Correlation Functions in Heavy-ion Collisions, and their Correction

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(YITP, Kyoto)

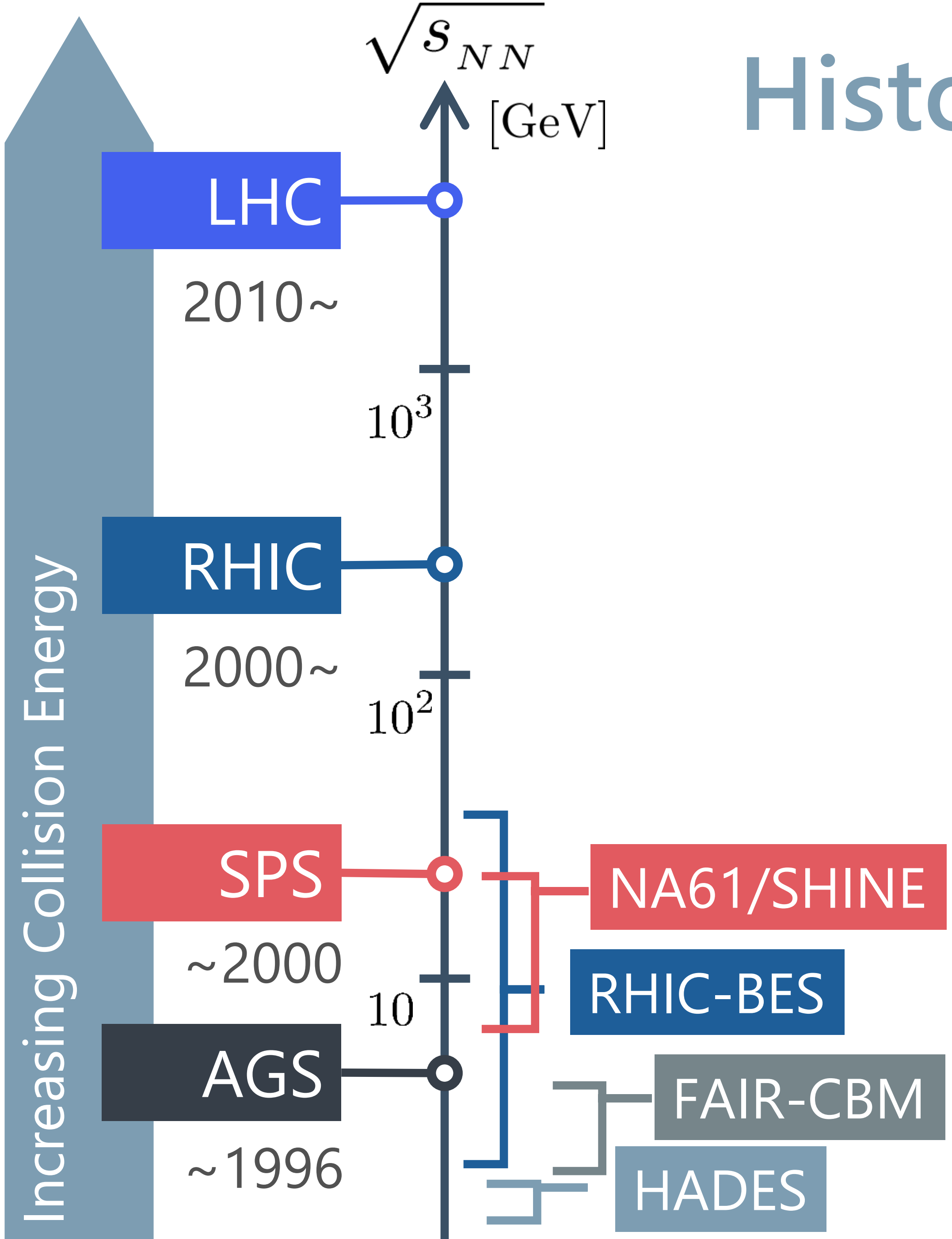
Contents

1. Overview of Relativistic Heavy-ion Collisions & Beam-energy Scan

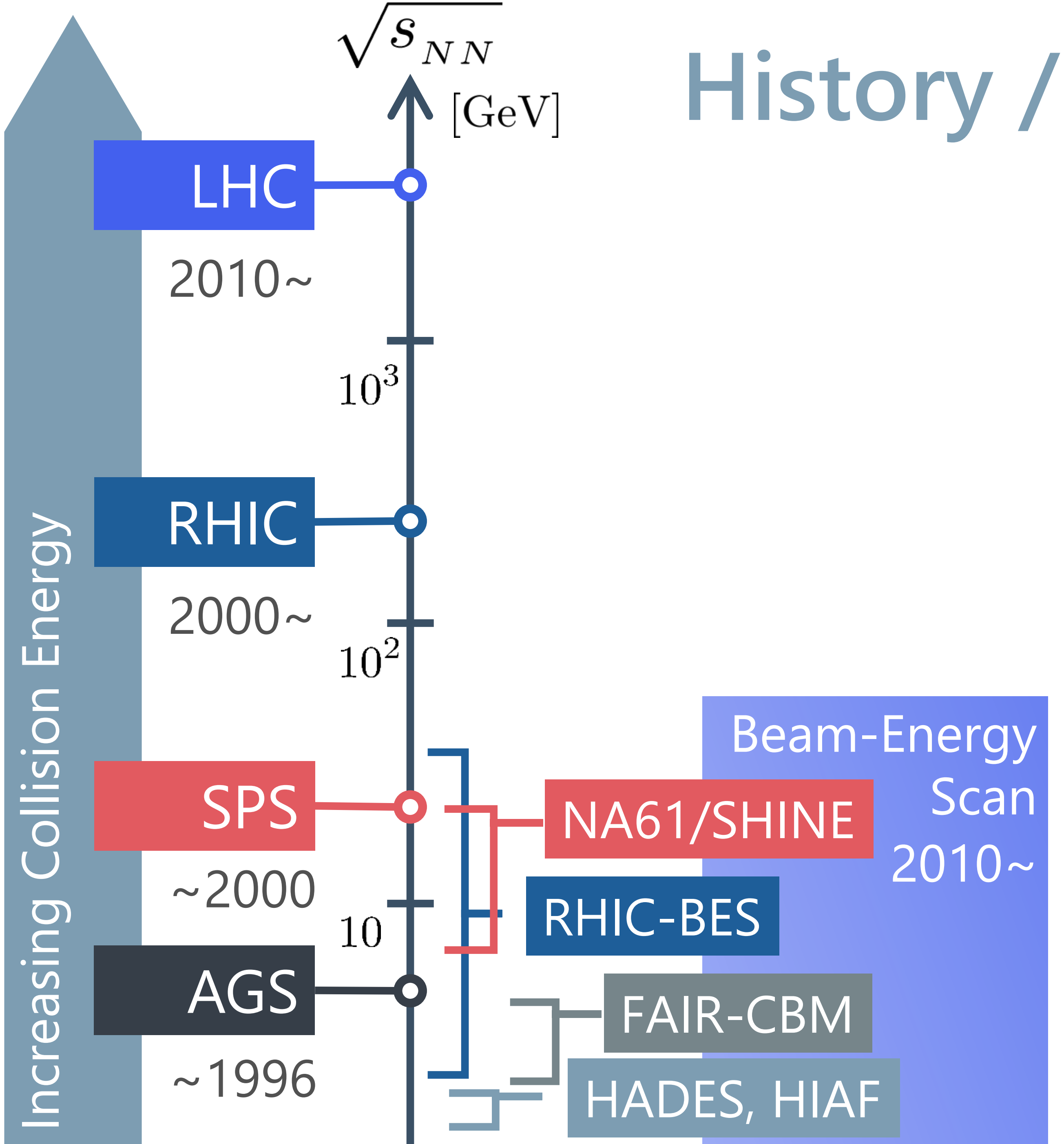
2. Efficiency Correction of Flow Correlations

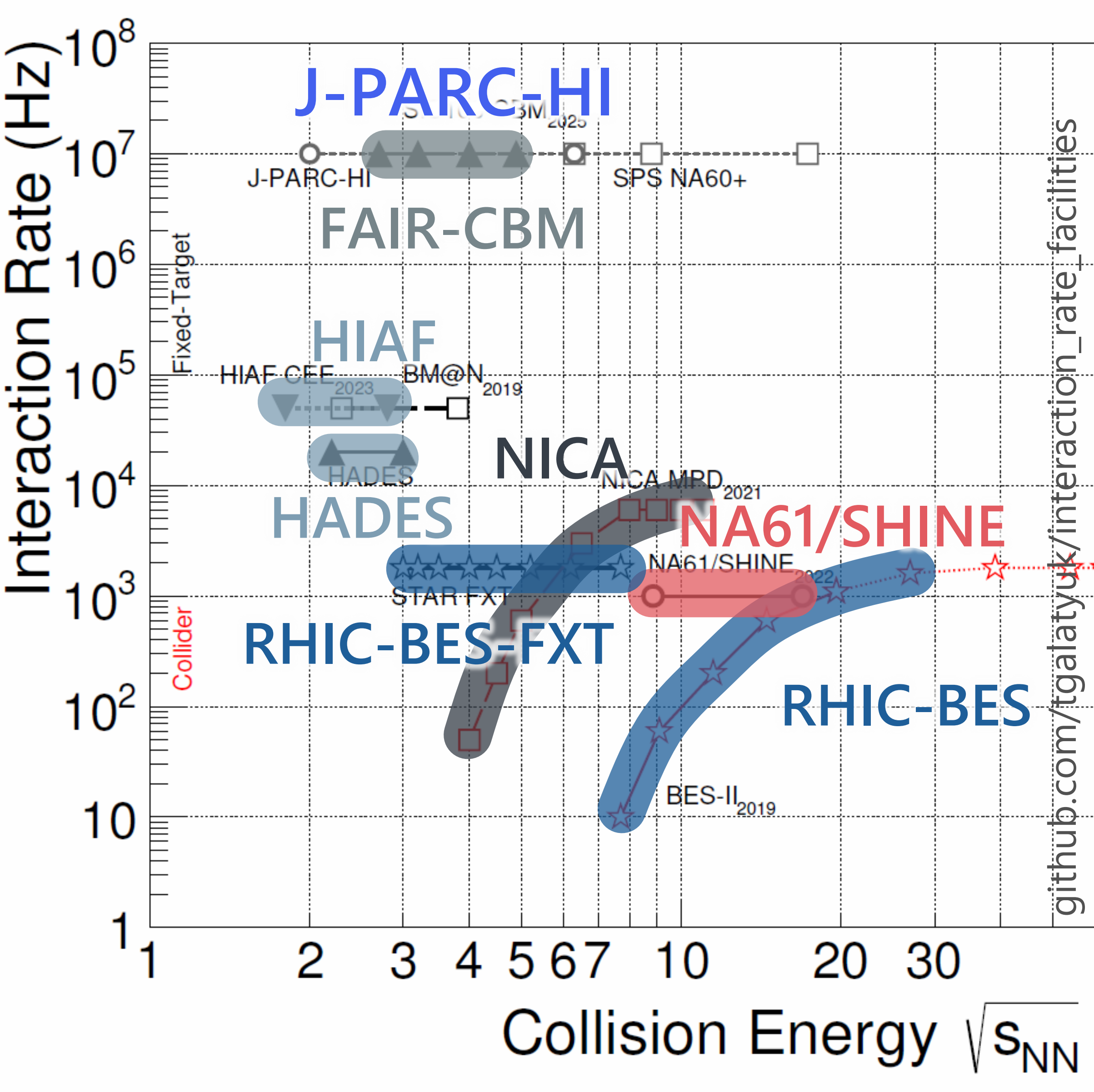
MK, Esumi, Niida, Nonaka, in prep.

History / Current Status of HIC



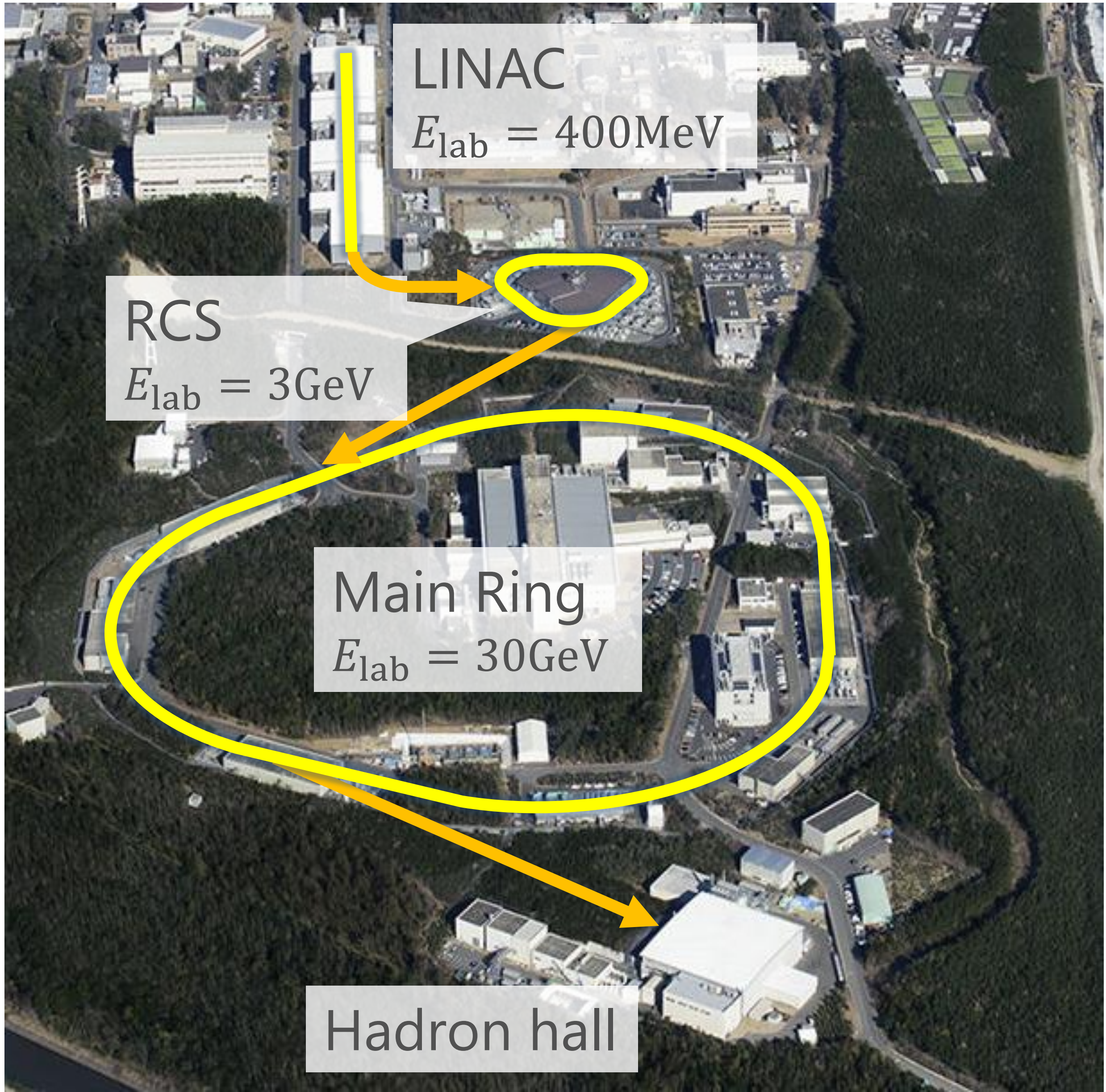
History / Current Status of HIC





J-PARC-HI = J-PARC Heavy-Ion Project

- New HI injector + existing accelerators (RCS, MR)
- Heavy-ion beams with world highest luminosity
- Realize various new experiments at J-PARC
- $\sqrt{s_{NN}} < 4.9\text{GeV}$



