

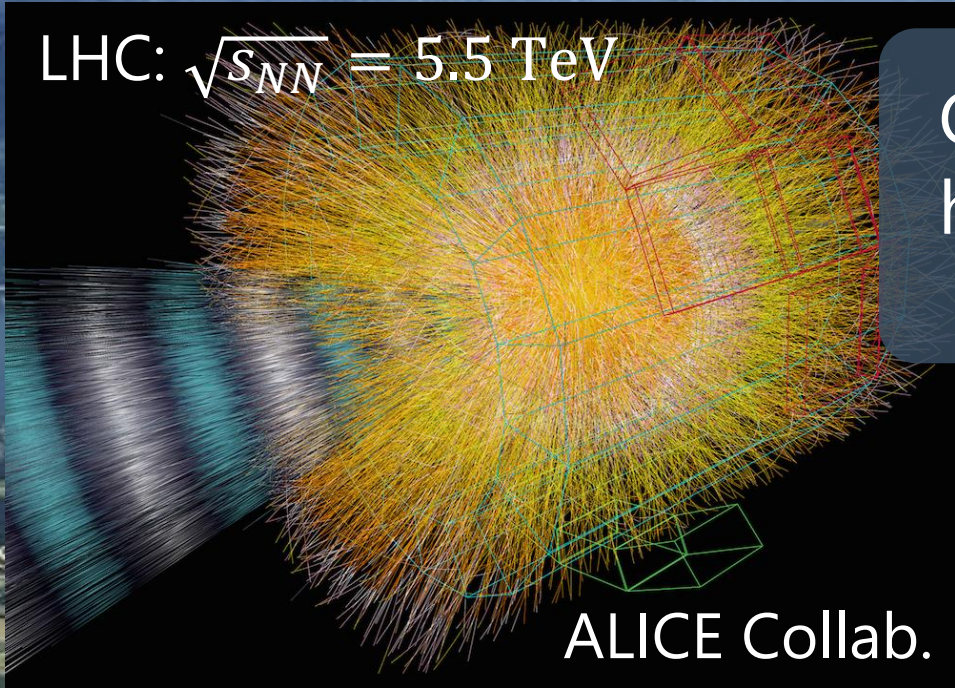
# Probing nuclear shapes in relativistic heavy-ion collisions

Masakiyo Kitazawa  
(YITP, Kyoto)

Hagino, MK, PRC 112 (2025) L041901 [2508.05125];  
MK, Esumi, Niida, Nonaka, PTEP (2026) in press, [2510.18383].

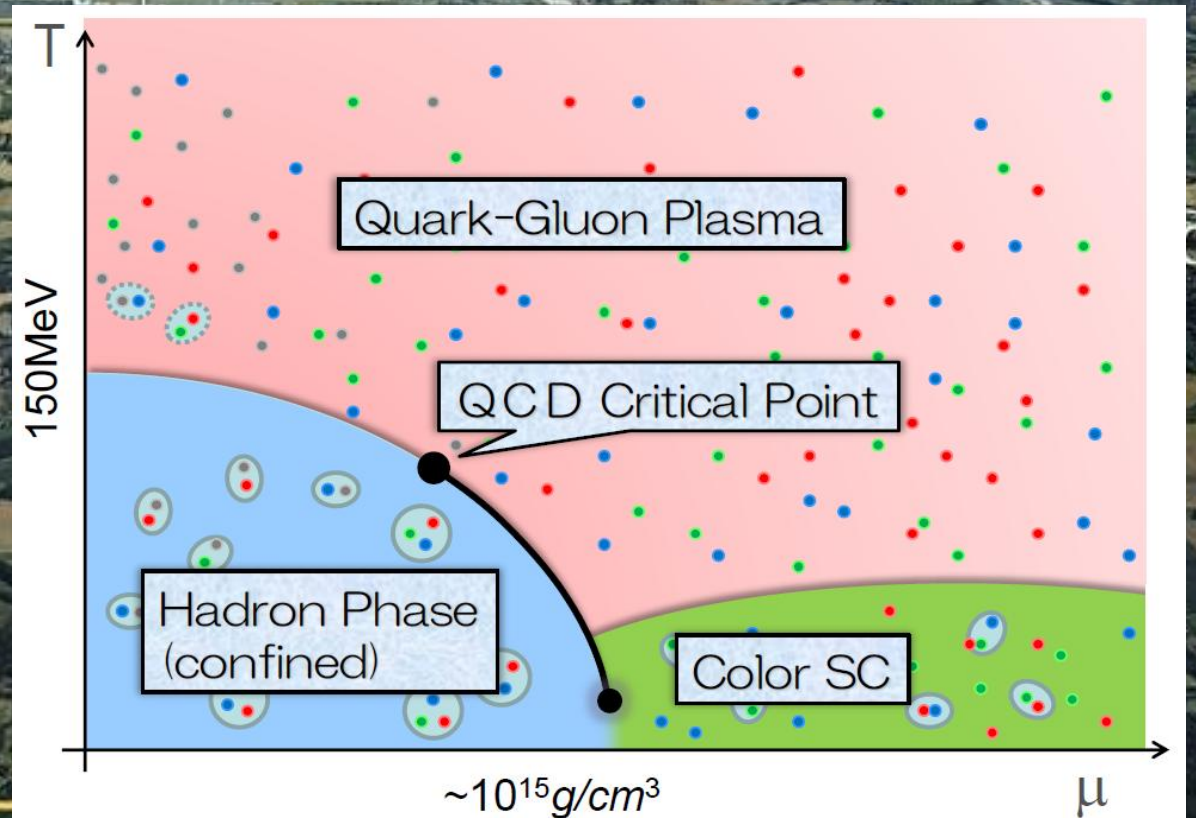
# Relativistic Heavy-Ion Collisions

LHC:  $\sqrt{s_{NN}} = 5.5 \text{ TeV}$

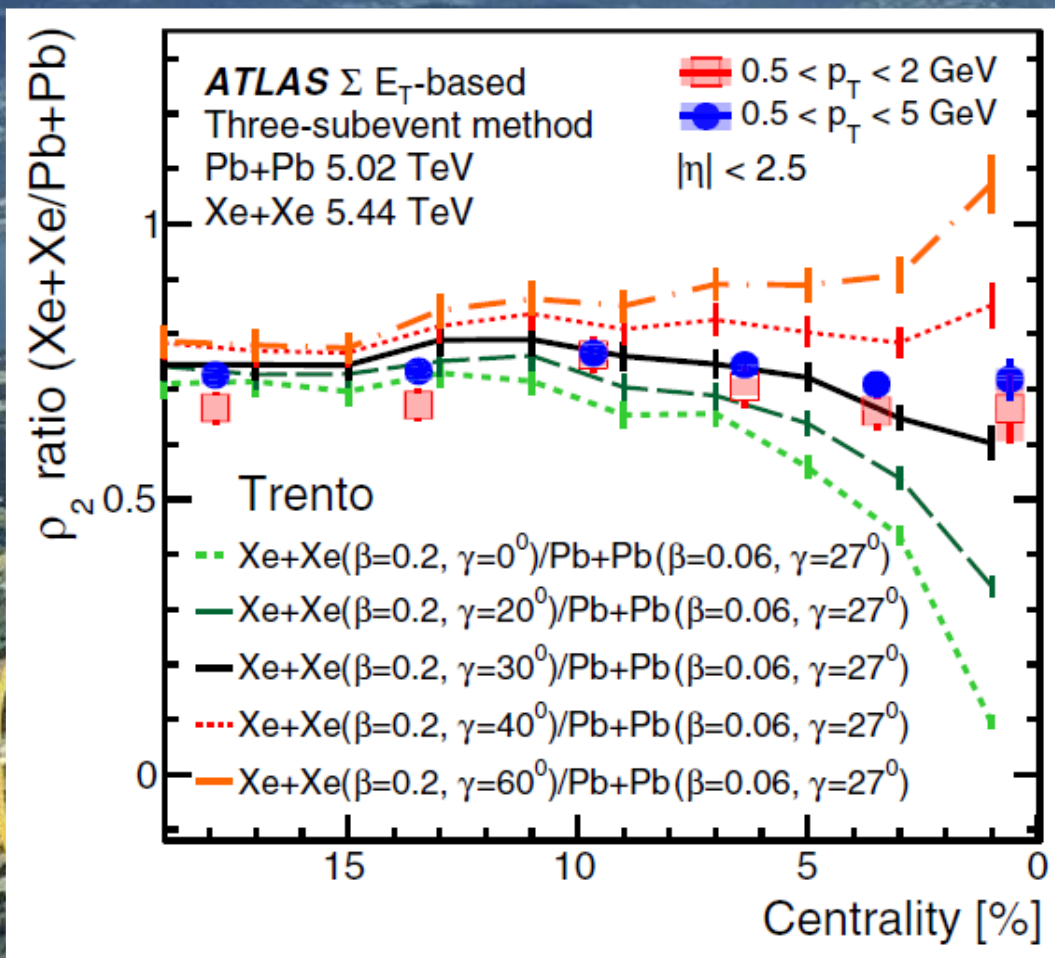


Creation of  
hot/dense matter

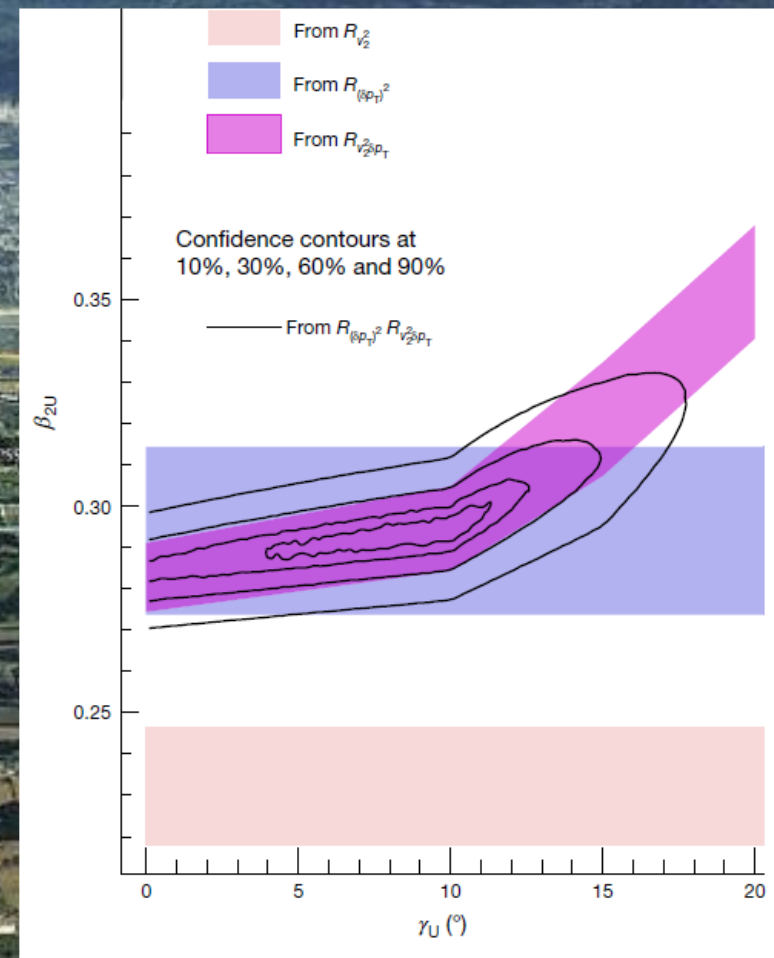
Study of  
—quark-gluon plasma  
—QCD phase diagram



# Probing Nuclear Shapes in RelHIC?!



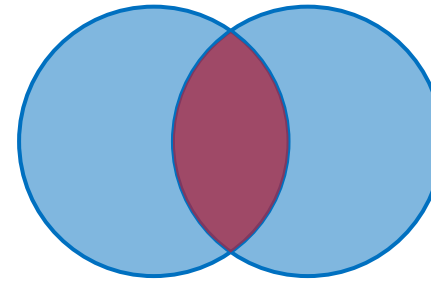
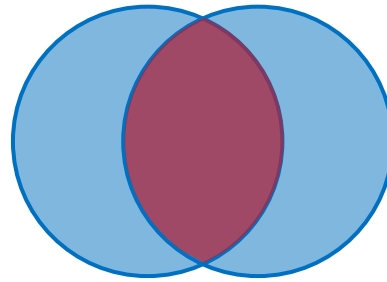
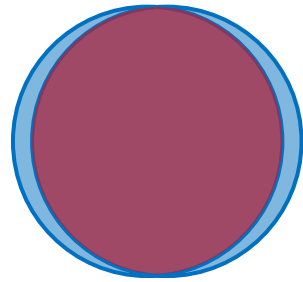
ATLAS, PRC 107, 054910 (2023)



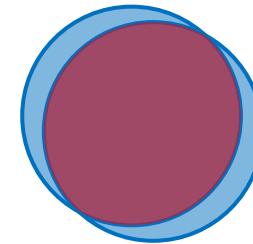
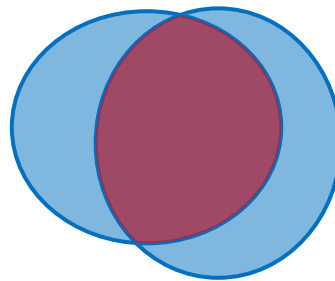
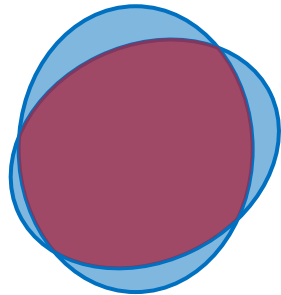
STAR, Nature 635, 67 (2024)

