Masakiyo Kitazawa (YITP)

Lattice study of stress-tensor distribution around the flux tube

R. Yanagihara+, Phys. Lett. **B789** (2019) 210 [arXiv: 1803.05656]

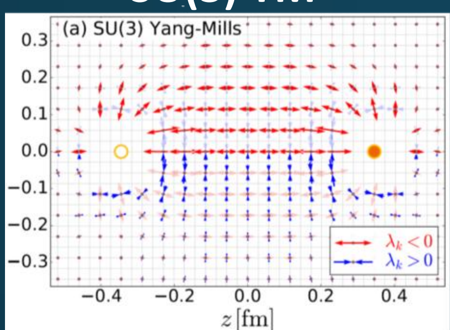
Flux tube in QQbar System

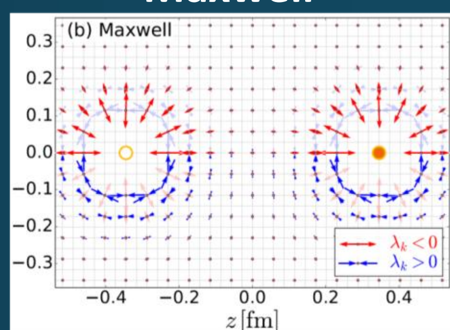
Our study

Lattice simulation of the **Stress-energy-momentum tensor distribution** in pure SU(3) Yang-Mills theory

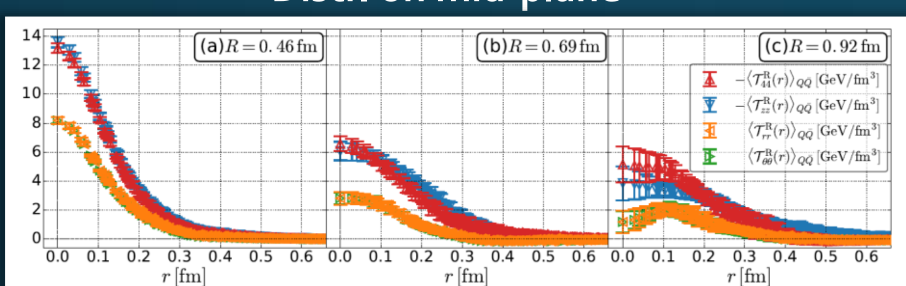
☐ Result

SU(3) YM **Maxwell**



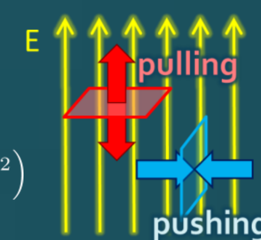


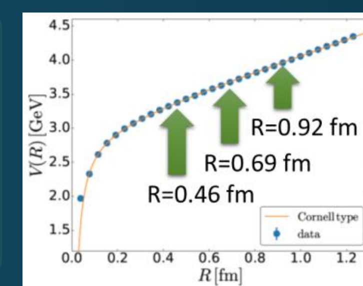
Distr. on mid-plane

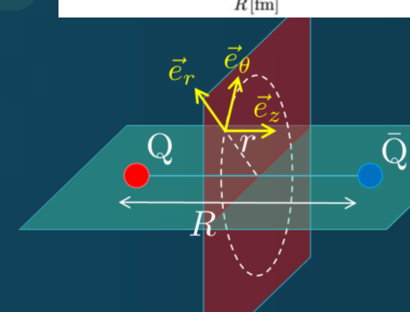


Maxwell Stress

$$\sigma_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$



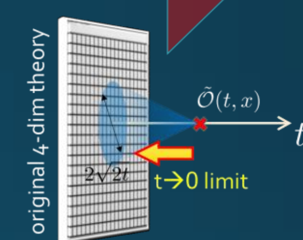




☐ Numerical Technique

Gradient flow and small-t expansion

$$\frac{\partial}{\partial t} A_\mu(t, x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$



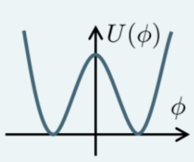
Energy-momentum tensor around the kink in 1+1d ϕ^4 theory

Ito, MK, JHEP**08** (2023) 033 [arXiv:2302.08762]

ϕ^4 model

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - U(\phi)$$

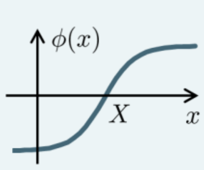
$$U(\phi) = \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2$$



