EMMI Workshop at the University of Wrocław - Aspects of Criticality II, 2 July, 2024, University of Wrocław

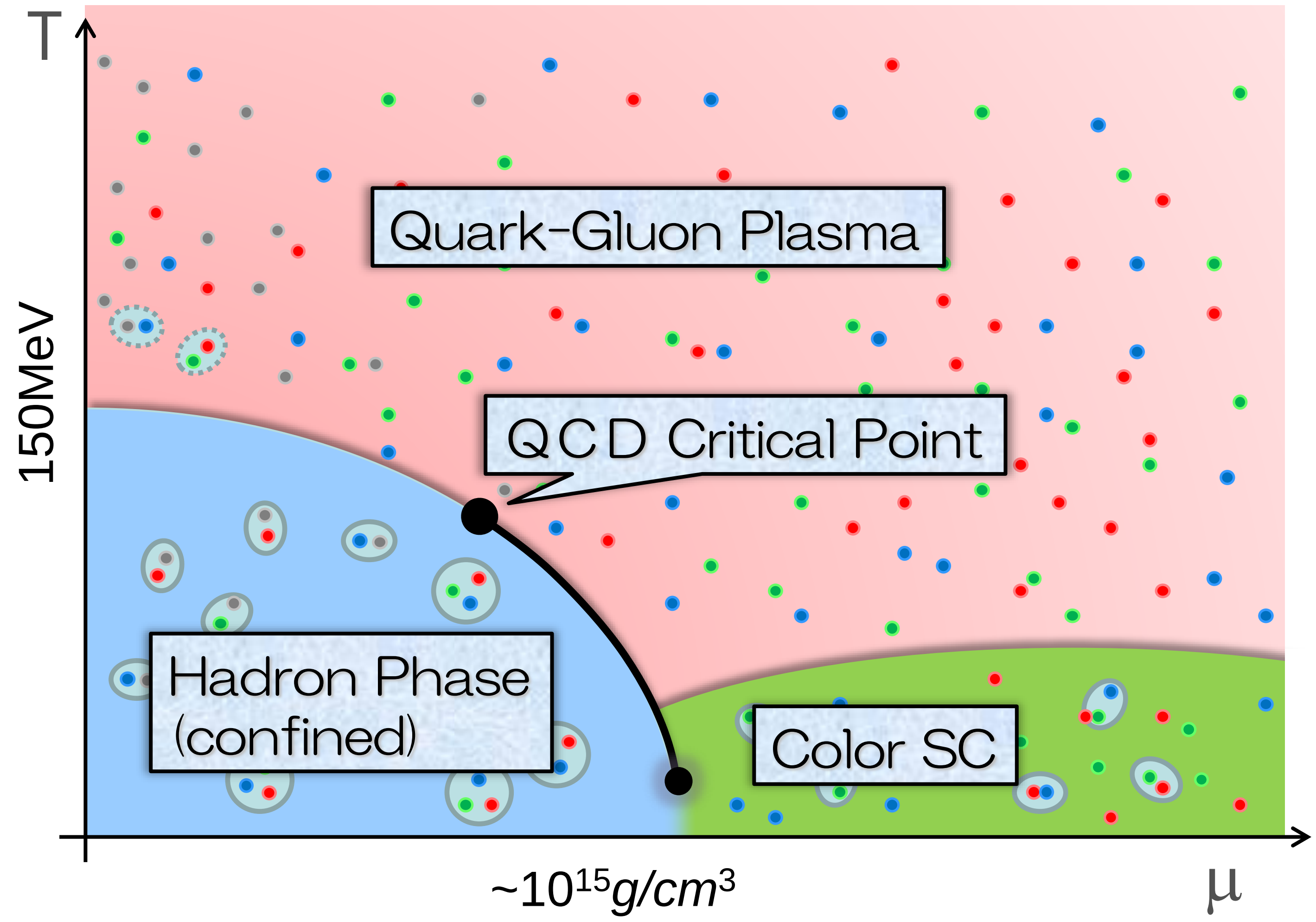
Electromagnetic Probes for Critical Fluctuations of Phase Transitions in Dense QCD

Masakiyo Kitazawa

(YITP, Kyoto)

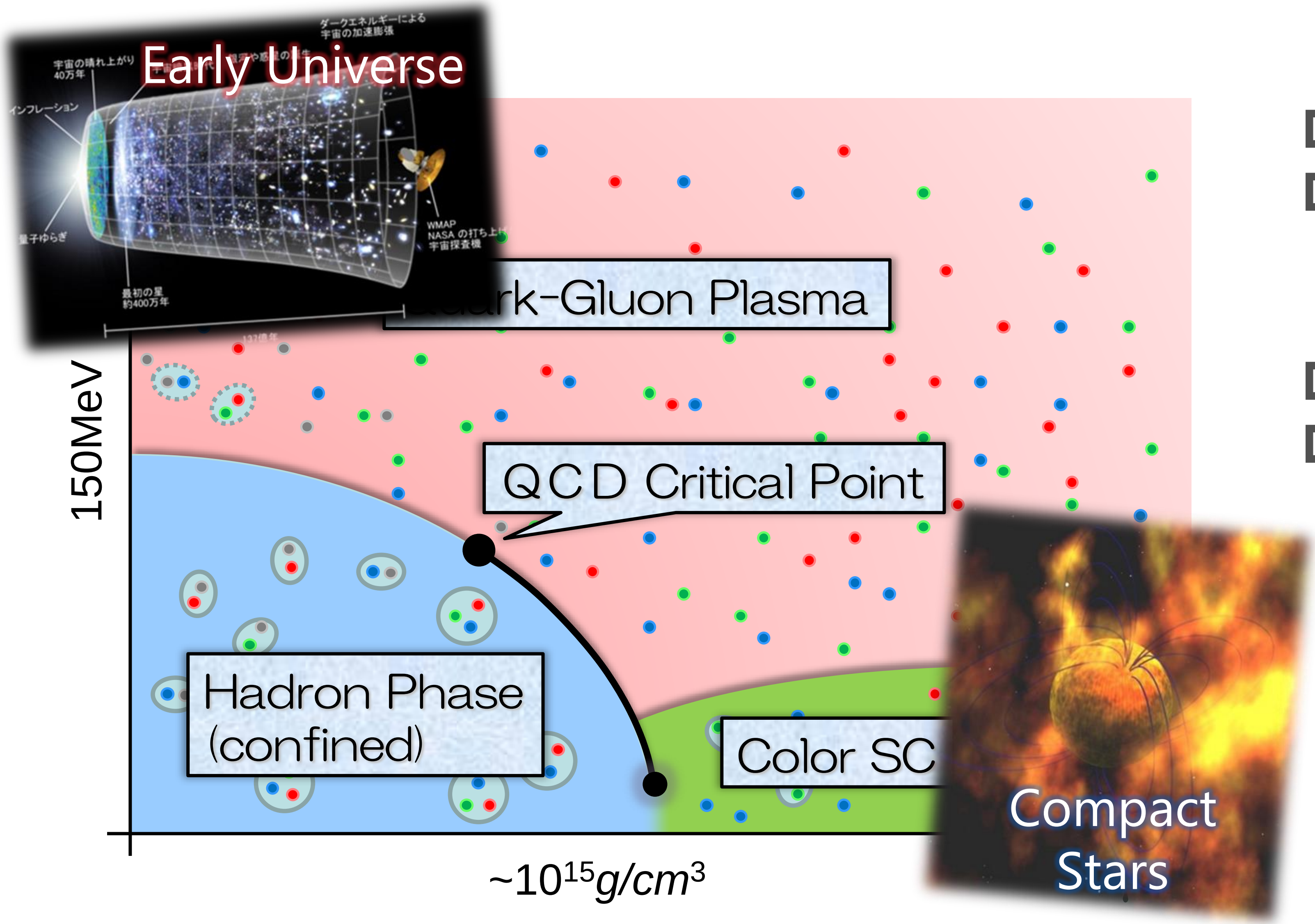
Taya, Jinno, MK, Nara, Nishimura, in preparation.
Nishimura, MK, Kunihiro, arXiv:2405.09240 [hep-ph]; PTEP 2023, 053D01; PTEP 2022, 093D02.

QCD Phase Diagram



- ☐ Crossover at $\mu = 0$
- ☐ Possible first-order transition and QCD critical point in dense region
- ☐ Multiple QCD-CP? MK+ ('02)
- ☐ Color superconducting phases in dense and cold quark matter

