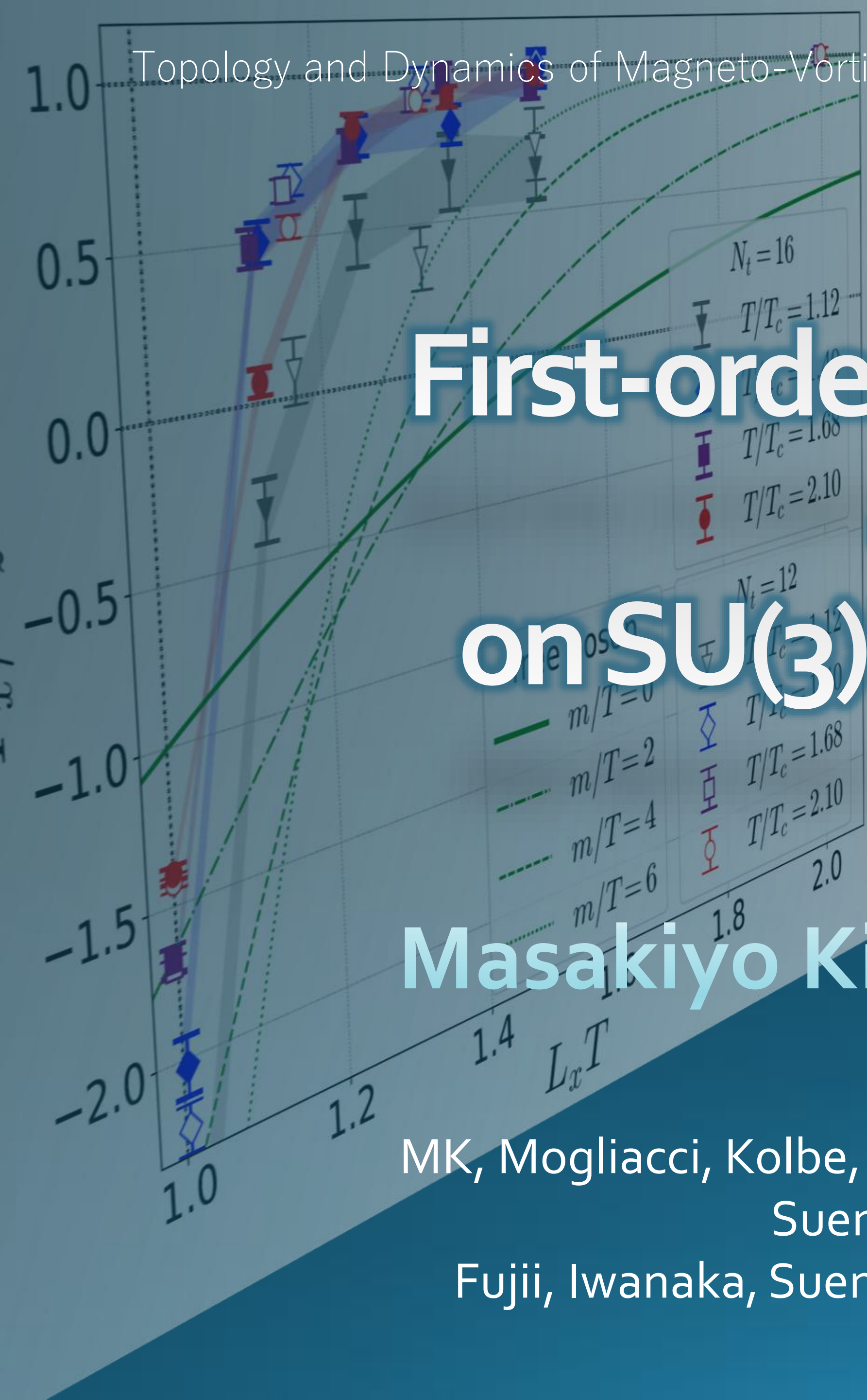


First-order Phase Transition and Critical Points on SU(3) Yang-Mills theory on $T^2 \times R^2$

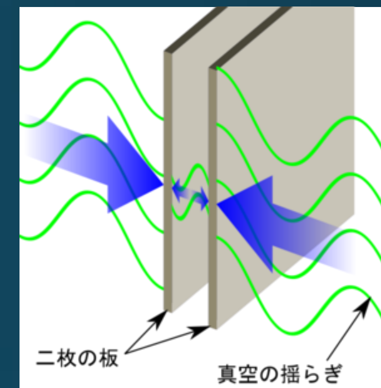
Masakiyo Kitazawa (YITP, Kyoto)

MK, Mogliacci, Kolbe, Horowitz, Phys. Rev. **D99** (2019) 094507
Suenaga, MK, Phys. Rev. **D107** (2023) 074502
Fujii, Iwanaka, Suenaga, MK, Phys.Rev. **D110** (2024) 094016

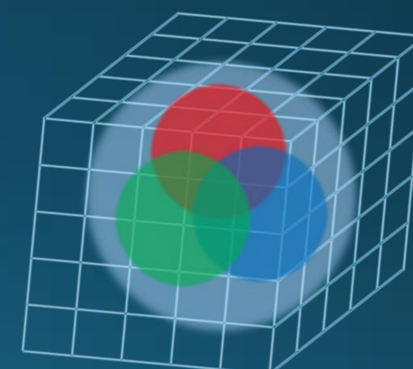
Boundary Conditions in QFT

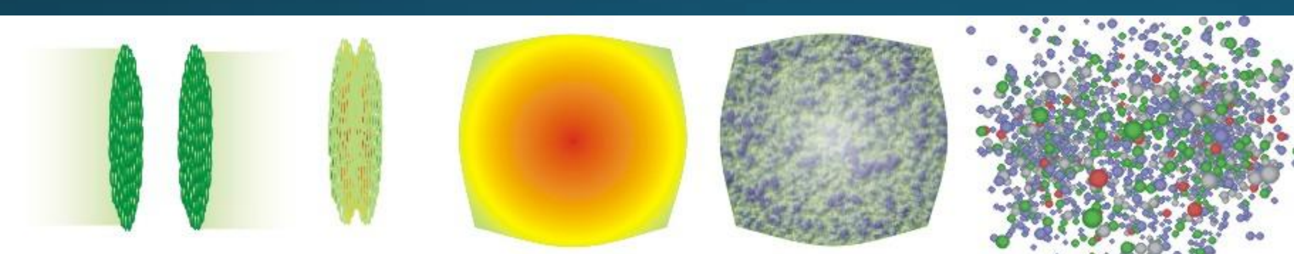
Many motivations

- ☐ Casimir effect
- ☐ Relativistic heavy-ion collisions
- ☐ Numerical simulations (ex. lattice QCD)
- ☐ Matsubara formalism for thermal systems



from wikipedia

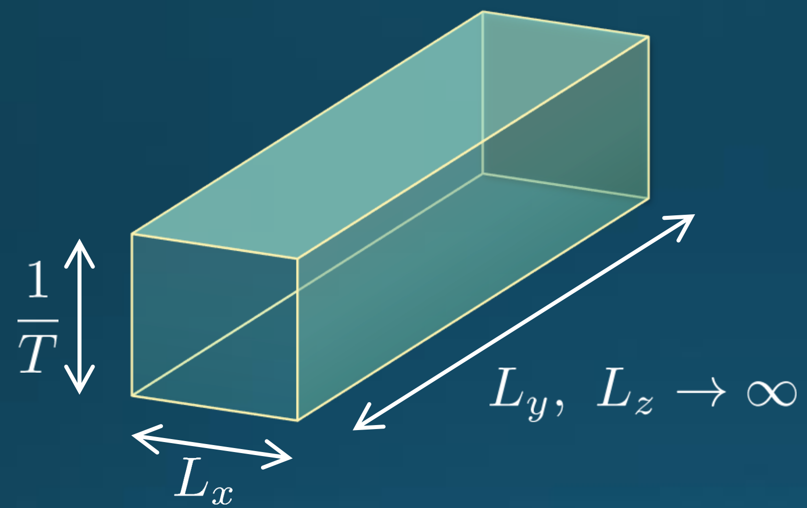




$$L_\tau = \frac{1}{T}$$

Our Purpose

Thermal SU(3) YM with PBC along x direction

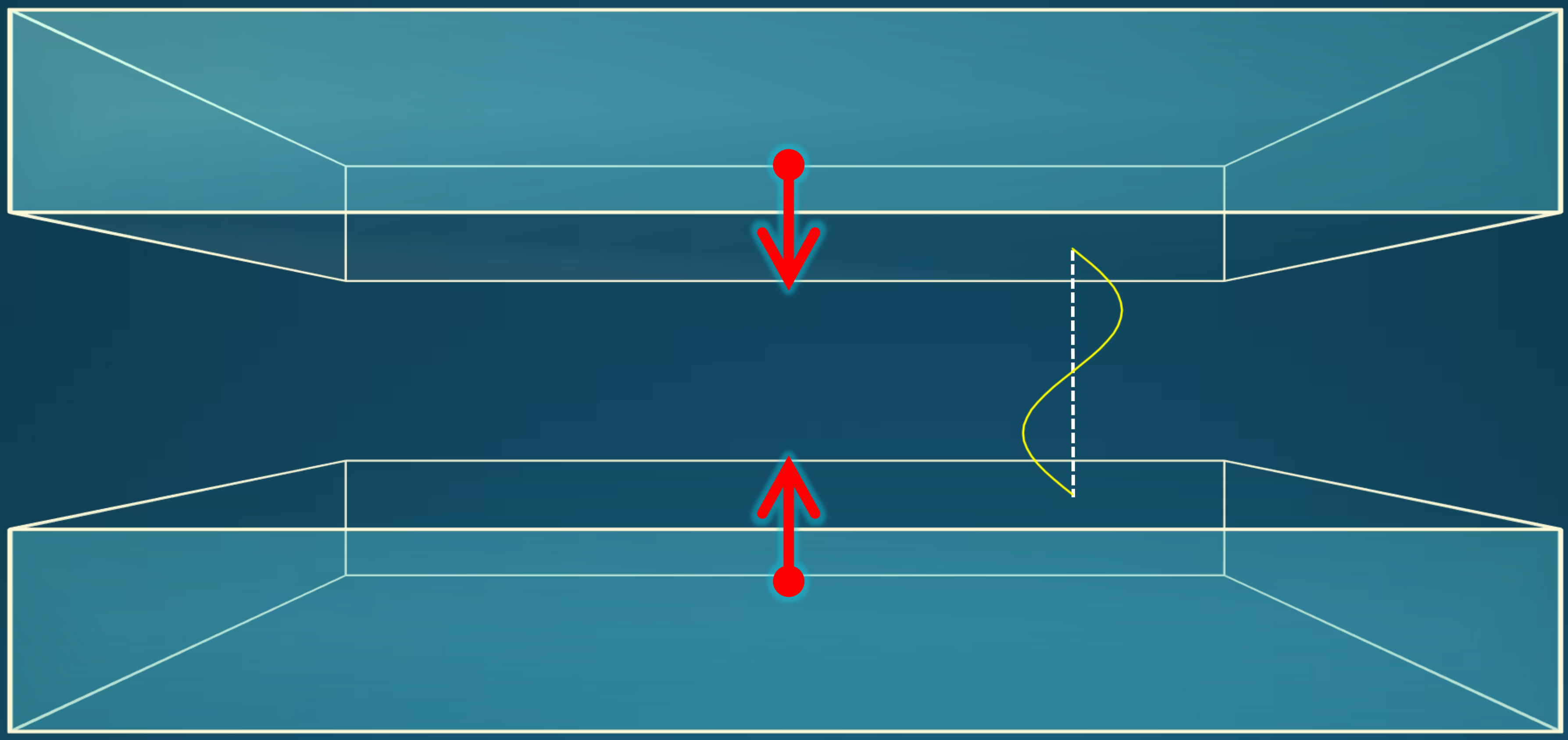


= QFT on Euclidean $T^2 \times R^2$

How does thermodynamics behave w.r.t. T and L_x?