# Rough Idea

**Spherical**



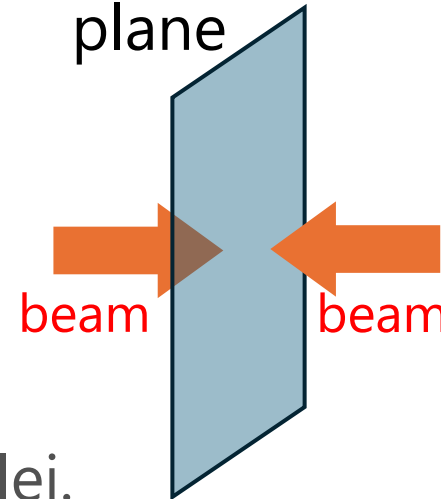
**Deformed**



beam axis



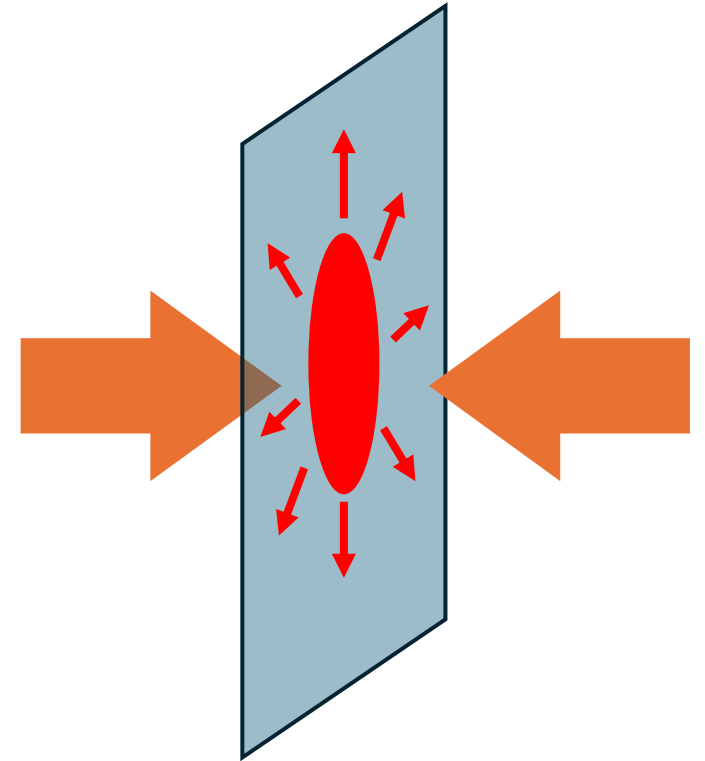
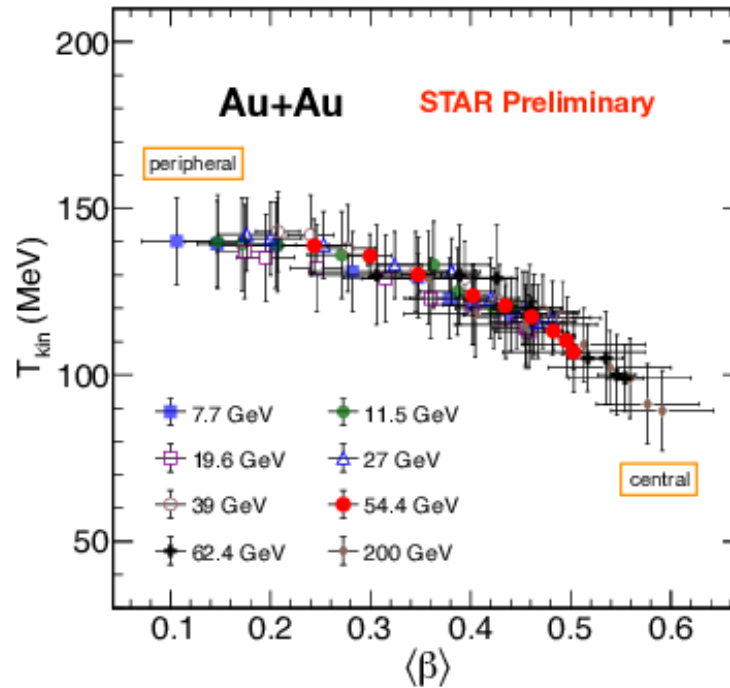
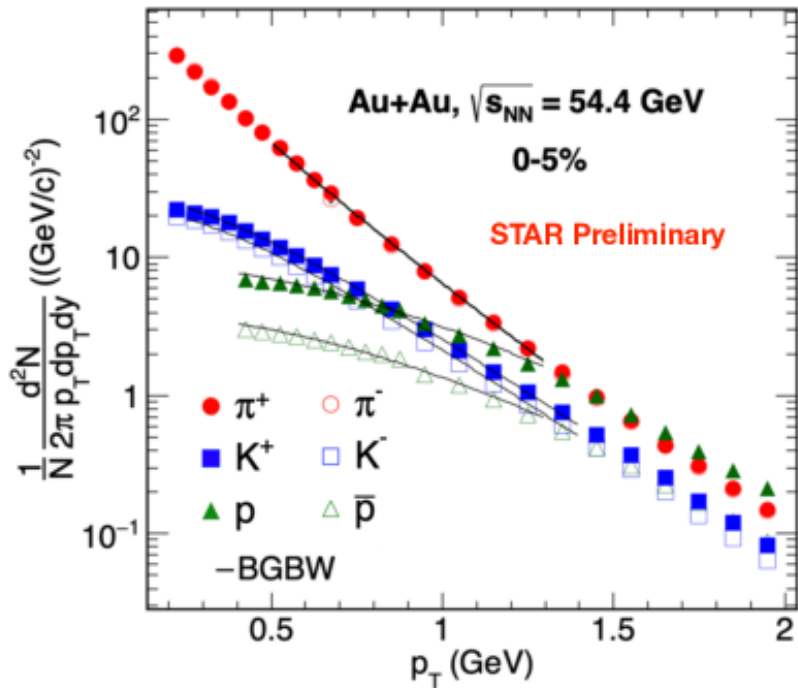
transverse plane



Different transverse shapes for the spherical and deformed nuclei.  
Distribution is reflected into anisotropic flows in the final state.

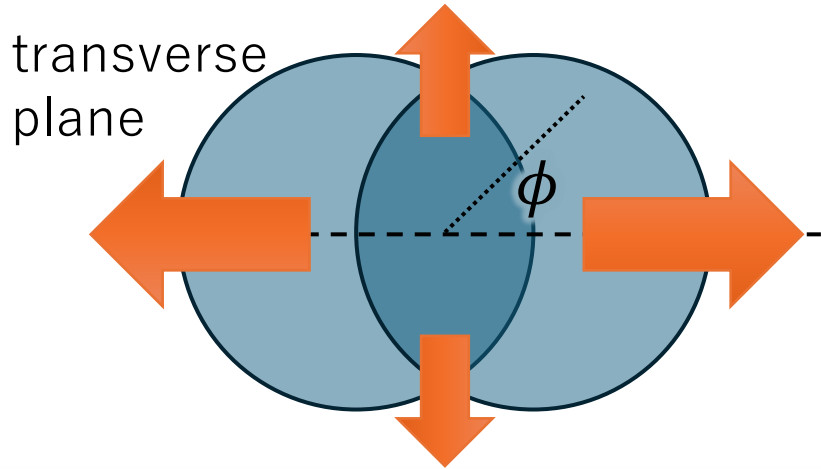
**High**-energy collisions → **snapshot** of the overlapping region of **intrinsic states**

# Preparation 1: Transverse Momenta $p_T$



- Blast-wave fit: thermal + radial expansion
- Radial expansion enhances  $[p_T]$  (radial flow)

# Preparation 2: Elliptic Flow / Eccentricity



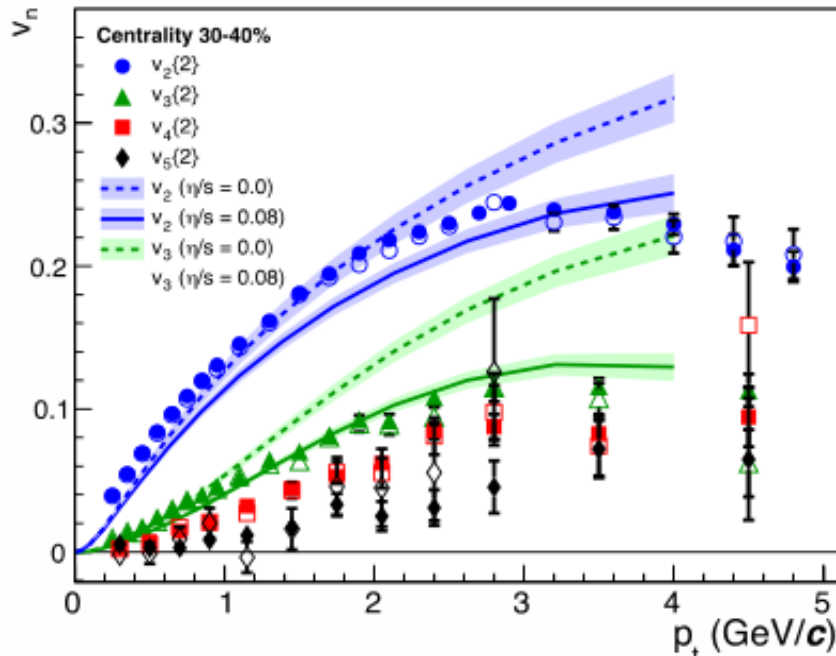
**Eccentricity:**  
(coordinate space)

$$\epsilon_n e^{i\Phi_n} \sim -\int d^2x_T e^{in\phi_s} w_n(r_T) \rho(\mathbf{x}_T)$$

$$\epsilon_2 = \frac{\langle x^2 \rangle - \langle y^2 \rangle}{\langle x^2 \rangle + \langle y^2 \rangle} \quad : \text{eccentricity}$$

**Anisotropic Flow:**  
(momentum space)

$$\frac{1}{N} \frac{dN}{d\phi} = \frac{1}{2\pi} \left[ 1 + 2 \sum_n v_n \cos n(\phi - \Psi_n) \right]$$



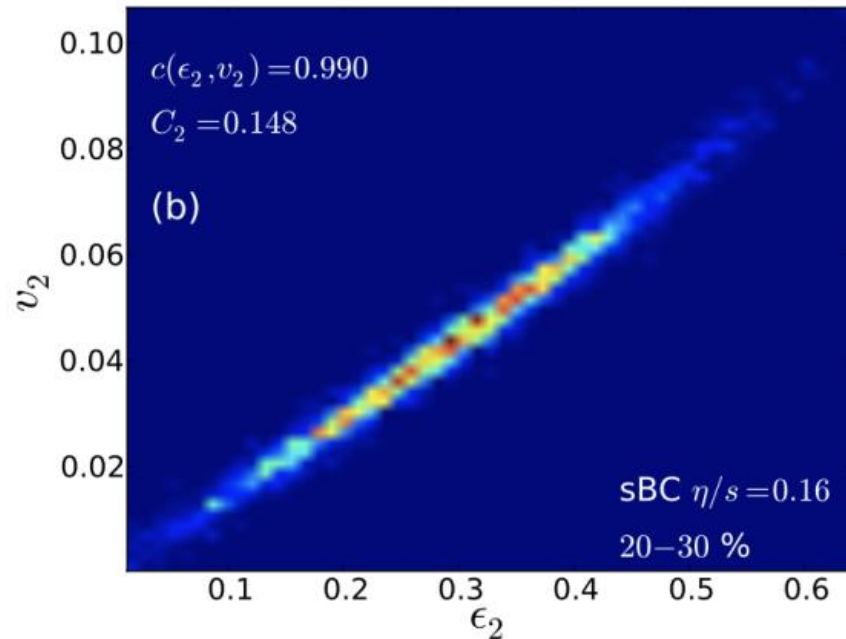
**Recent definition:**  
(multi-particle correlations)

$$v_n\{2\}^2 = \frac{1}{N_{pairs}} \sum_{i \neq k} e^{in\phi_i} e^{-in\phi_k}$$

- Initial anisotropy in coordinate space is converted into momentum one in the final state.
- Evidence of hydrodynamic response

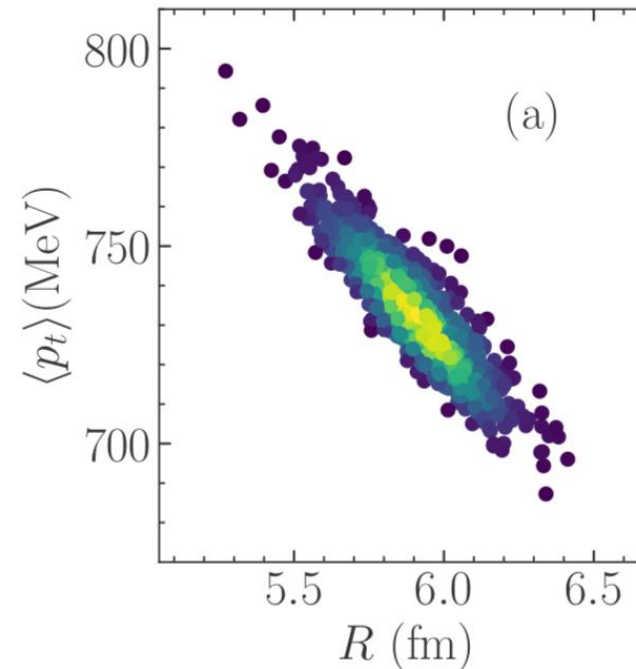
# Strong Correlation b/w Initial/Final

$\epsilon_2$  VS  $v_2$



Niemi+, PRC87, 054901('13)

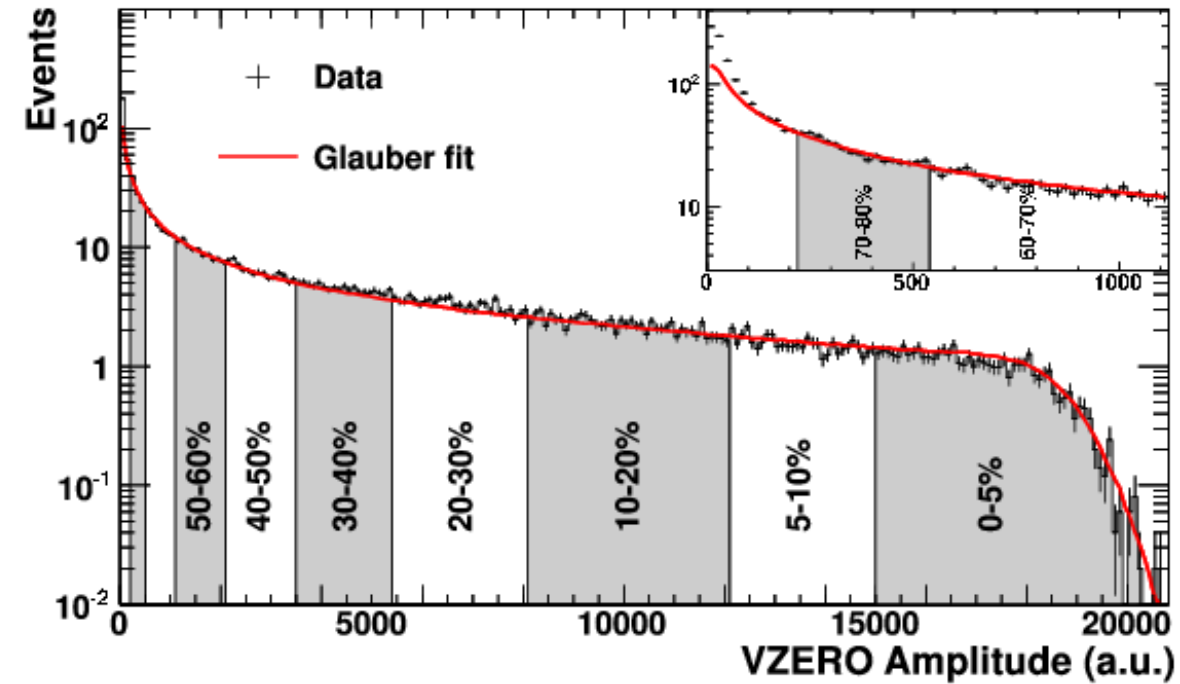
$R$  vs  $[p_T]$



Giaccalone+, PRC103, 024909

Strong e-v-e correlations between  $\epsilon_2$  and  $v_2$  /  $R$  and  $[p_T]$ .  
Information of the initial shape can be encoded in flow observables!

