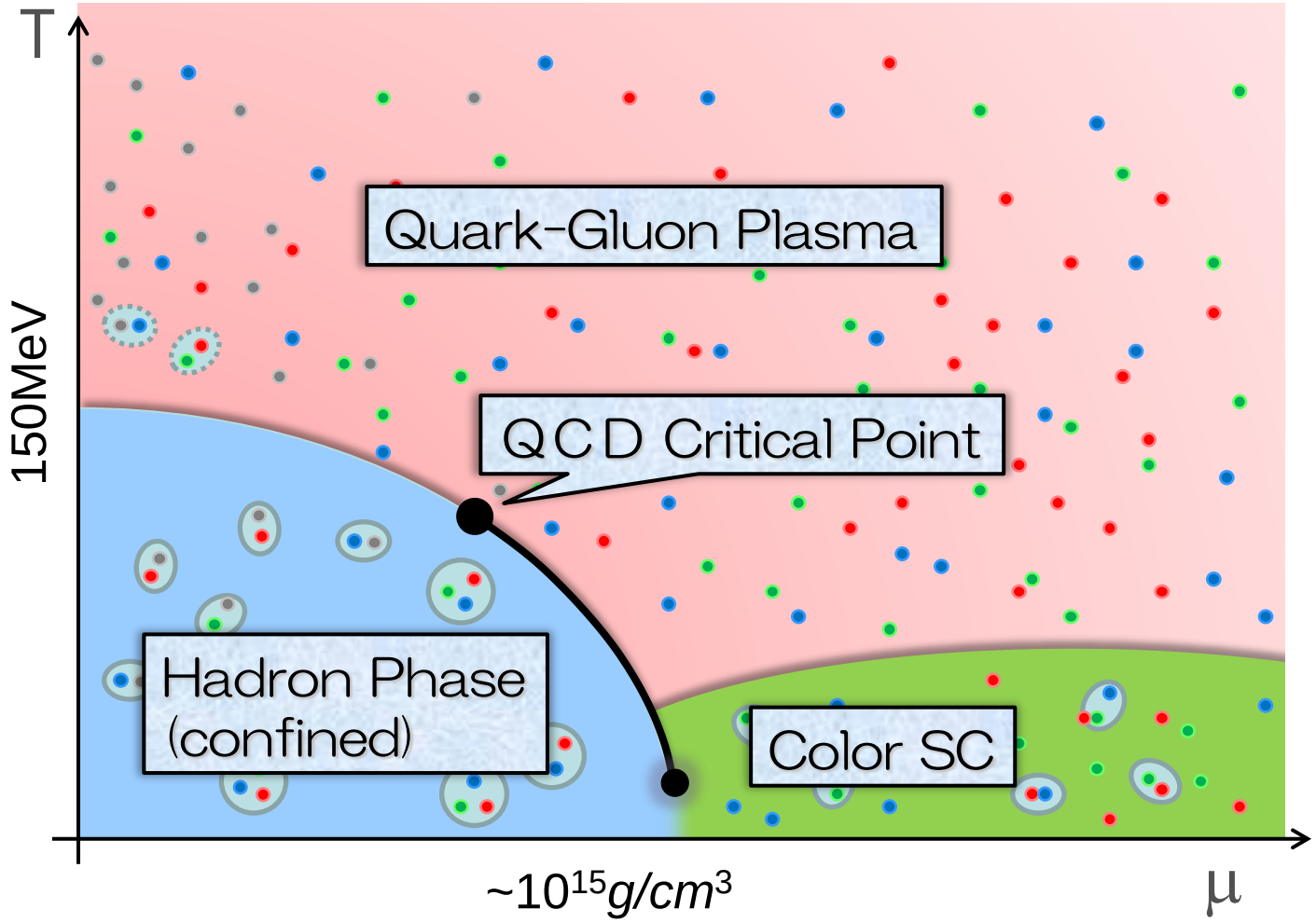
Seminar @ Wroclaw University, 13 Dec, 2024, Wroclaw, Poland

Three topics to explore matter under extreme conditions

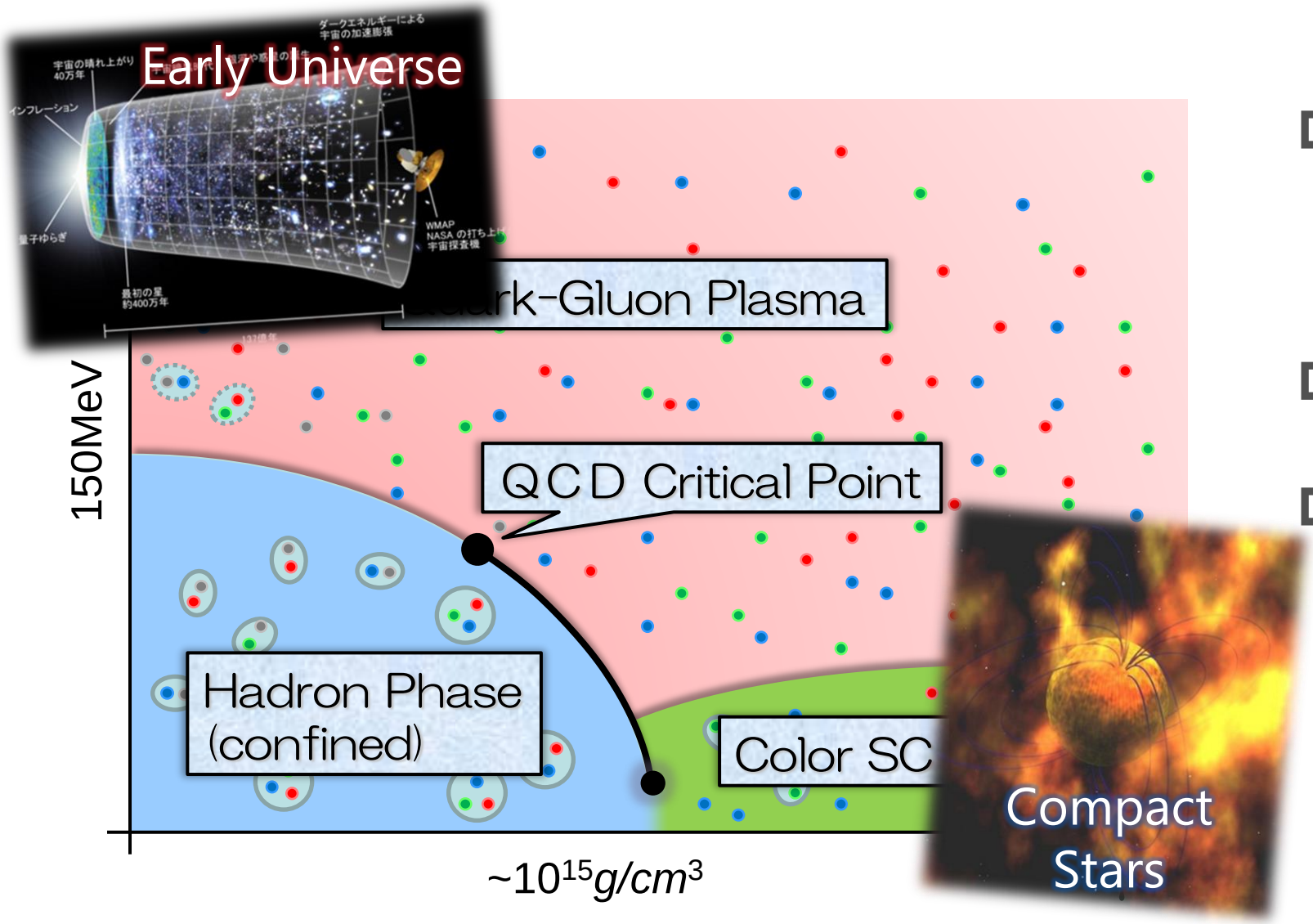
Masakiyo Kitazawa
(YITP, Kyoto)

QCD Phase Diagram



- Possible first-order transition and QCD critical point in dense region
- Multiple QCD-CP? MK+ ('02)
- Color superconducting phases in dense and cold quark matter

QCD Phase Diagram

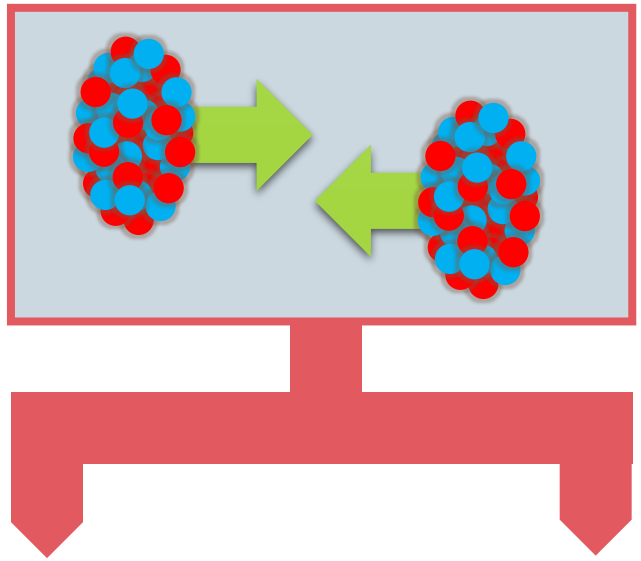


- Possible first-order transition and QCD critical point in dense region
- Multiple QCD-CP? MK+ ('02)
- Color superconducting phases in dense and cold quark matter

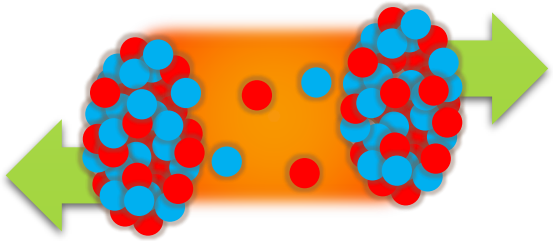
Contents

1. Optimal collision energy for studying dense matter in heavy-ion collisions
   Taya, Jinno, MK, Nara, 2409.07685
2. Dilepton production for a signal of phase transitions in dense matter
   Nishimura, MK, Kunihiro, Ann. Phys. 469 (2024) 169768; PTEP 2023, 053D01; PTEP 2022, 093D02
3. Novel critical points in SU(3) Yang-Mills theory with boundary conditions
   MK+, PRD99 (2019) 094507; Suenaga, MK, PRD107 (2023) 074502; Fujii, Iwanaka, Suenaga, MK, PRD110 (2024) 094016

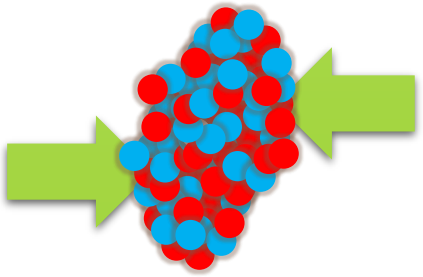
Beam-Energy Scan



High E

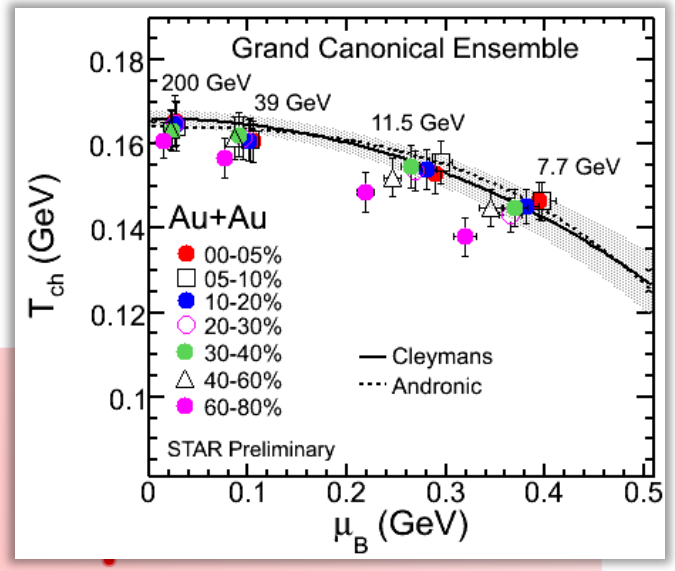


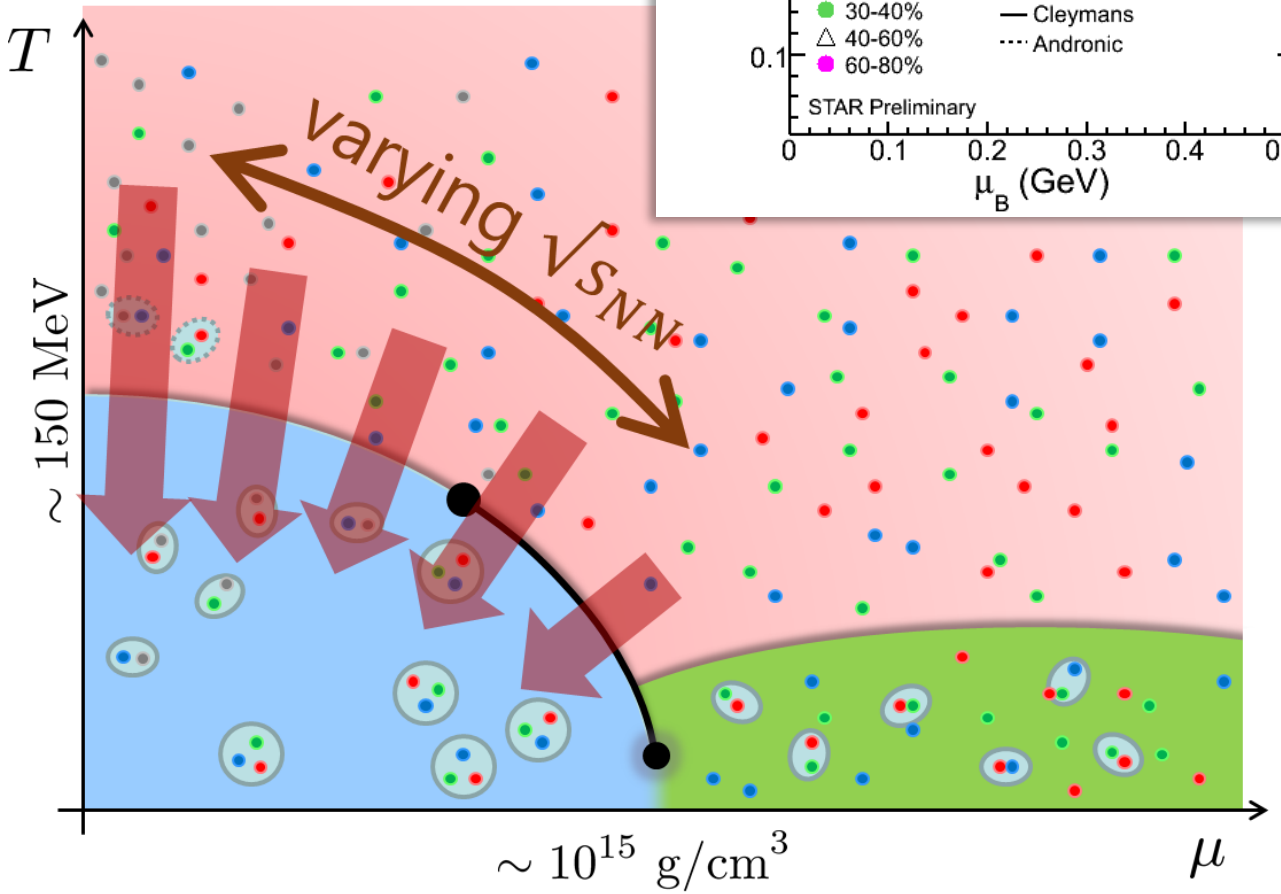
Low E



Baryon stopping

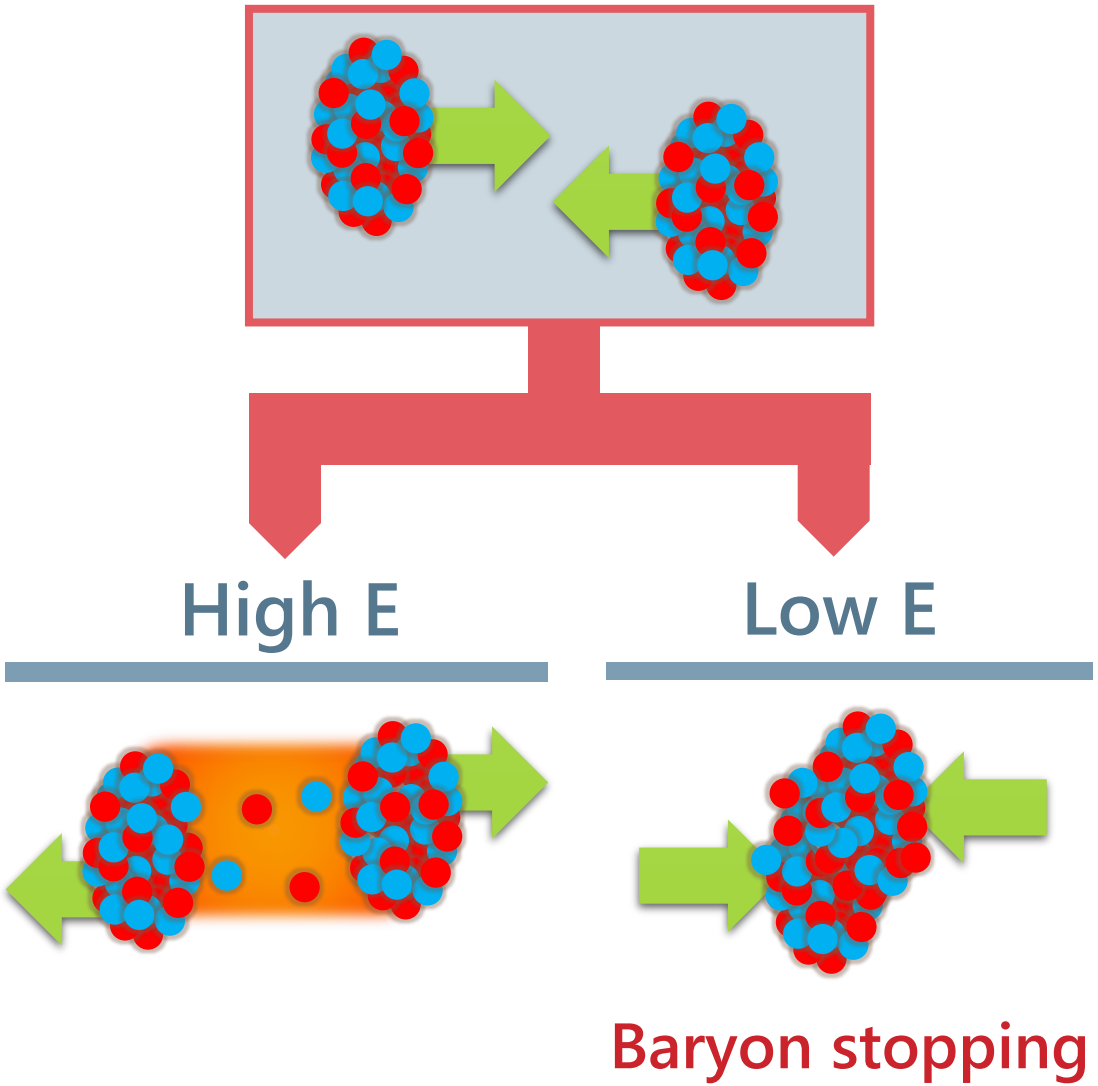
STAR, 2012

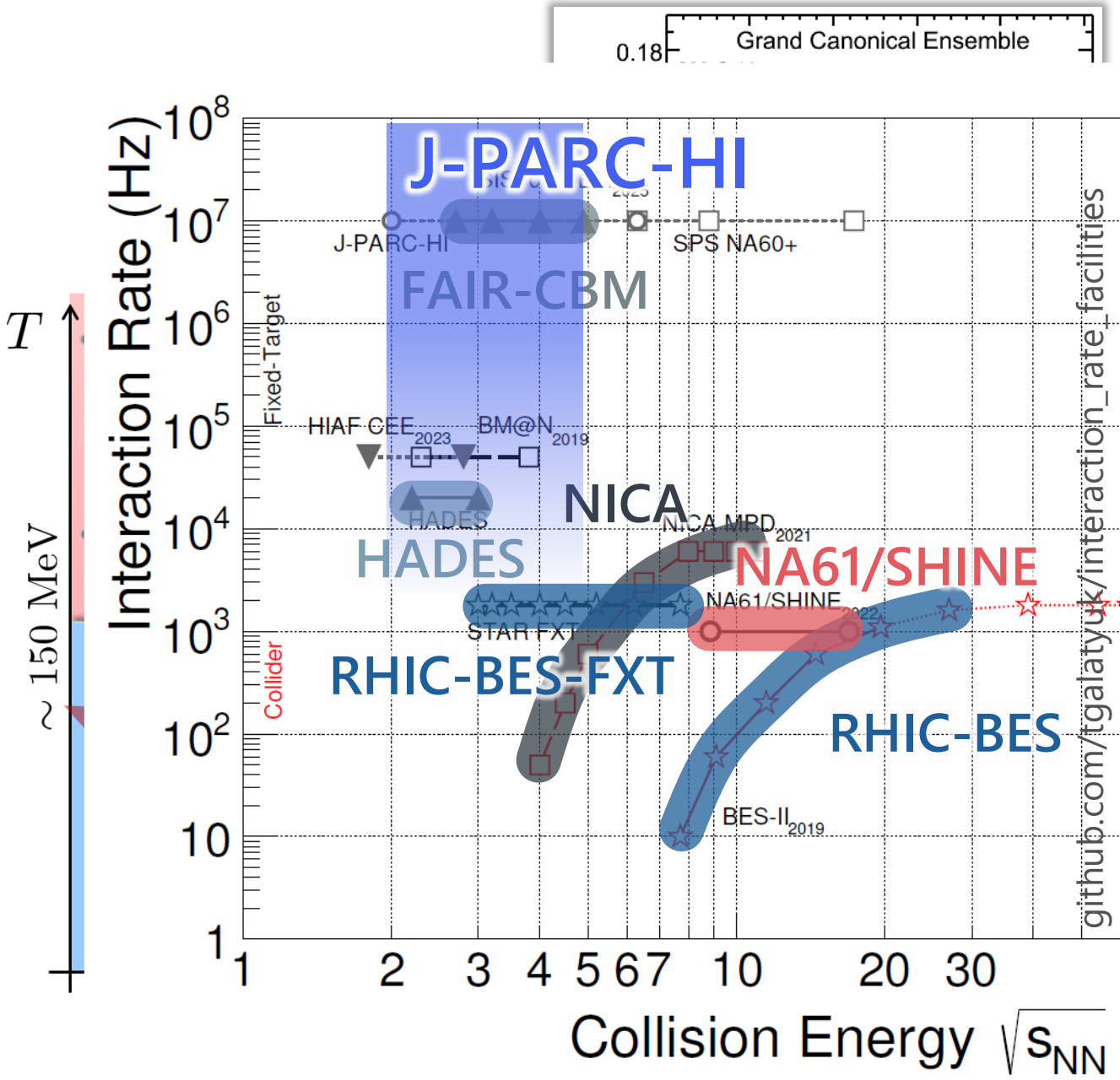




Beam-Energy Scan

STAR, 2012





Key Questions

— What is **the optimal collision energy** to explore the baryon-rich matter?

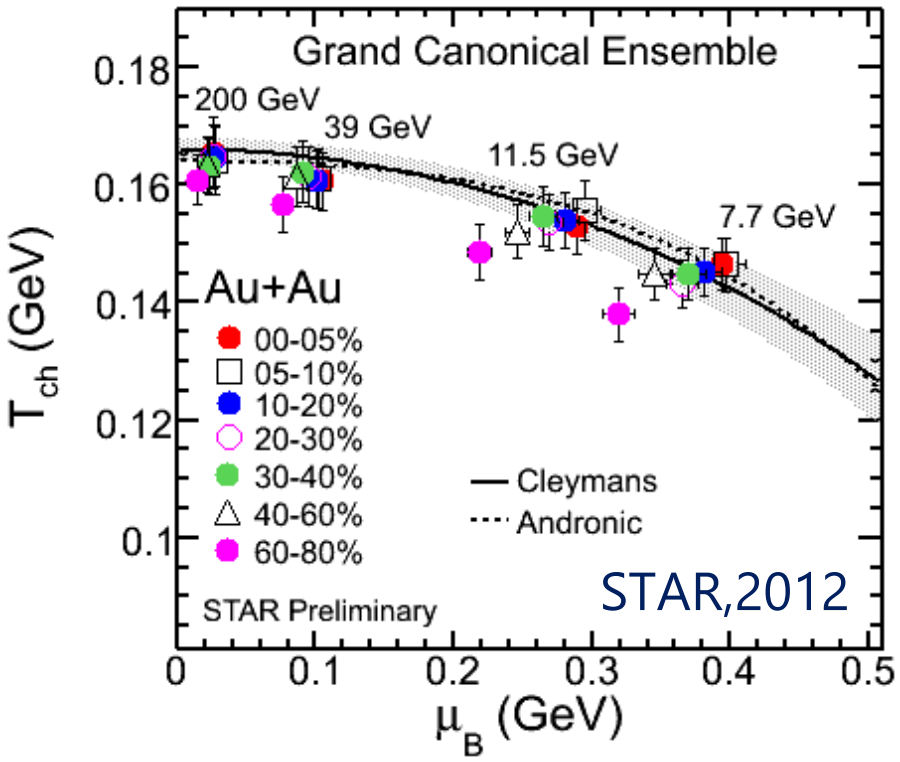
— **How high density** is accessible?