- ☐ Thermal Casimir effect in a non-perturbative system
- ☐ QCD phase diagram as a function of L_x
- ☐ Anisotropic pressure
- ☐ 2 Polyakov loops will play important roles

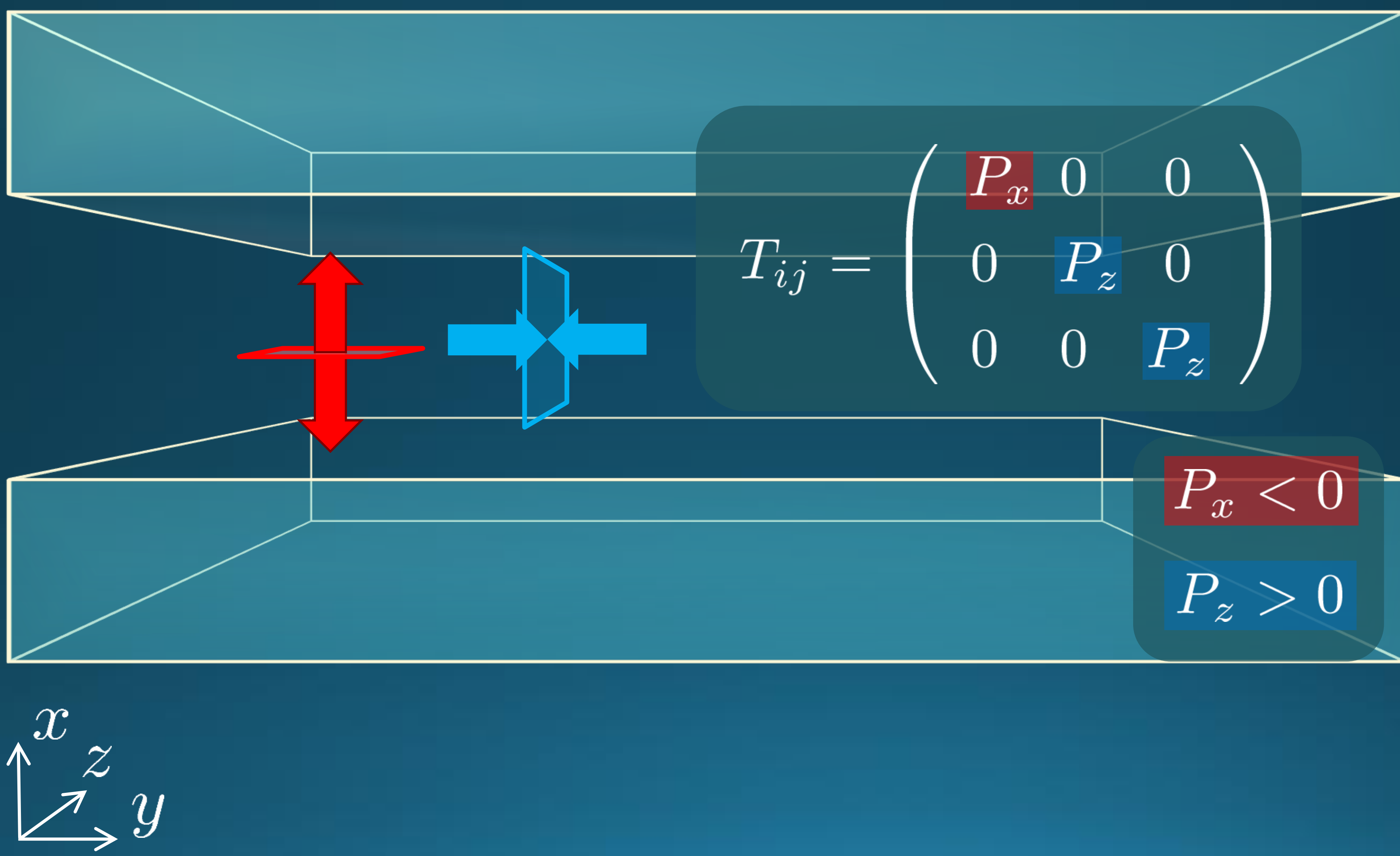
Casimir Effect



attractive force between two conductive plates

Casimir Effect

Brown, Maclay
1969



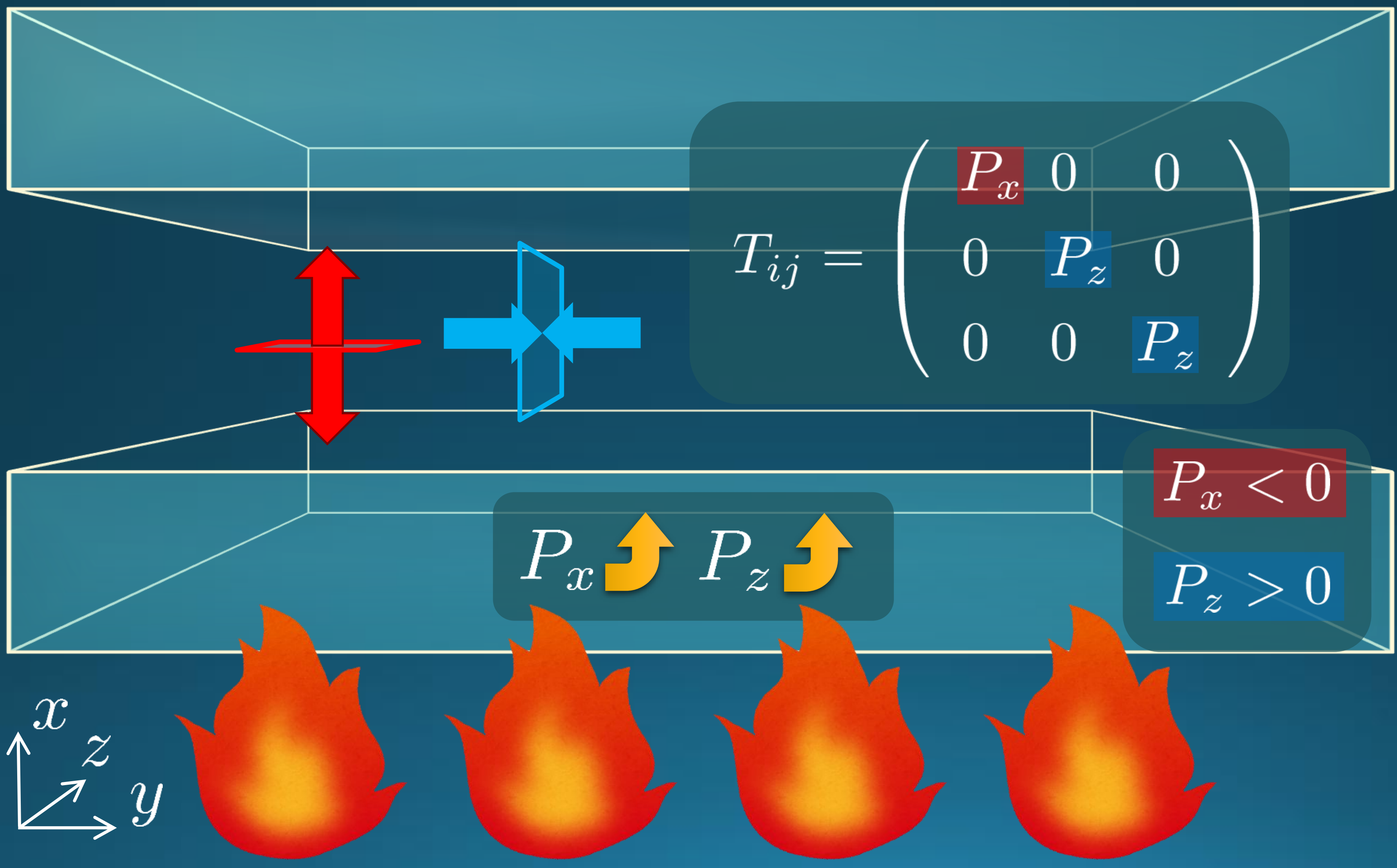
$$T_{ij} = \begin{pmatrix} P_x & 0 & 0 \\ 0 & P_z & 0 \\ 0 & 0 & P_z \end{pmatrix}$$

$$P_x < 0$$

$$P_z > 0$$

Casimir Effect

Brown, Maclay
1969



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Thermodynamics on the Lattice

Various Methods

- **Integral**, differential, moving frame, non-equilibrium, …
- rely on thermodynamic relations valid in V→∞

$$P = \frac{T}{V} \ln Z \qquad sT = \varepsilon + P$$

Not applicable to anisotropic systems

- We employ **Gradient Flow (SF*t*X) Method**

$$\varepsilon = \langle T_{00} \rangle \quad P = \langle T_{11} \rangle$$

Components of EMT are directly accessible!

Yang-Mills Gradient Flow

Luscher 2010
Narayanan, Neuberger, 2006
Luscher, Weiss, 2011

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

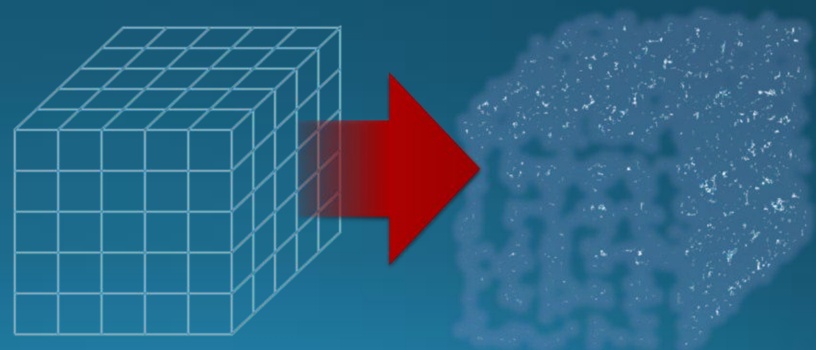
$$A_\mu(0,x) = A_\mu(x)$$

t: "flow time"
dim:[length^2]

leading

$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- diffusion equation in 4-dim space
- diffusion distance $d \sim \sqrt{8t}$
- "continuous" cooling/smearing
- No UV divergence at t>0



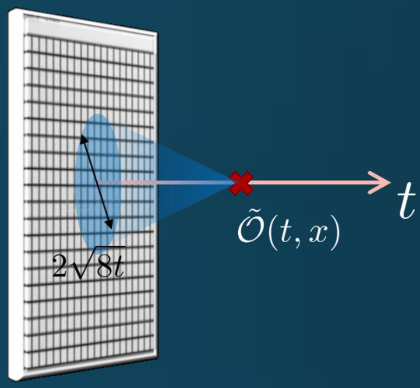
Constructing EMT

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)T^R_{\rho\rho}(x) + \mathcal{O}(t)$$

vacuum subtr.



