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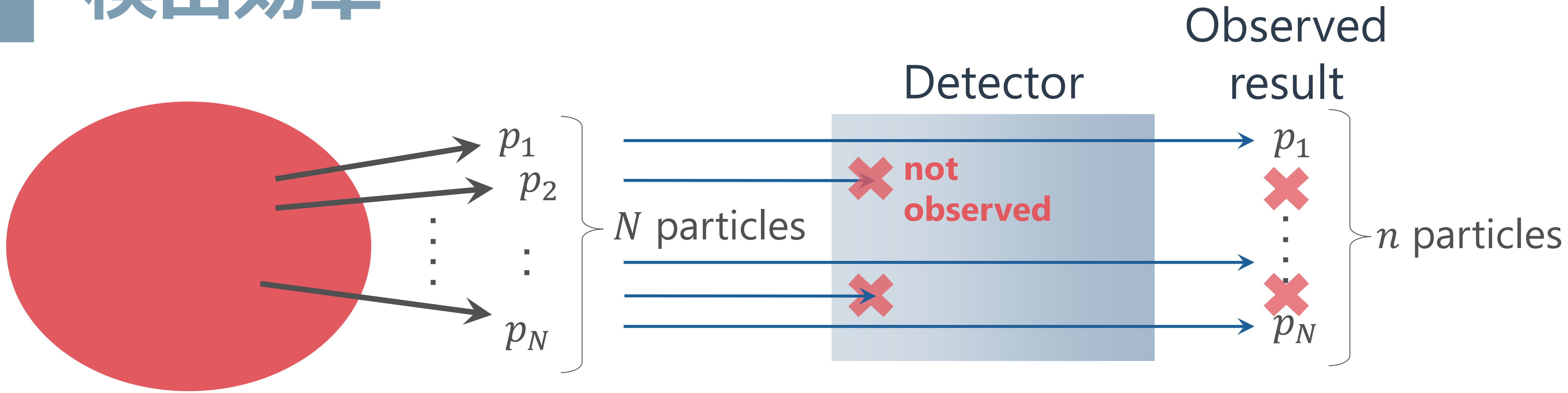
23pV1-1

粒子平均量の検出効率補正について

北沢正清（京大基研）
江角晋一、新井田貴文、野中俊宏

MK, Esumi, Niida, Nonaka, arXiv:2510.18383, PTEP (2026) in press.

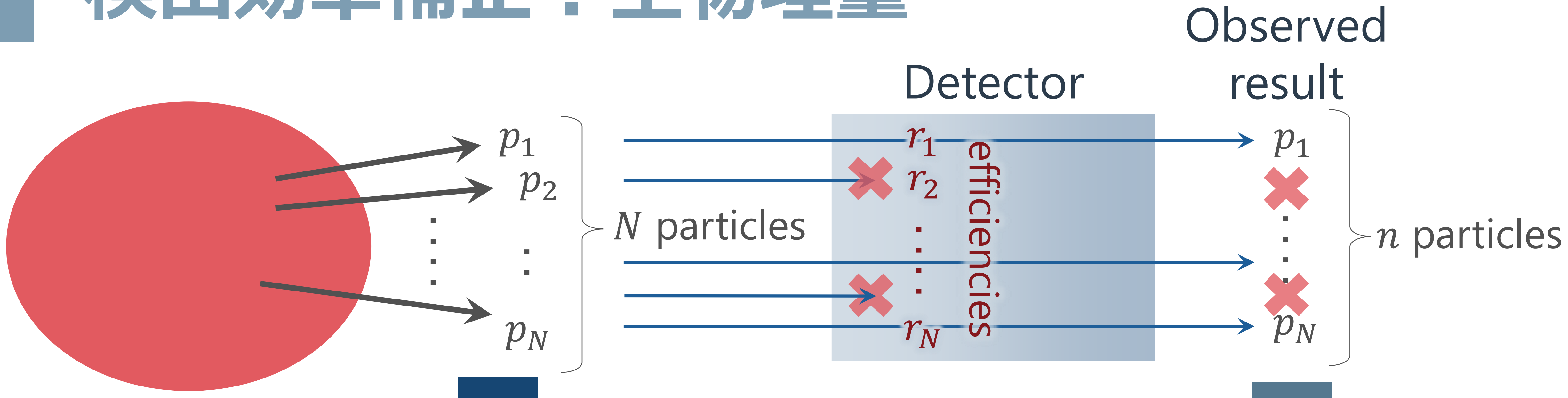
検出効率



Real detectors cannot detect all particles

Observed results are modified.
Effects must be corrected to obtain the true result.