# Preparation 3: Centrality



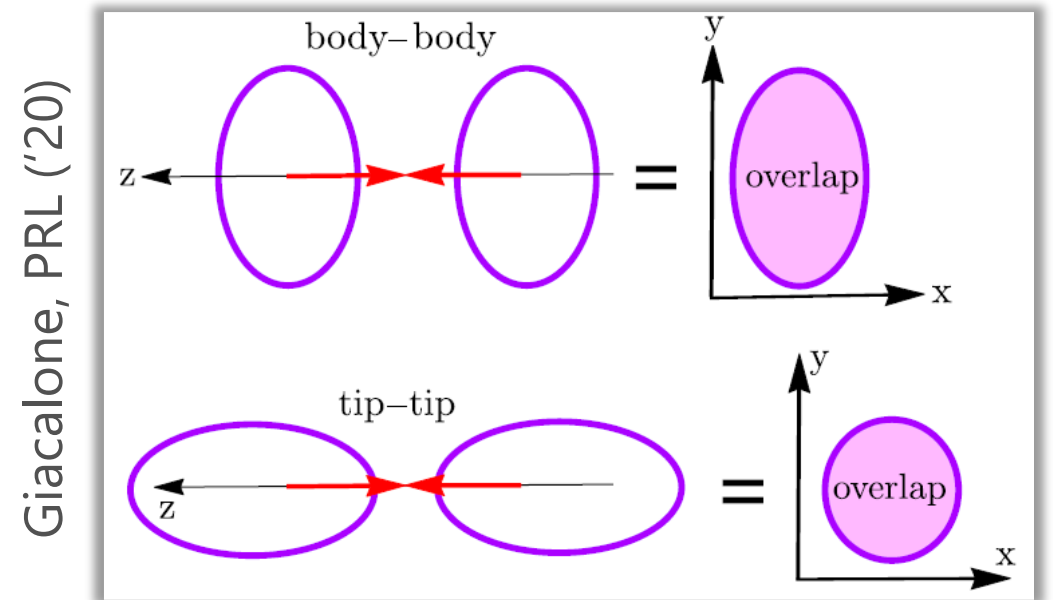
- The impact parameter is not observable in HIC.
- **Centrality:** Proxies of impact param., such as particle multiplicities
- Notice: no direct connection between impact parameter & centrality

# Ultra-Central Collisions (UCC)

UCC → Almost all particles participate in the collisions

## Collision of Prolate nuclei

	mean radius	anisotropy
tip-tip	small	small
body-body	large	large



# Ultra-Central Collisions (UCC)

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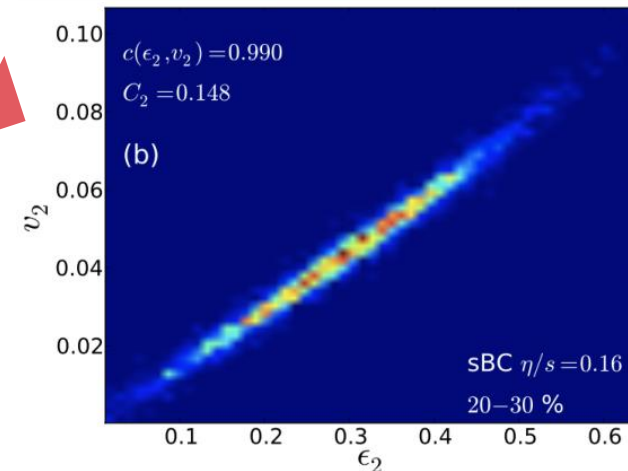
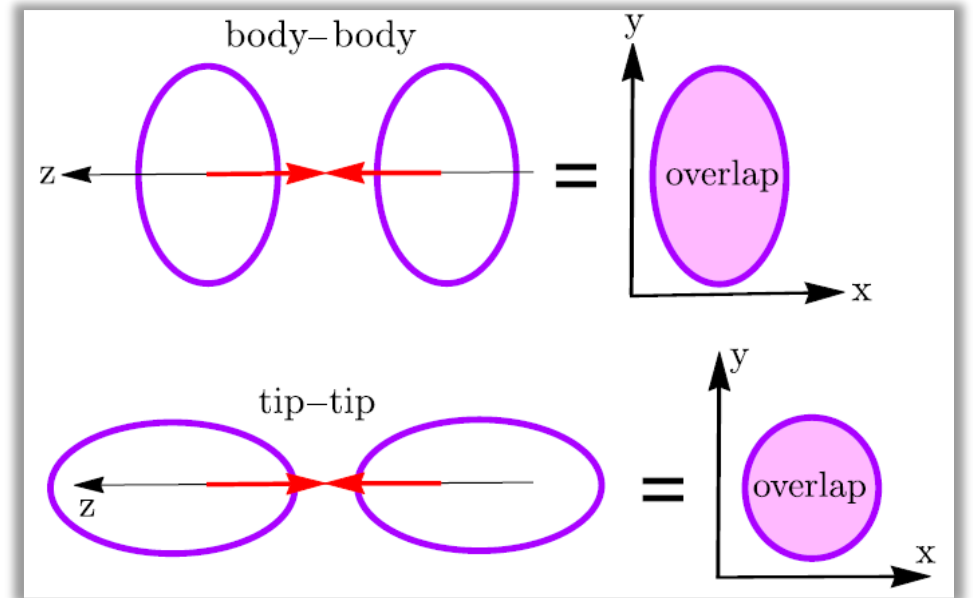


hydro. evolution

tip-tip	large $\bar{p}_T$ / small $v_2$
body-body	small $\bar{p}_T$ / large $v_2$

→ Inverse correlation of  $v_2$  &  $\bar{p}_T$

Giocalone, PRL ('20)



# Experimental Result @STAR

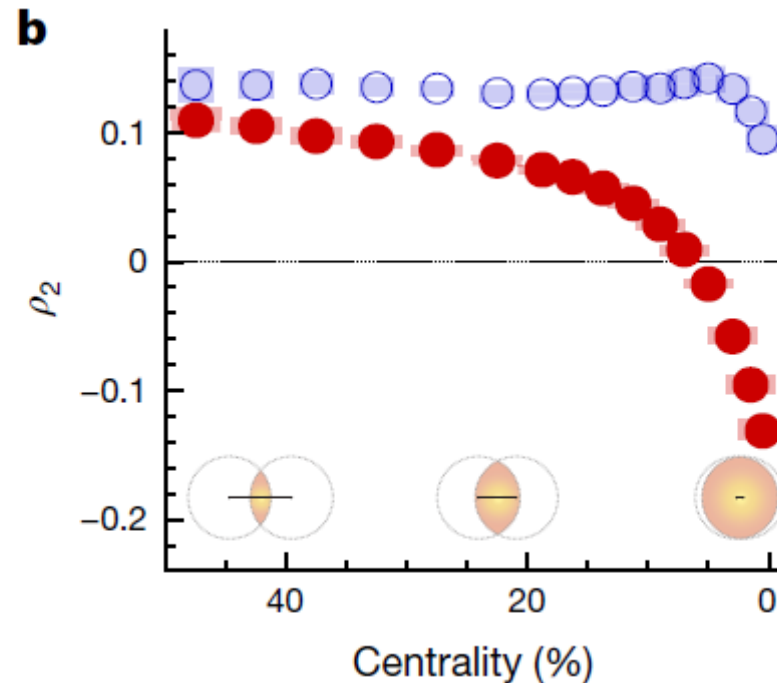
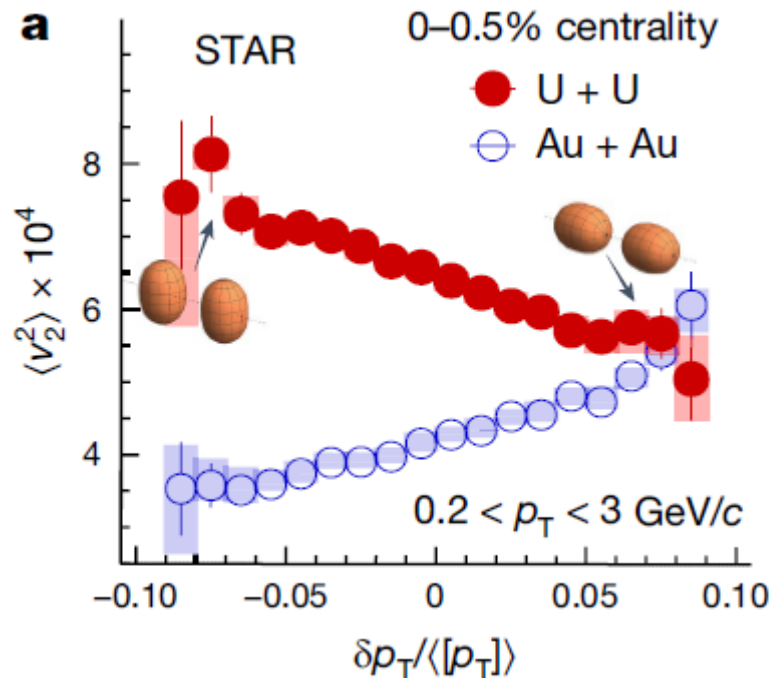
tip-tip	large $\bar{p}_T$ / small $v_2$
body-body	small $\bar{p}_T$ / large $v_2$

→ Inverse correlation of  $v_2$  &  $\bar{p}_T$

$v_n - p_T$  correlation:

$$\rho(v_n\{2\}^2, [p_\perp]) = \frac{\text{cov}(v_n\{2\}^2, [p_\perp])}{\sqrt{\text{Var}(v_n^2)_{\text{dyn}} C_{p_\perp}}}$$

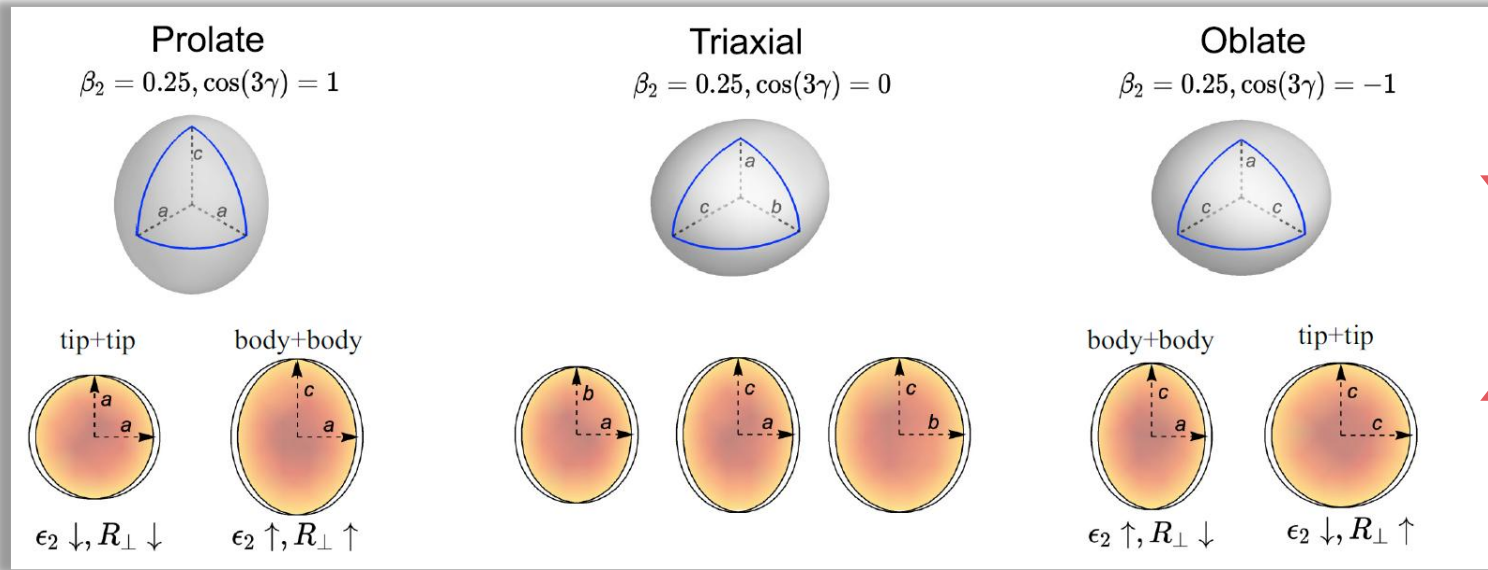
Bozek, PRC 93 ('16)



STAR, Nature 635 ('24)

# Triaxial Deformation

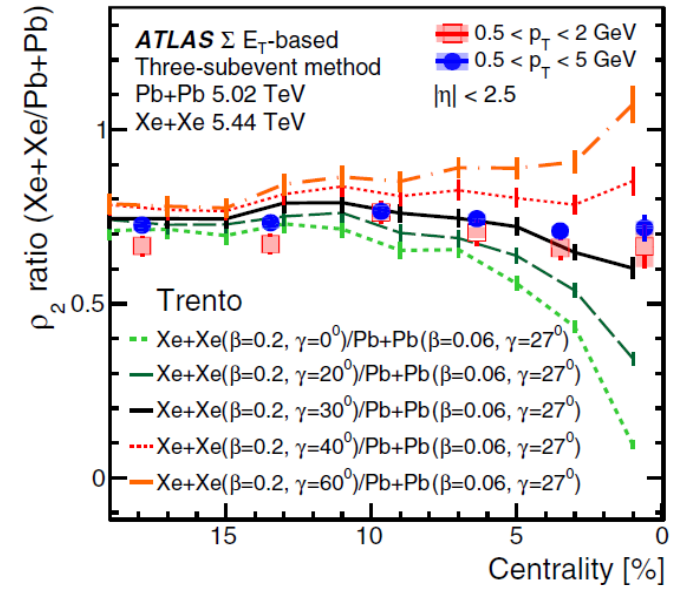
Fig.: Jia, PRC ('22)



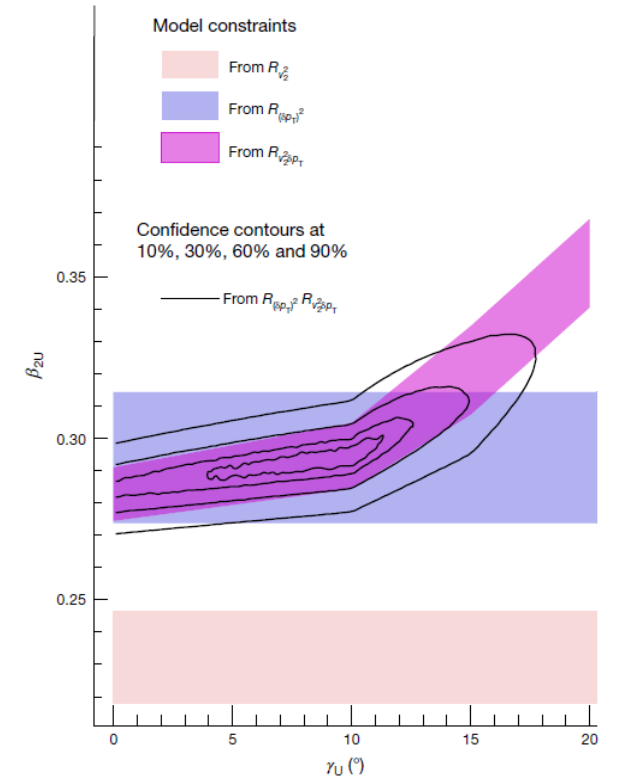
- $v_2$ - $p_T$  correlation is sensitive to deform. param.  $\gamma$ .
- Other correlations sensitive to  $\beta_2$  and  $\gamma$ .

$$\left\langle \varepsilon_2^2 \frac{\delta d_\perp}{d_\perp} \right\rangle = -\frac{3\sqrt{5}}{28\pi^{3/2}} \cos(3\gamma) \beta_2^3, \quad \left\langle \left( \frac{\delta d_\perp}{d_\perp} \right)^2 \right\rangle = \frac{1}{16\pi} \beta_2^2$$

ATLAS, PRC 107 (2023)



STAR, Nature 635 (2024)