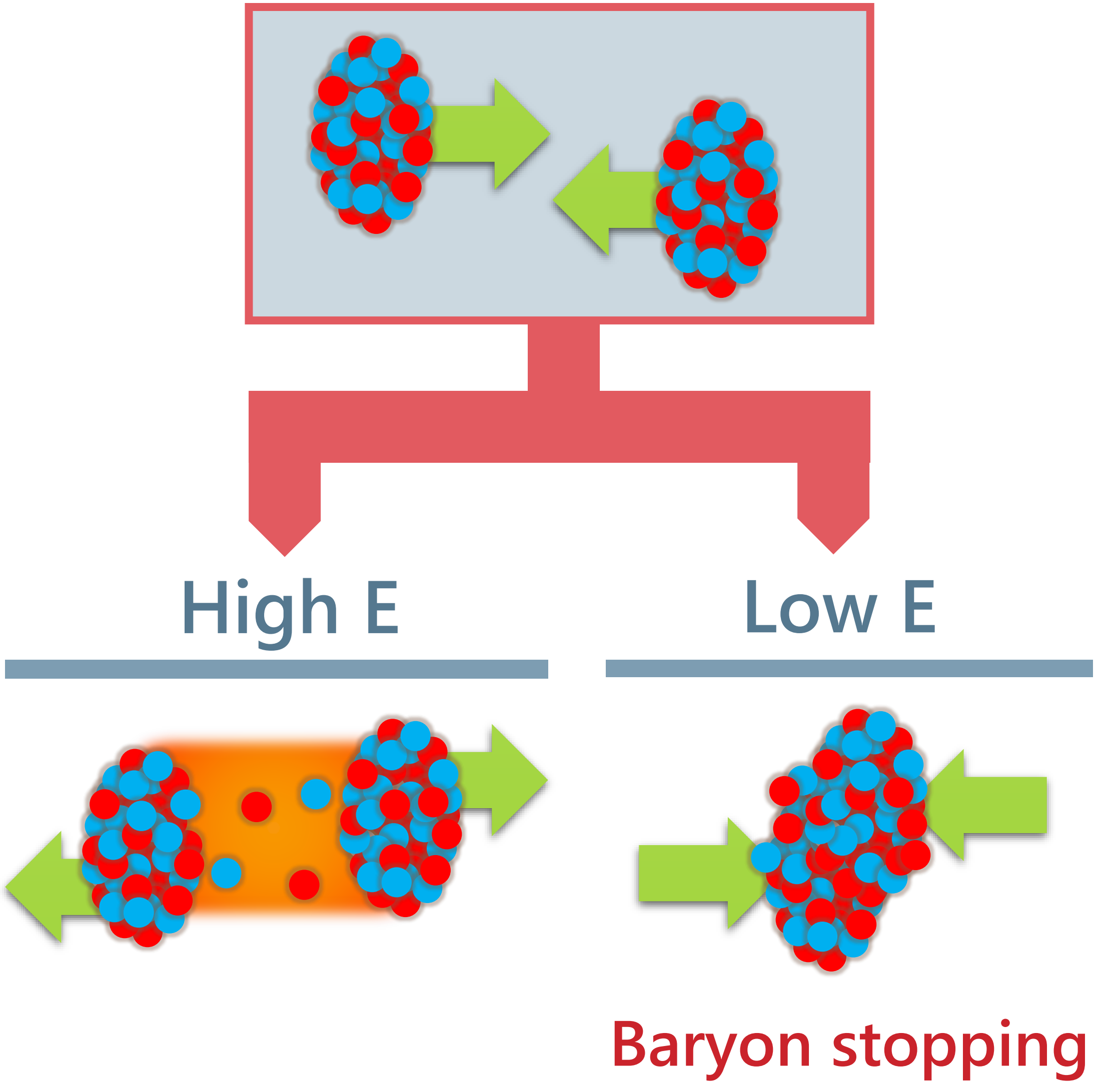
QCD Phase Diagram

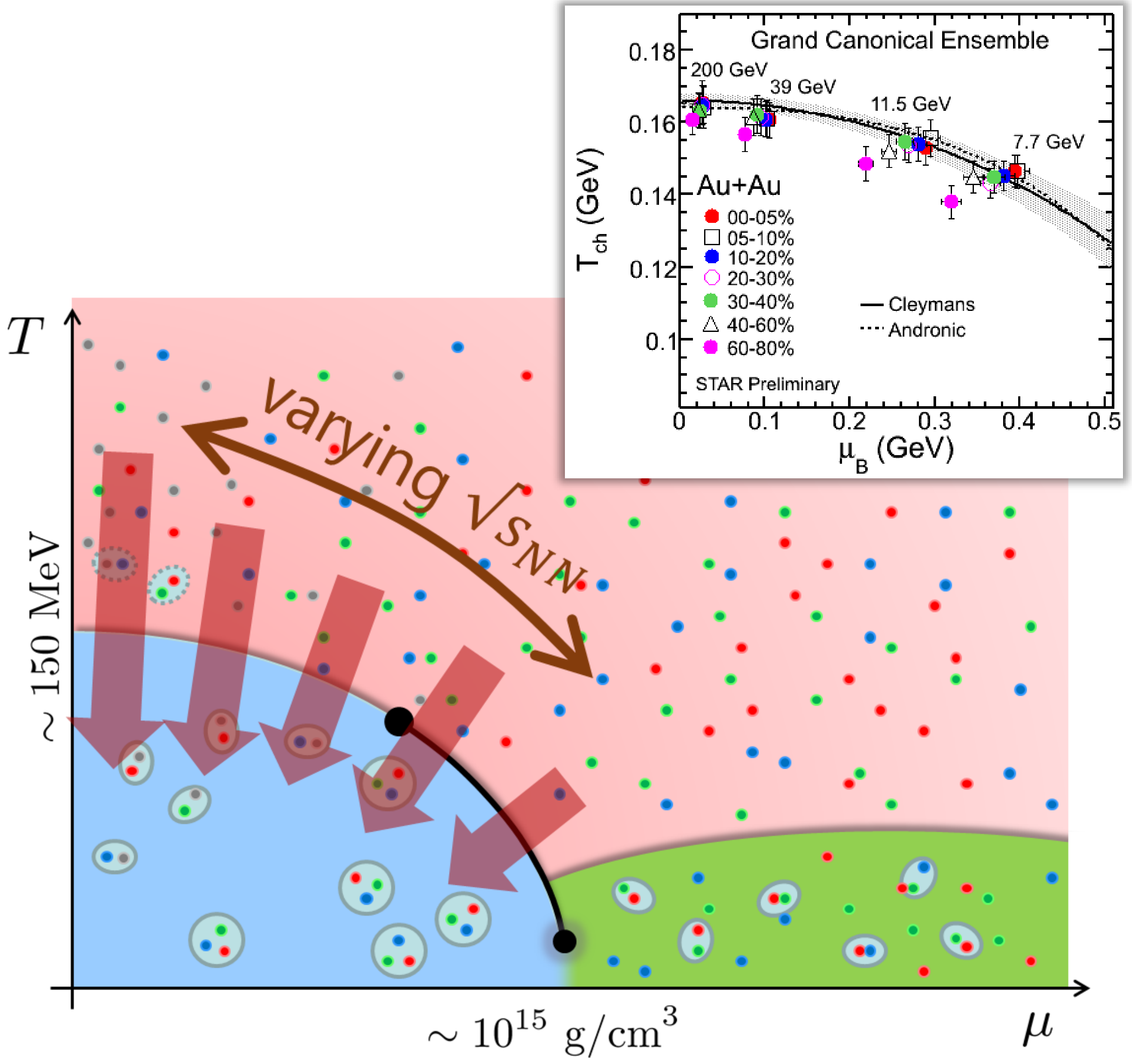


- Crossover at $\mu = 0$
- Possible first-order transition and QCD critical point in dense region
- Multiple QCD-CP? MK+ ('02)
- Color superconducting phases in dense and cold quark matter

Beam-Energy Scan

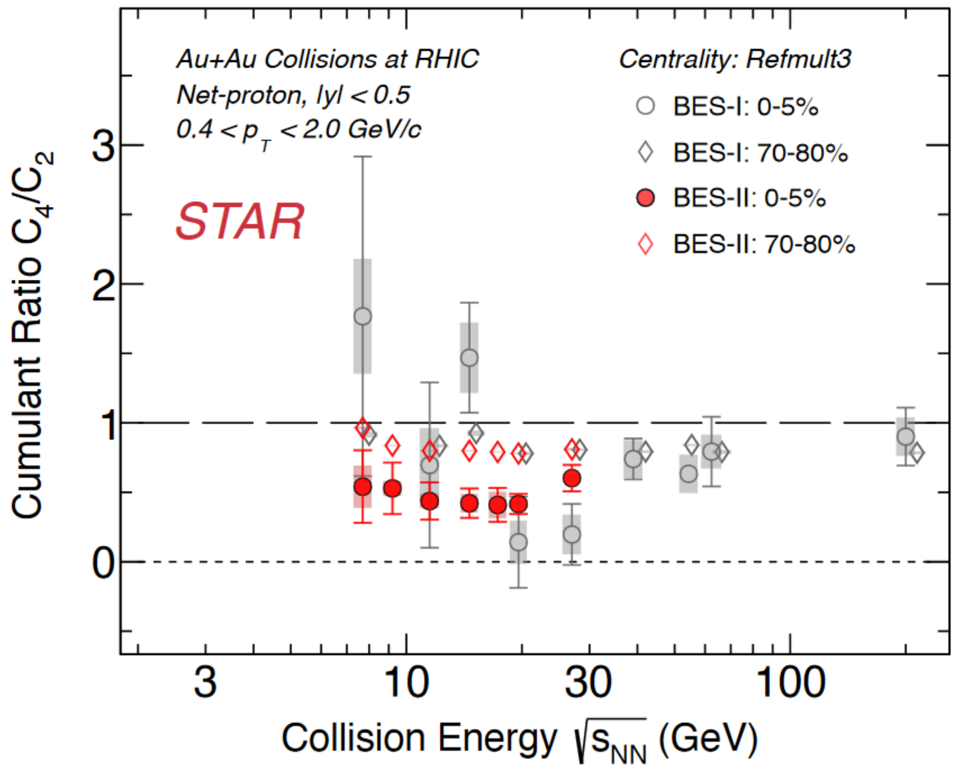
STAR, 2012

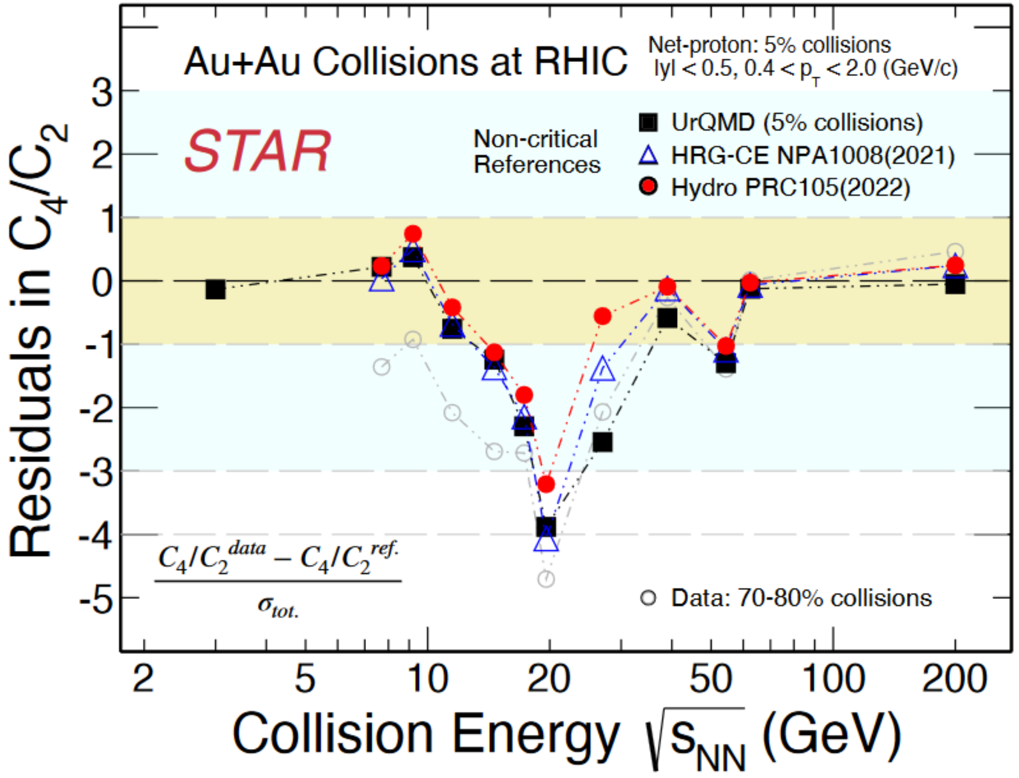




Kurtosis of Net-proton Number

A. Pandav, CPOD2024

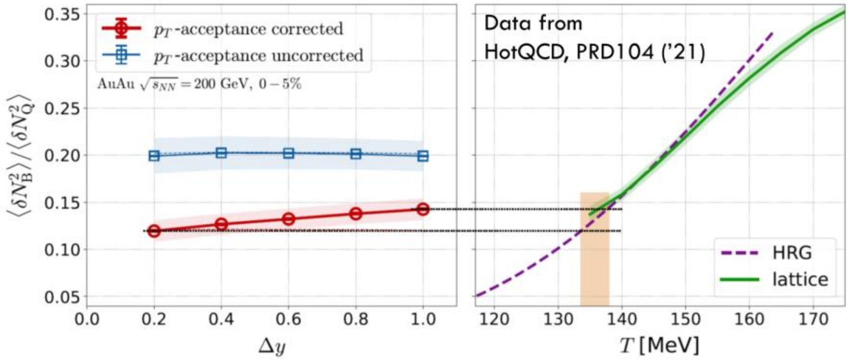




MK's Comments

1 Reconstruct baryon number cumulants
Correct momentum acceptance dependence

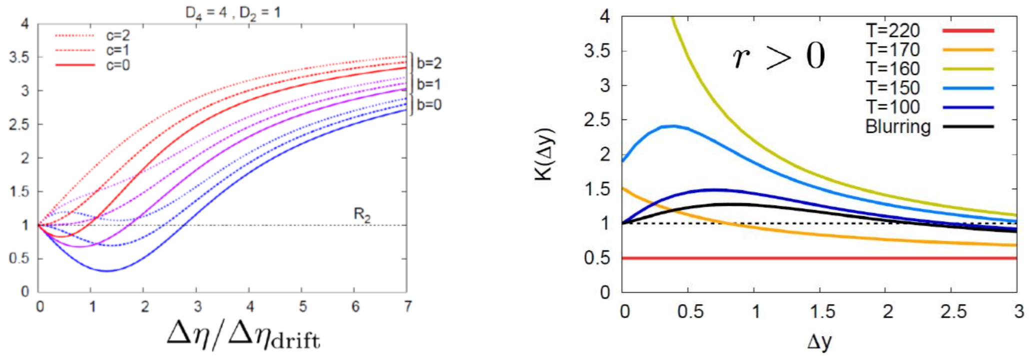
MK, Asakawa, '12; '12
MK, Esumi, Nonaka, '22



Partial measurement blurs the signal!

2 Experimental results
≠ thermal fluctuations at chemical freezeout

MK, Asakawa, Ono, '14
Sakaida, Asakawa, MK, '14
MK, '15
Sakaida, Asakawa, Fujii, MK, '17



Effect of diffusion will be crucial.
Investigate rapidity-window dep.

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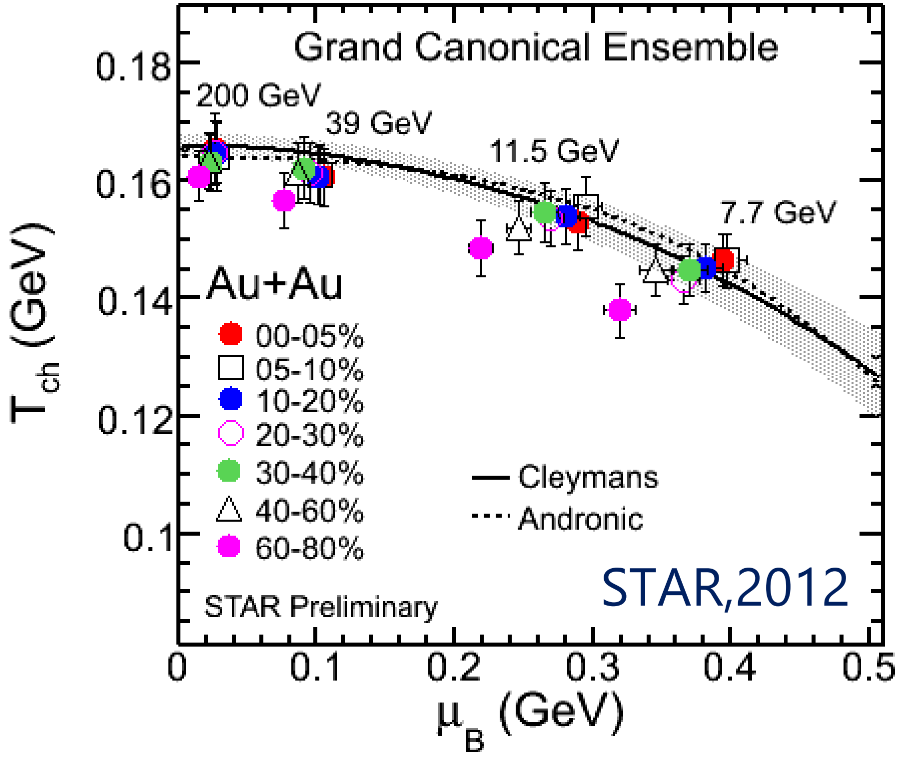
Key Questions

- What is **the best collision energy** to explore the baryon-rich matter?
- **How high density** are accessible?