検出効率補正：全物理量



true
total value: $\left\langle \sum_i p_i \right\rangle$

modified

observed
total value: $\left\langle\!\left\langle \sum_{i\in(\mathrm{obs})} p_i \right\rangle\!\right\rangle$

Correction Formula: $\left\langle \sum_i p_i \right\rangle = \left\langle\!\left\langle \sum_i \frac{p_i}{r_i} \right\rangle\!\right\rangle$

Moments (Cumulants) of Total Number

$$\left\langle \left(\sum_i p_i \right)^n \right\rangle$$

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Correction Procedure:

Use factorial moments/cumulants

$$\left\langle \left(\sum_i p_i \right)^n \right\rangle_{\mathrm{f}} = \left\langle\!\left\langle \left(\sum_i \frac{p_i}{r_i} \right)^n \right\rangle\!\right\rangle_{\mathrm{f}}$$

Assumption: efficiencies of individual particles are independent

Nonaka, MK, Esumi ('17)
Asakwa, MK, PPNP ('16); MK, Luo ('17)

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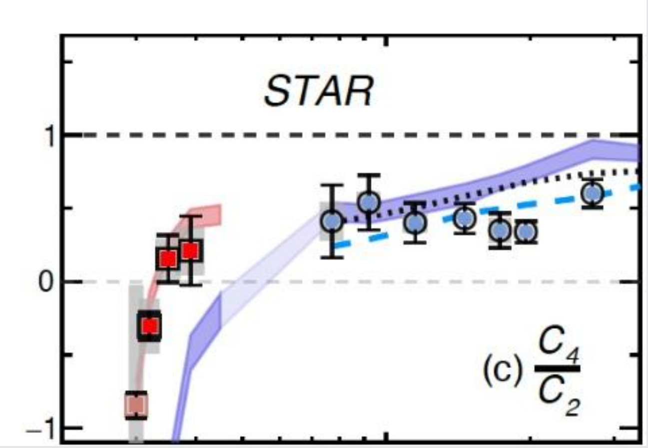
Note

Search for QCD-CP using conserved-charge fluctuations



Long history of efficiency correction:
MK, Asakawa ('12); Bzdak, Koch ('12,'15); Luo ('14); MK ('16); Nonaka+ ('16); Bzdak, Holtzman, Koch ('16); MK, Luo ('17); Nonaka, MK, Esumi ('17); ...

Particle-Averaged Quantities

$$\left\langle \tfrac{1}{N} \sum_i p_i \right\rangle, \quad \left\langle \left(\tfrac{1}{N} \sum_i p_i \right)^n \right\rangle, \quad \left\langle \tfrac{1}{N(N-1)} \sum_{i \neq j} p_i^{(1)} p_j^{(2)} \right\rangle$$

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Many fundamental observables in HIC are of this form!

mean p_T, flow anisotropy $v_n\{m\}$, $v_2 - p_T$ correlation, etc.

$$v_n\{2\} = \left\langle \frac{1}{N(N-1)} \sum_{i \neq j} e^{in(\phi_i - \phi_j)} \right\rangle$$