# Further Extension

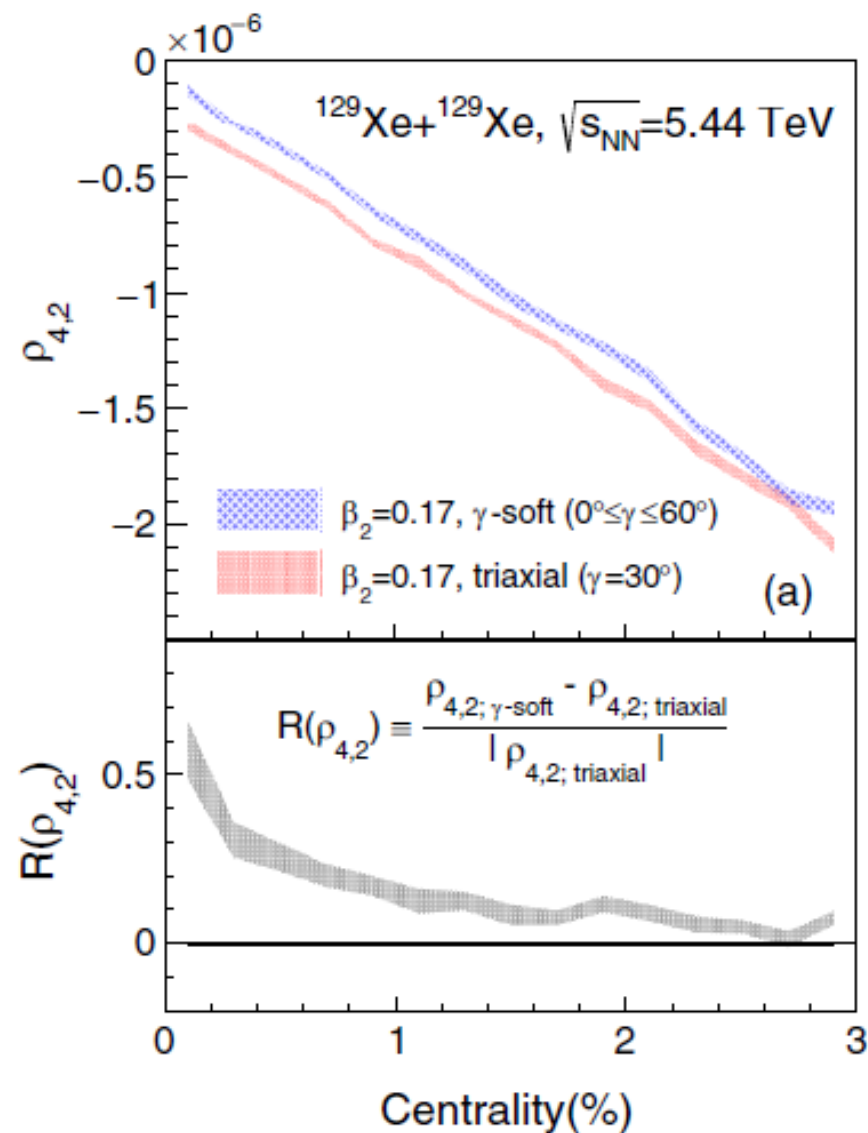
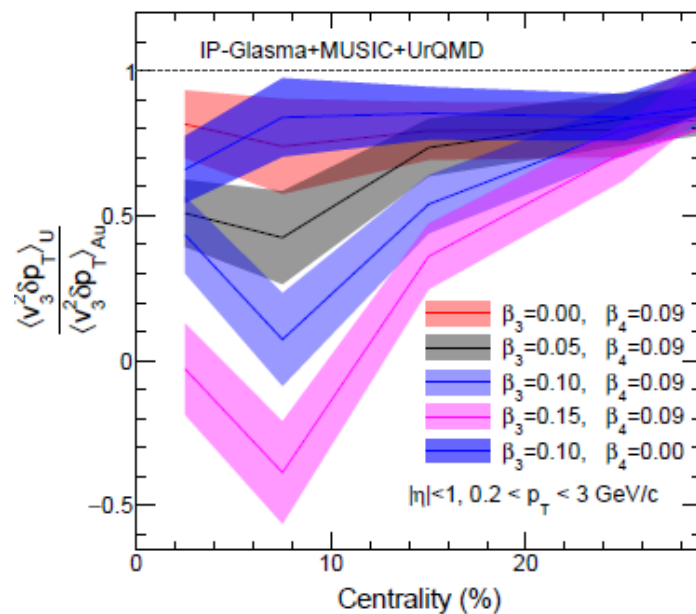
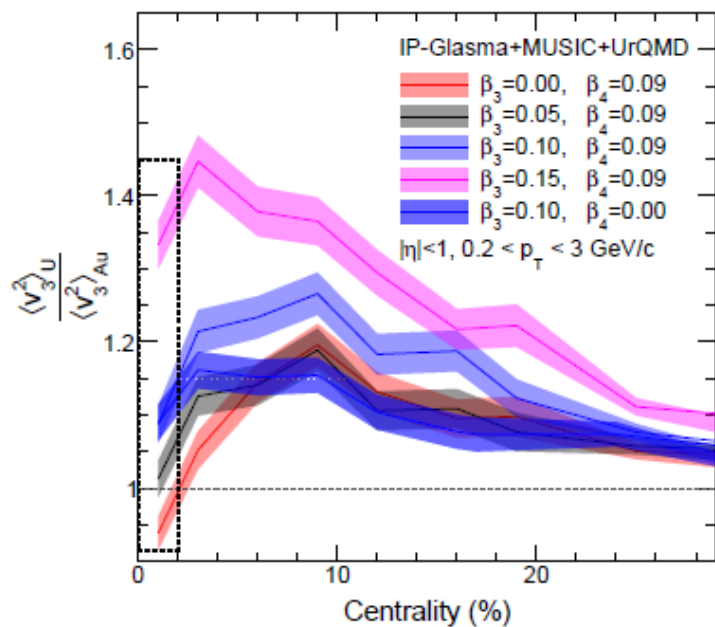
## Probing

### — Shape fluctuations

Zhao, Xu, Zhou, Liu, Song, PRL ('24); Hagino, MK, PRC ('25);  
Liu+, 2509.09376; ...

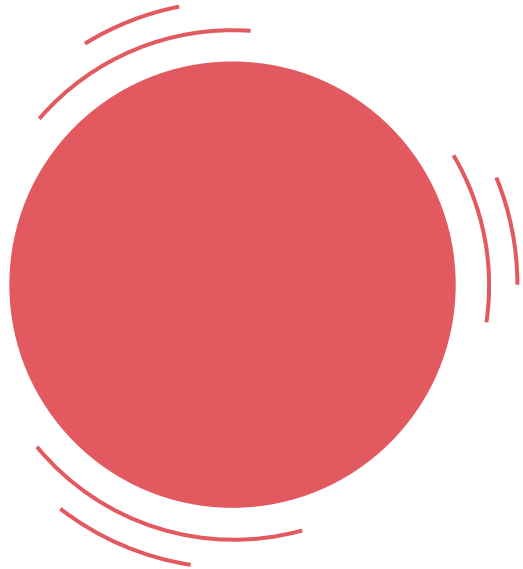
### — Octupole/hexadecapole deformation

Zhang+, 2504.15245; Xu+, 2504.19644; ...



# Quantum Surface Vibration

Hagino, MK, PRC ('25)

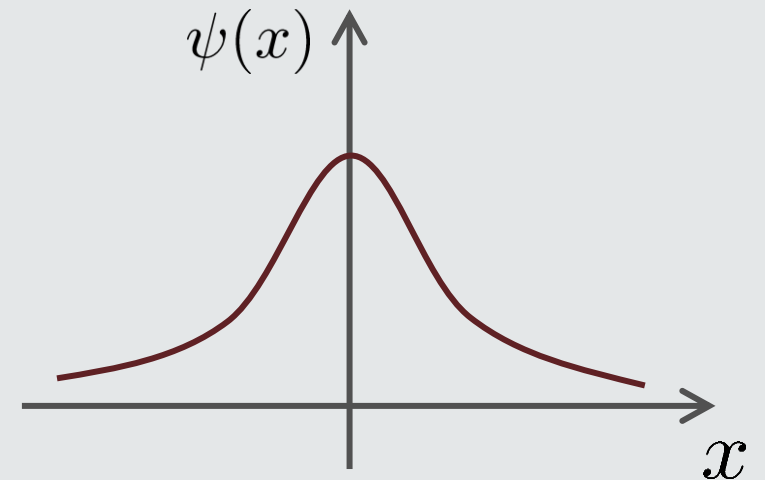


- Shape of a nucleus is **quantumly** vibrating even on the ground state.
- timescale of HIC  $\gg$  surface vibration

**➤ HIC takes a snapshot of shape fluctuation.**

## Harmonic Oscillators

ground state



$$\langle x \rangle = 0 \quad \langle x^2 \rangle \neq 0$$

See also

Zhao, Xu, Zhou, Liu, Song ('24)

Xu, Xu, Zhao, Zhao, Song, Wang ('25)

Liu+, 2509.09376

# Spherical Nuclei

Hagino, MK, PRC ('25)

Space-fixed coordinates

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R(\theta,\phi))/a}}$$

$$R(\theta, \phi) = R_0 \left( 1 - \frac{1}{4\pi} \sum_{\lambda, \mu} |\alpha_{\lambda\mu}|^2 + \sum_{\lambda, \mu} \alpha_{\lambda\mu} Y_{\lambda\mu}^*(\hat{r}) \right)$$

Harmonic-oscillator model for surface vib.

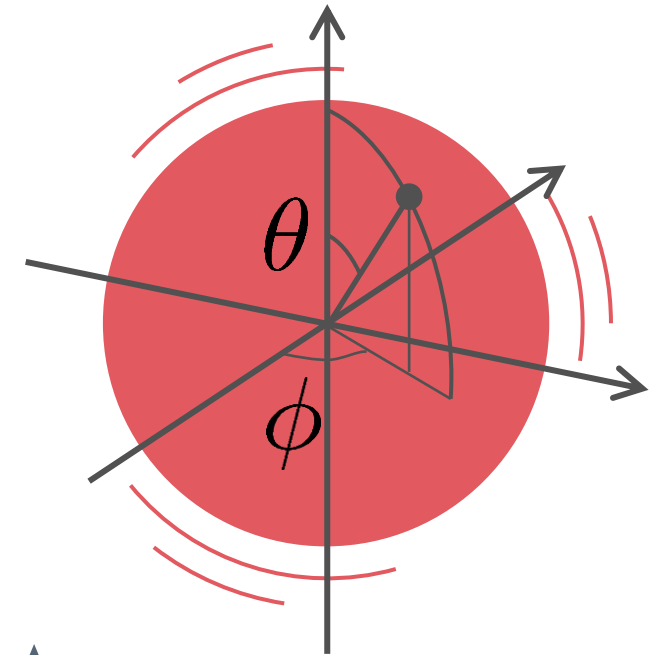
$$H = \frac{1}{2} \sum_{\lambda, \mu} (B_\lambda |\dot{\alpha}_{\lambda\mu}|^2 + C_\lambda |\alpha_{\lambda\mu}|^2) \quad \rightarrow \quad \left\langle \sum_{\mu} |\alpha_{\lambda\mu}|^2 \right\rangle = (\beta_\lambda)^2$$

Constraint from low- $E$  exp. of  $B(E\lambda)$

$$\beta_\lambda = \frac{4\pi}{3ZR_0^\lambda} \sqrt{\frac{B(E\lambda) \uparrow}{e^2}}$$

Hagino, Takigawa ('12)

Hagino, Ogata, Moro ('22)



More complicated in  
**body-fixed coordinates**

Treatment of surface vibration is apparent in the space-fixed coordinates.  
Deformation params.  $\beta_\lambda$  can be constrained from transition probability.

# Transverse Distribution

Hagino, MK, PRC ('25)

## Initial Transverse Distr.

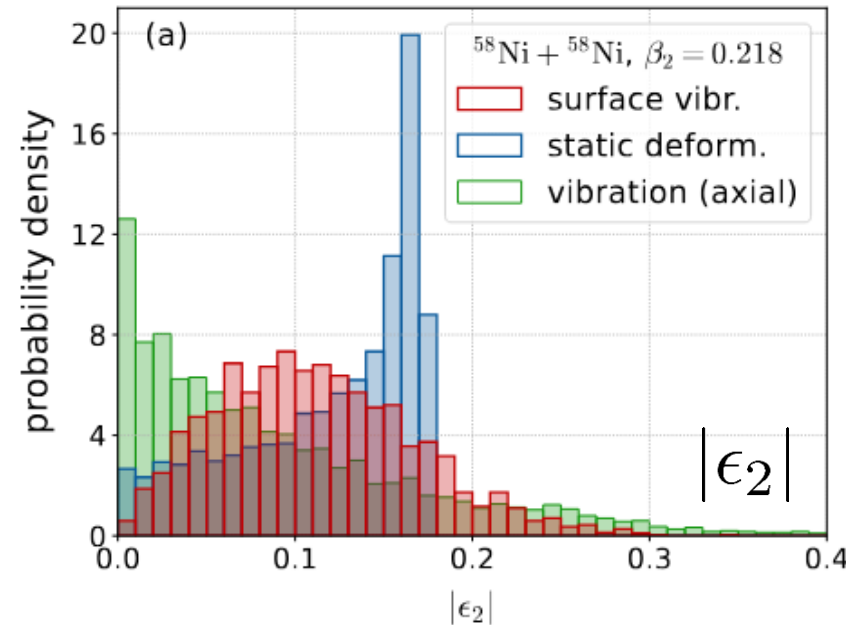
droplet full-overlap model

$$\rho^{(z)}(\mathbf{r}_\perp) = \int_{-\infty}^{\infty} dz \rho(\mathbf{r})$$

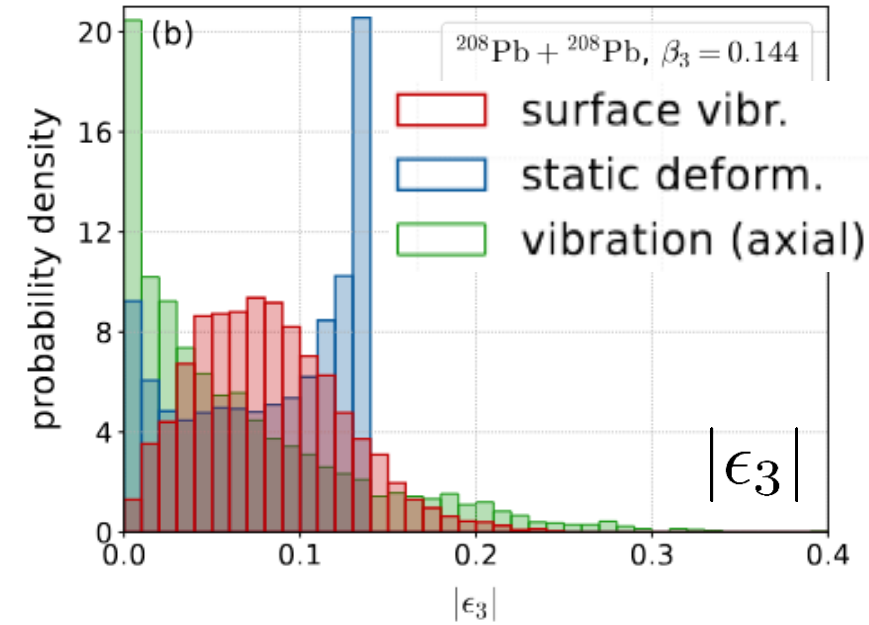
Jia ('22)

$$\epsilon_n = -\frac{\langle\langle (x - iy)^n \rangle\rangle}{\langle\langle (x^2 + y^2)^{n/2} \rangle\rangle}$$

## $^{58}\text{Ni}$ , Quadrupole



## $^{208}\text{Pb}$ , Octupole



- Distribution differs significantly between the surface vibration and static deformation.
- Axial deformation is insufficient to describe the surface vibration.

