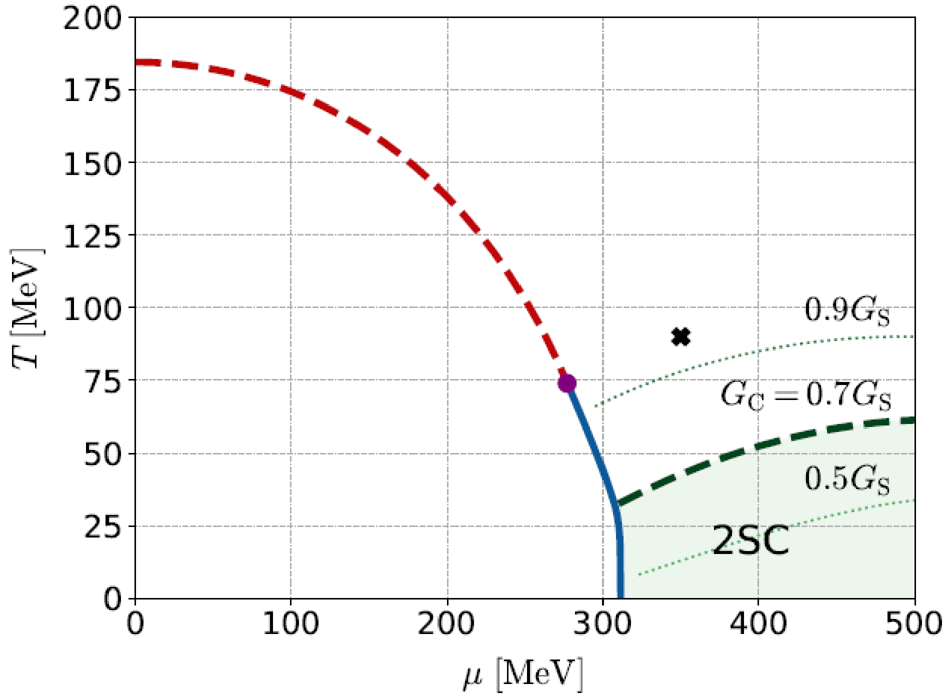
$G_C = 0.7G_s$, $T_c \simeq 45$ MeV

- Di-quark fluctuations give rise to large enhancement in the low energy region $\omega < 200$ MeV and $T < 1.5T_c$.
- Anomalous enhancement is not sensitive to T.

Invariant-Mass Spectrum

Nishimura, MK, Kunihiro ('22)





- ☐ Strong enhancement at low invariant mass.
- ☐ **Observable in the HIC?**

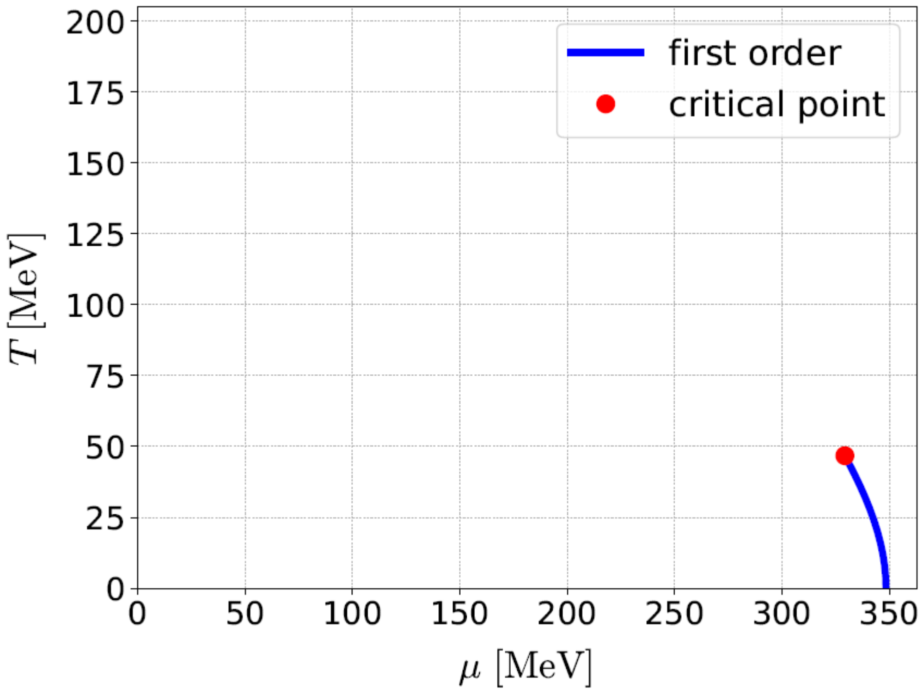
Dileptons from QCD Critical Point

NJL model (2-flavor)

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G_S\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2\right)$$

Parameters

$$G_S = 5.5\ \mathrm{GeV}^{-2},\ \Lambda = 631\mathrm{MeV},\ m_q = 5.5\ \mathrm{MeV}$$

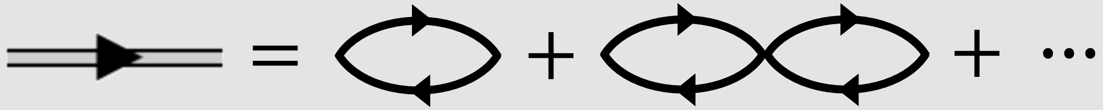


Soft Mode of QCD-CP

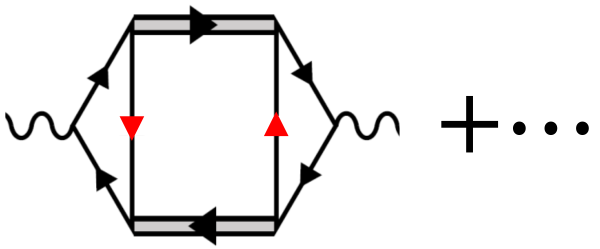
= fluctuation of scalar ($\bar{q}q$) channel