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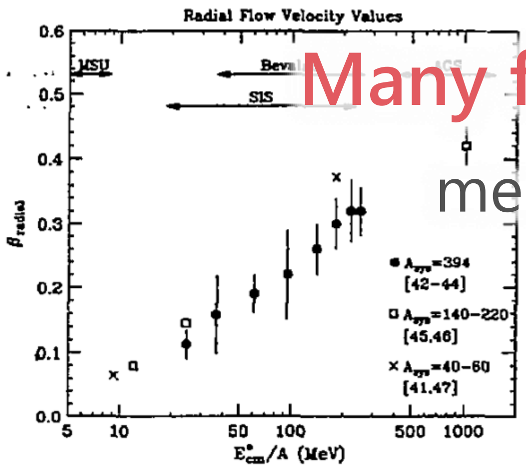
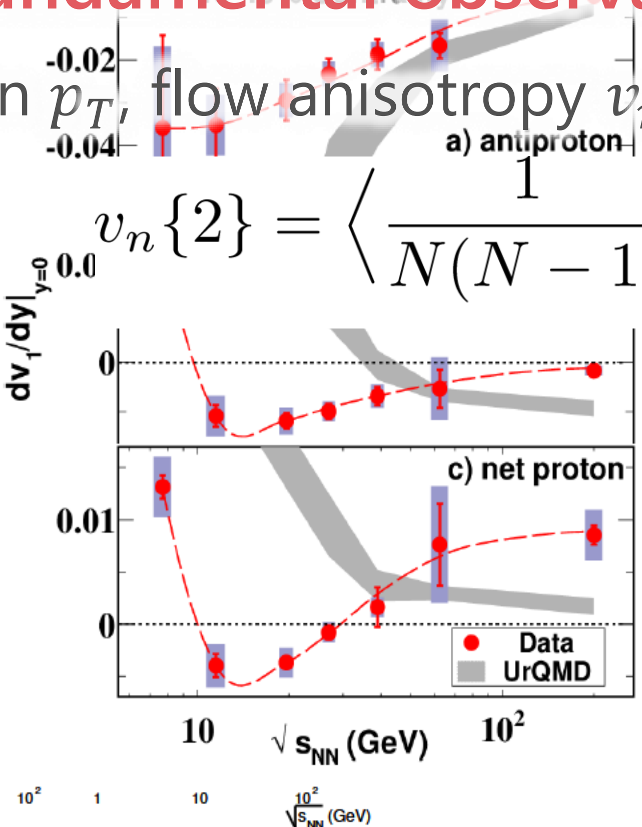
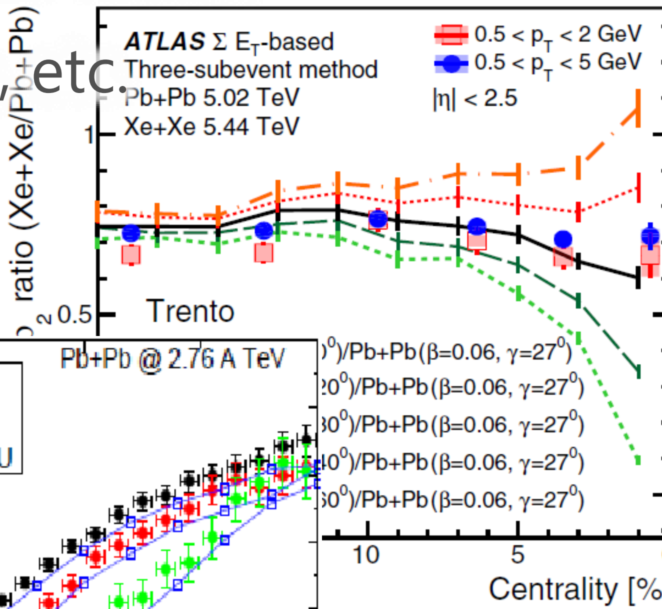
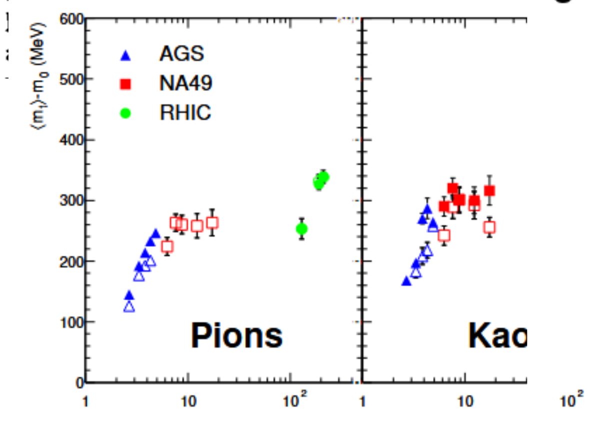
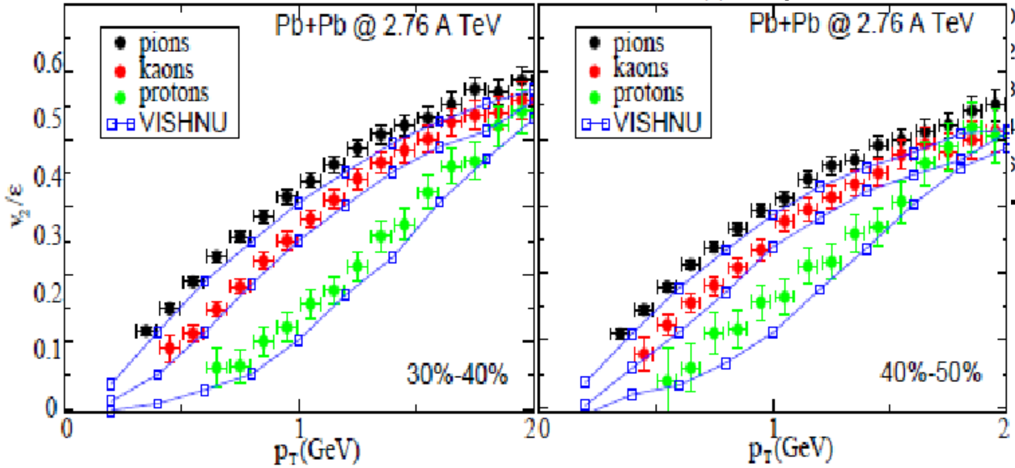