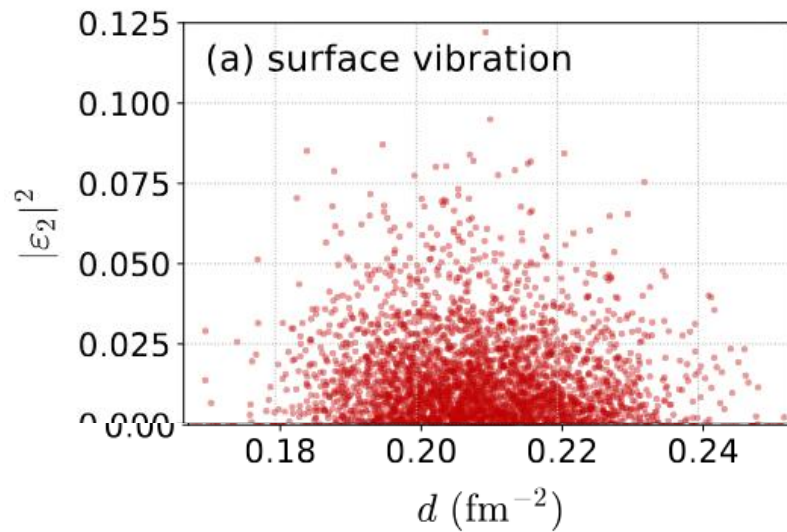
		mean	std. dev.	skewness	kurtosis
$^{58}\text{Ni}$ , $ \epsilon_2 $	SV	0.112(1)	0.0554(7)	0.49(3)	-0.02(11)
	SD	0.119(1)	0.0500(5)	-0.79(3)	-0.62(6)
	SV-A	0.090(1)	0.0816(13)	1.22(4)	1.12(20)
$^{208}\text{Pb}$ , $ \epsilon_3 $	SV	0.0822(8)	0.0416(5)	0.55(4)	0.15(11)
	SD	0.0821(8)	0.0461(4)	-0.38(3)	-1.29(3)
	SV-A	0.0650(12)	0.0649(11)	1.35(5)	1.49(22)

# Transverse Distribution 2

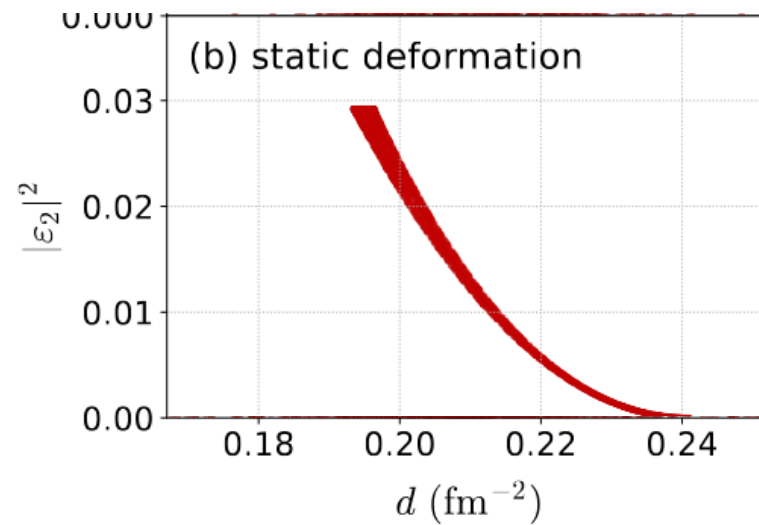
Hagino, MK, PRC ('25)

$^{58}\text{Ni}$ , Quadrupole,  $\beta_2 = 0.218$

Surface Vibration



Static Deform.



inverse mean radius

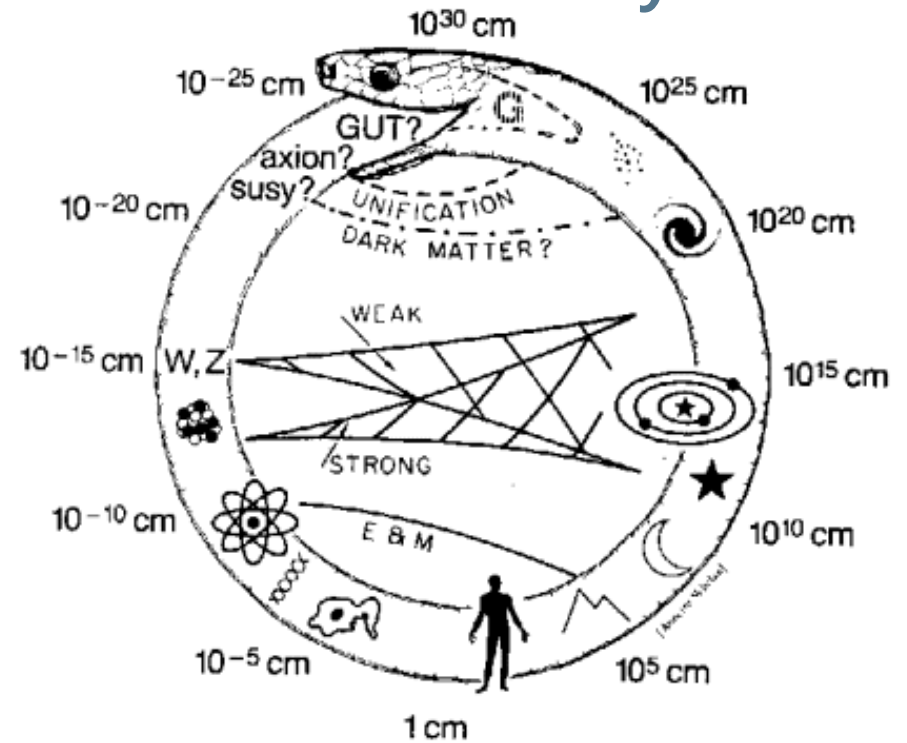
$$\epsilon_n = -\frac{\langle\langle (x - iy)^n \rangle\rangle}{\langle\langle (x^2 + y^2)^{n/2} \rangle\rangle},$$
$$d = \frac{1}{\sqrt{\langle\langle x^2 \rangle\rangle \langle\langle y^2 \rangle\rangle}}$$

## Short Summary

- ❑ Surface vibration and static deformation are discriminable through the distributions of  $\epsilon_2$ ,  $d$ .
- ❑ Space-fixed prescription is more convenient in treating the surface vibration of spherical nuclei.

# Uroboros in Nuclear Physics

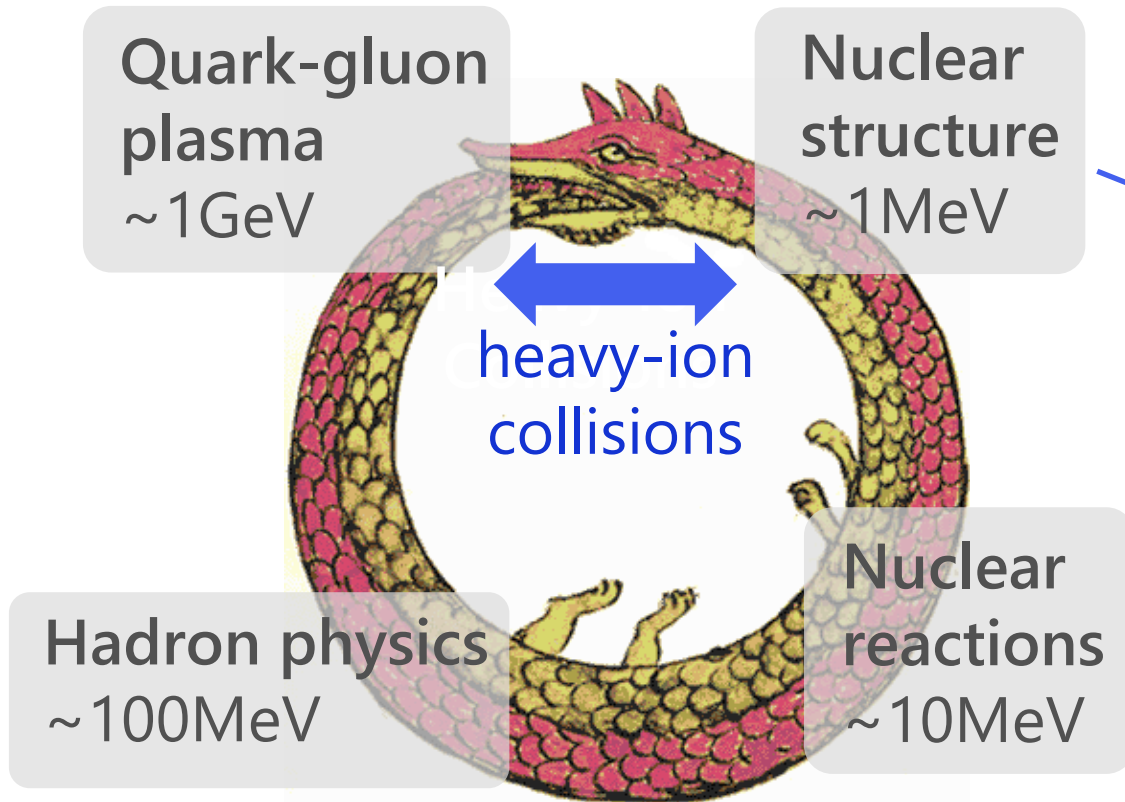
## Uroboros of Physics



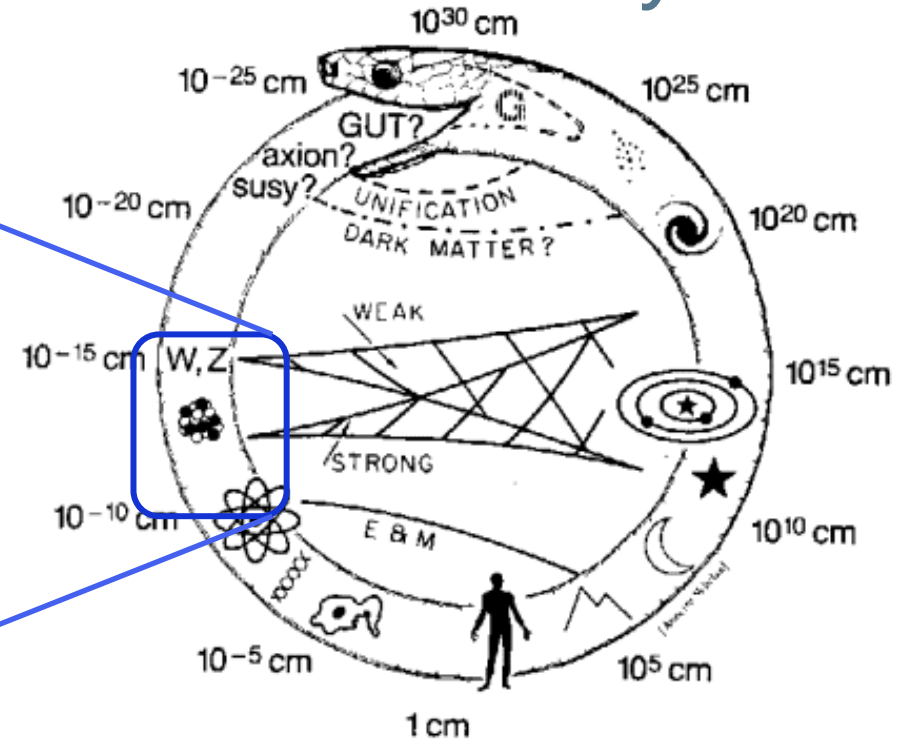
Glashow (1982)

- High-Energy HIC provides us with info. of nuclear structure.
- Nuclear structure is necessary for understanding Relativistic HIC.

# Uroboros in Nuclear Physics



## Uroboros of Physics



Glashow (1982)

- High-Energy HIC provides us with insights into nuclear structure.
- Nuclear structure is necessary for understanding relativistic HIC.

# Workshop in April @ Kyoto

The screenshot shows a web browser displaying the Indico page for the workshop. The browser's address bar shows 'Public' and 'Asia/Tokyo'. The page title is 'Intersection of nuclear structure and high-energy nuclear collisions 2026'. Below the title, it says 'Apr 13–24, 2026' and 'Yukawa Institute for Theoretical Physics, Asia/Tokyo timezone'. There is a search bar with the placeholder text 'Enter your search term'. On the left, there is a navigation menu with 'Overview', 'Timetable', 'Registration', and 'Participant List'. The main content area has an 'Overview' section with the following text: 'The purpose of this workshop is to review recent progress at the interface between the physics of low-energy nuclear structure and high-energy nuclear collisions, and to explore future directions in this rapidly developing interdisciplinary field. We will bring together theorists and experimentalists from both communities to foster mutual understanding and initiate collaborations.' Below this, it says 'The workshop will be held in Yukawa Institute for Theoretical Physics, Kyoto University. (Access map)'. There is a 'Schedule' section with two bullet points: '1st week (Apr. 13-17): Discussion week' and '2nd week (Apr. 20-24): Workshop week'. Finally, there is an 'Invited Speakers' section with a list of names and affiliations.

Public Asia/Tokyo

## Intersection of nuclear structure and high-energy nuclear collisions 2026

Apr 13–24, 2026  
Yukawa Institute for Theoretical Physics  
Asia/Tokyo timezone

Enter your search term

- Overview
- Timetable
- Registration
- Participant List

### Overview

The purpose of this workshop is to review recent progress at the interface between the physics of low-energy nuclear structure and high-energy nuclear collisions, and to explore future directions in this rapidly developing interdisciplinary field. We will bring together theorists and experimentalists from both communities to foster mutual understanding and initiate collaborations.

The workshop will be held in Yukawa Institute for Theoretical Physics, Kyoto University. ([Access map](#))

### Schedule

- **1st week (Apr. 13-17): Discussion week**  
1~2 seminars per day and discussions
- **2nd week (Apr. 20-24): Workshop week**  
4~10 talks, including 1~2 seminars, per day, and discussions

### Invited Speakers

- Thomas Duguet (CEA Saclay)
- Reyes Alemany Fernandez (CERN)
- Wataru Horiuchi (Osaka Metropolitan U.)
- Weiyao Ke (CCNU)
- Takayuki Miyagi (Tsukuba)
- Oscar Garcia Montero (Santiago)
- Koichi Murase (RCNP)
- Shunji Nishimura (RIKEN)
- Björn Schenke (BNL)
- Daisuke Suzuki (Tokyo)
- Kenichi Yoshida (RCNP)
- Chunqiang Zhang (Fudan U.)

Intersection of nuclear structure and high-energy nuclear collisions

April 13-24

YITP, Kyoto University

**Deadline: Mar. 13**

<https://indico.yukawa.kyoto-u.ac.jp/event/75/>

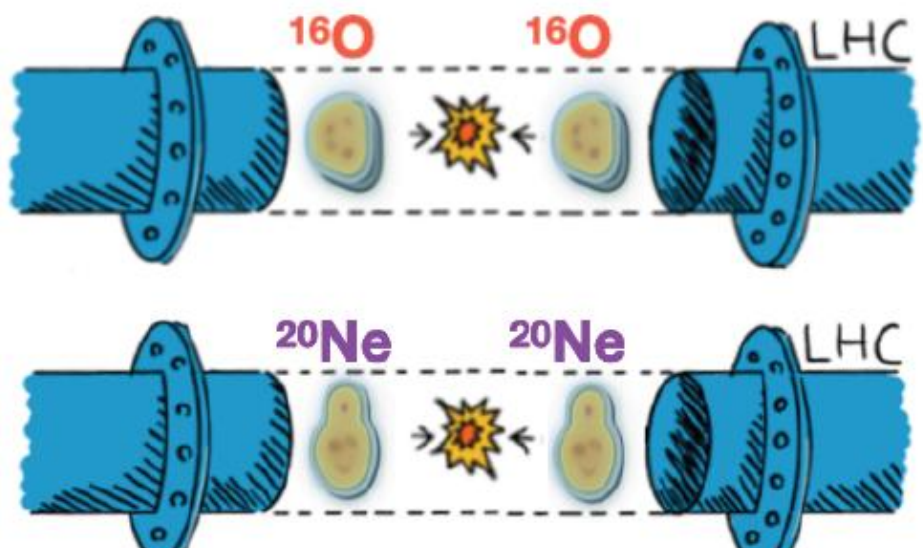


# Workshop in April

Intersection of nuclear structure and high-energy nuclear collisions 2026

Further investigations  
OO, NeNe collision @ LHC, 2025  
Motivations for other nuclei?

**July, 2025!**



The diagram illustrates two collision scenarios at the LHC. The top scenario shows two  $^{16}\text{O}$  nuclei (represented as blue cylinders) colliding head-on, with a central starburst indicating the collision point. The bottom scenario shows two  $^{20}\text{Ne}$  nuclei colliding head-on, also with a central starburst. The LHC structure is shown as blue cylindrical components.

