

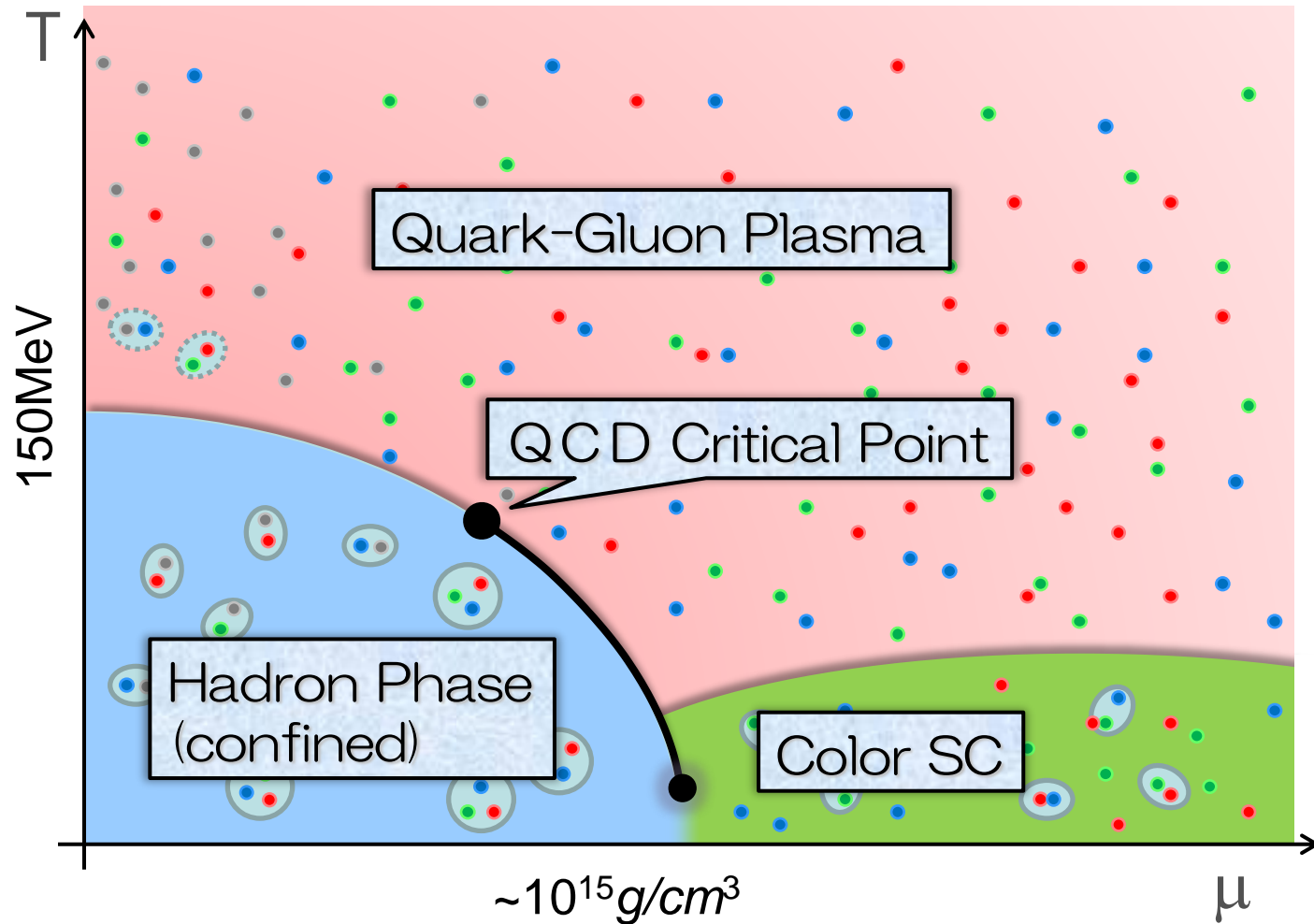
# Two topics on high density matter and heavy-ion physics

Masakiyo Kitazawa  
(YITP, Kyoto)

Taya, Jinno, MK, Nara, 2409.07685.

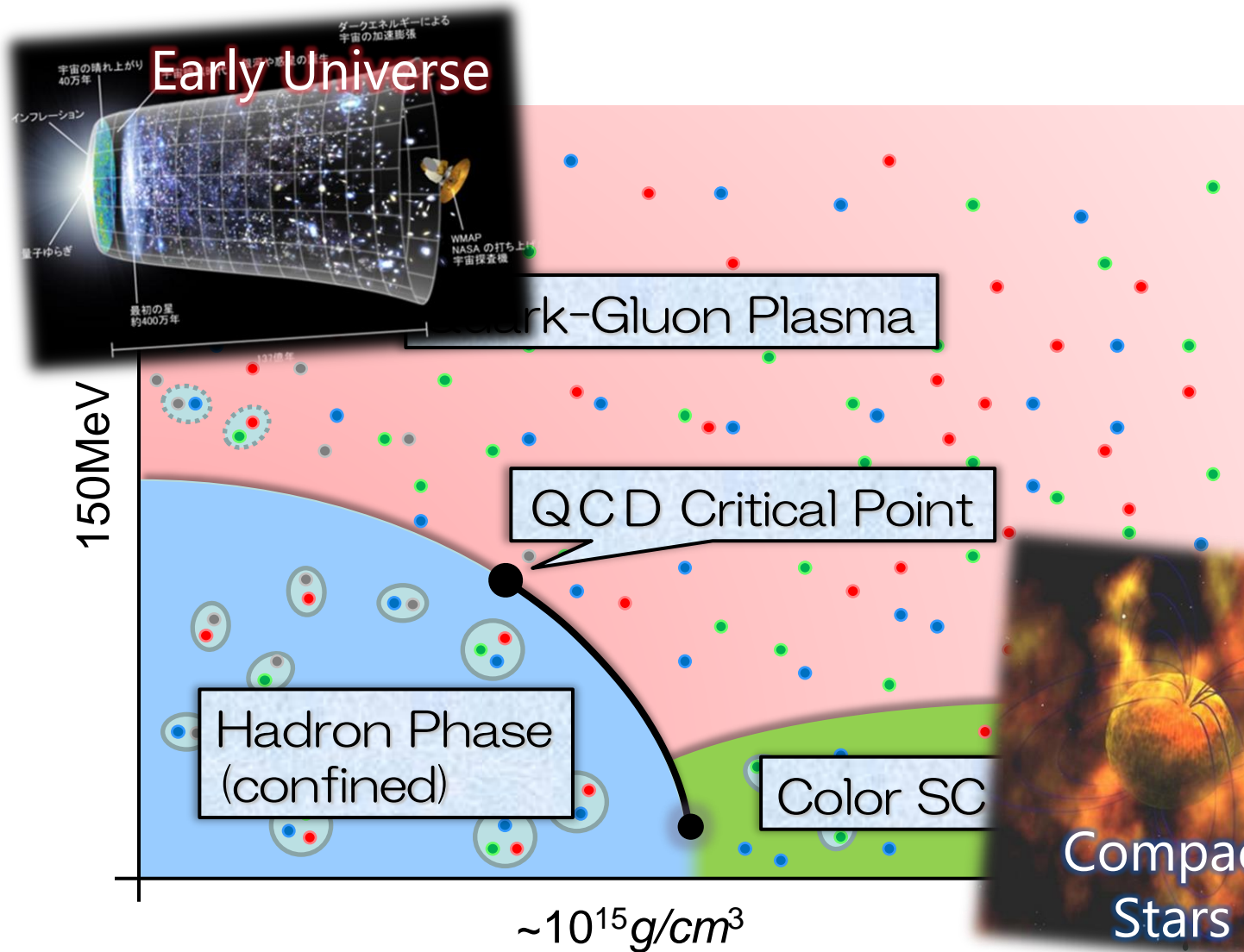
Nishimura, MK, Kunihiro, Ann. Phys. **469**, 169768 (2024); PTEP **2023**, 053D01; PTEP **2022**, 093D02.

# QCD Phase Diagram



- ❑ Possible first-order transition and QCD critical point in dense region
- ❑ Multiple QCD-CP? [MK+ \('02\)](#)
- ❑ Color superconducting phases in dense and cold quark matter

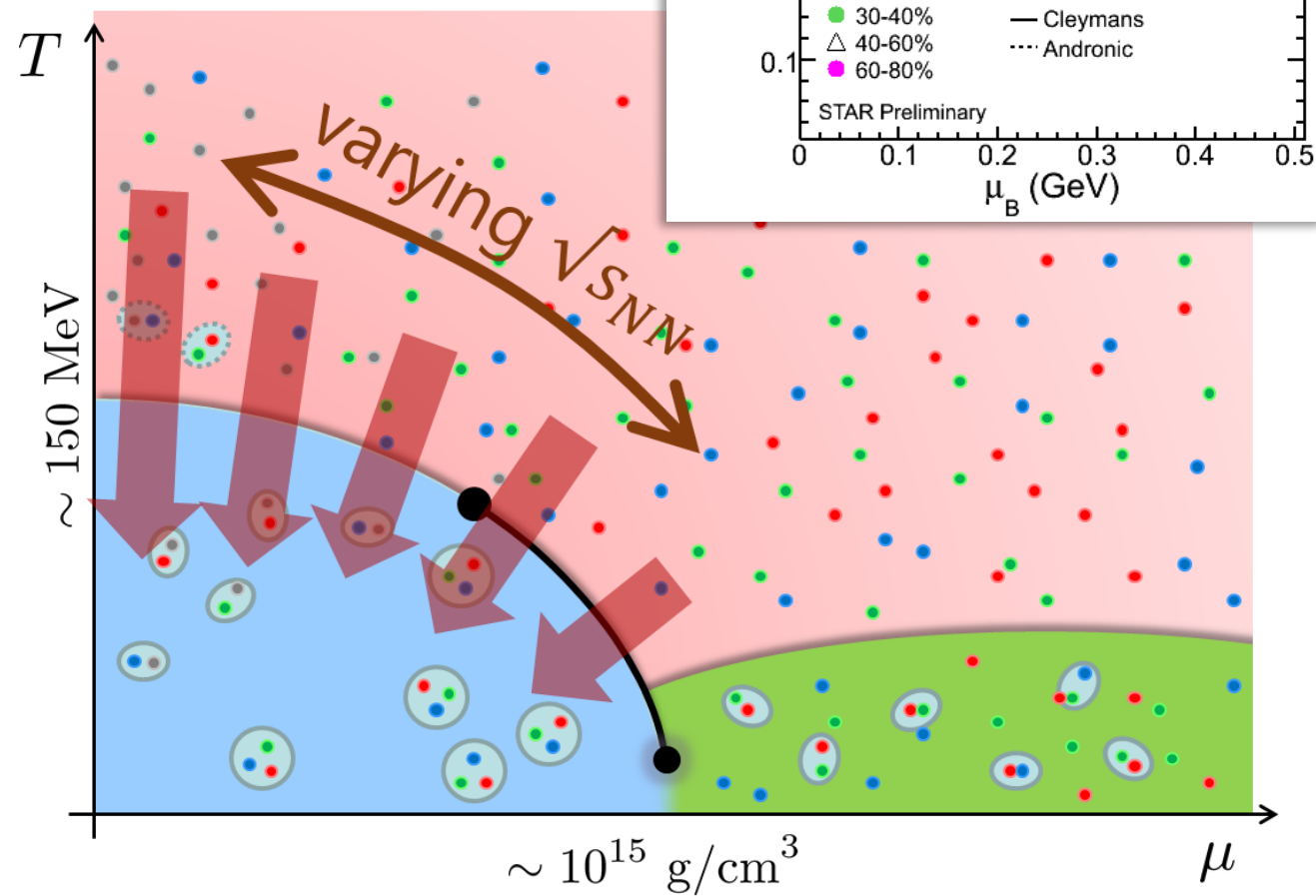
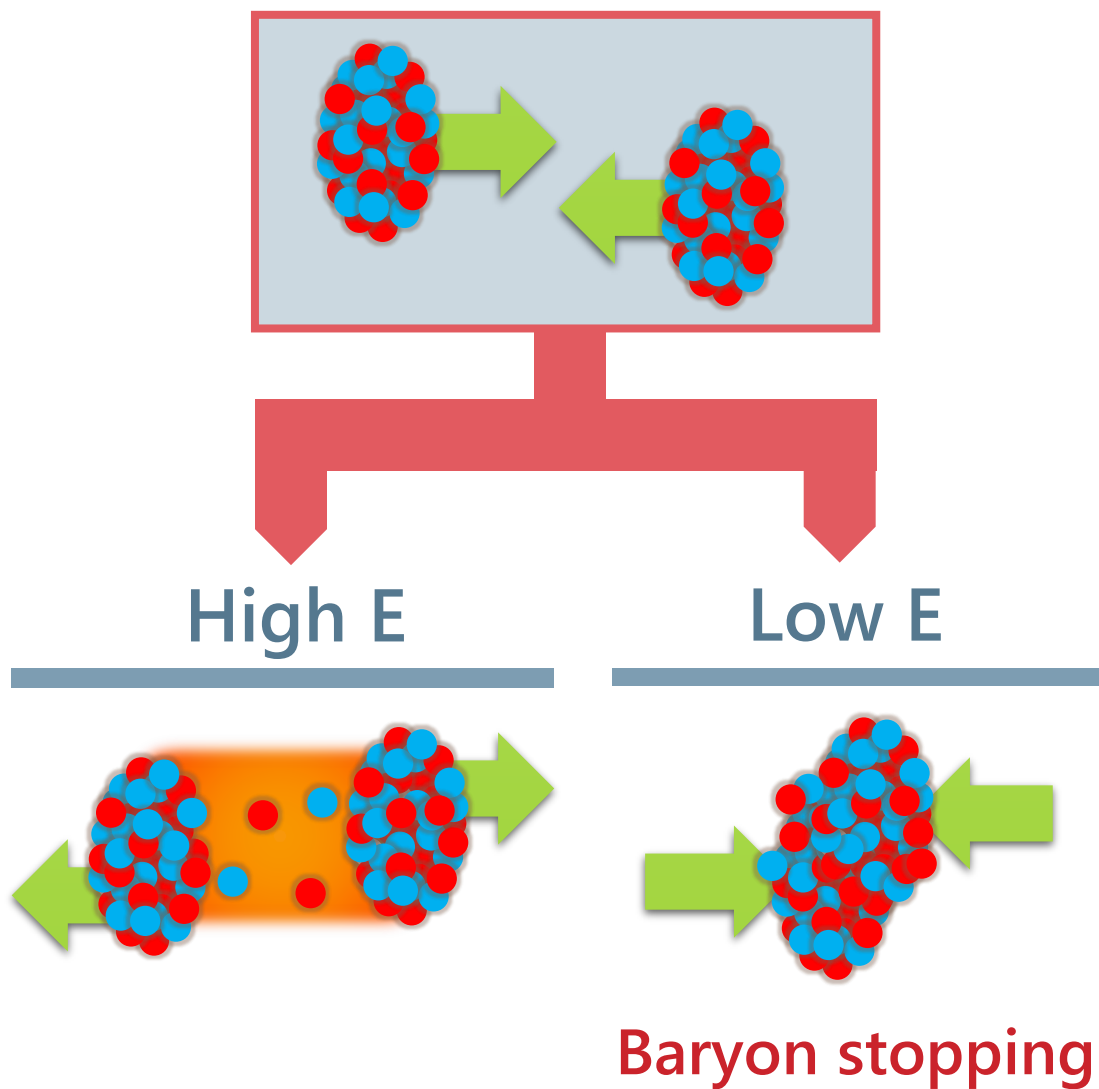
# QCD Phase Diagram