$$D^R(x) = \langle[\bar{\psi}\psi(x), \bar{\psi}\psi(0)]\rangle\theta(t) =$$

- Random Phase Approximaion



Modification of dilepton production through

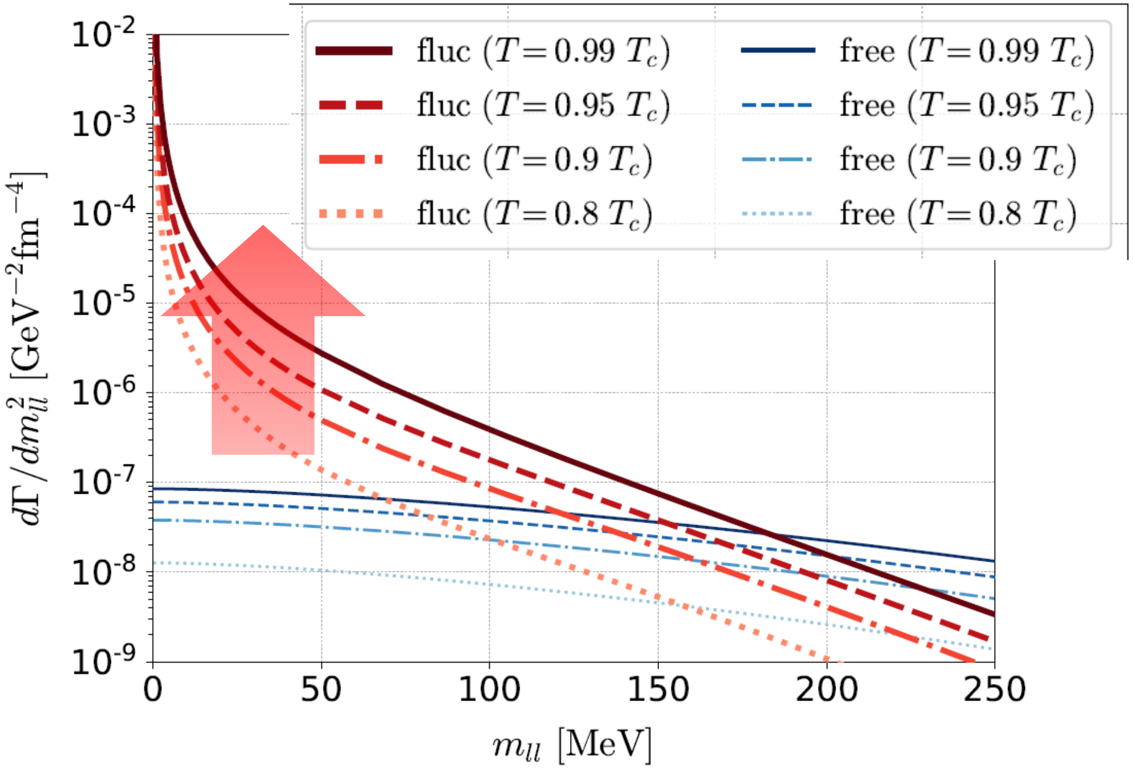


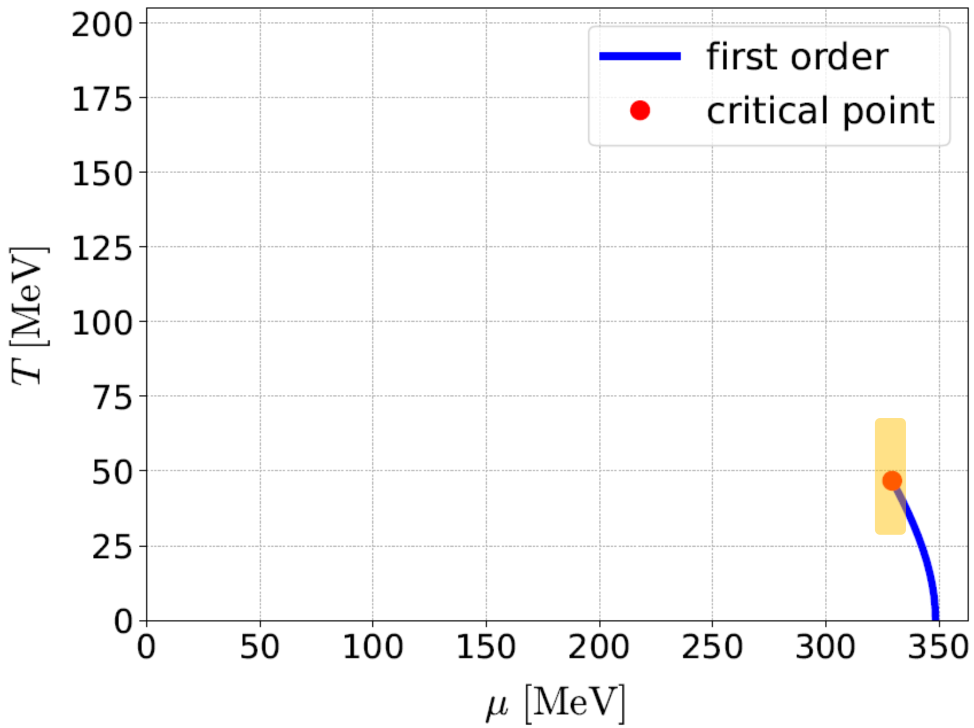
$+\cdots$ MT, DoS

Dilepton production rate near QCD-CP

Nishimura, MK, Kunihiro ('23)