Our Answer

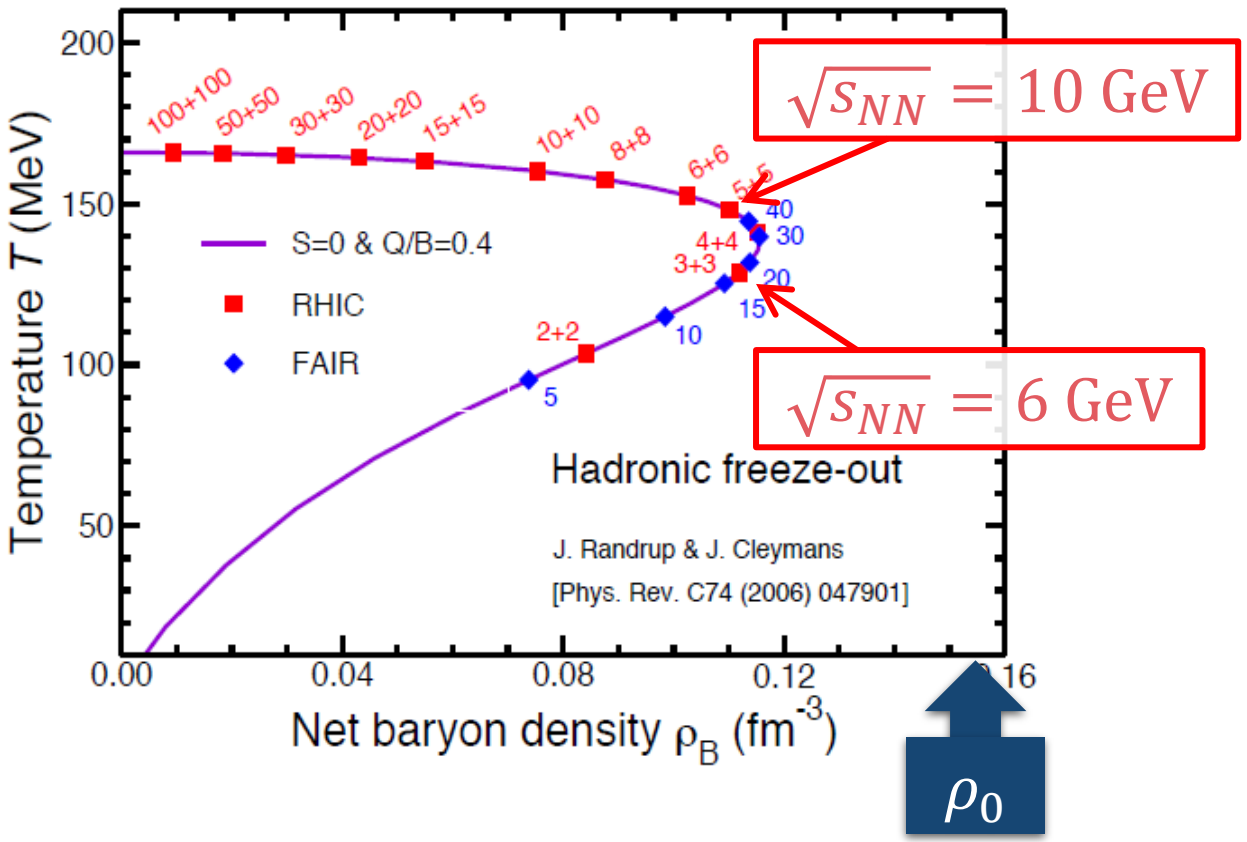
– $\sqrt{s_{NN}} = \mathbf{3\ GeV}$ is enough to study $\rho = 3\rho_0$.

– $\boldsymbol{\rho/\rho_0 = 4\sim5}$ may be accessible with $\sqrt{s_{NN}} = \mathbf{3\sim6\ GeV}$.

Chemical Freezeout

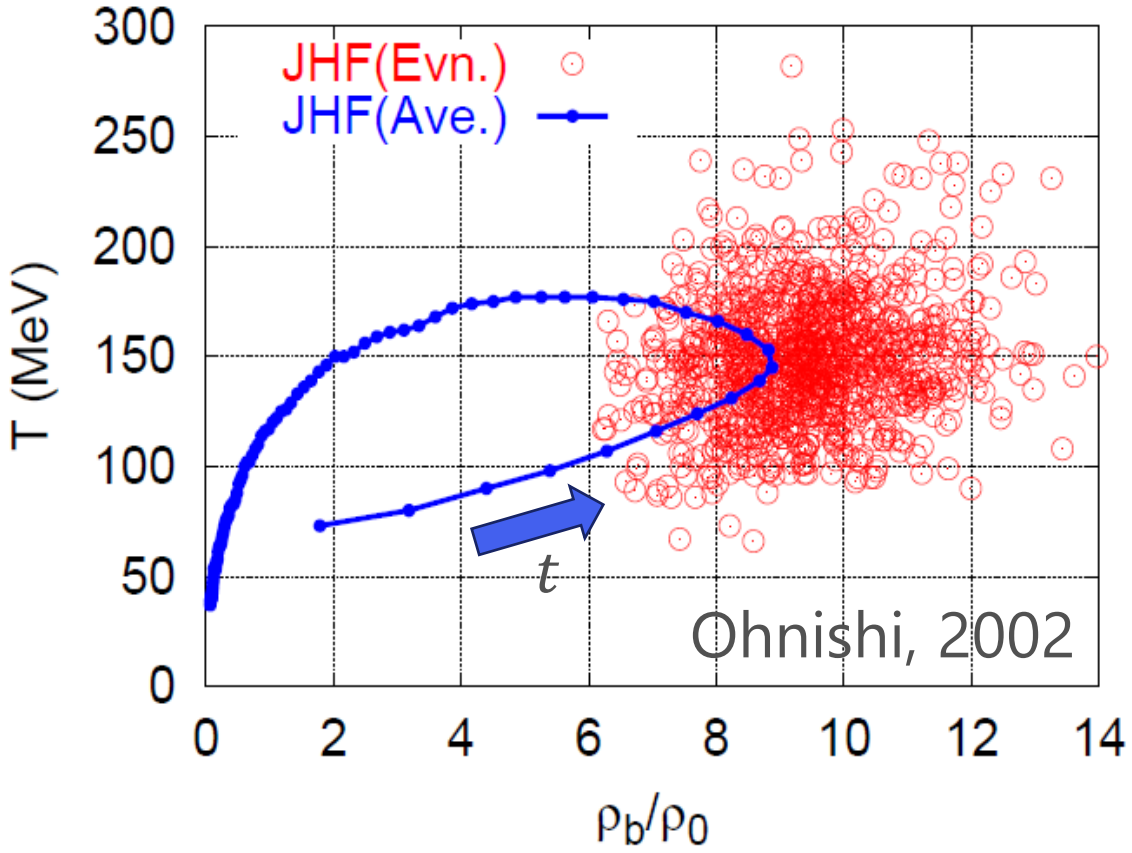


Randrup, Cleymans, '06



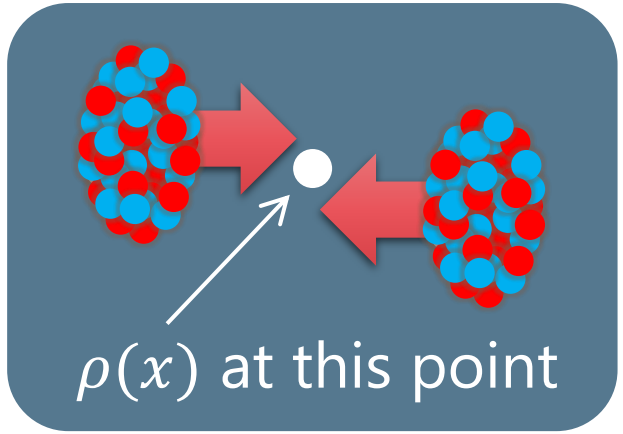
- Highest baryon density **at chemical freezeout** at $\sqrt{s_{NN}} \simeq 6 - 10$ GeV?
- Not the highest density in the early stage.
- Density in earlier stage? ➤ Analysis in dynamical models

Baryon Density at Collision Point



Simulation by JAM

$E/A = 20\mathrm{GeV},\ \sqrt{s_{NN}} \simeq 6\mathrm{GeV}$



— Maximum baryon density exceeds $\rho/\rho_0 \simeq 8$!
— Large event-by-event fluctuations
— How large is the high-density region? How long is the lifetime?

Volume of Dense Region

Taya, Jinno, MK, Nara, 2409.07685

Volume where the local baryon density is larger than a threshold value $\rho_{\rm th}$

$$V_3(\rho_{\rm th}, t) = \int_{\rho(x)>\rho_{\rm th}} d^3\boldsymbol{x}\gamma$$

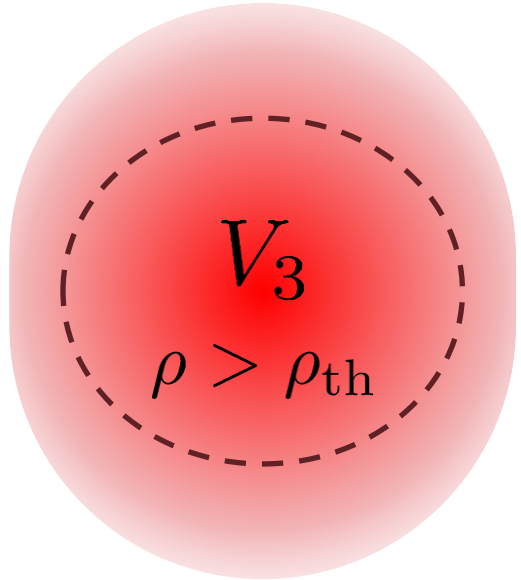
Baryon current $J^\mu(x)$

Baryon density $\rho(x) = \sqrt{J^\mu(x) J_\mu(x)}$

Lorentz factor $\gamma = (1 - (\boldsymbol{J}/J_0)^2)^{-1/2}$

Note:

- Event-by-event basis / no event average
- Directly calculable in a dynamical model
- We do not care about local thermalization.
  - V_3 is the upper limit of thermalized volume.
  - Even non-thermal, dense region is interesting!



Simulation Setup in JAM

- Au+Au collision for $2.4 \leq \sqrt{s_{NN}} \leq 20$ GeV
- Impact parameter $b \leq 3$ fm : top 5% centrality

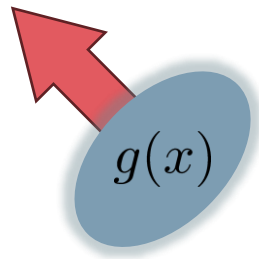
- Momentum-dependent mean field (MF2) Nara, Ohnishi, 2022
  - Setup reproducing $\sqrt{s_{NN}}$ dep. of $dv_1/d\eta$ and v_2

Smeared baryon current

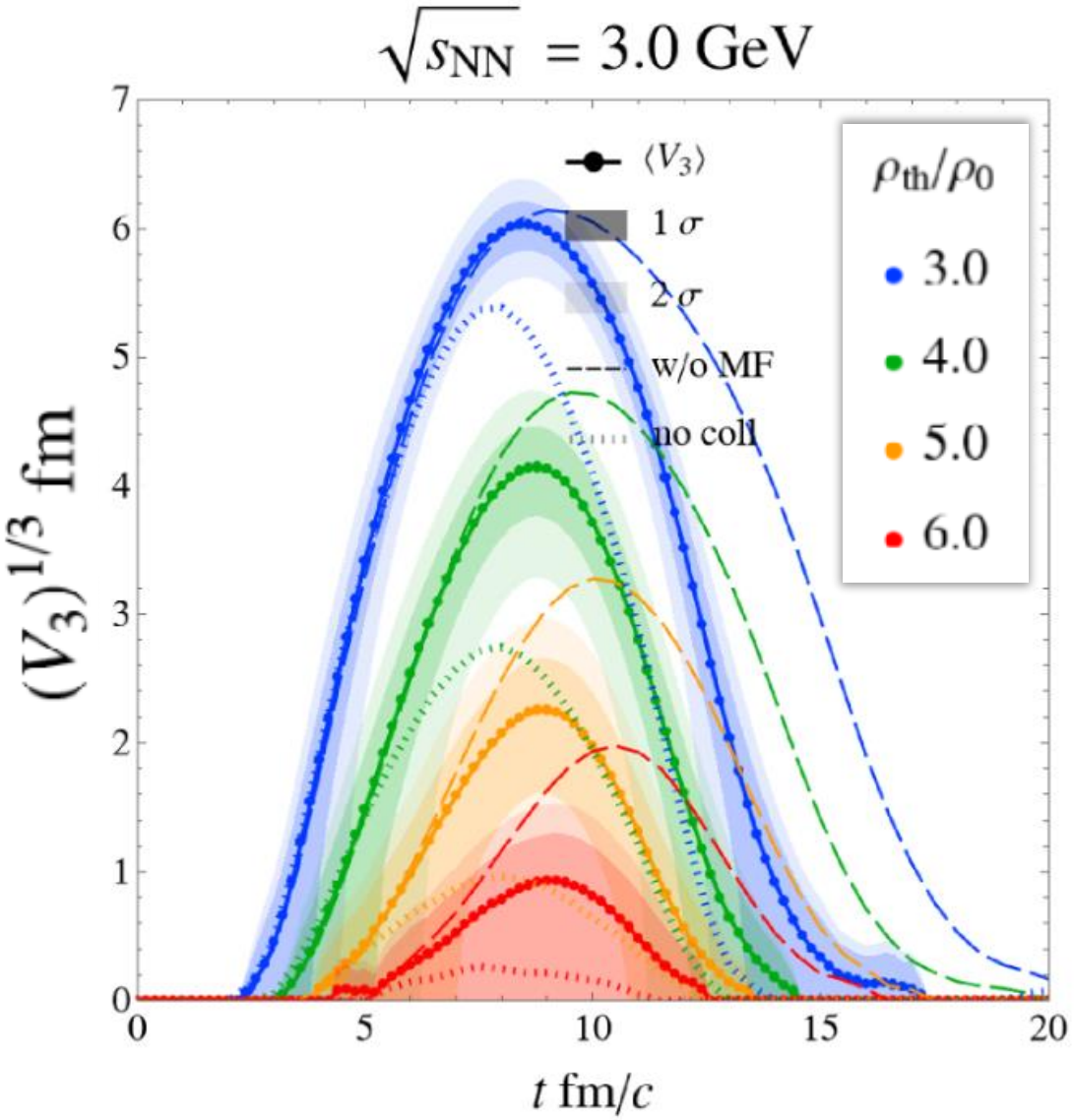
discrete particle distribution ➔ continuous current by smearing

$$J^\mu(x) = \sum_{i \in \text{baryons}} B_i g(x; X_i, P_i) \frac{P_i^\mu}{P_i^0}$$

$$g(x; X, P) := \frac{\gamma}{(\sqrt{2\pi} r)^3} \mathrm{e}^{-\frac{|\boldsymbol{x}-\boldsymbol{X}|^2 + (\gamma \boldsymbol{V}\cdot(\boldsymbol{x}-\boldsymbol{X}))^2}{2r^2}} \qquad r = 1 \text{ fm}$$



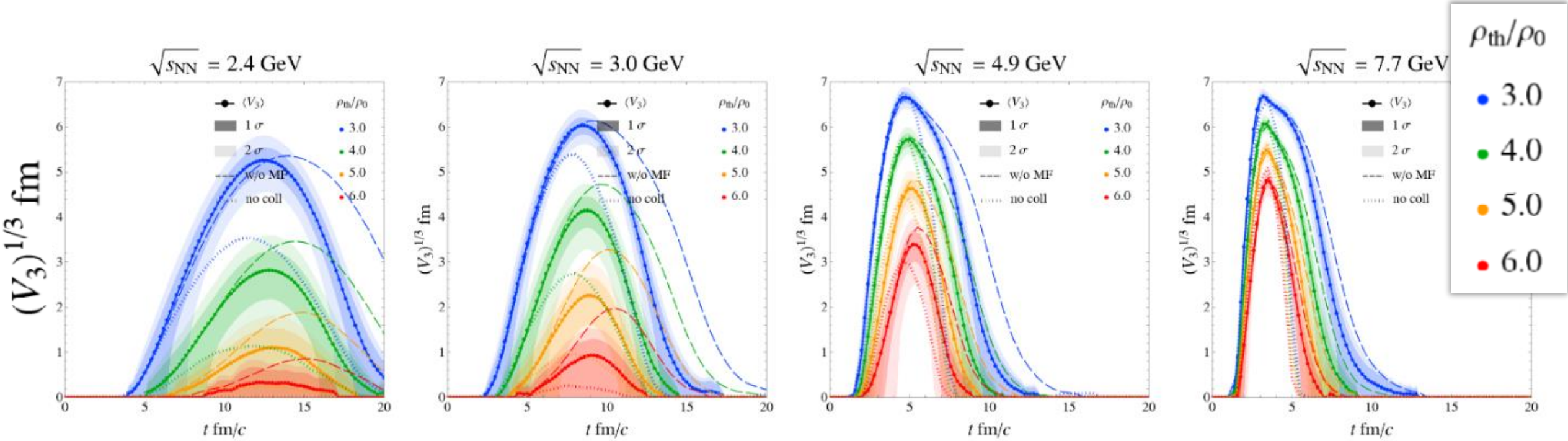
V_3 in JAM



— **solid: JAM+MF** Nara, Ohnishi, 2022
— shaded band: 1σ and 2σ e-v-e fluct.
— dashed: JAM cascade mode
— dotted: no-collision

- Formation of dense region:
  - $V_3(3\rho_0, t) = (6\ \mathrm{fm})^3$
  - $V_3(4\rho_0, t) = (4\ \mathrm{fm})^3$
- Large e-v-e fluctuations
  - ➔separable by event selection?
- Repulsive MF ➔ weaker compression
- Compression owing to interaction

V_3 for various $\sqrt{s_{NN}}$



As $\sqrt{s_{NN}}$ becomes larger,

- $\max V_3(\rho_{\mathrm{th}}, t)$ becomes larger.
- The lifetime of dense region becomes shorter.
- E-v-e fluctuations are more suppressed.

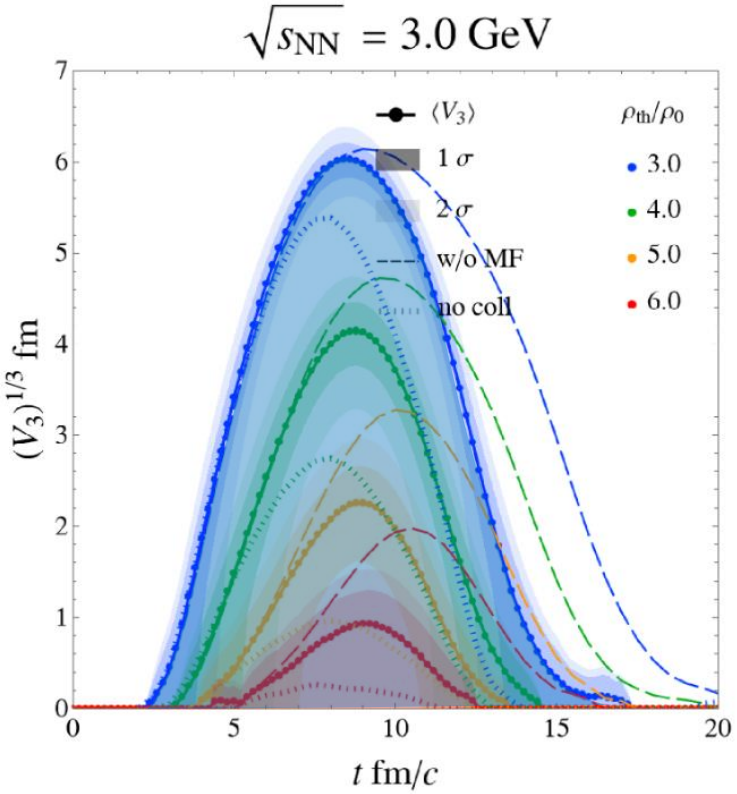
Four-Volume / Lifetime

Four Volume

$$V_4(\rho_{\rm th}) = \int_{-\infty}^{\infty} dt \int_{\rho(x)>\rho_{\rm th}} d^3\boldsymbol{x}$$

Lifetime

$$\tau(\rho_{\rm th}) = \frac{V_4(\rho_{\rm th})}{\max V_3(\rho_{\rm th}, t)}$$

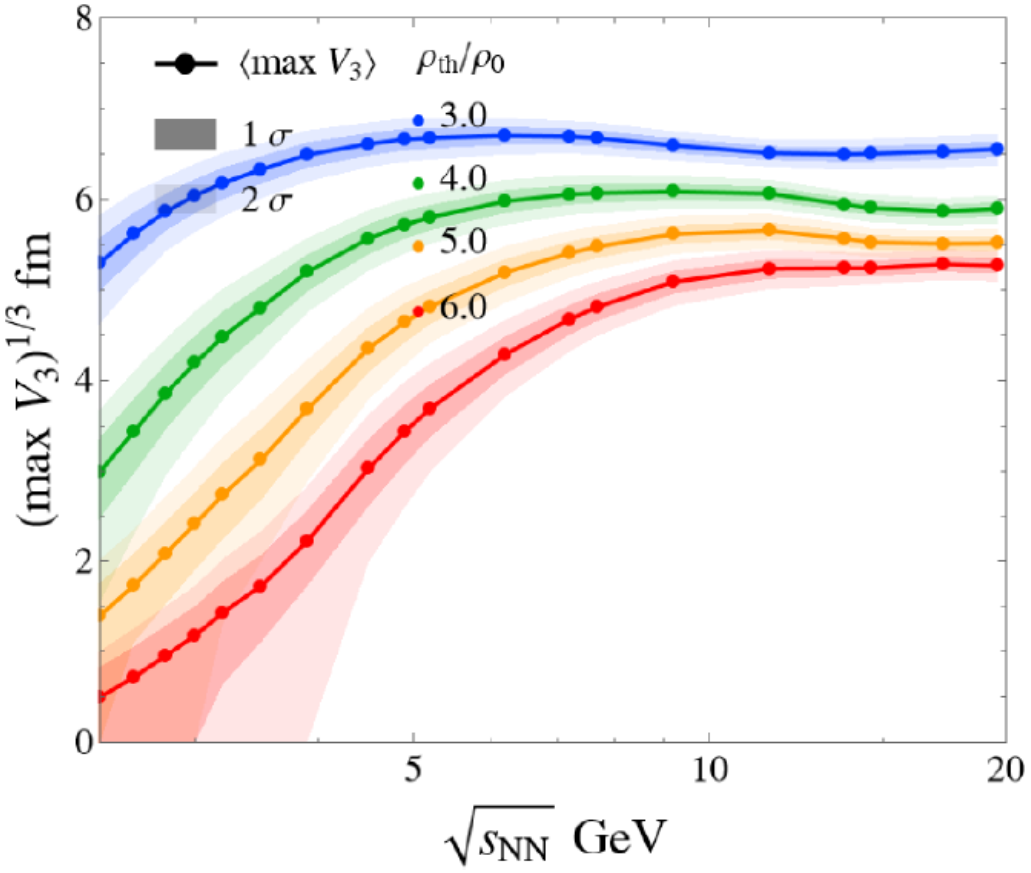


Note

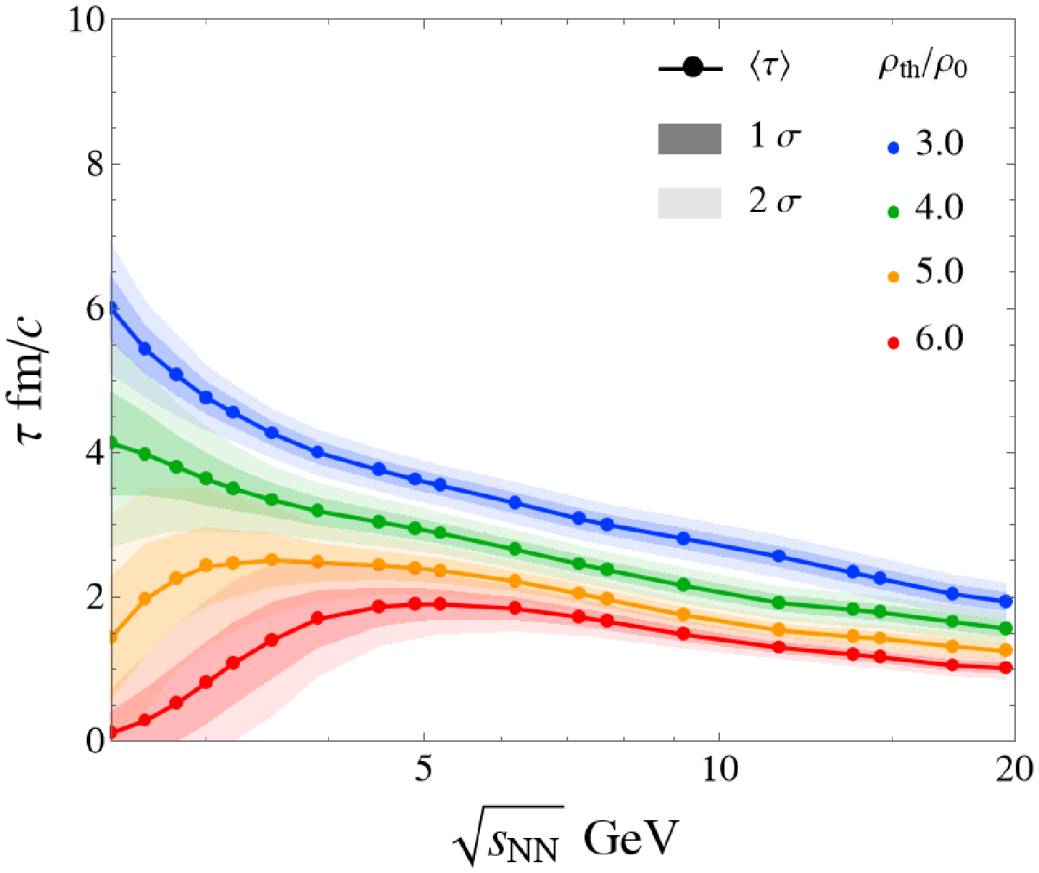
V_4 may be relevant for the dilepton production rate.