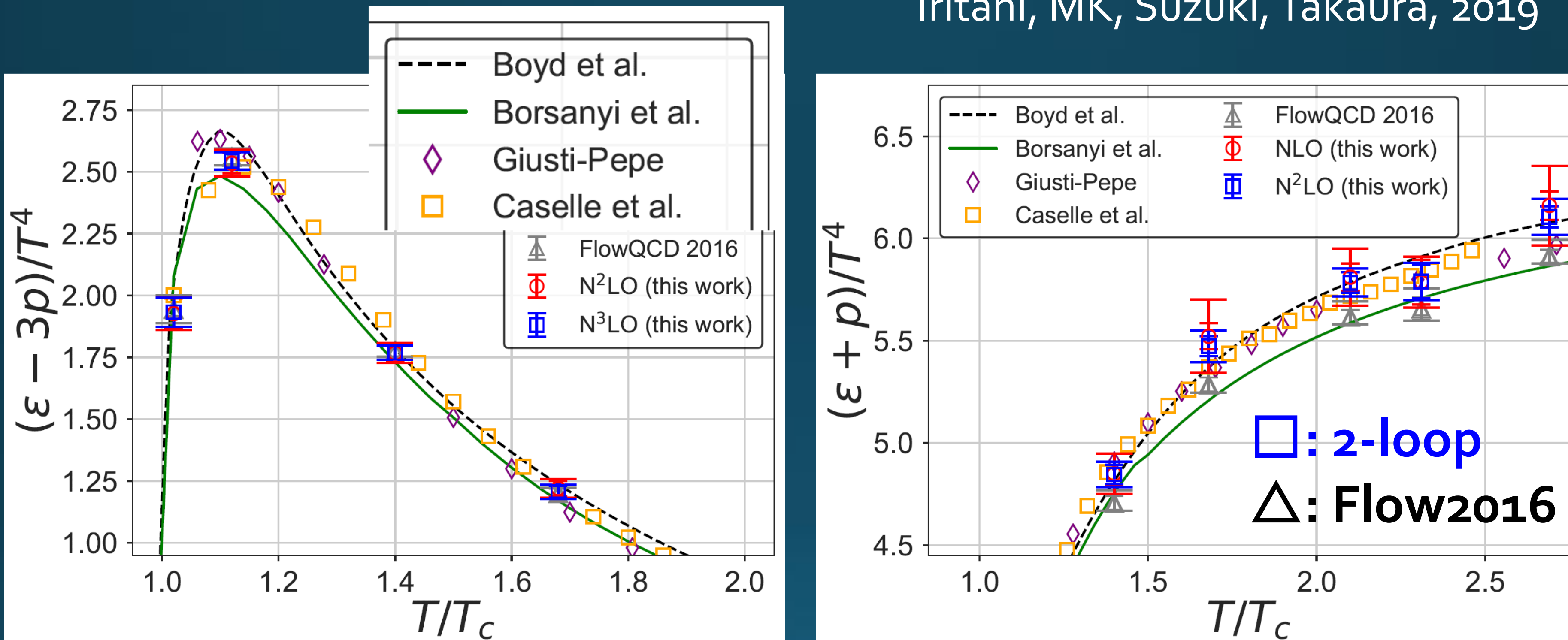
Remormalized EMT

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[c_1(t)U_{\mu\nu}(t,x) + \delta_{\mu\nu}c_2(t)E(t,x)_{\text{subt.}}\right]$$

Perturbative coefficient:
Suzuki (2013); Makino, Suzuki (2014); Harlander+ (2018); Iritani, MK, Suzuki, Takaura (2019)

Thermodynamics

Iritani, MK, Suzuki, Takaura, 2019



Systematic error: μ_0 or μ_d, Λ, $t \to 0$ function, fit range

- Good agreement within **1% level**
- Our method can deal with the anisotropic pressure

Numerical Setup

- SU(3) YM theory
- Wilson gauge action

- $N_t = 16, 12$
- $N_z/N_t = 6$
- 2000~4000 confs.
- Even N_x
- No Continuum extrap.

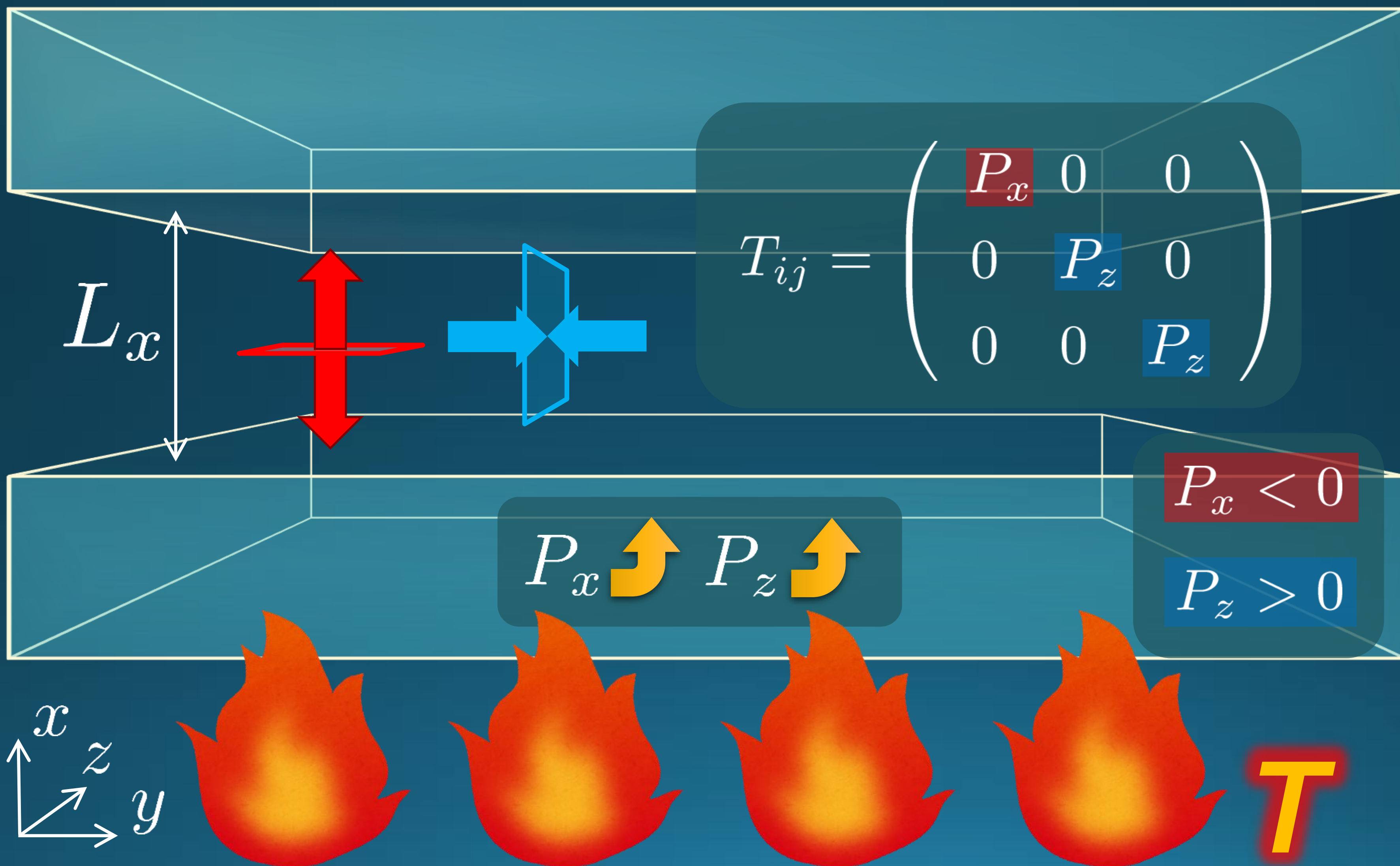
T/T_c	β	N_z	N_τ	N_x	$N_{\rm vac}$
1.12	6.418	72	12	12, 14, 16, 18	64
	6.631	96	16	16, 18, 20, 22, 24	96
1.40	6.582	72	12	12, 14, 16, 18	64
	6.800	96	16	16, 18, 20, 22, 24	128
1.68	6.719	72	12	12, 14, 16, 18, 24	64
	6.719	96	12	14, 18	64
	6.941	96	16	16, 18, 20, 22, 24	96
2.10	6.891	72	12	12, 14, 16, 18, 24	72
	7.117	96	16	16, 18, 20, 22, 24	128
2.69	7.086	72	12	12, 14, 16, 18	-
$\simeq 8.1$	8.0	72	12	12, 14, 16, 18	-
$\simeq 25$	9.0	72	12	12, 14, 16, 18	-

- Same System volume
  - $12 \times 72^2 \times 12 \sim 16 \times 96^2 \times 16$
  - $18 \times 72^2 \times 12 \sim 24 \times 96^2 \times 16$