Invariant mass spectrum





for fixed chem. pot.: $\mu = \mu_c$

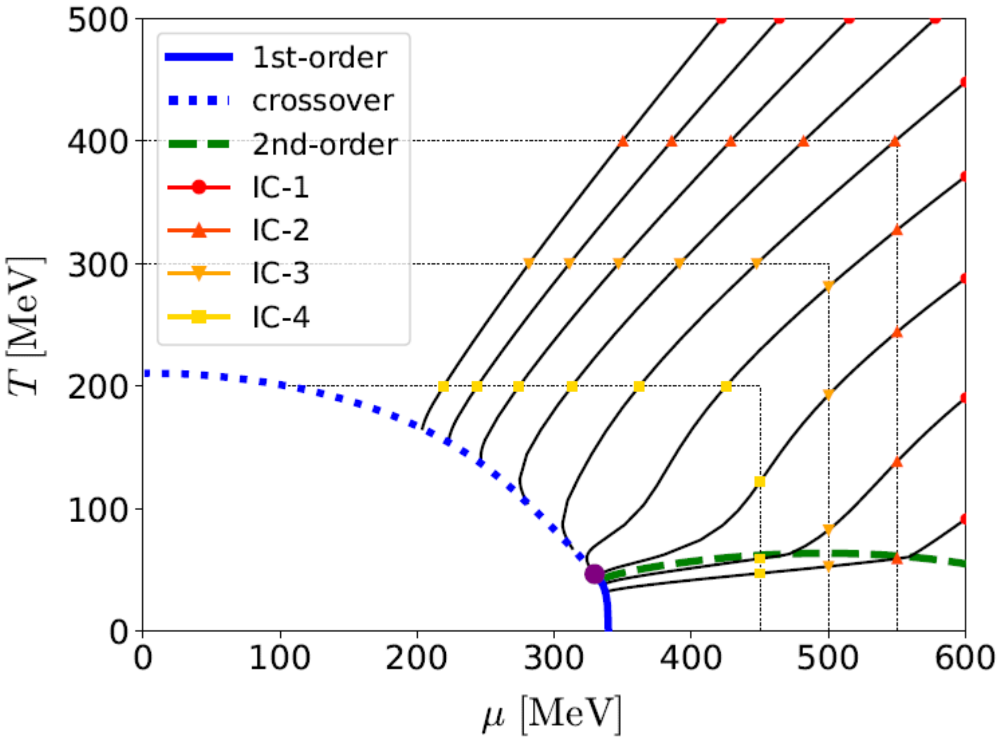
- ☐ Enhancement at low M_{ll} region near QCD-CP
- ☐ Distinguishment from diquark soft mode may be difficult.

Dilepton Yields: Beam-Energy Scan

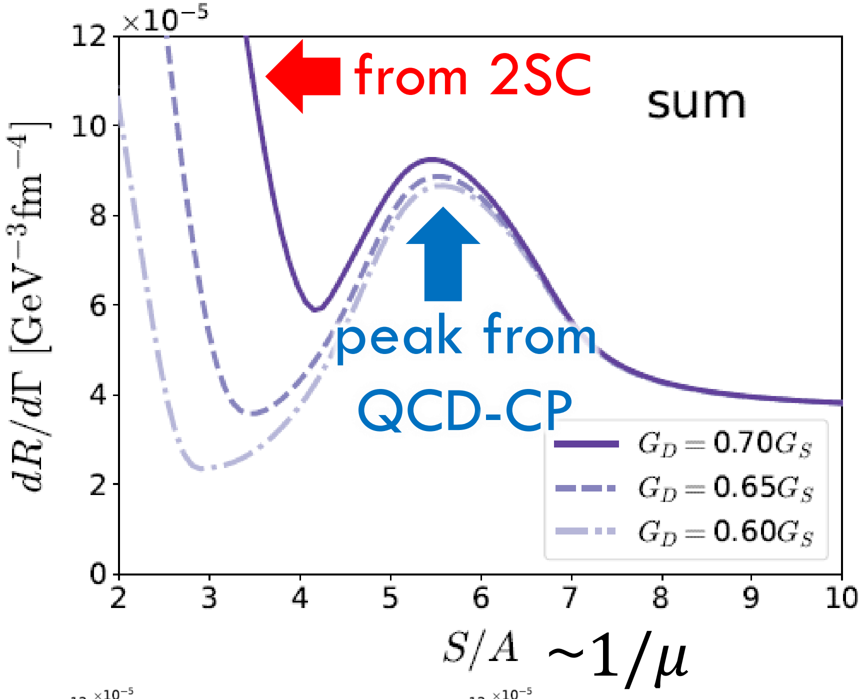
Dilepton yields
$\simeq$ integrated rate along isentropic lines

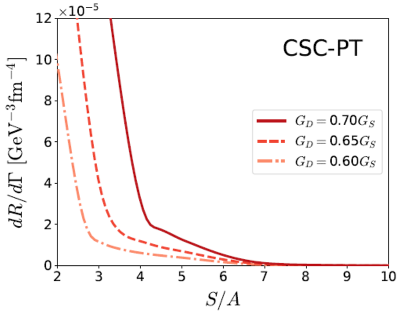
Nishimura, Nara, Steinheimer, arXiv:2311.14135

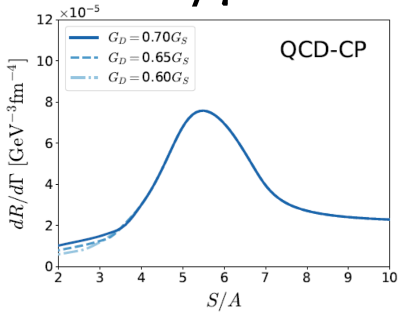
Isentropic lines in NJL model



Dilepton Yields $50 < M < 100$ MeV







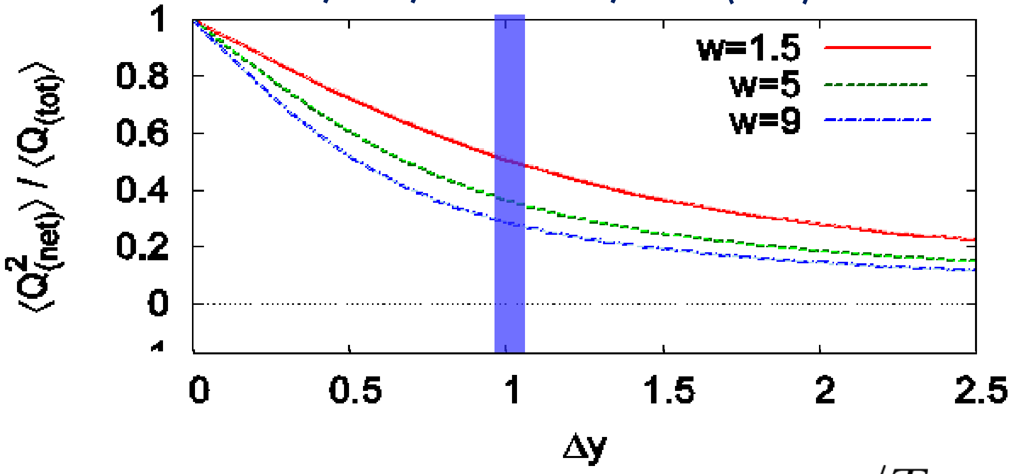
Summary

- A rich phase structure exists in dense QCD.
  - QCD critical point, color superconductivity, etc.
- The beam-energy scan will reveal it.