$\sqrt{s_{NN}}$ Dependence

max V_3

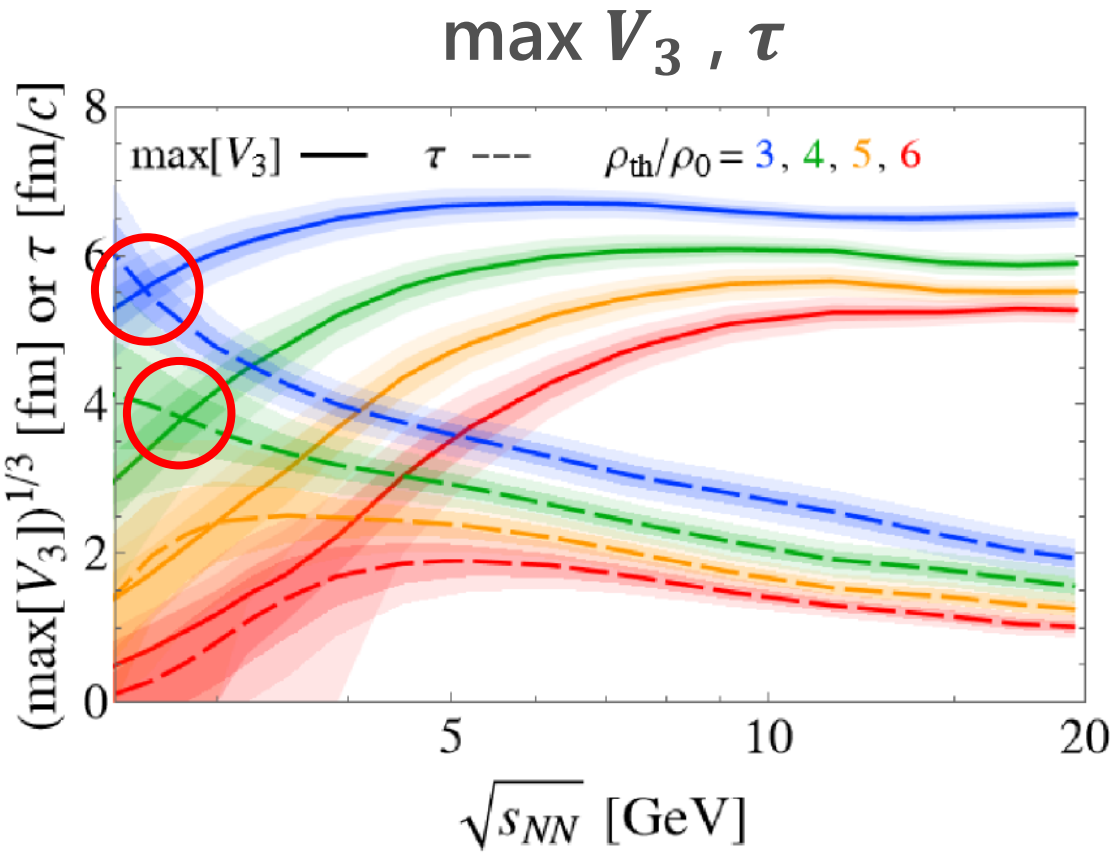


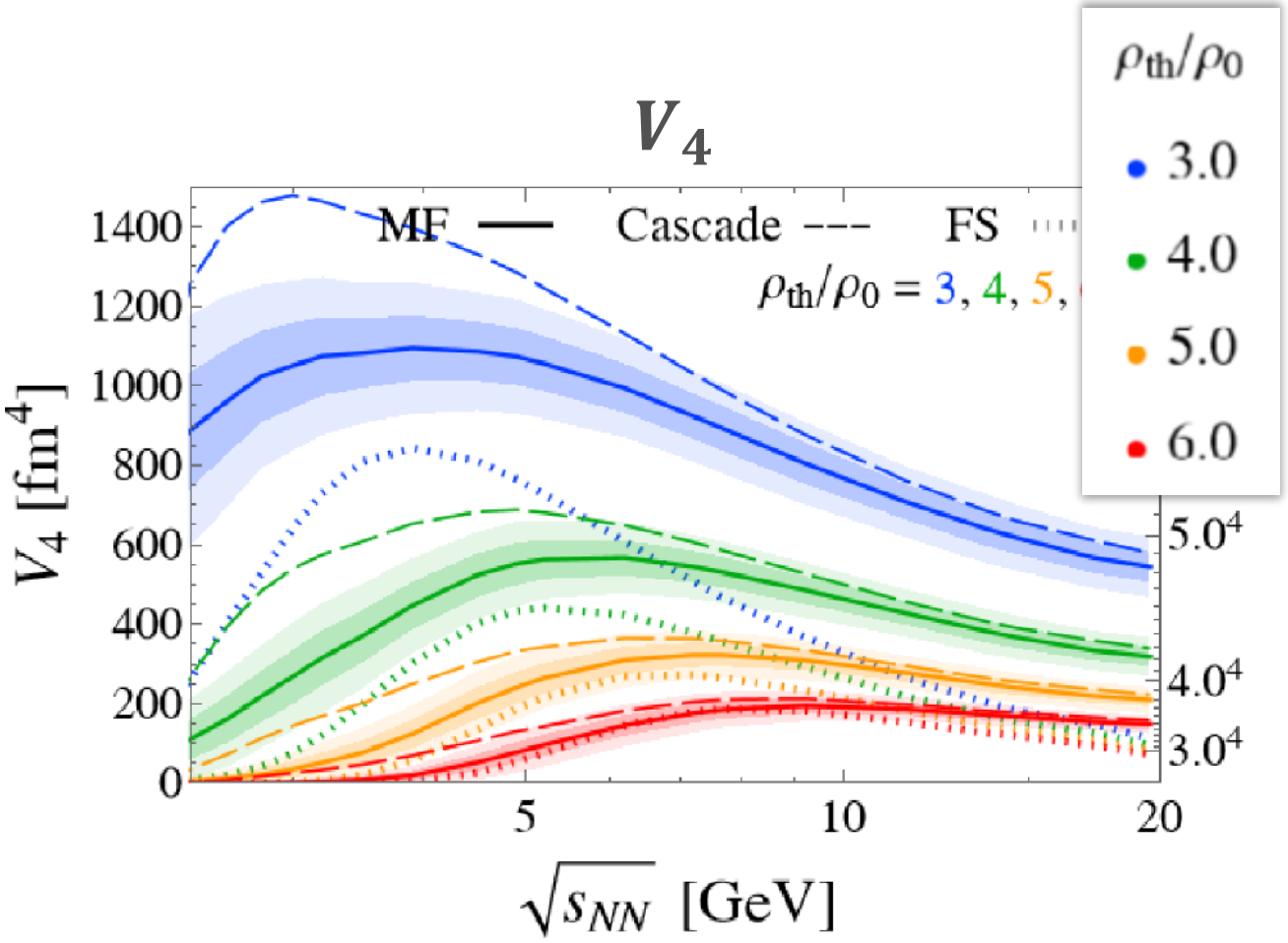
Lifetime



ρ_{th}/ρ_0
- 3.0
- 4.0
- 5.0
- 6.0

$\sqrt{s_{NN}}$ Dependence

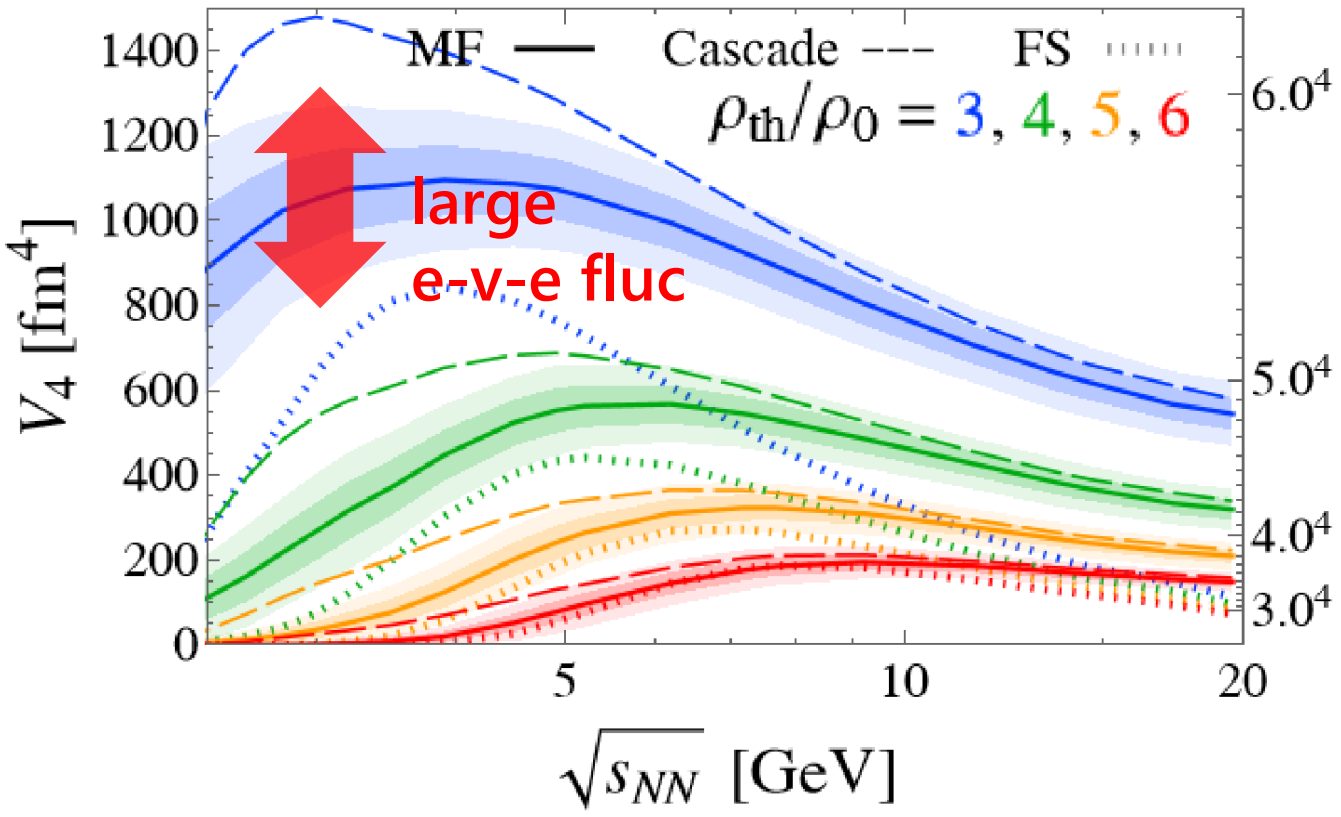




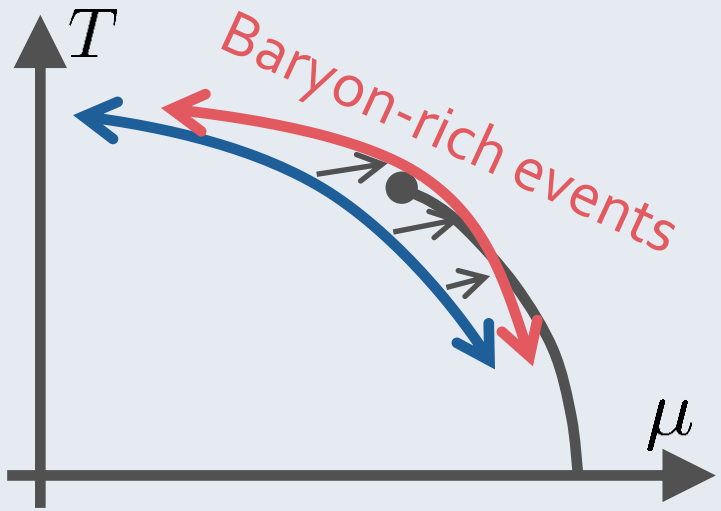
- $\sqrt{s_{NN}} \simeq 3$ GeV would be the best energy to create $\rho = 3{\sim}4\rho_0$ with large V_3 and τ.
- Lower $\sqrt{s_{NN}}$ is suitable to create colder matter.

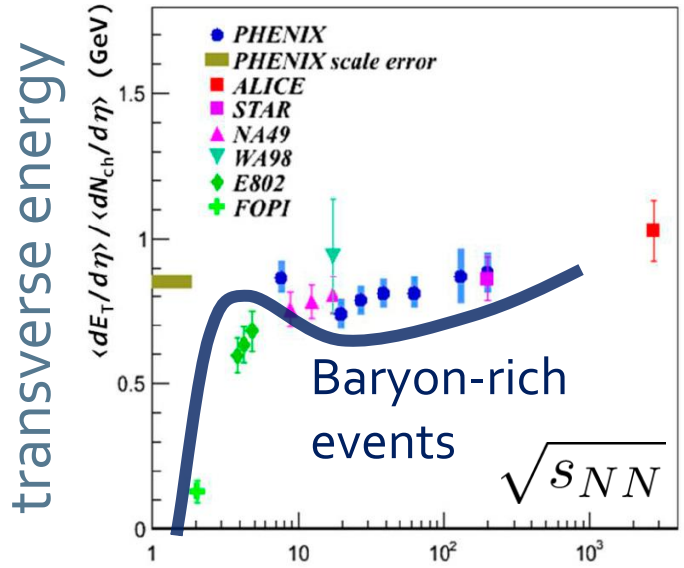
Event Selection

Taya, Jinno, MK, Nara, 2409.07685



Density Scan?





- Event selections via highest baryon/energy density will allow us a detailed study of QCD phase diagram.

Summary

- $\sqrt{s_{NN}} = \mathbf{3\ GeV}$ is enough to study $\rho = 3\rho_0$.
- $\boldsymbol{\rho/\rho_0 = 4\sim5}$ may be accessible with $\sqrt{s_{NN}} = \mathbf{3\sim6\ GeV}$.

Future

- Check model independence
  - Analyses in various models
- Experiments at the sweet spot $\sqrt{s_{NN}} = \mathbf{2.5\sim6\ GeV}$
  - Future exps. at FAIR, NICA, HIAF, & J-PARC-HI

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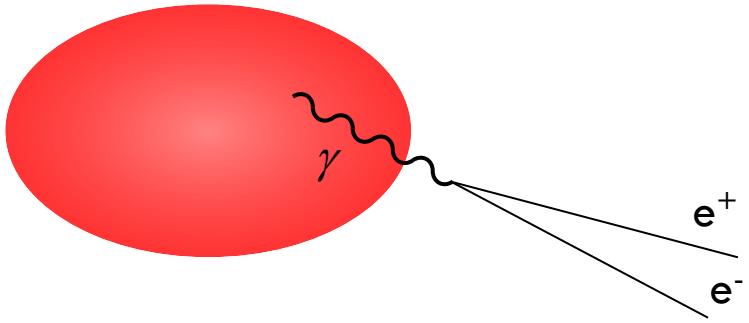
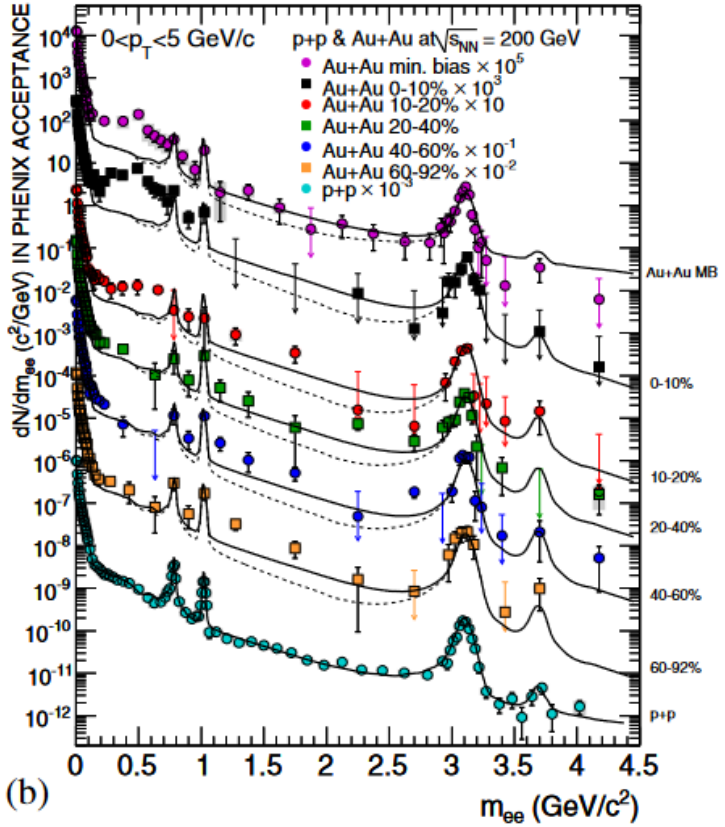
Dilepton Production Rate



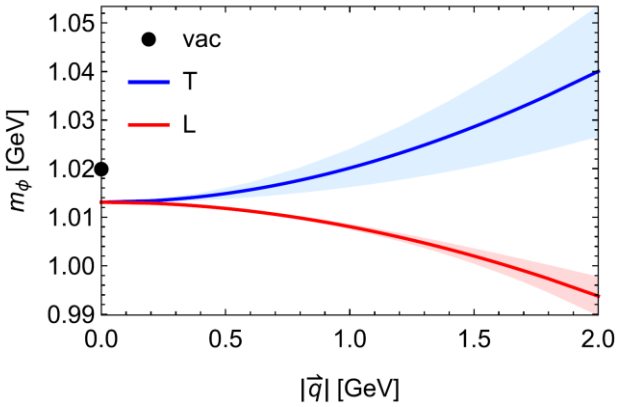


PHENIX, '10

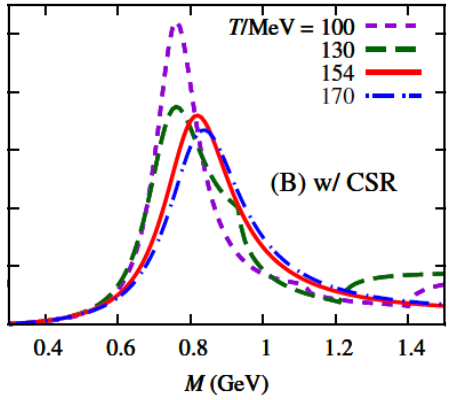
- Generated by the decay of virtual photons
- Carry information of primordial medium

Physics accessible with DPR

- Medium temperature
- Dispersion relations
- Chiral mixing by chiral restoration
- Signal of phase transitions



Kim, Gubler, 2020



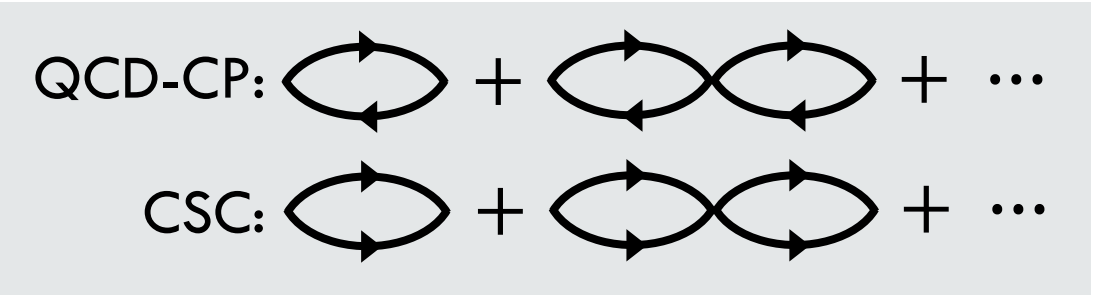
Sakai+, 2024

Soft Modes of Second-order Phase Transitions

Soft modes

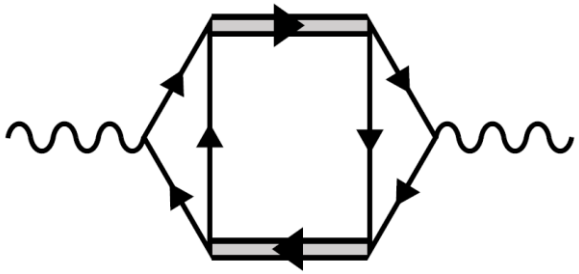
- Divergence of the order-parameter fluctuations at a 2nd-order transition.
- Collective fluctuations become massless there.

- QCD-CP : density-density fluctuations
- CSC : diquark-pair field



Coupling of soft modes with dynamical observables

- Ex.: dilepton production rate

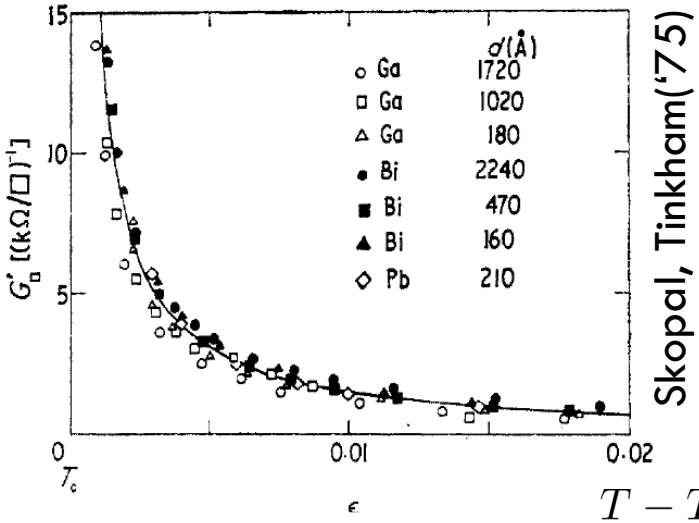


Precursor in Metallic Superconductors

- **Anomalous behavior of observables near but above Tc of SC**
  - electric conductivity
  - magnetic susceptibility
  - pseudogap

- Enhanced pair fluctuations is one of the origins of precursory phenomena.
- More significant phenomena in strongly-coupled systems.

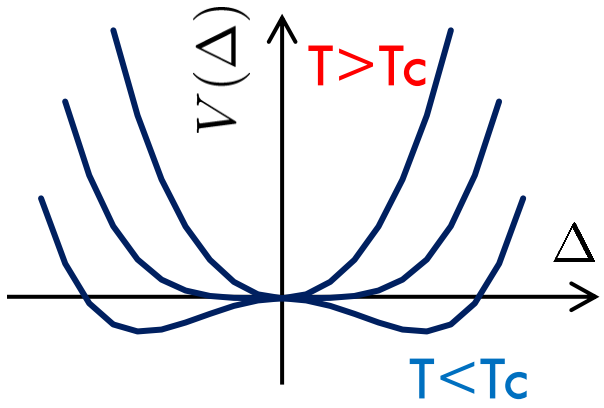
Electric conductivity



Skopal, Tinkham('75)

$$\varepsilon = \frac{T - T_c}{T_c}$$

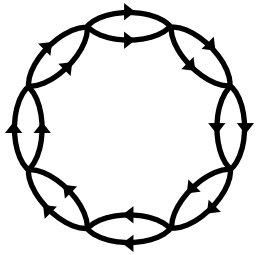
Landau's free energy



Precursor of Color Superconductivity

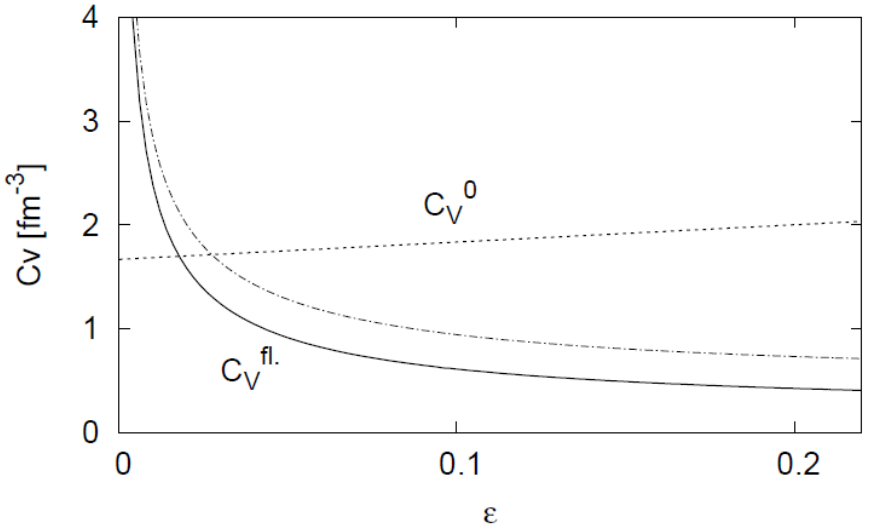
MK, Koide, Kunihiro, Nemoto, '03, '05

☐Thermodynamic Potential

$\Omega =$ 

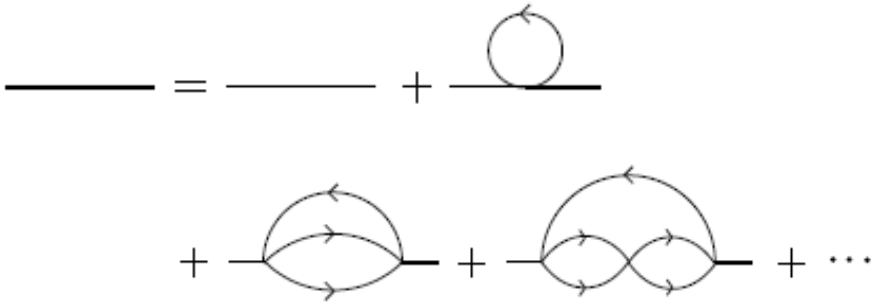
Specific heat

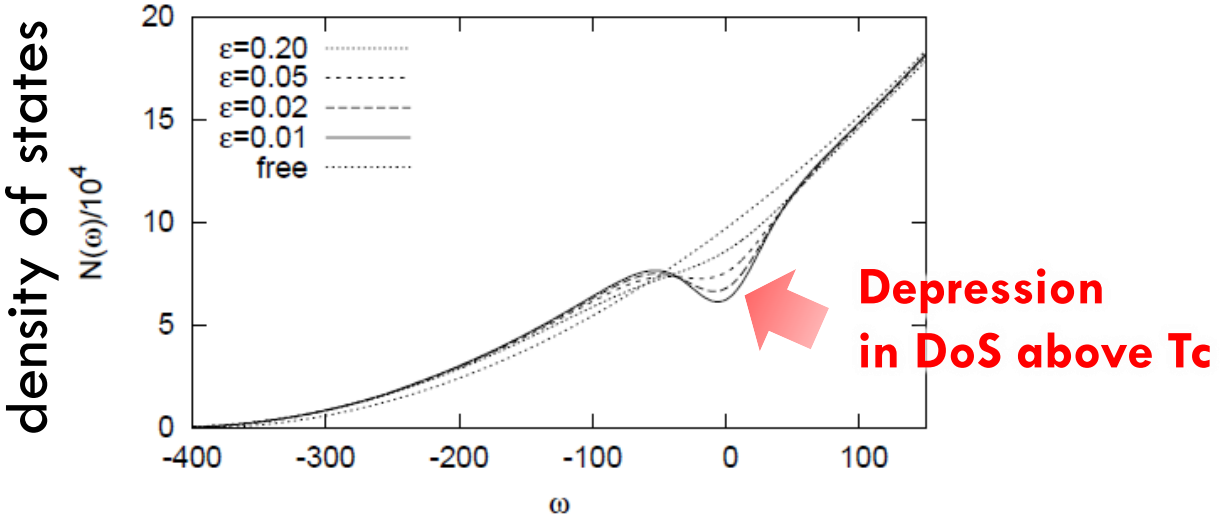
$$c = -T\frac{\partial^2 \Omega}{\partial T^2}$$



$$\varepsilon = \frac{T - T_c}{T_c}$$

☐Pseudogap





Depression in DoS above Tc

Model

NJL model (2-flavor)

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \mathcal{L}_S + \mathcal{L}_C$$

$$\mathcal{L}_S = G_S \left((\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \tau \psi)^2 \right)$$

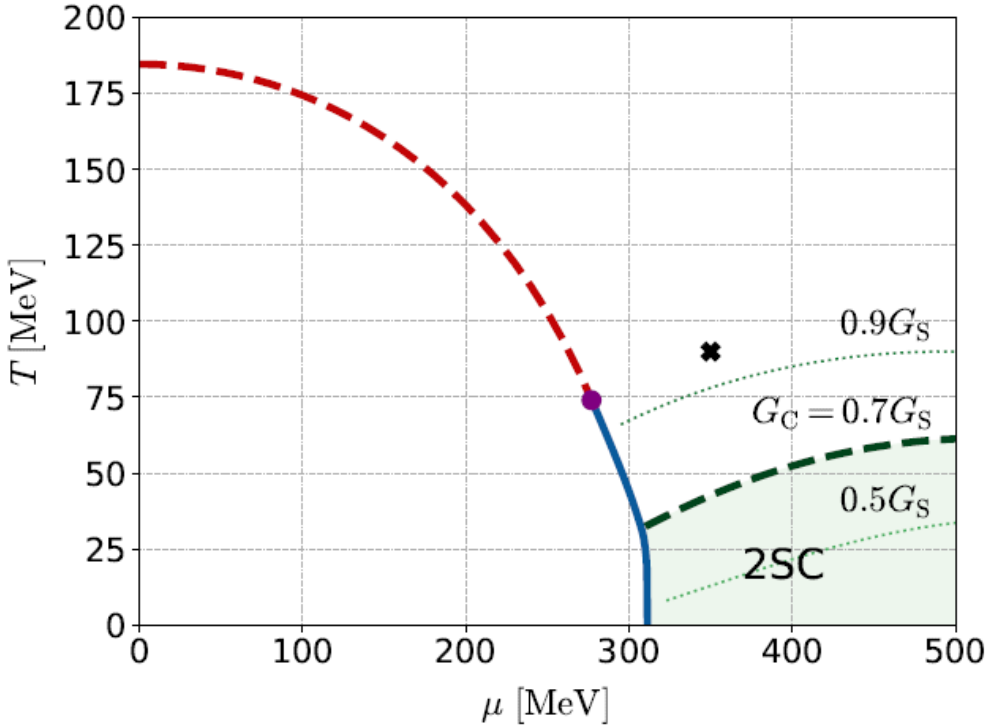
$$\mathcal{L}_C = G_C \left((\bar{\psi} i \gamma_5 \tau_A \lambda_A \psi^C)(\text{h.c.}) \right)$$

diquark interaction

Parameters

$$G_S = 5.01\ \text{GeV}^{-2}, \quad \Lambda = 650\text{MeV}, \quad m_q = 0$$

Phase Diagram in MFA



- Order of phase transition
  - 2nd in the MFA
  - can be 1st due to gauge fluctuation