Simulations on OCTOPUS/Reedbush

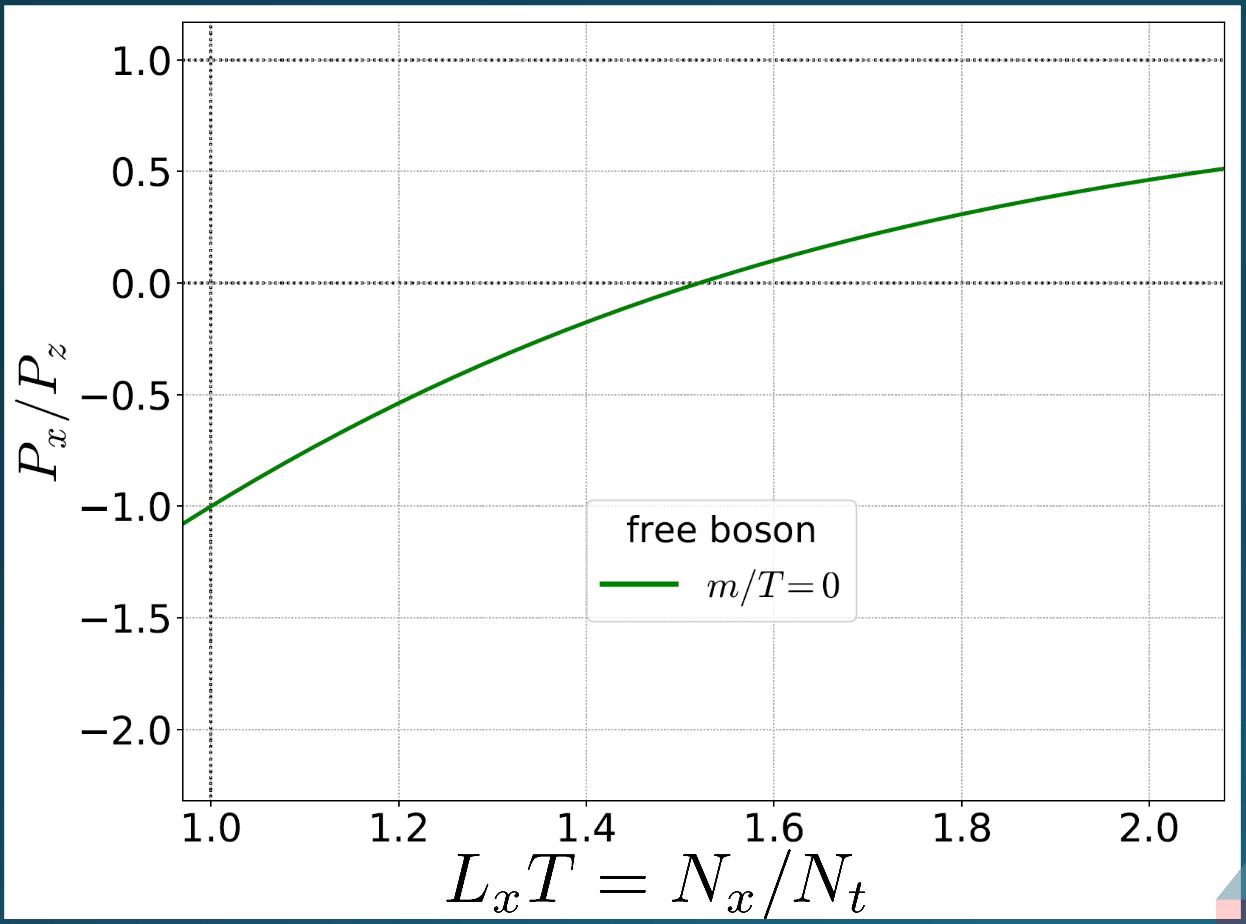
Casimir Effect

Brown, Maclay
1969



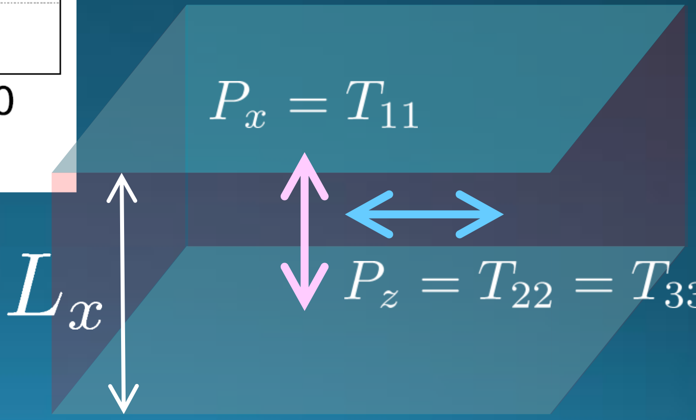
Pressure Anisotropy @ $T \neq 0$

MK, Mogliacci, Kolbe, Horowitz ('21)



Free scalar field

- ☐ $L_2=L_3=\infty$
- ☐ Periodic BC



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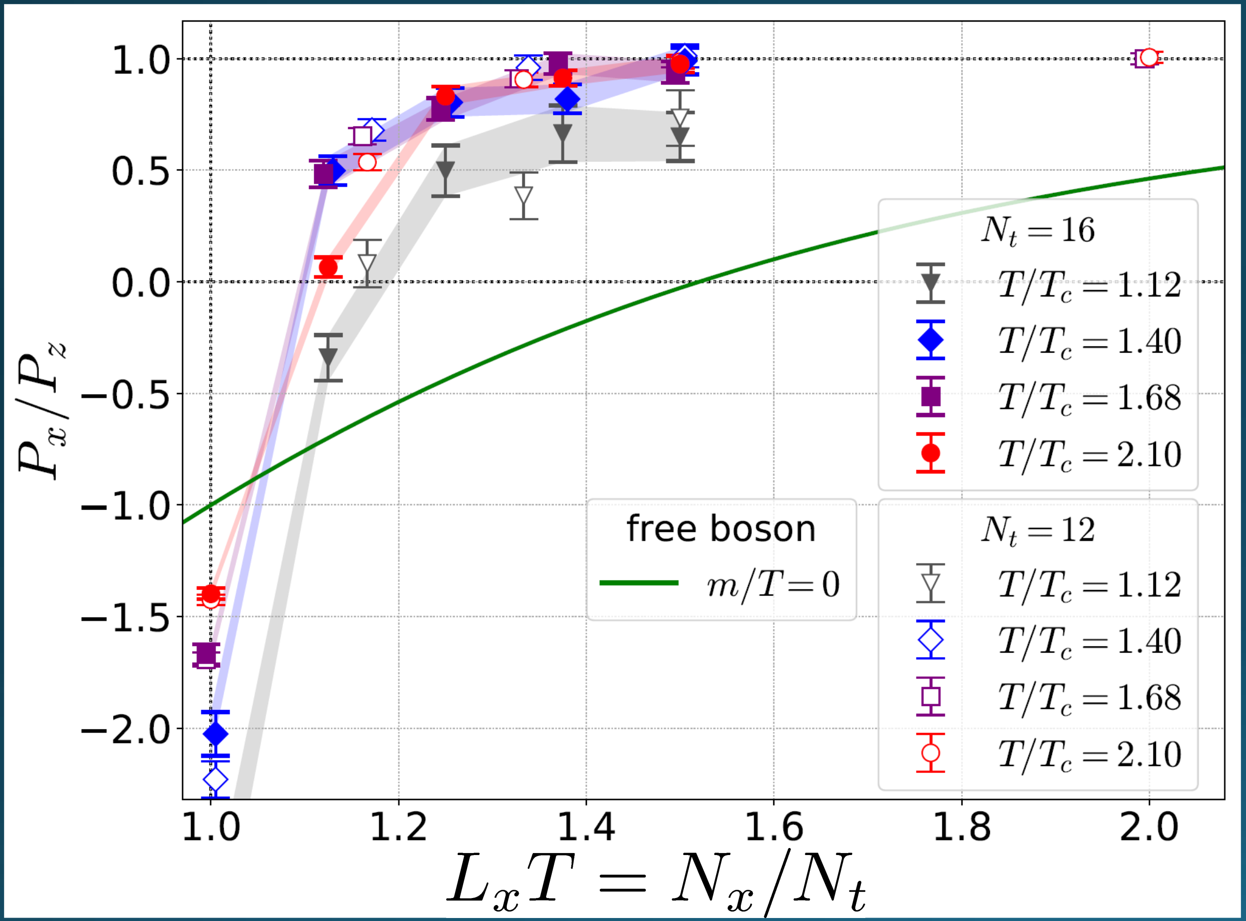


Free scalar field

- $L_2=L_3=\infty$
- Periodic BC

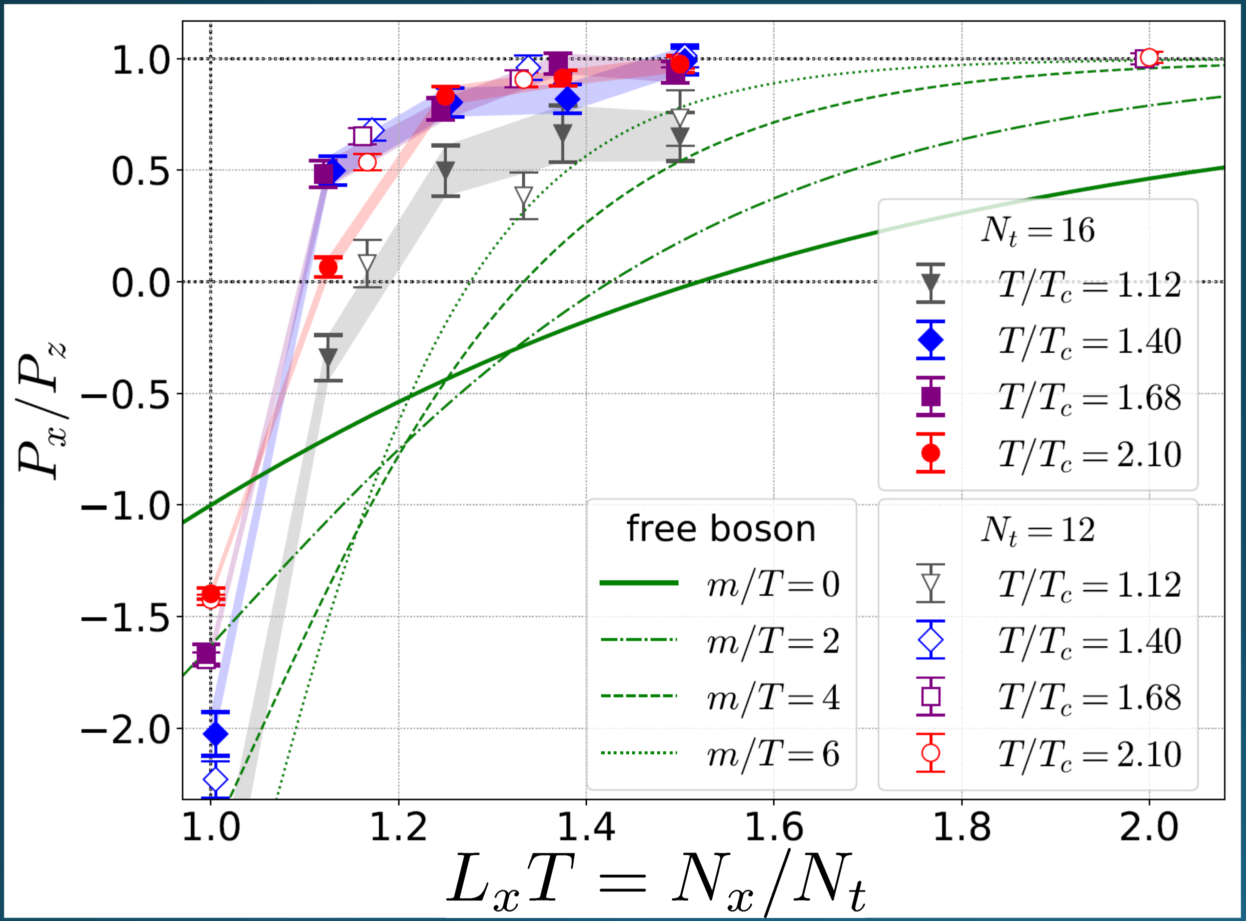
Lattice result

- Periodic BC
- Only t→0 limit
- Error: stat.+sys.