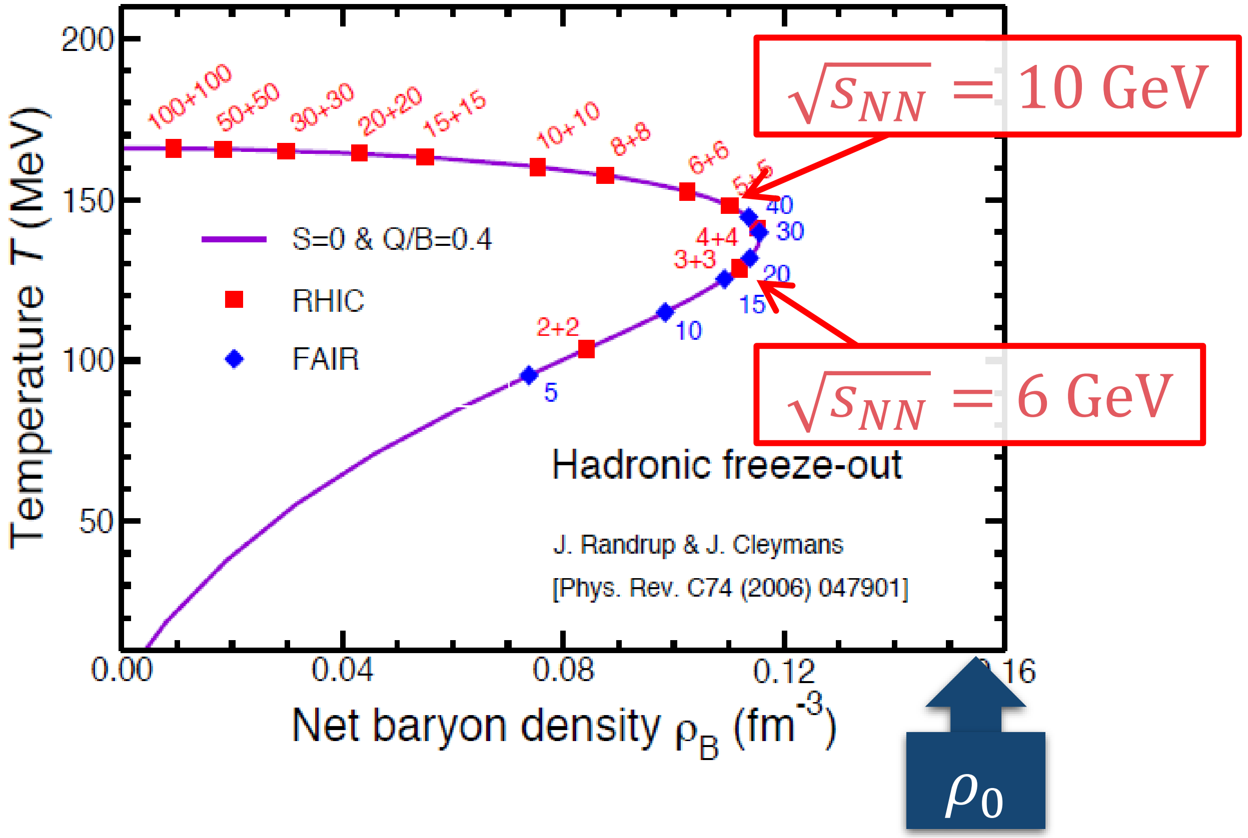
Our Answer

- $\sqrt{s_{NN}} = 3$ **GeV** is enough to study $\rho = 3\rho_0$.
- $\boldsymbol{\rho/\rho_0 = 4 \sim 5}$ may be accessible with $\sqrt{s_{NN}} = 3 \sim 6$ **GeV**.

Chemical Freezeout

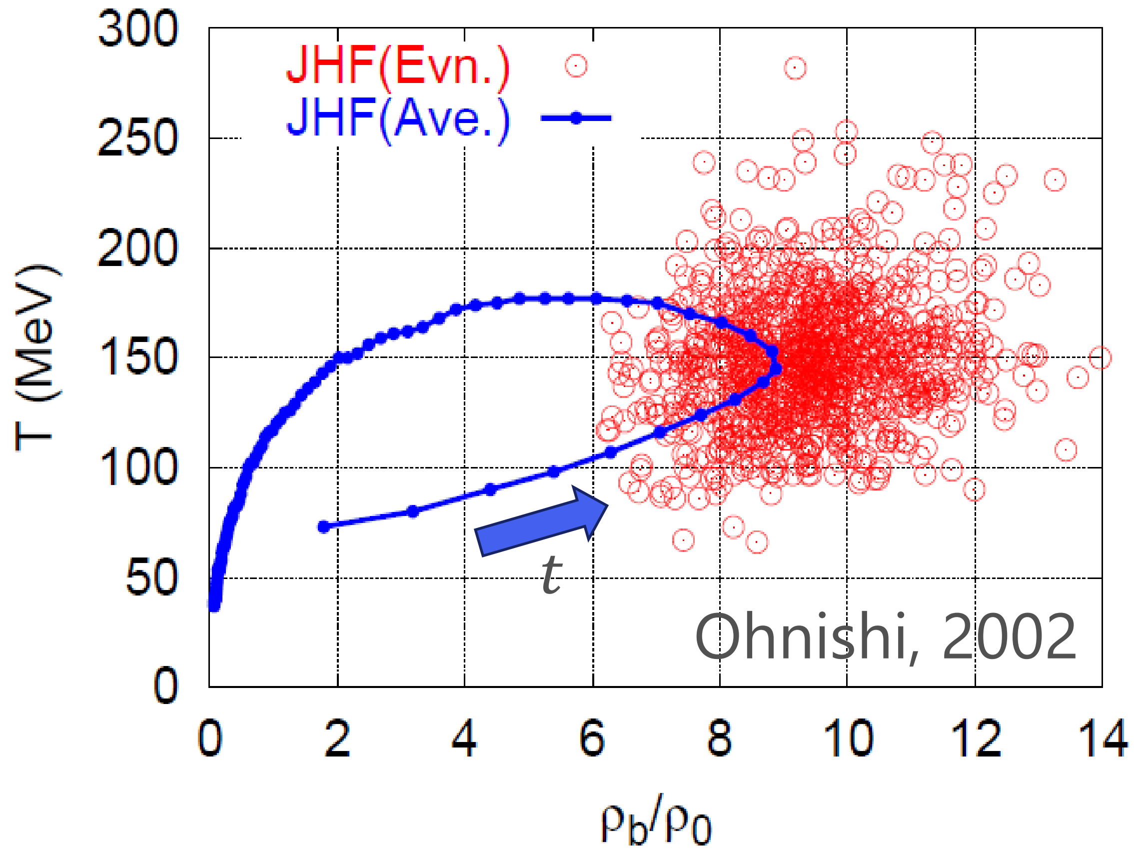


Randrup, Cleymans, '06



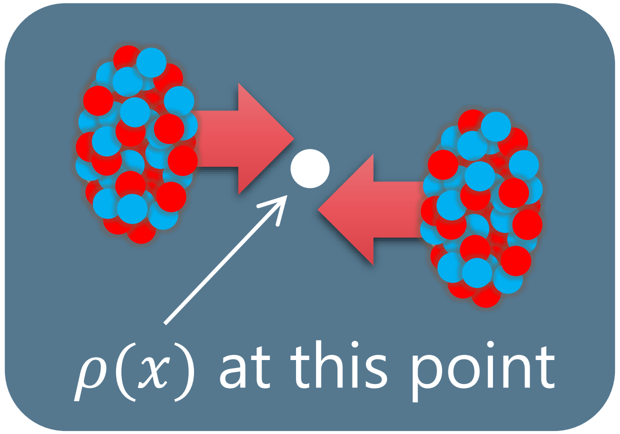
- Highest baryon density **at chemical freezeout** at $\sqrt{s_{NN}} \simeq 6 - 10$ GeV?
- Not the highest density at the early stage.
- Density at earlier stage? ➤ Analysis in dynamical models

Baryon Density at Collision Point



Simulation by JAM

$E/A = 20\mathrm{GeV},\ \sqrt{s_{NN}} \simeq 6\mathrm{GeV}$



— Maximum baryon density exceeds $\rho/\rho_0 \simeq 8$!

— Large event-by-event fluctuations

— How large is the high-density region? How long is the lifetime?

Volume of Dense Region

Volume where the local baryon density is larger than a threshold value $\rho_{\rm th}$

$$V_3(\rho_{\rm th}, t) = \int_{\rho(x)>\rho_{\rm th}} d^3\boldsymbol{x}\gamma$$

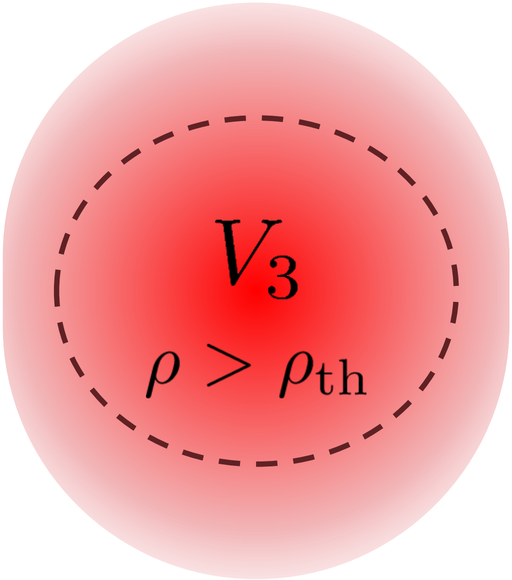
Baryon current $J^\mu(x)$

Baryon density $\rho(x) = \sqrt{J^\mu(x)J_\mu(x)}$

Lorentz factor $\gamma = (1 - (\boldsymbol{J}/J_0)^2)^{-1/2}$

Note:

— Event-by-event basis / no event average
— Directly calculable in a dynamical model
— We do not care about local thermalization.
  — V_3 is the upper limit of thermalized volume.
  — Even non-thermal, dense region is interesting!



Simulation Setup in JAM

- Au+Au collision for $2.4 \le \sqrt{s_{NN}} \le 20$ GeV
- Impact parameter $b \le 3$ fm : top 5% centrality

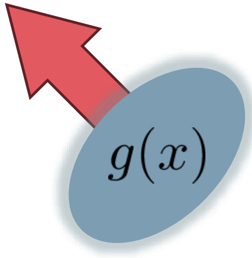
- Momentum-dependent mean field (MF2) Nara, Ohnishi, 2022
  - Setup reproducing $\sqrt{s_{NN}}$ dep. of $dv_1/d\eta$ and v_2

Smeared baryon current

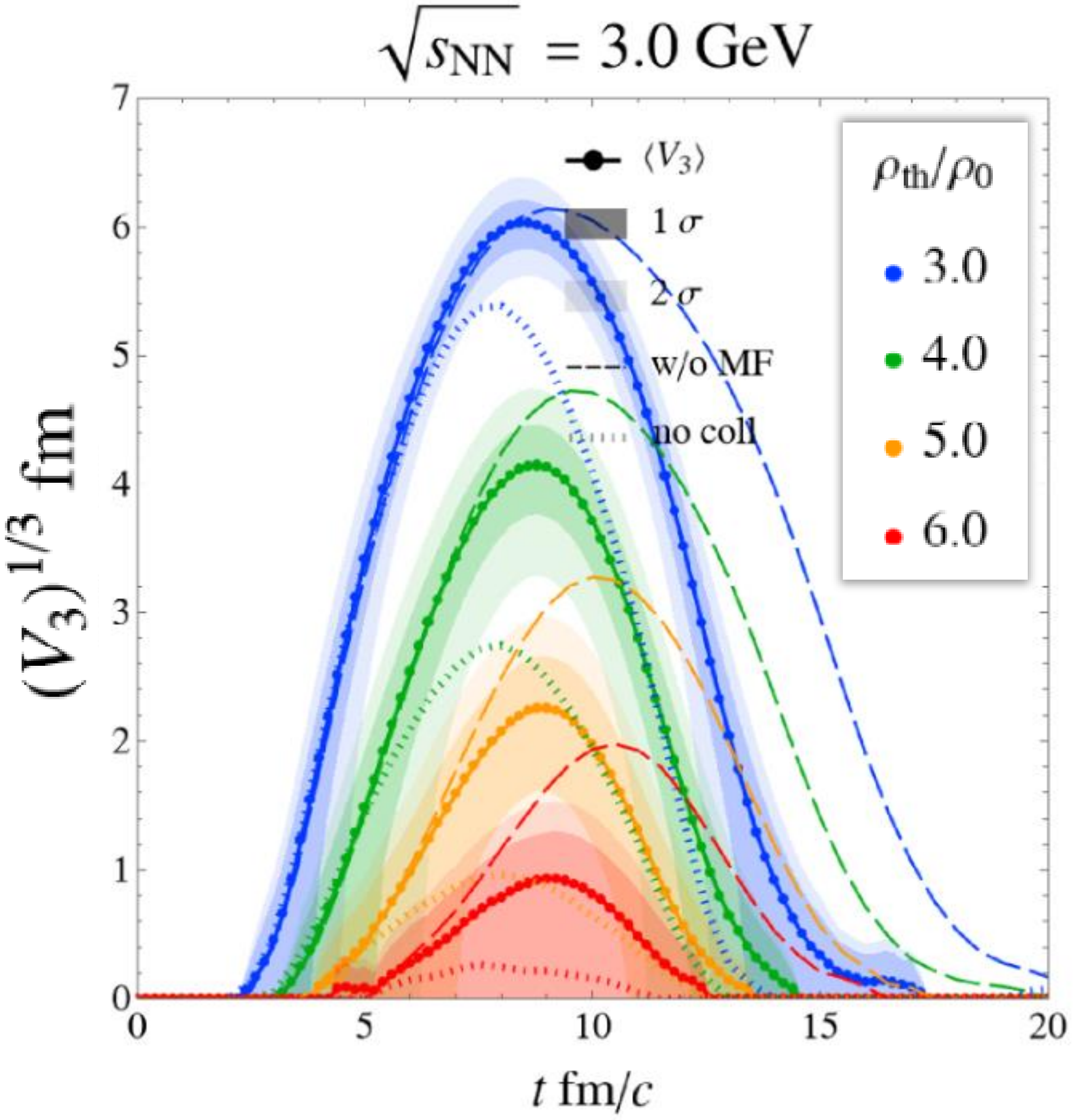
discrete particle distribution ➔ continuous current by smearing

$$J^\mu(x) = \sum_{i \in \text{baryons}} B_i g(x; X_i, P_i) \frac{P_i^\mu}{P_i^0}$$

$$g(x; X, P) := \frac{\gamma}{(\sqrt{2\pi} r)^3} \mathrm{e}^{-\frac{|\boldsymbol{x}-\boldsymbol{X}|^2 + (\gamma \boldsymbol{V}\cdot(\boldsymbol{x}-\boldsymbol{X}))^2}{2r^2}} \qquad r = 1 \text{ fm}$$