Particle-Averaged Quantities

$$\left\langle \frac{1}{N}\sum_i p_i \right\rangle, \quad \left\langle \left(\frac{1}{N}\sum_i p_i \right)^n \right\rangle, \quad \left\langle \frac{1}{N(N-1)}\sum_{i\neq j} p_i^{(1)} p_j^{(2)} \right\rangle$$

Many fundamental observables in HIC are of this form!

"Conventional" Correction Formulas

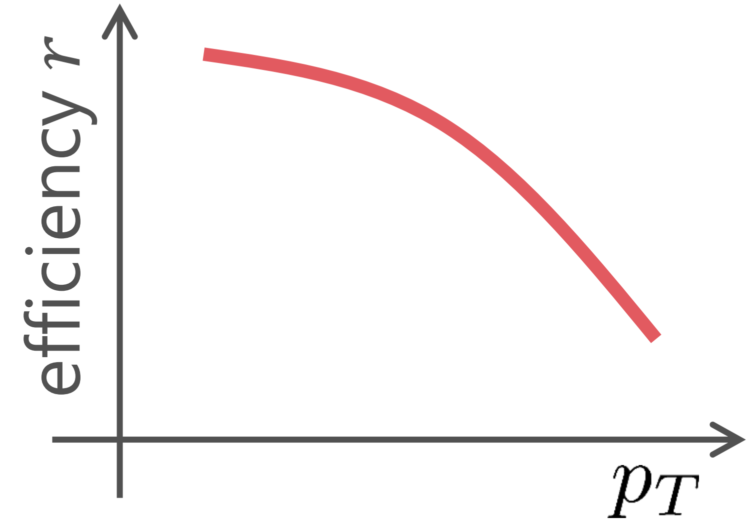
$$\left\langle \frac{1}{N(N-1)}\sum_{i\neq j} p_i^{(1)} p_j^{(2)} \right\rangle = \left\langle\!\left\langle \frac{\sum_{i\neq j} p_i^{(1)} p_j^{(2)} / r_i r_j}{\sum_{i\neq j} 1/r_i r_j} \right\rangle\!\right\rangle$$

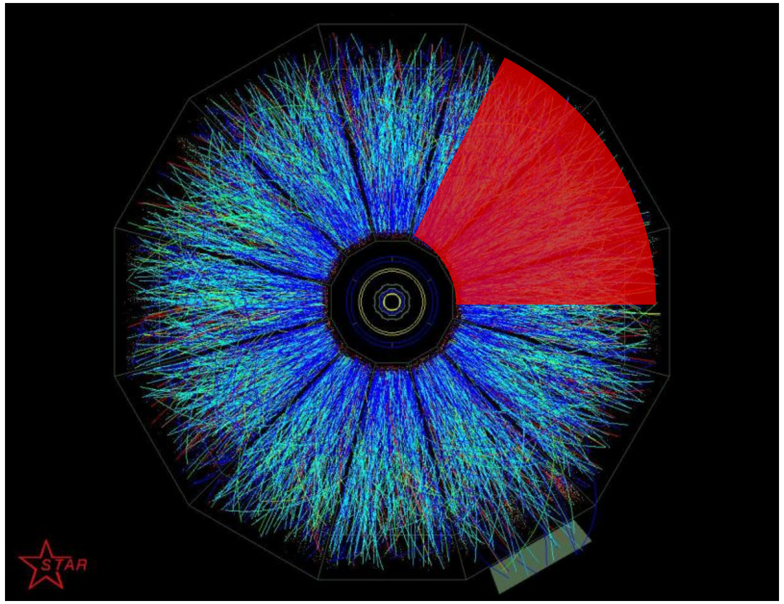
e.g. ATLAS, PR**C107**, 054910 ('23); STAR, Nature **635**, 67 ('24)

Question: Are these formulas correct?