Medium near T_c is remarkably insensitive to finite size!

Pressure Anisotropy @ $T \neq 0$

MK, Mogliacci, Kolbe, Horowitz ('21)



Free scalar field

- $L_2=L_3=\infty$
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Lattice result

- Periodic BC
- Only t→0 limit
- Error: stat.+sys.

Medium near T_c is remarkably insensitive to finite size!

Higher *T*

High-*T* limit: massless free gluons
How does the anisotropy approach this limit?

Difficulties

- ☐ Vacuum subtraction requires large-volume simulations.
- ☐ Lattice spacing not available → $c_1(t)$, $c_2(t)$ are not determined.

Higher T

High-T limit: massless free gluons
How does the anisotropy approach this limit?

Difficulties

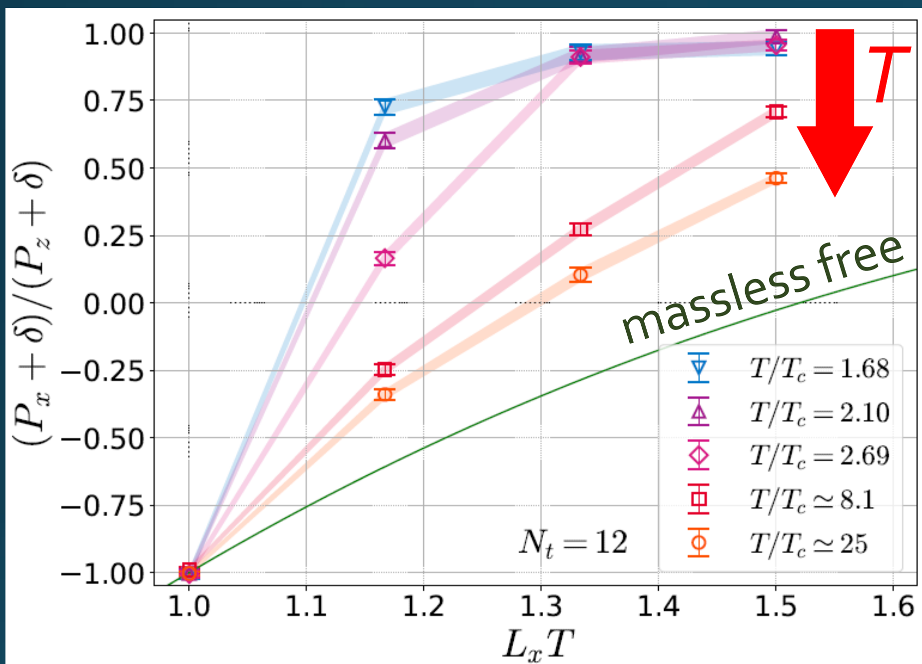
- ☐ Vacuum subtraction requires large-volume simulations.
- ☐ Lattice spacing not available → $c_1(t)$, $c_2(t)$ are not determined.

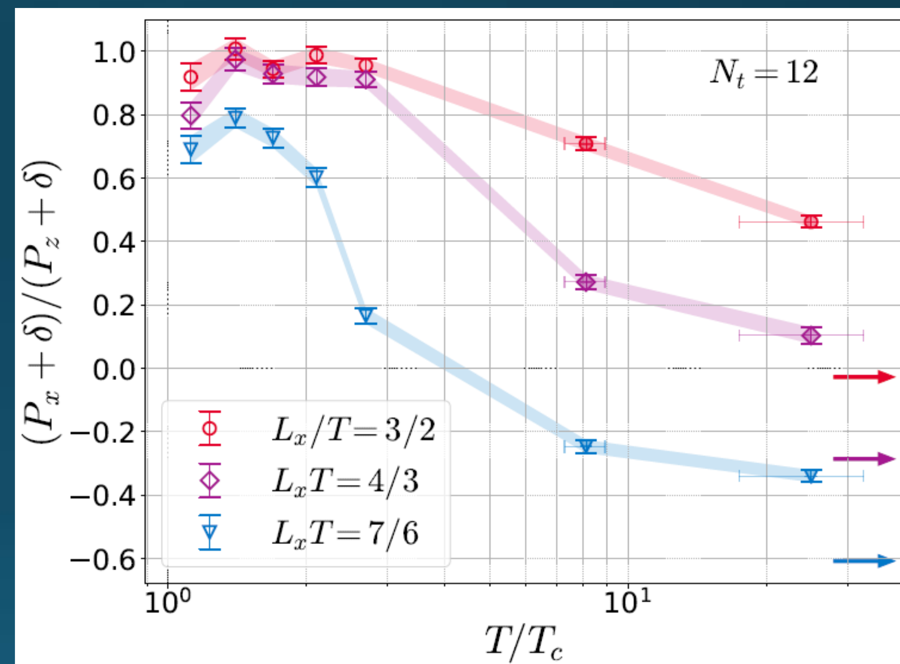
We study

$$R = \frac{P_x + \delta}{P_z + \delta} \qquad \delta = -\frac{1}{4}\sum_{\mu} T^{\mathrm{E}}_{\mu\mu}$$

No vacuum subtr. nor Suzuki coeffs. necessary!

$$\frac{P_x + \delta}{P_z + \delta}$$





$T/T_c \simeq 8.1$ $(\beta = 8.0)$, $T/T_c \simeq 25$ $(\beta = 9.0)$

- ☐ Ratio approaches the asymptotic value for large T.
- ☐ But, large deviation exists even at $T/T_c \simeq 25$.
- ☐ 1st-order phase transition??

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Polyakov-loop Effective Models

Meisinger+, PRD (2003)

General Idea

Constant Polyakov loop P as dynamical variable

$$P = \mathrm{Tr}\left[\mathcal{P}\exp\left(i\int_0^{L_\tau} A_\tau d\tau\right)\right]$$

- $P = 0$: confinement
- $P \neq 0$: deconfinement

Free Energy

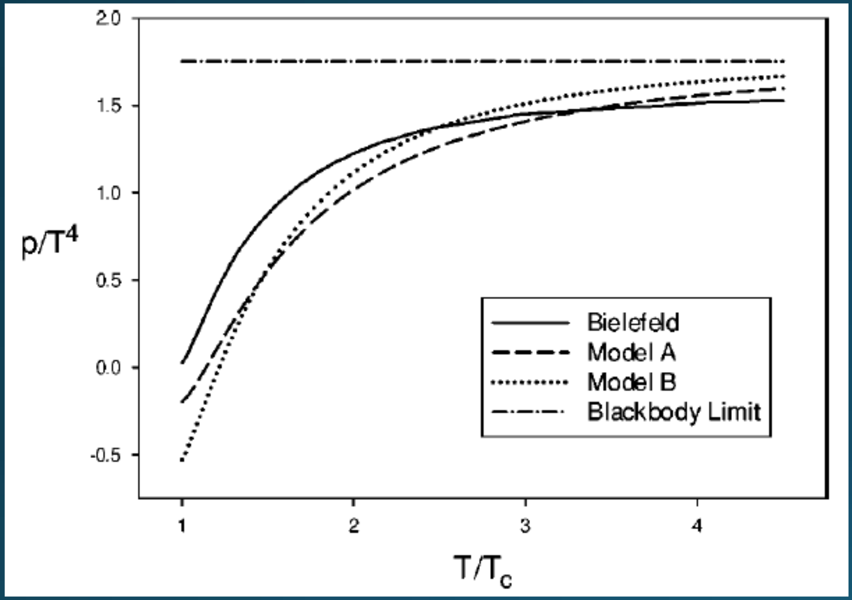
$$F(T;P) = F_{\mathrm{pert.}}(T;P) + F_{\mathrm{pot.}}(T;P)$$

- $F_{\mathrm{pert.}}$: massless free gluons with constant $A_0(x)$
- $F_{\mathrm{pot.}}$: Phenomenological potential term

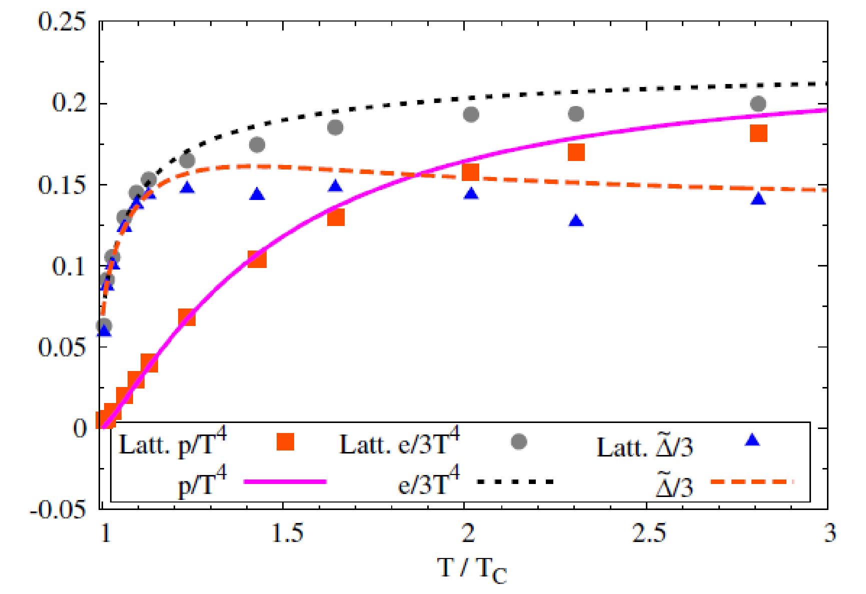
→ $\langle P \rangle$ is determined to minimize $F(T;P)$.

Thermodynamics

Meisinger+, PRD ('03)



Dumitru+, PRD ('12)



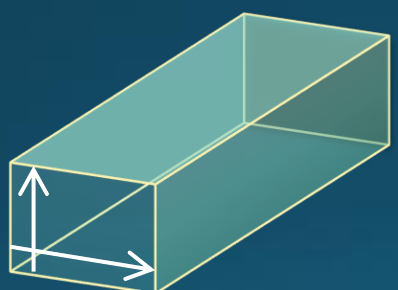
Qualitative behavior of lattice thermodynamics near and above T_c is well reproduced.