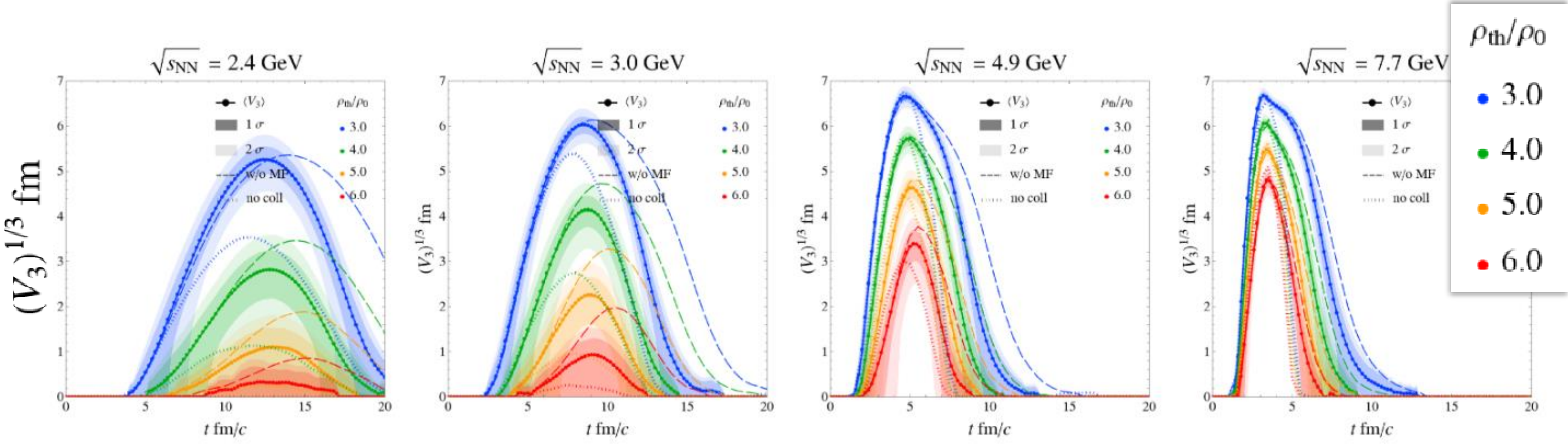
V_3 in JAM



— **solid: JAM+MF** Nara, Ohnishi, 2022
— shaded band: 1σ and 2σ e-v-e fluct.
— dashed: JAM cascade mode
— dotted: no-collision

- Formation of dense region:
  - $V_3(3\rho_0, t) = (6\text{ fm})^3$
  - $V_3(4\rho_0, t) = (4\text{ fm})^3$
- Large e-v-e fluctuations
  ➔separable by event selection?
- Repulsive MF ➔ weaker compression
- Compression owing to interaction

V_3 for various $\sqrt{s_{NN}}$



As $\sqrt{s_{NN}}$ becomes larger,

- ☐ max $V_3(\rho_{th}, t)$ becomes larger.
- ☐ The lifetime of dense region becomes shorter.
- ☐ E-v-e fluctuations are more suppressed.

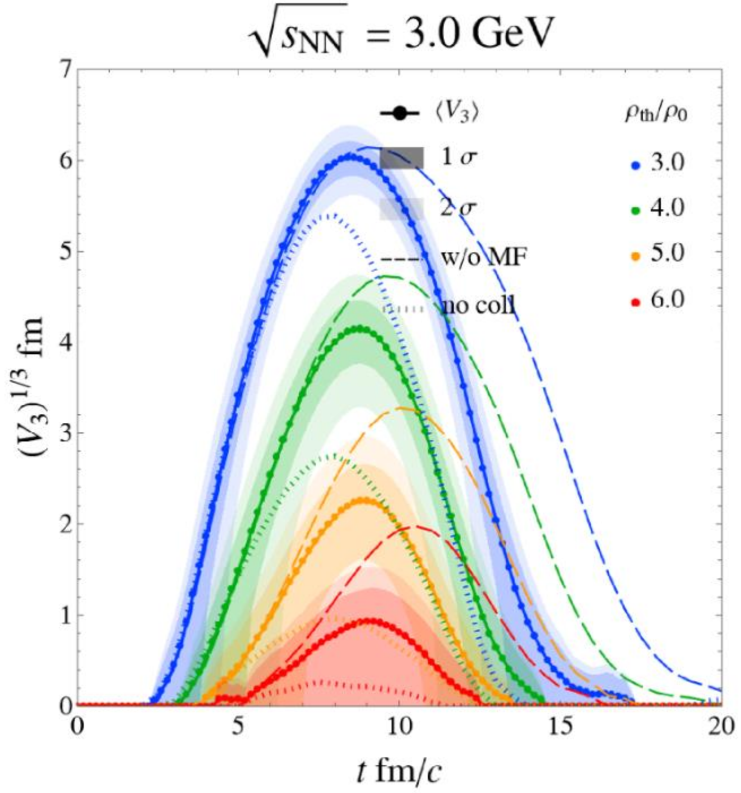
Four-Volume / Lifetime

Four Volume

$$V_4(\rho_{\mathrm{th}}) = \int_{-\infty}^{\infty} dt \int_{\rho(x)>\rho_{\mathrm{th}}} d^3\boldsymbol{x}$$

Lifetime

$$\tau(\rho_{\mathrm{th}}) = \frac{V_4(\rho_{\mathrm{th}})}{\max V_3(\rho_{\mathrm{th}}, t)}$$

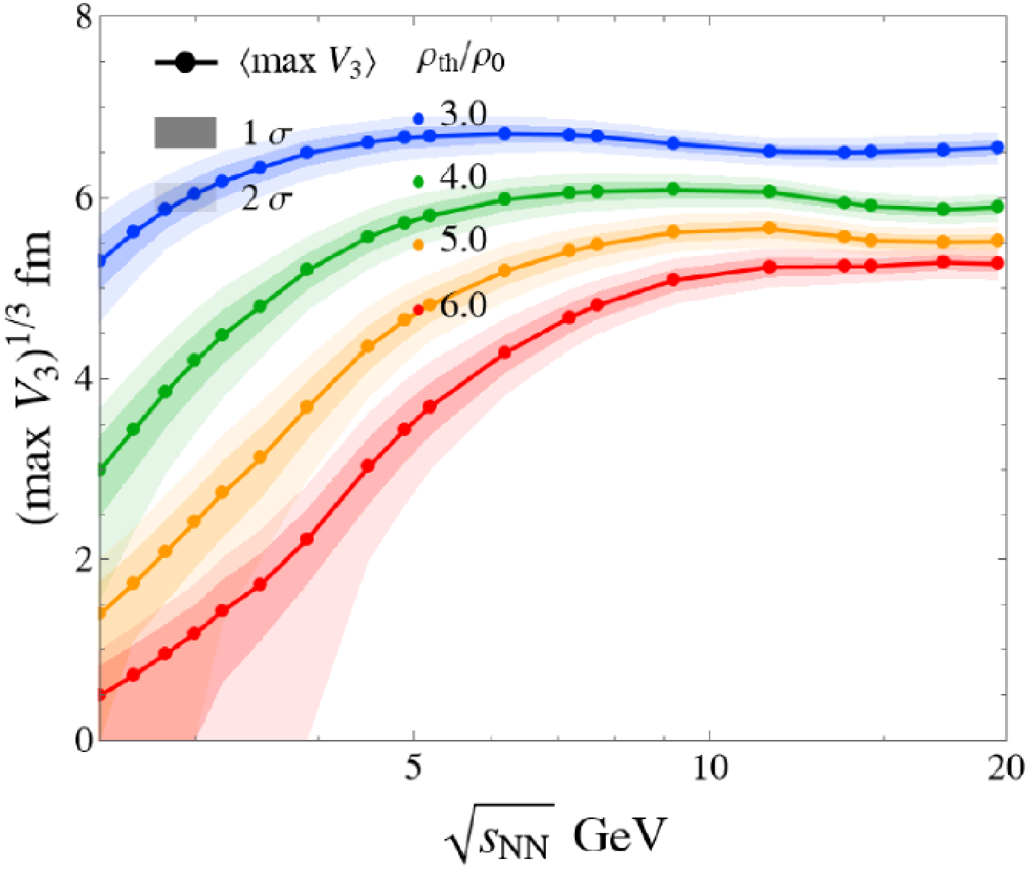


Note

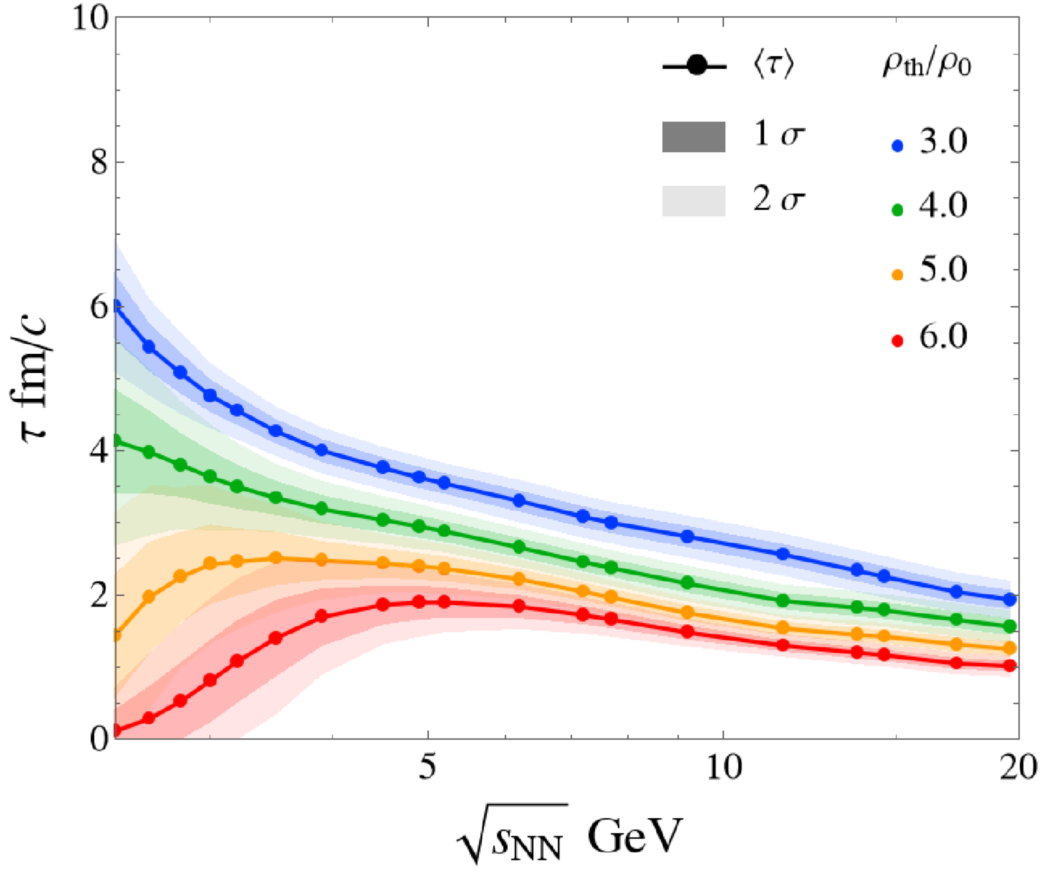
V_4 may be relevant for the dilepton production rate.