## Intersection of nuclear structure and high-energy nuclear collisions

April 13-24

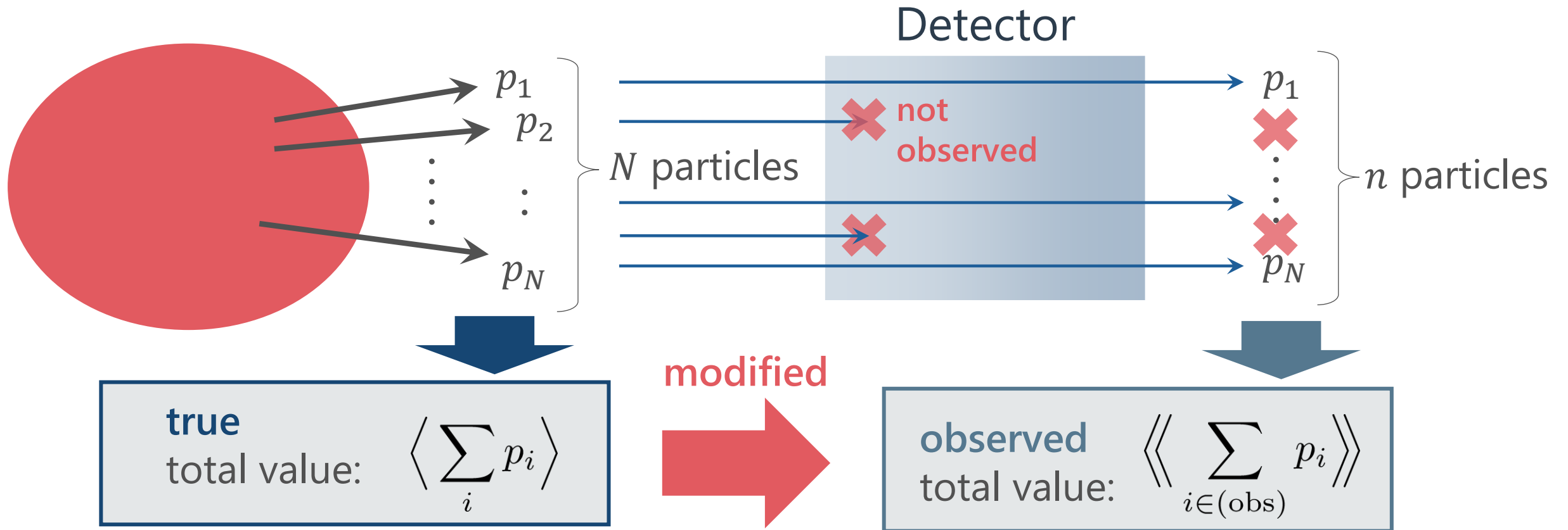
YITP, Kyoto University

**Deadline: Mar. 13**

<https://indico.yukawa.kyoto-u.ac.jp/event/75/>



# Detector Efficiency

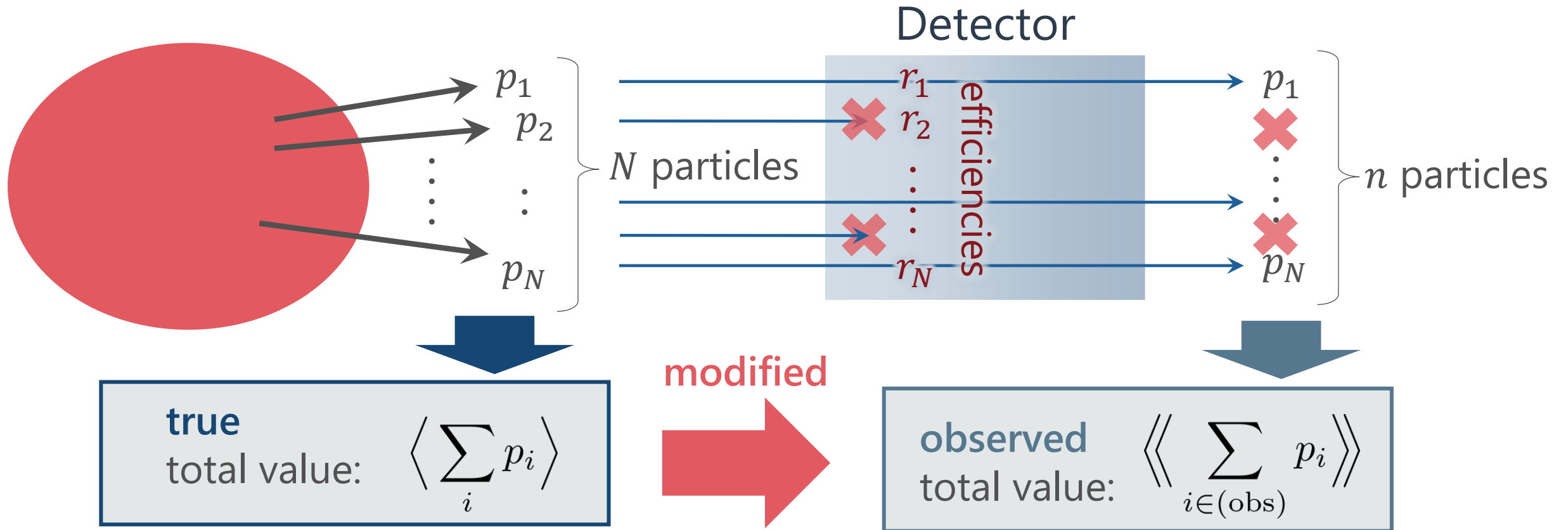


Real detectors lose some particles



Observed results are modified.  
Effects must be corrected to obtain the true result.

# Efficiency Correction: Total Number



Correction Formula:

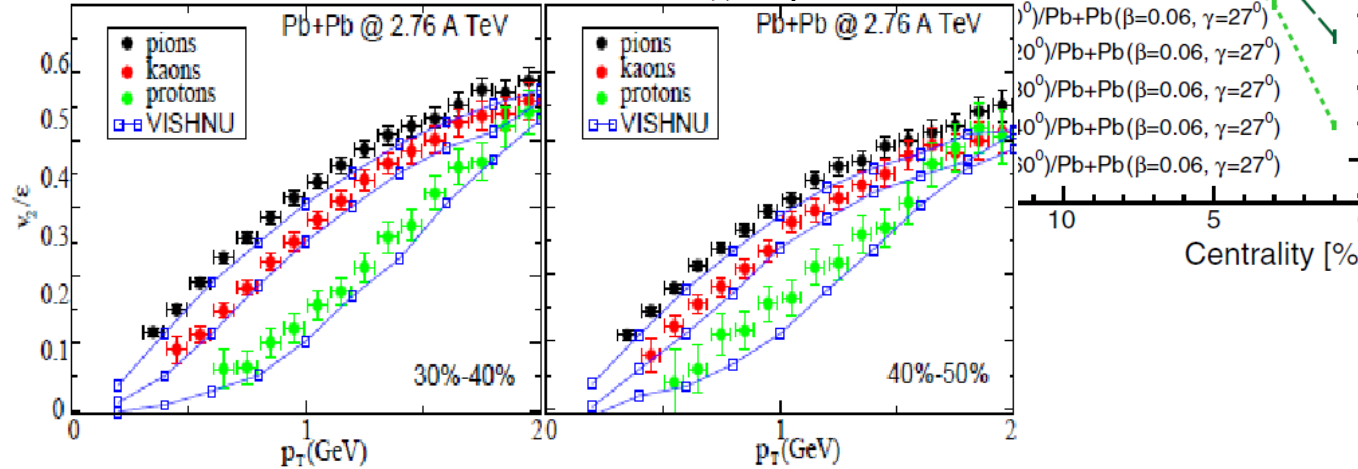
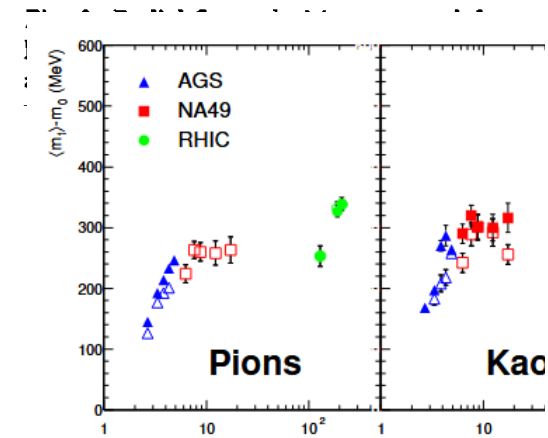
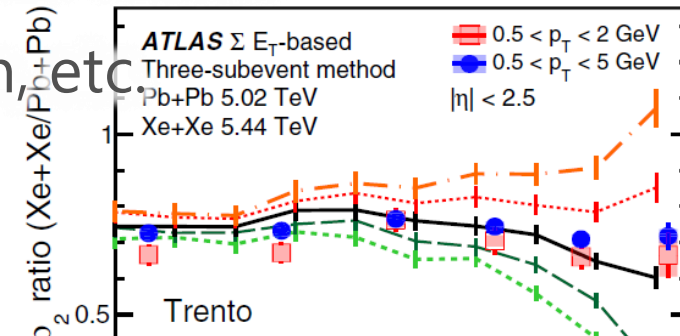
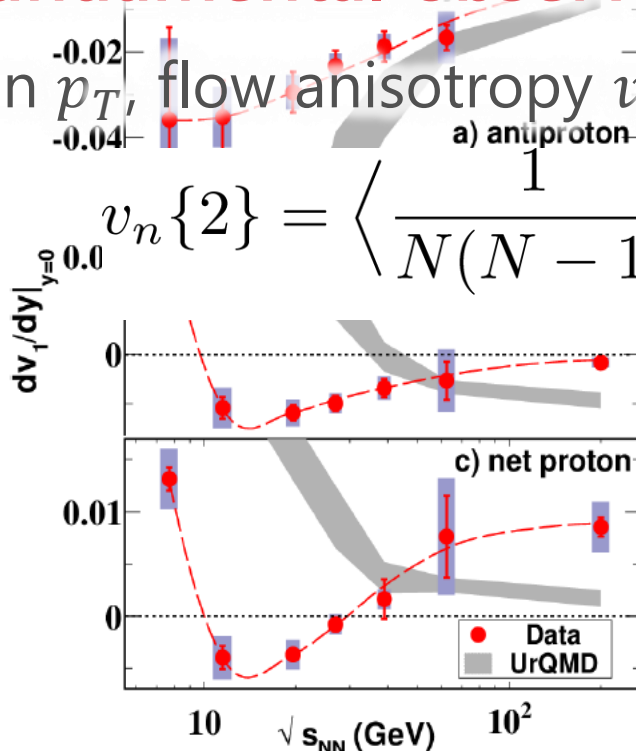
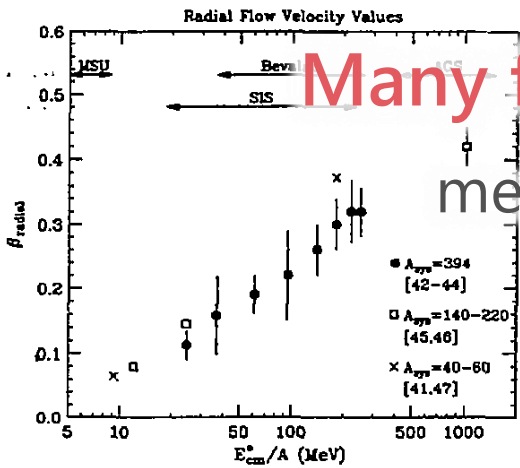
$$\langle \sum_i p_i \rangle = \langle\langle \sum_i \frac{p_i}{r_i} \rangle\rangle$$

# Particle-Averaged Quantities

$$\left\langle \frac{1}{N} \sum_i p_i \right\rangle, \quad \left\langle \left( \frac{1}{N} \sum_i p_i \right)^n \right\rangle, \quad \left\langle \frac{1}{N(N-1)} \sum_{i \neq j} p_i^{(1)} p_j^{(2)} \right\rangle$$

Many fundamental observables in HIC are of this form!

mean  $p_T$ , flow anisotropy  $v_n\{m\}$ ,  $v_2 - p_T$  correlation, etc



# Particle-Averaged Quantities

$$\left\langle \frac{1}{N} \sum_i p_i \right\rangle, \quad \left\langle \left( \frac{1}{N} \sum_i p_i \right)^n \right\rangle, \quad \left\langle \frac{1}{N(N-1)} \sum_{i \neq j} p_i^{(1)} p_j^{(2)} \right\rangle$$

Many fundamental observables in HIC are of this form!

“Conventional” Correction Formulas

$$\left\langle \frac{1}{N(N-1)} \sum_{i \neq j} p_i^{(1)} p_j^{(2)} \right\rangle = \left\langle \left\langle \frac{\sum_{i \neq j} p_i^{(1)} p_j^{(2)} / r_i r_j}{\sum_{i \neq j} 1 / r_i r_j} \right\rangle \right\rangle$$

e.g. ATLAS, PRC107, 054910 ('23); STAR, Nature 635, 67 ('24)

Question: Are these formulas correct?

# Correct Correction Formulas

MK, Esumi, Niida, Nonaka, arXiv:25010.13838

## Mean

$$\left\langle \frac{Q}{N} \right\rangle = \left\langle \left\langle \sum_{i=1}^n \xi_i k_i \right\rangle \right\rangle_{n \neq 0}$$

$$k_i = \frac{1}{r_i} \int_0^1 d\sigma \prod_{j \neq i} \frac{\sigma + r_j \alpha_j}{r_j}$$

## 2nd Order

$$\left\langle \frac{\{Q_1 Q_2\}}{N(N-1)} \right\rangle = \left\langle \left\langle \sum_{i \neq j} q_{1,i} q_{2,j} k_{2;i,j} \right\rangle \right\rangle_{n \neq 0,1}$$

$$k_{2;i,j} = \frac{1}{r_i r_j} \int_0^1 d\sigma' \int_0^{\sigma'} d\sigma \prod_{l \neq i, l \neq j} \left( \frac{\sigma}{r_l} + \alpha_l \right)$$