- Many interesting observables to explore the QCD phase diagram:
  - Event-by-event fluctuations of conserved charges
  - Dilepton production at ultra-low-mass-region
  - …

Effect of Diffusion and Rapidity Conversion

Ohnishi, MK, Asakawa, PRC ('16)



$w = m/T$

- **☐** Blurring due to diffusion & rapidity conversion ($Y \to y$)
    - Stronger modification in Q than B
- **☐** Resonance Decays
    - About 30% charged particles come from RD
    - Enhancement of charged particles

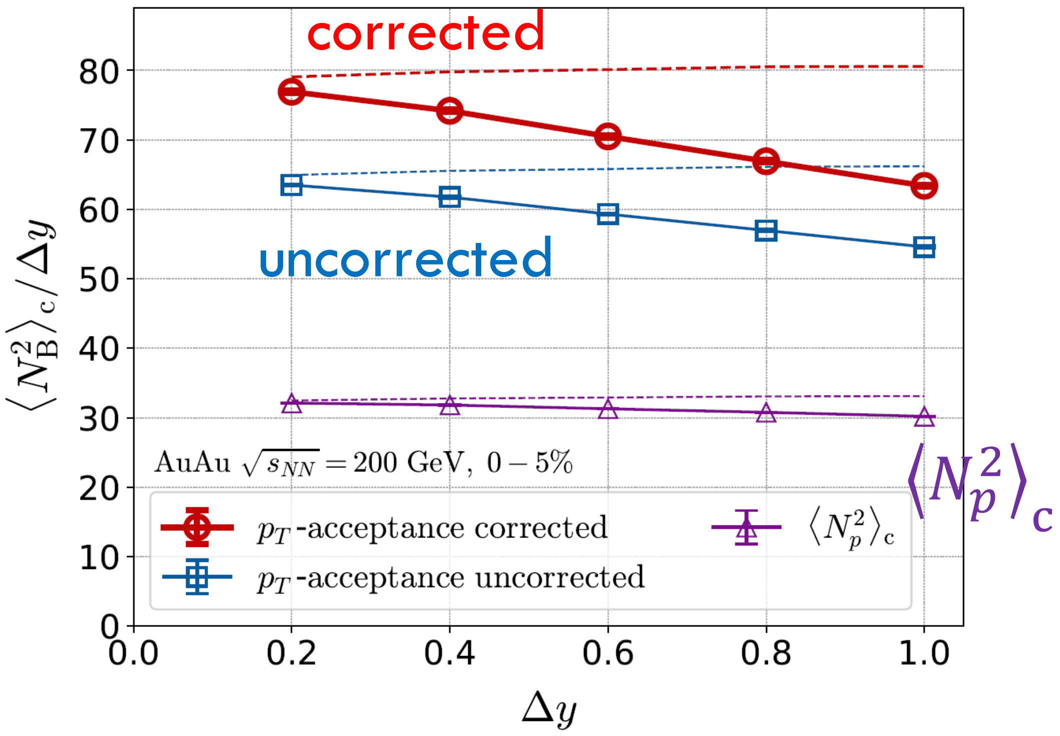
⇒
- ☐ Increase $\langle N_Q^2 \rangle$
- ☐ Reduce $\langle N_B^2 \rangle_c / \langle N_Q^2 \rangle_c$

These effects will be more important for higher order cumulants!

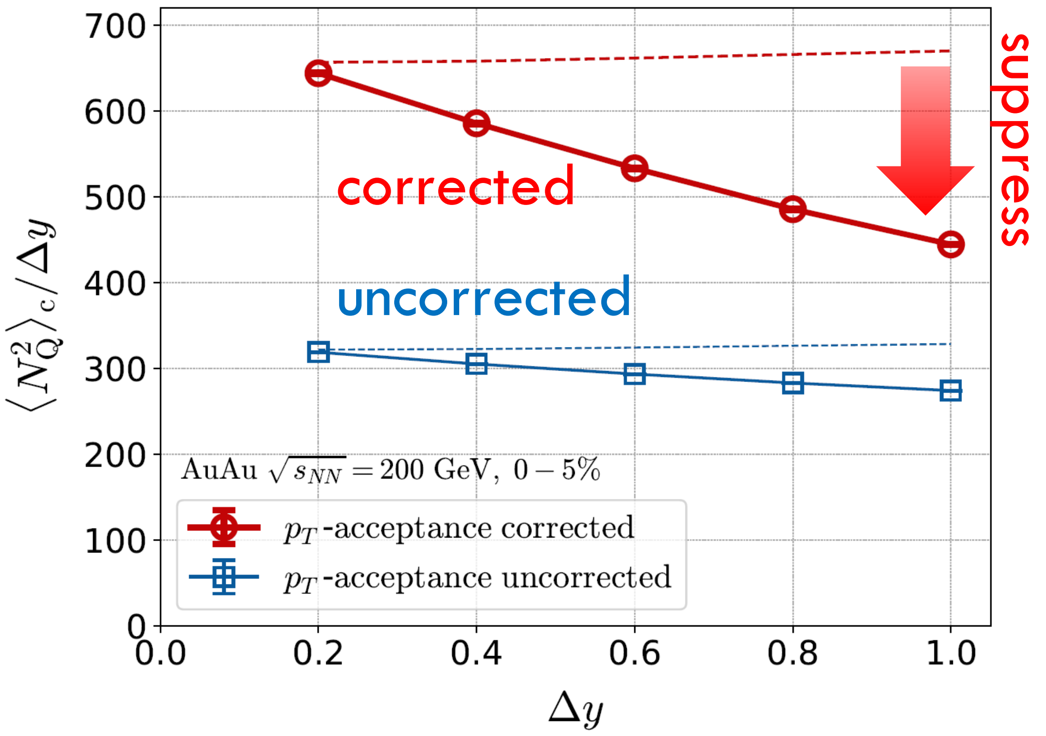
Cumulants: Proton→Baryon & Acceptance Correction