$\sqrt{s_{NN}}$ Dependence

max V_3



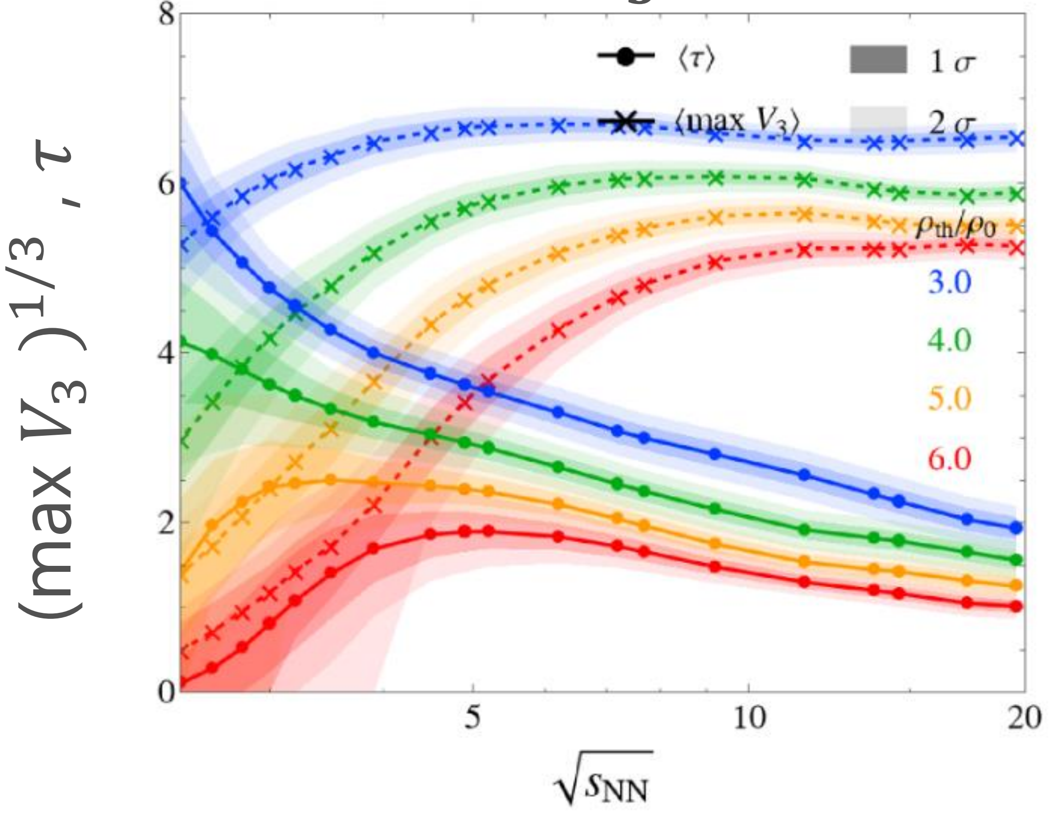
Lifetime



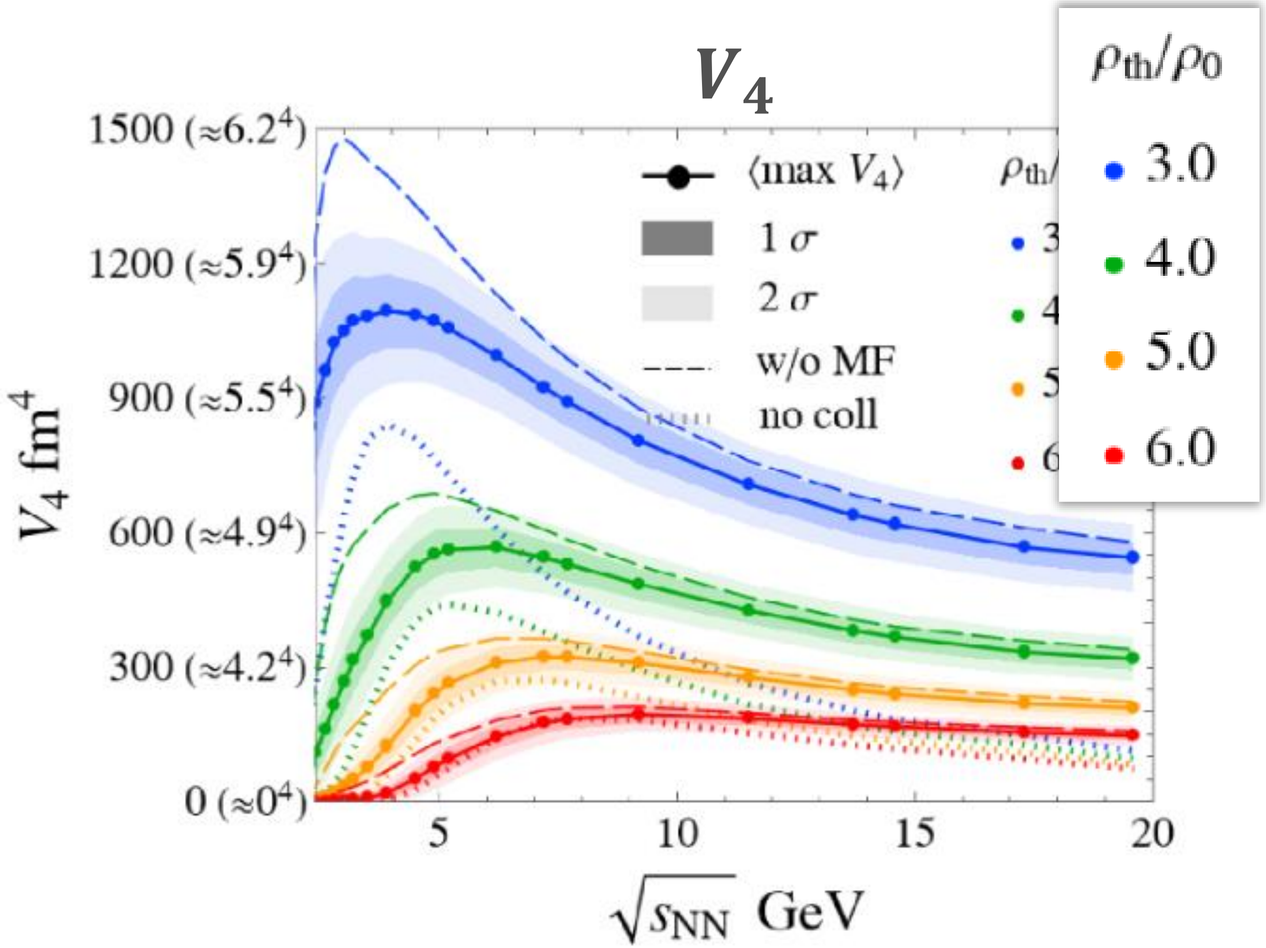
$\rho_{\rm th}/\rho_0$
- 3.0
- 4.0
- 5.0
- 6.0

$\sqrt{s_{NN}}$ Dependence

max V_3 , τ



V_4



- $\sqrt{s_{NN}} \simeq 3$ GeV would be the best energy to create $\rho = 3\sim 4\rho_0$ with large V_3 and τ.
- Lower $\sqrt{s_{NN}}$ is suitable to create colder matter.
- All results have large e-v-e fluctuations ➔ Event selection by density?

Key Questions

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- **How high density** are accessible?

Our Answer

- $\sqrt{s_{NN}} = 3$ **GeV** is enough to study $\rho = 3\rho_0$.
- $\rho/\rho_0 = 4\sim5$ may be accessible with $\sqrt{s_{NN}} = 3\sim6$ **GeV**.

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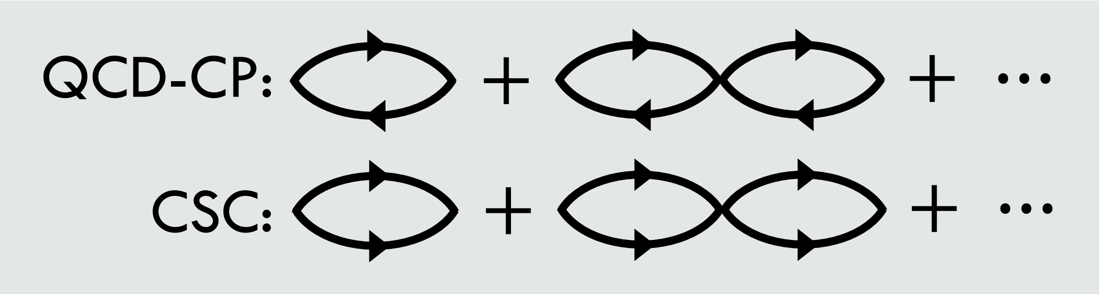
Soft Modes of Second-order Phase Transitions

□ Soft modes

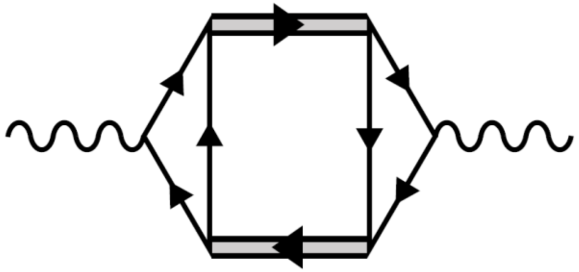
- □ Divergence of the order-parameter fluctuations at a 2nd-order transition.
- □ Collective fluctuations become massless there.

- □ QCD-CP : density-density fluctuations
- □ CSC : diquark-pair field



□ Coupling of soft modes with dynamical observables

- □ Ex.: dilepton production rate

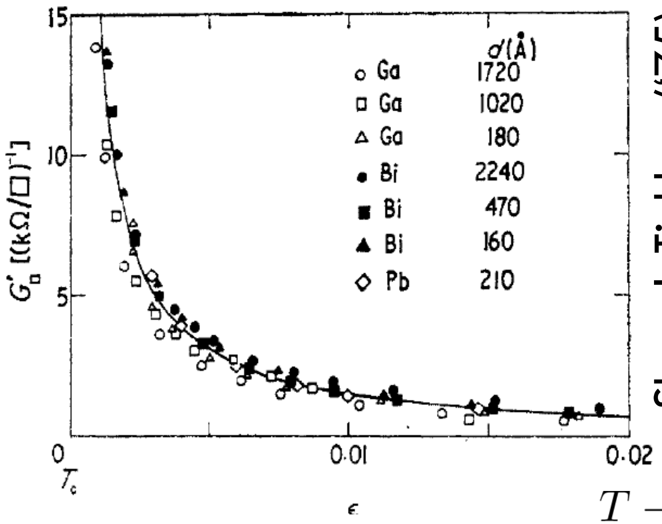


Precursor in Metallic Superconductors

- **Anomalous behavior of observables near but above Tc of SC**
    - electric conductivity
    - magnetic susceptibility
    - pseudogap

- Enhanced pair fluctuations is one of the origins of precursory phenomena.
- More significant phenomena in strongly-coupled systems.

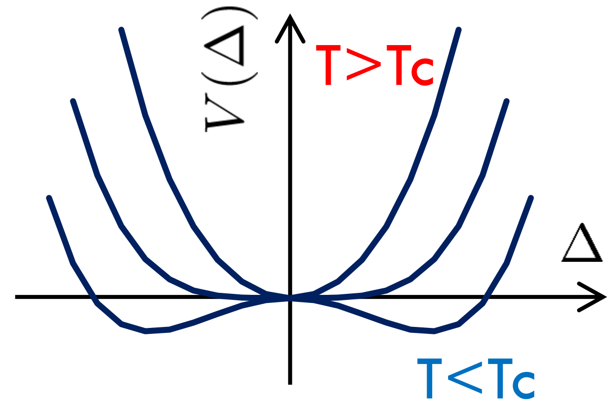
Electric conductivity



Skopal, Tinkham('75)