- ❑ Possible first-order transition and QCD critical point in dense region
- ❑ Multiple QCD-CP? [MK+ \('02\)](#)
- ❑ Color superconducting phases in dense and cold quark matter

# Beam-Energy Scan

STAR, 2012





# Contents

## 1. Optimal collision energy for investigating dense matter

Taya, Jinno, MK, Nara, 2409.07685

## 2. Dilepton production for the signal of phase transitions

Nishimura, MK, Kunihiro, Ann. Phys. 469, 169768; PTEP 2023, 053D01; PTEP 2022, 093D02

# Key Questions

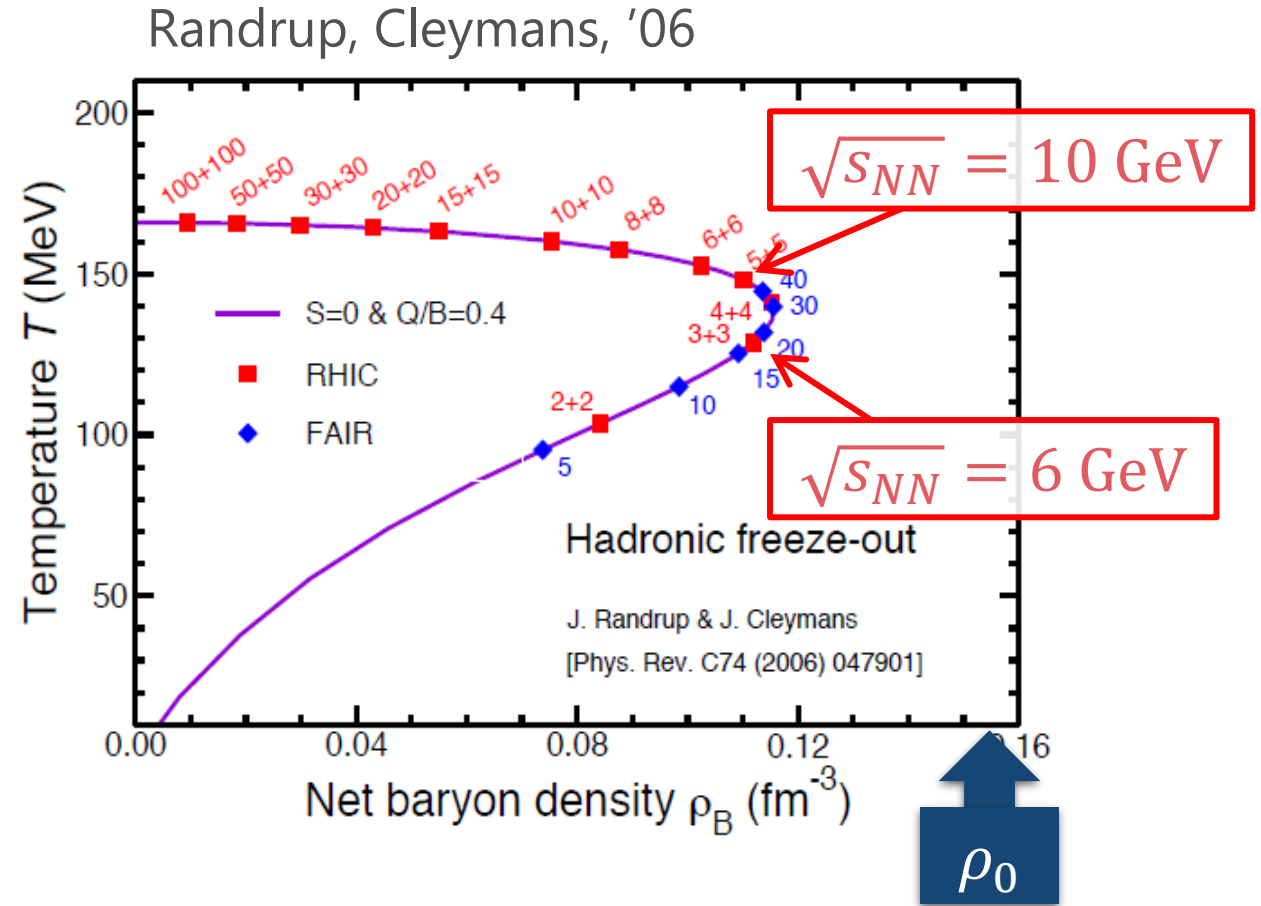
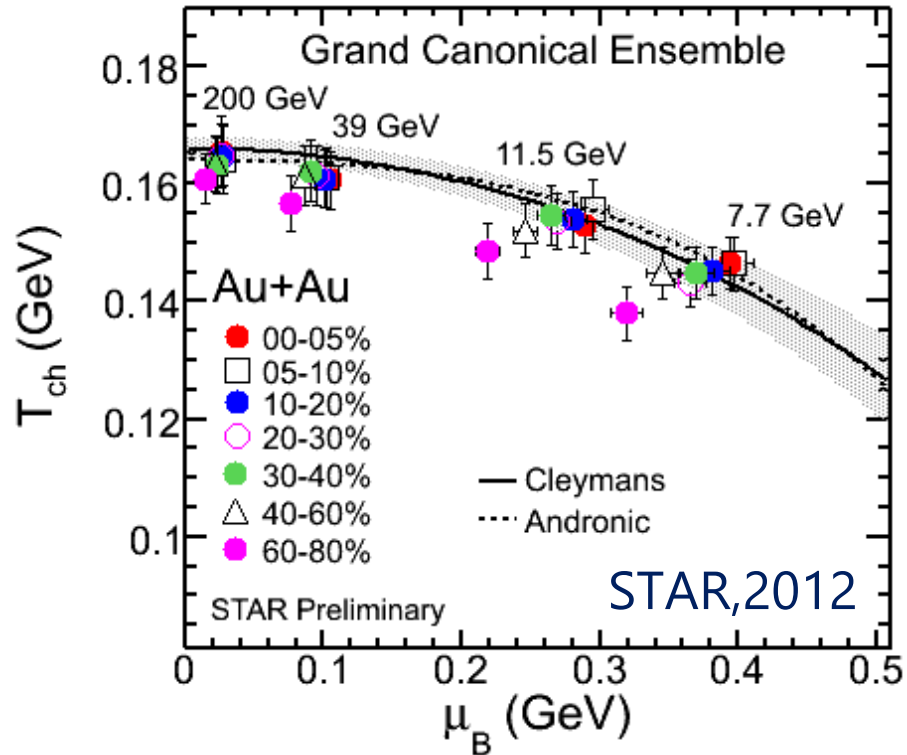
- What is **the optimal collision energy** to explore the baryon-rich matter?
- **How high density** is accessible?

## Our Answer

- $\sqrt{s_{NN}} = 3 \text{ GeV}$  is enough to study  $\rho = 3\rho_0$ .
- $\rho/\rho_0 = 4\sim 5$  may be accessible with  $\sqrt{s_{NN}} = 3\sim 6 \text{ GeV}$ .



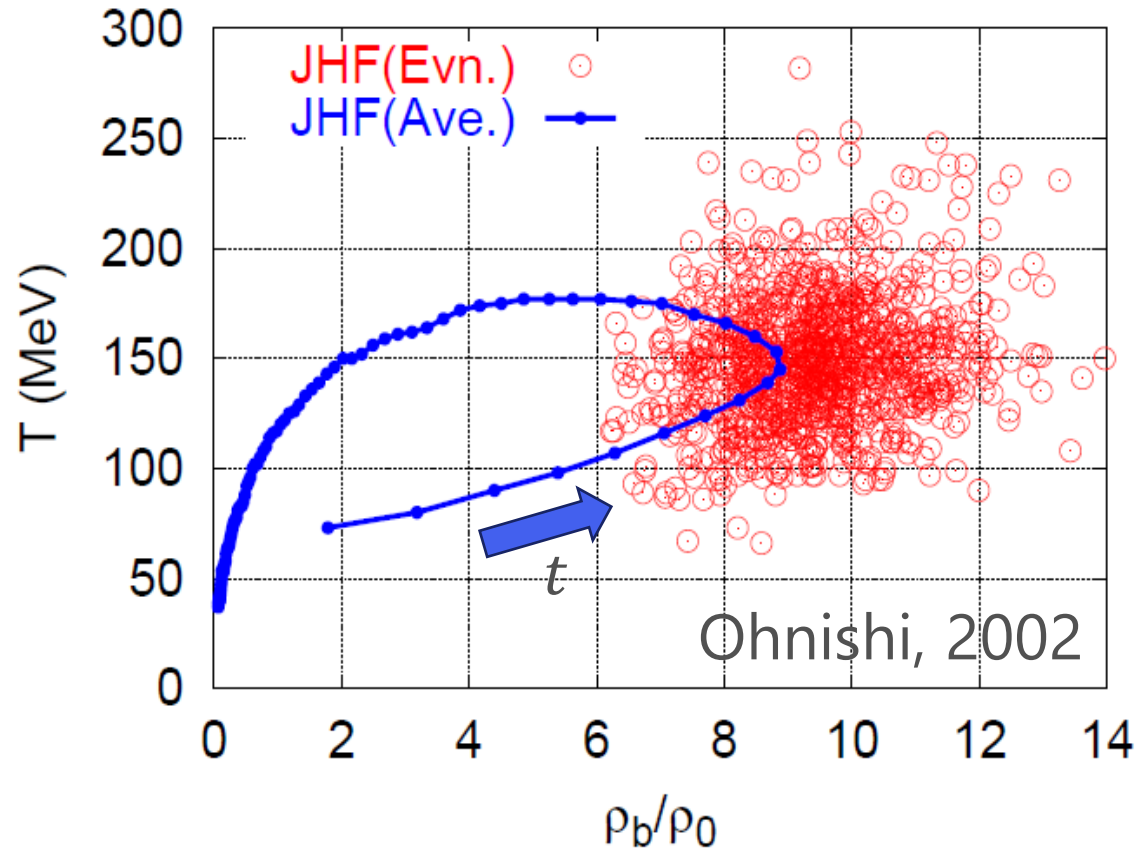
# Chemical Freezeout



- Highest baryon density **at chemical freezeout** at  $\sqrt{s_{NN}} \simeq 6 - 10 \text{ GeV}$ ?
- Not the highest density in the early stage.
- Density in earlier stage? Analysis in dynamical models

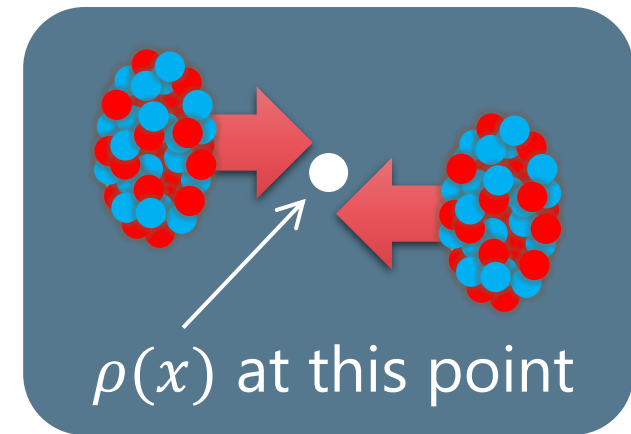


# Baryon Density at Collision Point



Simulation by JAM

$$E/A = 20\text{GeV}, \quad \sqrt{s_{NN}} \simeq 6\text{GeV}$$



- Maximum baryon density exceeds  $\rho/\rho_0 \simeq 8$ !
- Large event-by-event fluctuations
- How large is the high-density region? How long is the lifetime?