Kink (Soliton)

$$\phi_{\rm kink} = \pm\frac{m}{\sqrt{\lambda}} \tanh\frac{m(x-X)}{\sqrt{2}}$$

- classical solution
- stable
- local



Energy-Momentum Tensor

Classical $\mathcal{O}(\lambda^{-1})$

$$\begin{cases} T_{00} = \dfrac{m^4}{\lambda}\cosh^{-4}\dfrac{m(x-X)}{\sqrt{2}} \\ T_{01} = T_{11} = 0 \end{cases}$$

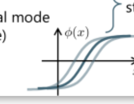
Purpose: quantum correction

Diagonalization

$$\left(-\partial_x^2 - m^2 + 3\lambda\phi_{\rm kink}^2\right)\psi_n = \omega_n^2\psi_n$$

ex) Rajaraman, "Solitons & Instantons"

Eigenvals	Eigenfuncs	
$\omega_q^2 = q^2 + 2m^2$	$\psi_q(x) \to \exp\left[i\left(qx \pm \frac{1}{2}\delta\right)\right]$ (δ: phase shift)	scattering states
$\omega_1^2 = \frac{3}{2}m^2$	$\psi_1(x)$: vibrational mode	bound states
$\omega_0^2 = 0$	$\psi_0(x) \sim \partial_x\phi_{\rm kink}(x)$: translational mode (zero mode)	bound states



Fluctuations around the Kink

Model: ϕ^4 theory (1+1d)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{\lambda}{4}\left(\phi - \frac{m^2}{\lambda}\right)^2$$

$\phi(x)$: real scalar

Kink

$$\phi_{\rm kink} = \frac{m}{\sqrt{\lambda}}\tanh\frac{m(x-X)}{\sqrt{2}}$$

Expansion around $\phi_{\rm kink}(x)$

$$\phi(x) = \phi_{\rm kink}(x) + \eta(x)$$

$$S[\eta] = S_{\rm cl} + \int d^2x\left\{\frac{1}{2}(\partial_0\eta)^2 - \frac{1}{2}\eta\left(-\partial_x^2 - m^2 + 3\lambda\phi_{\rm kink}^2\right)\eta - \lambda\phi_{\rm kink}\eta^3 - \frac{\lambda}{4}\eta^4\right\}$$

quadratic ➡ diagonalize; $\mathcal{O}(\lambda^{1/2})$

- First-order terms can be eliminated by the partial integral & EoM.

Vacuum Subtraction (Dashen+, '74)

Rebhan and Nieuwenhuizen ('97)
Rajaraman, "solitons & instantons"

Mode Number Cutoff

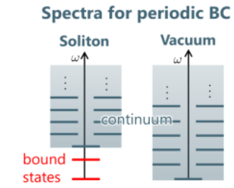
- System with finite length L
  → Scattering modes become discrete.
- Perform the vacuum subtraction with the same mode number N.
- Take $N \to \infty$, and then $L \to \infty$.

Scattering modes $\psi_q(x)$ satisfy

- Soliton sector: $q_n + 2\delta(q_n) = 2n\pi/L$
- Vacuum sector: $q_n = 2n\pi/L$

➡ $$E_{\rm kink} = \frac{2\sqrt{2}}{3}\frac{m^3}{\lambda} + \left(\frac{\sqrt{3}}{6\sqrt{2}} - \frac{3}{\sqrt{2}\pi}\right)m$$

Spectra for periodic BC

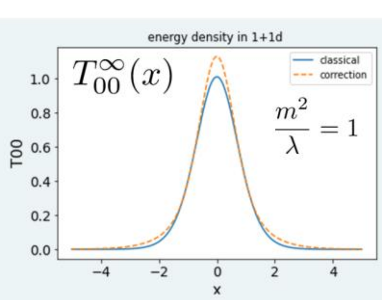


Results

$$T_{00}(x) = T_{00}^\infty(x) - \frac{3\sqrt{2}m}{2\pi}\frac{1}{L}$$

$$T_{11}(x) = \frac{3\sqrt{2}m}{2\pi}\frac{1}{L}$$

- T_{00}^∞: finite & local function
- L: spatial length



Collective Coordinate Method

Zero mode gives rise to IR divergence!

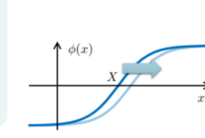
Gervais, Sakita '74
Gervais, Jevicki, Sakita, '75
Tomboulis, '75; Christ, Lee, '75

CCM: Basic idea

zero mode = translational mode

$$\psi_0(x) \sim \partial_x\phi_{\rm kink}(x)$$

➡
- Eliminate the zero mode
- Promote X to a dynamical val.



$$\phi(x,t) = \phi_{\rm kink}(x - X(t)) + \tilde{\eta}(x - X(t))$$

$X(t)$: dynamical

constraint: $\int dx\,\tilde{\eta}(x)\psi_0(x) = 0$