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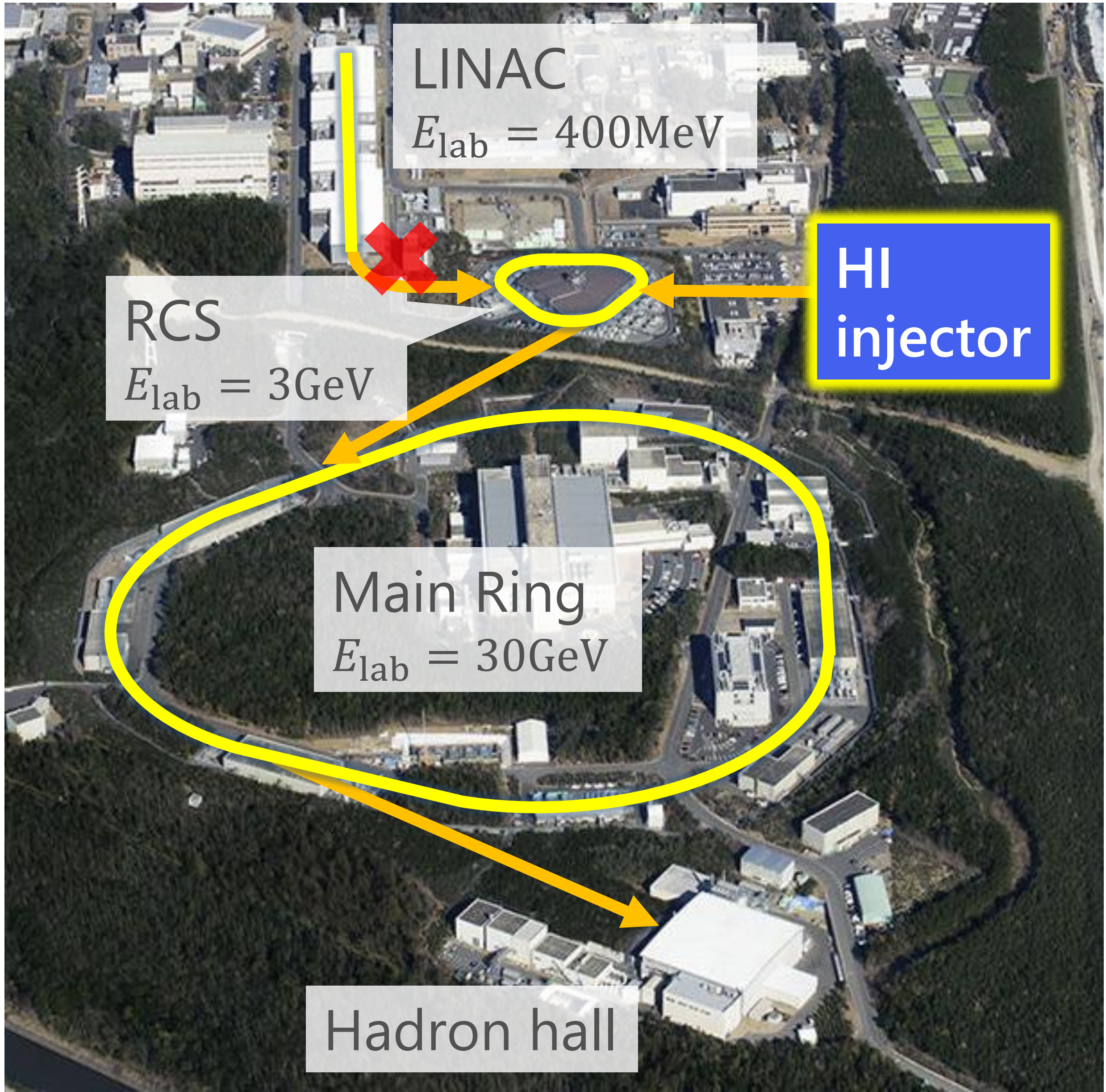
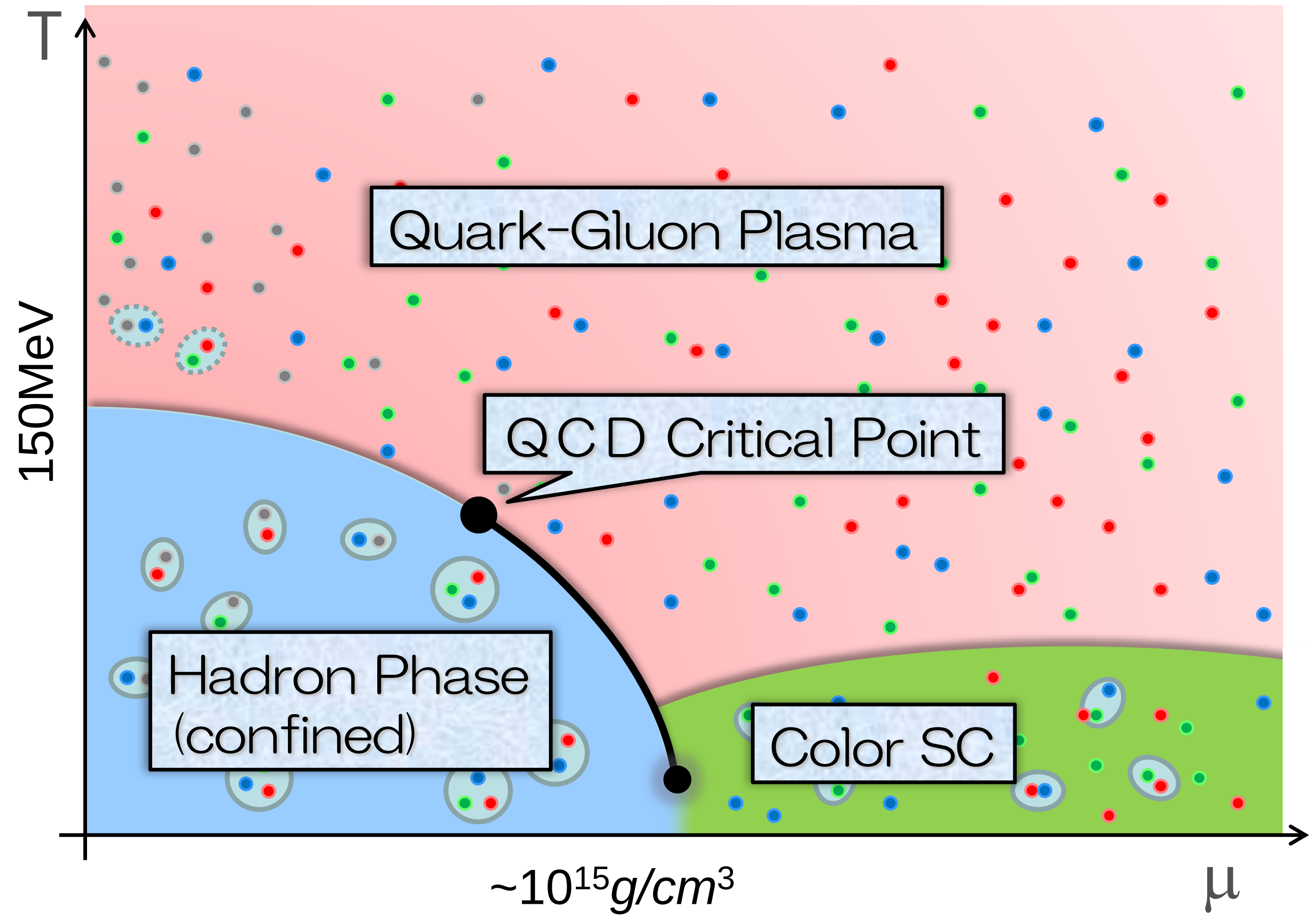
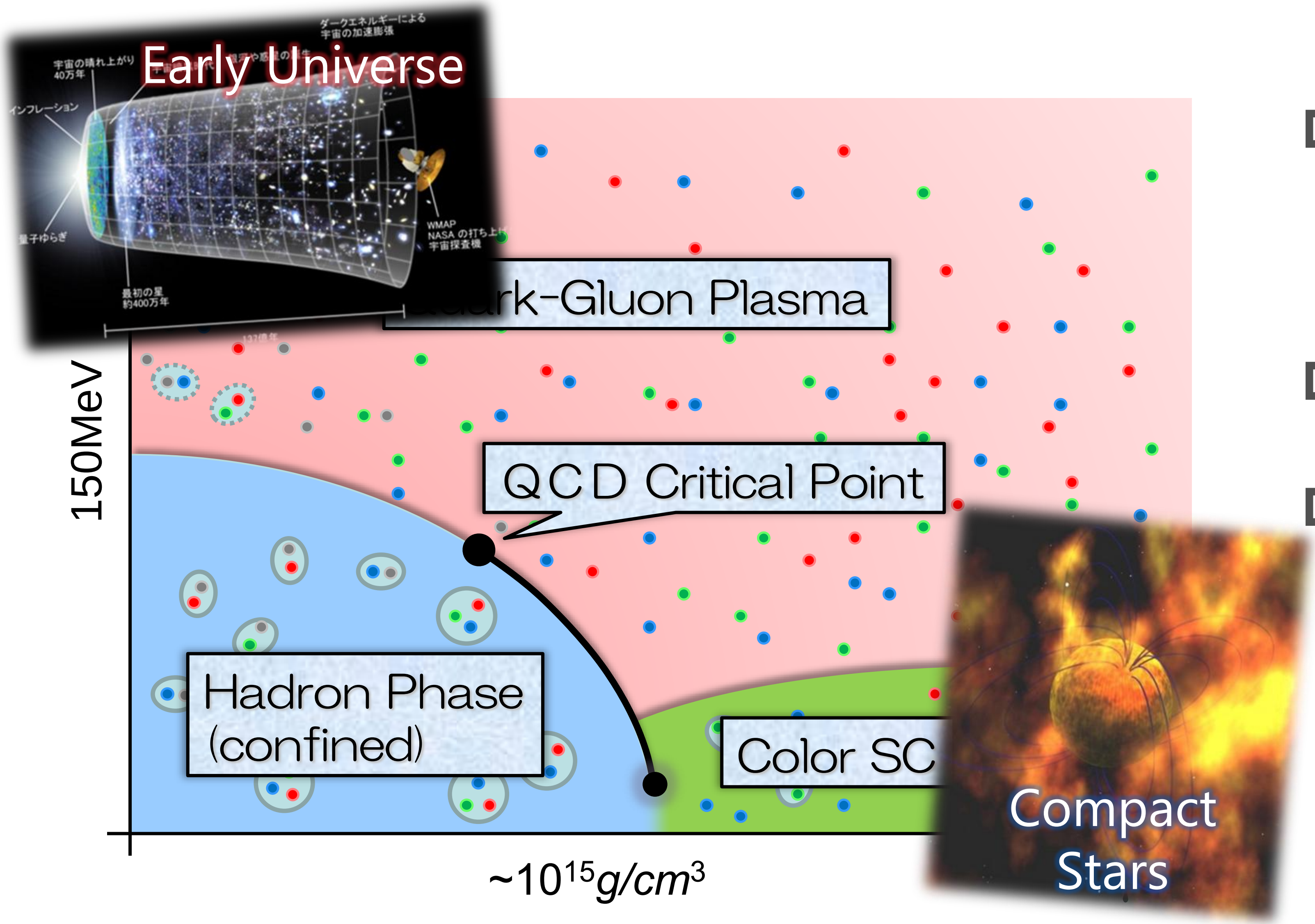


QCD Phase Diagram



- Possible first-order transition and QCD critical point in dense region
- Multiple QCD-CP? MK+ ('02)
- Color superconducting phases in dense and cold quark matter