# Volume of Dense Region

Taya, Jinno, MK, Nara, 2409.07685

Volume where the local baryon density is larger than a threshold value  $\rho_{\text{th}}$

$$V_3(\rho_{\text{th}}, t) = \int_{\rho(x) > \rho_{\text{th}}} d^3 \mathbf{x} \gamma$$

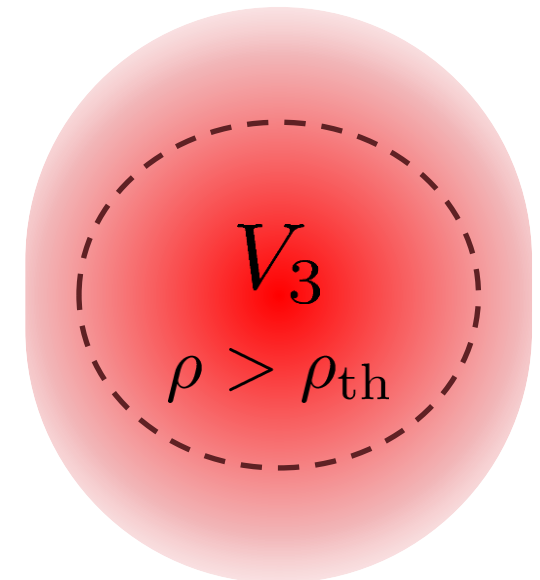
Baryon current  $J^\mu(x)$

Baryon density  $\rho(x) = \sqrt{J^\mu(x) J_\mu(x)}$

Lorentz factor  $\gamma = (1 - (\mathbf{J}/J_0)^2)^{-1/2}$

## Note:

- Event-by-event basis / no event average
- Directly calculable in a dynamical model
- We do not care about local thermalization.
  - $V_3$  is the upper limit of thermalized volume.
  - Even non-thermal, dense region is interesting!



# Simulation Setup in JAM

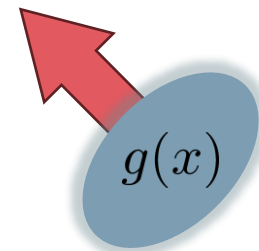
- Au+Au collision for  $2.4 \leq \sqrt{s_{NN}} \leq 20$  GeV
- Impact parameter  $b \leq 3$  fm : top 5% centrality
- Momentum-dependent mean field (MF2) Nara, Ohnishi, 2022
  - Setup reproducing  $\sqrt{s_{NN}}$  dep. of  $dv_1/d\eta$  and  $v_2$

## Smeared baryon current

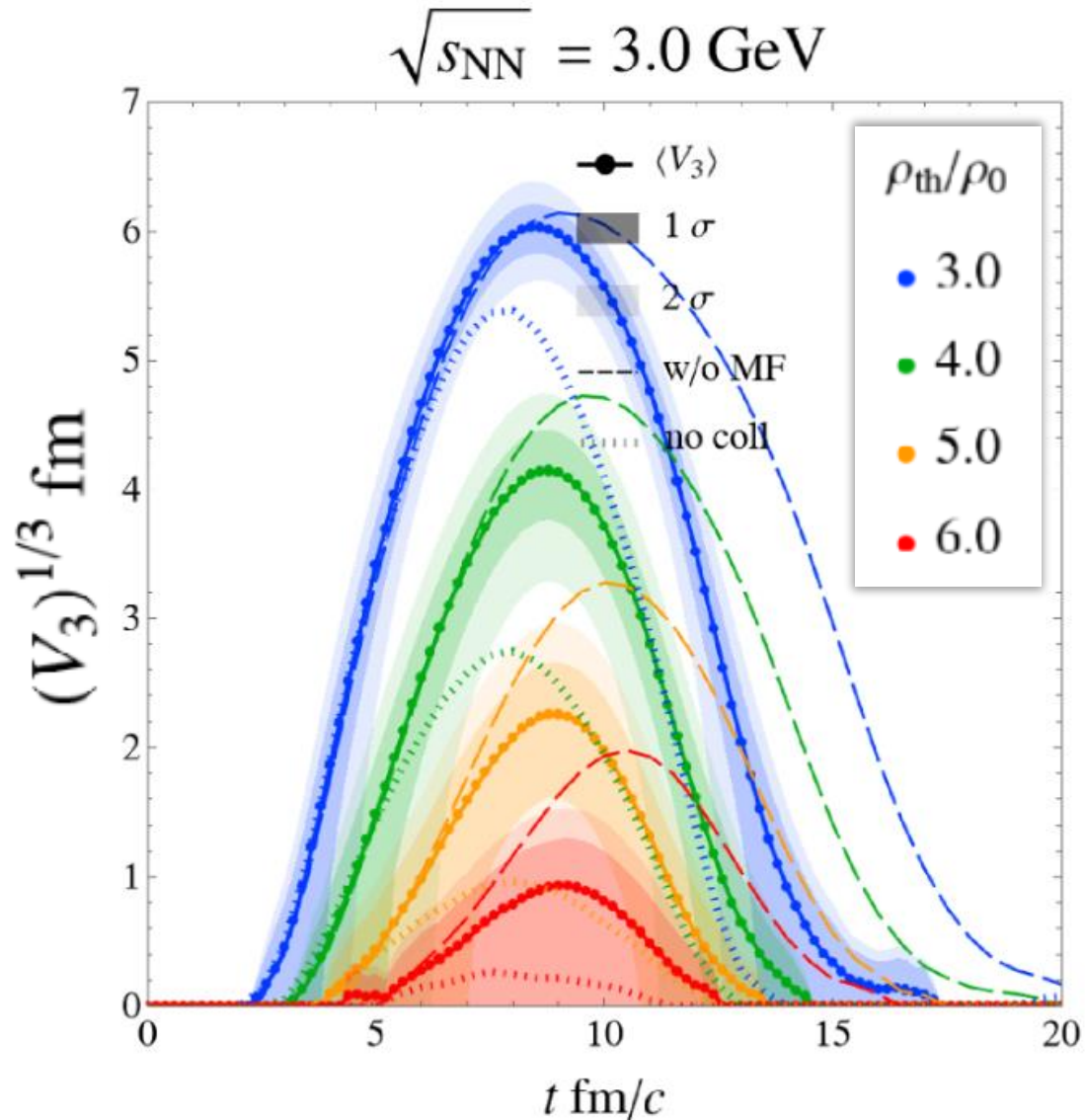
discrete particle distribution  $\rightarrow$  continuous current by smearing

$$J^\mu(x) = \sum_{i \in \text{baryons}} B_i g(x; X_i, P_i) \frac{P_i^\mu}{P_i^0}$$

$$g(x; X, P) := \frac{\gamma}{(\sqrt{2\pi}r)^3} e^{-\frac{|\mathbf{x}-\mathbf{X}|^2 + (\gamma\mathbf{V} \cdot (\mathbf{x}-\mathbf{X}))^2}{2r^2}} \quad r = 1 \text{ fm}$$



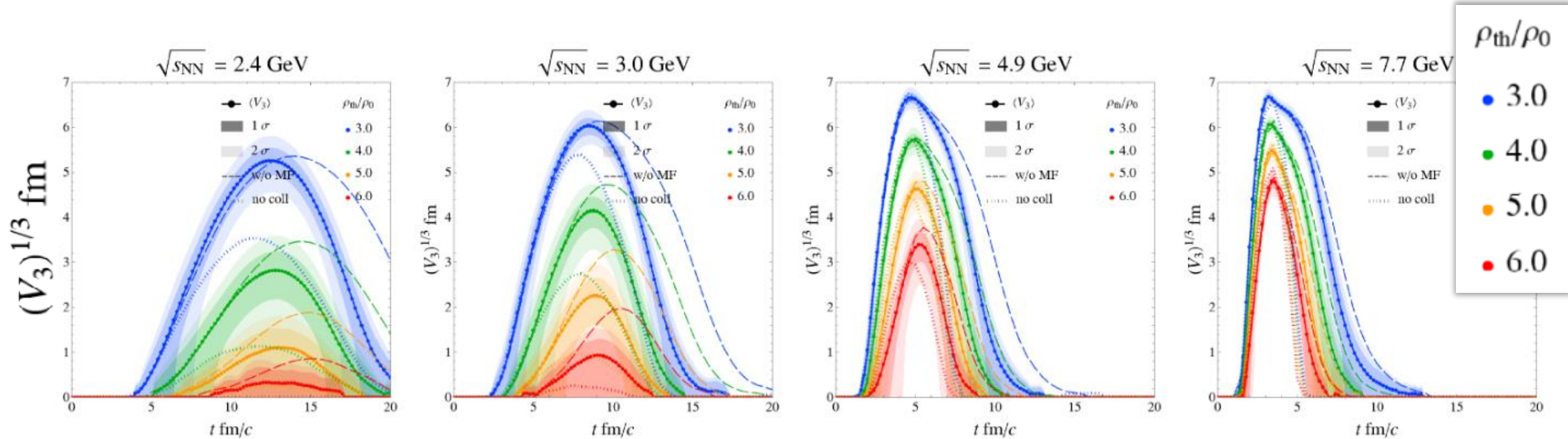
# $V_3$ in JAM



- solid: JAM+MF Nara, Ohnishi, 2022
- shaded band: 1 $\sigma$  and 2 $\sigma$  e-v-e fluct.
- dashed: JAM cascade mode
- dotted: no-collision

- Formation of dense region:
  - $V_3(3\rho_0, t) = (6 \text{ fm})^3$
  - $V_3(4\rho_0, t) = (4 \text{ fm})^3$
- Large e-v-e fluctuations
  - separable by event selection?
- Repulsive MF → weaker compression
- Compression owing to interaction

# $V_3$ for various $\sqrt{s_{NN}}$



As  $\sqrt{s_{NN}}$  becomes larger,

- $\max V_3(\rho_{th}, t)$  becomes larger.
- The lifetime of dense region becomes shorter.
- E-v-e fluctuations are more suppressed.

# Four-Volume / Lifetime

## Four Volume

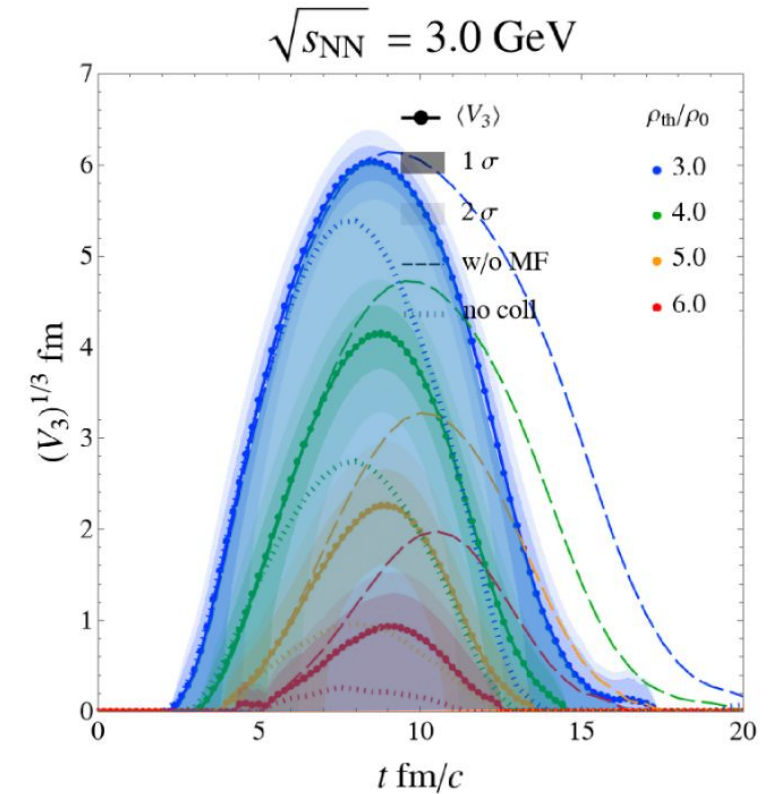
$$V_4(\rho_{\text{th}}) = \int_{-\infty}^{\infty} dt \int_{\rho(x) > \rho_{\text{th}}} d^3 \mathbf{x}$$

## Lifetime

$$\tau(\rho_{\text{th}}) = \frac{V_4(\rho_{\text{th}})}{\max V_3(\rho_{\text{th}}, t)}$$

## Note

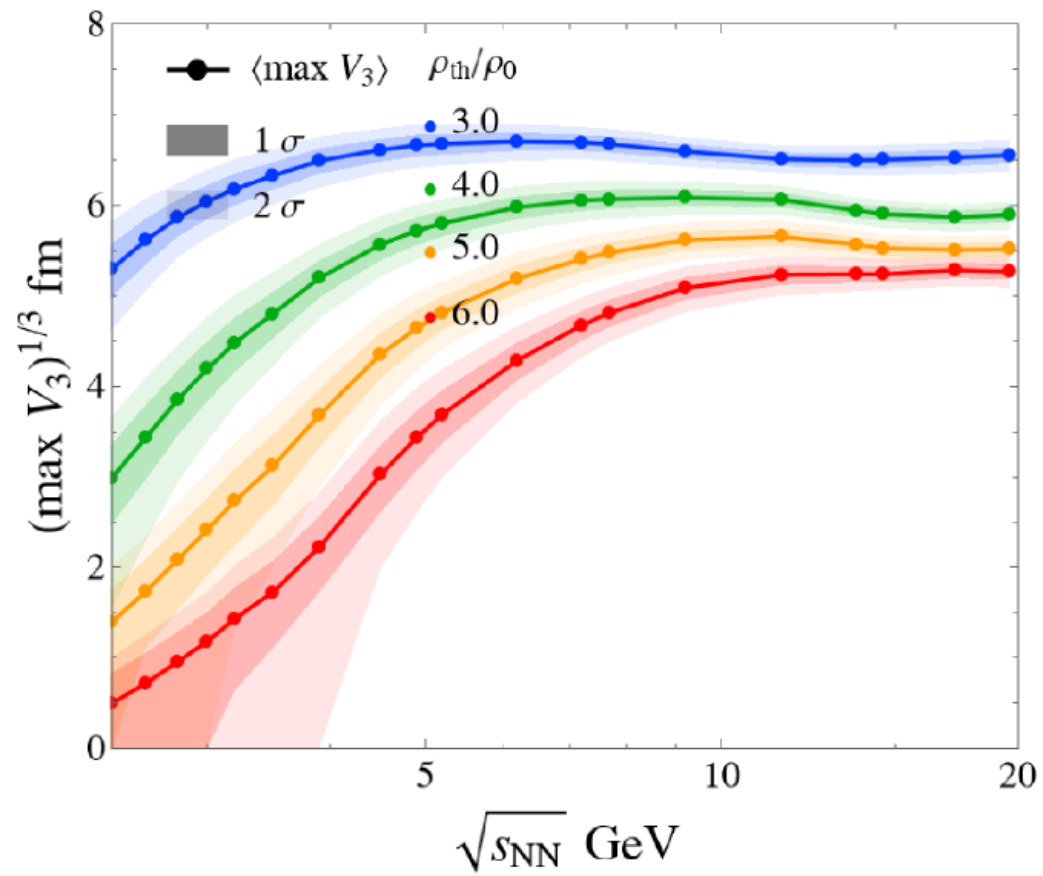
$V_4$  may be relevant for the dilepton production rate.