Data from STAR, '19, '21

$\langle N_B^2 \rangle_c / \Delta y$



$\langle N_Q^2 \rangle_c / \Delta y$



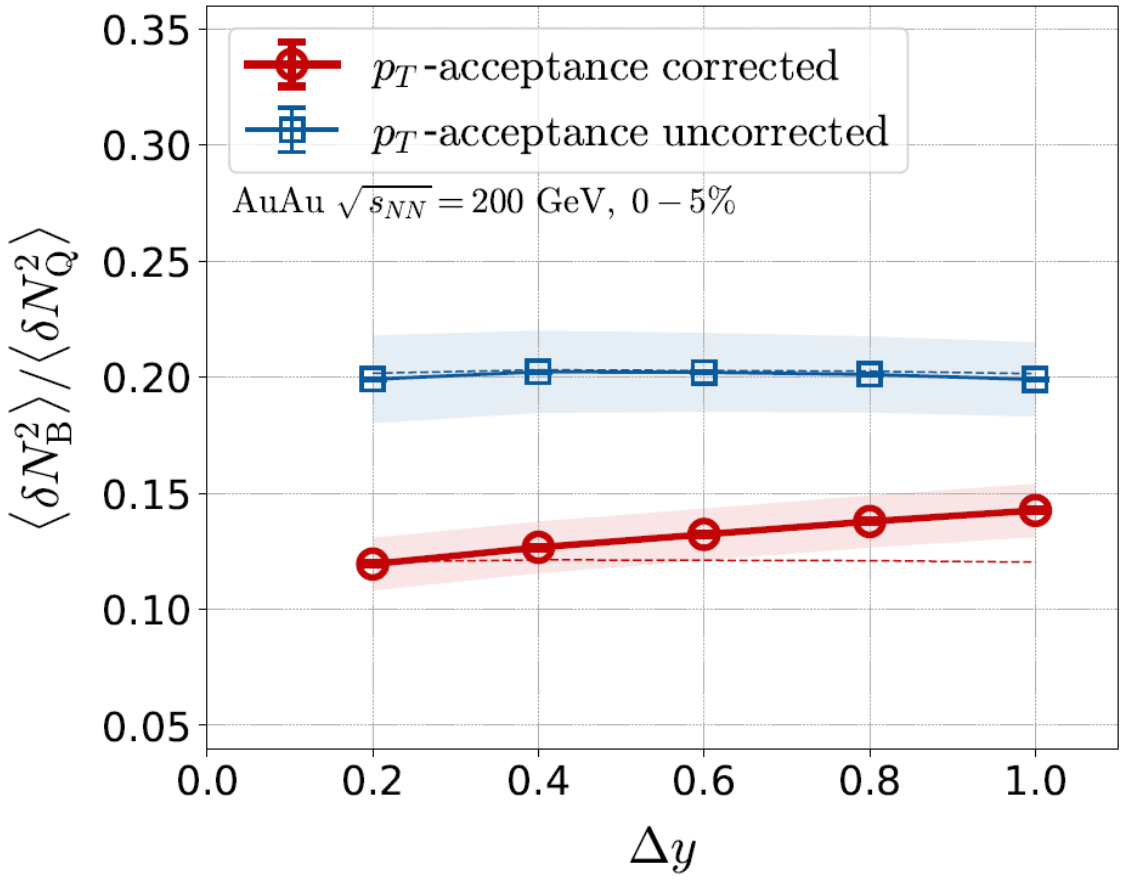
dashed: $\langle N^{\mathrm{tot}} \rangle$ Poisson

- ☐ Deviation from Poissonian is clarified by the acceptance correction.

$$\langle N_{\mathrm{net}}^2 \rangle_c^{\mathrm{corrected}} = \frac{1}{p^2} \Big(\langle n_{\mathrm{net}}^2 \rangle_c - (1-p) \langle n_{\mathrm{tot}} \rangle_c \Big)$$