QCD Phase Diagram



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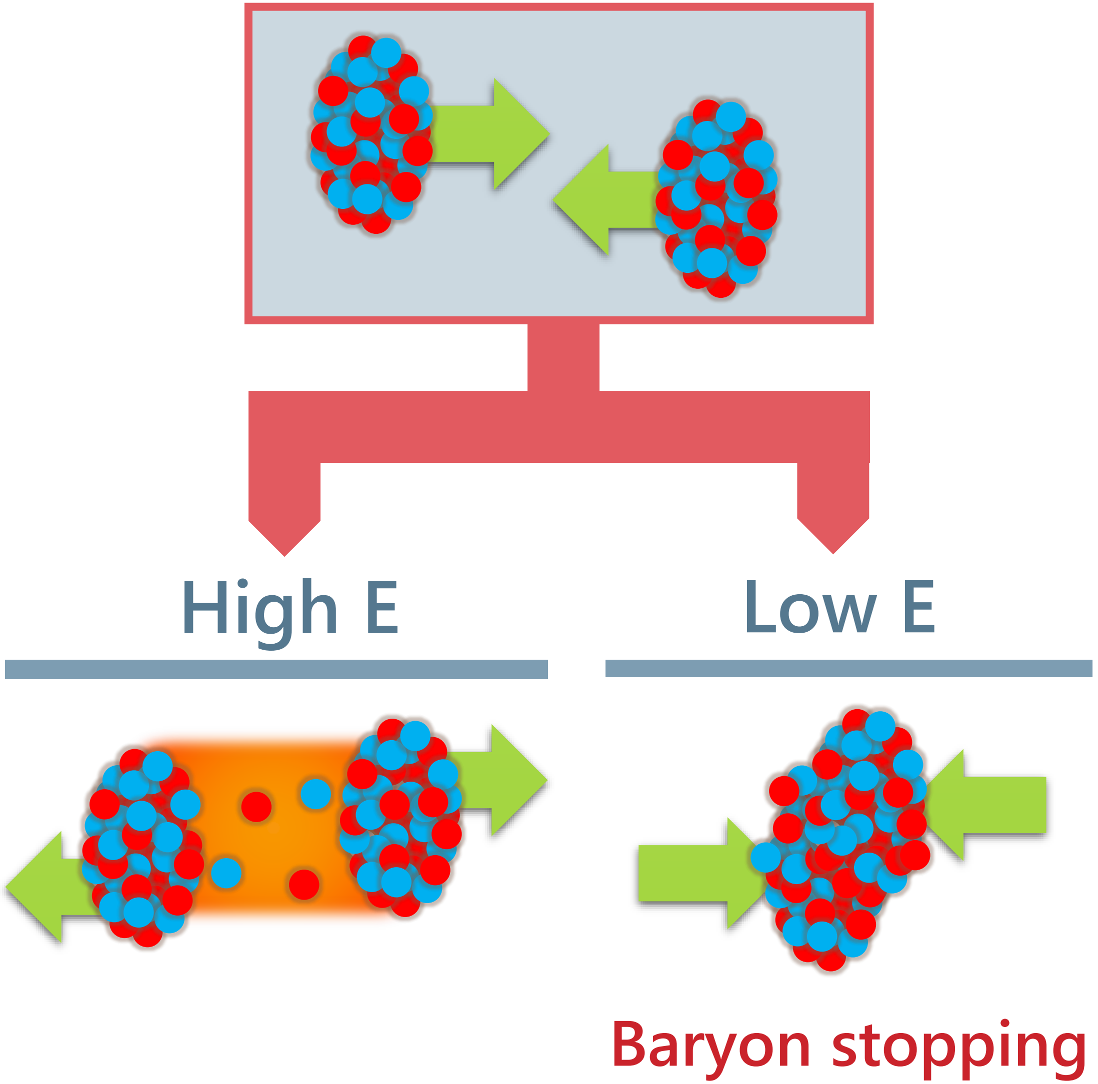
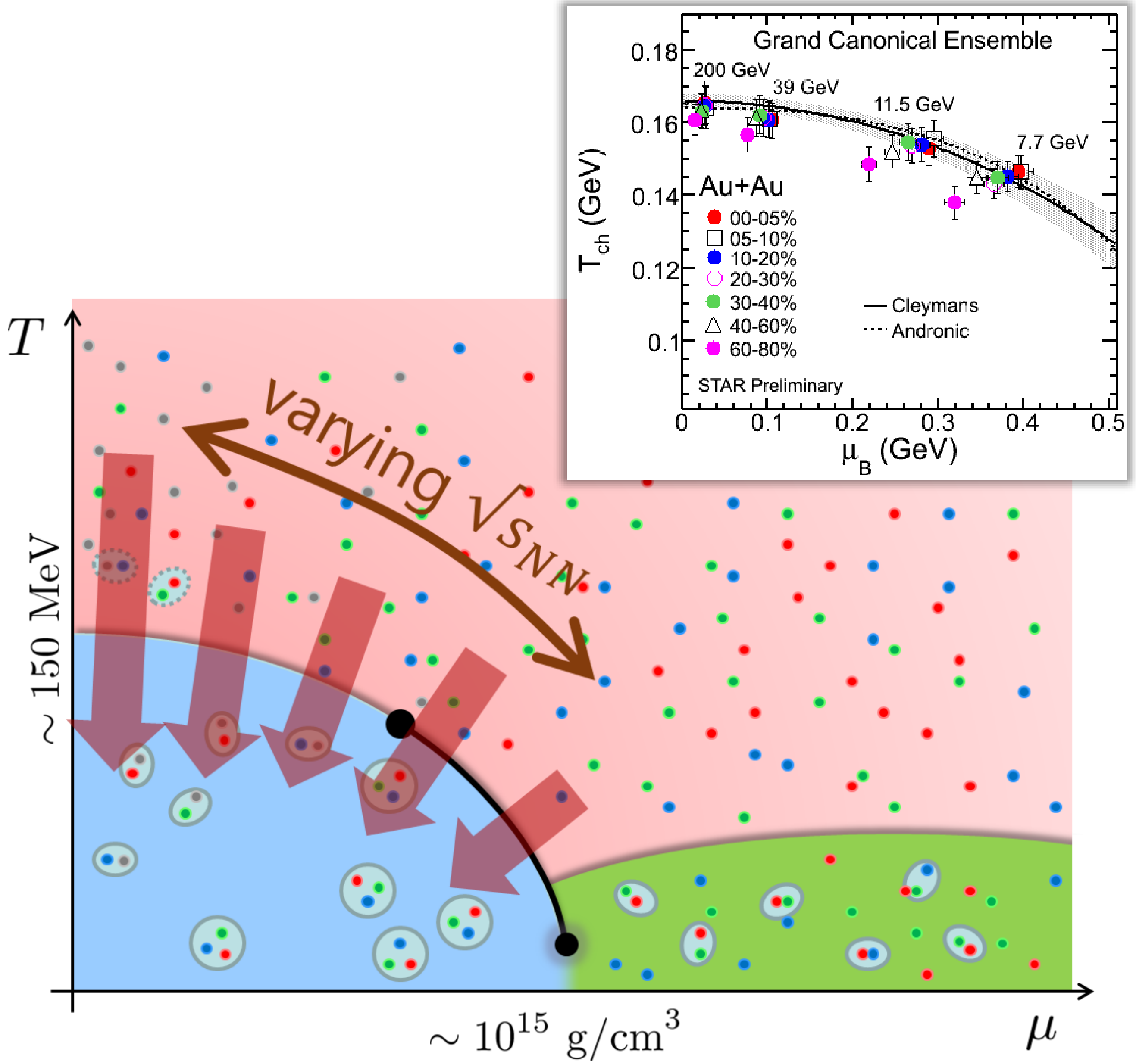
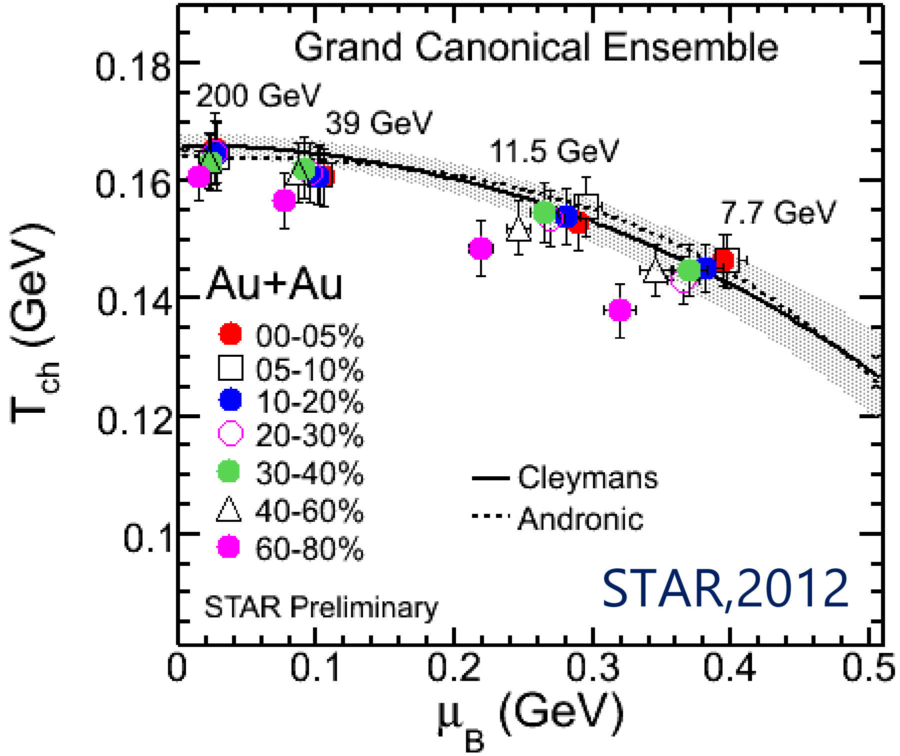
Beam-Energy Scan



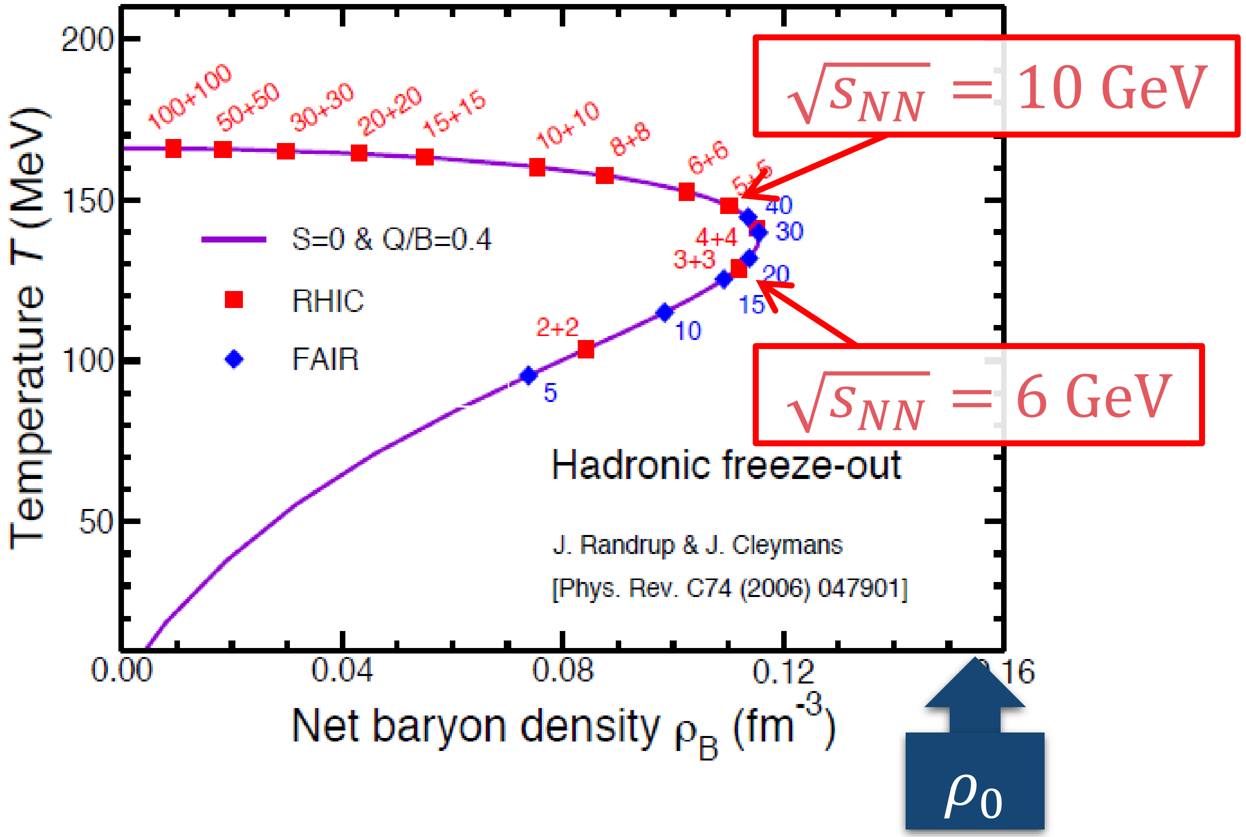
STAR, 2012



Chemical Freezeout



Randrup, Cleymans, '06



- Highest baryon density **at chemical freezeout** at $\sqrt{s_{NN}} \simeq 6 - 10$ GeV?
- Not the highest density in the early stage.
- Density in earlier stage? ➤ Analysis in dynamical models

Volume of Dense Region

Taya, Jinno, MK, Nara, 2409.07685

Volume where the local baryon density is larger than a threshold value $\rho_{\rm th}$

$$V_3(\rho_{\rm th}, t) = \int_{\rho(x)>\rho_{\rm th}} d^3\boldsymbol{x}\,\gamma$$

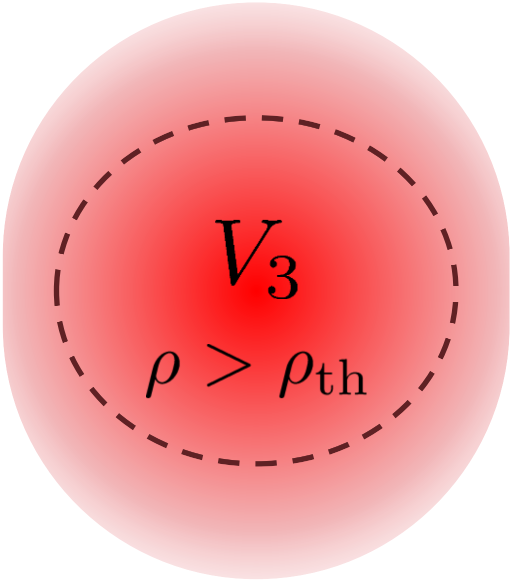
Baryon current $J^\mu(x)$

Baryon density $\rho(x) = \sqrt{J^\mu(x) J_\mu(x)}$

Lorentz factor $\gamma = (1 - (\boldsymbol{J}/J_0)^2)^{-1/2}$

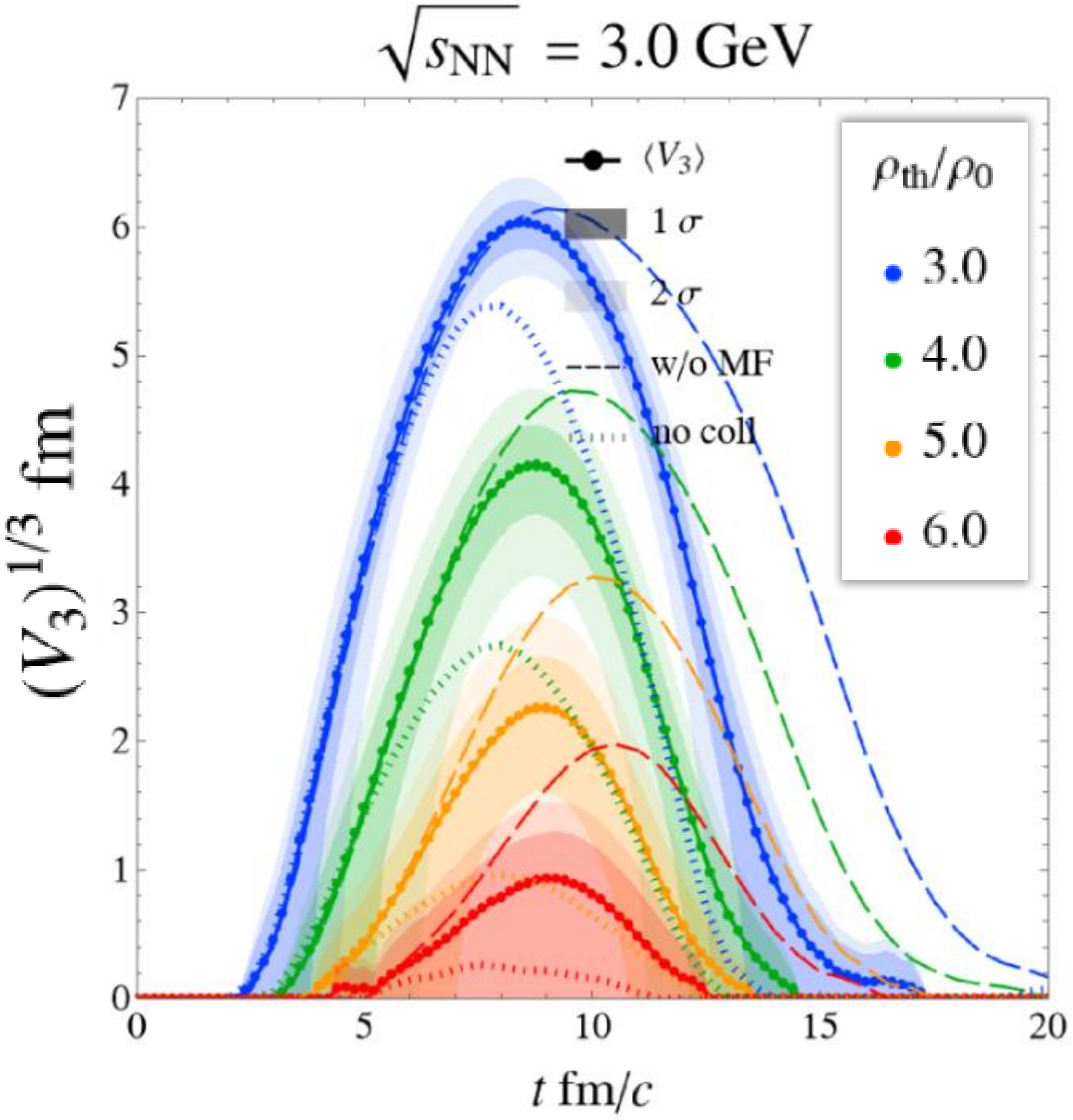
Note:

— Event-by-event basis / no event average
— Directly calculable in a dynamical model
— We do not care about local thermalization.
  — V_3 is the upper limit of thermalized volume.
  — Even non-thermal, dense region is interesting!