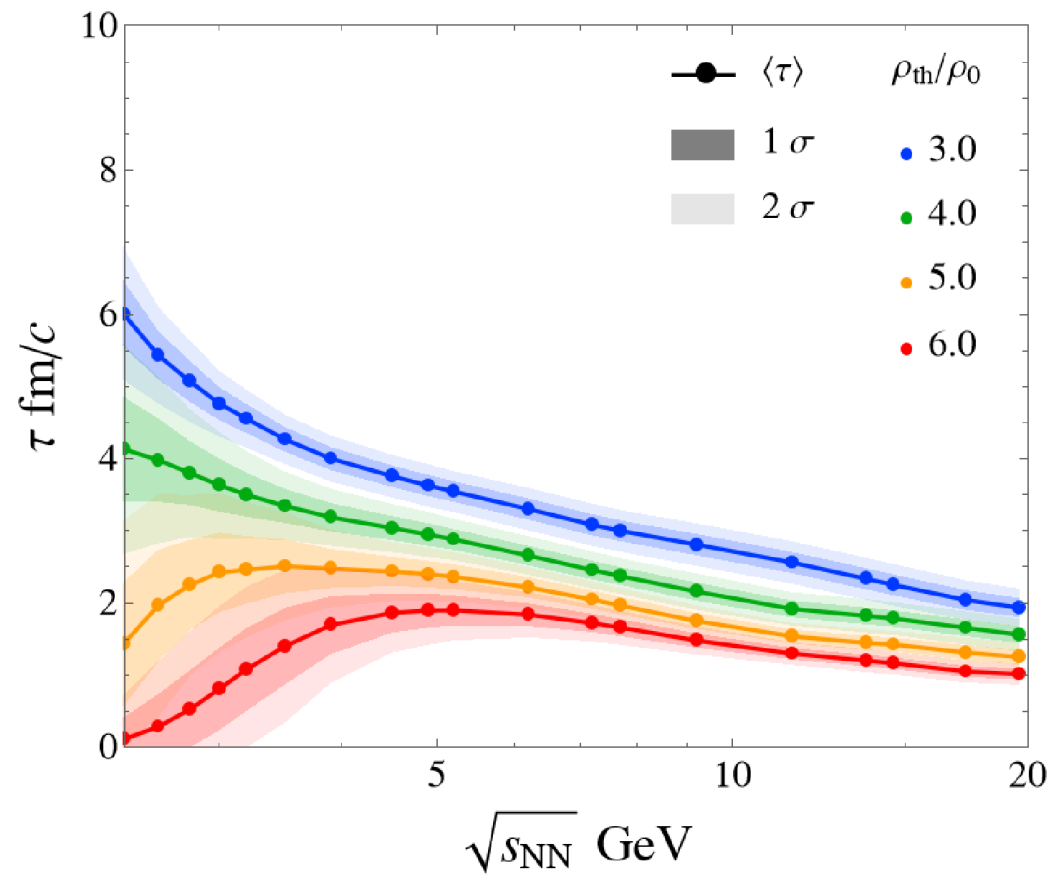


# $\sqrt{s_{NN}}$ Dependence

## max $V_3$



## Lifetime



$\rho_{th}/\rho_0$

3.0

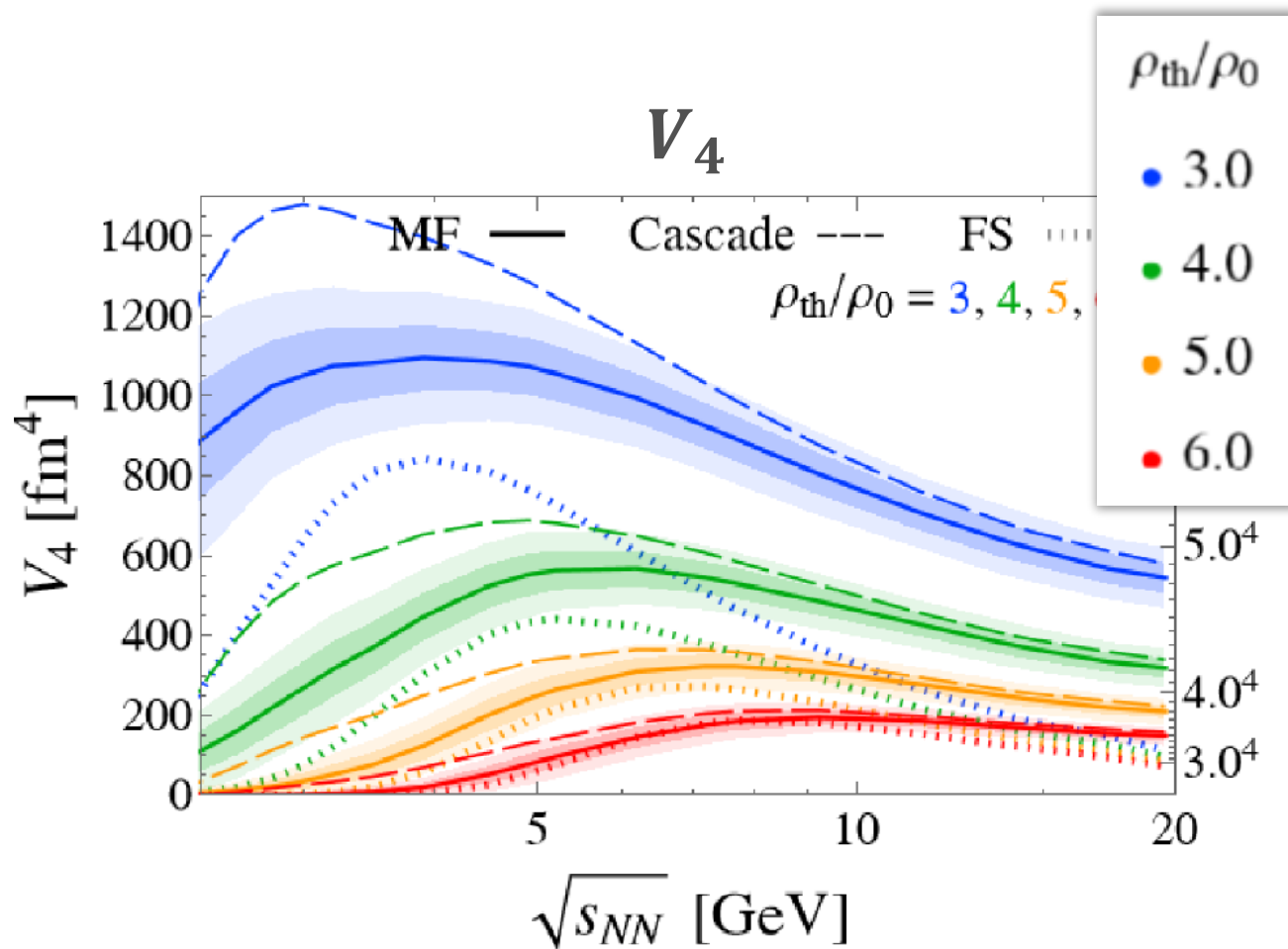
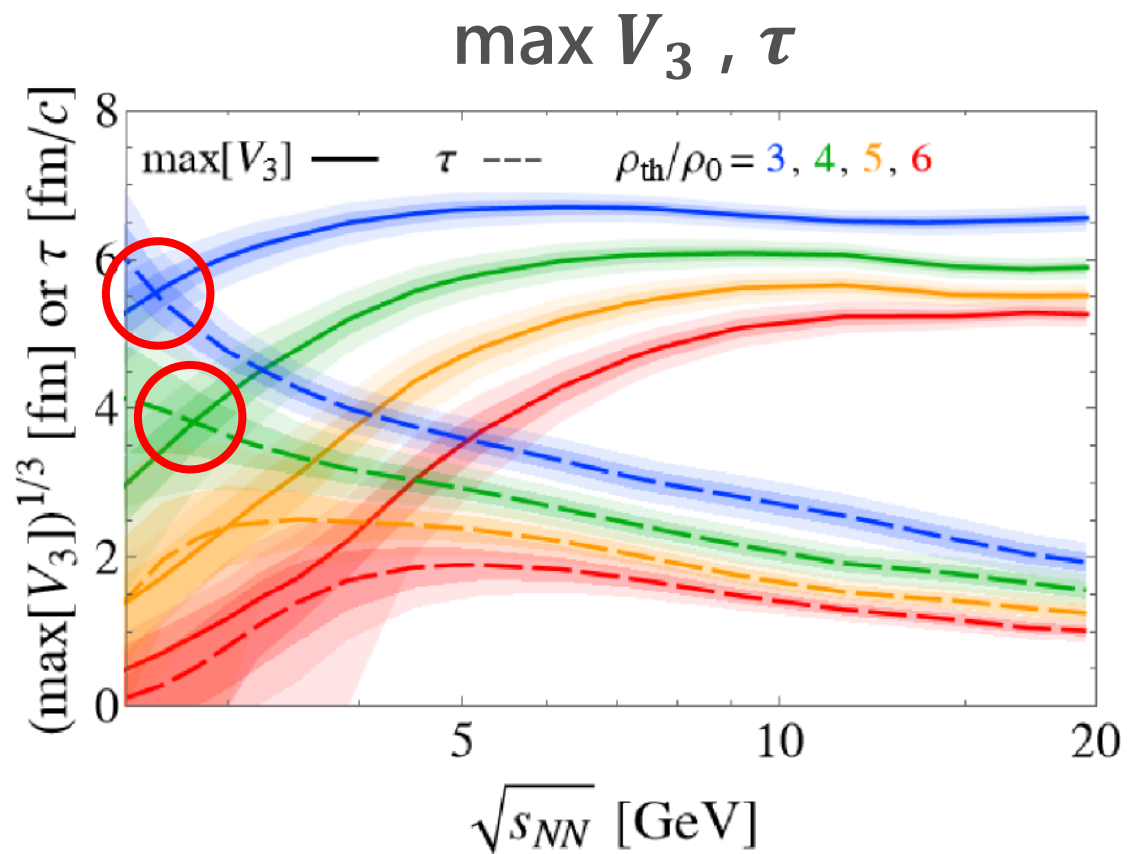
4.0