MK, Asakawa, '12, '12

$\langle N_B^2 \rangle_c / \langle N_Q^2 \rangle_c$



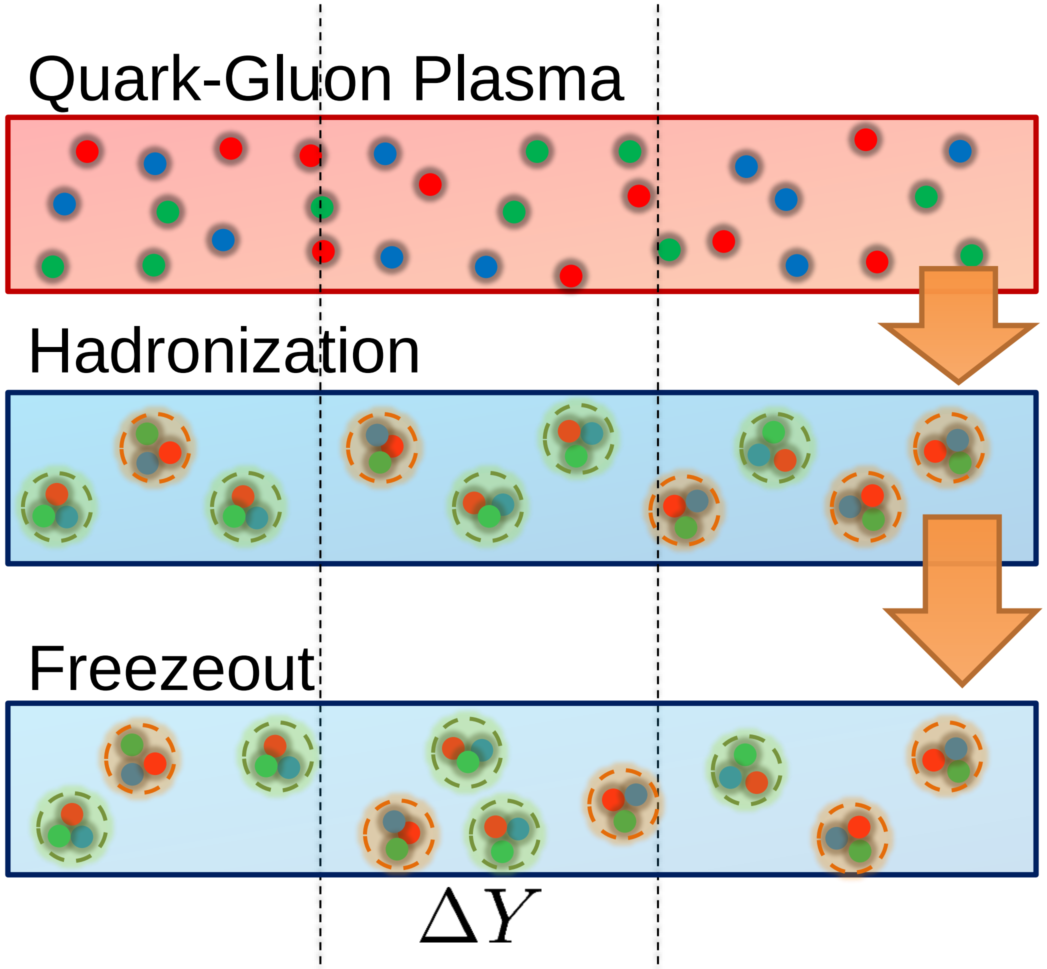
p_T-acceptance **un**corrected

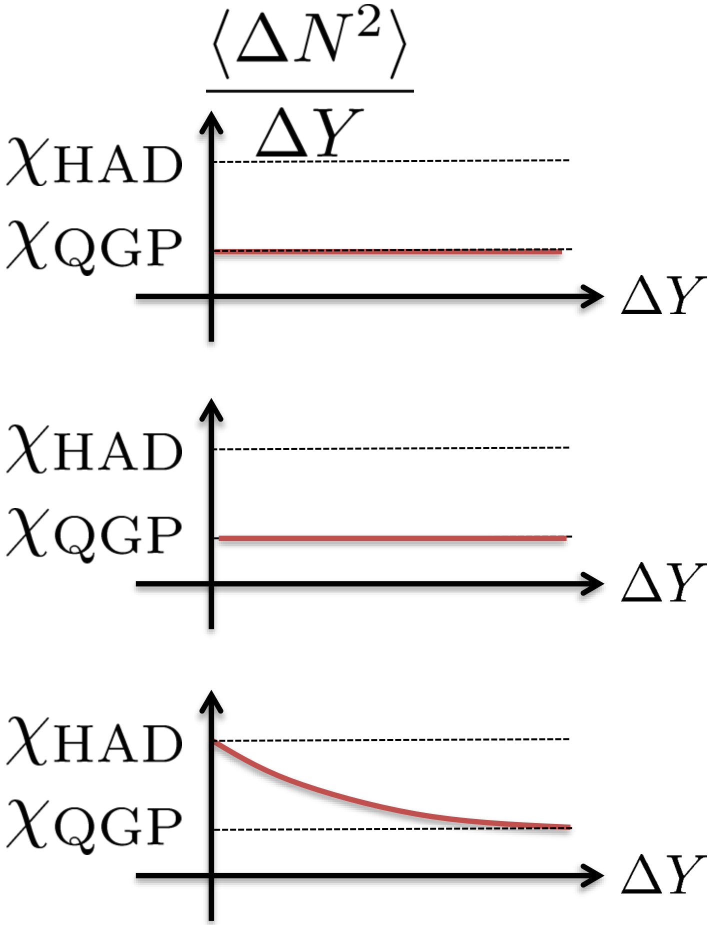
p_T-acceptance corrected

dashed: Poisson

- $\langle N_B^2 \rangle_c / \langle N_Q^2 \rangle_c$ becomes smaller due to the p_T-acceptance correction.
- Clear Δy dependence ➡ non-thermal effects behind fluctuations

Evolution of Conserved-charge Fluctuations





Fluctuations of CC are modified by the diffusion.

Relaxation time becomes longer as ΔY →large.

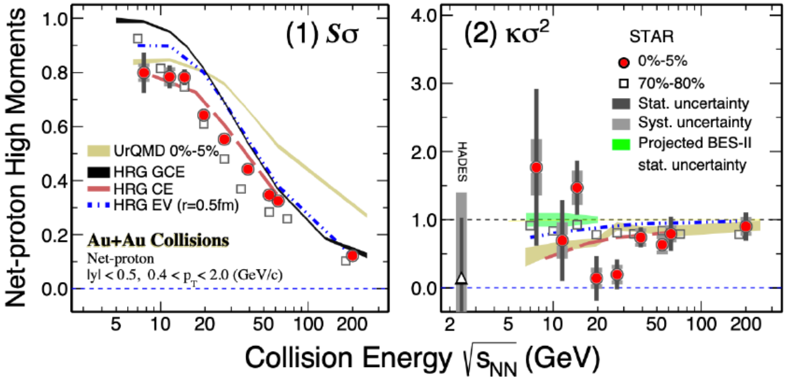
Experiments on $\langle N_Q^2 \rangle$

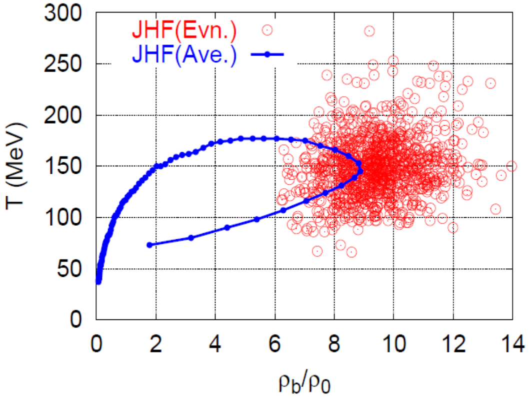
- ☐ No QGP signal @ RHIC ('02, '03)
- ☐ QGP signal? @ ALICE ('12)

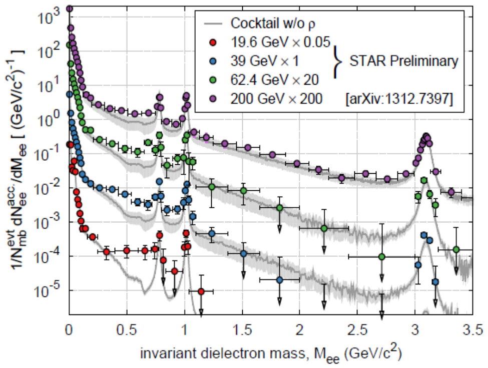
Asakawa, Heinz, Muller, 2000
Jeon, Koch, 2000; Shuryak, Stephanov, 2001