Caution: Detector's effect is more pronounced for higher-order correlations!

Correction is Necessary!!





p_T-dependent efficiency

alter mean p_T

Azimuthally nonuniform efficiency

produce unphysical $v_n\{m\}$

More serious effects on higher-order correlations!

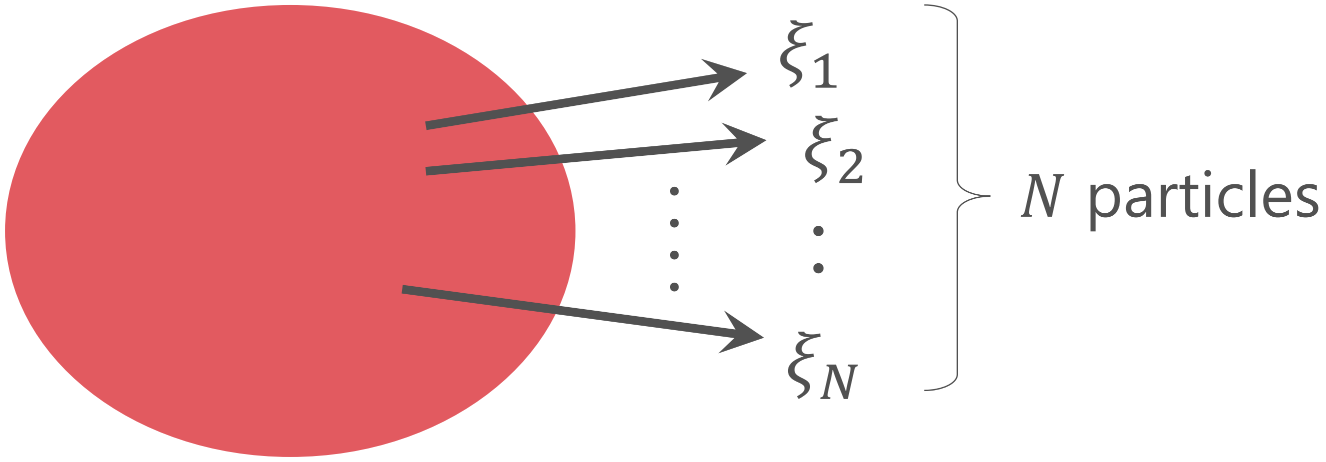
Derivation of Correction Formulas

Assumptions

1. Particle production is described by a classical prob. distr. func. $P(N;\vec{p}_T)$.
2. **Probs. to observe individual particles are independent.**
3. **For each observed particle, the value of efficiency r_i can be specified.**
4. Other detectors' effects are not considered.

True distr. func.

$$P(N;\vec{\xi})$$



Connecting True/Observed Distr. Funcs.

True distr. func.

$$P(N;\vec{\xi})$$

Observed distr. func.

$$\tilde{P}(n;q)$$

- n: observed particle number
- $q = \Sigma_{i\in(\text{obs.})}\xi_i$: observed sum

Probability Distr. of Observed Quantities (uniform r)

$$\tilde{P}(n;q) = \sum_{N=1}^{\infty}\int d\vec{\xi}\sum_{\{b_i\}}\left[\prod_{i=1}^{N}(1-r)^{1-b_i}r^{b_i}\right]\delta_{n,\sum_i b_i}\delta(q-\Sigma_i b_i\xi_i)P(N;\vec{\xi})$$

$$b_i = 0,1$$

Generating Function

MK, Esumi, Niida, Nonaka, PTEP2026

Prob. distr. func: $\tilde{P}(n;q) = \sum_{N=1}^{\infty} \int d\vec{\xi} \sum_{\{b_i\}} \left[\prod_{i=1}^{N} (1-r)^{1-b_i} r^{b_i} \right] \delta_{n,\sum_i b_i} \delta(q - \Sigma_i b_i \xi_i) P(N;\vec{\xi})$