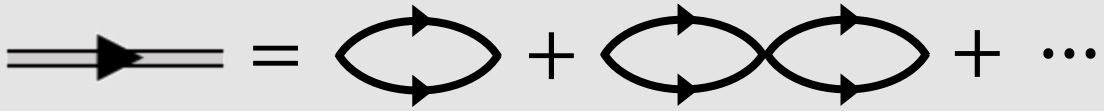
Matsuura+('04), Giannakis+('04)
Noronha+('06), Fejos, Yamamoto('19)

Di-quark Fluctuations

Diquark Propagator

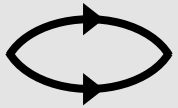
$$D^R(x) = \langle [\Delta^\dagger(x), \Delta(0)] \rangle \theta(t) =$$

Random Phase Approximaion



$$= \frac{Q^R(\boldsymbol{k}, \omega)}{1 + G_C Q^R(\boldsymbol{k}, \omega)}$$

$$Q^R(\boldsymbol{k}, \omega) =$$

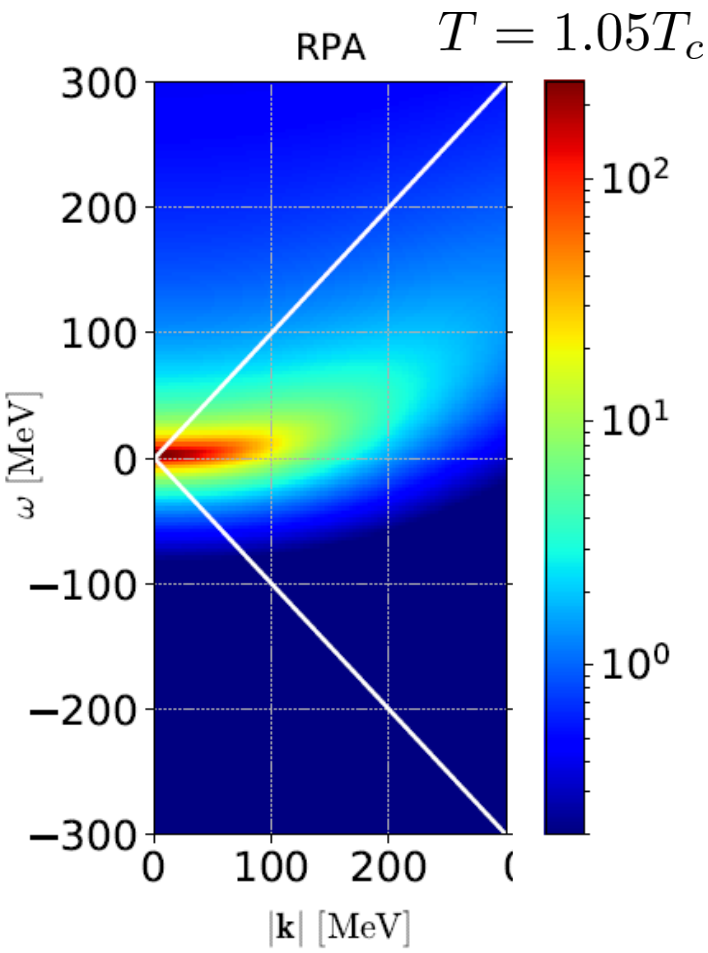


- Diquark field becomes massless at T=Tc
- Soft mode of CSC transition
- Strength in the space-like region

MK, Koide, Kunihiro, Nemoto, '01,'05

Dynamical Structure Factor

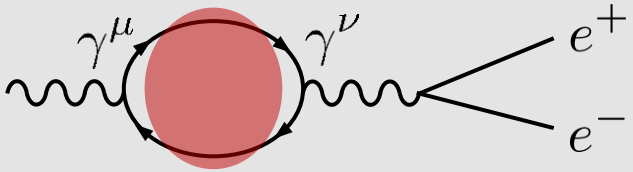
$$S(\boldsymbol{k}, \omega) = -\frac{1}{\pi} \frac{1}{1 - e^{-\beta\omega}} \mathrm{ImD^R(k, \omega)}$$



Photon Self-Energy: Precursor of CSC

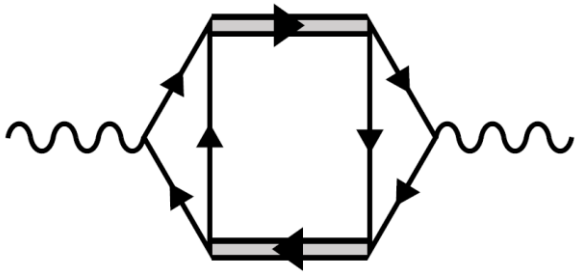
☐ Dilepton Production Rate

$$\frac{d^4\Gamma}{dk^4} = \frac{\alpha}{12\pi^4}\frac{1}{k^2}\frac{1}{e^{\beta\omega-1}}\mathrm{Im}\Pi_\mu^{R\mu}(k)$$

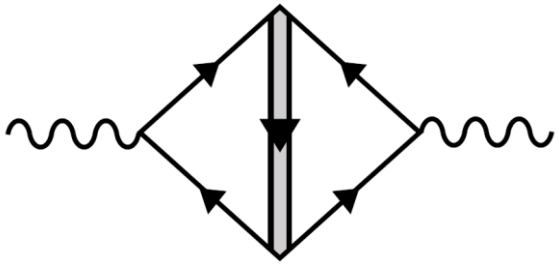


☐ Effect of Di-quarks on $\Pi^{\mu\nu}(k)$

Aslamasov-Larkin term



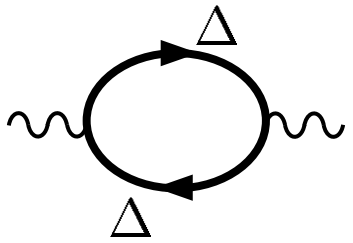
Maki-Thompson term

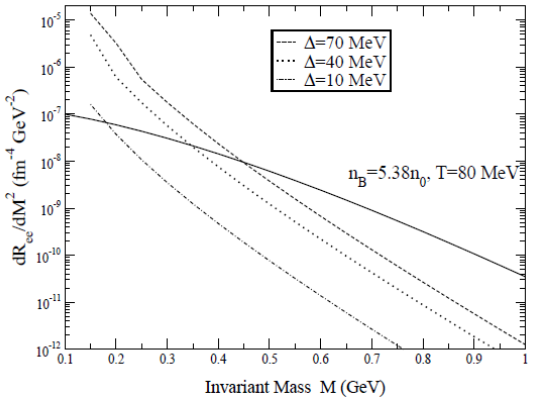


Well-known diagrams in metallic SC for describing paraconductivity

☐ DPR from CFL phase

Jaikumar, Rapp, Zahed ('02)

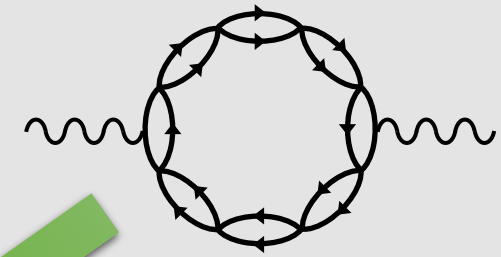


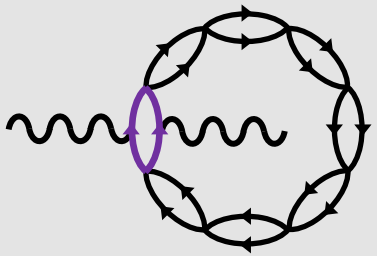


Gauge-Invariant Construction of $\Pi_{\mu\nu}(k)$

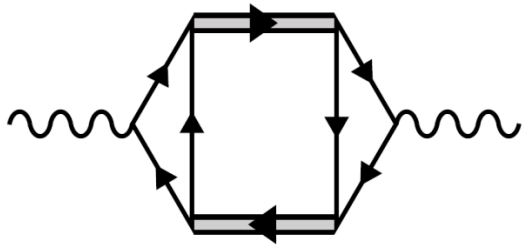
Insert two photon vertices in thermodynamic potential

$$\Pi^{\mu\nu}(k) =$$

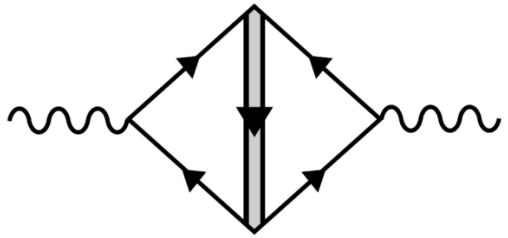




Aslamasov-Larkin (AL)



Maki-Thompson (MT)



Density of States (DoS)

- WT identity $k_\mu \Pi^{\mu\nu}(k) = 0$ is satisfied with AL, MT and DoS terms.

(Modified) Time-Dependent Ginzburg-Landau Approximation

TDGL approximation for T-matrix

$$\Xi^R(\boldsymbol{k},\omega) = \frac{G_C}{1+G_C Q^R(\boldsymbol{k},\omega)} \simeq \frac{1}{c\omega + \Xi^R(\boldsymbol{k},0)^{-1}} \qquad c = \left.\frac{\partial(\Xi^R)^{-1}}{\partial\omega}\right|_{\omega=0}$$

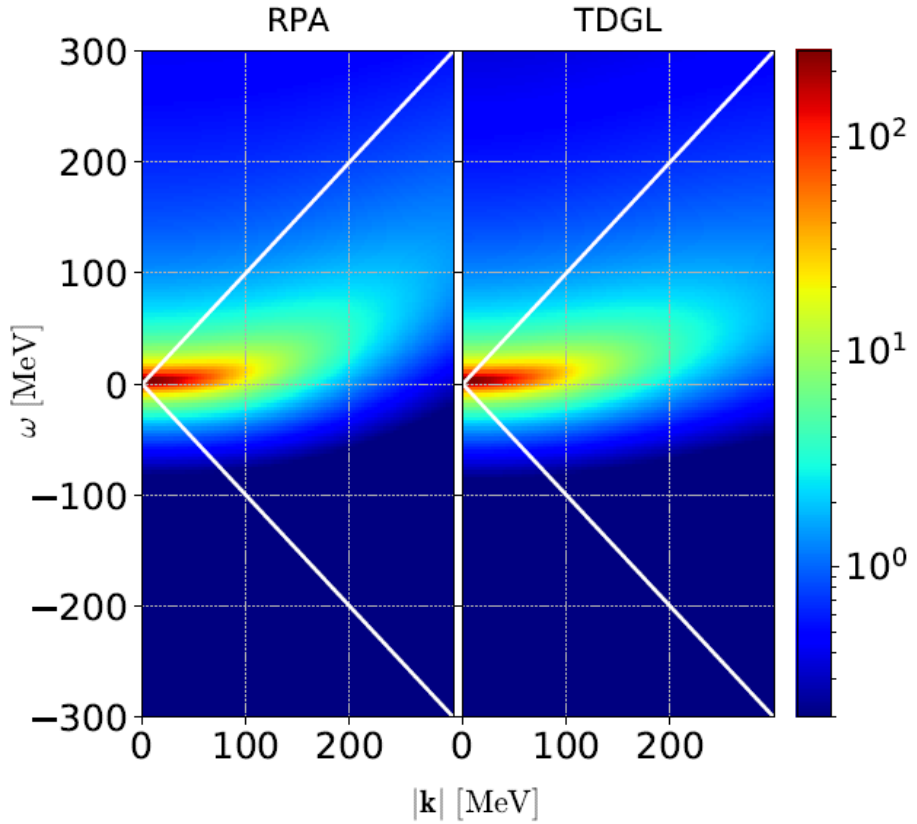
$$\Xi^R(\boldsymbol{k},\omega) = \frac{G_C}{Q^R(\boldsymbol{k},\omega)} D^R(\boldsymbol{k},\omega)$$

Note:

- Valid in low energy region
- $[\Xi^R(0,0)]^{-1} = 0$ at $T = T_c$
- We do not expand w.r.t. k

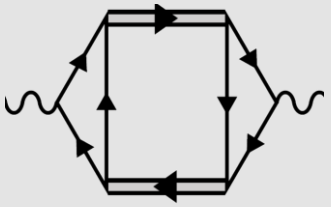
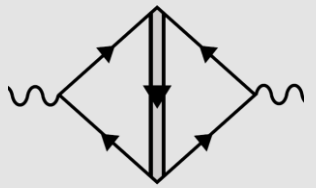
$$\Xi^R(\boldsymbol{k},\omega) \simeq \frac{1}{c\omega + \Xi^R(\boldsymbol{k},0)^{-1}} \simeq \frac{1}{c\omega + a + b\boldsymbol{k}^2}$$

⇔ TDGL equation: $ic\frac{\partial}{\partial t}\Delta + a\Delta - b\nabla^2\Delta = 0$



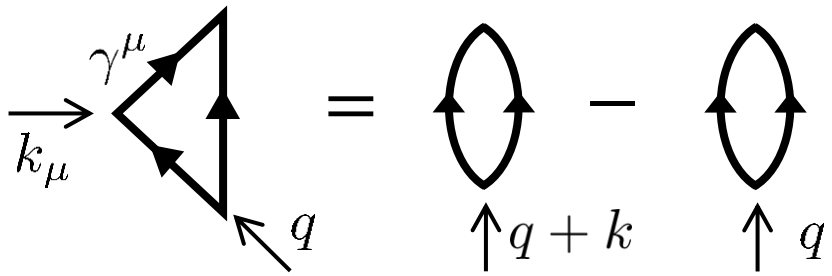
Vertices

Vertices must be determined to be consistent with the TDGL approx.

$\Pi^{\mu\nu}_{\mathrm{AL}}(k) =$  $\Pi^{\mu\nu}_{\mathrm{MT}}(k) =$ 

☐ WT identity for AL vertex

$$k_\mu \Gamma^\mu(q, q+k) = \Xi^{-1}(q+k) - \Xi^{-1}(k)$$

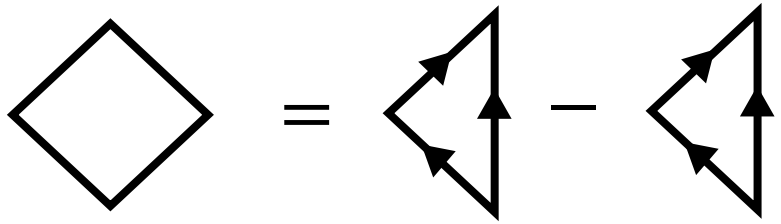


At the lowest order in k

$$\begin{cases} \Gamma^0 = e_\Delta c \\ \Gamma^i = e_\Delta \dfrac{\partial^2 \Xi(q)^{-1}}{\partial q^2}(2q^i + k^i) \end{cases}$$

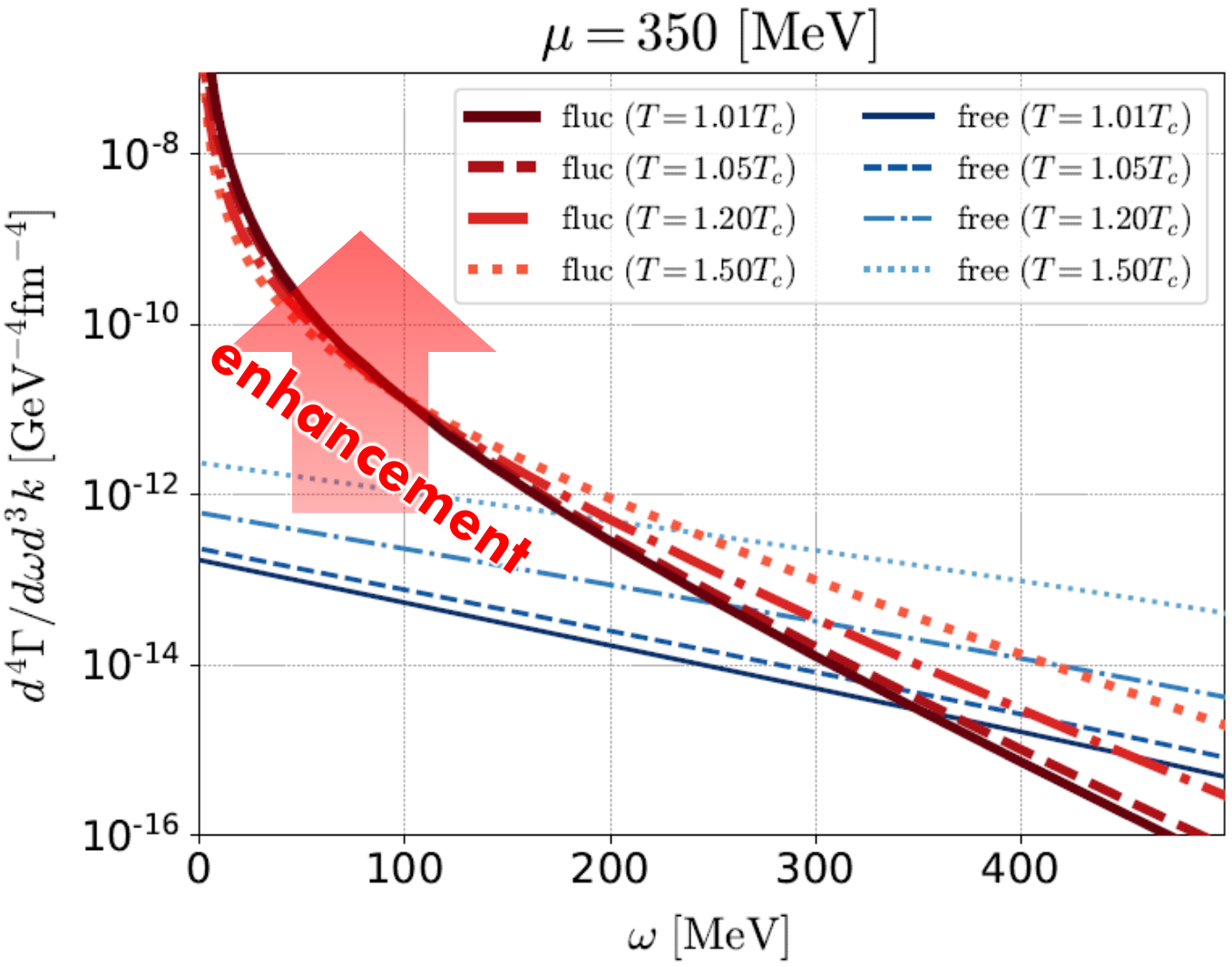
e_Δ: electric charge of diquarks

☐ MT+DoS



Similar formula for MT+DoS vertex

Production Rate at $k = 0$



Nishimura, MK, Kunihiro ('22)

Red: fluctuation contribution

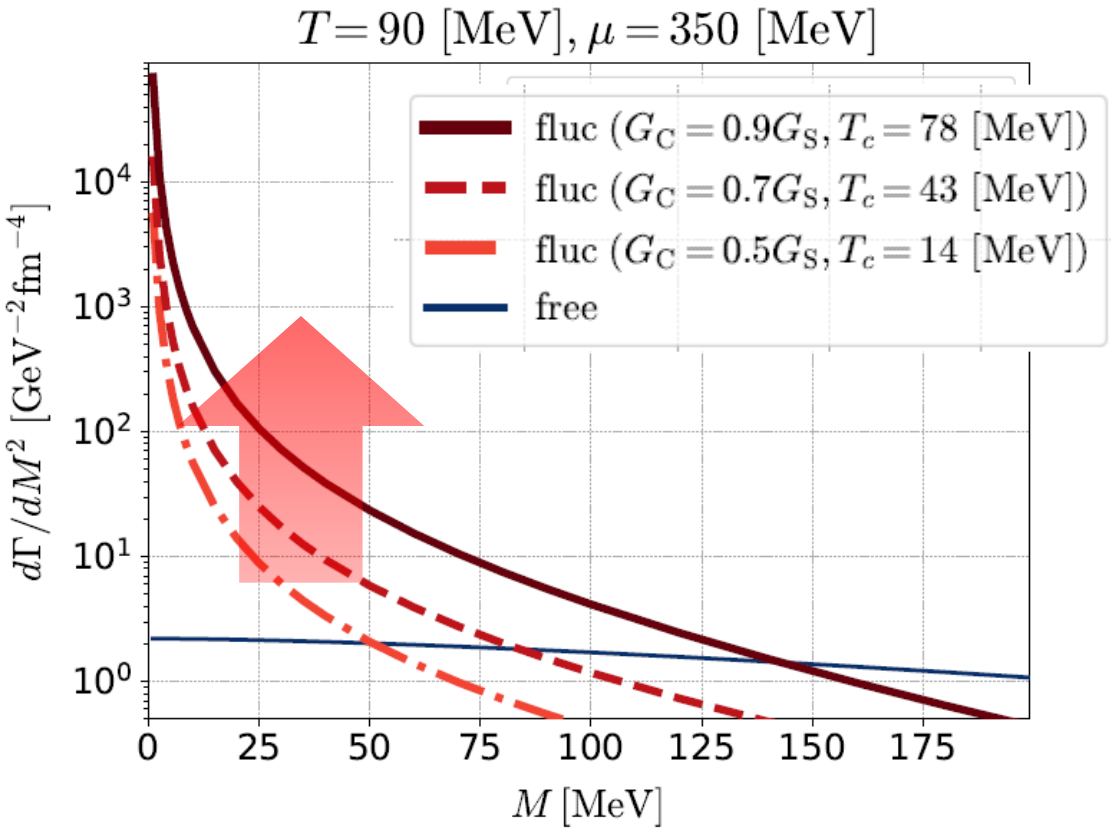
Blue: free quarks

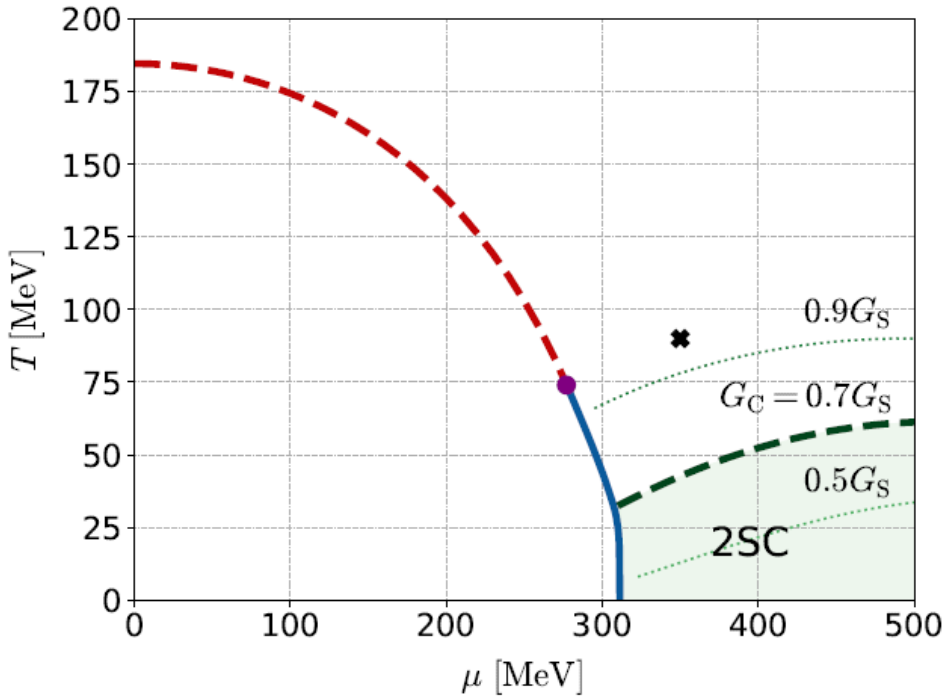
$G_C = 0.7G_s$, $T_c \simeq 45$ MeV

- Di-quark fluctuations give rise to large enhancement in the low energy region $\omega < 200$ MeV and $T < 1.5T_c$.
- Anomalous enhancement is not sensitive to T.

Invariant-Mass Spectrum

Nishimura, MK, Kunihiro ('22)





- Strong enhancement at low invariant mass.
- **Observable in the HIC?**

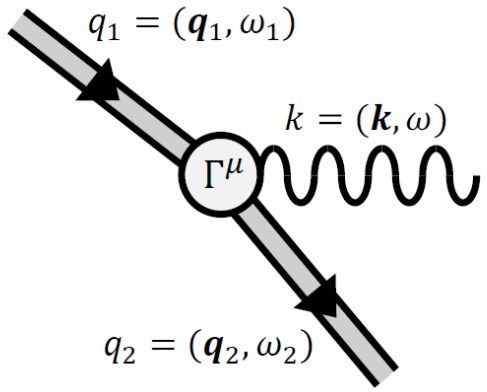
Production Mechanism of Virtual Photons

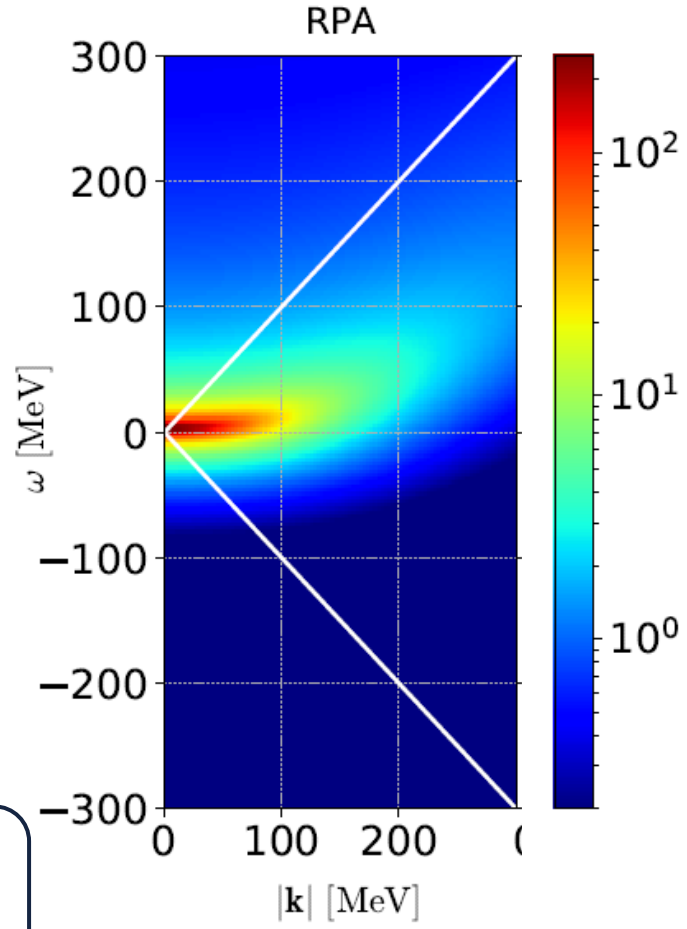
☐ Production mechanism

- ☐ scattering of diquarks
- ☐ diquarks: space-like region

$$\omega = \omega_1 - \omega_2$$

$$\boldsymbol{k} = \boldsymbol{q}_1 - \boldsymbol{q}_2$$



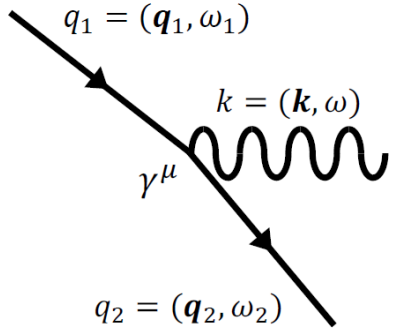


➡ Production in the time-like region is possible.