Exploring Dense Medium

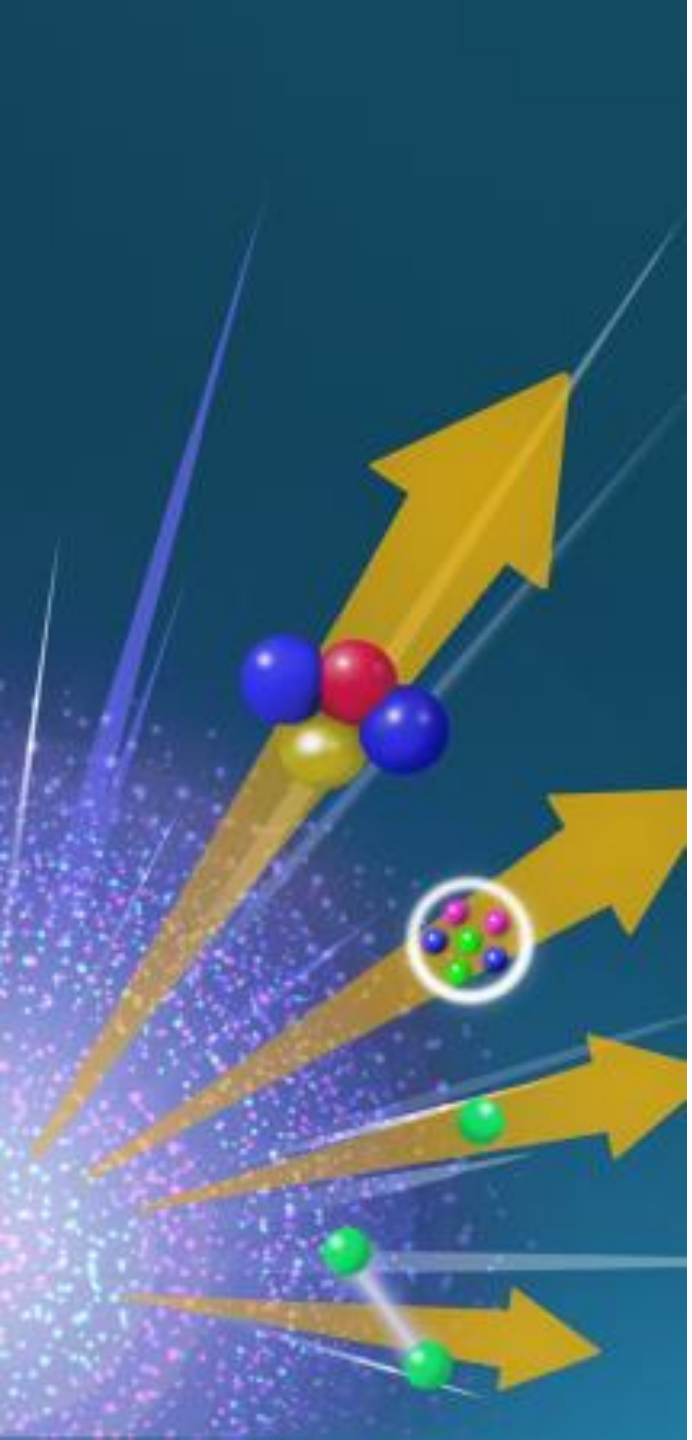
- Equation of state
- QCD critical point /
  1st order transition /
  Color superconductivity
- Dilepton production rate
- Event selection /
  Higher correlations