5.0

6.0



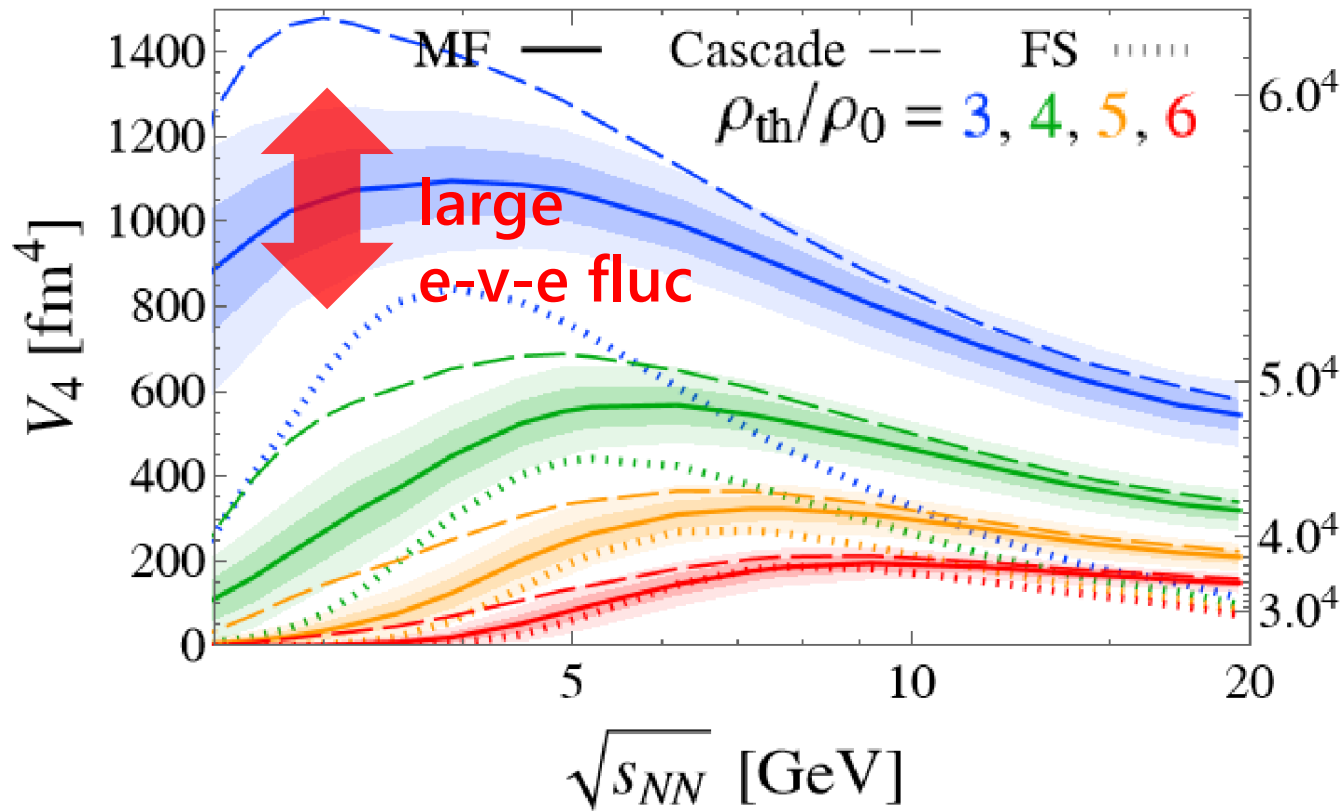
# $\sqrt{s_{NN}}$ Dependence



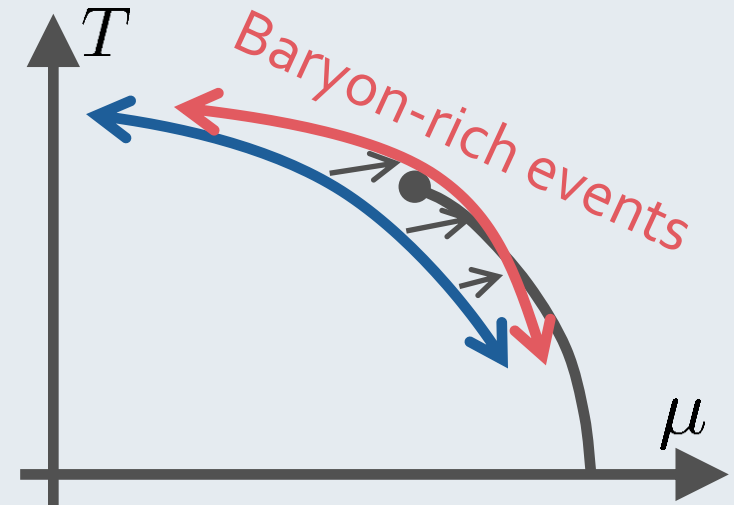
- $\sqrt{s_{NN}} \approx 3$  GeV would be the best energy to create  $\rho = 3 \sim 4\rho_0$  with large  $V_3$  and  $\tau$ .
- Lower  $\sqrt{s_{NN}}$  is suitable to create colder matter.

# Event Selection

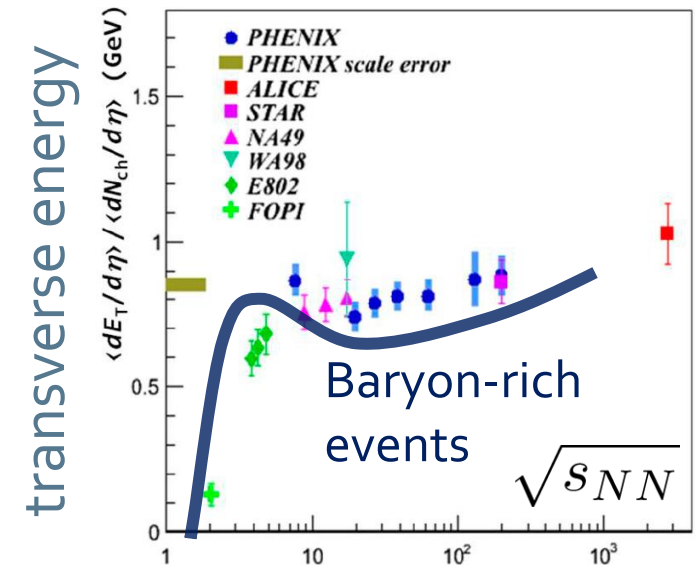
Taya, Jinno, MK, Nara, 2409.07685



## Density Scan?



■ Event selections via highest baryon/energy density will allow us a detailed study of QCD phase diagram.



# Short Summary

- $\sqrt{s_{NN}} = 3 \text{ GeV}$  is enough to study  $\rho = 3\rho_0$ .
- $\rho/\rho_0 = 4\sim 5$  may be accessible with  $\sqrt{s_{NN}} = 3\sim 6 \text{ GeV}$ .

## Future

- Check model independence
  - Analyses in various models
- Experiments at the sweet spot  $\sqrt{s_{NN}} = 2.5\sim 6 \text{ GeV}$ 
  - Future exps. at FAIR, NICA, HIAF, & J-PARC-HI

# Contents

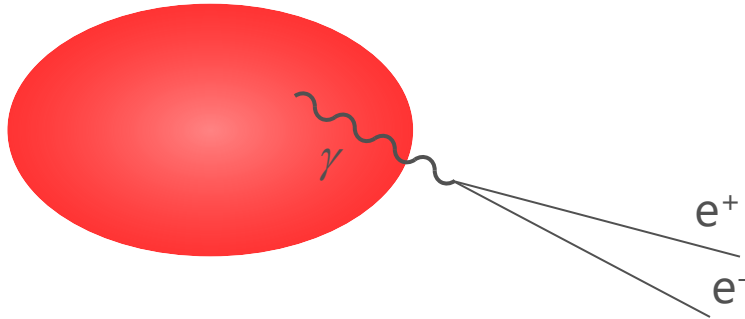
## 1. Optimal collision energy for investigating dense matter

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## 2. Dilepton production for the signal of phase transitions

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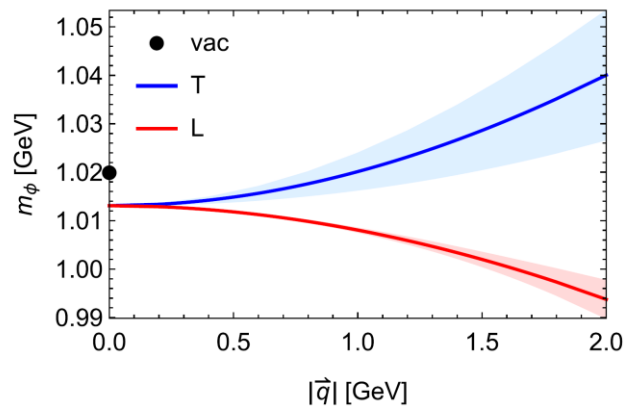
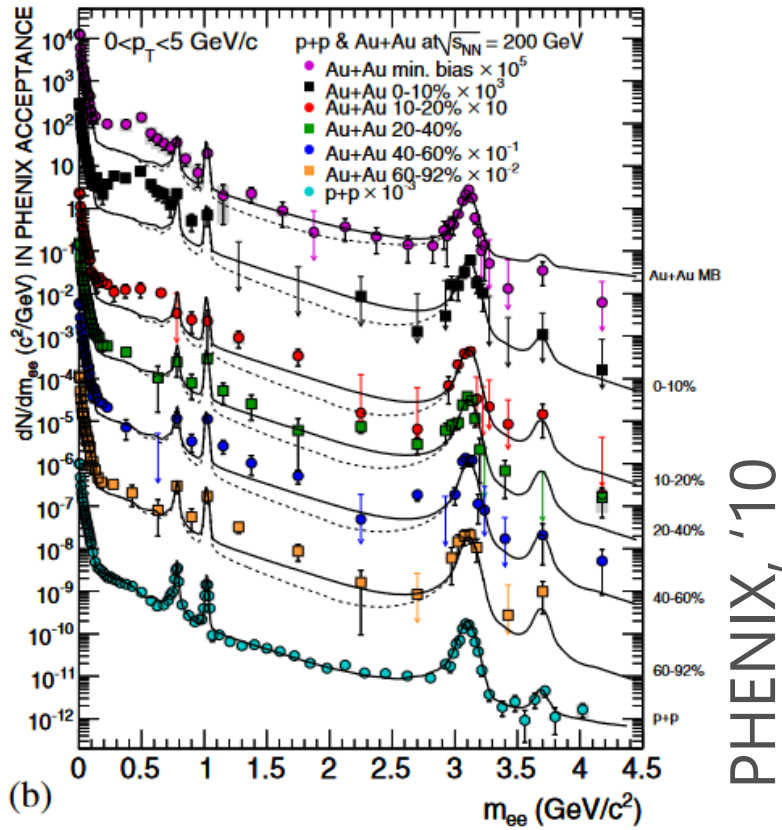
# Dilepton Production Rate



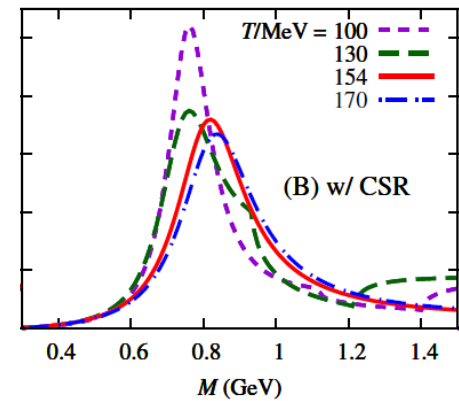
- Generated by the decay of virtual photons
- Carry information of primordial medium

## Physics accessible with DPR

- Medium temperature
- Dispersion relations
- Chiral mixing by chiral restoration
- Signal of phase transitions



Kim, Gubler, 2020



Sakai+, 2024

# Soft Modes of Second-order Phase Transitions

## □ Soft modes

- Divergence of the order-parameter fluctuations at a 2nd-order transition.
- **Collective fluctuations become massless** there.

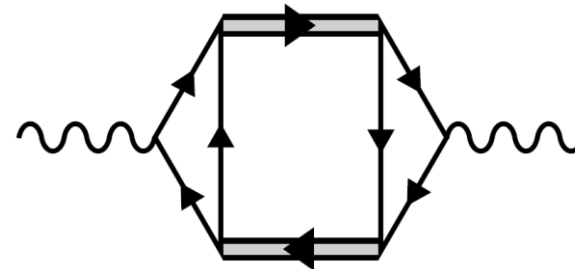
- QCD-CP : density-density fluctuations
- CSC : diquark-pair field

$$\begin{aligned} \text{QCD-CP: } & \text{loop} + \text{two-loop} + \dots \\ \text{CSC: } & \text{loop} + \text{two-loop} + \dots \end{aligned}$$



## □ Coupling of soft modes with dynamical observables

- Ex.: dilepton production rate



# Precursor in Metallic Superconductors

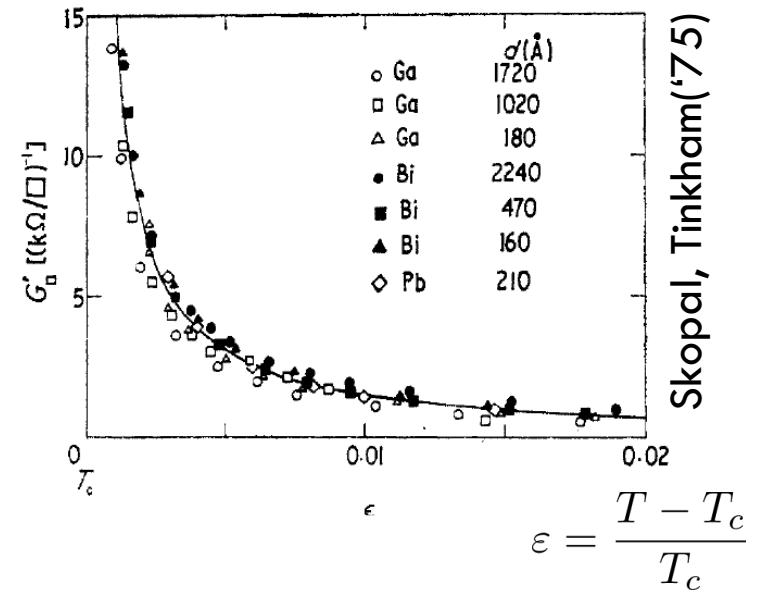
## □ Anomalous behavior of observables near but above $T_c$ of SC

- electric conductivity
- magnetic susceptibility
- pseudogap

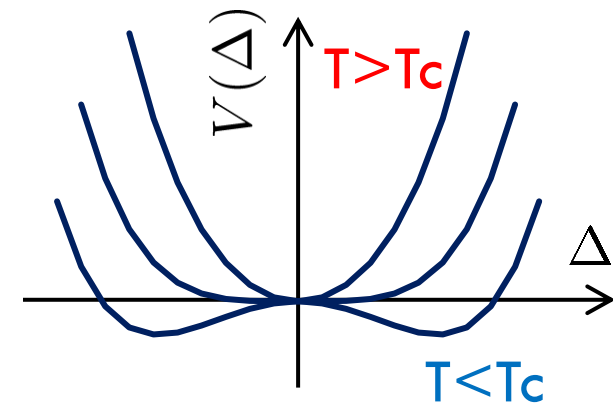
□ Enhanced pair fluctuations is one of the origins of precursory phenomena.

□ More significant phenomena in strongly-coupled systems.