Extension to $T^2 \times R^2$

Suenaga, MK ('23); Fujii+ ('24)

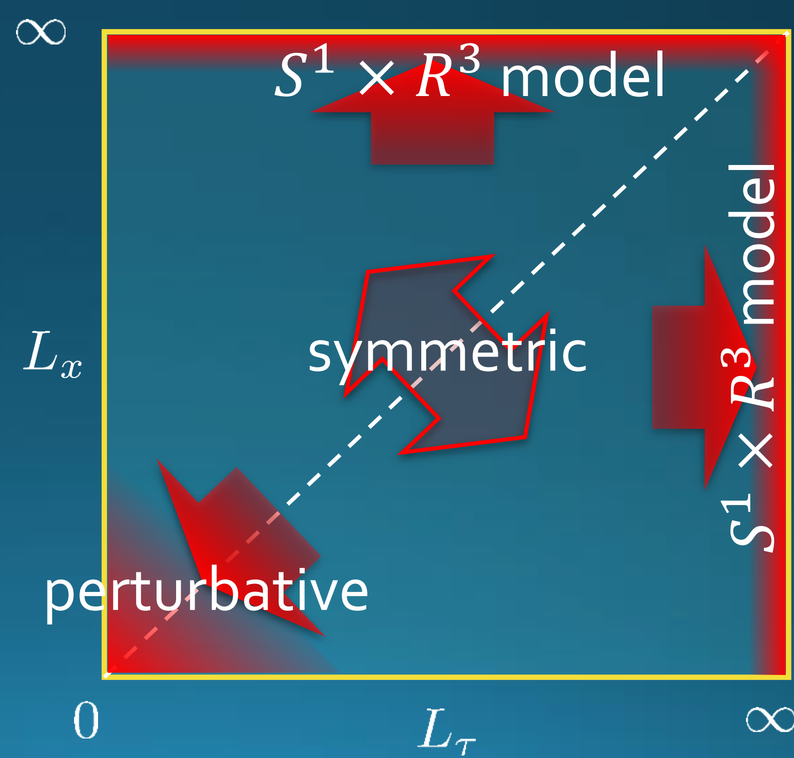
2 Polyakov loops along τ and x directions

$$P_\tau = \mathrm{Tr}\left[\mathcal{P}\exp\left(i\int_0^{L_\tau} A_\tau d\tau\right)\right] \qquad P_x = \mathrm{Tr}\left[\mathcal{P}\exp\left(i\int_0^{L_\tau} A_x d\tau\right)\right]$$



Free Energy

- ☐ Function of 2 Polyakov loops.
- ☐ Constructed under constraints in various limits and symmetries



Polyakov-loop Potential Term

Fujii+ (2024)

$$F_{\rm pot} = F_{\rm sep} + F_{\rm cross}$$

$$F_{\rm sep}(P_\tau, P_x; L_\tau, L_x) = F_{\rm pot}^{S^1\times R^3}(P_\tau, L_\tau) + F_{\rm pot}^{S^1\times R^3}(P_x, L_x)$$

Potential on $S^1 \times R^3$
from Dmitru+('12)

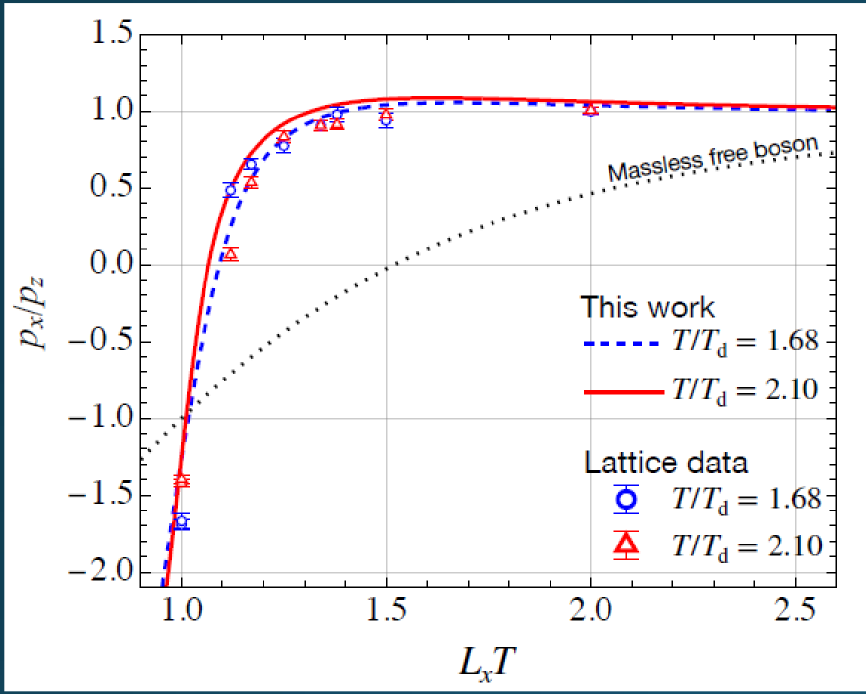
$$F_{\rm cross} = g(L_\tau, L_x)\Big[c_4 {\rm Tr}[P_\tau^\dagger P_\tau]{\rm Tr}[P_x^\dagger P_x] + c_5\left({\rm Tr}[P_\tau^\dagger P_\tau]{\rm Tr}[P_x^3] + {\rm Tr}[P_\tau^3]{\rm Tr}[P_x^\dagger P_x]\right) + c_6 {\rm Tr}[P_\tau^3]{\rm Tr}[P_x^3]\Big]$$

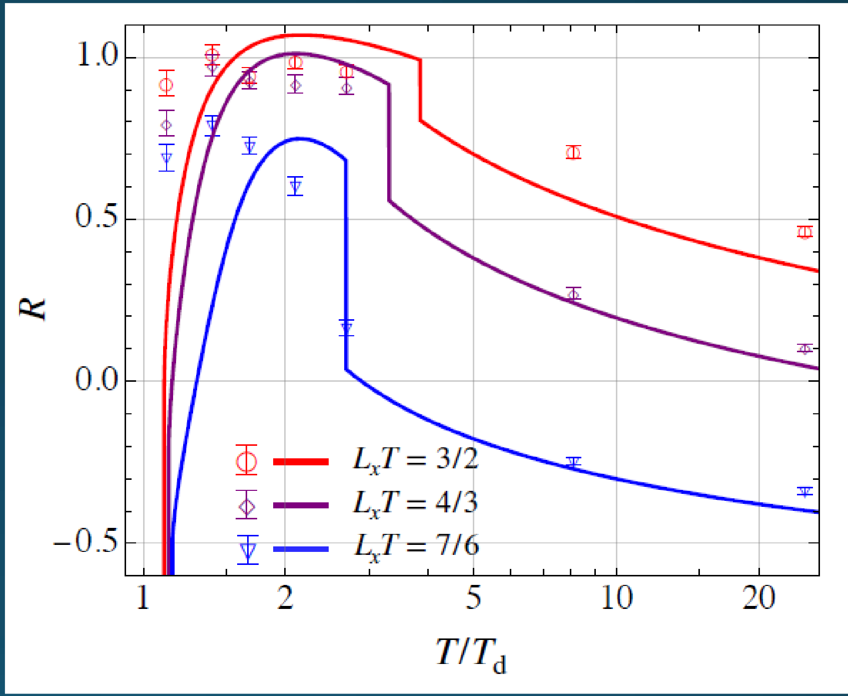
$$g(L_\tau, L_x) = T_c^4\Big((T_c L_\tau)^2 + (T_c L_x)^2\Big)^{-n}$$

$c_4,\ c_5,\ c_6,\ n$: parameters in the model

Result

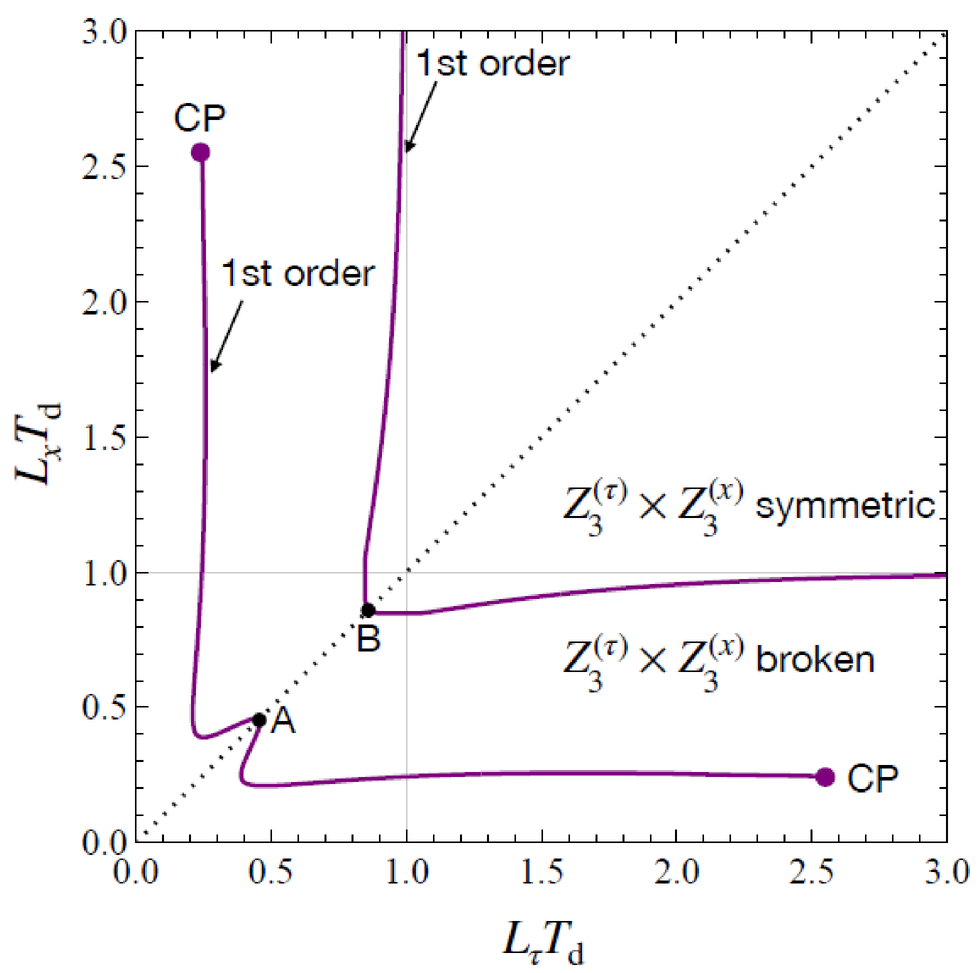
$$R = \frac{P_x + \delta}{P_z + \delta}$$

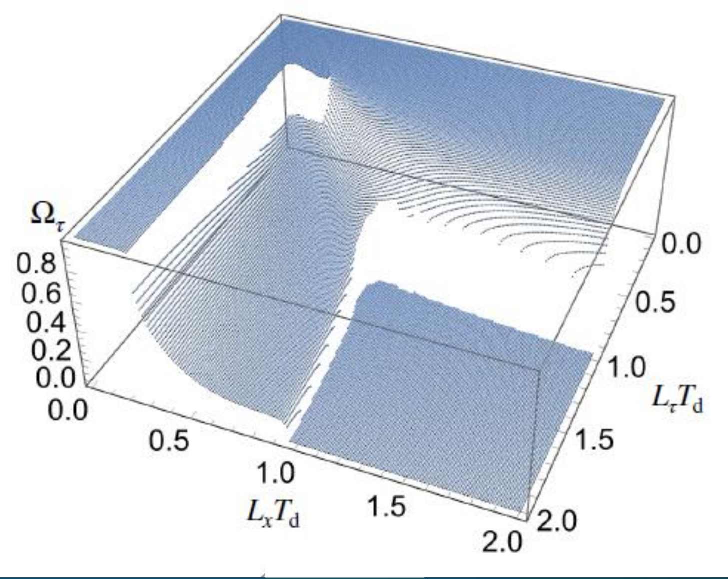




- Lattice results for $T/T_c > 1.5$ are well reproduced.
- No parameters to fit the results for $T/T_c = 1.4, 1.12$.
- Appearance of discontinuity = 1st-order PT

Phase Diagram





2 first-order tranitions!