⬌

☐ C.f.) Scattering of free quarks produces virtual photons only in the space-like region.

$$|\boldsymbol{q}_1 - \boldsymbol{q}_2| \geq \omega_1 - \omega_2$$



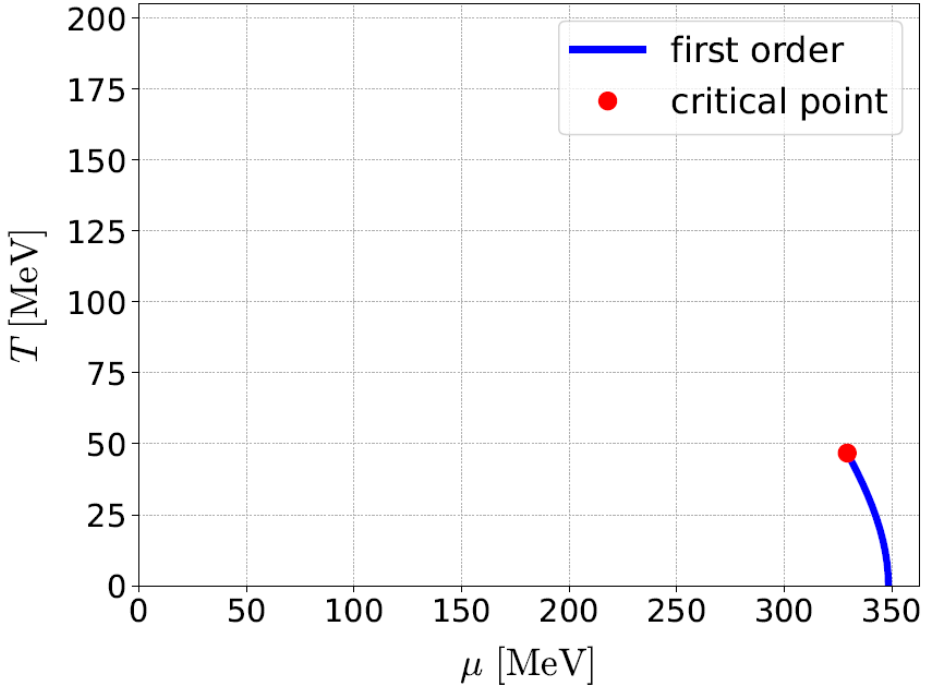
Dileptons from QCD Critical Point

NJL model (2-flavor)

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G_S\left((\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \tau \psi)^2\right)$$

Parameters

$$G_S = 5.5\ \mathrm{GeV}^{-2},\ \Lambda = 631\mathrm{MeV},\ m_q = 5.5\ \mathrm{MeV}$$



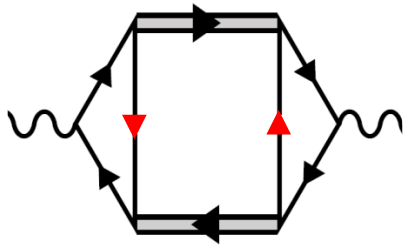
Soft Mode of QCD-CP

= fluctuation of scalar ($\bar{q}q$) channel

$$D^R(x) = \langle[\bar{\psi}\psi(x), \bar{\psi}\psi(0)]\rangle\theta(t) =$$

☐ Random Phase Approximaion

Modification of dilepton production through

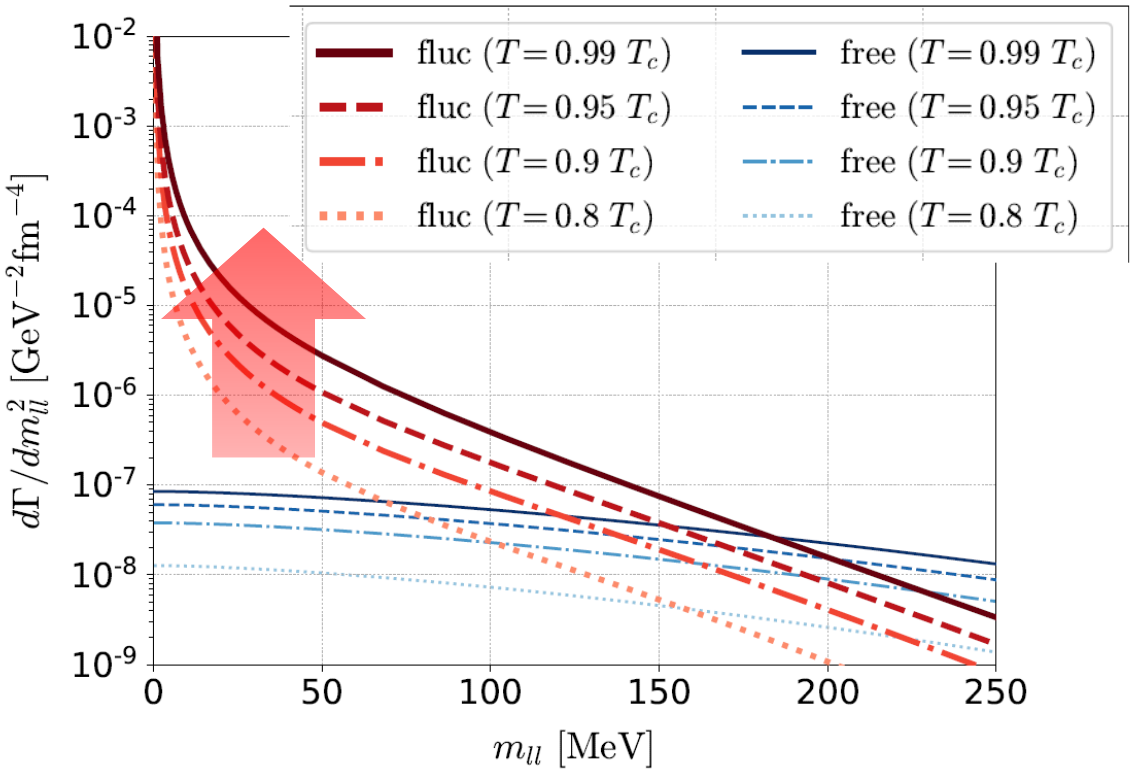


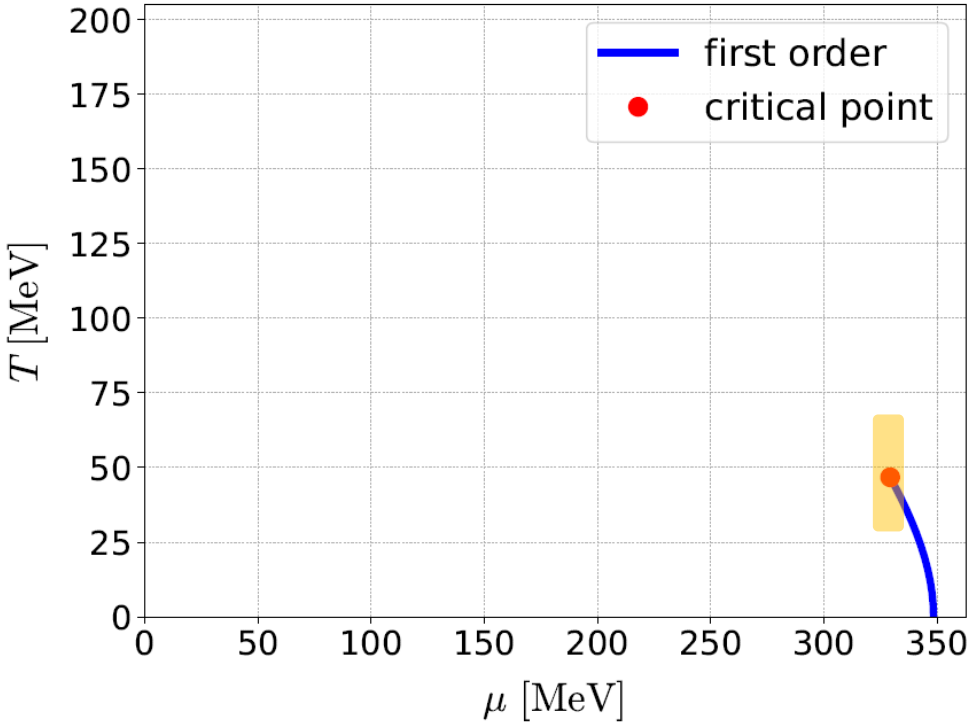
+ ··· MT, DoS

Dilepton production rate near QCD-CP

Nishimura, MK, Kunihiro ('23)

Invariant mass spectrum

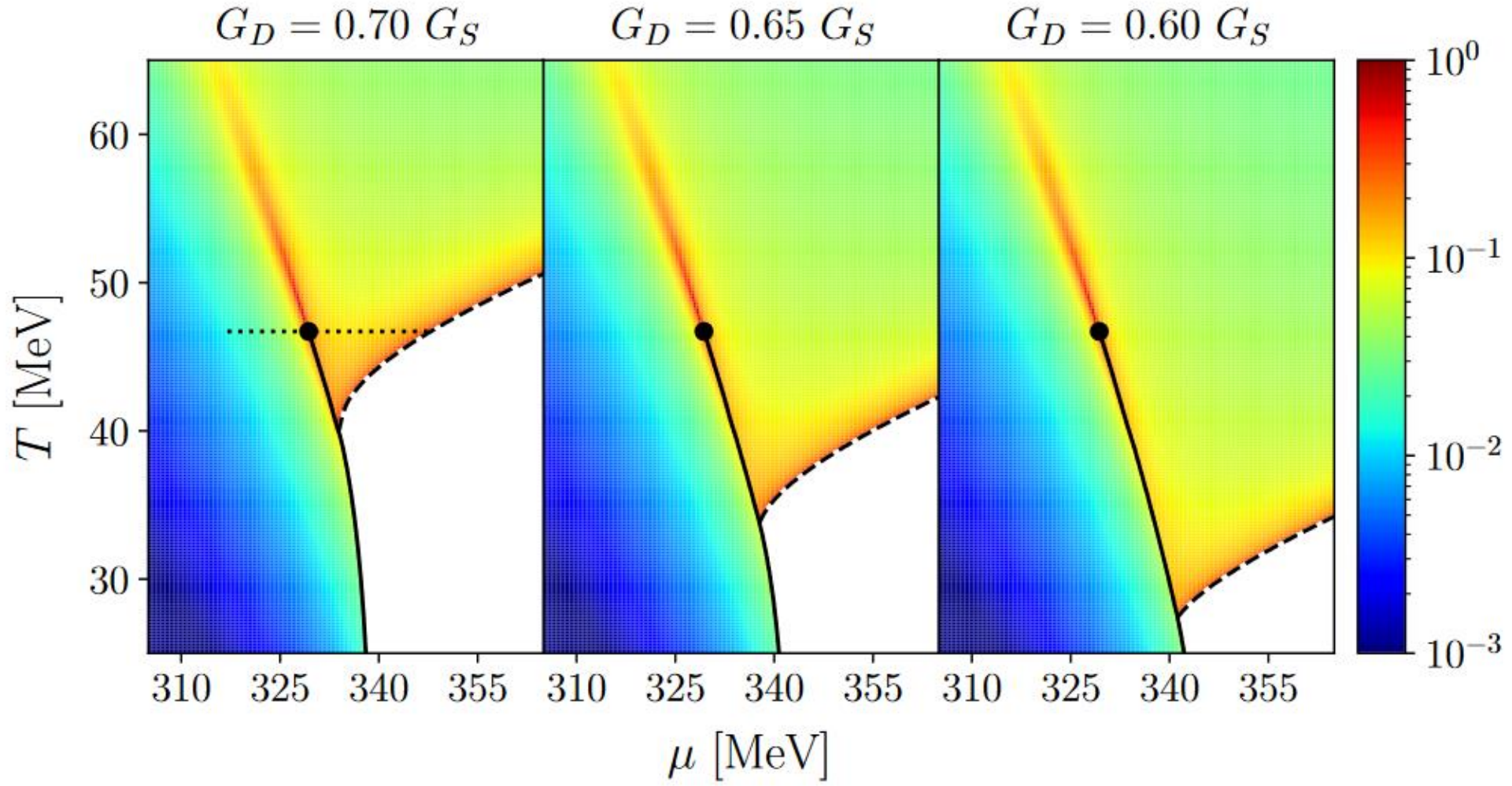




for fixed chem. pot.: $\mu = \mu_c$

- Enhancement at low M_{ll} region near QCD-CP
- Distinguishment from diquark soft mode may be difficult.

Electric Conductivity on QCD Phase Diagram



Nishimura, MK, Kunihiro ('24)

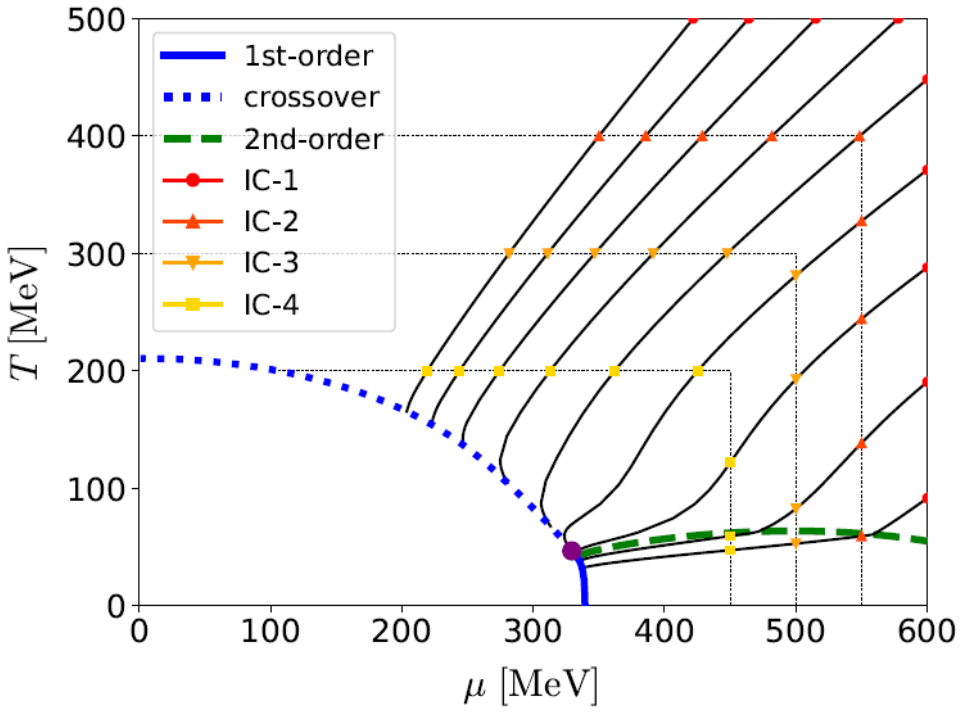
- DPR in the low-energy limit = electric conductivity
- Two "hot spots" on the T-μ plane

Dilepton Yields: Beam-Energy Scan

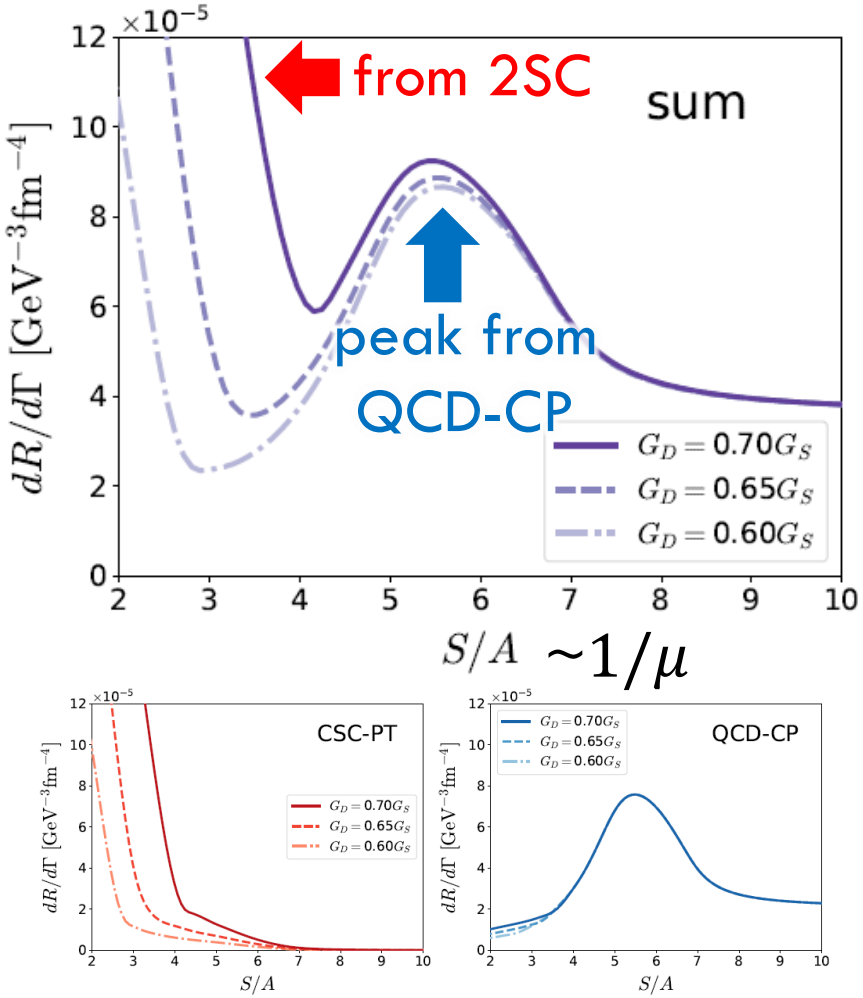
Dilepton yields
≃ integrated rate along isentropic lines

Nishimura, Nara, Steinheimer, Eur.Phys.J.A 60, 2024

Isentropic lines in NJL model



Dilepton Yields $50 < M < 100$ MeV



Effect of 1st-tr on evolution: Savchuk+ 2209.05267

Summary

- Divergence of fluctuations at 2nd-order phase transition
  - ➔Soft modes near the CSC phase transitions & QCD-CP
  - ➔Modification of medium property near the phase transitions

- Anomalous enhancement of the dilepton production in the ultrasoft invariant-mass region.
  - ➔Experimental signature to measure CSC and QCD-CP?

Future

- Competition with Dalitz decay
- DPR from CSC phases

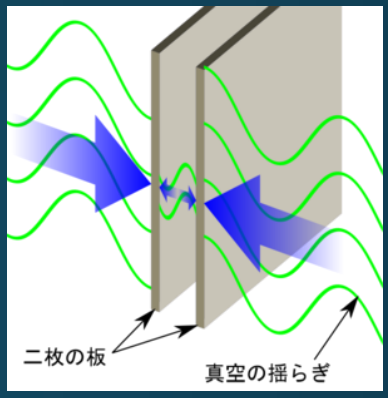
Contents

1. Optimal collision energy for studying dense matter in heavy-ion collisions
   Taya, Jinno, MK, Nara, 2409.07685
2. Dilepton production for a signal of phase transitions in dense matter
   Nishimura, MK, Kunihiro, Ann. Phys. 469 (2024) 169768; PTEP 2023, 053D01; PTEP 2022, 093D02
3. Novel critical points in SU(3) Yang-Mills theory with boundary conditions
   MK+, PRD99 (2019) 094507; Suenaga, MK, PRD107 (2023) 074502; Fujii, Iwanaka, Suenaga, MK, PRD110 (2024) 094016

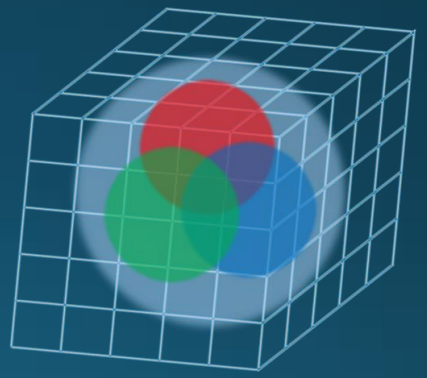
Boundary Conditions in QFT

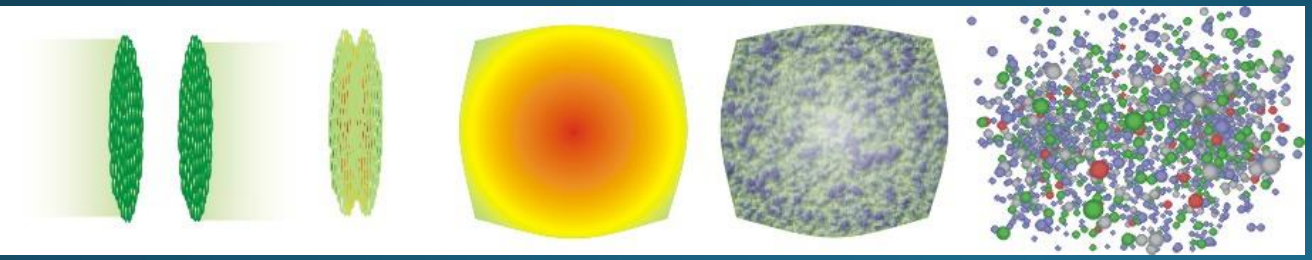
Many motivations

- Casimir effect
- Relativistic heavy-ion collisions
- Numerical simulations (ex. lattice QCD)
- Matsubara formalism for thermal systems



from wikipedia

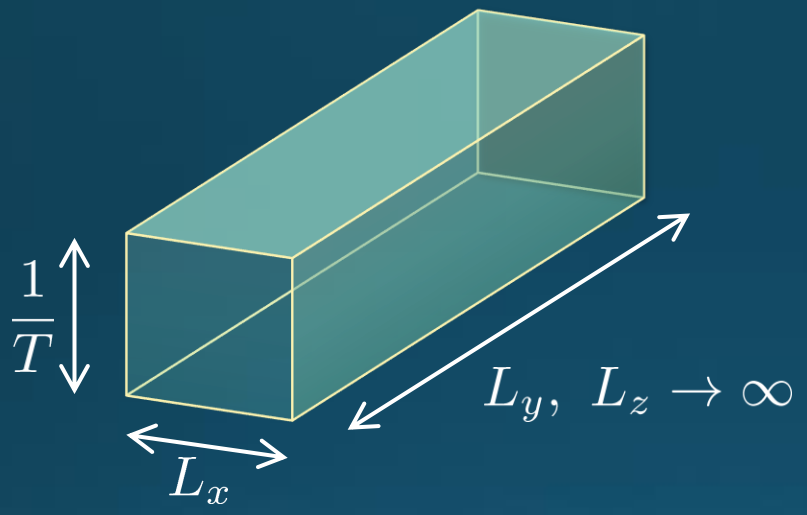




$$L_\tau = \frac{1}{T}$$

Purpose

Thermal SU(3) YM with PBC along x direction

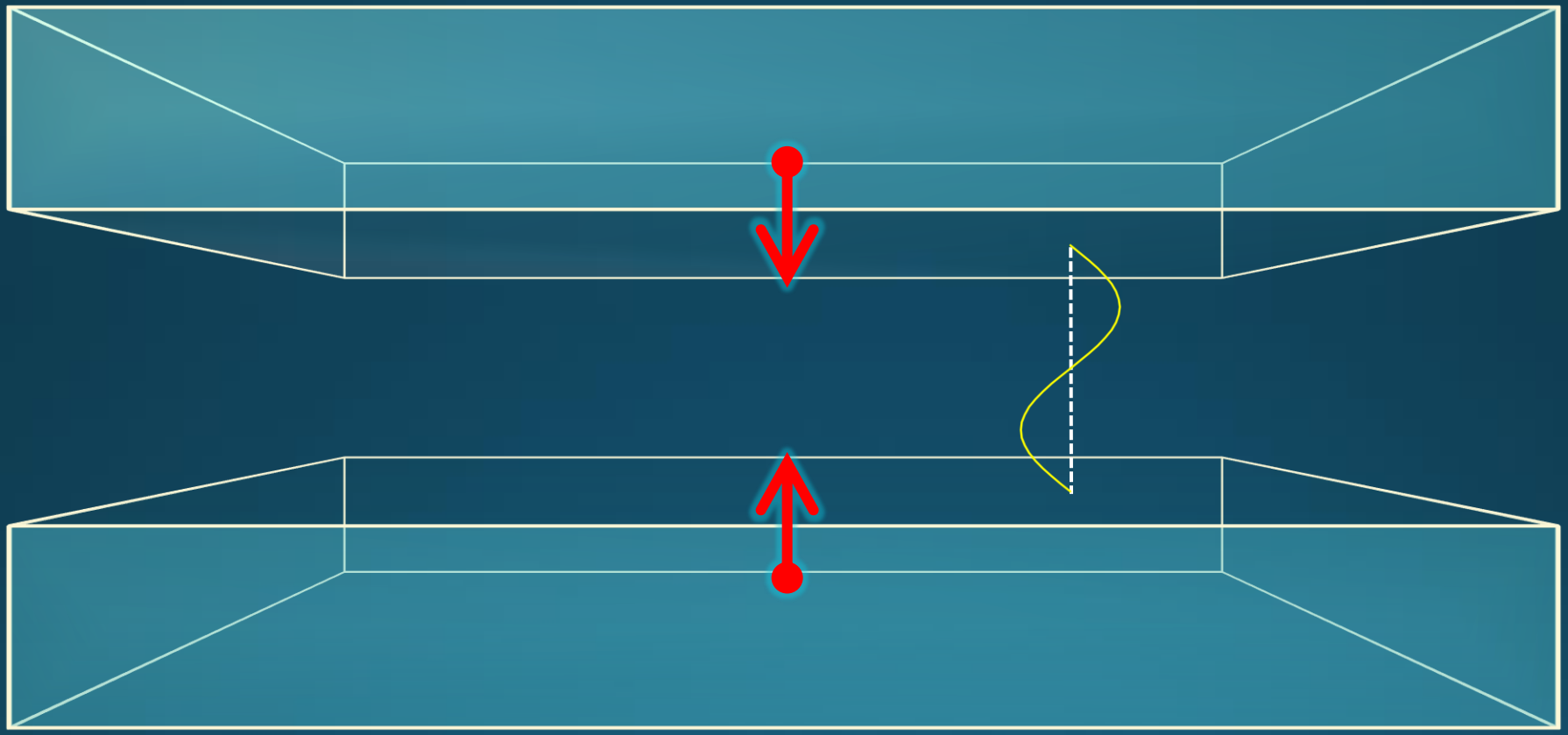


$=$ QFT on Euclidean $\boldsymbol{T^2 \times R^2}$

How does thermodynamics behave w.r.t. T and L_x?