V_3 in JAM

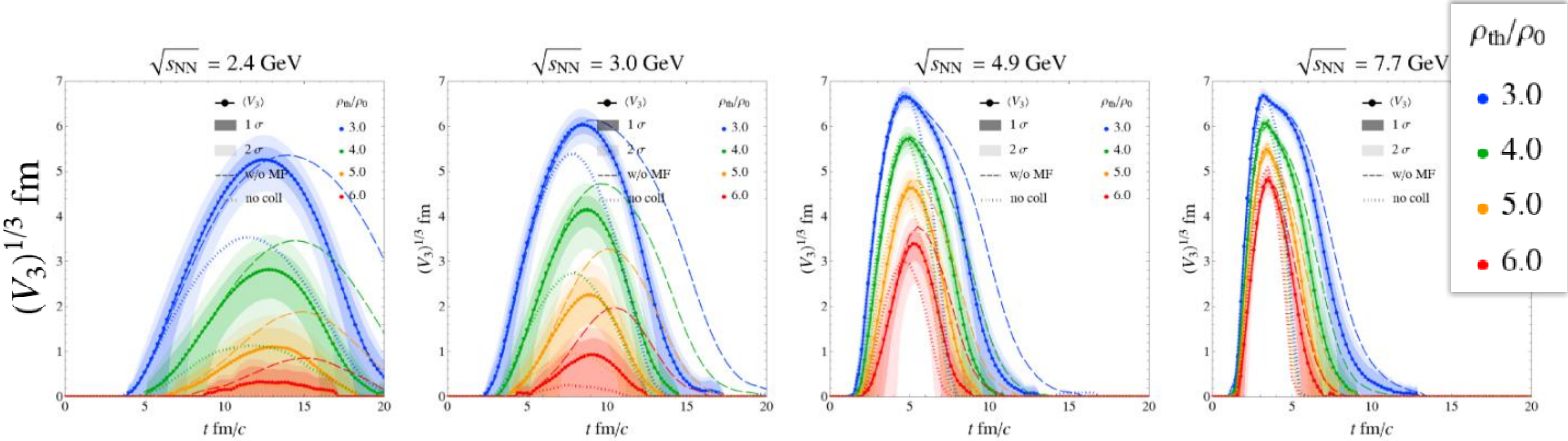
Taya, Jinno, MK, Nara, 2409.07685



— **solid: JAM+MF** Nara, Ohnishi, 2022
— shaded band: 1σ and 2σ e-v-e fluct.
— dashed: JAM cascade mode
— dotted: no-collision

- Formation of dense region:
  - $V_3(3\rho_0, t) = (6\ \mathrm{fm})^3$
  - $V_3(4\rho_0, t) = (4\ \mathrm{fm})^3$
- Large e-v-e fluctuations
  ➔separable by event selection?
- Repulsive MF ➔ weaker compression
- Compression owing to interaction

V_3 for various $\sqrt{s_{NN}}$



As $\sqrt{s_{NN}}$ becomes larger,

- ☐ $\max V_3(\rho_{\mathrm{th}}, t)$ becomes larger.
- ☐ The lifetime of dense region becomes shorter.
- ☐ E-v-e fluctuations are more suppressed.

Four-Volume / Lifetime

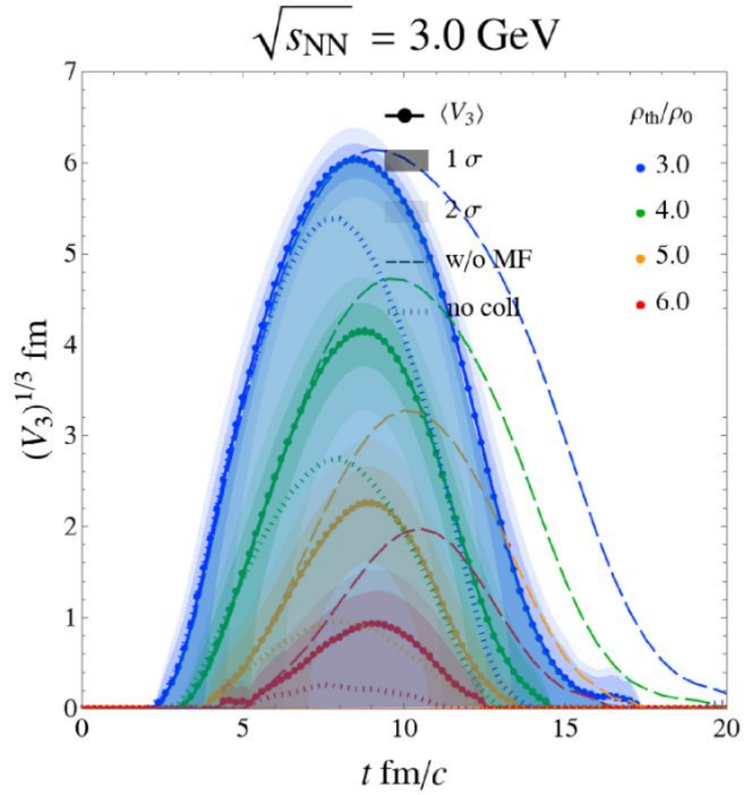
Taya, Jinno, MK, Nara, 2409.07685

Four Volume

$$V_4(\rho_{\rm th}) = \int_{-\infty}^{\infty} dt \int_{\rho(x)>\rho_{\rm th}} d^3\boldsymbol{x}$$

Lifetime

$$\tau(\rho_{\rm th}) = \frac{V_4(\rho_{\rm th})}{\max V_3(\rho_{\rm th}, t)}$$



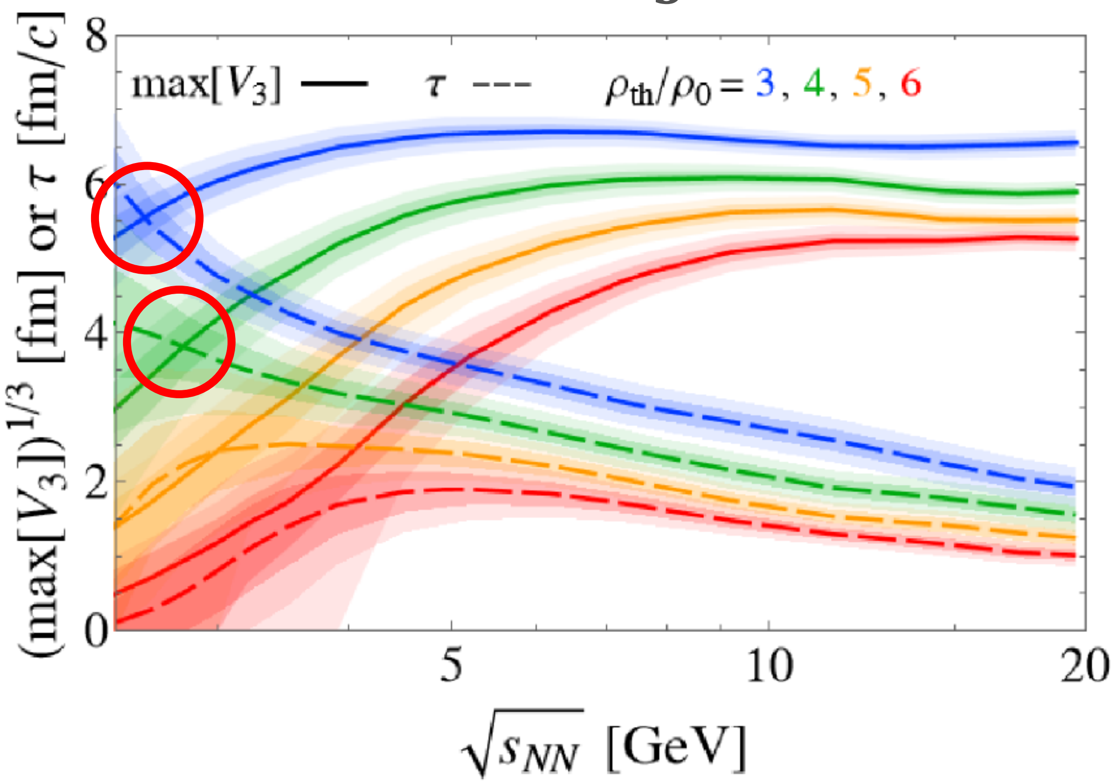
Note

V_4 may be relevant for the dilepton production rate.

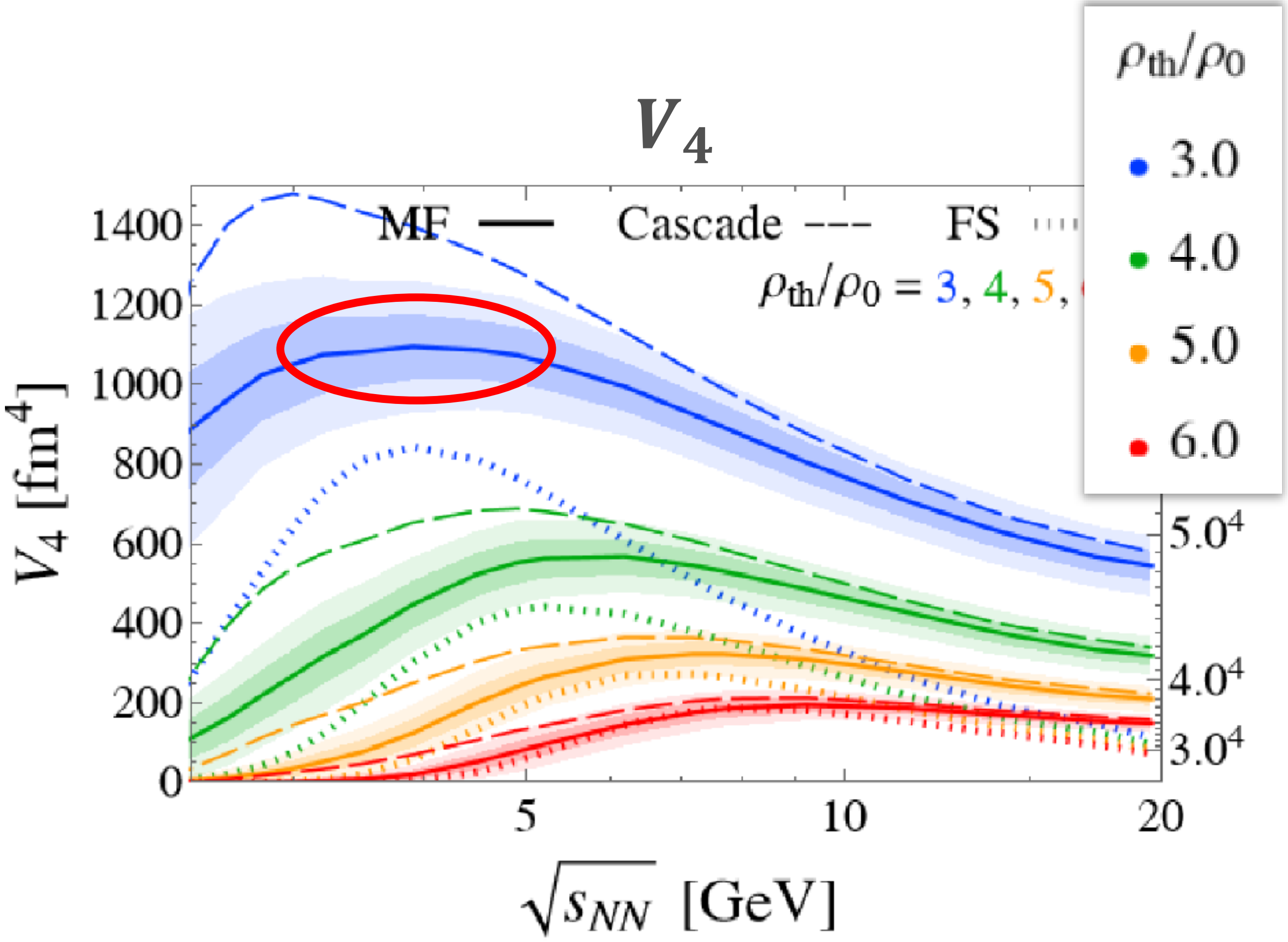
Taya, Jinno, MK, Nara, 2409.07685

$\sqrt{s_{NN}}$ Dependence

max V_3 , τ



V_4

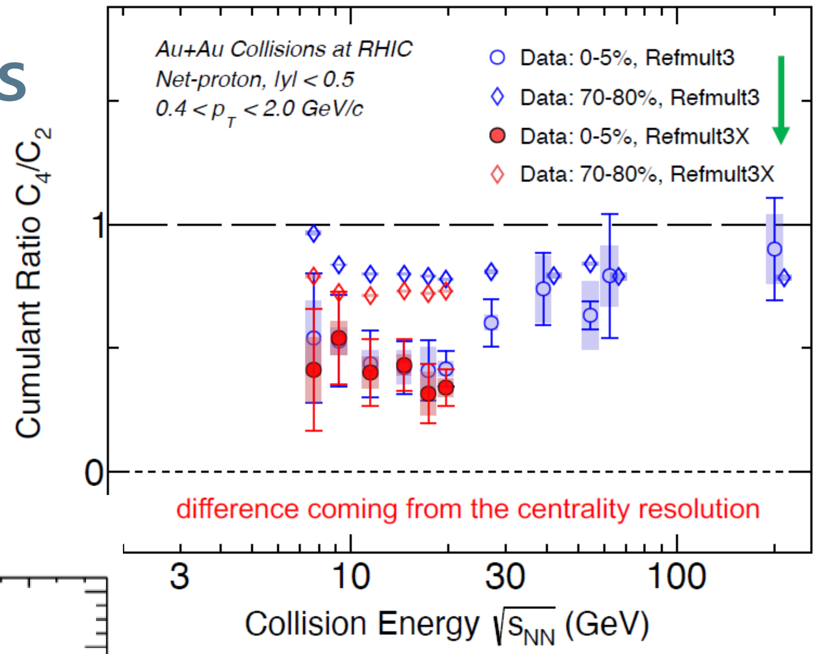


- $\sqrt{s_{NN}} \simeq 3$ GeV would be the best energy to create $\rho = 3\sim4\rho_0$ with large V_3 and τ.
- Lower $\sqrt{s_{NN}}$ is suitable to create colder matter.