$$\varepsilon = \frac{T - T_c}{T_c}$$

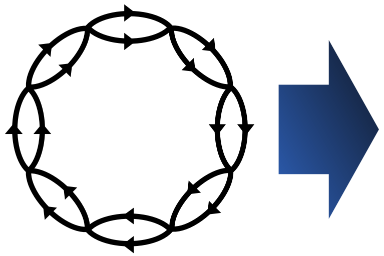
Landau's free energy



Precursor of Color Superconductivity

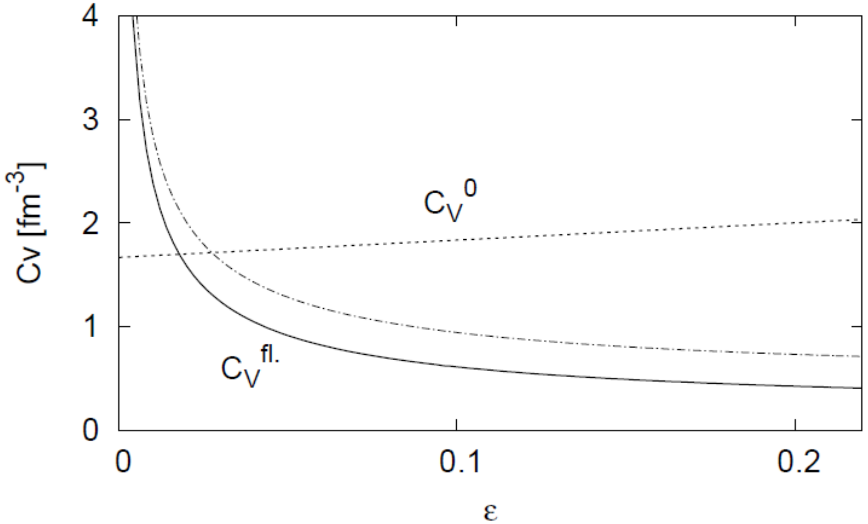
MK, Koide, Kunihiro, Nemoto, '03, '05

□Thermodynamic Potential

$\Omega =$ 

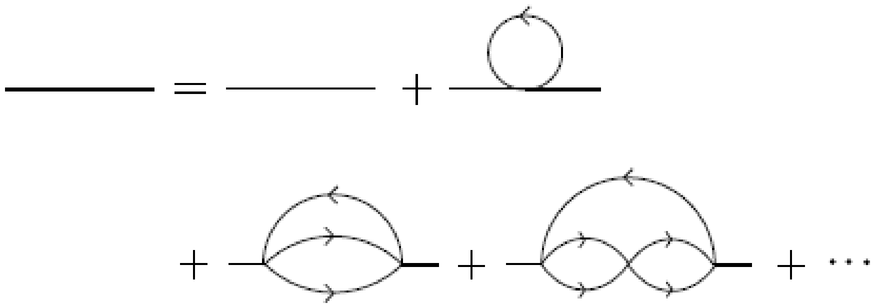
Specific heat

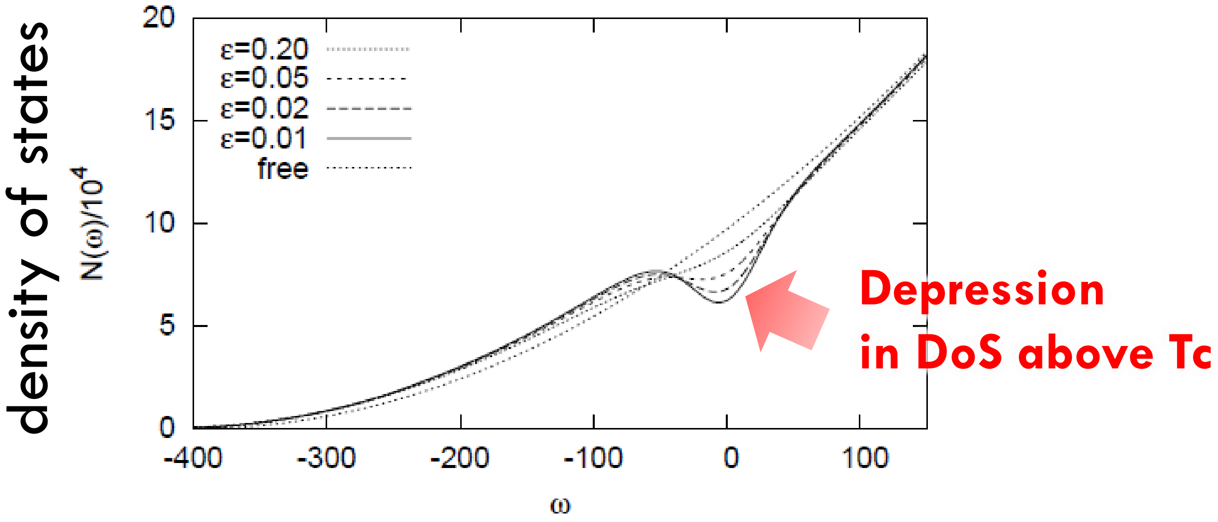
$$c = -T\frac{\partial^2\Omega}{\partial T^2}$$



$$\varepsilon = \frac{T - T_c}{T_c}$$

□Pseudogap





Model

NJL model (2-flavor)

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \mathcal{L}_S + \mathcal{L}_C$$

$$\mathcal{L}_S = G_S\left((\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \tau \psi)^2\right)$$

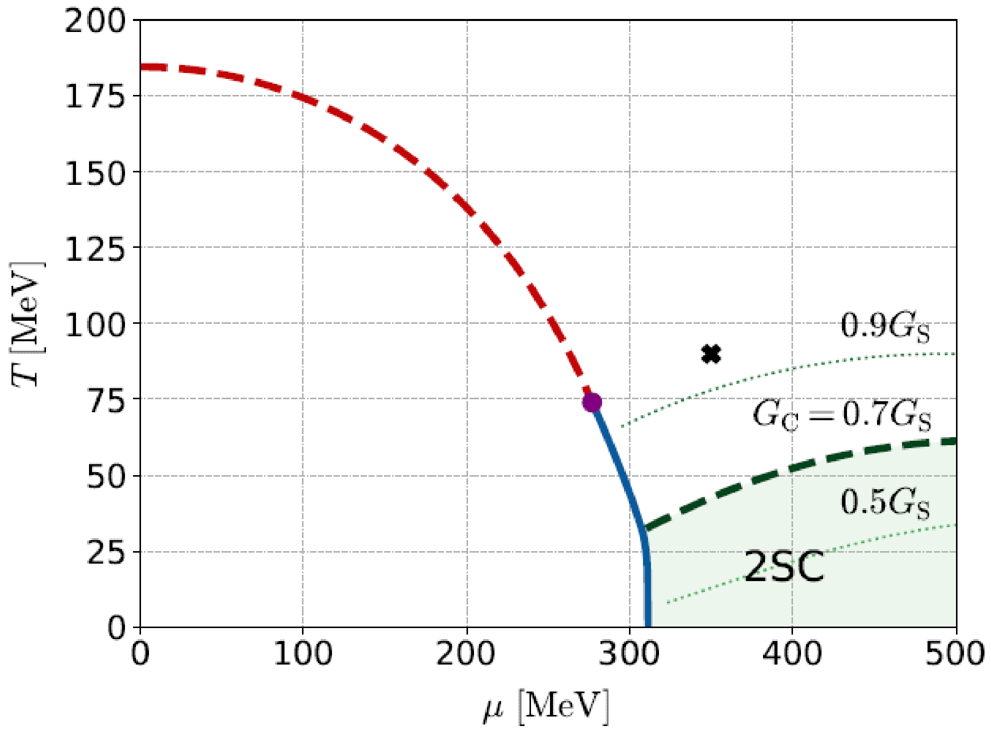
$$\mathcal{L}_C = G_C\left((\bar{\psi} i\gamma_5 \tau_A \lambda_A \psi^C)(\text{h.c.})\right)$$

diquark interaction

Parameters

$$G_S = 5.01\ \text{GeV}^{-2}, \quad \Lambda = 650\text{MeV}, \quad m_q = 0$$

Phase Diagram in MFA



- Order of phase transition
  - 2nd in the MFA
  - can be 1st due to gauge fluctuation

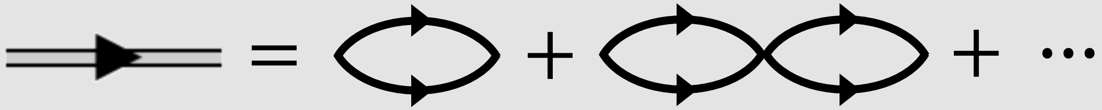
Matsuura+('04), Giannakis+('04)
Noronha+('06), Fejos, Yamamoto('19)

Di-quark Fluctuations

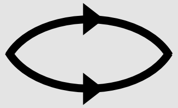
Diquark Propagator

$$D^R(x) = \langle[\Delta^\dagger(x), \Delta(0)]\rangle\theta(t) =$$ 

- Random Phase Approximaion



$$= \frac{Q^R(\boldsymbol{k},\omega)}{1 + G_C Q^R(\boldsymbol{k},\omega)}$$

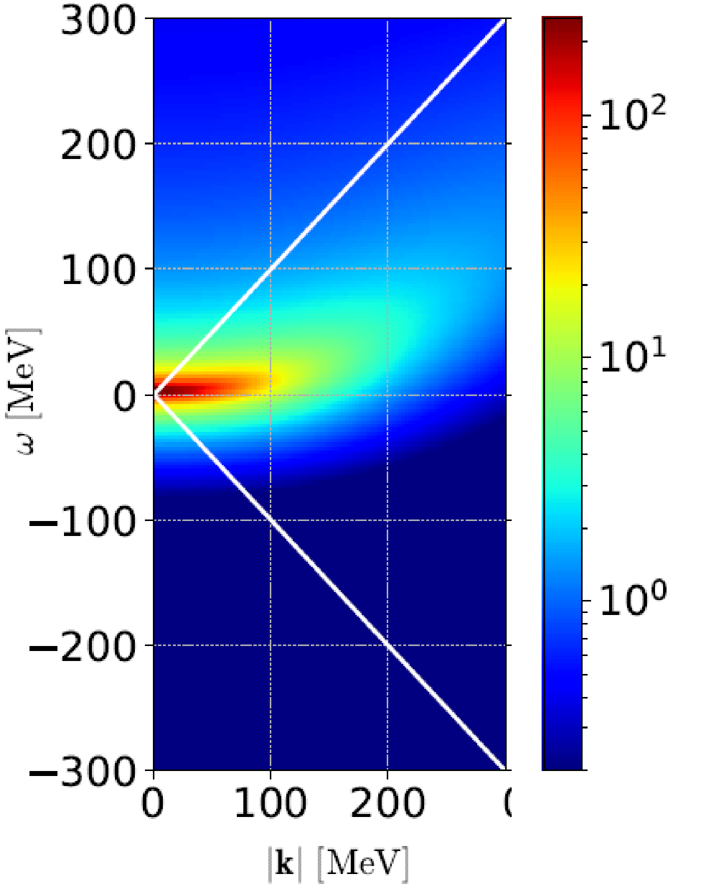
$$Q^R(\boldsymbol{k},\omega) =$$ 

- Diquark field becomes massless at T=Tc
- Soft mode of CSC transition
- Strength in the space-like region

MK, Koide, Kunihiro, Nemoto, '01,'05

Dynamical Structure Factor

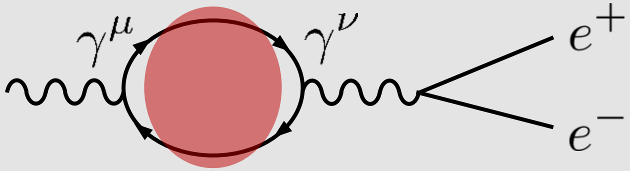
$$S(\boldsymbol{k},\omega) = -\frac{1}{\pi}\frac{1}{1-e^{-\beta\omega}}\mathrm{Im}D^{\mathrm{R}}(\mathrm{k},\omega)$$



Photon Self-Energy: Precursor of CSC

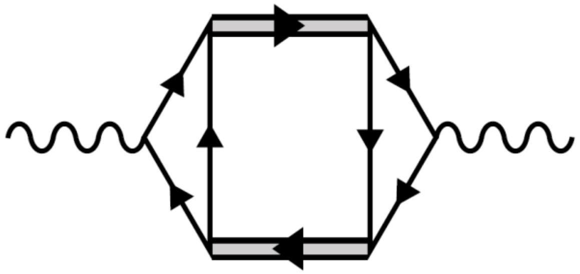
☐ Dilepton Production Rate

$$\frac{d^4\Gamma}{dk^4} = \frac{\alpha}{12\pi^4}\frac{1}{k^2}\frac{1}{e^{\beta\omega-1}}\mathrm{Im}\Pi^{R\mu}_{\mu}(k)$$