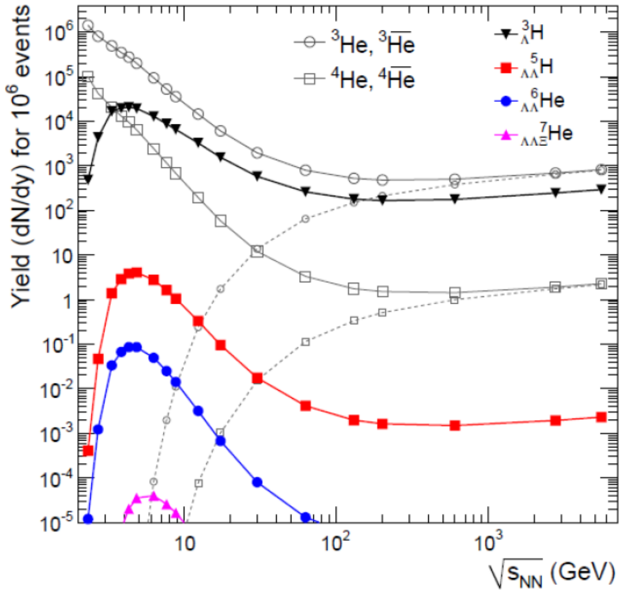




Hadron/Hypernuclear Physics

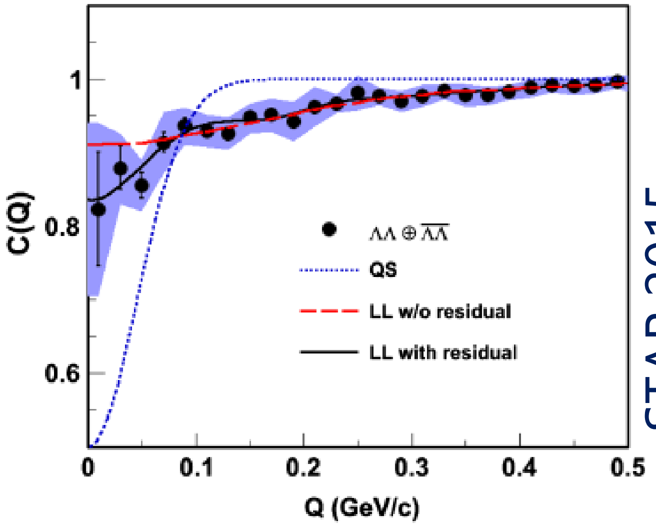


Hypernuclei

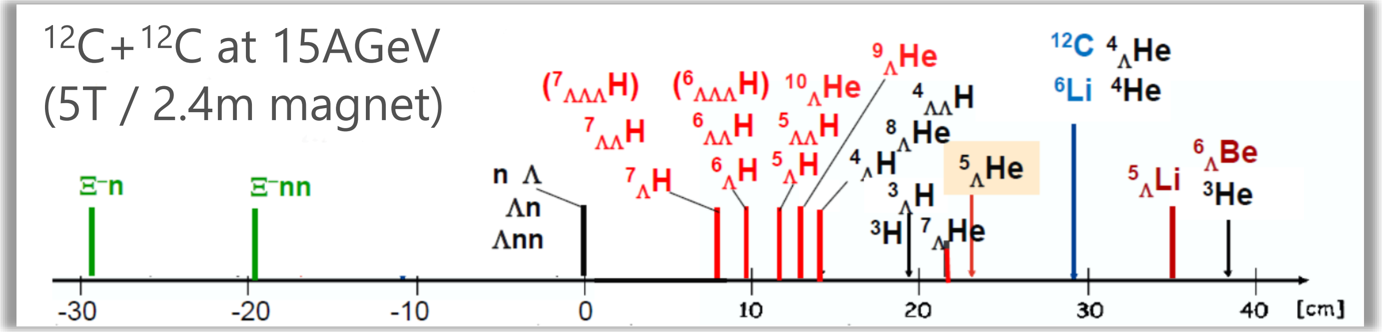


Correlation functions

→hadron interaction



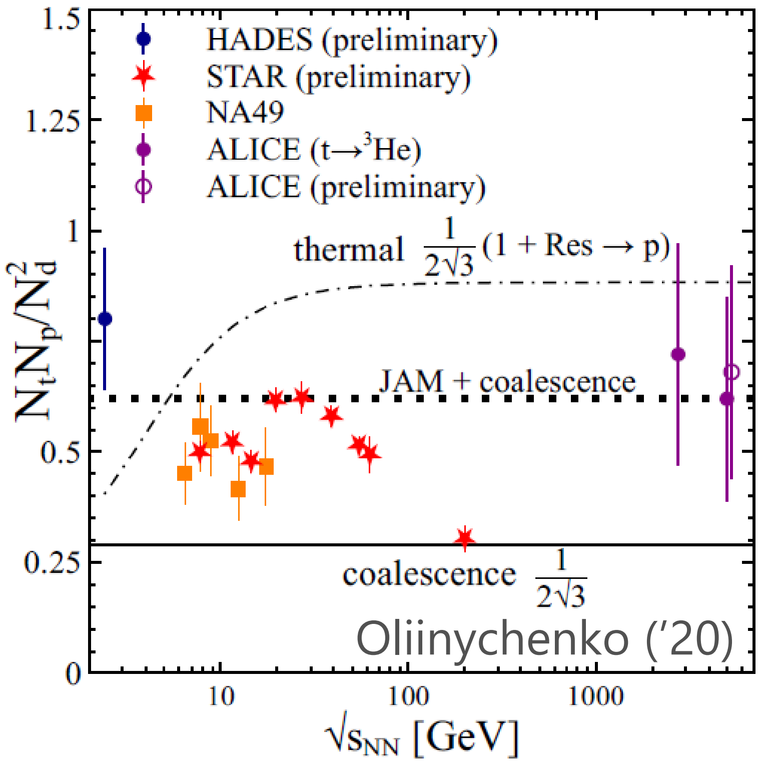
STAR,2015



Production Mechanism

Light-nuclei production

as a signal of QCD critical point

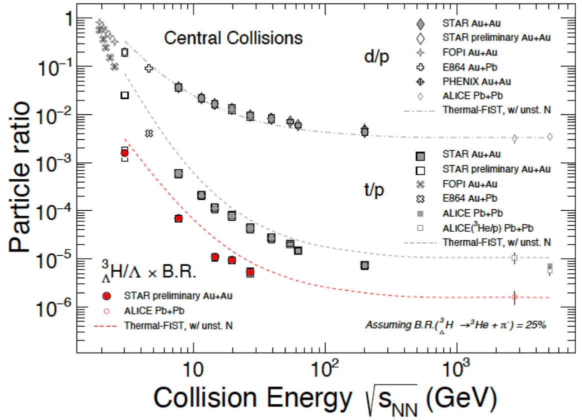


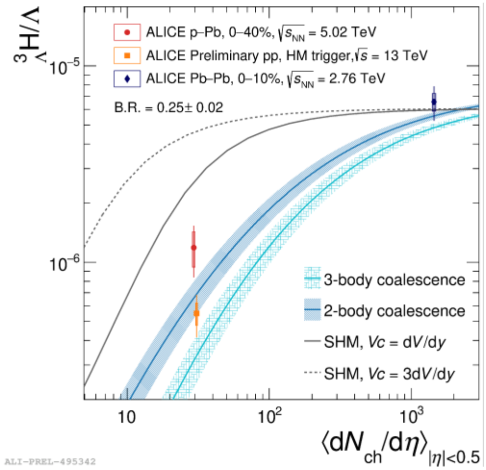
baryon fluctuations

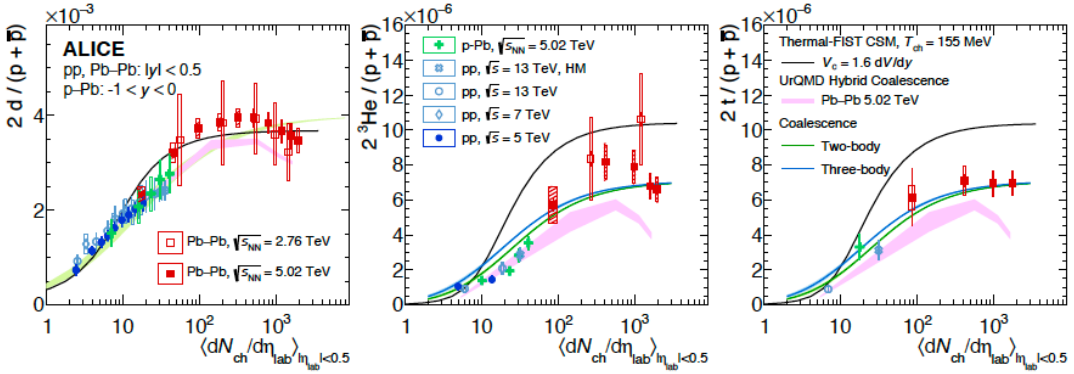
→ enhancement of light nuclei?

Sun+ ('18)

Measurement of light/hyper-nuclei







from QM2023

Precise data will lead us to a better understanding of production mechanism