- Thermal Casimir effect in a non-perturbative system
- QCD phase diagram as a function of L_x
- Anisotropic pressure
- 2 Polyakov loops will play important roles

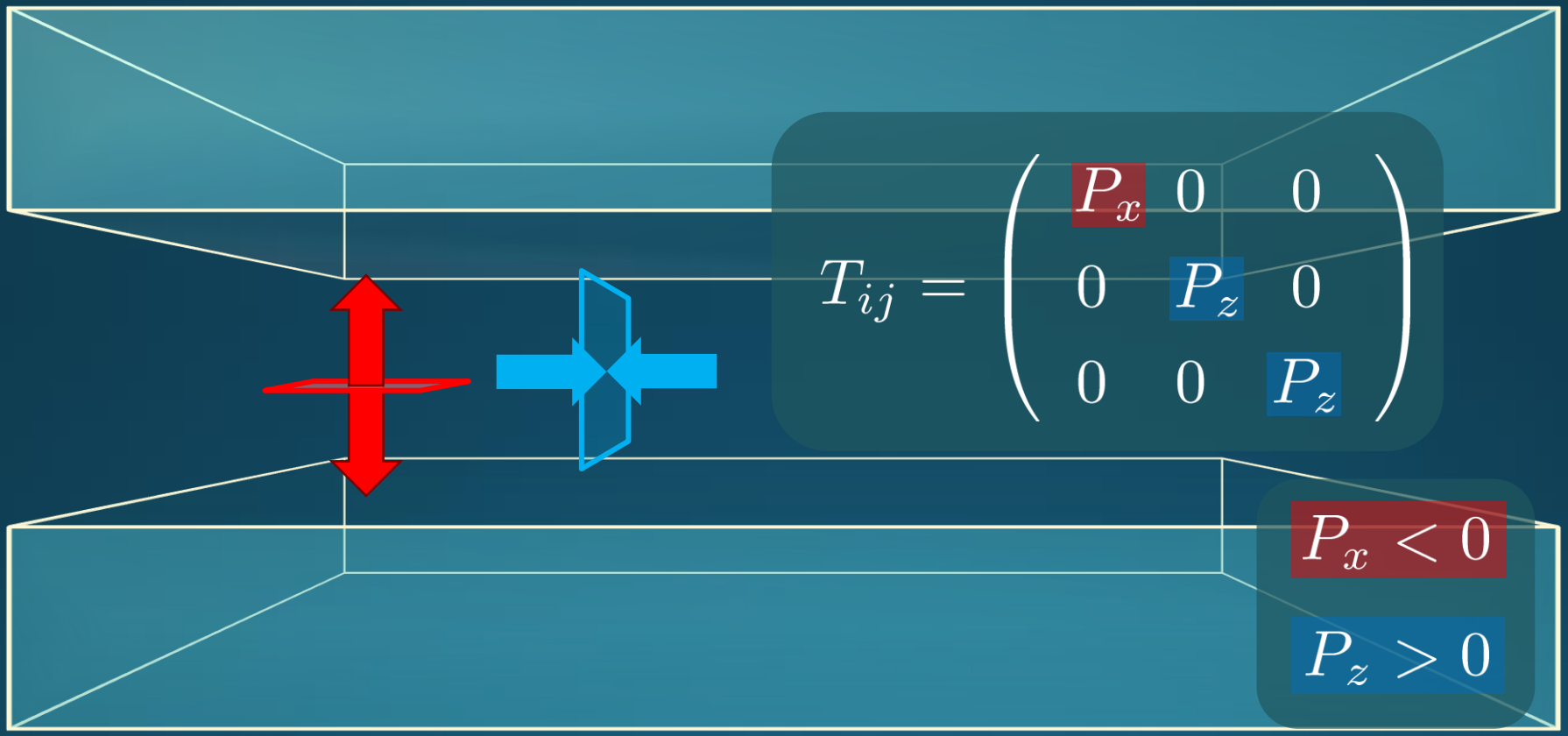
Casimir Effect



attractive force between two conductive plates

Casimir Effect

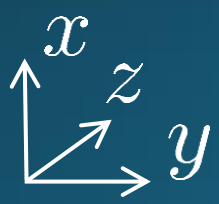
Brown, Maclay
1969



$$T_{ij} = \begin{pmatrix} P_x & 0 & 0 \\ 0 & P_z & 0 \\ 0 & 0 & P_z \end{pmatrix}$$

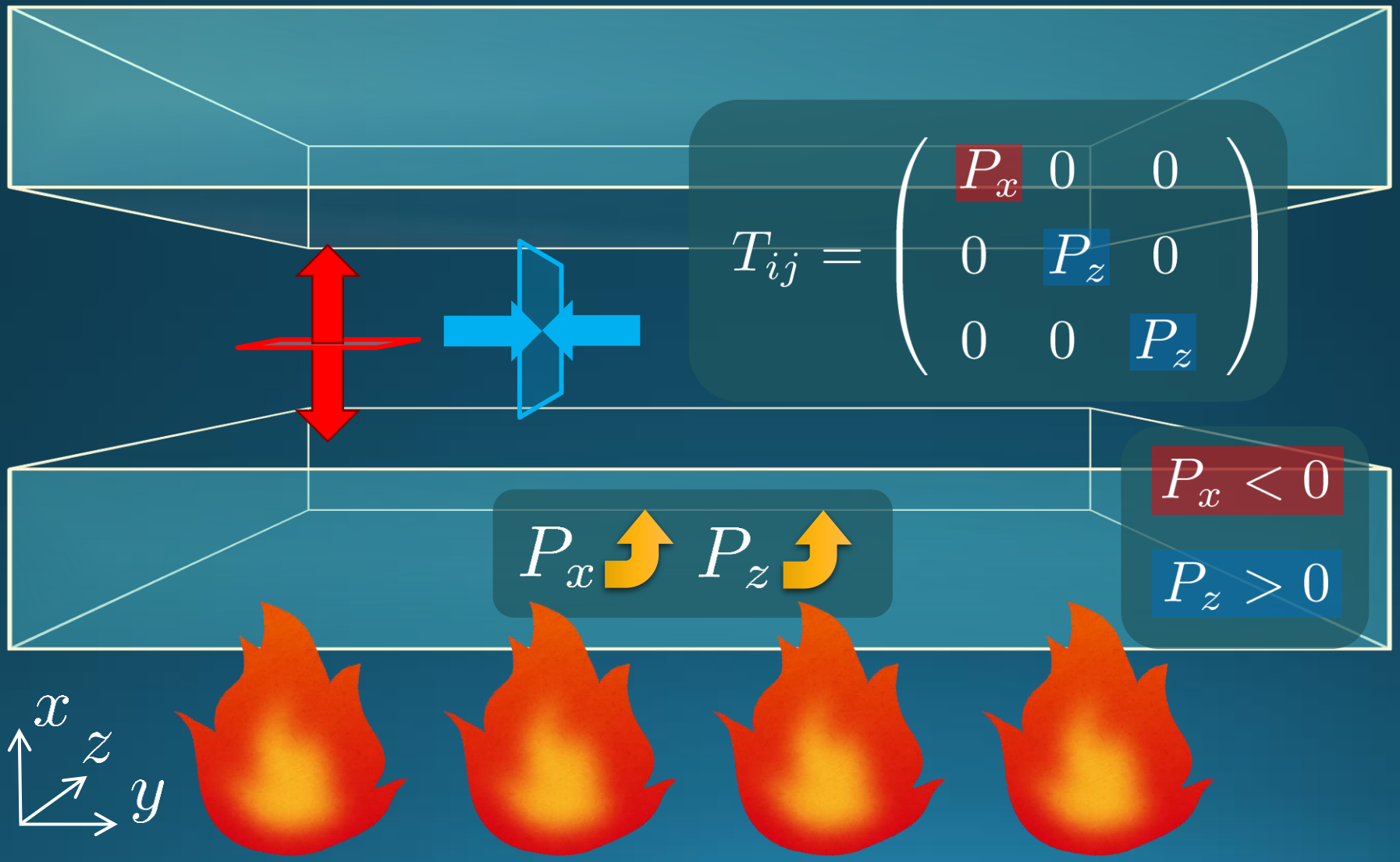
$P_x < 0$

$P_z > 0$



Casimir Effect

Brown, Maclay
1969



Contents

1. Lattice study

MK+, Phys. Rev. **D99** (2019) 094507

2. Model analyses

Suenaga, MK, Phys. Rev. **D107** (2023) 074502

D. Fujii, A. Iwanaka, D. Suenaga, MK, arXiv:2404.07899

Thermodynamics on the Lattice

Various Methods

- **Integral**, differential, moving frame, non-equilibrium, …
- rely on thermodynamic relations valid in V→∞

$$P = \frac{T}{V} \ln Z \qquad sT = \varepsilon + P$$

Not applicable to anisotropic systems

- We employ **Gradient Flow (SF*t*X) Method**

$$\varepsilon = \langle T_{00} \rangle \quad P = \langle T_{11} \rangle$$

Components of EMT are directly accessible!

Yang-Mills Gradient Flow

Luscher 2010
Narayanan, Neuberger, 2006
Luscher, Weiss, 2011

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

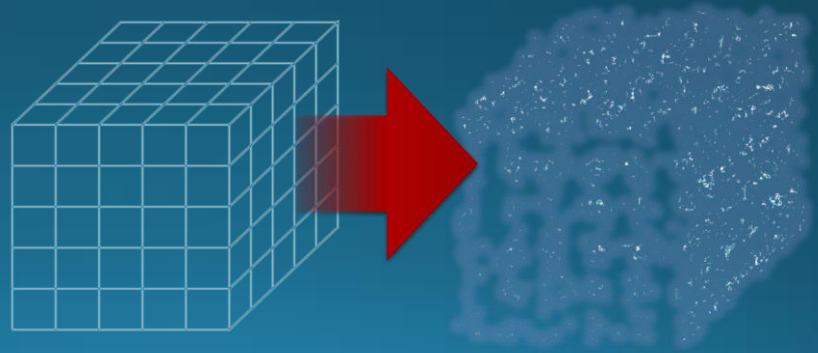
$$A_\mu(0,x) = A_\mu(x)$$

t: "flow time"
dim:[length^2]

leading

$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- diffusion equation in 4-dim space
- diffusion distance $d \sim \sqrt{8t}$
- "continuous" cooling/smearing
- No UV divergence at t>0



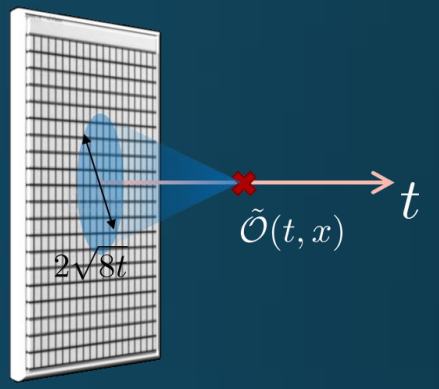
Constructing EMT

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)T^R_{\rho\rho}(x) + \mathcal{O}(t)$$

vacuum subtr.



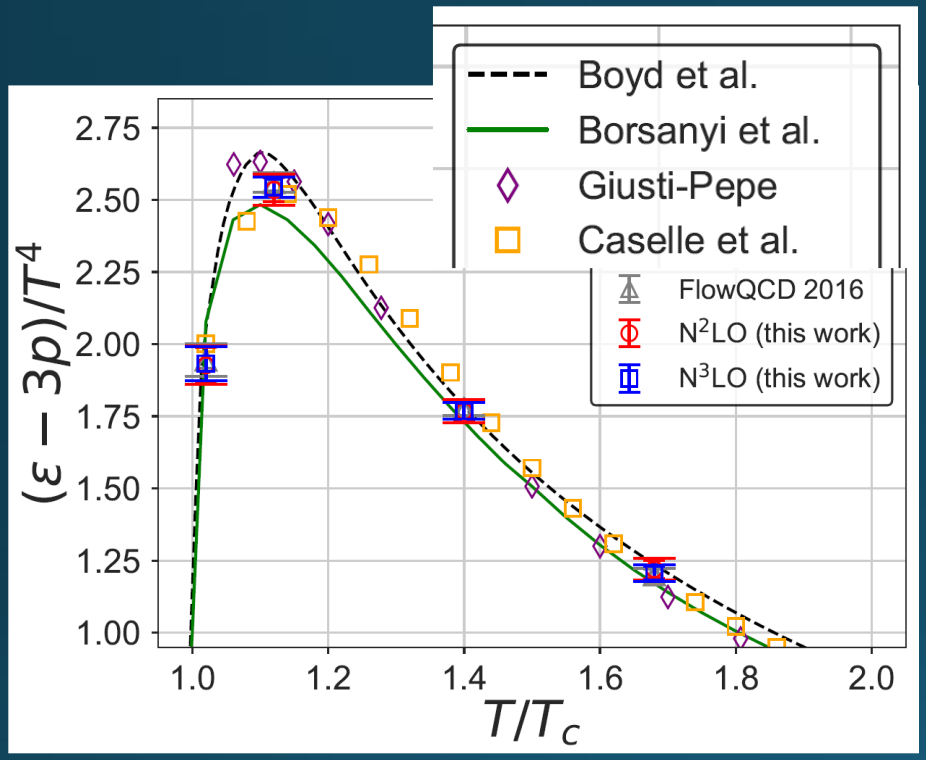
Remormalized EMT

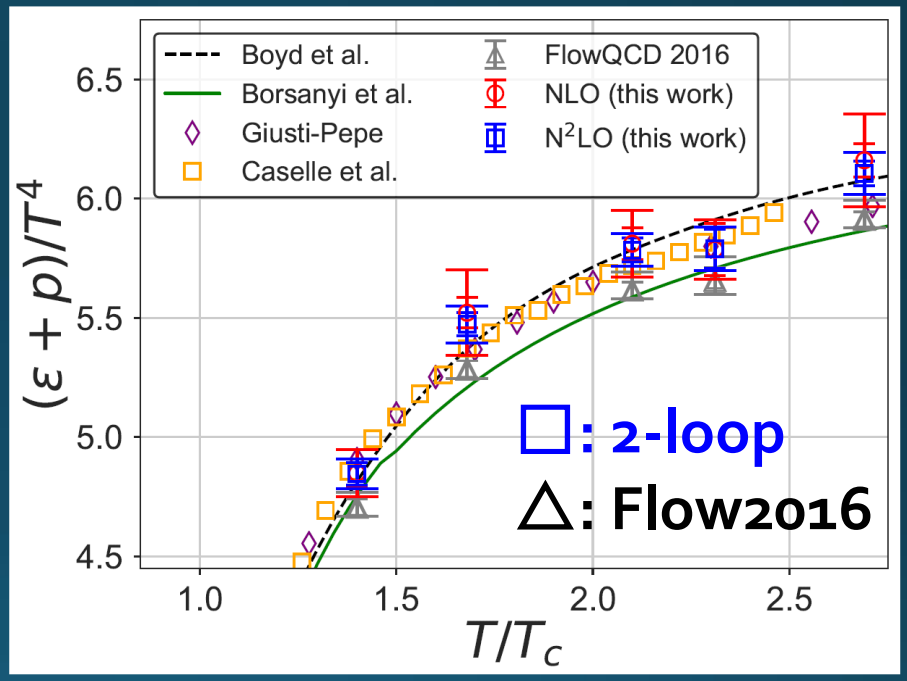
$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[c_1(t)U_{\mu\nu}(t,x) + \delta_{\mu\nu}c_2(t)E(t,x)_{\text{subt.}}\right]$$

Perturbative coefficient:
Suzuki (2013); Makino, Suzuki (2014); Harlander+ (2018); Iritani, MK, Suzuki, Takaura (2019)

Thermodynamics

Iritani, MK, Suzuki, Takaura, 2019





Systematic error: μ_0 or μ_d, Λ, t→0 function, fit range

- Good agreement within **1% level**
- Our method can deal with the anisotropic pressure

Numerical Setup

- SU(3) YM theory
- Wilson gauge action

- $N_t = 16, 12$
- $N_z/N_t = 6$
- 2000~4000 confs.
- Even N_x
- No Continuum extrap.

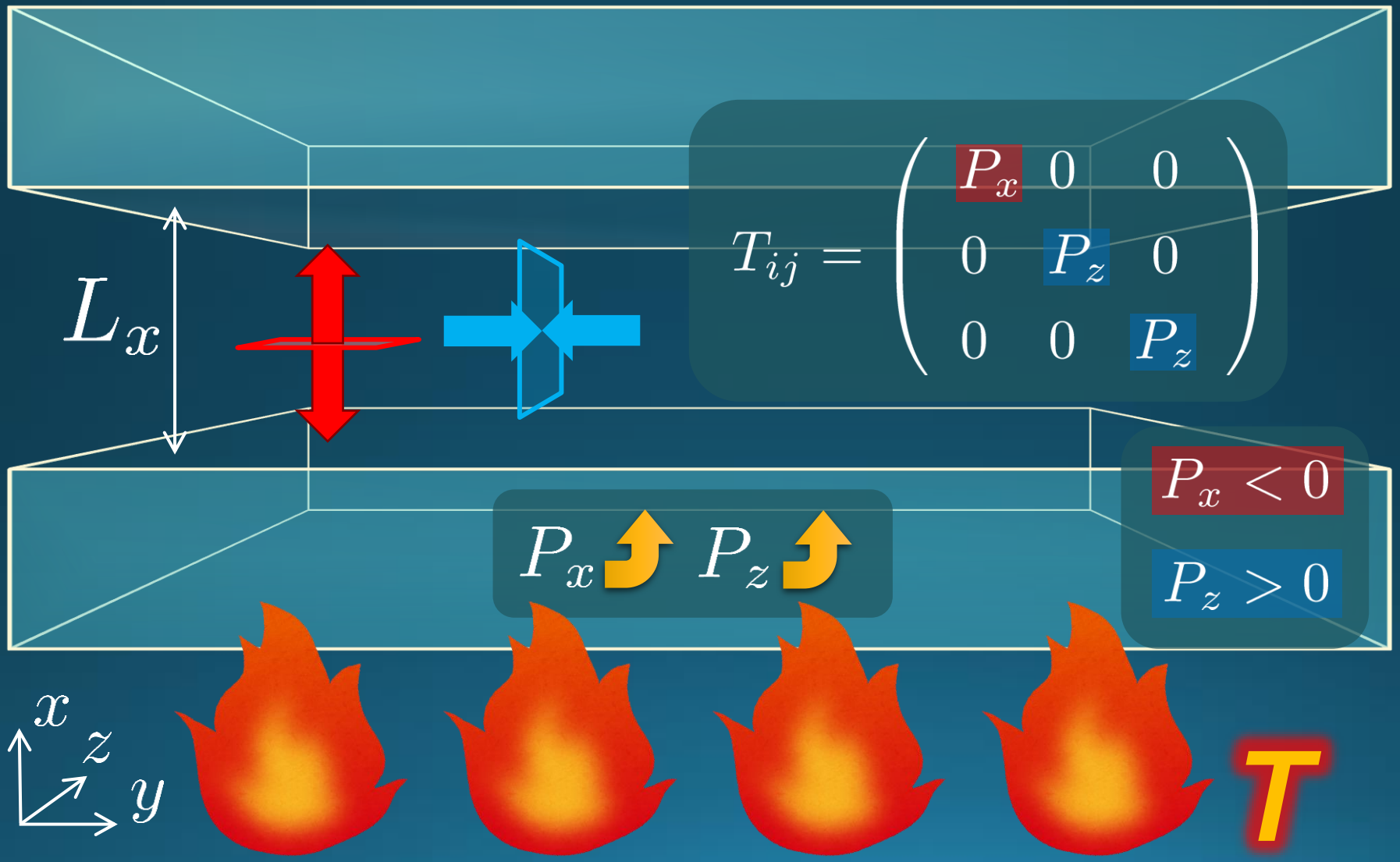
T/T_c	β	N_z	N_τ	N_x	$N_{\rm vac}$
1.12	6.418	72	12	12, 14, 16, 18	64
	6.631	96	16	16, 18, 20, 22, 24	96
1.40	6.582	72	12	12, 14, 16, 18	64
	6.800	96	16	16, 18, 20, 22, 24	128
1.68	6.719	72	12	12, 14, 16, 18, 24	64
	6.719	96	12	14, 18	64
	6.941	96	16	16, 18, 20, 22, 24	96
2.10	6.891	72	12	12, 14, 16, 18, 24	72
	7.117	96	16	16, 18, 20, 22, 24	128
2.69	7.086	72	12	12, 14, 16, 18	-
$\simeq 8.1$	8.0	72	12	12, 14, 16, 18	-
$\simeq 25$	9.0	72	12	12, 14, 16, 18	-

- Same System volume
  - $12 \times 72^2 \times 12 \sim 16 \times 96^2 \times 16$
  - $18 \times 72^2 \times 12 \sim 24 \times 96^2 \times 16$

Simulations on OCTOPUS/Reedbush

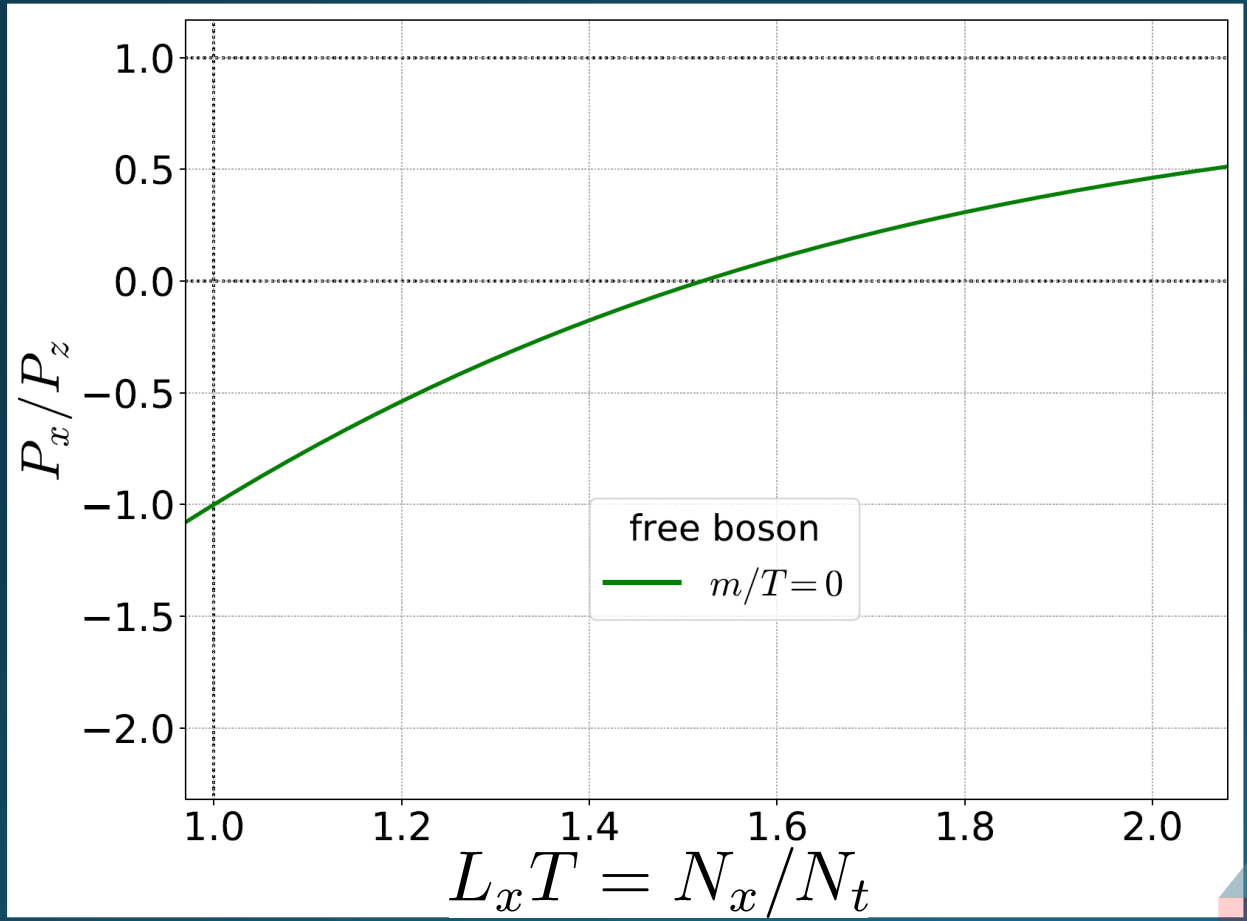
Casimir Effect

Brown, Maclay
1969



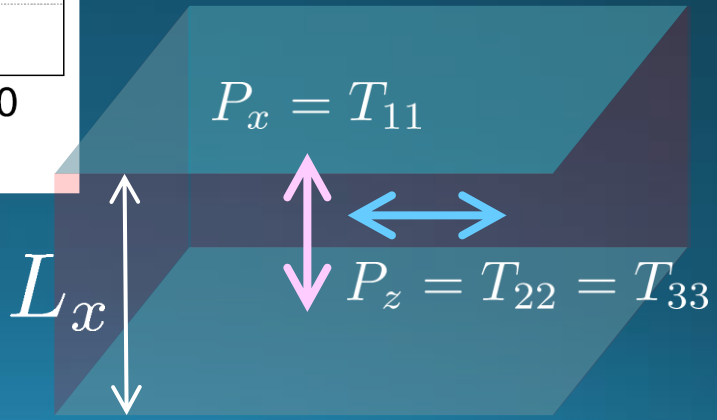
Pressure Anisotropy @ $T \neq 0$

MK, Mogliacci, Kolbe, Horowitz ('21)



Free scalar field

- $L_2=L_3=\infty$
- Periodic BC



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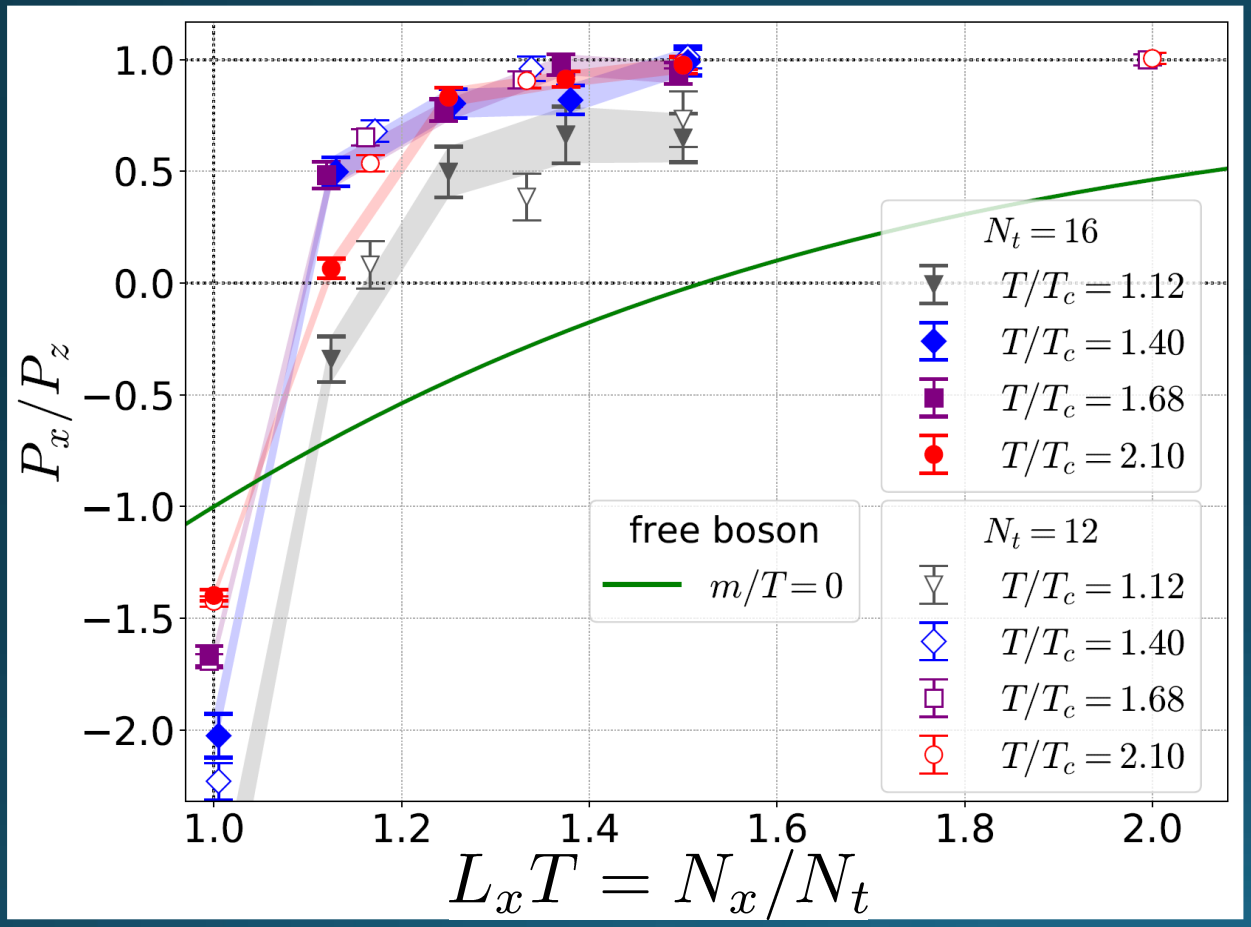

Free scalar field

- L_2=L_3=∞
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Lattice result

- Periodic BC
- Only t→0 limit
- Error: stat.+sys.