Electric conductivity



Landau's free energy

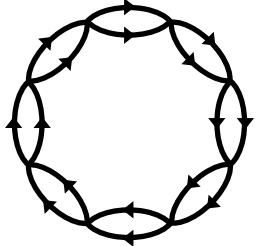


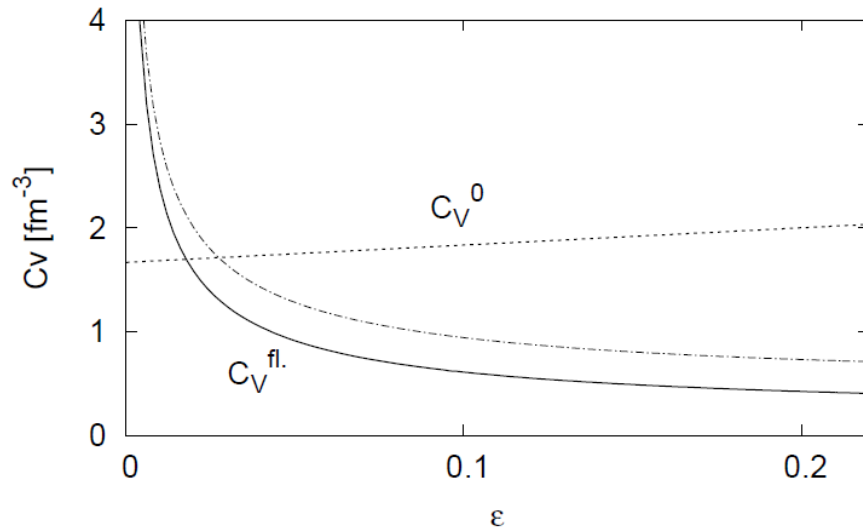


# Precursor of Color Superconductivity

MK, Koide, Kunihiro, Nemoto, '03, '05

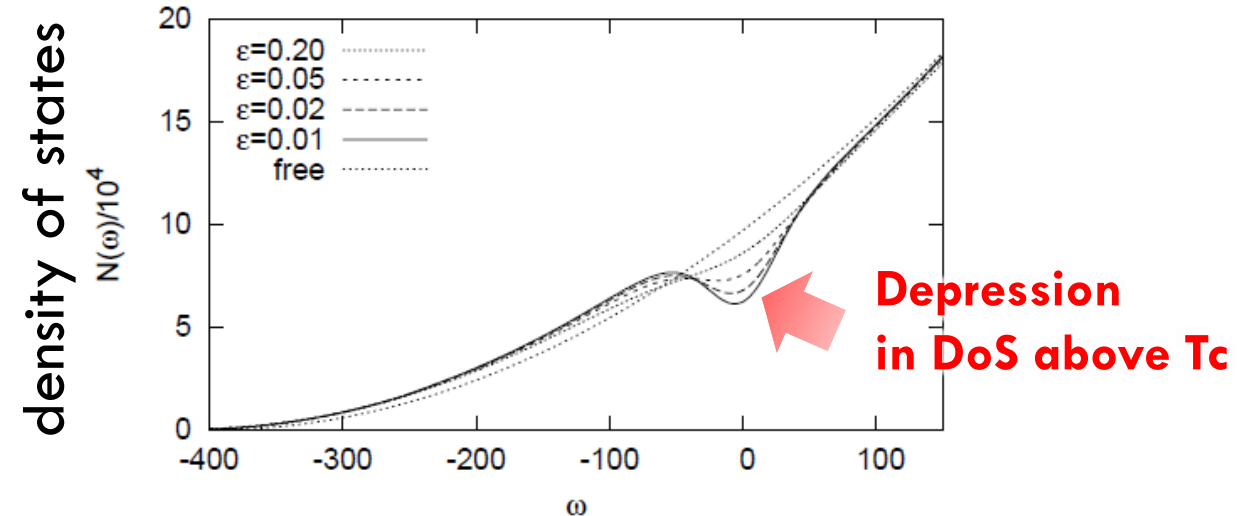
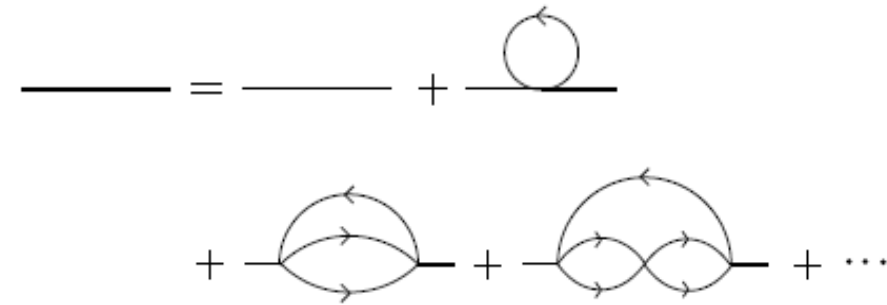
## □ Thermodynamic Potential

$\Omega =$ 

 $\rightarrow$ 
 Specific heat
 
$$c = -T \frac{\partial^2 \Omega}{\partial T^2}$$



$$\varepsilon = \frac{T - T_c}{T_c}$$

## □ Pseudogap



## NJL model (2-flavor)

$$\mathcal{L} = \bar{\psi}i\not{\partial}\psi + \mathcal{L}_S + \mathcal{L}_C$$

$$\mathcal{L}_S = G_S((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2)$$

$$\mathcal{L}_C = G_C((\bar{\psi}i\gamma_5\tau_A\lambda_A\psi^C)(\text{h.c.}))$$

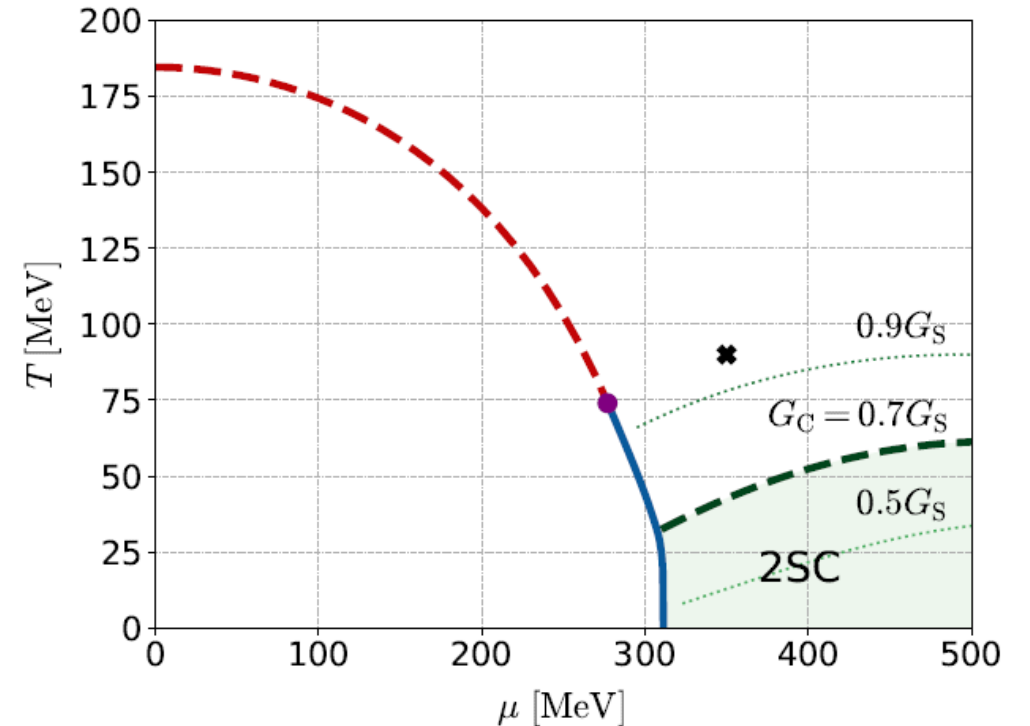
diquark interaction

## Parameters

$$G_S = 5.01 \text{ GeV}^{-2}, \quad \Lambda = 650\text{MeV}, \quad m_q = 0$$



## Phase Diagram in MFA



- Order of phase transition
  - 2nd in the MFA
  - can be 1st due to gauge fluctuation

Matsuura+('04), Giannakis+('04)  
Noronha+('06), Fejos, Yamamoto('19)

# Di-quark Fluctuations

## □ Diquark Propagator

$$D^R(x) = \langle [\Delta^\dagger(x), \Delta(0)] \rangle \theta(t) = \Rightarrow \Rightarrow$$

## □ Random Phase Approximation

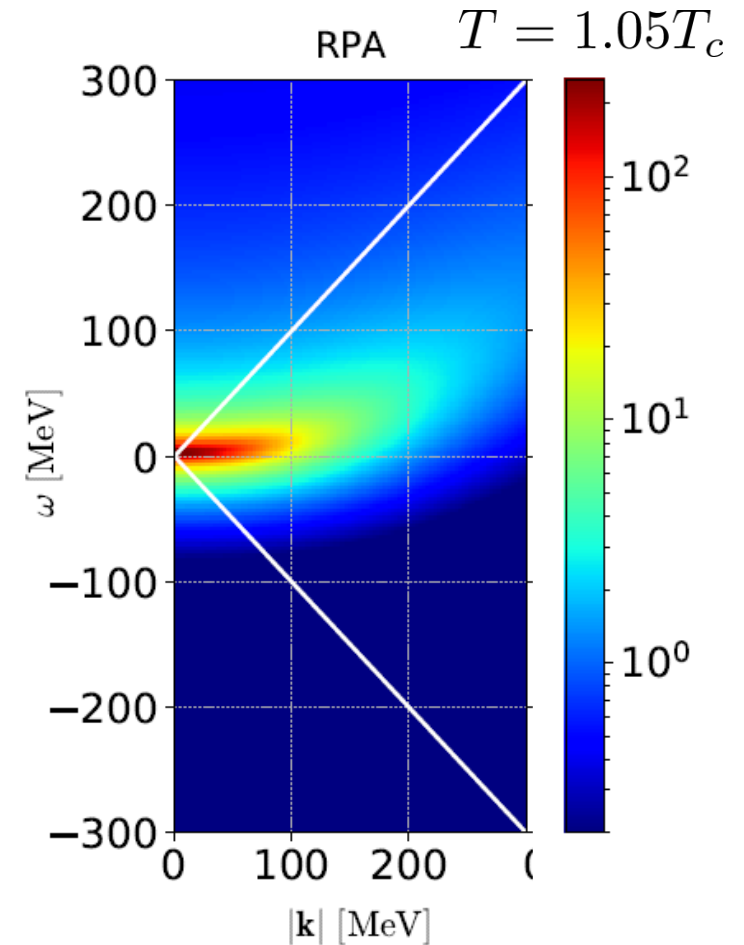
$$\begin{aligned} \Rightarrow \Rightarrow &= \text{loop} + \text{two loops} + \dots \\ &= \frac{Q^R(\mathbf{k}, \omega)}{1 + G_C Q^R(\mathbf{k}, \omega)} \\ Q^R(\mathbf{k}, \omega) &= \text{loop} \end{aligned}$$

- Diquark field becomes massless at  $T=T_c$
- Soft mode of CSC transition
- Strength in the space-like region

MK, Koide, Kunihiro, Nemoto, '01,'05

## Dynamical Structure Factor

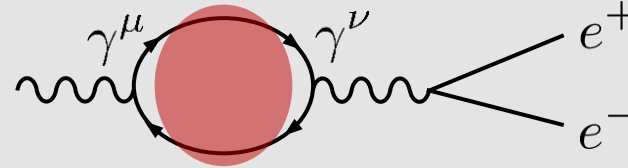
$$S(\mathbf{k}, \omega) = -\frac{1}{\pi} \frac{1}{1 - e^{-\beta\omega}} \text{Im} D^R(\mathbf{k}, \omega)$$



# Photon Self-Energy: Precursor of CSC

## □ Dilepton Production Rate

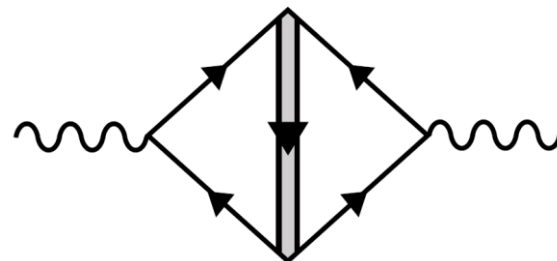
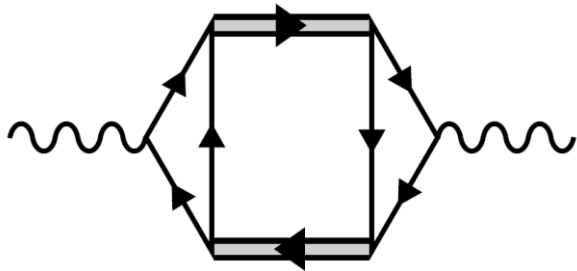
$$\frac{d^4\Gamma}{dk^4} = \frac{\alpha}{12\pi^4} \frac{1}{k^2} \frac{1}{e^{\beta\omega}-1} \text{Im}\Pi^{R\mu}{}_{\mu}(k)$$



## □ Effect of Di-quarks on $\Pi^{\mu\nu}(k)$

Aslamasov-Larkin term

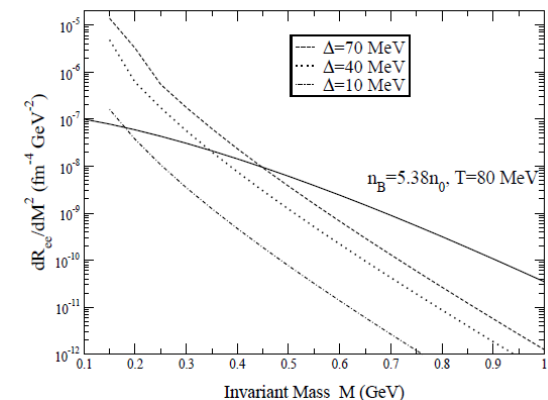
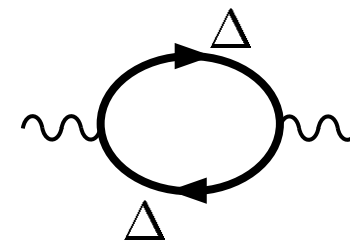
Maki-Thompson term