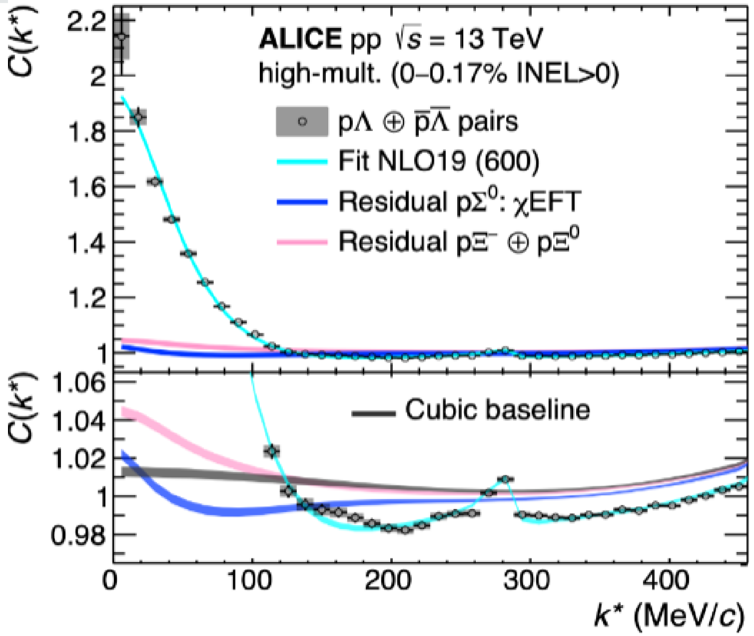
Higher-order Correlations

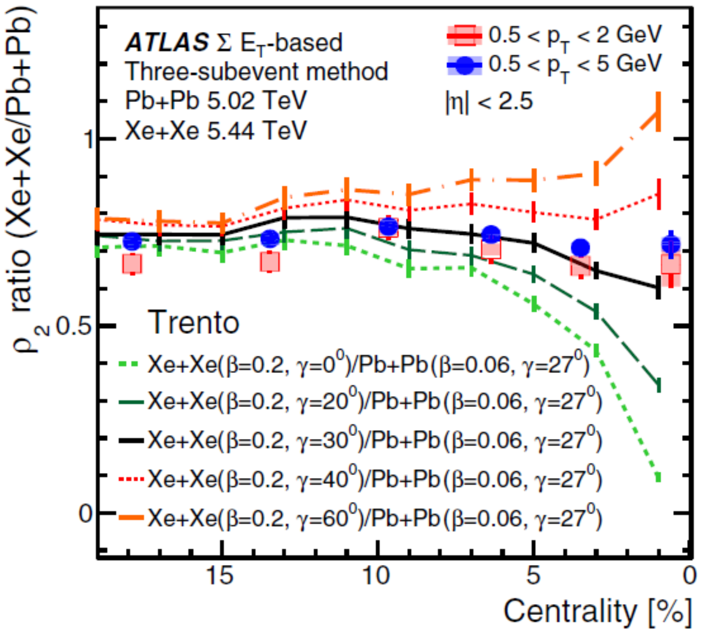
= central observables in the future experiments
that fully make use of their high statistics.

—Search for the QCD critical point
—Femtoscopy → Hadron interaction
—Nuclear structure

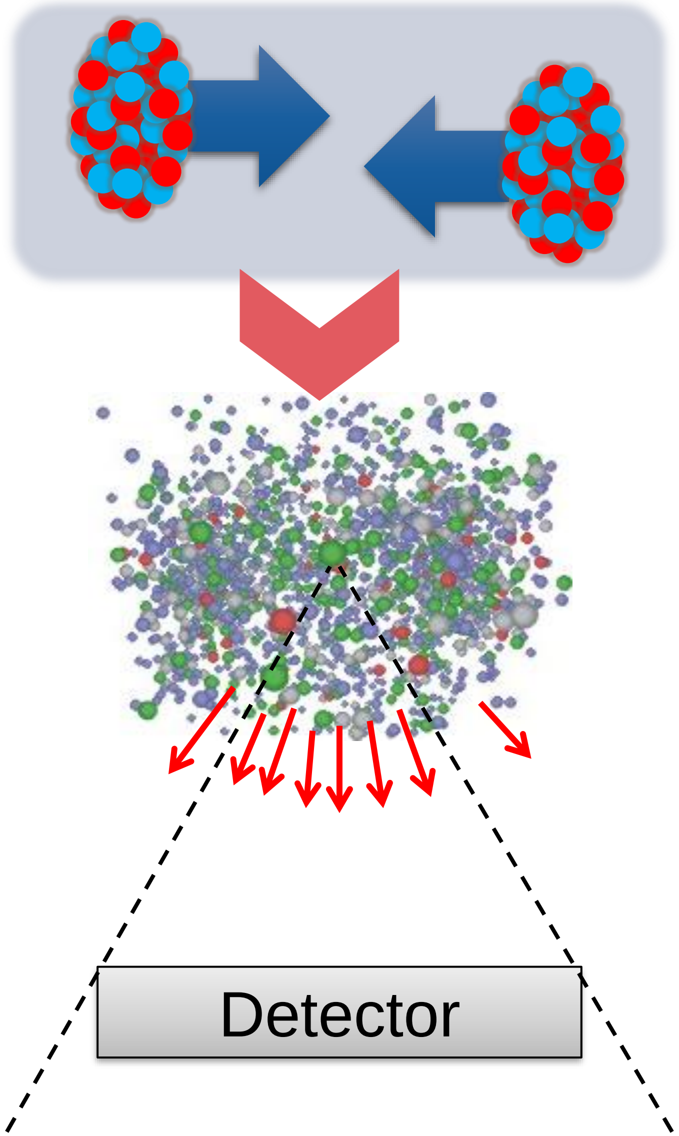


pΛ correlation function
ALICE Coll. PLB 833 (2022), 137272

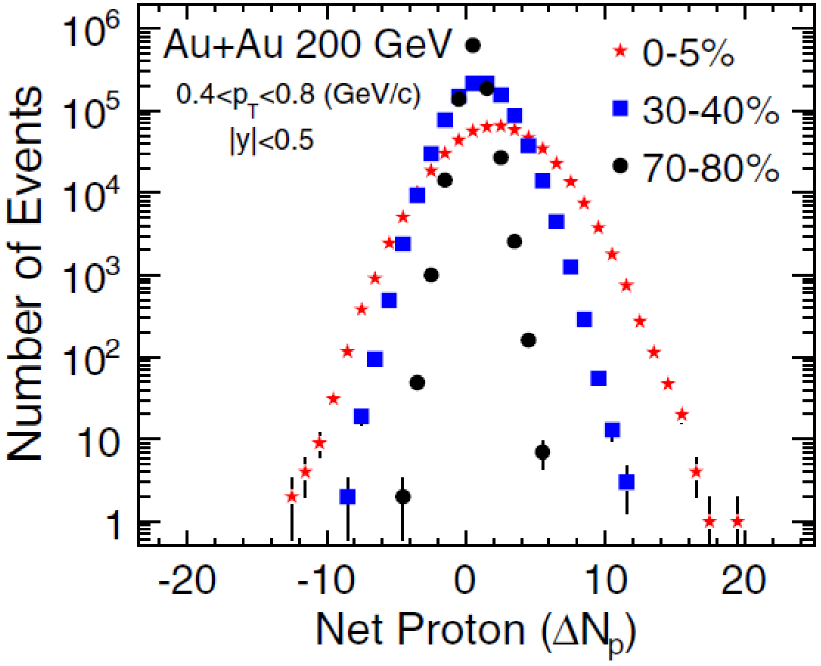




Event-by-event Fluctuations



Particle-number Distribution



STAR, PRL**105** (2010)

Cumulants

$$\langle N^2\rangle_c = \langle(\delta N^2)\rangle,$$
$$\langle N^3\rangle_c = \langle(\delta N^3)\rangle,$$
$$\langle N^4\rangle_c = \cdots$$

Review: Asakawa, MK, PPNP **90** (2016)

A Coin Game

① Bet 25 Euro
② You get head coins of

A. 50 x 1 Euro

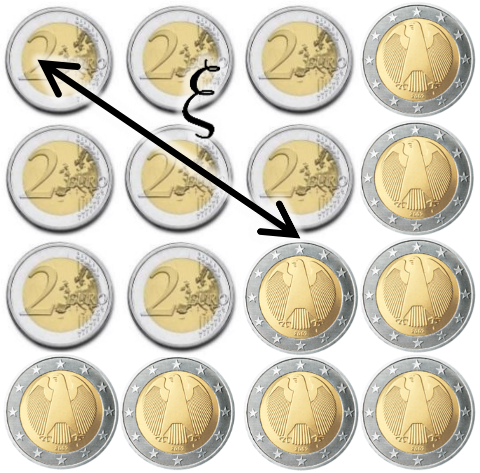
B. 25 x 2 Euro

C. 1 x 50 Euro

Same expectation value.
But, different fluctuation.

Fluctuations in HIC @ 2nd Order

Search for QCD CP



Fluctuation
increases

Stephanov, Rajagopal, Shuryak, 1998; 1999

Onset of QGP

Fluctuation
decreases

Asakawa, Heinz, Muller, 2000;
Jeon, Koch, 2000

Higher-Order Cumulants

A. 50 x 1 Euro

B. 25 x 2 Euro

$$2\langle \delta€^2 \rangle_{\text{1€}} = \langle \delta€^2 \rangle_{\text{2€}}$$

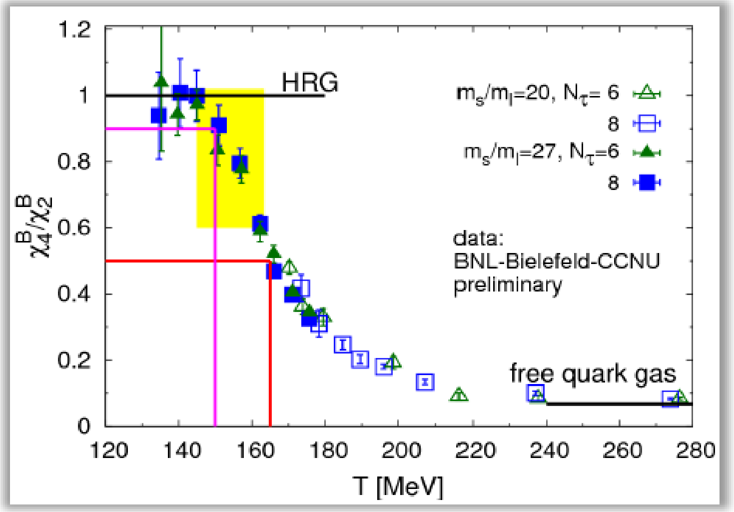
$$4\langle \delta€^3 \rangle_{\text{1€}} = \langle \delta€^3 \rangle_{\text{2€}}$$

$$8\langle €^4 \rangle_{c\,\text{1€}} = \langle €^4 \rangle_{c\,\text{2€}}$$

Asakawa, MK,
PPNP 90, 299 (2016)

Non-Gaussian Fluctuations

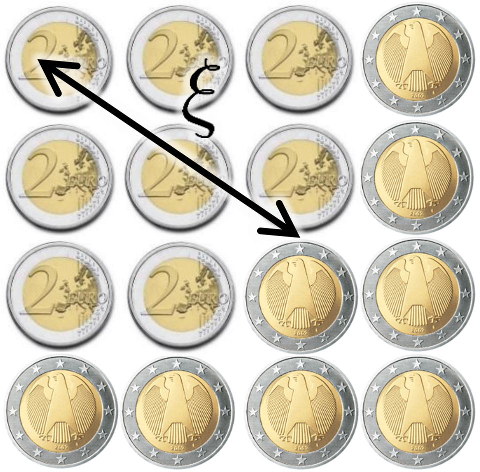
Onset of QGP



Fluctuation
decreases

Ejiri, Karsch, Redlich, 2006

Search for QCD CP



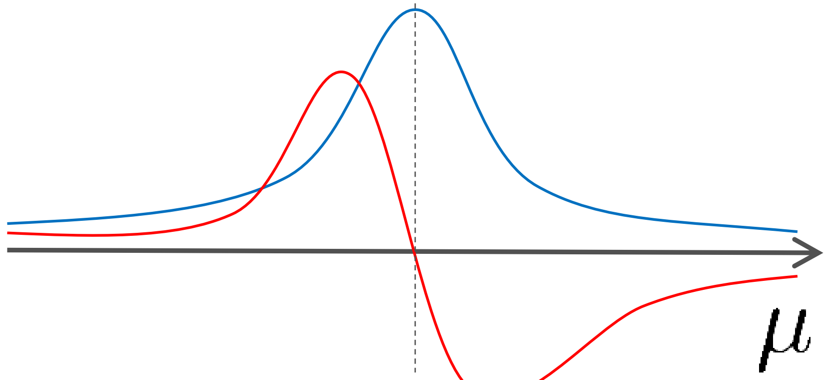
Fluctuation
increases

Stephanov, 2009

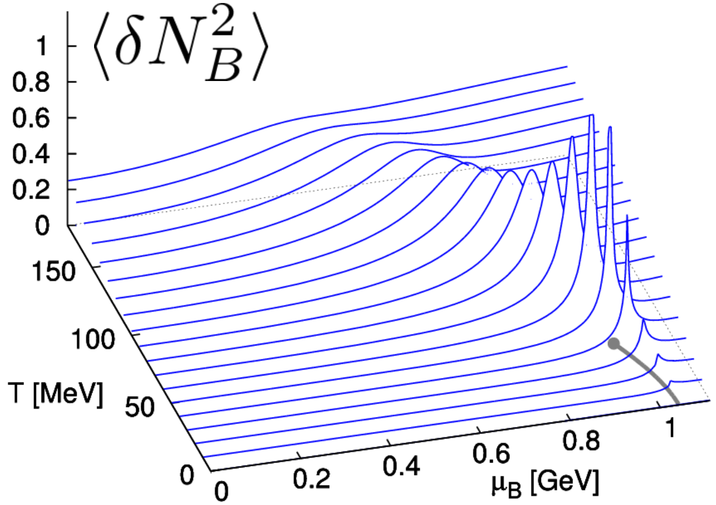
Sign of Higher-Order Cumulants

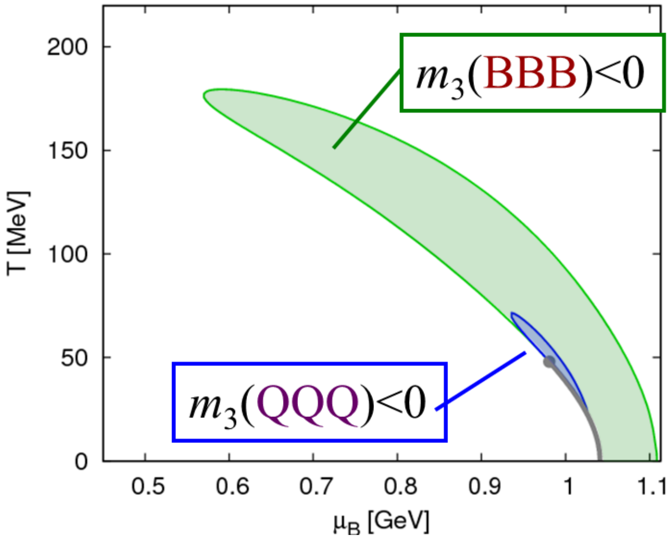
Geometric Interpretation

$$\langle \delta N^3 \rangle = T \frac{\partial \langle \delta N^2 \rangle}{\partial \mu}$$