Medium near T_c is remarkably insensitive to finite size!

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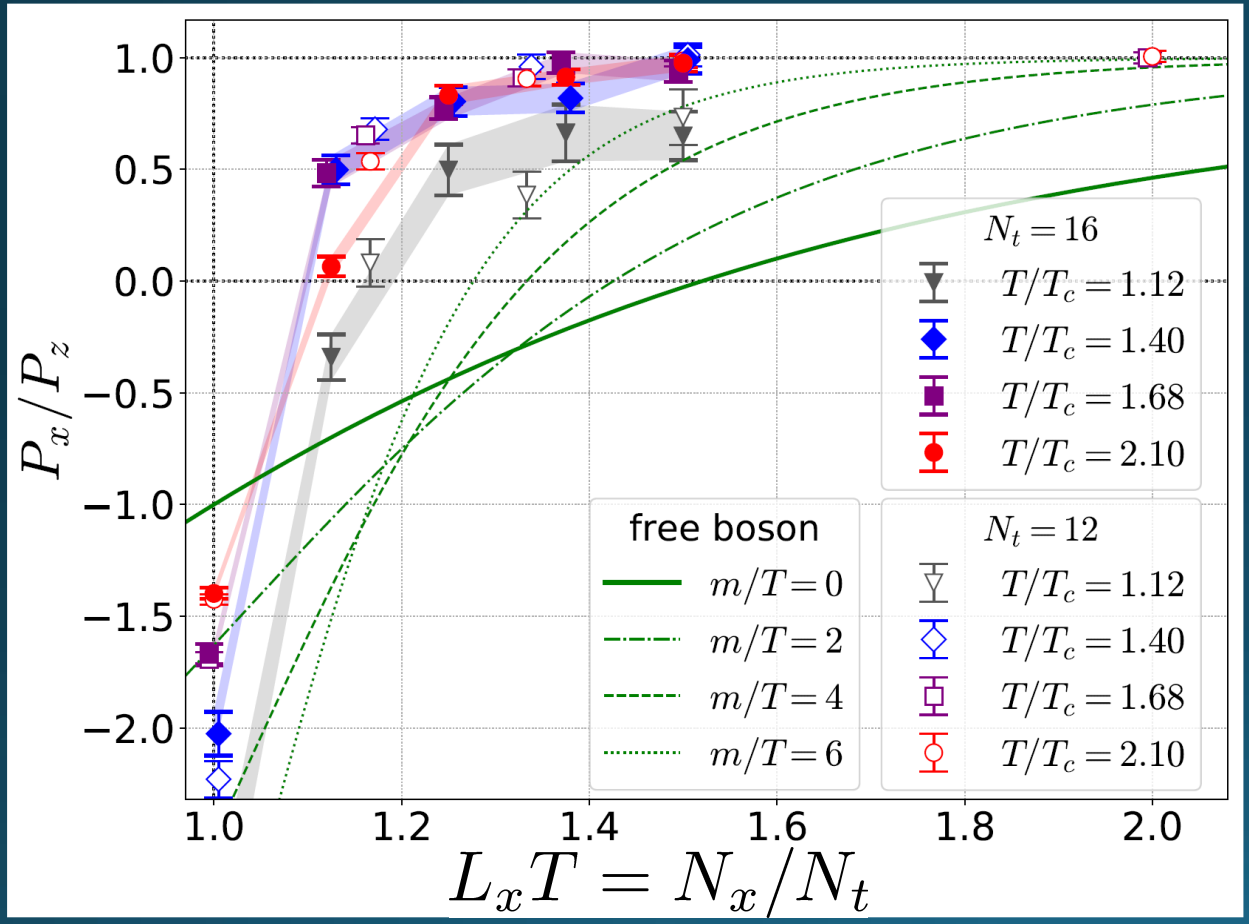

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Higher *T*

High-*T* limit: massless free gluons

How does the anisotropy approach this limit?

Difficulties

- Vacuum subtraction requires large-volume simulations.
- Lattice spacing not available → $c_1(t)$, $c_2(t)$ are not determined.

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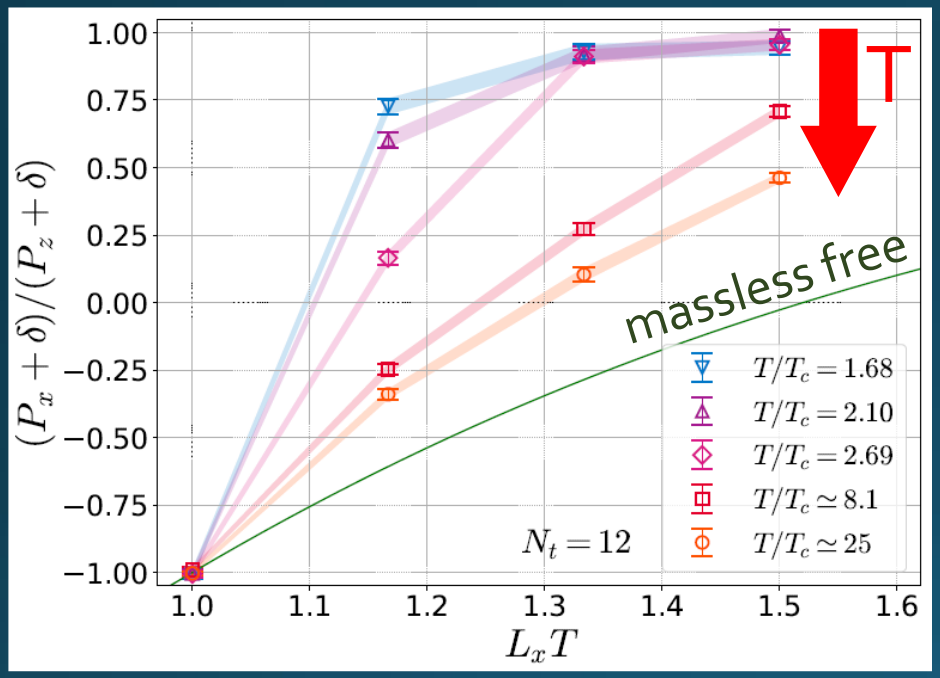
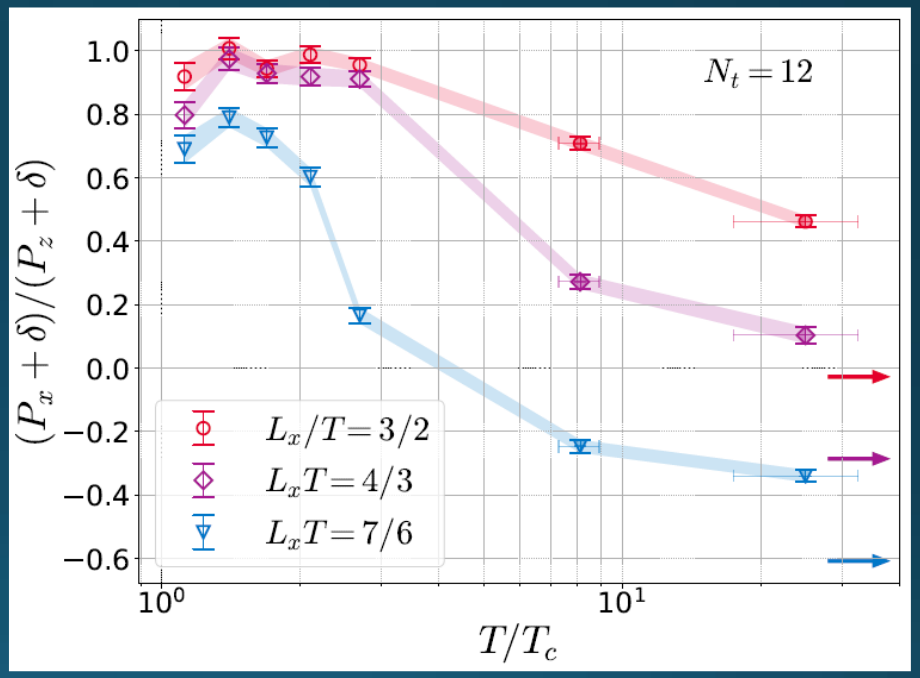
We study

$$R = \frac{P_x + \delta}{P_z + \delta} \qquad \delta = -\frac{1}{4}\sum_{\mu} T^{\mathrm{E}}_{\mu\mu}$$

No vacuum subtr.
nor Suzuki coeffs.
necessary!

$$\frac{P_x + \delta}{P_z + \delta}$$





$T/T_c \simeq 8.1\ (\beta = 8.0),\ T/T_c \simeq 25\ (\beta = 9.0)$

- Ratio approaches the asymptotic value for large T.
- But, large deviation exists even at $T/T_c \simeq 25$.
- 1st-order phase transition??

Contents

1. Lattice study

MK+, Phys. Rev. **D99** (2019) 094507

2. Model analyses

Suenaga, MK, Phys. Rev. **D107** (2023) 074502

D. Fujii, A. Iwanaka, D. Suenaga, MK, arXiv:2404.07899

Polyakov-loop Effective Models

Meisinger+, PRD (2003)

General Idea

Constant Polyakov loop P as dynamical variable

$$P = \mathrm{Tr}\left[\mathcal{P}\exp\left(i\int_0^{L_\tau} A_\tau d\tau\right)\right]$$

- $P = 0$: confinement
- $P \neq 0$: deconfinement

Free Energy

$$F(T;P) = F_{\mathrm{pert.}}(T;P) + F_{\mathrm{pot.}}(T;P)$$

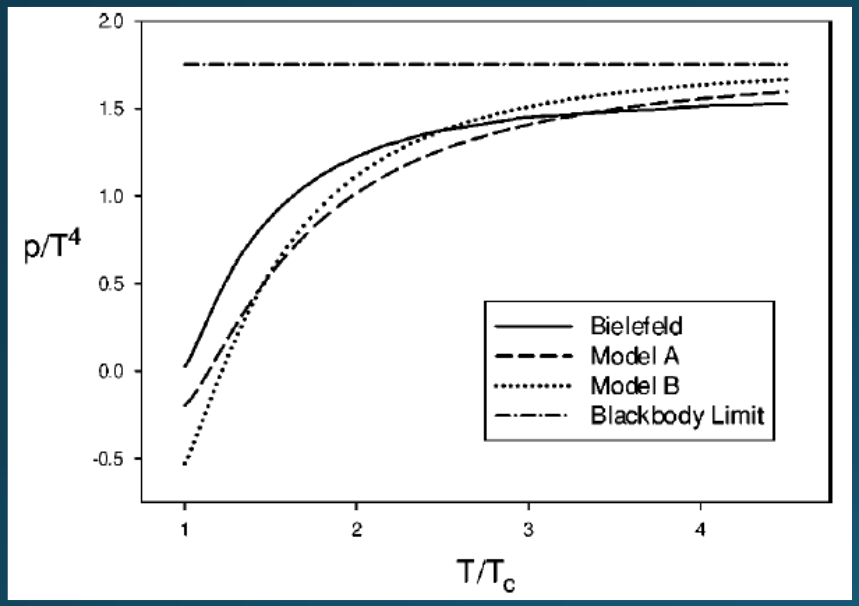
massless free gluons with constant $A_0(x)$

Phenomenological potential term

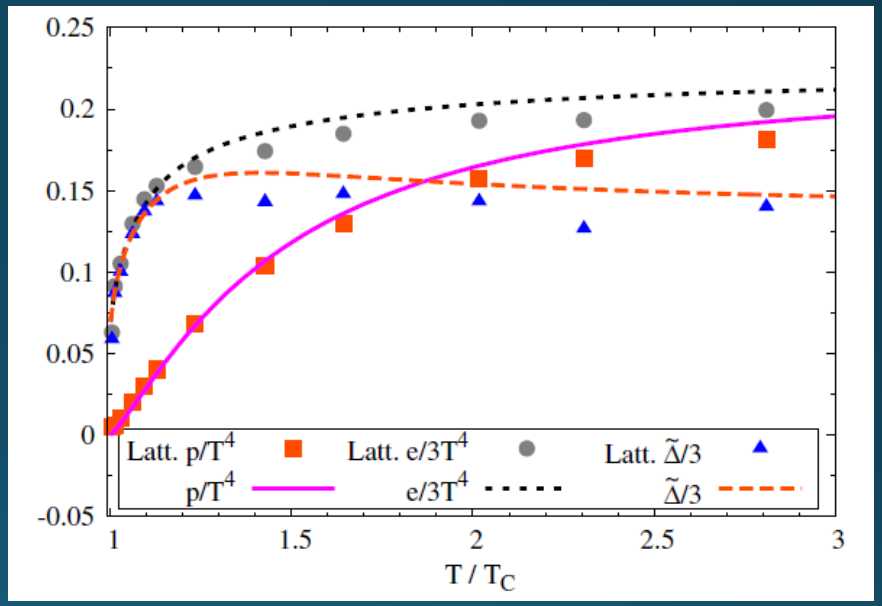
$\langle P \rangle$ is determined to minimize $F(T;P)$.

Thermodynamics

Meisinger+, PRD ('03)



Dumitru+, PRD ('12)



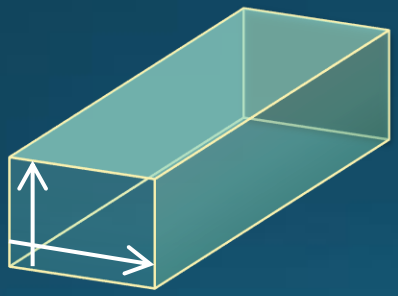
Qualitative behavior of lattice thermodynamics near and above T_c is well reproduced.

Extension to $T^2 \times R^2$

Suenaga, MK ('23); Fujii+ ('24)

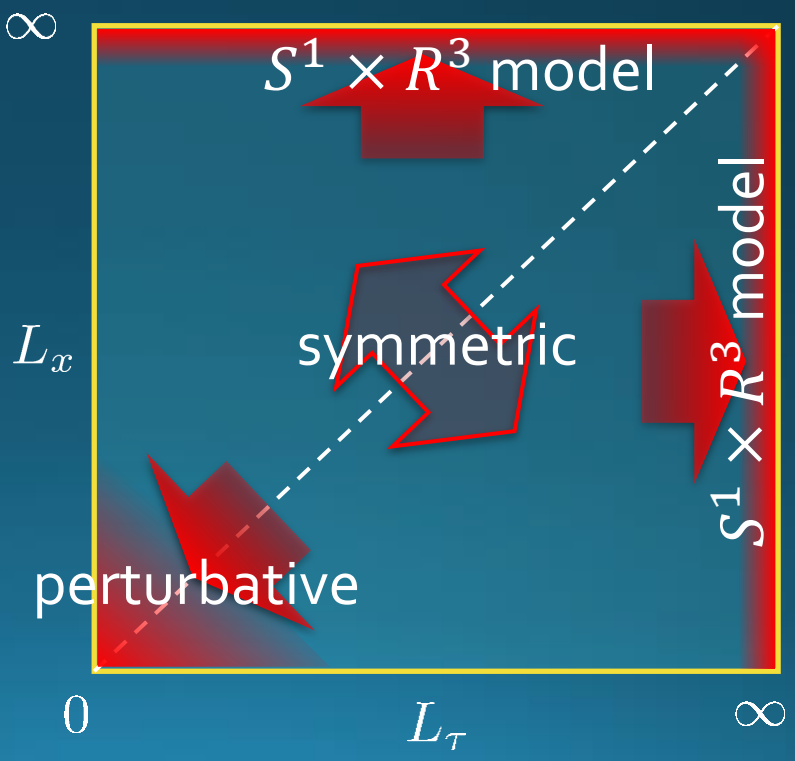
2 Polyakov loops along τ and x directions

$$P_\tau = \mathrm{Tr}\left[\mathcal{P}\exp\left(i\int_0^{L_\tau} A_\tau d\tau\right)\right] \qquad P_x = \mathrm{Tr}\left[\mathcal{P}\exp\left(i\int_0^{L_\tau} A_x d\tau\right)\right]$$



Free Energy

- Function of 2 Polyakov loops.
- Constructed under constraints in various limits and symmetries



Polyakov-loop Potential Term

Fujii+ (2024)

$$F_{\rm pot} = F_{\rm sep} + F_{\rm cross}$$

$$F_{\rm sep}(P_\tau, P_x; L_\tau, L_x) = F_{\rm pot}^{S^1\times R^3}(P_\tau, L_\tau) + F_{\rm pot}^{S^1\times R^3}(P_x, L_x)$$

Potential on $S^1 \times R^3$
from Dmitru+('12)

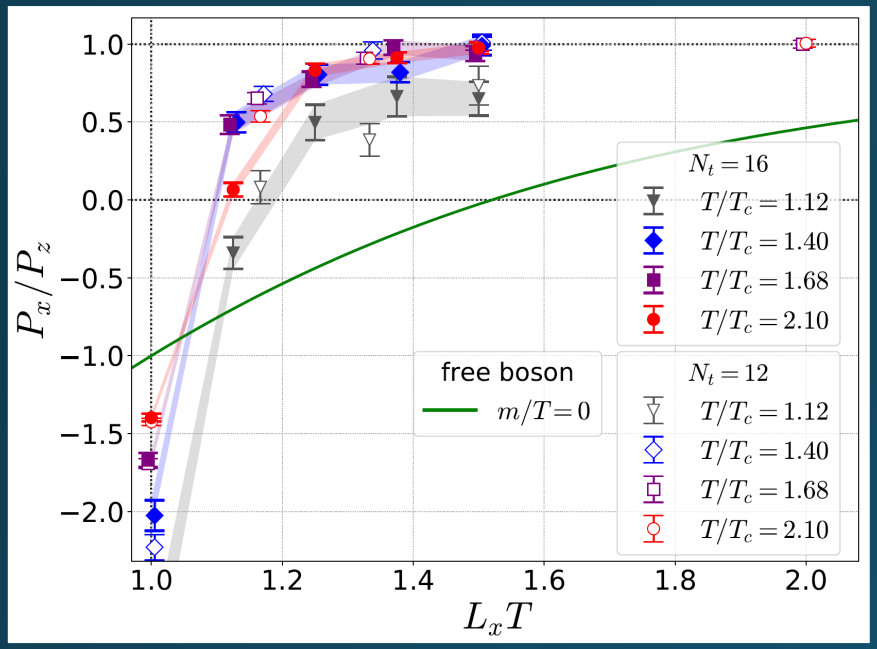
$$\begin{aligned} F_{\rm cross} = g(L_\tau, L_x)\Big[& c_4 {\rm Tr}[P_\tau^\dagger P_\tau]{\rm Tr}[P_x^\dagger P_x] \\ & + c_5({\rm Tr}[P_\tau^\dagger P_\tau]{\rm Tr}[P_x^3] + {\rm Tr}[P_\tau^3]{\rm Tr}[P_x^\dagger P_x]) \\ & + c_6 {\rm Tr}[P_\tau^3]{\rm Tr}[P_x^3]\Big] \end{aligned}$$

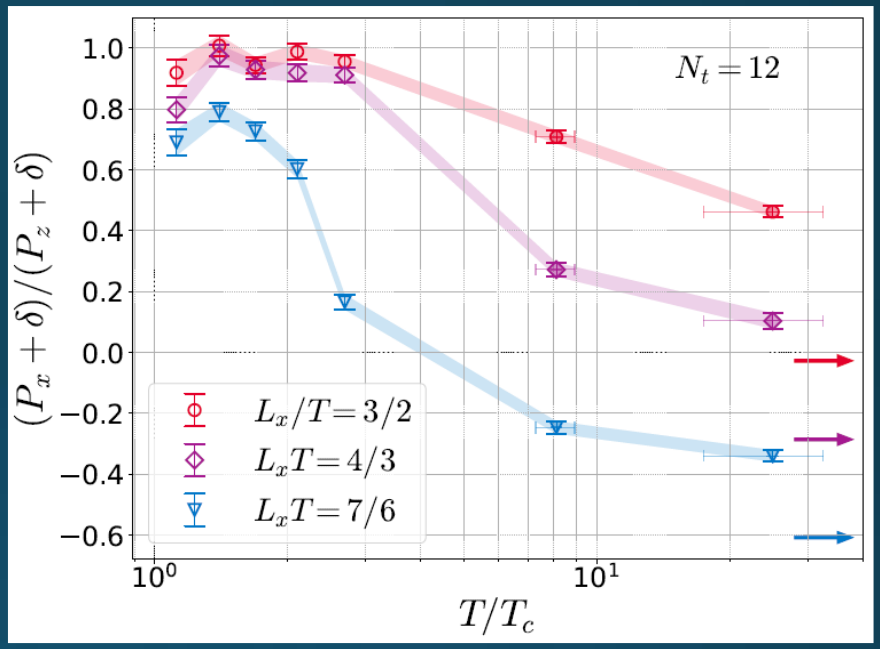
$$g(L_\tau, L_x) = T_c^4\Big((T_c L_\tau)^2 + (T_c L_x)^2\Big)^{-n}$$

$c_4,\ c_5,\ c_6,\ n$: parameters in the model

Result

$$R = \frac{P_x + \delta}{P_z + \delta}$$

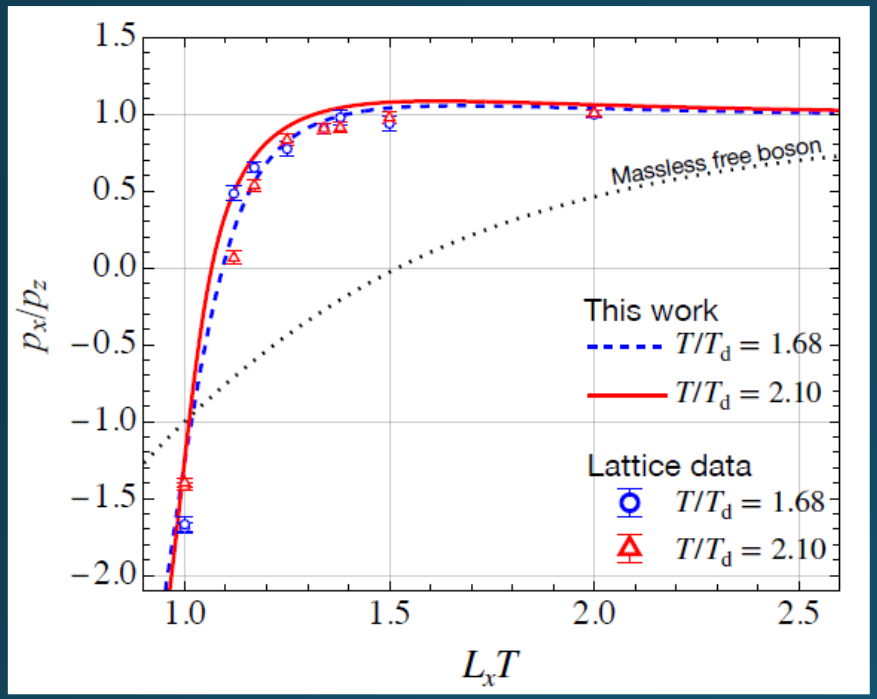


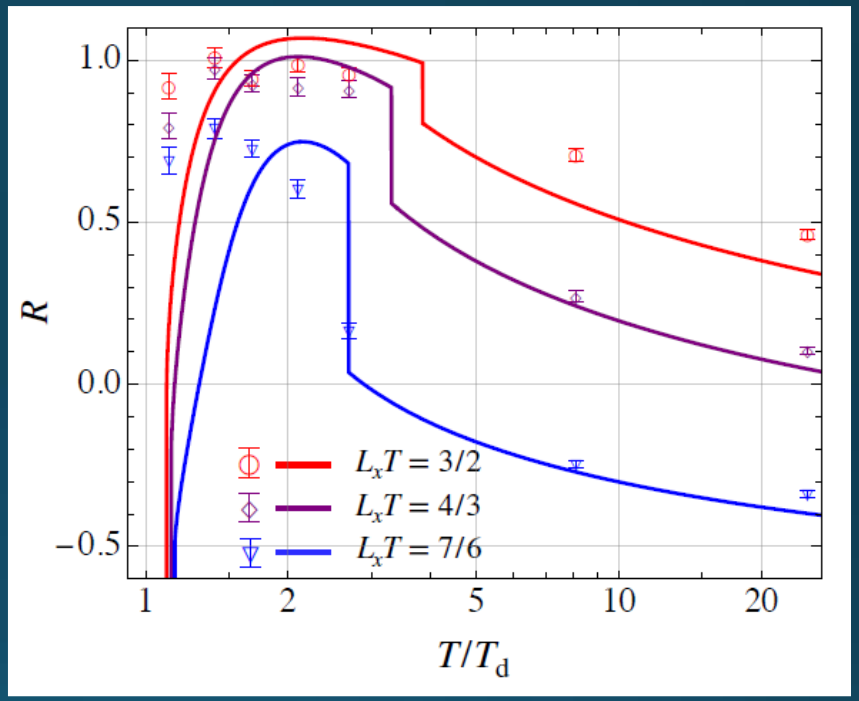


- Lattice results for $T/T_c > 1.5$ are well reproduced.
- No parameters to fit the results for $T/T_c = 1.4, 1.12$.
- Appearance of discontinuity = 1st-order PT