B: connected to deconf. tr. on $S^1 \times R^3$

A: new phase transition

Novel 1st-tr & CP induced by interplay between 2 Polyakov loops

Summary

Lattice thermodynamics in SU(3) YM on $T^2 \times R^2$ has peculiar behaviors:

- Medium at $1.4<T/T_c<2.1$ is remarkably insensitive to the boundary.
- Slow approach to the SB limit at small L_τ, L_x.

Model analysis with two Polyakov loops explains the lattice results for $T \geq 1.5T_c$ qualitatively:

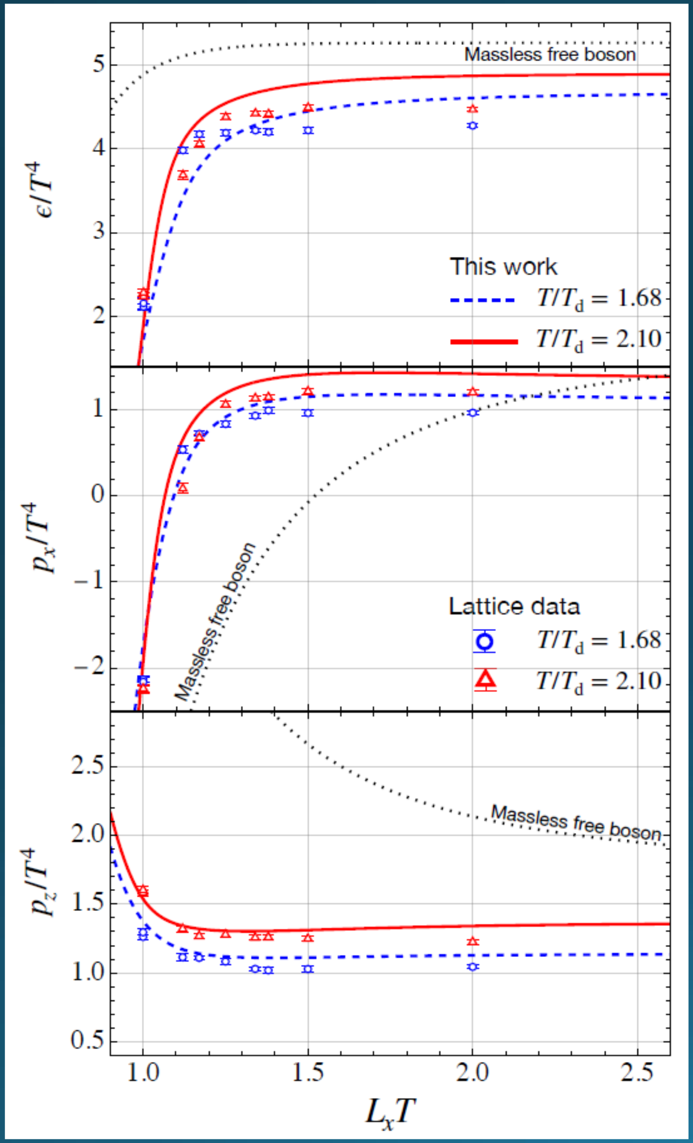
- Interplay b/w two Polyakov loops plays a crucial role.
- Appearance of new 1st-PT & CP is predicted.

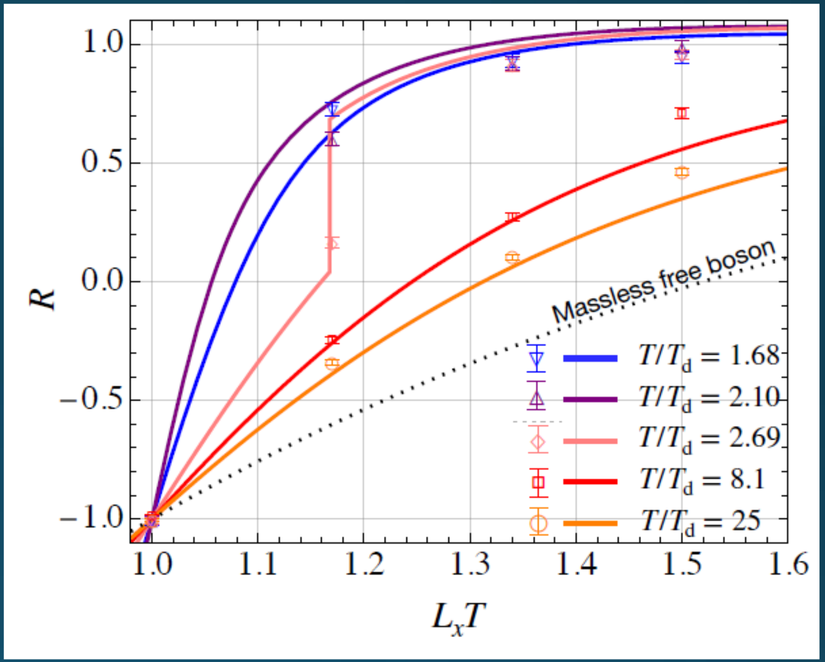
Future

- More lattice results to confirm the existence of the 1st PT
- Anti-periodic / Dirichlet BCs, BC for two directions, below T_c, …

backup

Numerical Results



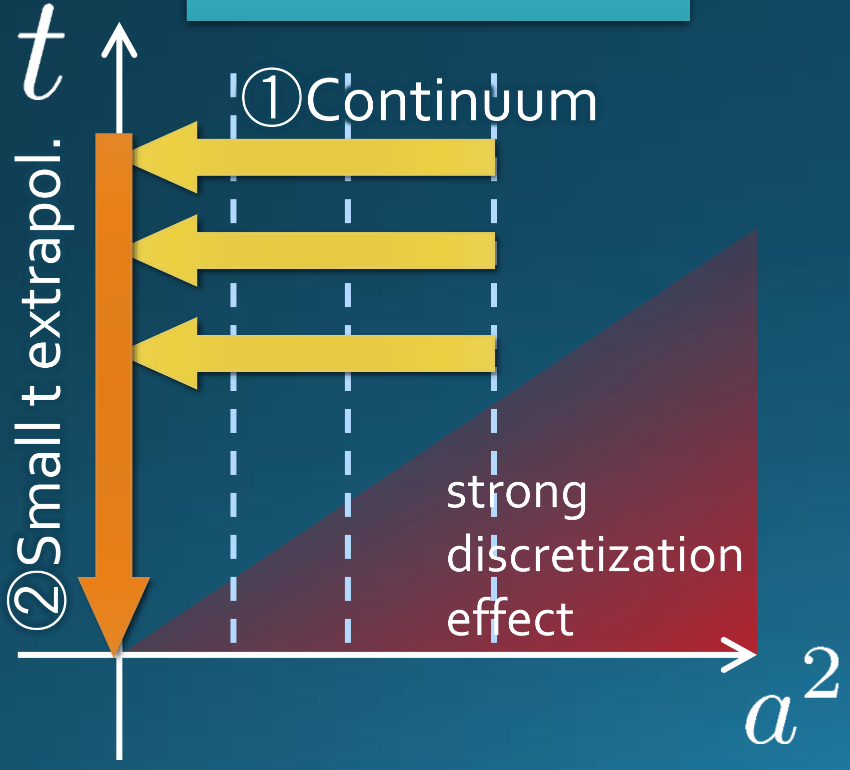


Extrapolations t→0, a→0

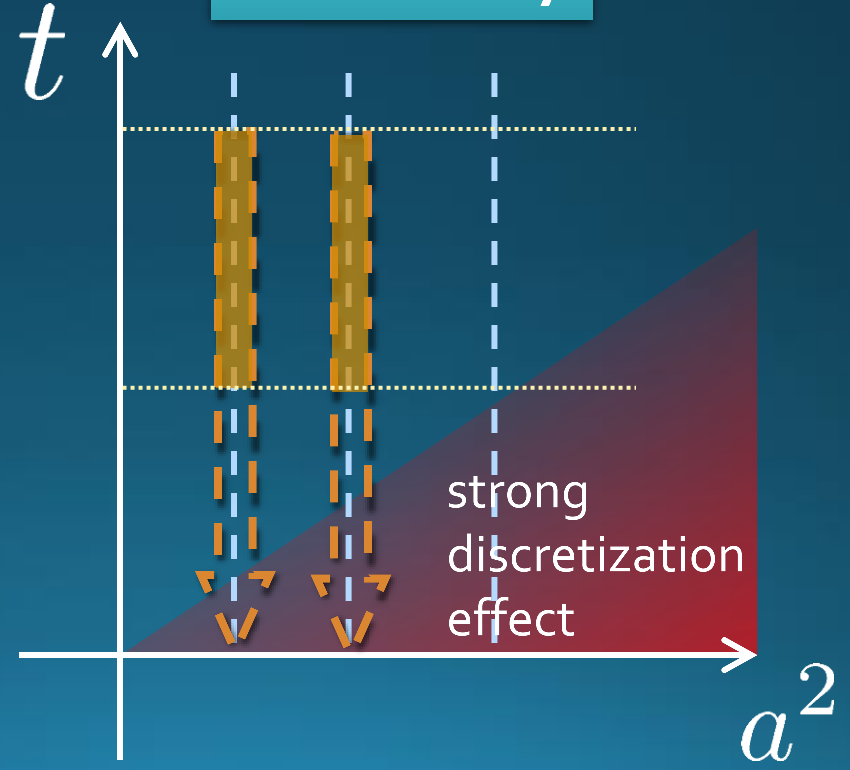
$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu}t + D_{\mu\nu}(t)\frac{a^2}{t}$$