Asakawa, Ejiri, MK, 2009





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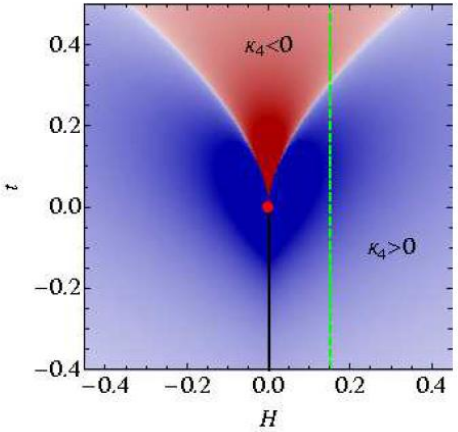
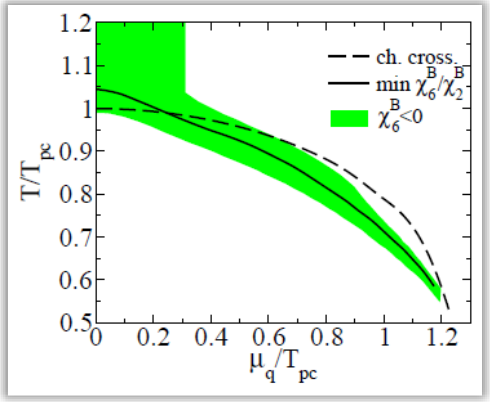
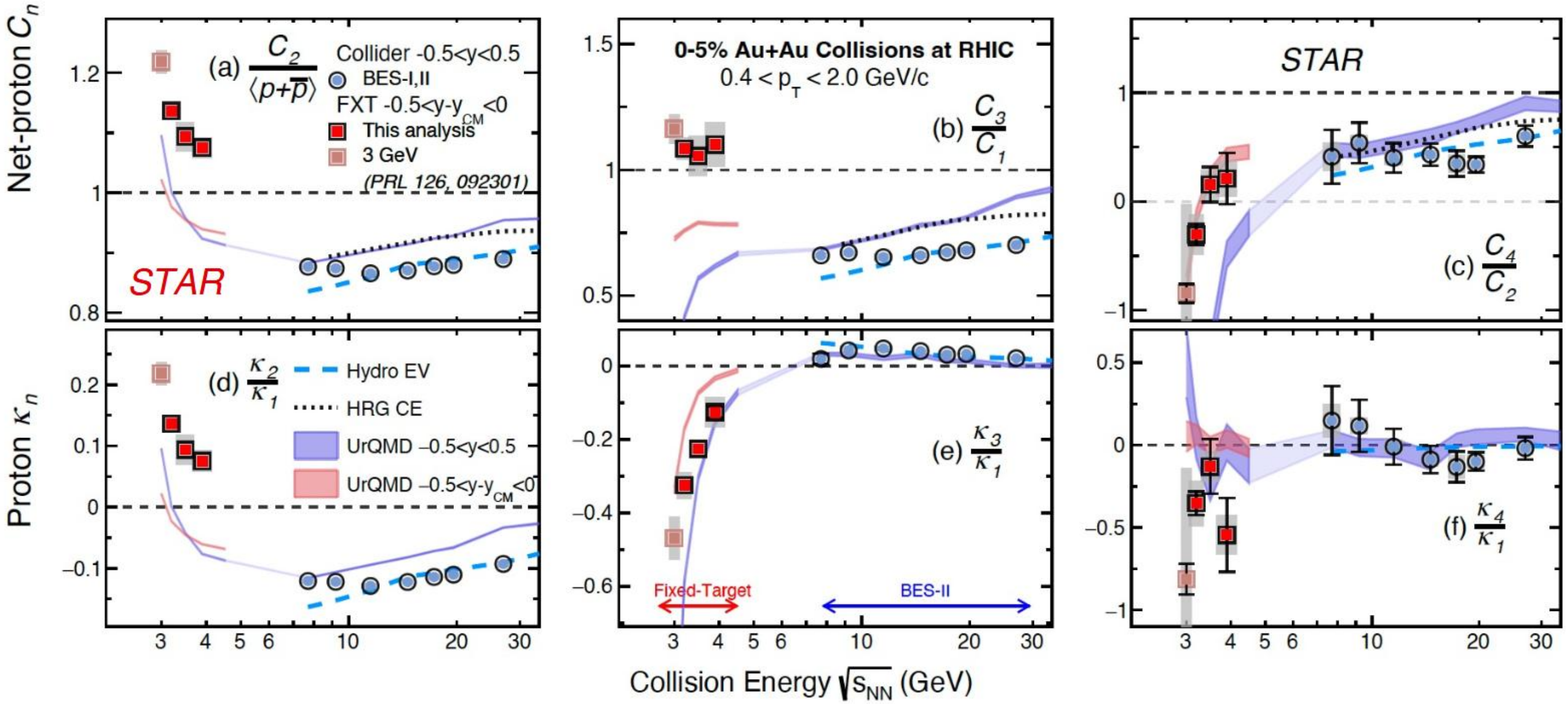




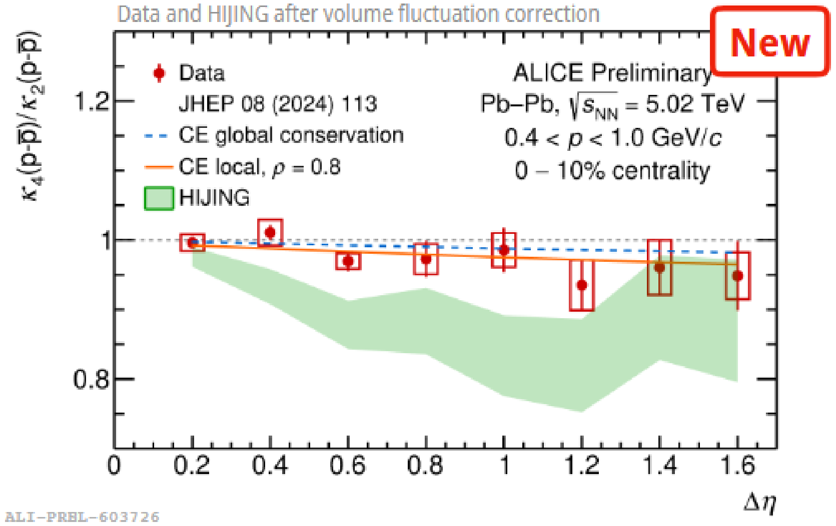
Stephanov, 2011;
Friman, Karsch, Redlich, Skokov, 2011; ...

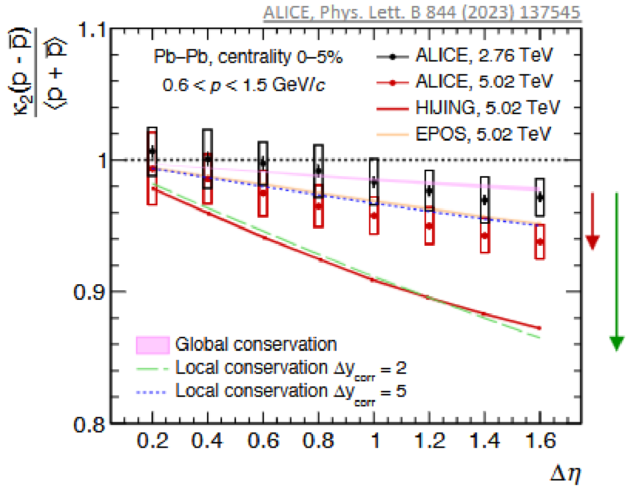
Latest Experimental Results @ QM2025

STAR (Esumi)



ALICE (Arslandok)





- Dip and/or enhancement in $\sqrt{s_{NN}}$ dep. of higher-order cumulants?
- No non-Poisson signal in the ALICE result.

Nuclear Shape from Relativistic HIC

Giacalone, PRL 124, 202301 (2020)
Jia, PRC 105, 044905 (2022); …

— High-E HIC takes a snapshots of the collision.

— Distribution of the initial shape is sensitive to the shape of colliding nuclei.

Initial state

$$\langle \epsilon_2^2 \delta d_\perp \rangle, \quad \langle (\delta d)^2 \rangle, \cdots$$

experimentally accessible using flow correlations

$$\langle v_2^2 \delta p_T \rangle, \quad \langle (\delta p_T)^2 \rangle, \cdots$$

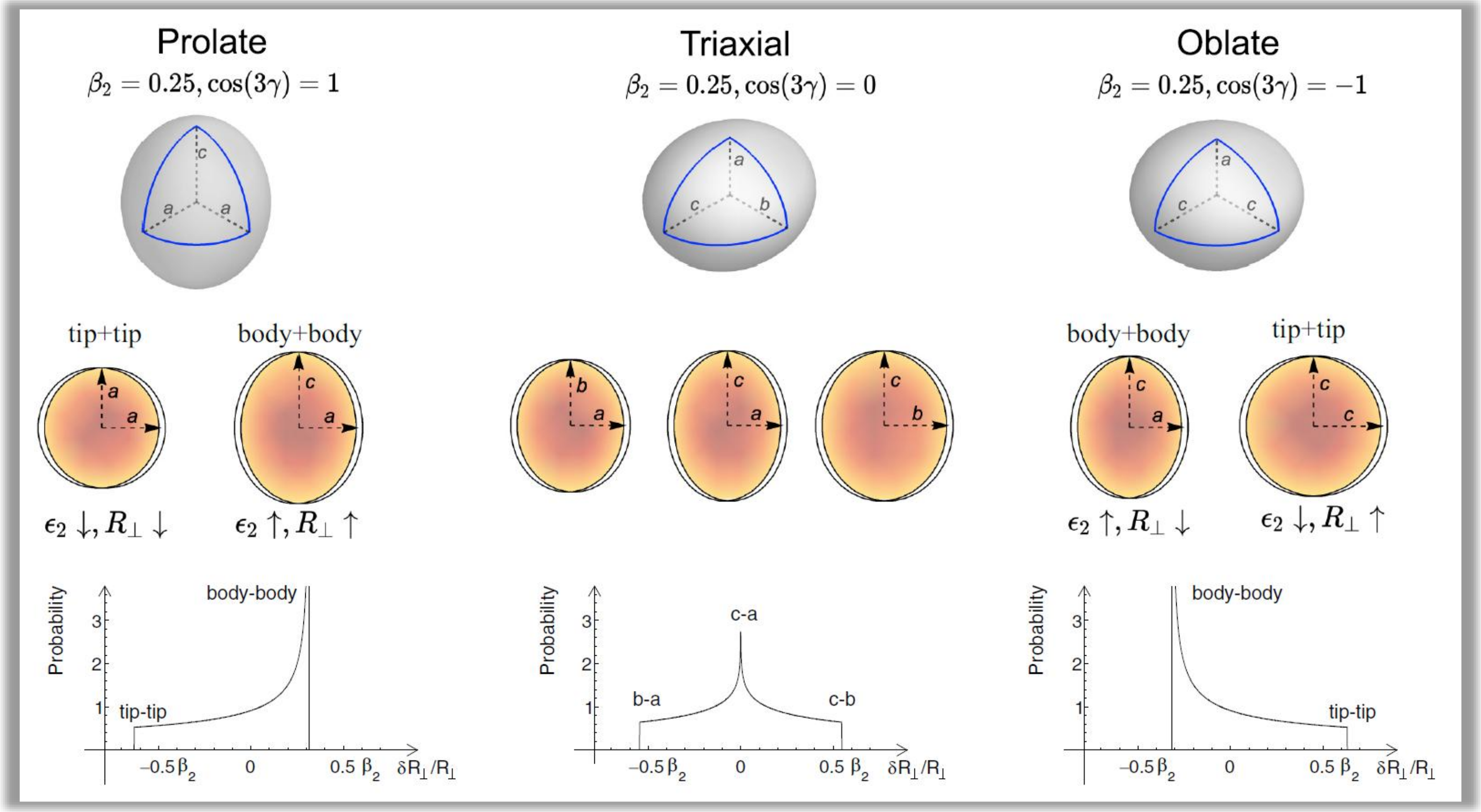
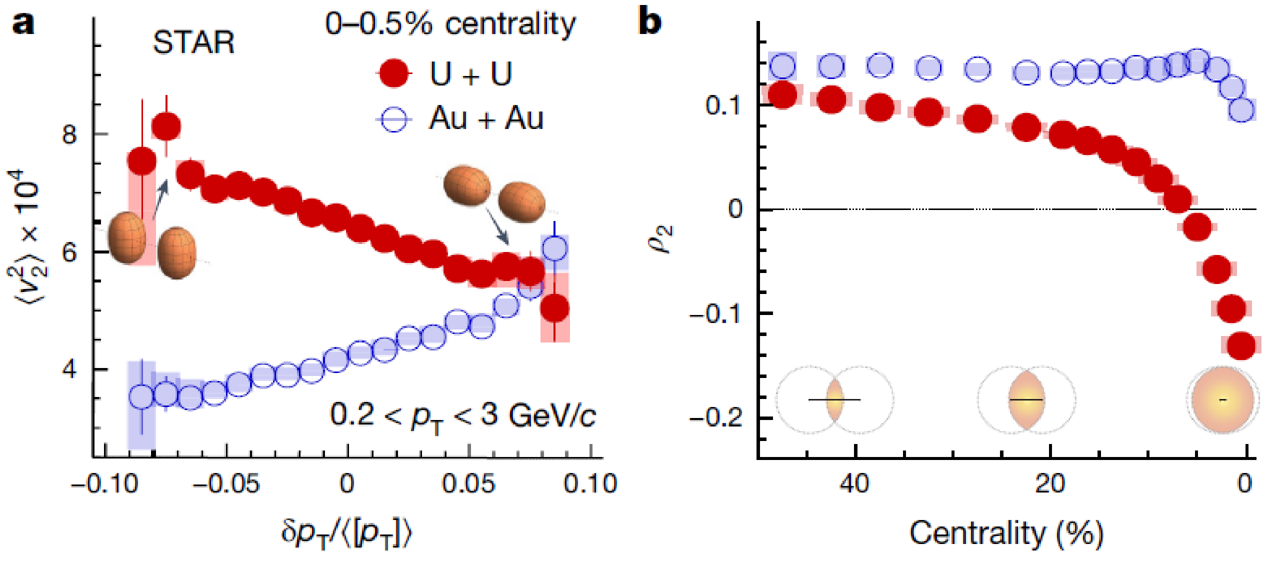