Generating func: $\tilde{G}(s,t) = \sum_n \int dq \tilde{P} s^n t^q = \sum_N \int d\vec{\xi} P \prod_i (1 - r + rst^{\xi_i})$

Represent the quantity that you want to express by the derivative of the generating function.
Then, represent it in terms of the observed variables.

$$\left\langle \frac{\sum_i \xi_i}{N} \right\rangle_{\mathrm{true}} = \int_\alpha^1 ds \frac{r}{s} \left[\partial_t \tilde{G}(s,t) \right]_{t=1} = \left\langle \frac{\sum_i \xi_i}{n} \left(1 - \alpha^n\right) \right\rangle_{\mathrm{obs}} \qquad \alpha = \frac{r-1}{r}$$

Note: $\left\langle \frac{\sum_i \xi_i}{N} \right\rangle_{\mathrm{true}} \neq \left\langle \frac{\sum_i \xi_i}{n} \right\rangle_{\mathrm{obs}}$ α^n term compensates the $n = 0$ contribution.

Results: Correction Formulas

MK, Esumi, Niida, Nonaka, PTEP2026

Mean

$$\left\langle \frac{Q}{N} \right\rangle = \left\langle\!\left\langle \sum_{i=1}^{n} \xi_i k_i \right\rangle\!\right\rangle_{n\neq 0}$$

$$k_i = \frac{1}{r_i}\int_0^1 d\sigma \prod_{j\neq i} \frac{\sigma + r_j\alpha_j}{r_j}$$

2nd Order

$$\left\langle \frac{\{Q_1Q_2\}}{N(N-1)} \right\rangle = \left\langle\!\left\langle \sum_{i\neq j} q_{1,i}q_{2,j}k_{2;i,j} \right\rangle\!\right\rangle_{n\neq 0,1}$$

$$k_{2;i,j} = \frac{1}{r_ir_j}\int_0^1 d\sigma' \int_0^{\sigma'} d\sigma \prod_{l\neq i, l\neq j} \left(\frac{\sigma}{r_l} + \alpha_l\right)$$

- Correction formulas are written in forms including integral.
- This formula can reproduce the correct result for the previous simple model.

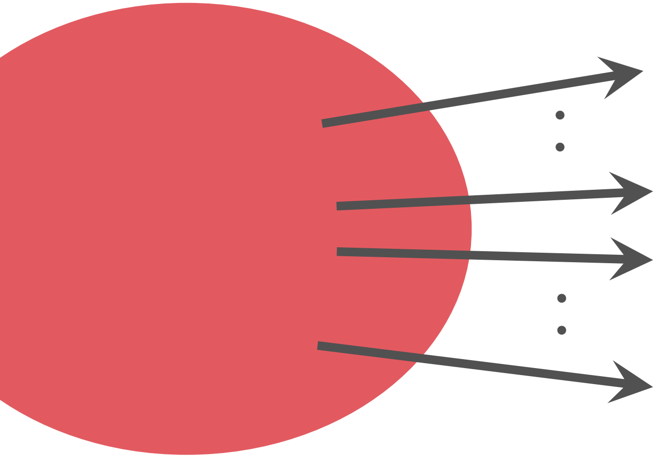
$$\{Q_{w_1}Q_{w_2}\} \equiv \sum_{i\neq j} \xi_i^{(w_1)}\xi_j^{(w_2)}$$