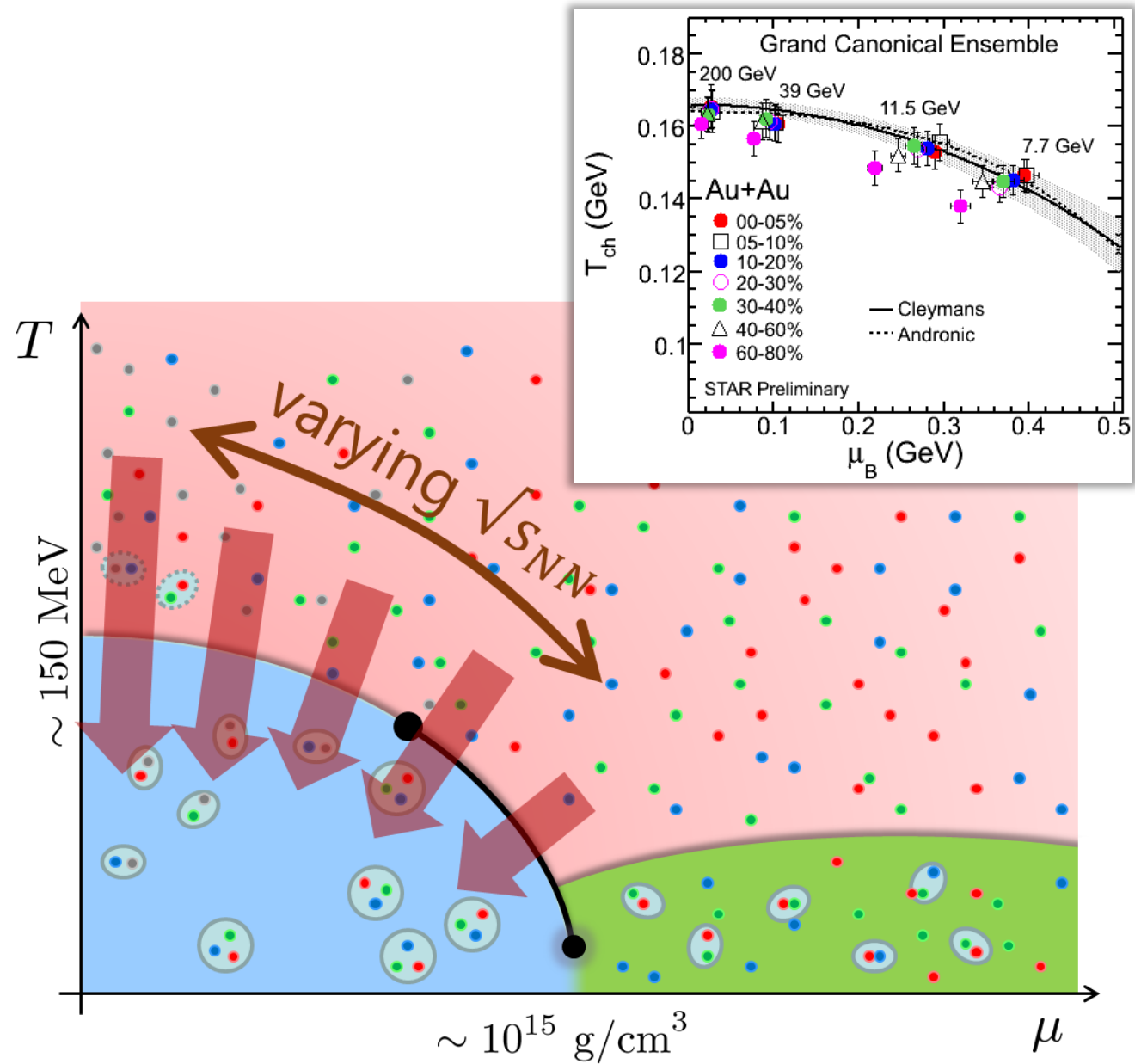
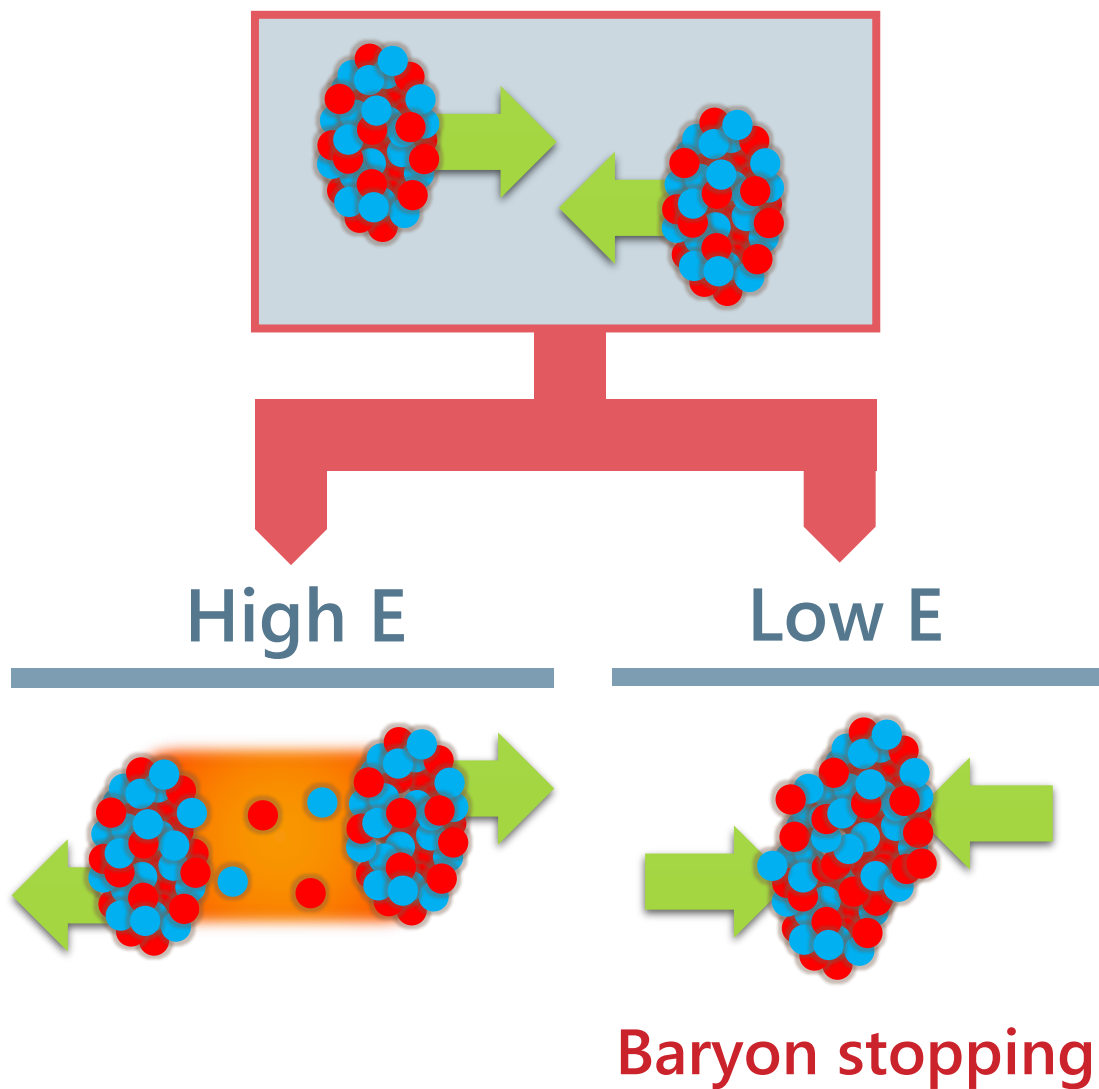
- Correction formulas are written in forms including integral.
- This formula can reproduce the correct result for the previous simple model.

$$\{Q_{w_1} Q_{w_2}\} \equiv \sum_{i \neq j} \xi_i^{(w_1)} \xi_j^{(w_2)}$$

$$\alpha_i = \frac{1 - r_i}{r_i}$$

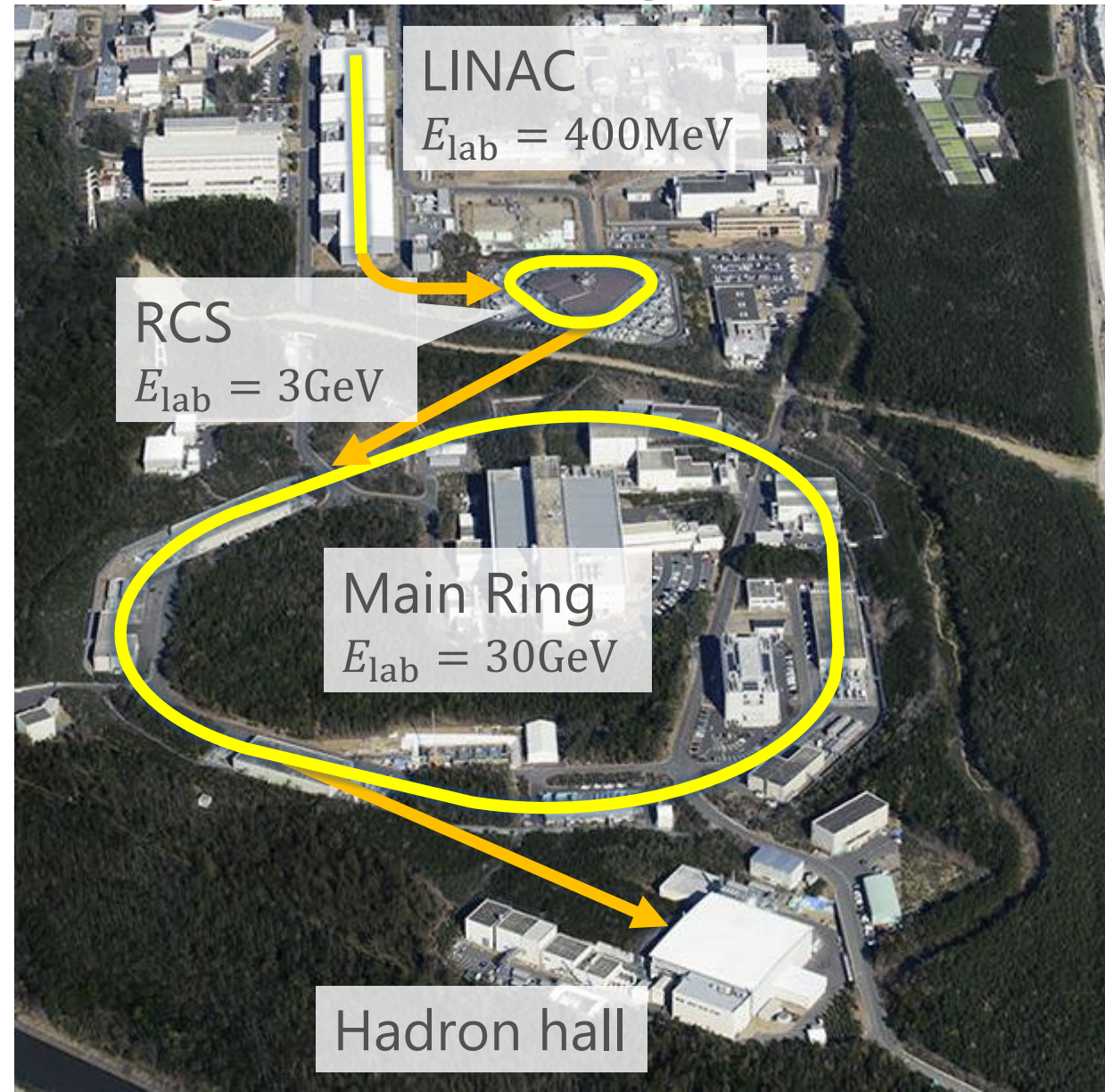
# Beam-Energy Scan

STAR, 2012



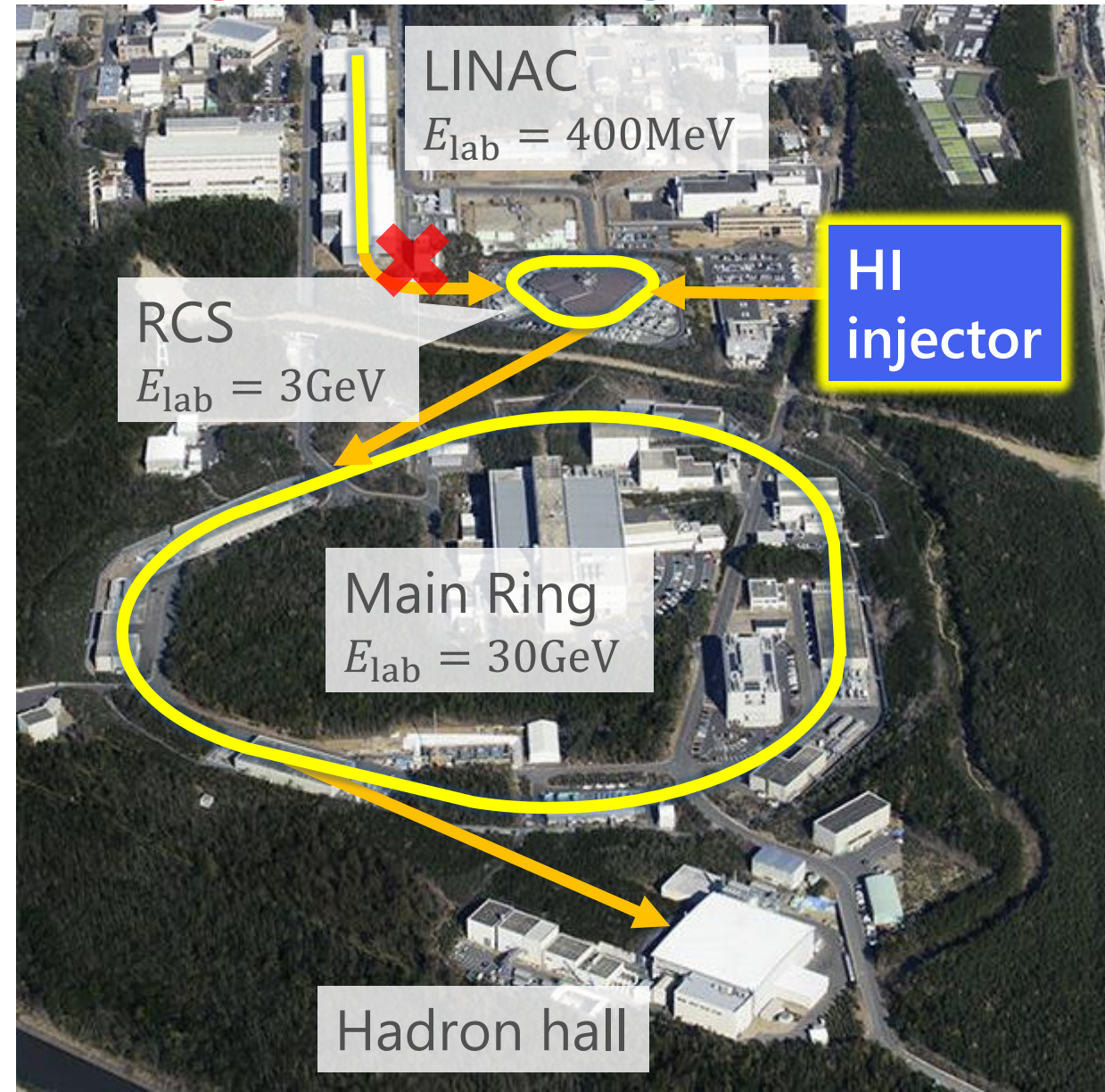
# J-PARC-HI = J-PARC Heavy-Ion Project

- **New HI injector** + existing accelerators (**RCS, MR**)
- Heavy-ion beams with **world highest luminosity**
- Realize various new experiments at J-PARC
- $\sqrt{s_{NN}} < 4.9\text{GeV}$



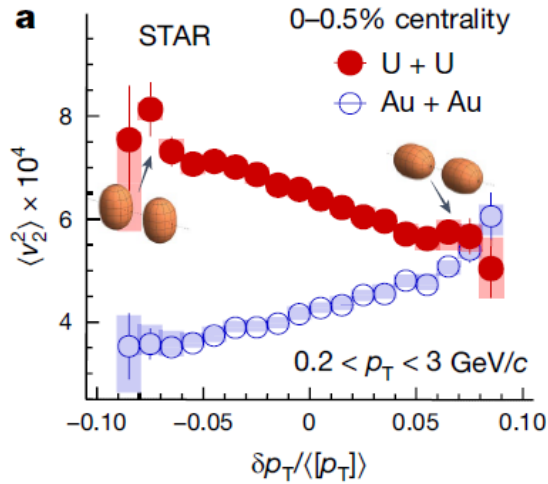
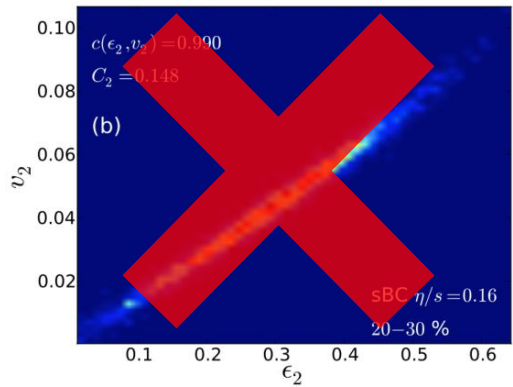
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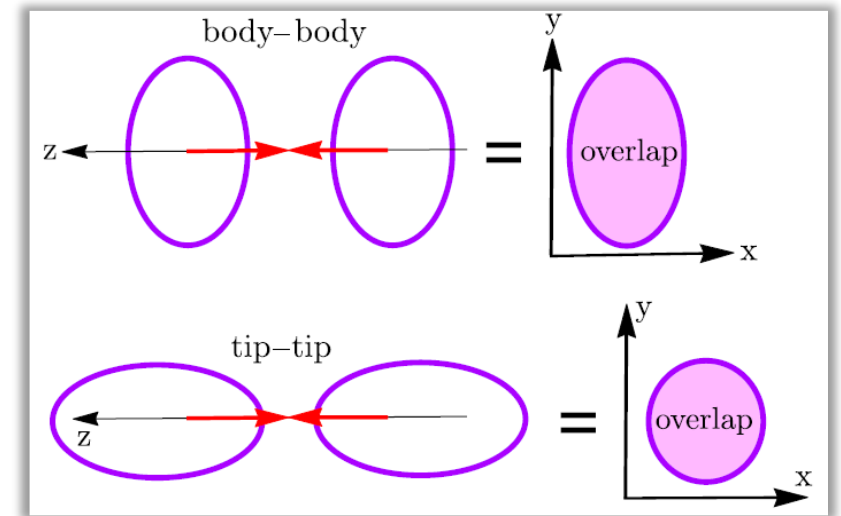


# Further Utilization of Deformation at low collision-energy region

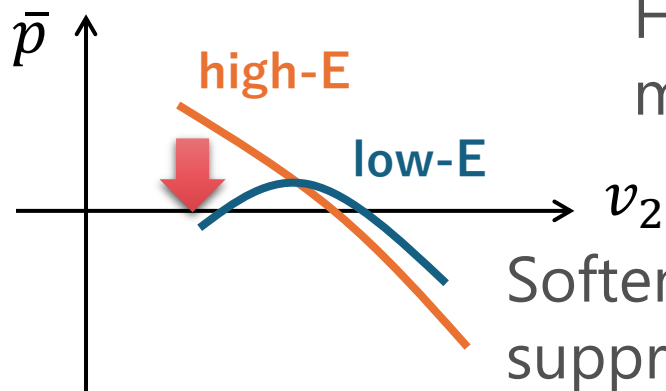
Violation of hydro. picture / EOS Change @ 1st-order tr.