Result

$$R = \frac{P_x + \delta}{P_z + \delta}$$





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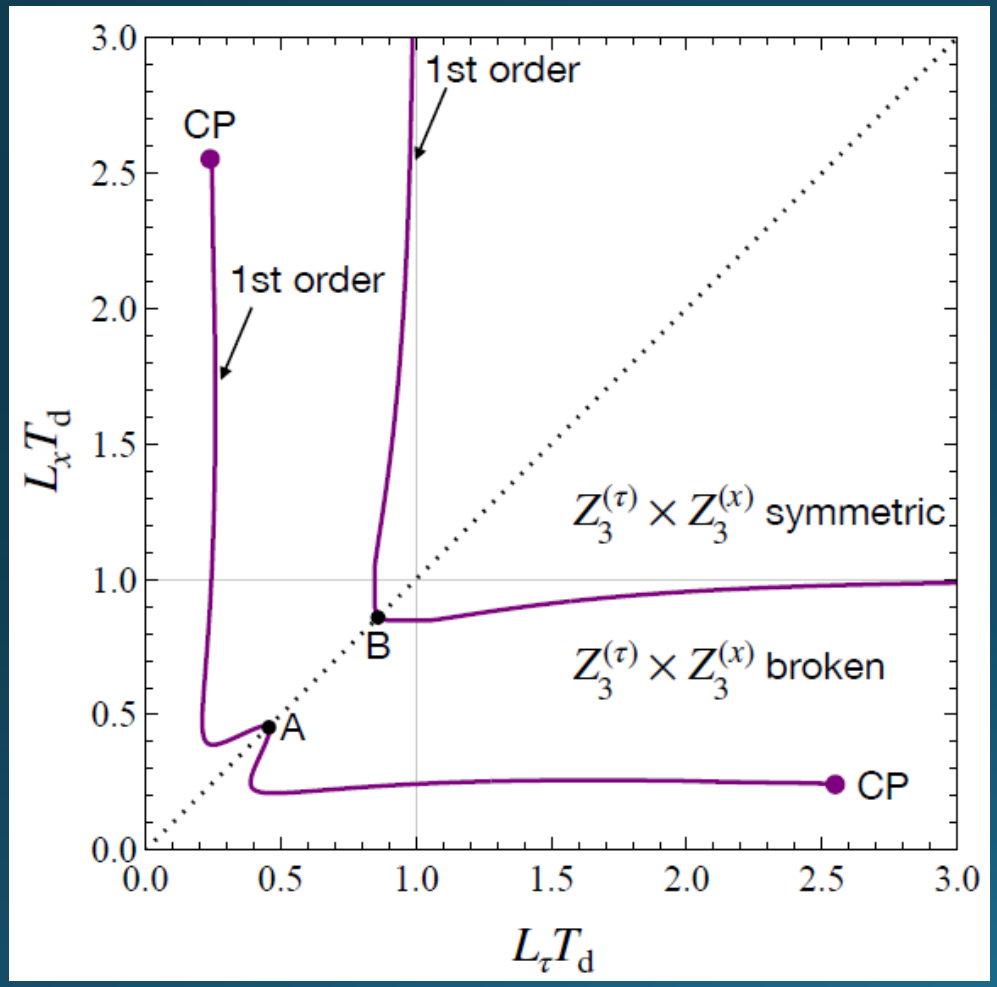
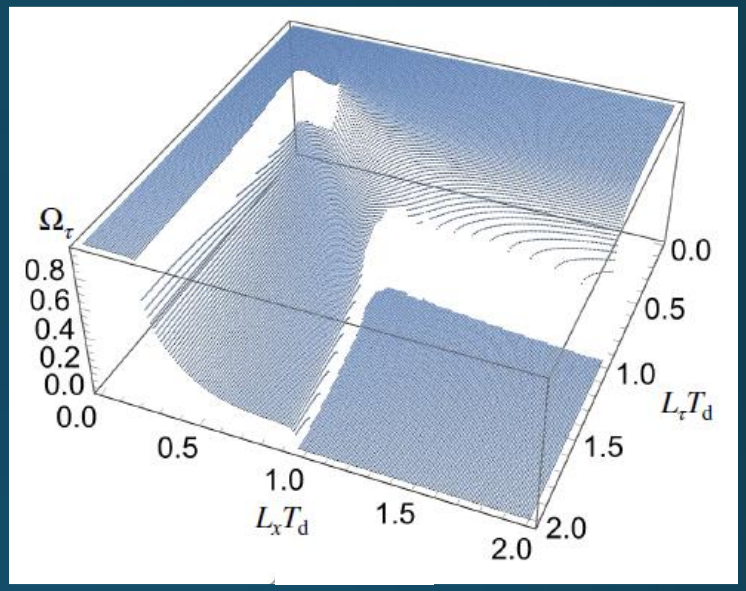
Phase Diagram





2 first-order tranitions!

B: connected to deconf. tr. on $S^1 \times R^3$

A: new phase transition

Novel 1st-tr & CP induced by interplay between 2 Polyakov loops

Summary

Lattice thermodynamics in SU(3) YM on $T^2 \times R^2$ has peculiar behaviors:

- Medium at $1.4 < T/T_c < 2.1$ is remarkably insensitive to the boundary.
- Slow approach to the SB limit at small L_τ, L_x.

Model analysis with two Polyakov loops explains the lattice results for $T \geq 1.5T_c$ qualitatively:

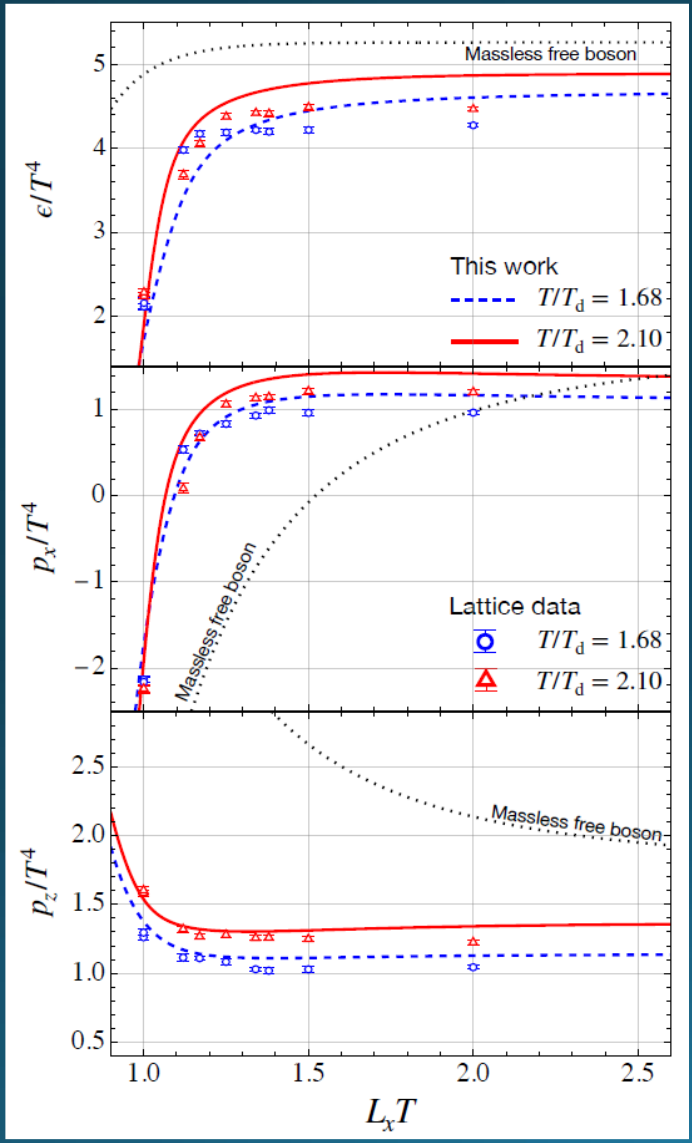
- Interplay b/w two Polyakov loops plays a crucial role.
- Appearance of new 1st-PT & CP is predicted.

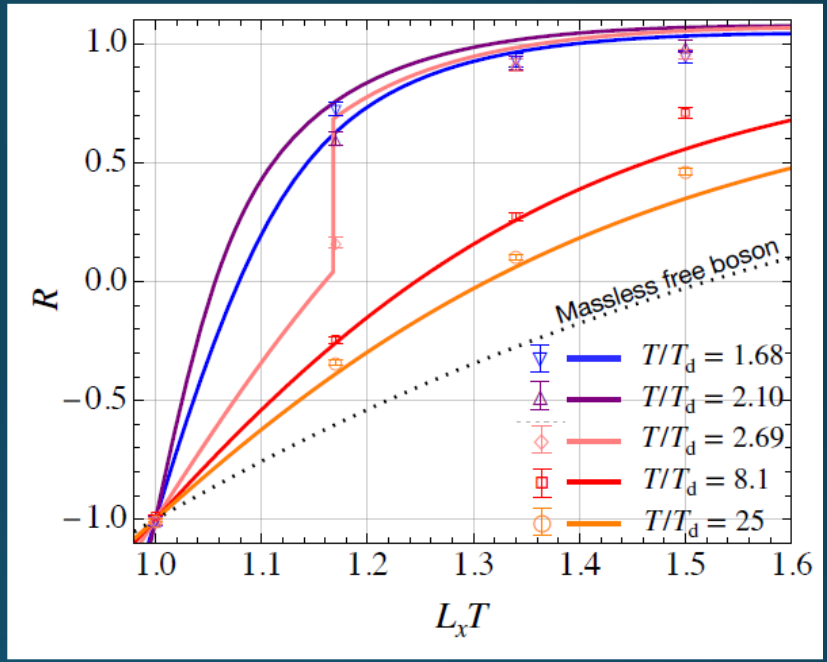
Future

- More lattice results to confirm the existence of the 1st PT
- Anti-periodic / Dirichlet BCs, BC for two directions, below T_c, ...

backup

Numerical Results

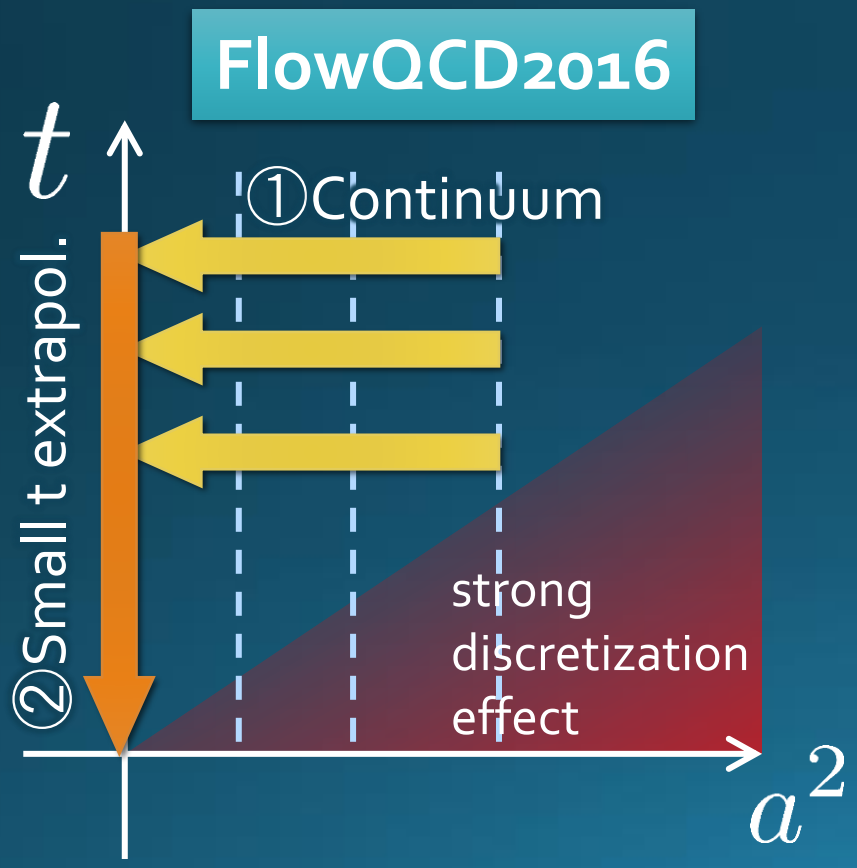


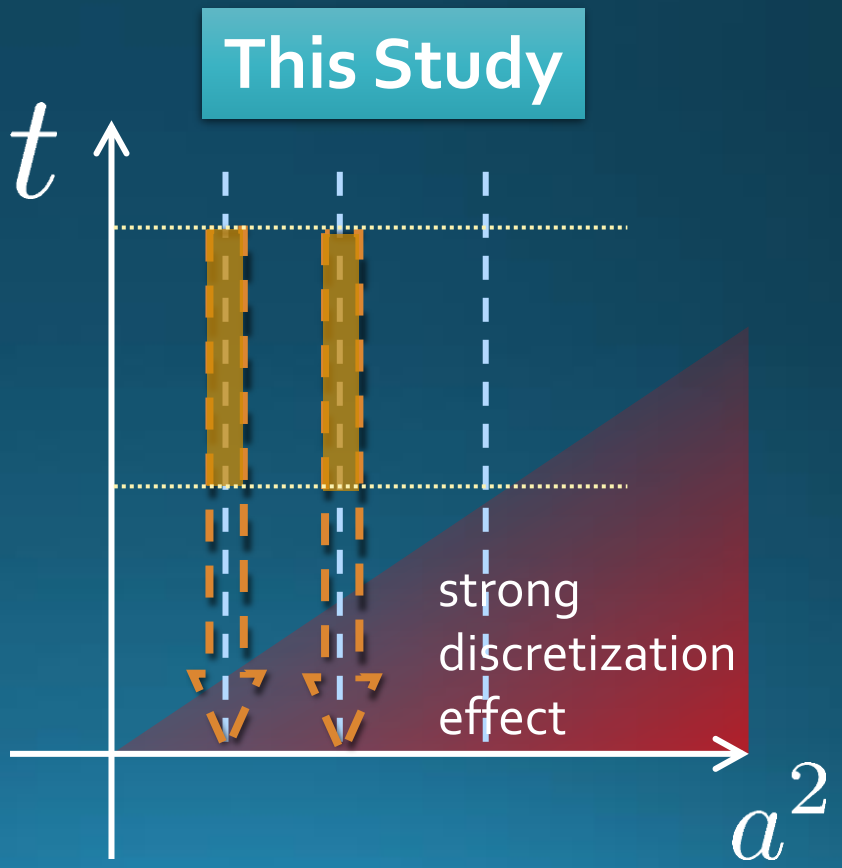


Extrapolations $t \to 0$, $a \to 0$

$$\langle T_{\mu\nu}(t) \rangle_{\text{latt}} = \langle T_{\mu\nu}(t) \rangle_{\text{phys}} + C_{\mu\nu} t + D_{\mu\nu}(t) \frac{a^2}{t}$$

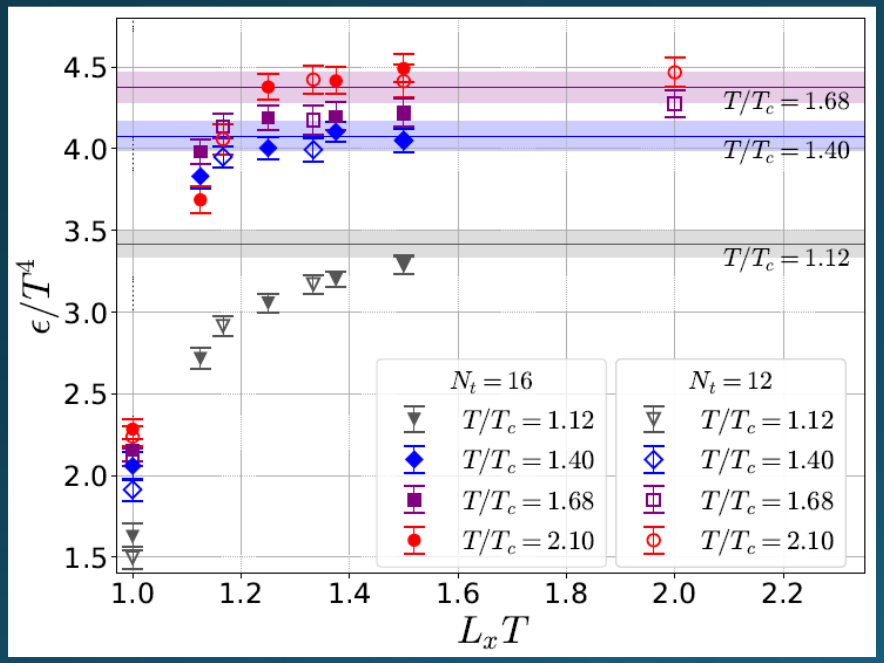
O(t) terms in SFTE lattice discretization



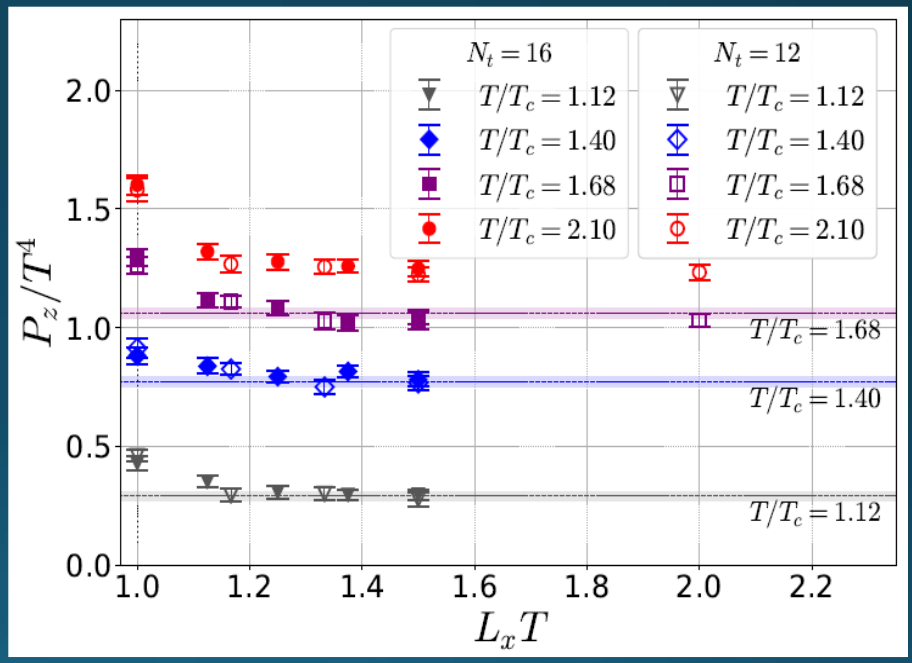


energy densty / transverse P

Energy Density

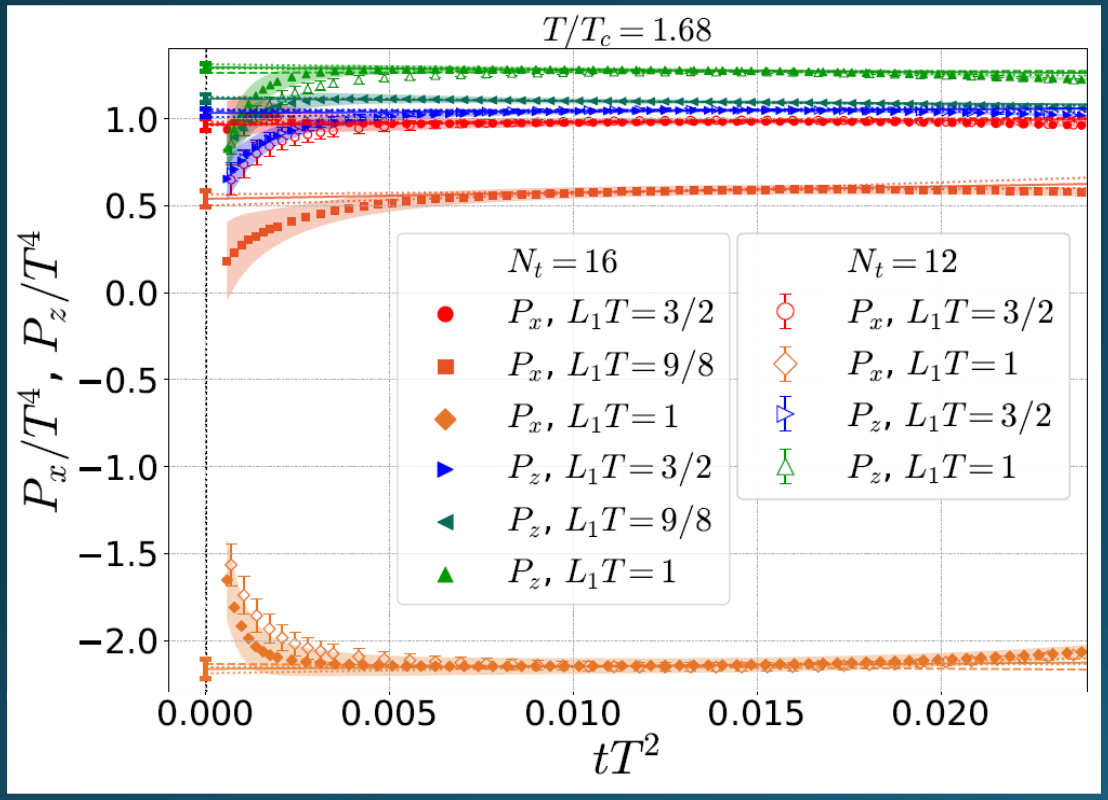


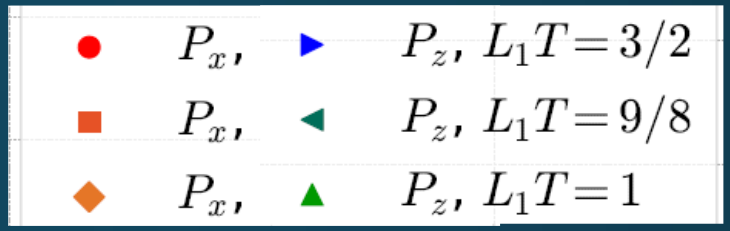
Transverse Pressure P_z



Small-t Extrapolation

$T/T_c = 1.68$





Filled: N_t=16 / Open: N_t=12

Small-t extrapolation

- Solid: N_t=16, Range-1
- Dotted: N_t=16, Range-2,3
- Dashed: N_t=12, Range-1

- Stable small-t extrapolation
- No N_t dependence within statistics for $L_x T$=1, 1.5

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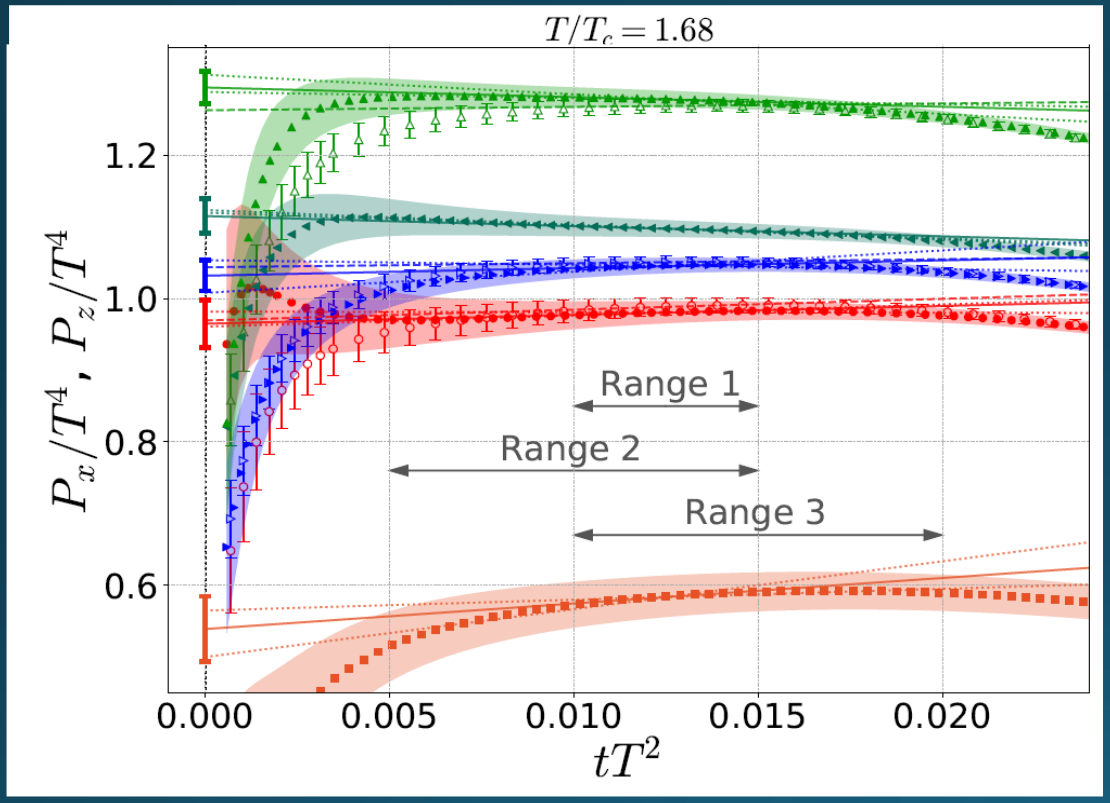

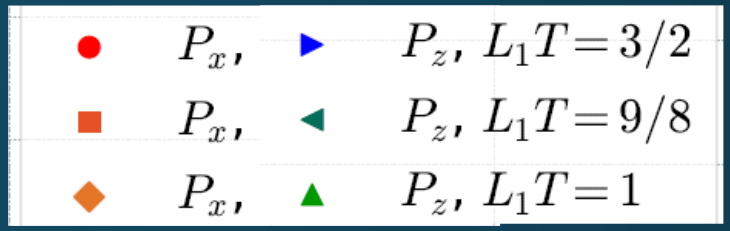

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