Well-known diagrams in metallic SC  
for describing paraconductivity

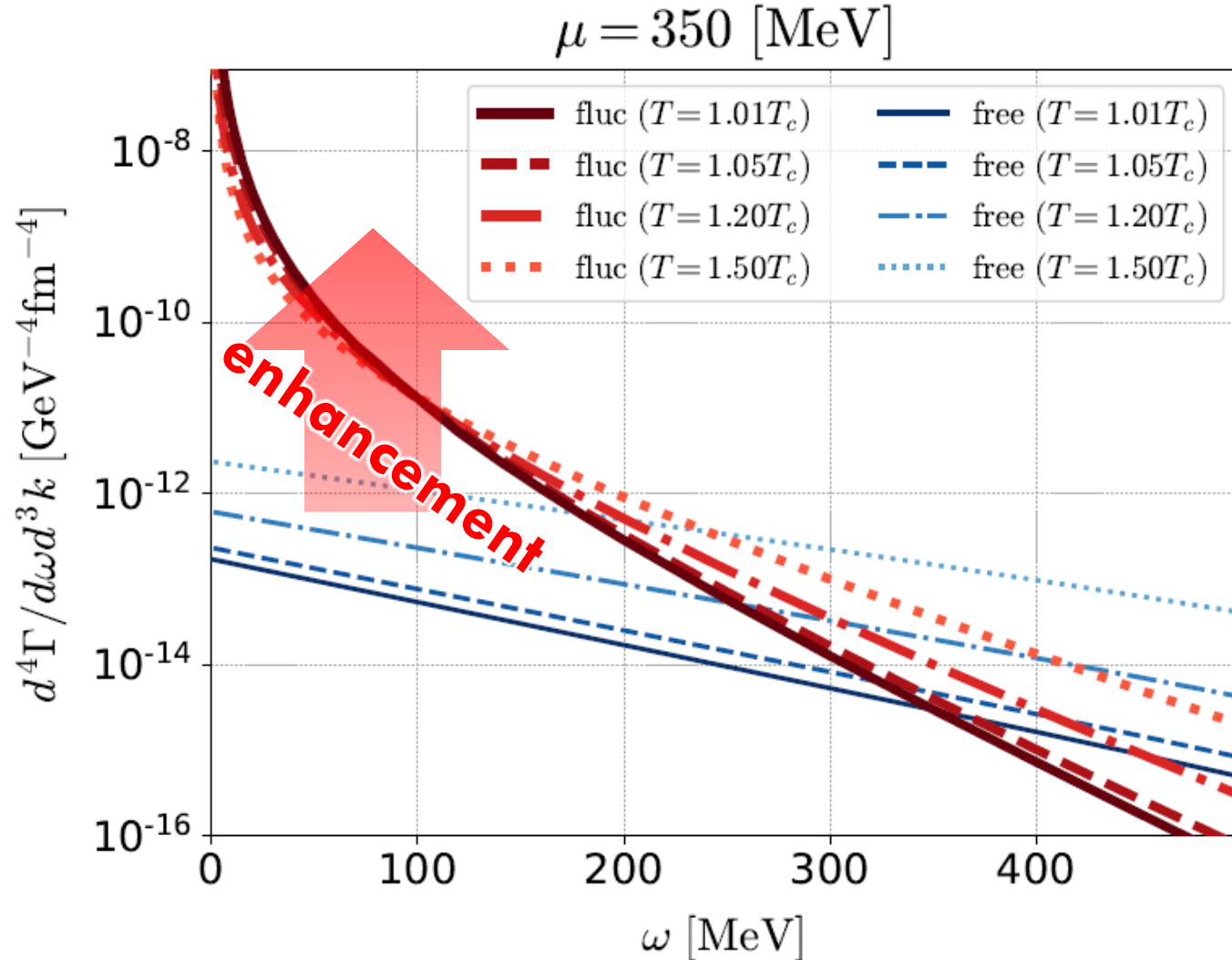
## □ DPR from CFL phase

Jaikumar, Rapp, Zahed ('02)



# Production Rate at $k = 0$

Nishimura, MK, Kunihiro ('22)



**Red:** fluctuation contribution

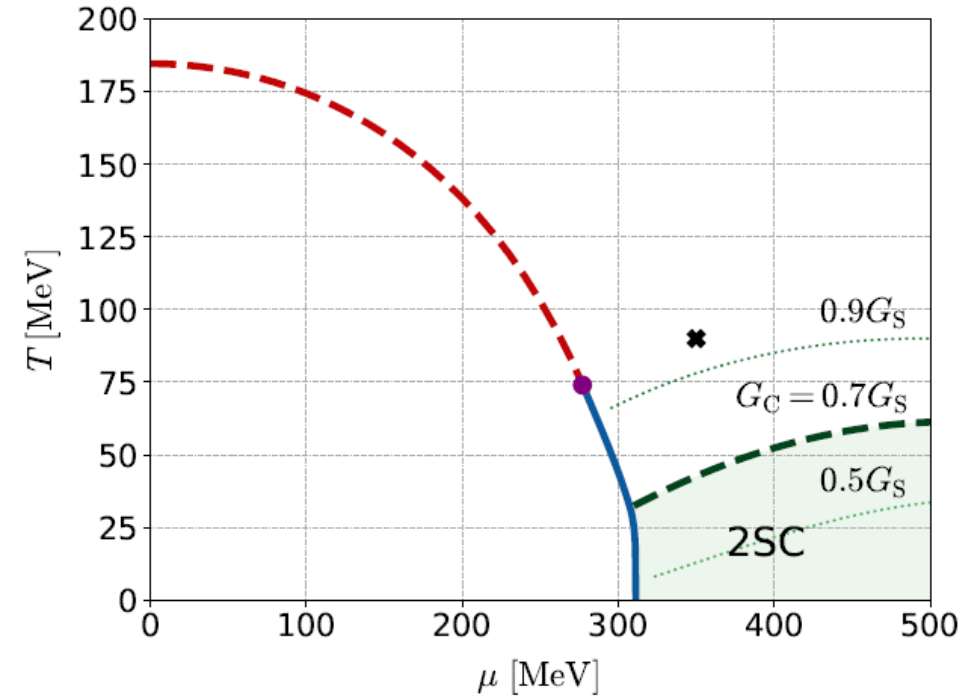
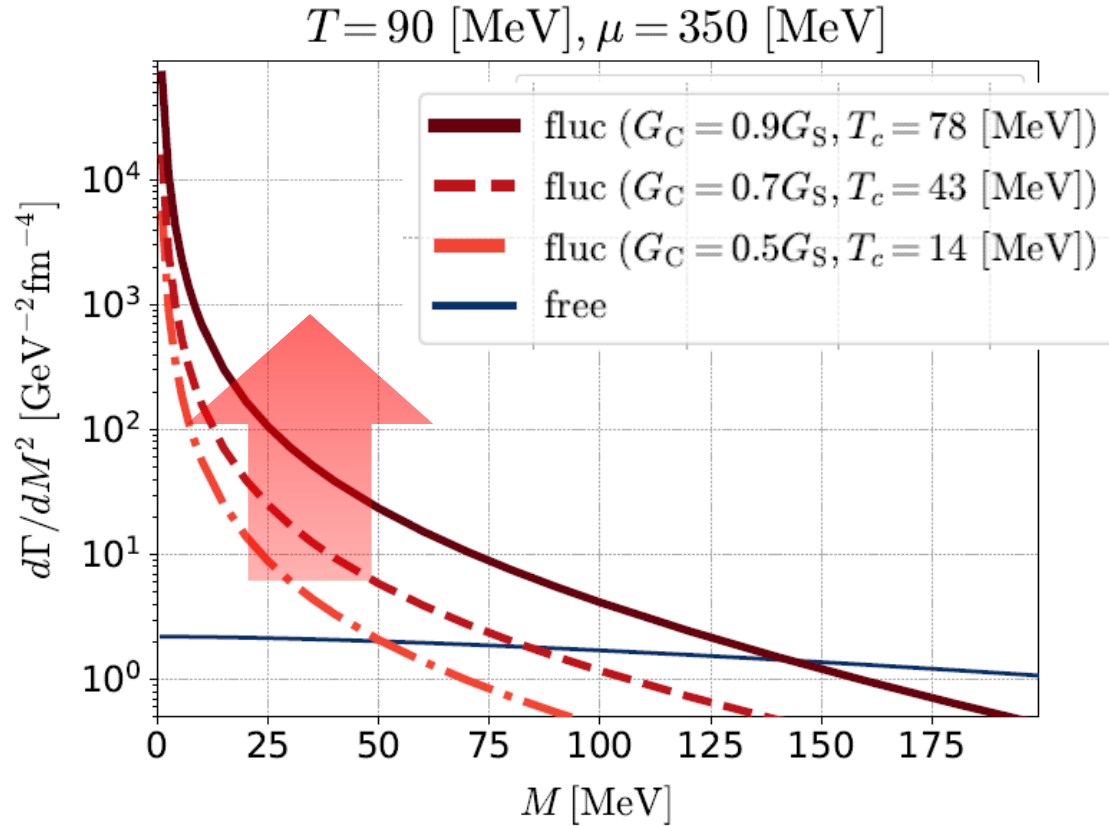
**Blue:** free quarks

$$G_C = 0.7G_S, T_C \simeq 45 \text{ MeV}$$

- Di-quark fluctuations give rise to large enhancement in the low energy region  $\omega < 200$  MeV and  $T < 1.5T_c$ .
- Anomalous enhancement is not sensitive to  $T$ .

# Invariant-Mass Spectrum

Nishimura, MK, Kunihiro ('22)



- ❑ Strong enhancement at low invariant mass.
- ❑ **Observable in the HIC?**

# Dileptons from QCD Critical Point

## NJL model (2-flavor)

$$\mathcal{L} = \bar{\psi}(i\partial - m)\psi + G_S((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2)$$

## Parameters

$$G_S = 5.5 \text{ GeV}^{-2}, \quad \Lambda = 631 \text{ MeV}, \quad m_q = 5.5 \text{ MeV}$$

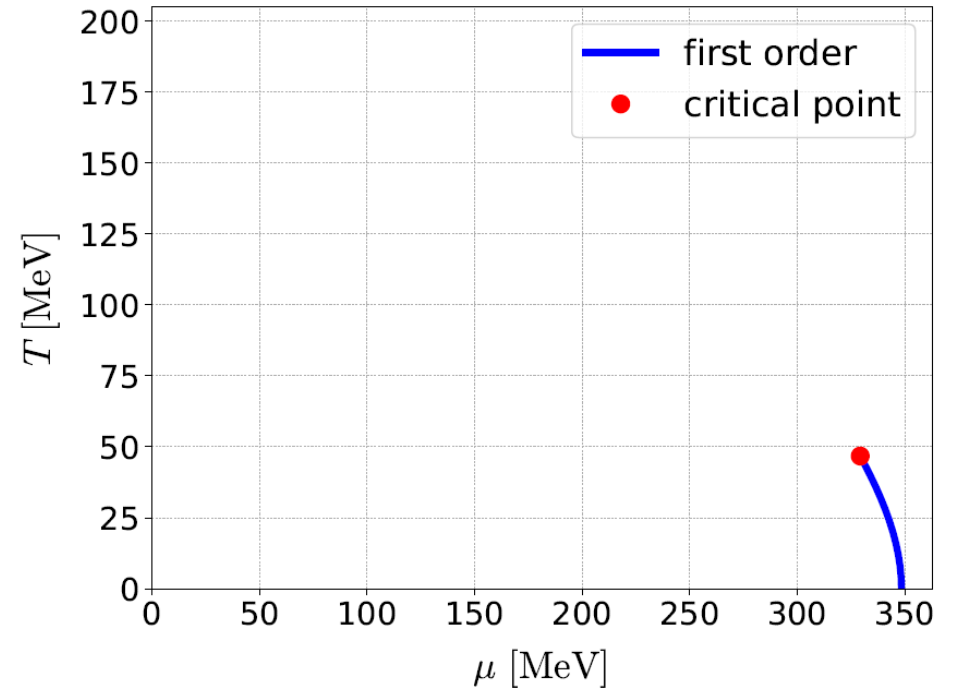
## Soft Mode of QCD-CP

= fluctuation of scalar ( $\bar{q}q$ ) channel

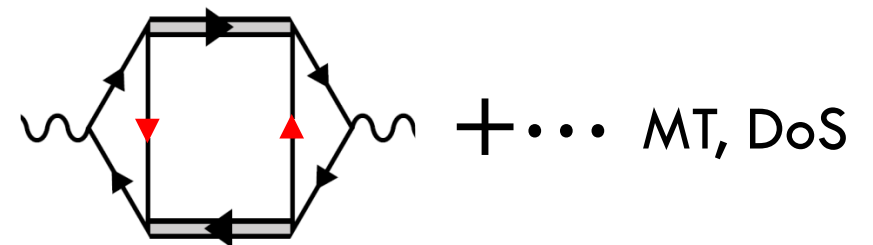
$$D^R(x) = \langle [\bar{\psi}\psi(x), \bar{\psi}\psi(0)] \rangle \theta(t) = \Rightarrow \Rightarrow$$