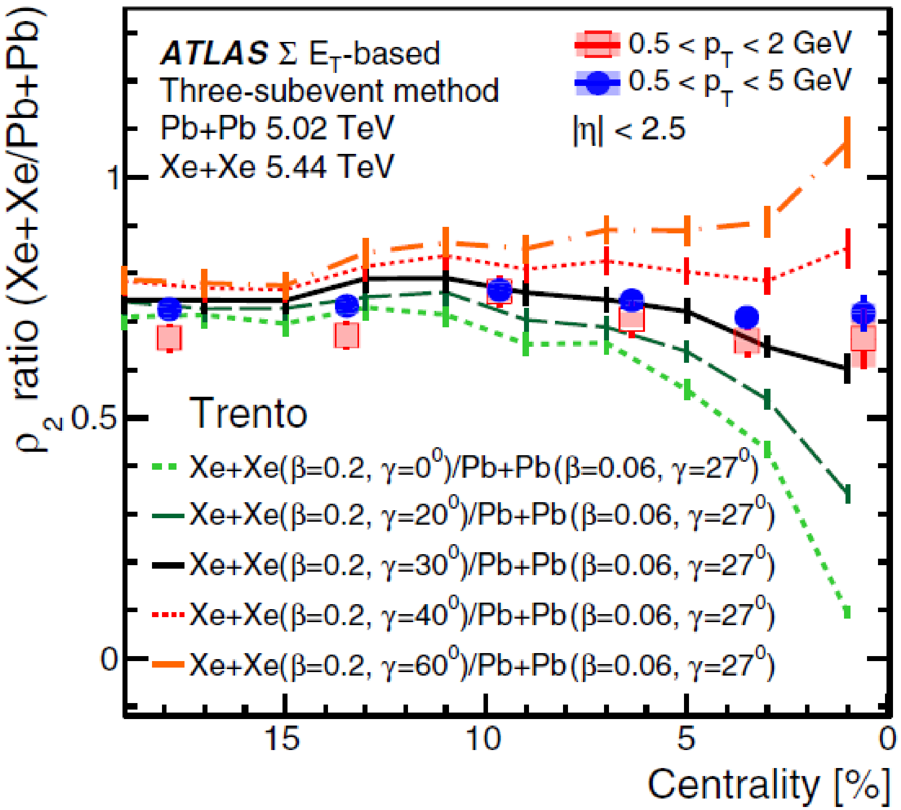
Fig. from Jia, PRC105

Experimental Results

STAR, Nature 635, 67 (2024)



ATLAS, PRC 107, 054910 (2023)



Information of the nuclear shape is reflected in flow observables.

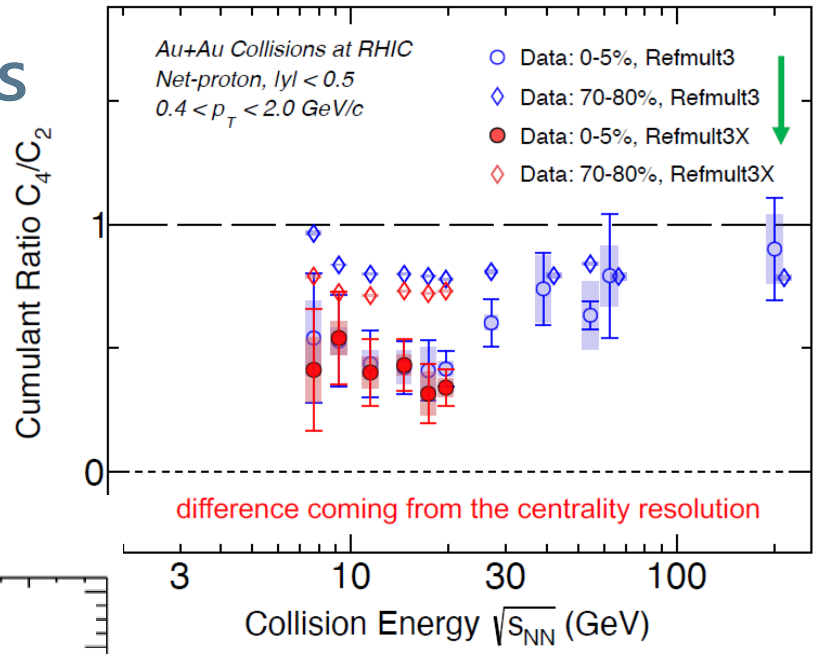
$v_n - p_T$ correlation:

$$\rho(v_n\{2\}^2, [p_\perp]) = \frac{\mathrm{cov}(v_n\{2\}^2, [p_\perp])}{\sqrt{\mathrm{Var}(v_n^2)_{\mathrm{dyn}} C_{p_\perp}}}$$

Higher-order Correlations

= **central observables in the future experiments**
that fully make use of their high statistics.

—Search for the QCD critical point
—Femtoscopy → Hadron interaction
—Nuclear structure



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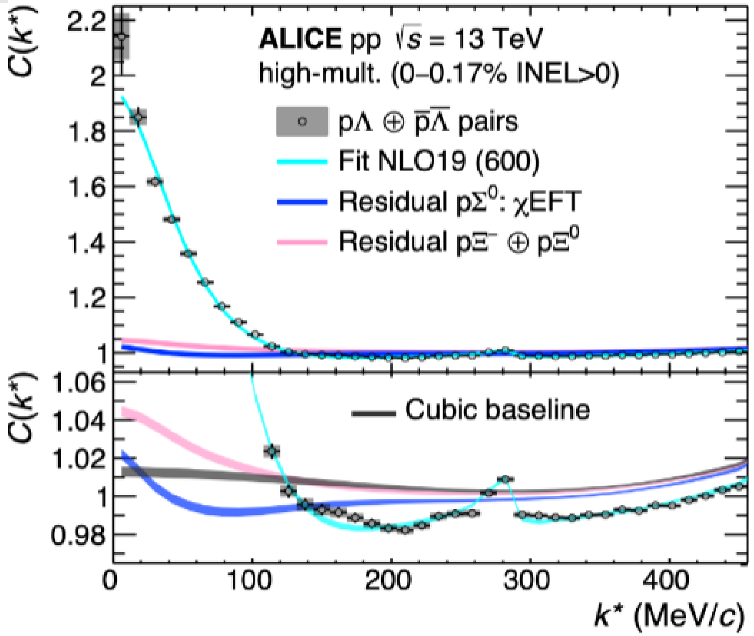

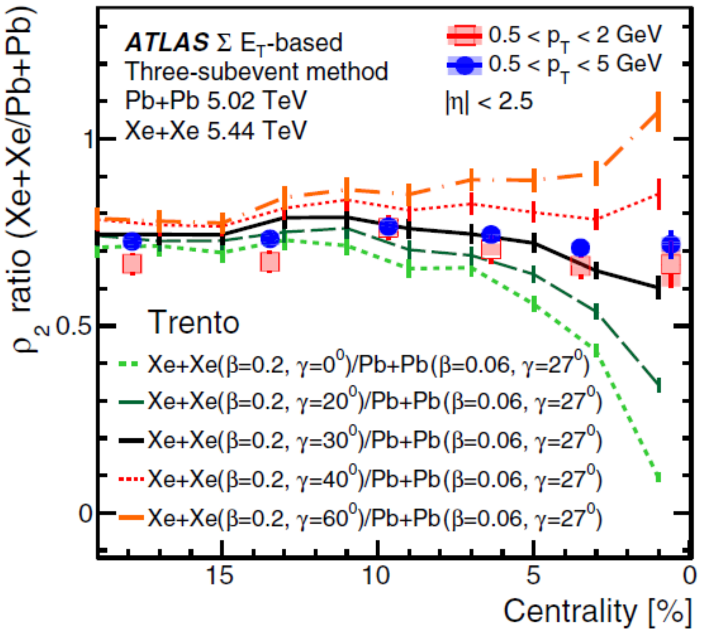

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Imperfect Particle Measurements

Real detectors cannot measure all particles, but lose some of them.

- Experimentally-observed results are modified from the true ones. We must correct the effects to obtain the true result.

Conventional Correction Formula

$$\mathrm{cov}_n = \left\langle \frac{\sum_{i,j,k,i\neq j\neq k} w_i w_j w_k e^{in(\phi_i-\phi_j)}(p_{\mathrm{T,k}} - \langle [p_{\mathrm{T}}] \rangle)}{\sum_{i,j,k,i\neq j\neq k} w_i w_j w_k} \right\rangle$$

$$w = (\mathrm{efficiency})^{-1}$$

e.g.
ATLAS, PR**C107**, 054910 ('23)
STAR, Nature **635**, 67 ('24)

Question: How to derive (or justify) the formula?

A Counterexample

All events emit 20 particles

- 10 particles with $p_T = 1\ \mathrm{GeV}$
- 10 particles with $p_T = 2\ \mathrm{GeV}$

Perfect measurement

$$\bar{p_T}^2 = \left\langle \frac{\sum_{i\neq j}(p_{T,i} - \langle p_T \rangle)(p_{T,j} - \langle p_T \rangle)}{N(N-1)} \right\rangle = 0$$

no fluctuations

Measurement w/ efficiency r

+

Correction formula

$$\bar{p_T}^2 = \left\langle \frac{\sum_{i\neq j} w_i w_j (p_{T,i} - \langle p_T \rangle)(p_{T,j} - \langle p_T \rangle)}{\sum_{i\neq j} w_i w_j} \right\rangle \neq 0$$

fluctuating

The conventional formula doesn't perfectly reproduce the true value!

論語　衛霊公第十五

工欲善其事、必先利其器

To make an establishment excellent,　first polish up tools

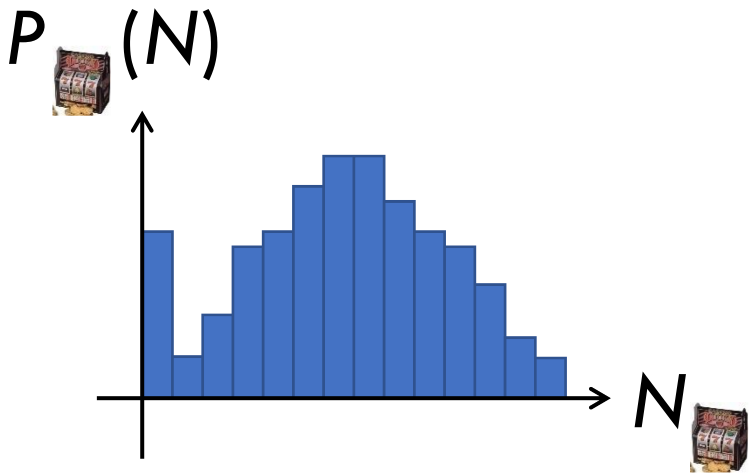
Simpler Example: Particle Number Fluc.

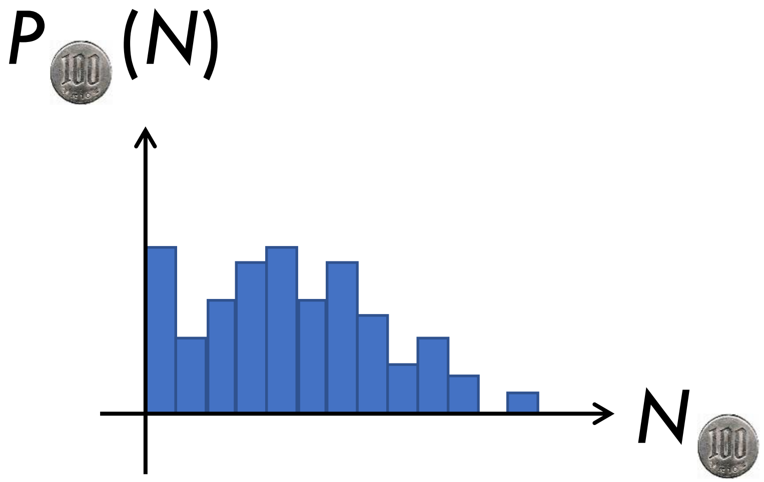
How can we obtain the cumulants of the true distribution only from observed information on $\tilde{P}(n)$?