Creating the highest baryon density medium



MK's conjecture



How is this behavior modified?

Softening of EOS suppresses tip-tip side??

Tip-tip collisions are obviously suitable to create denser matter!

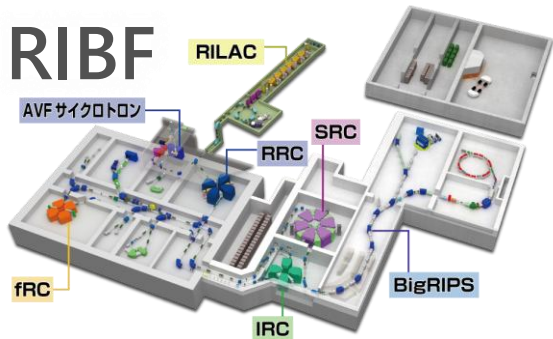
How to measure it?

# Physics of Heavy-Ion Collisions

Various physical picture / research purposes / theoretical methods depending on the collision E



**Nuclear reaction**      **Nuclear liquid-gas transition**      **compressed baryonic matter**      **QGP**  
**Fragmentation**      **Nuclear Deformation**      **Quark deconfinement**  
**Heavy elements**      **EOS**      **hydro. models**  
**TDHF**      **Langevin**      **AMD**      **BUU**      **hadron cascade**

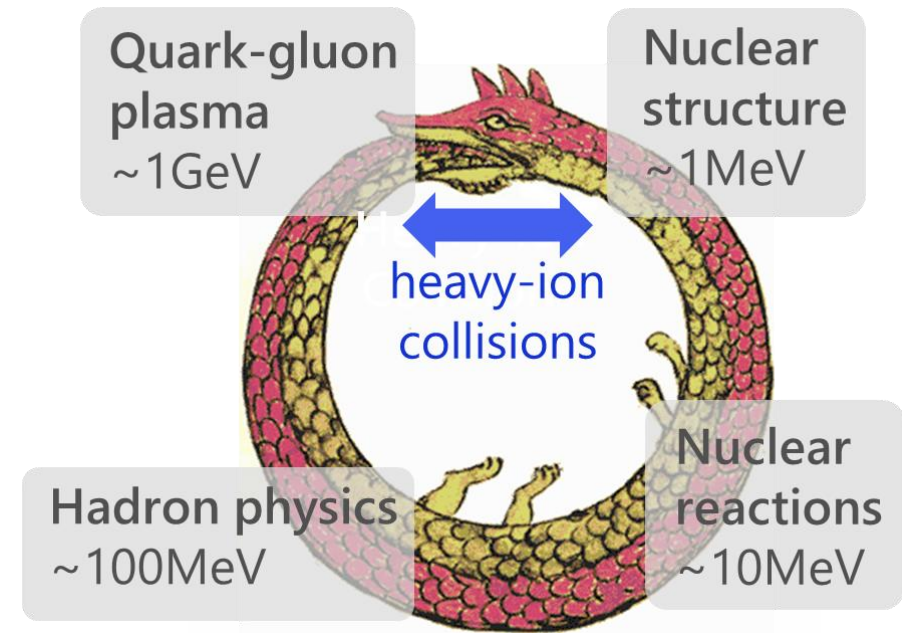


# Summary

High-energy heavy-ion collisions provide a novel experimental probe for studying nuclear structure. Extraction of  $\beta$ ,  $\gamma$ , shape fluctuations, etc.

There are many more usage of nuclear deformation in HIC, especially at intermediate energies.

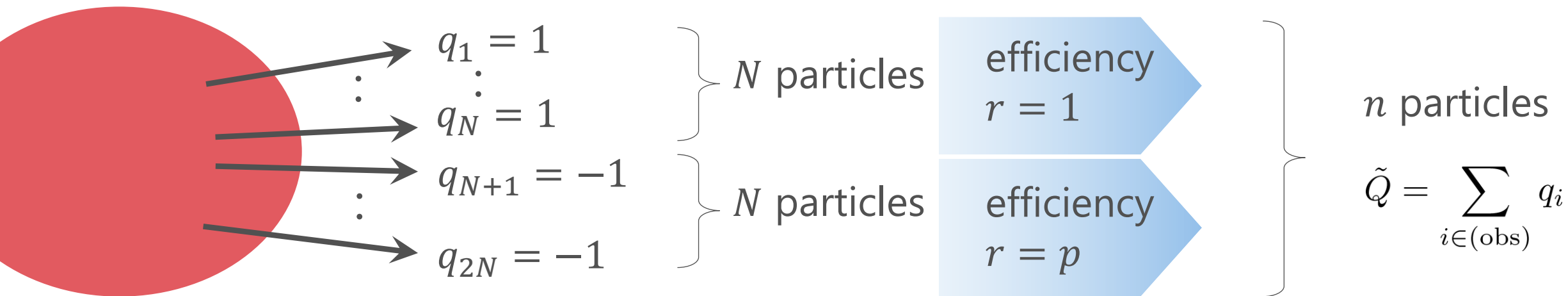
Let's promote interdisciplinary discussions in terms of the keyword, "nuclear deformation" and "heavy-ion collisions".



drawn by ChatGPT

# Check in a Simple Model

$2N$ : fixed for all events



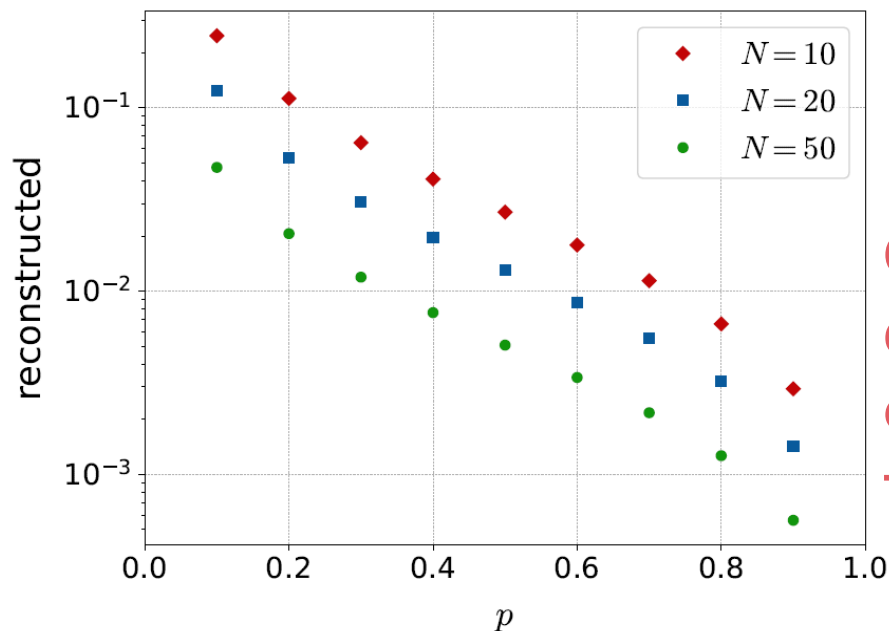
Mean:

True result

$$\left\langle \frac{Q}{N} \right\rangle = 0$$

Reconstructed

$$\left\langle \frac{\sum_i q_i / r_i}{\sum_i 1 / r_i} \right\rangle$$



Conventional formula  
does not reproduce the  
correct result even for  
the mean!!

# Moments (Cumulants) of Total Number

$$\left\langle \left( \sum_i p_i \right)^n \right\rangle$$

## Correction Procedure:

Use factorial moments/cumulants

$$\left\langle \left( \sum_i p_i \right)^n \right\rangle_f = \left\langle \left\langle \left( \sum_i \frac{p_i}{r_i} \right)^n \right\rangle \right\rangle_f$$

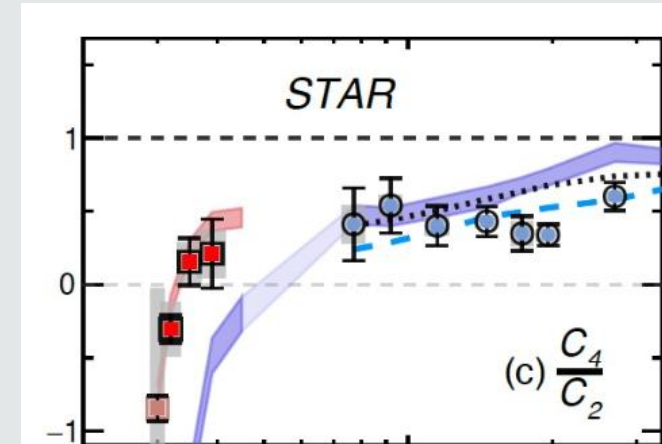
Assumption: efficiencies of individual particles are independent

Nonaka, MK, Esumi ('17)

Asakawa, MK, PPNP ('16); MK, Luo ('17)

## Note

Search for QCD-CP using conserved-charge fluctuations



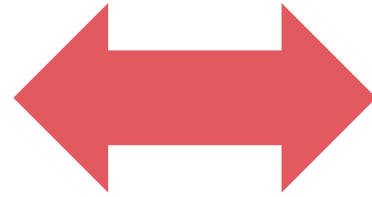
Long history of efficiency correction:

MK, Asakawa ('12); Bzdak, Koch ('12,'15); Luo ('14); MK ('16); Nonaka+ ('16); Bzdak, Holtzman, Koch ('16); MK, Luo ('17); Nonaka, MK, Esumi ('17); ...

# Connecting True/Observed Distr. Funcs.

**True** distr. func.

$$P(N; \vec{\xi})$$



**Observed** distr. func.

$$\tilde{P}(n; q)$$

- $n$ : observed particle number
- $q = \sum_{i \in (\text{obs.})} \xi_i$ : observed sum

## Probability Distr. of Observed Quantities (uniform $r$ )

$$\tilde{P}(n; q) = \sum_{N=1}^{\infty} \int d\vec{\xi} \sum_{\{b_i\}} \left[ \prod_{i=1}^N (1-r)^{1-b_i} r^{b_i} \right] \delta_{n, \sum_i b_i} \delta(q - \sum_i b_i \xi_i) P(N; \vec{\xi})$$

$$b_i = 0, 1$$

# Generating Function

MK, Esumi, Niida, Nonaka, arXiv:25010.13838

$$\text{Prob. distr. func: } \tilde{P}(n; q) = \sum_{N=1}^{\infty} \int d\vec{\xi} \sum_{\{b_i\}} \left[ \prod_{i=1}^N (1-r)^{1-b_i} r^{b_i} \right] \delta_{n, \sum_i b_i} \delta(q - \sum_i b_i \xi_i) P(N; \vec{\xi})$$

$$\text{Generating func: } \tilde{G}(s, t) = \sum_n \int dq \tilde{P} s^n t^q = \sum_N \int d\vec{\xi} P \prod_i (1-r + r s t^{\xi_i})$$

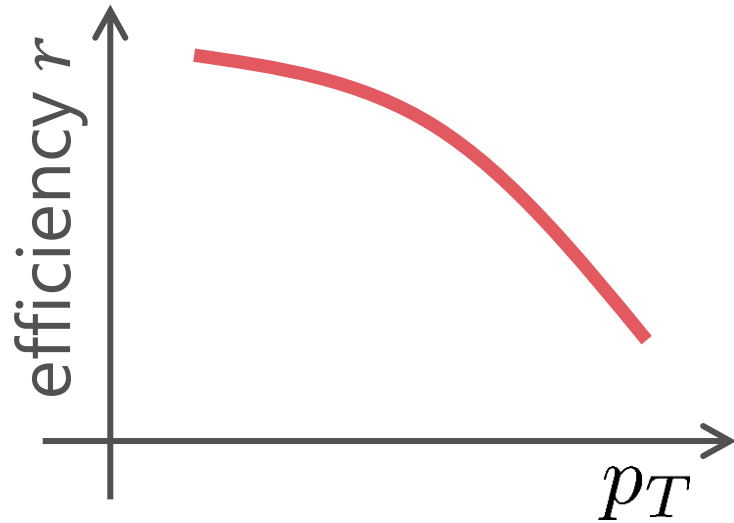
Represent the quantity that you want to express by the derivative of the generating function.

Then, represent it in terms of the observed variables.

$$\left\langle \frac{\sum_i \xi_i}{N} \right\rangle_{\text{true}} = \int_{\alpha}^1 ds \frac{r}{s} [\partial_t \tilde{G}(s, t)]_{t=1} = \left\langle \frac{\sum_i \xi_i}{n} (1 - \alpha^n) \right\rangle_{\text{obs}} \quad \alpha = \frac{r-1}{r}$$

**Note:**  $\left\langle \frac{\sum_i \xi_i}{N} \right\rangle_{\text{true}} \neq \left\langle \frac{\sum_i \xi_i}{n} \right\rangle_{\text{obs}}$   $\alpha^n$  term compensates the  $n = 0$  contribution.

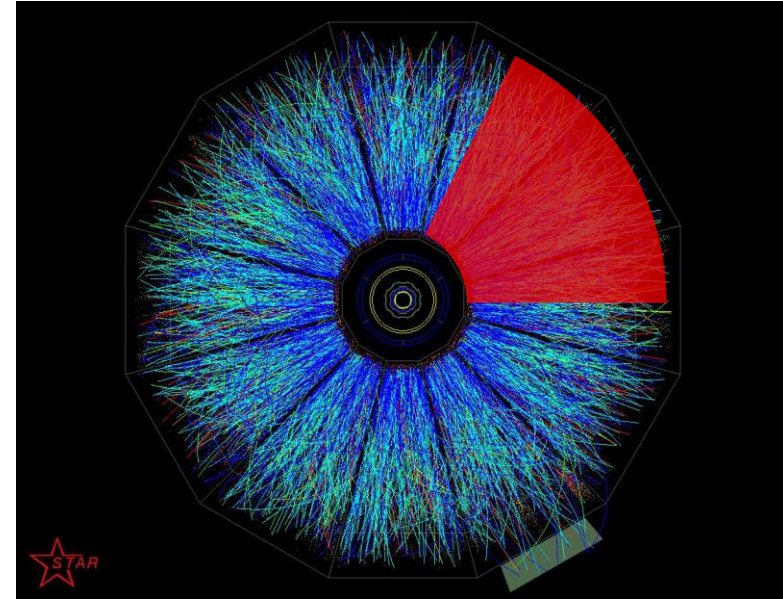
# Correction is Necessary!!



$p_T$ -dependent efficiency



alter mean  $p_T$



Azimuthally nonuniform efficiency



produce unphysical  $v_n\{m\}$

More serious effects on higher-order correlations!

# Derivation of Correction Formulas

## Assumptions

1. Particle production is described by a classical prob. distr. func.  $P(N; \vec{p}_T)$ .
2. Probs. to observe individual particles are independent.
3. For each observed particle, the value of efficiency  $r_i$  can be specified.
4. Other detectors' effects are not considered.

**True** distr. func.

$$P(N; \vec{\xi})$$