O(t) terms in SFTE　lattice discretization

FlowQCD2016

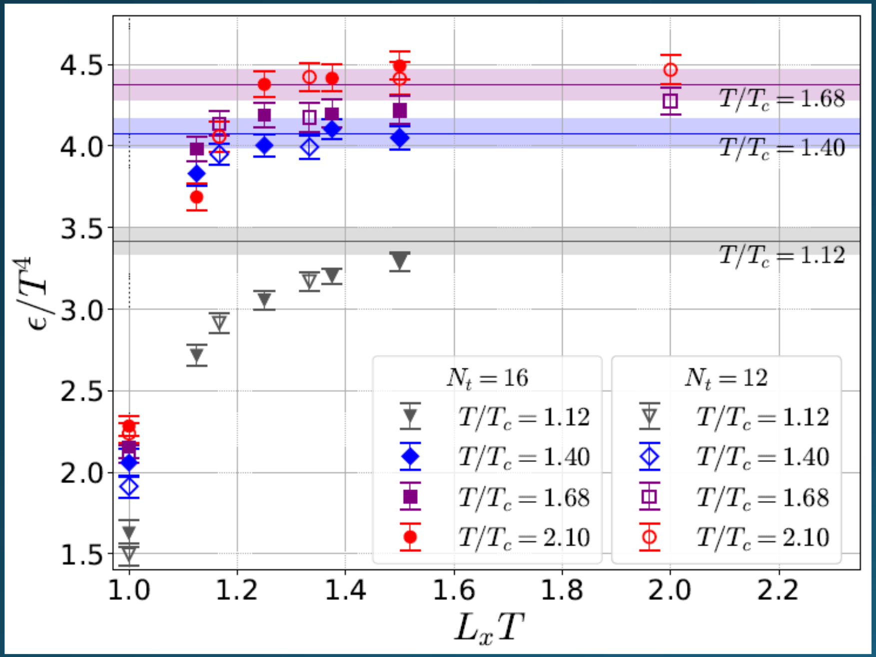


This Study

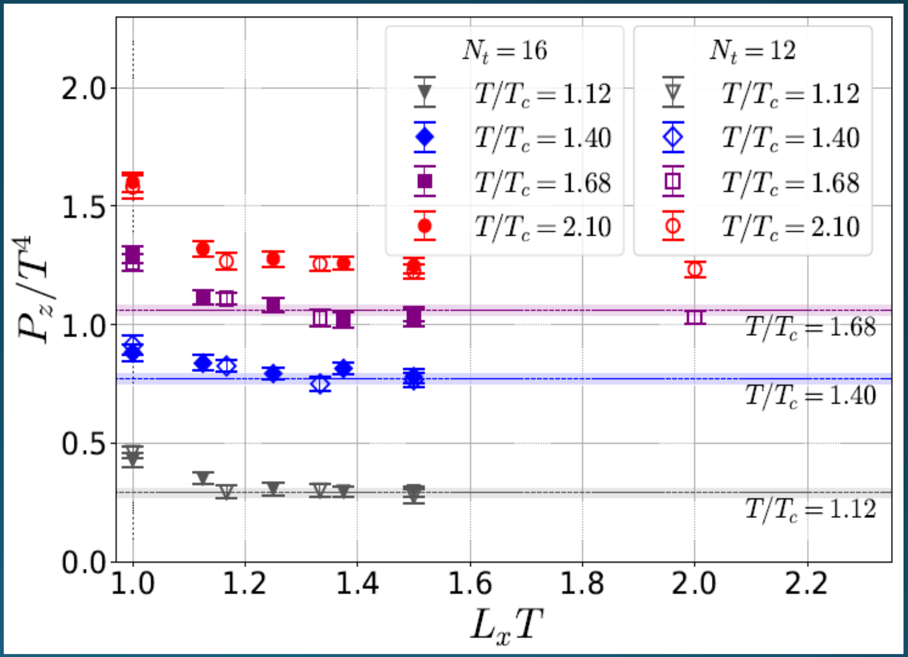


energy densty / transverse P

Energy Density

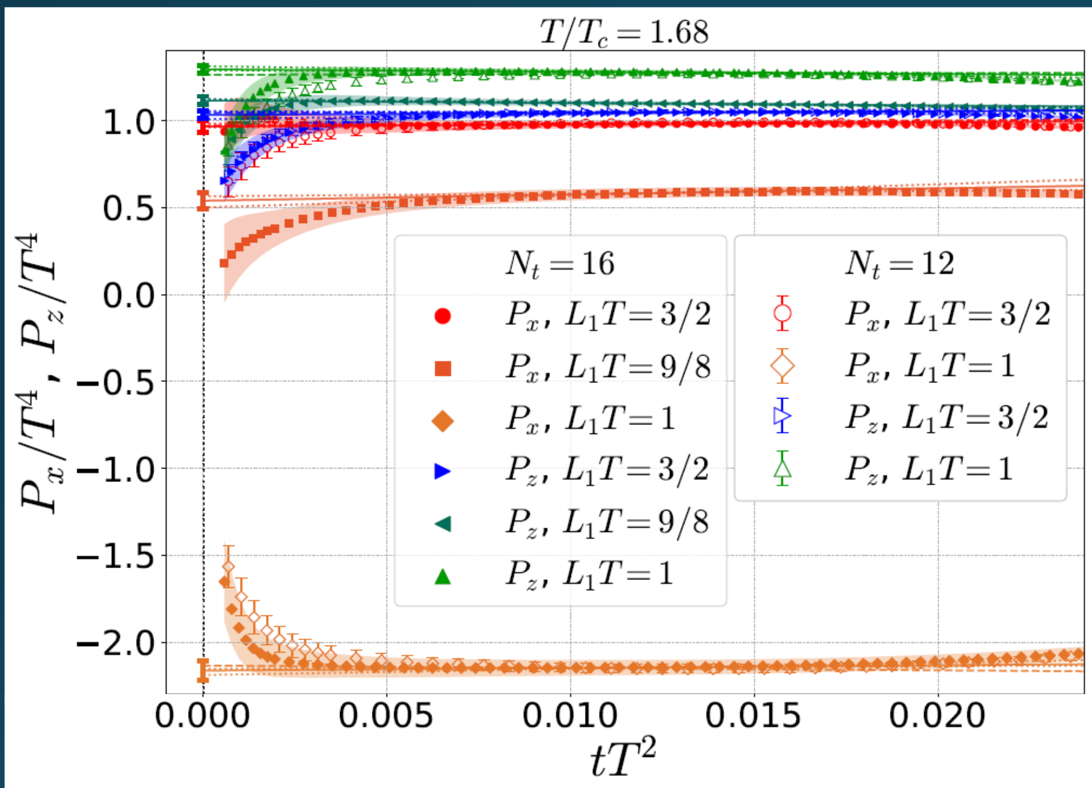


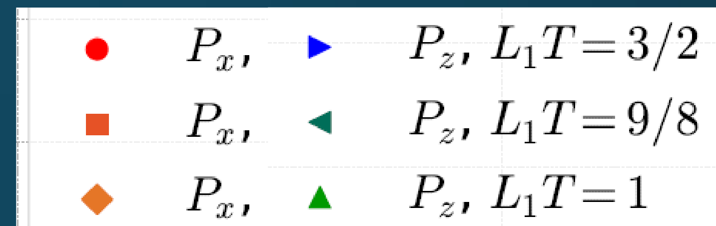
Transverse Pressure P_z



Small-t Extrapolation

$T/T_c = 1.68$





Filled: N_t=16 / Open: N_t=12

Small-t extrapolation

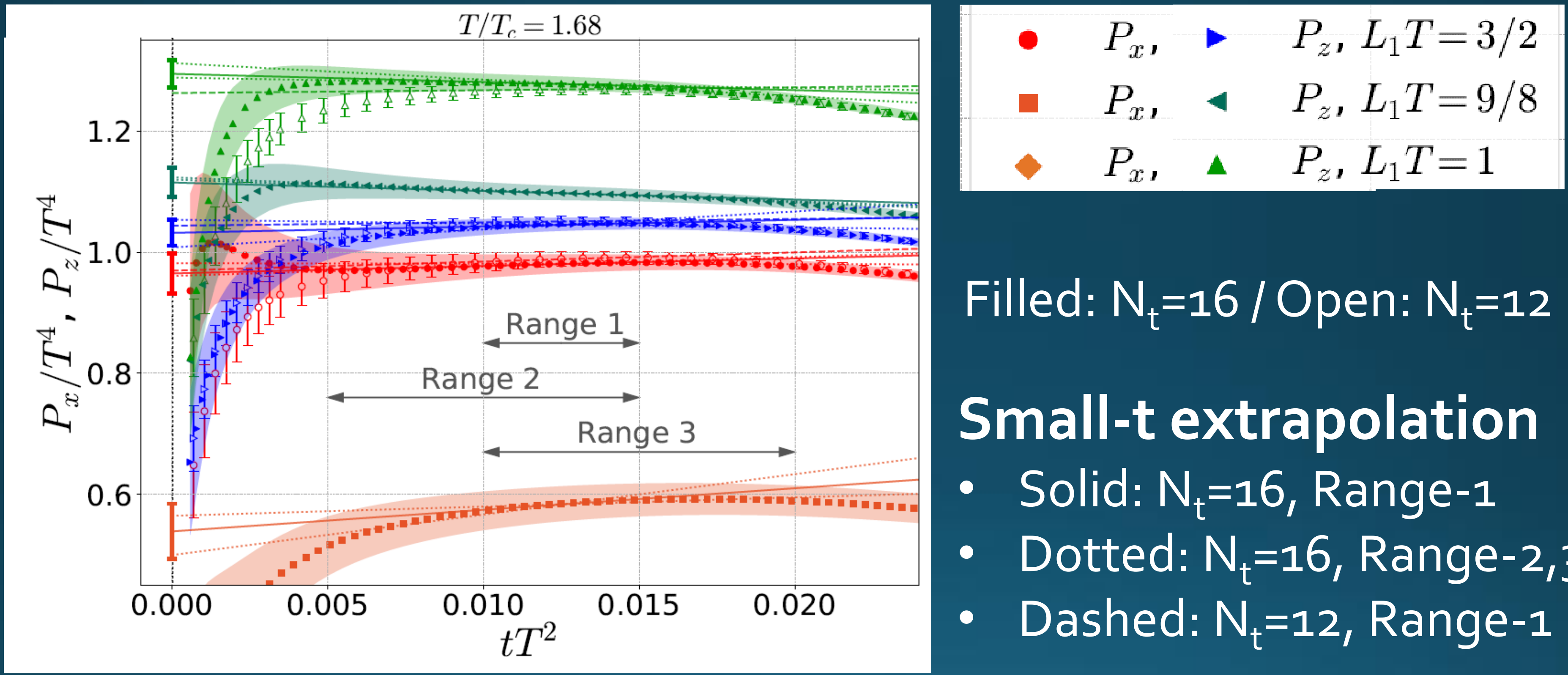
- Solid: N_t=16, Range-1
- Dotted: N_t=16, Range-2,3
- Dashed: N_t=12, Range-1

☐ Stable small-t extrapolation

☐ No N_t dependence within statistics for L_xT=1, 1.5

Small-t Extrapolation

$T/T_c = 1.68$



Filled: N_t=16 / Open: N_t=12

Small-t extrapolation

- Solid: N_t=16, Range-1
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- ☐ Stable small-t extrapolation
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