MK, Asakawa, 2012

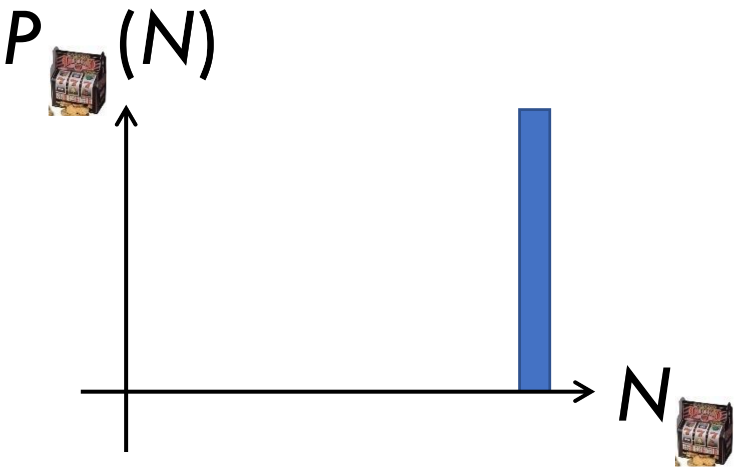
MK, Asakawa, 2012;2012

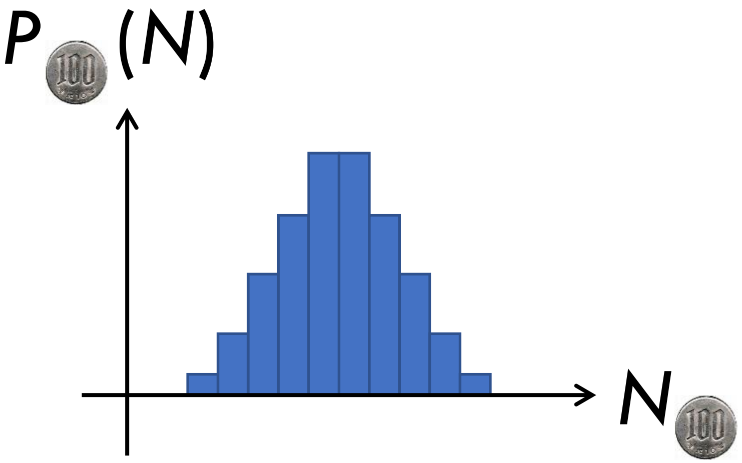
Slot Machine Analogy



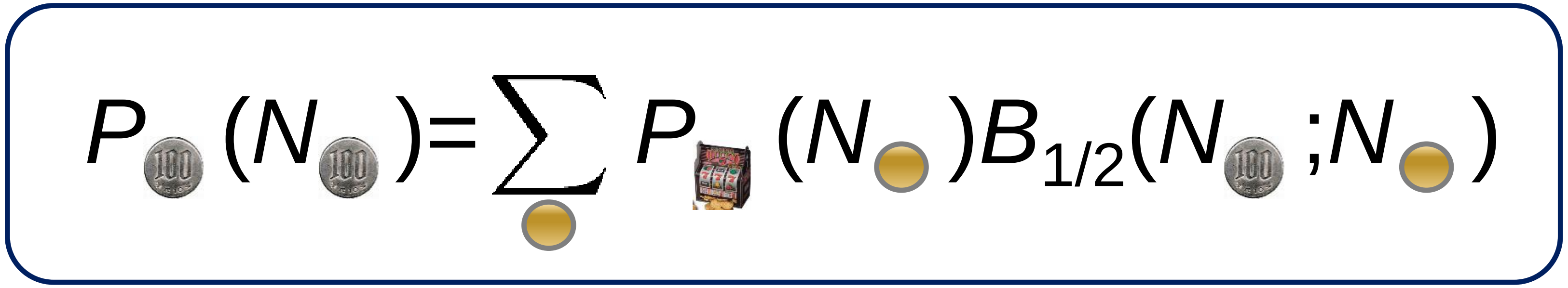


Fixed # of coins





Reconstructing Total Coin



- Example

$$
\begin{cases}
2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \cdots
\end{cases}
$$

genuine info. | Poisson noise

Note: Higher order cumulants are more fragile.

MK, Asakawa, 2012;2012

Derivation

generating funcs.: Asakawa, MK, 2016
factorial cumulants: MK, Luo, 2017

Use Generating Function

$$G(\theta) = \sum_N e^{\theta N} P(N)$$

$$\partial_\theta^m G = \sum_N N^m e^{\theta N} P(N) \langle N^m \rangle$$

$$\langle N^m \rangle = \partial_\theta^m G(0)$$

Efficiency correction for the binomial model

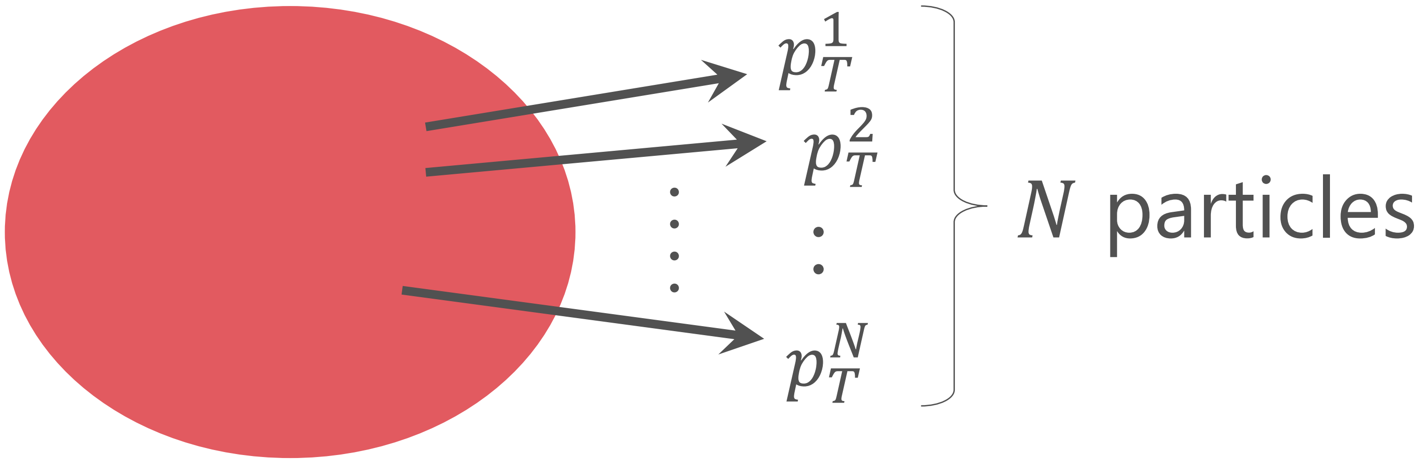
$$\tilde{P}(n) = \sum_n B_r(n;N) P(N)$$

$$\tilde{G}(\theta) = \sum_n e^{\theta n} \tilde{P}(n) = \sum_N (1 - r + r e^{\theta})^N P(N)$$

$$\langle N \rangle_{\text{true}} = \frac{\partial_\theta}{r} = \frac{1}{r} \langle n \rangle_{\text{obs}}$$

Modelling Imperfect Measurement

Each collision event:



True distr. func.

$$P(N; \vec{p}_T)$$

Observed distr. func.

$$\tilde{P}(n; \bar{p}_T)$$

Modelling Imperfect Measurement

True distr. func.

$$P(N;\vec{p}_T)$$

Observed distr. func.

$$\tilde{P}(N;\vec{p}_T)$$

Assumption

All particles are observed with the common (independent) efficiency r

Probability Distr. of Observed Quantities

$$\tilde{P}(n;\bar{p}) = \sum_{N=1}^{\infty} \int d\vec{p}_T \sum_{\{b_i\}} \left[\prod_{i=1}^{N} (1-r)^{1-b_i} r^{b_i} \right] \delta_{n,\sum_i b_i} \delta(\bar{p} - \Sigma_i b_i p_T^{(i)}) P(N;\vec{p}_T)$$

$$b_i = 0,1$$

Reconstructing Mean p_T

Prob. distr. func: $\tilde{P}(n;\bar{p}) = \sum_{N=1}^{\infty} \int d\vec{p} \sum_{\{b_i\}} \left[\prod_{i=1}^{N} (1-r)^{1-b_i} r^{b_i} \right] \delta_{n,\sum_i b_i} \delta(\bar{p} - \Sigma_i b_i p_i) P(N;\vec{p})$

Generating func: $\tilde{G}(s,t) = \sum_n \int d\bar{p} \tilde{P} s^n t^{\bar{p}} = \sum_N \int d\vec{p} P \prod_i (1 - r + rst^{p_i})$

Result

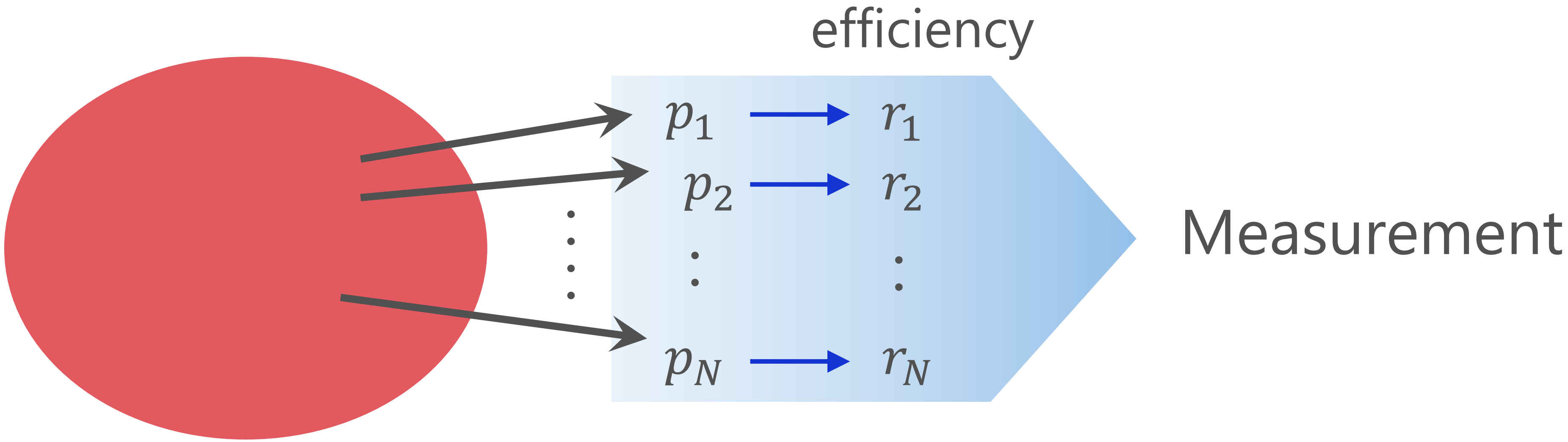
$$\left\langle \frac{\sum_i p_i}{N} \right\rangle_{\text{true}} = \int_\alpha^1 ds \frac{r}{s} \left[\partial_t \tilde{G}(s,t) \right]_{t=1} = \left\langle \frac{\sum_i p_i}{n} \left(1 - \alpha^n\right) \right\rangle_{\text{obs}} \qquad \alpha = \frac{r-1}{r}$$

$$\left\langle \frac{\sum_{i\neq j} p_i p_j}{N(N-1)} \right\rangle_{\text{true}} = \left\langle \frac{\sum_{i\neq j} p_i p_j}{n(n-1)} \left(1 - \alpha^n - n\alpha(1-\alpha^{n-1})\right) \right\rangle_{\text{obs}}$$

Note: $\left\langle \frac{\sum_i p_i}{N} \right\rangle_{\text{true}} \neq \left\langle \frac{\sum_i p_i}{n} \right\rangle_{\text{obs}}$

α^n term compensates the $n = 0$ contribution.

Multiple Efficiencies



My Answer

$$\left\langle \frac{\sum_i p_i}{N} \right\rangle_{\mathrm{true}} = \left\langle \sum_i p_i C_i \right\rangle_{\mathrm{obs}} \qquad C_i = \int_0^1 d\sigma \prod_{j \neq i} \left(1 - \frac{\sigma}{r_j}\right)$$

... and systematic derivation for higher-order correlations

Should Self-correlations be Eliminated?

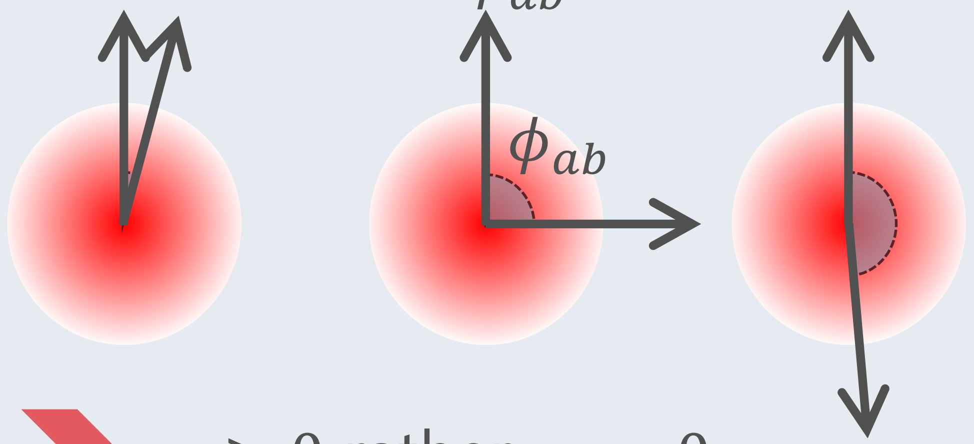
Flow correlations: $v_2^2 = \left\langle \frac{\sum_{i \neq j} e^{i(\phi_i - \phi_j)}}{N(N-1)} \right\rangle$

The "self correlation" terms are usually neglected. **Why?**

Argument 1:

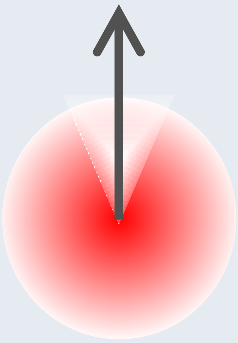
Emission of 2 independent particles

ϕ_{ab}: random



$v_2 > 0$ rather $v_2 = 0$

Argument 2:



Emission of a particle take away density

suppress probability to emit particles to same direction

Summary

- The beam-energy scan will reveal rich structures of the QCD phase diagram. Detailed measurements of higher-order correlations will be realized in the future experiments, such as HIAF, J-PARC-HI, etc.

- Quantitative analysis of the size and lifetime of the dense region:
  - $\sqrt{s_{NN}} \simeq 3\ \mathrm{GeV}$ may be an optimal energy to study $\rho = 3{\sim}4\rho_0$.

- The conventional formula for the efficiency correction of flow observables is reinvestigated. A new derivation is proposed.

Simulation Setup in JAM

- Au+Au collision for $2.4 \leq \sqrt{s_{NN}} \leq 20$ GeV
- Impact parameter $b \leq 3$ fm : top 5% centrality

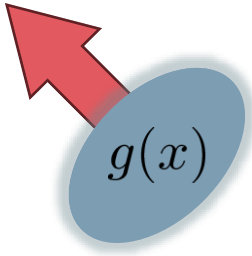
- Momentum-dependent mean field (MF2) Nara, Ohnishi, 2022
  - Setup reproducing $\sqrt{s_{NN}}$ dep. of $dv_1/d\eta$ and v_2

Smeared baryon current

discrete particle distribution ➔ continuous current by smearing

$$J^\mu(x) = \sum_{i \in \text{baryons}} B_i g(x; X_i, P_i) \frac{P_i^\mu}{P_i^0}$$

$$g(x; X, P) := \frac{\gamma}{(\sqrt{2\pi} r)^3} \mathrm{e}^{-\frac{|\boldsymbol{x}-\boldsymbol{X}|^2 + (\gamma \boldsymbol{V} \cdot (\boldsymbol{x}-\boldsymbol{X}))^2}{2r^2}} \qquad r = 1 \text{ fm}$$



Reconstruction of $\langle 1/N \rangle$

$$\tilde{P}(n) = \sum_n B_r(n;N)P(N)$$

$$\tilde{G}_{\mathrm{f}}(s) = \sum_n s^n \tilde{P}(n) = \sum_N (1-r+rs)^N P(N)$$: factorial-moment generating func.

$$\int_{(r-1)/r}^{1} ds \frac{r}{1-r+rs} \tilde{G}_{\mathrm{f}}(s) = \left\langle \frac{1}{N} \right\rangle_{\mathrm{true}} = \left\langle \int_{(r-1)/r}^{1} \frac{s^n}{1-r+rs} \right\rangle_{\mathrm{obs}}$$

Reconstruction of $\langle 1/N \rangle$ is possible in this case!

(RHS is divergent. One needs a regularization to obtain a finite result.)