$$\alpha_i = \frac{1-r_i}{r_i}$$

MK, Esumi, Niida, Nonaka,
PTEP2026

Check in a Simple Model

$2N$: fixed for all events



$q_1 = 1$
$\vdots$
$q_N = 1$

$q_{N+1} = -1$
$\vdots$
$q_{2N} = -1$

N particles: efficiency $r = 1$

N particles: efficiency $r = p$

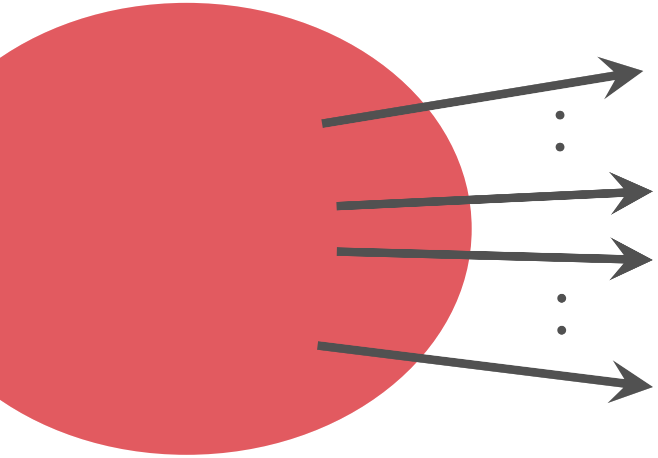
n particles

$$\tilde{Q} = \sum_{i \in (\mathrm{obs})} q_i$$

MK, Esumi, Niida, Nonaka, PTEP2026

Check in a Simple Model

$2N$: fixed for all events



$q_1 = 1$, ..., $q_N = 1$ } N particles → efficiency $r = 1$

$q_{N+1} = -1$, ..., $q_{2N} = -1$ } N particles → efficiency $r = p$

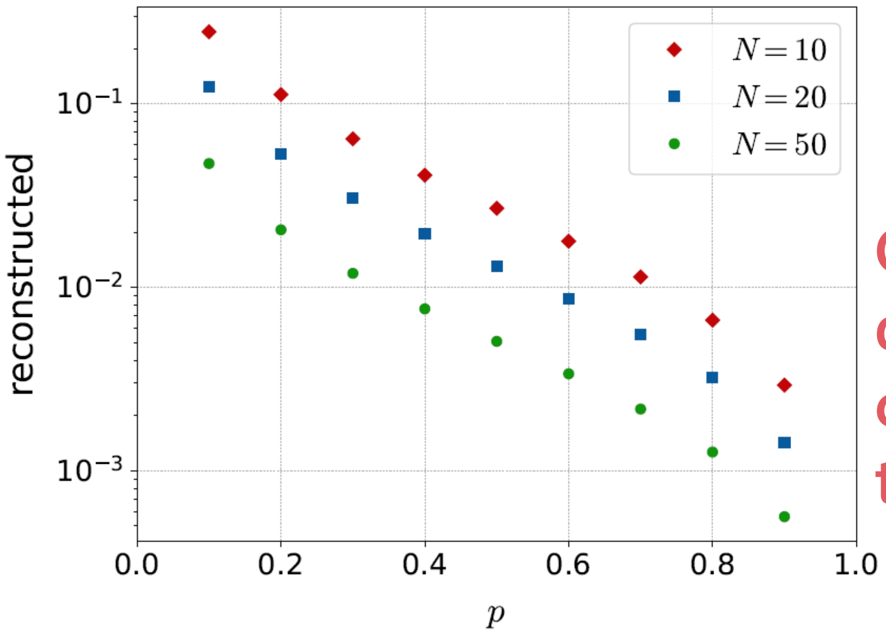
} n particles

$$\tilde{Q} = \sum_{i \in (\mathrm{obs})} q_i$$

Mean:

True result $$\left\langle \frac{Q}{N} \right\rangle = 0$$

Reconstructed $$\left\langle \frac{\sum_i q_i / r_i}{\sum_i 1/r_i} \right\rangle$$



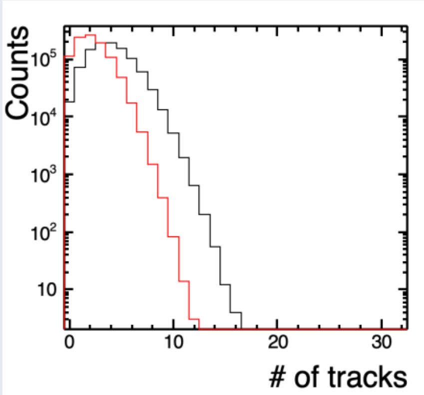
Conventional formula does not reproduce the correct result even for the mean!!

Test in Toy Models

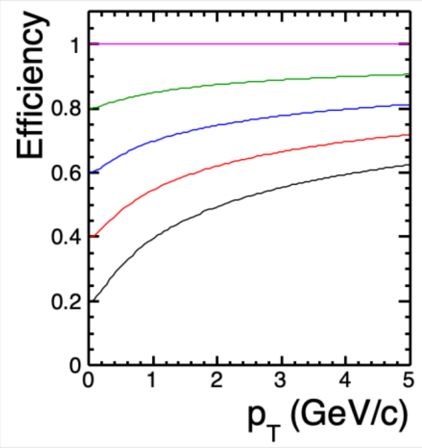
Nonaka+, in prep.