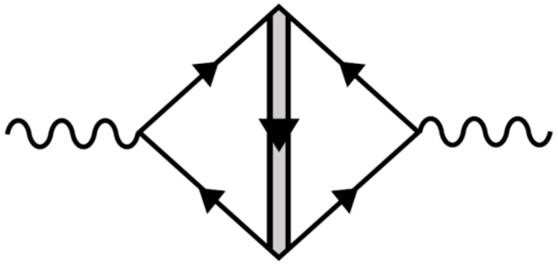


☐ Effect of Di-quarks on $\Pi^{\mu\nu}(k)$

Aslamasov-Larkin term



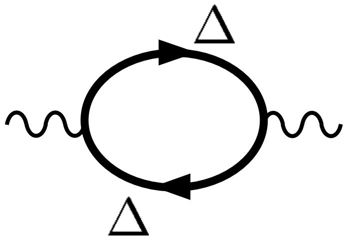
Maki-Thompson term

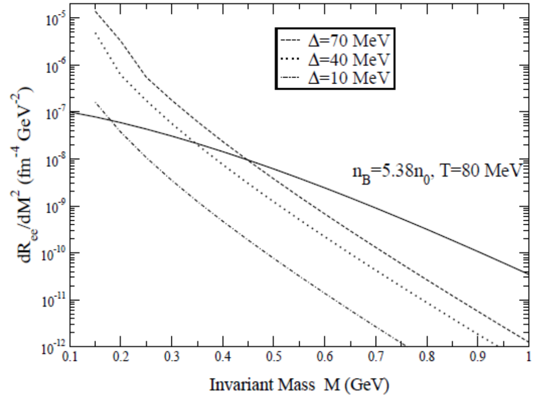


Well-known diagrams in metallic SC
for describing paraconductivity

☐ DPR from CFL phase

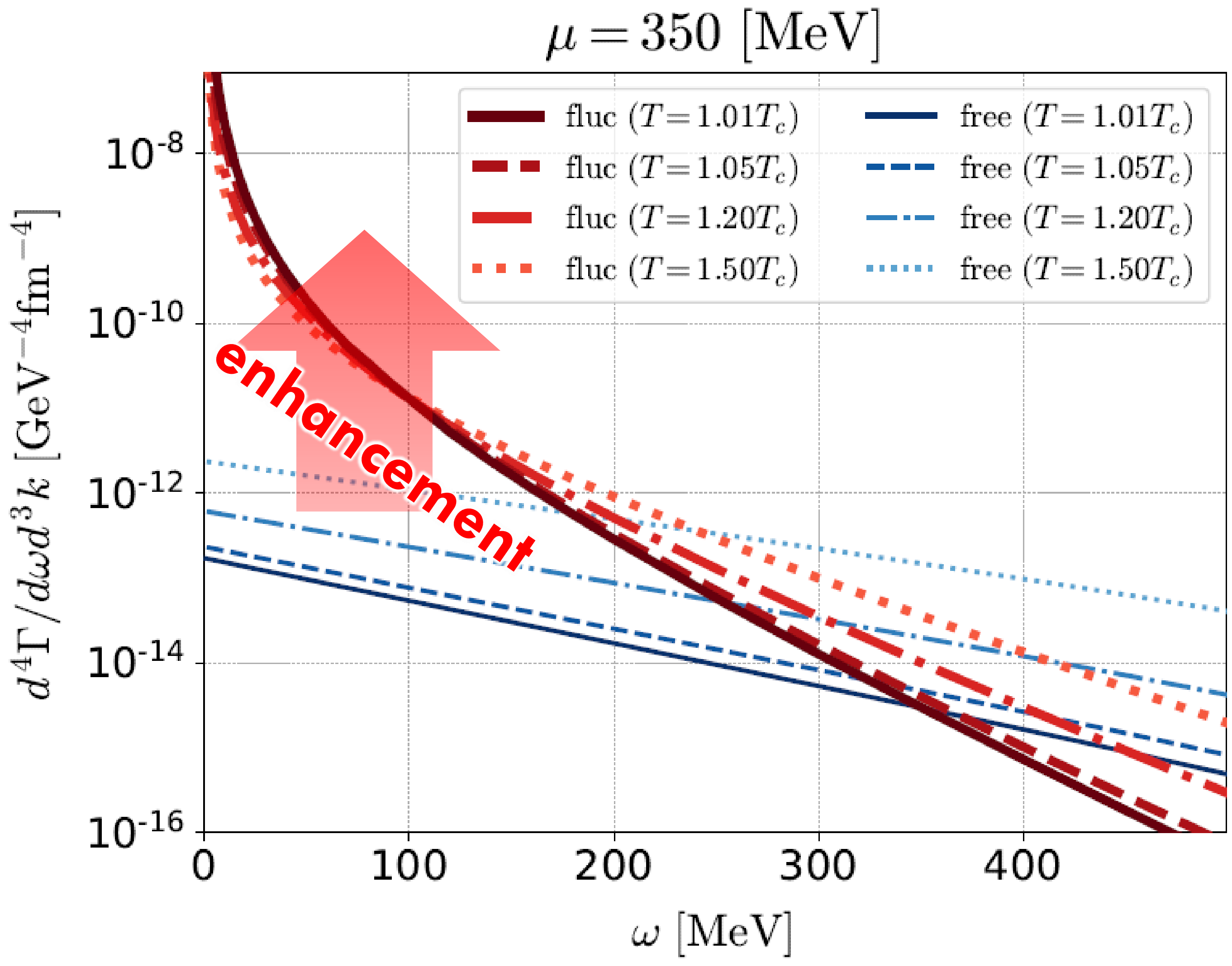
Jaikumar, Rapp, Zahed ('02)





Production Rate at $\boldsymbol{k} = 0$

Nishimura, MK, Kunihiro ('22)



Red: fluctuation contribution

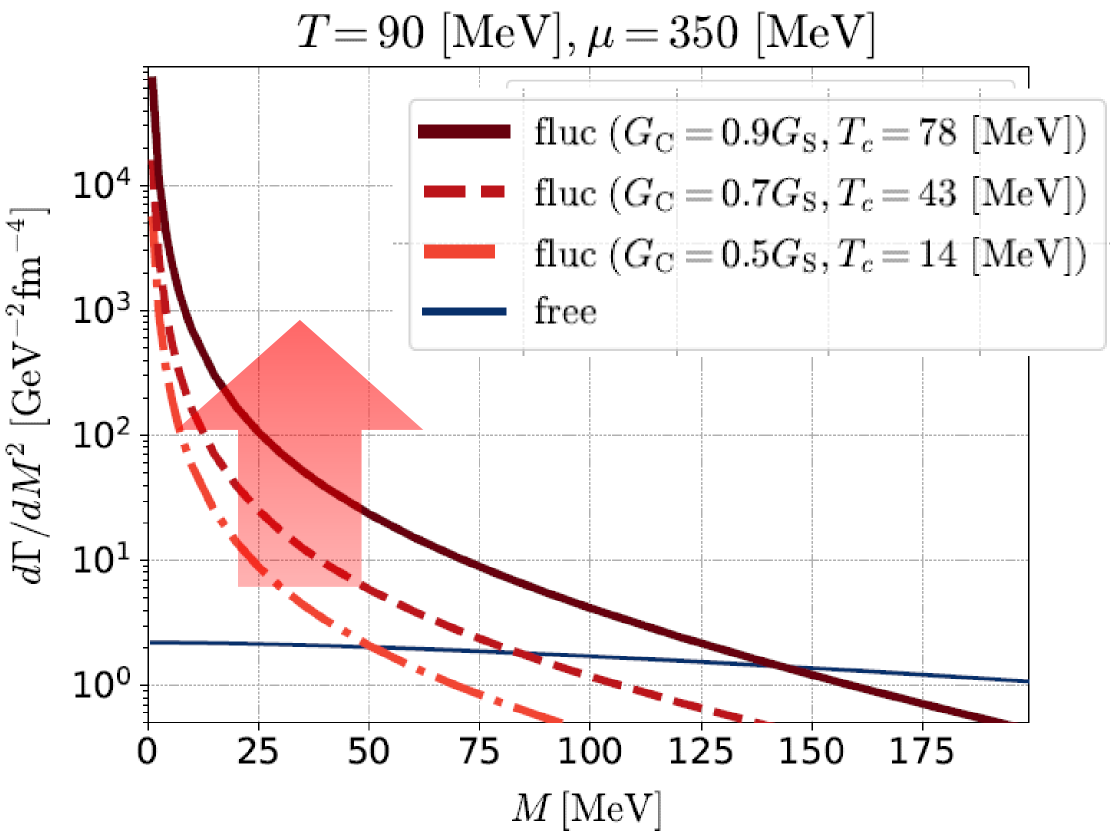
Blue: free quarks

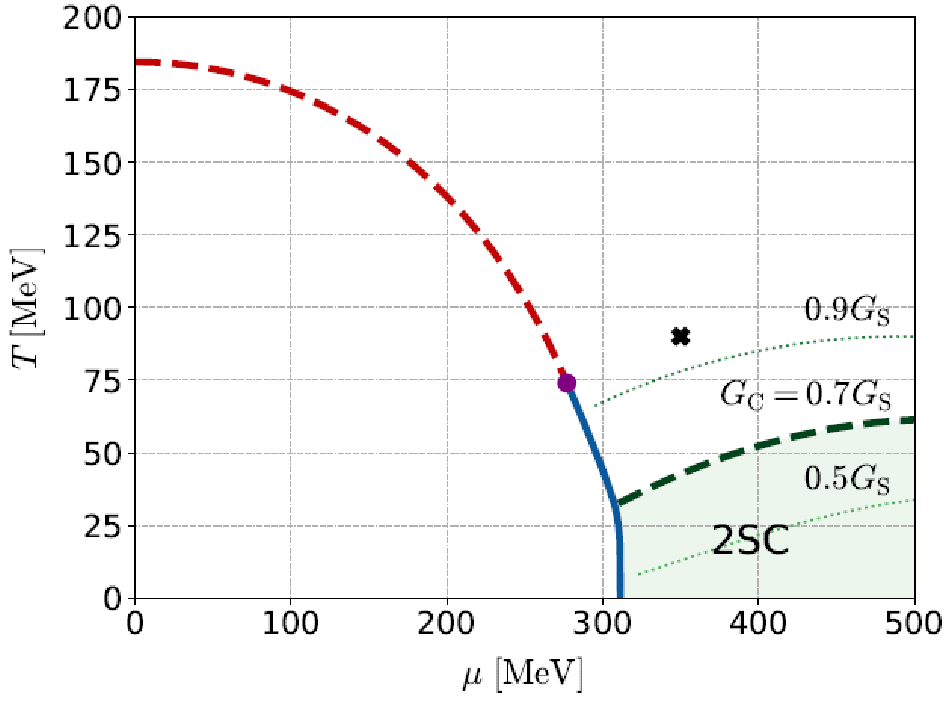
$G_C = 0.7G_s$, $T_c \simeq 45$ MeV

- Di-quark fluctuations give rise to large enhancement in the low energy region $\omega < 200$ MeV and $T < 1.5T_c$.
- Anomalous enhancement is not sensitive to T.

Invariant-Mass Spectrum

Nishimura, MK, Kunihiro ('22)





- ☐ Strong enhancement at low invariant mass.
- ☐ **Observable in the HIC?**

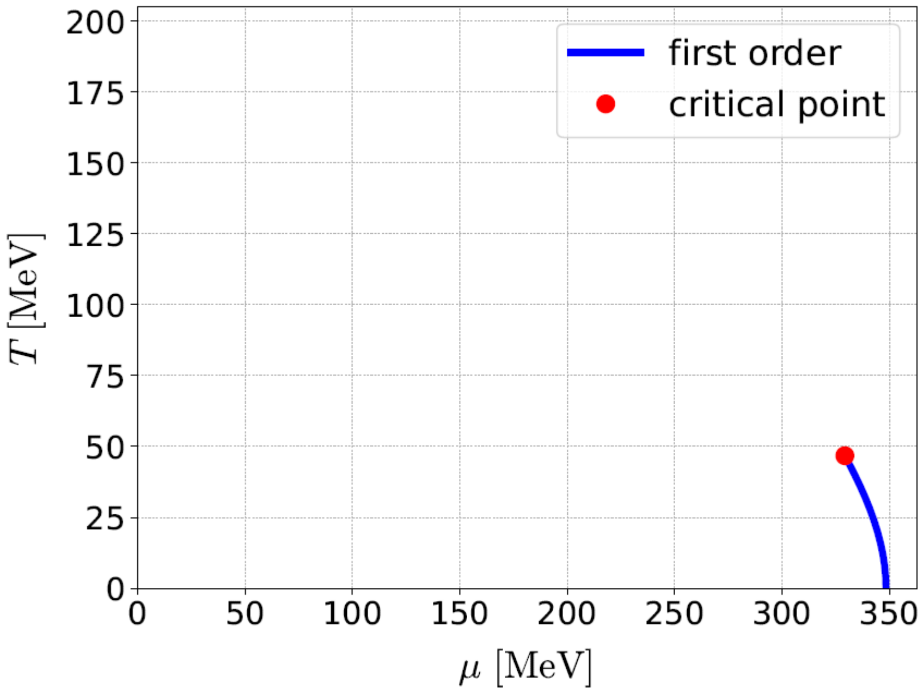
Dileptons from QCD Critical Point

NJL model (2-flavor)

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G_S\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2\right)$$

Parameters

$$G_S = 5.5 \text{ GeV}^{-2},\ \Lambda = 631\text{MeV},\ m_q = 5.5 \text{ MeV}$$

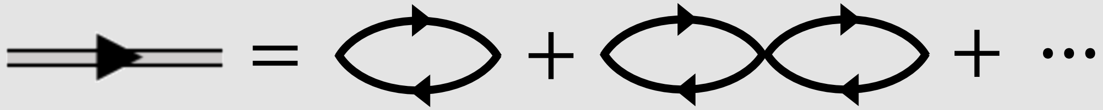


Soft Mode of QCD-CP

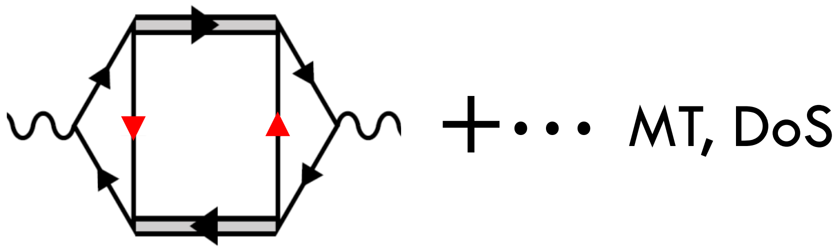
= fluctuation of scalar $(\bar{q}q)$ channel

$$D^R(x) = \langle[\bar{\psi}\psi(x), \bar{\psi}\psi(0)]\rangle\theta(t) =$$

- Random Phase Approximaion



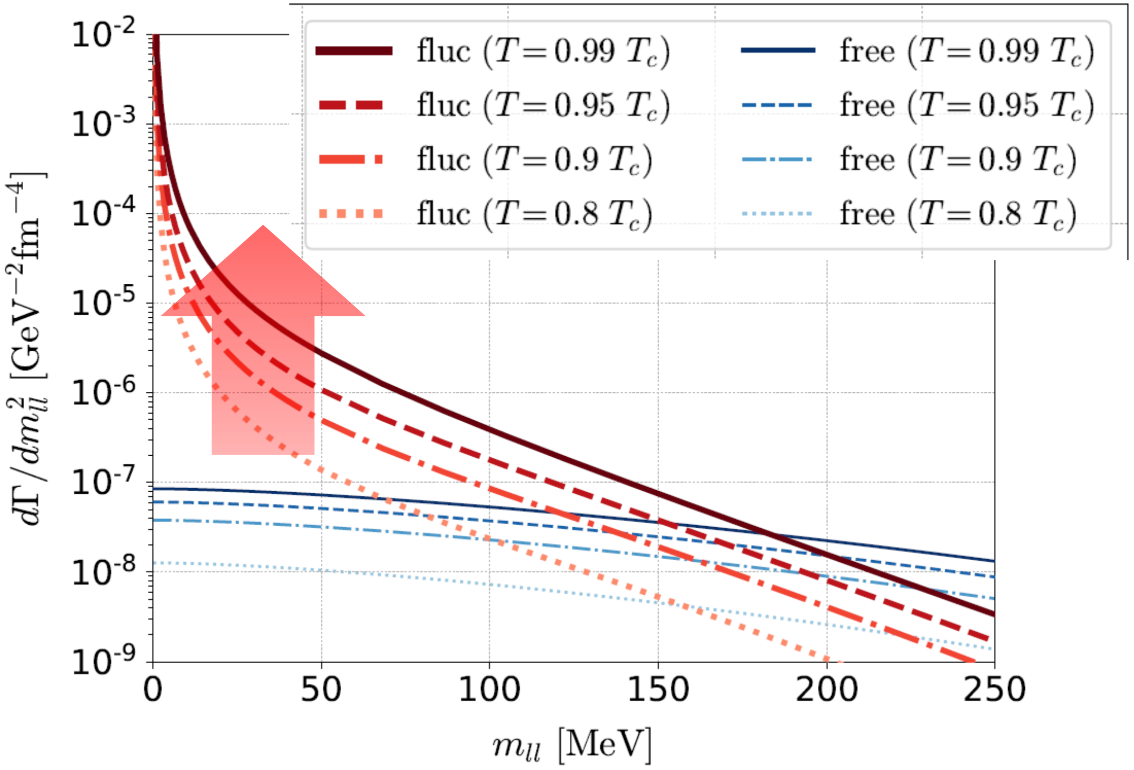
Modification of dilepton production through