□ Random Phase Approximation

$$\Rightarrow \Rightarrow = \text{loop} + \text{two-loop} + \dots$$



**Modification of dilepton production through**

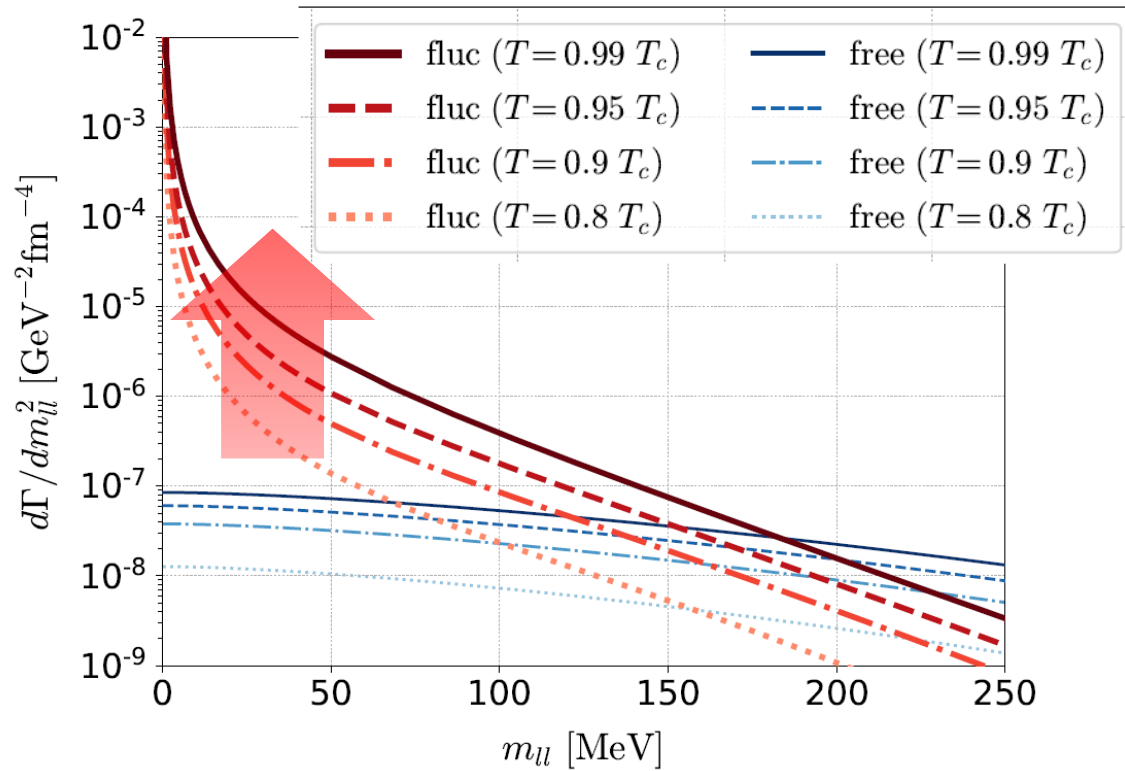




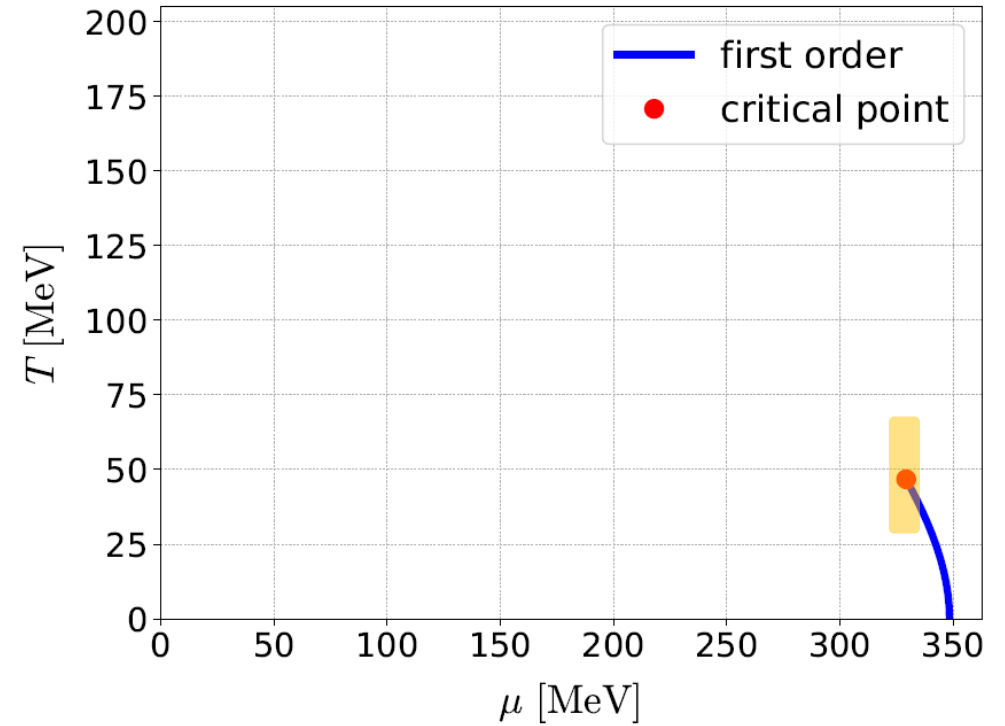
# Dilepton production rate near QCD-CP

Nishimura, MK, Kunihiro ('23)

## Invariant mass spectrum

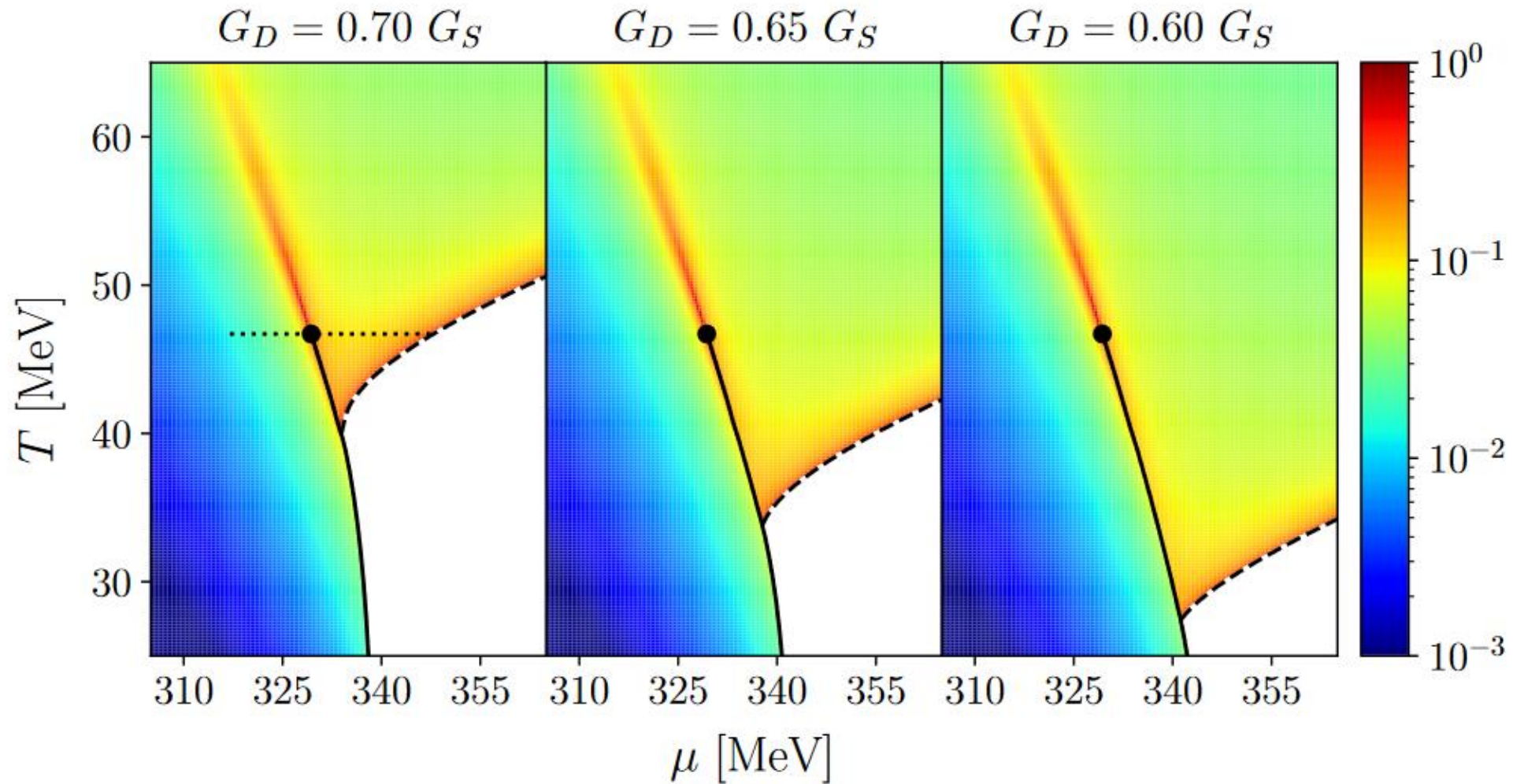


for fixed chem. pot.:  $\mu = \mu_c$



- Enhancement at low  $M_{\ell\ell}$  region near QCD-CP
- Distinguishment from diquark soft mode may be difficult.

# Electric Conductivity on QCD Phase Diagram



- DPR in the low-energy limit = electric conductivity
- Two “hot spots” on the  $T$ - $\mu$  plane

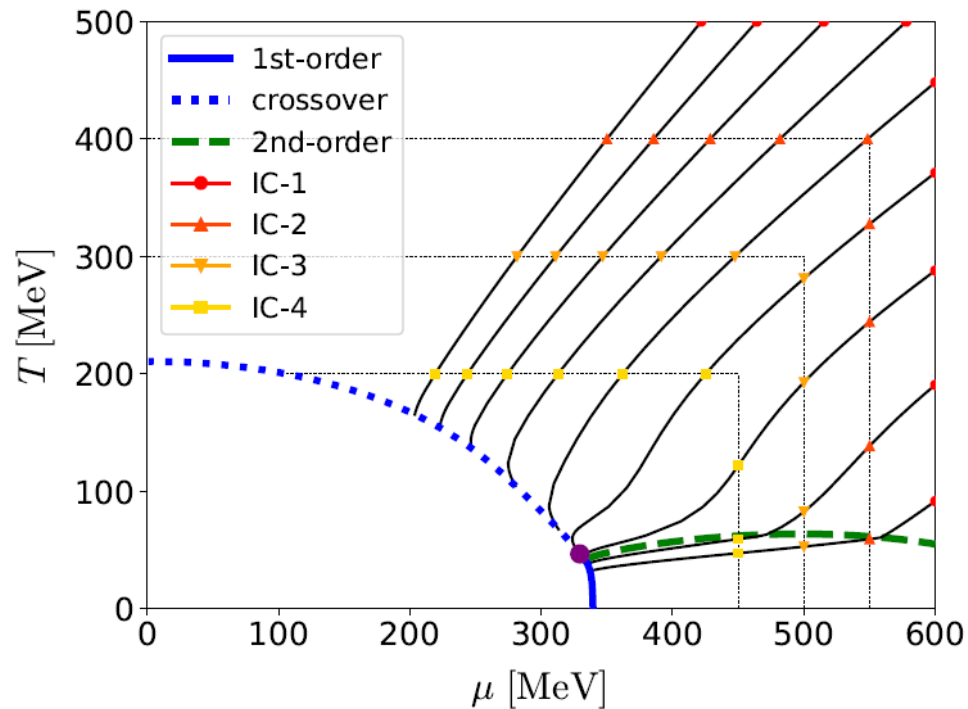
# Dilepton Yields: Beam-Energy Scan

Nishimura, Nara, Steinheimer, Eur.Phys.J.A 60, 2024

Dilepton yields

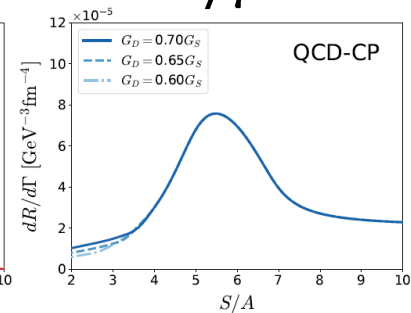
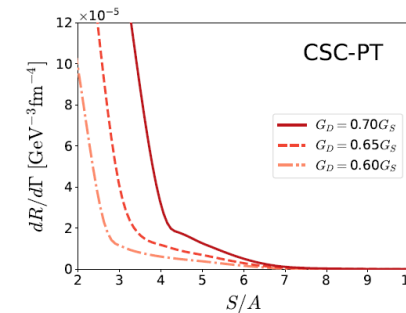
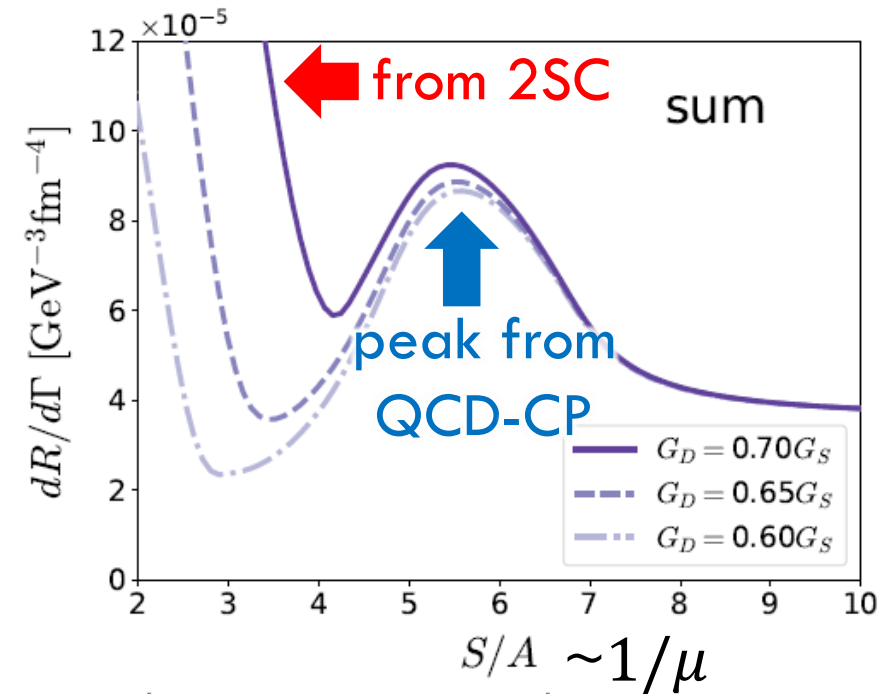
$\approx$  integrated rate along isentropic lines

## Isentropic lines in NJL model



Effect of 1st-tr on evolution: Savchuk+ 2209.05267

## Dilepton Yields $50 < M < 100$ MeV



# Summary

- The beam-energy scan will reveal rich structures on QCD phase diagram, such as the QCD critical point and color superconductivity.
- Quantitative analysis of the size and lifetime of the dense region:
  - $\sqrt{s_{NN}} \simeq 3 \text{ GeV}$  may be an optimal energy to study  $\rho = 3 \sim 4 \rho_0$ .
- Phase transitions in dense quark matter may be detectable through the enhancement of the dilepton production rate at ultra-low-mass-region.