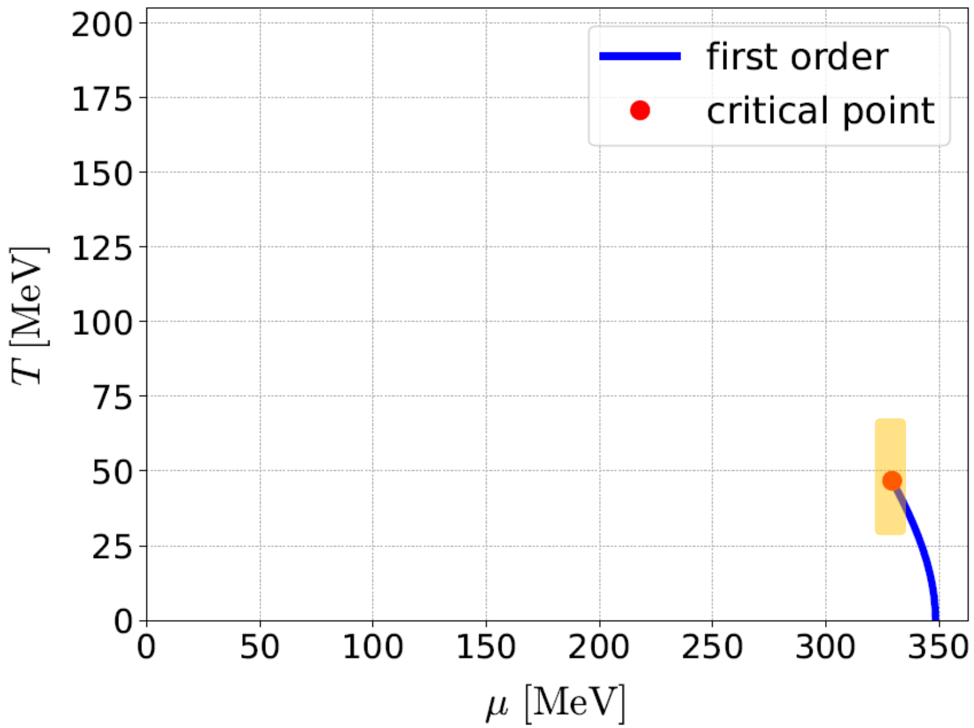


Dilepton production rate near QCD-CP

Nishimura, MK, Kunihiro ('23)

Invariant mass spectrum

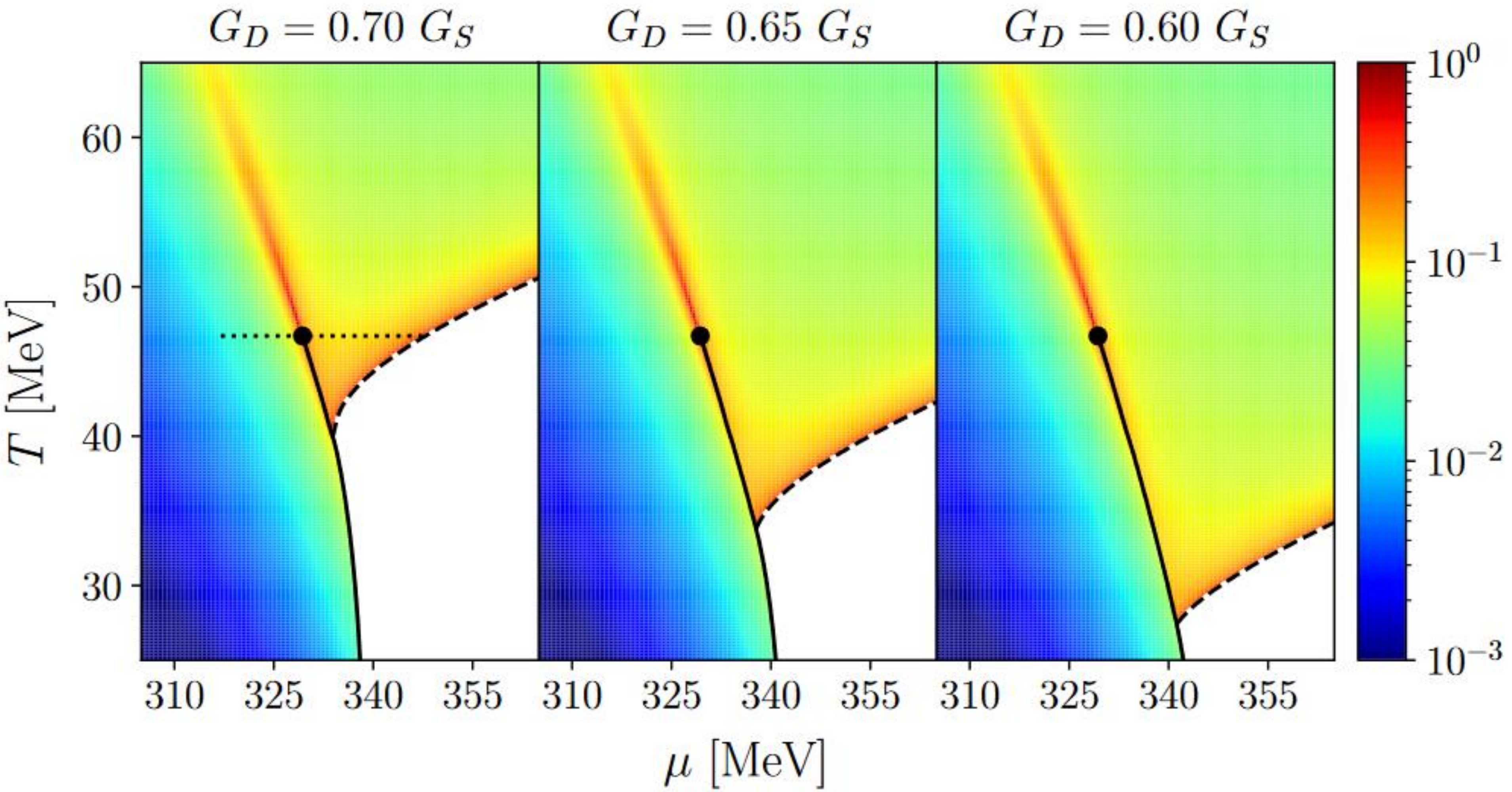




for fixed chem. pot.: $\mu = \mu_c$

- ☐ Enhancement at low M_{ll} region near QCD-CP
- ☐ Distinguishment from diquark soft mode may be difficult.

Electric Conductivity on QCD Phase Diagram



Nishimura, MK, Kunihiro ('24)

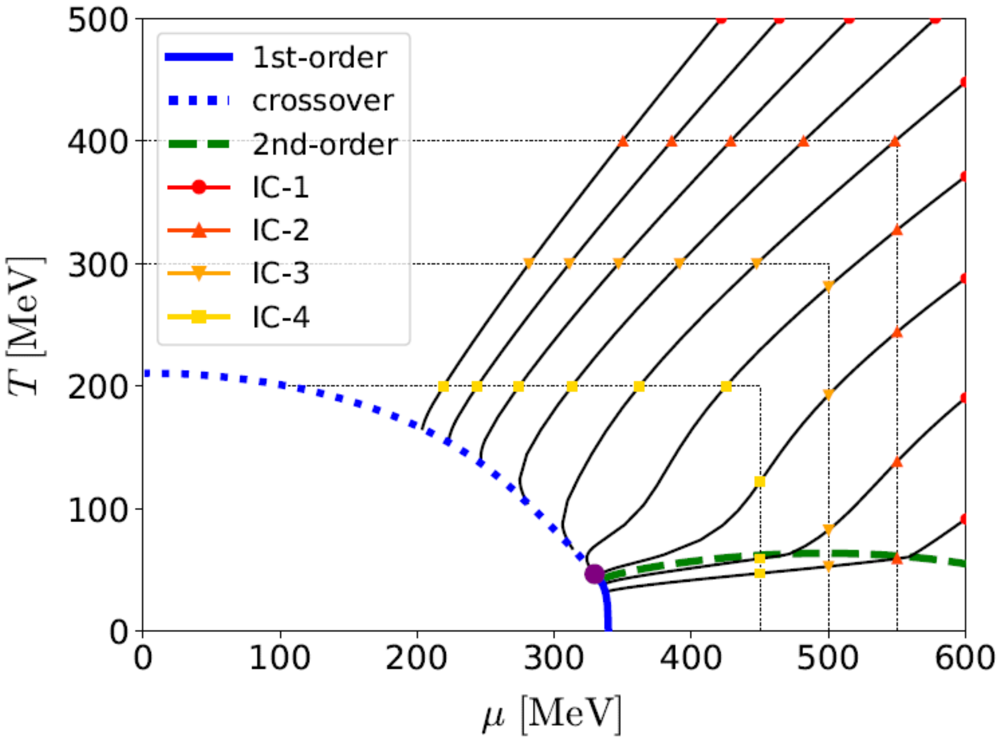
- ☐ DPR in the low-energy limit = electric conductivity
- ☐ Two "hot spots" on the T-μ plane

Dilepton Yields: Beam-Energy Scan

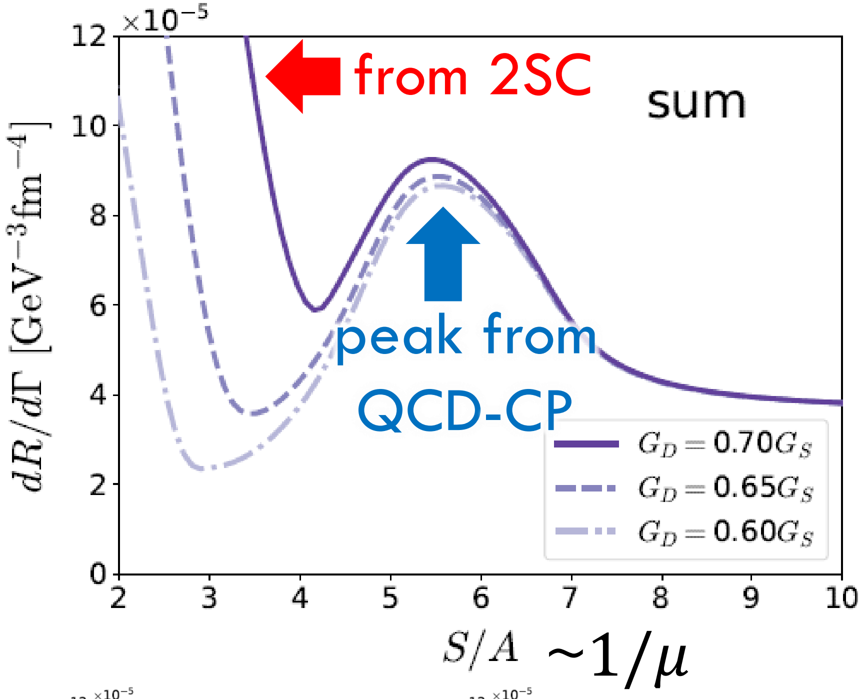
Nishimura, Nara, Steinheimer, Eur.Phys.J.A 60, 2024

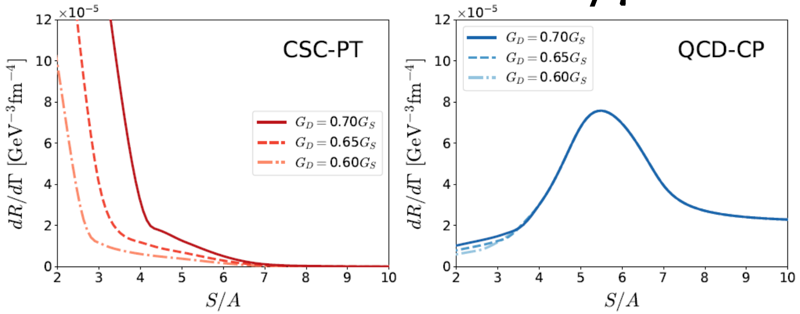
Dilepton yields
$\simeq$ integrated rate along isentropic lines

Isentropic lines in NJL model



Dilepton Yields $50 < M < 100$ MeV





Summary

- QCD phase diagram has rich structures especially in the high-density region, such as the QCD critical point and color superconductivity.
- The beam-energy scan will reveal them.

- We performed a quantitative analysis of the size and lifetime of the dense region.
  - $\sqrt{s_{NN}} \simeq 3\ \mathrm{GeV}$ may be an optimal energy to study $\rho = 3{\sim}4\rho_0$.

- Phase transitions in dense quark matter may be detectable through the enhancement of the dilepton production rate at ultra-low-mass-region.