Simulation Procedure

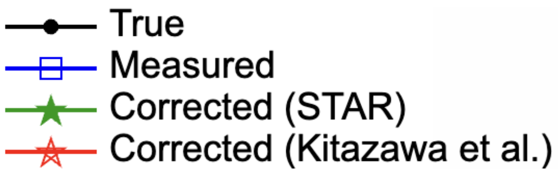
Generate
p_T distr.

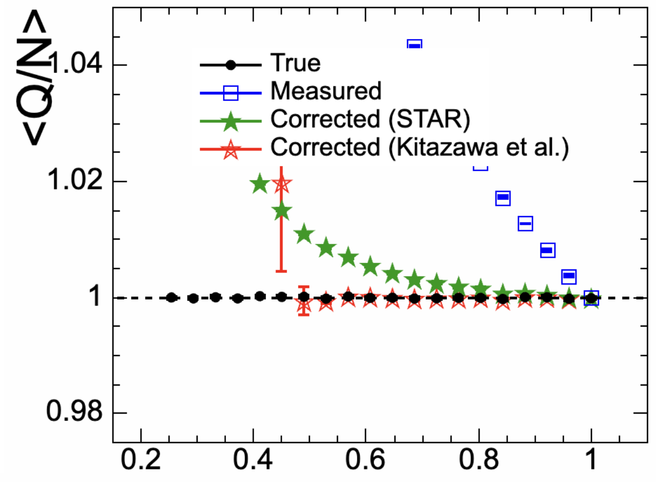


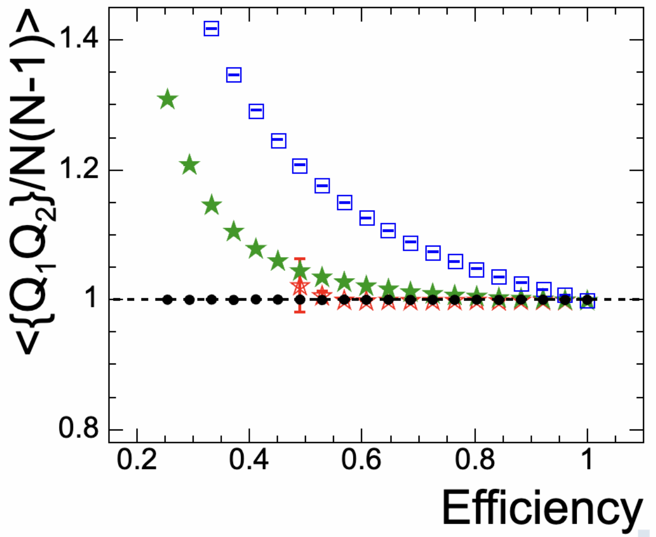
“Observation”
with
p_T dependent
efficiency

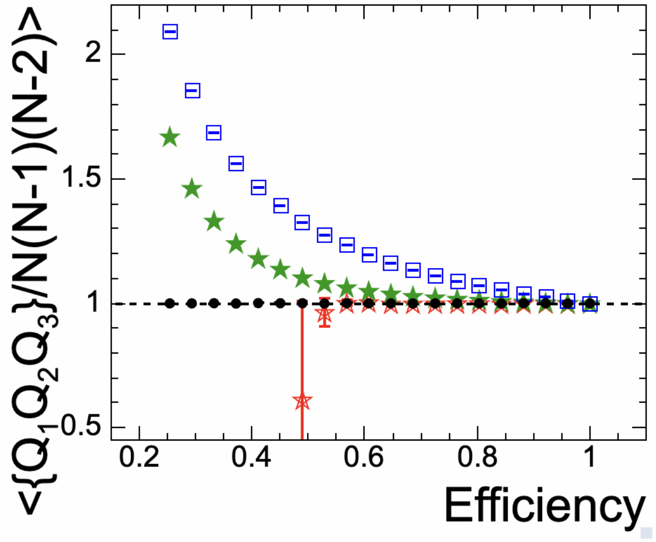


Results (mean p_T)









検出ロスの測定への影響は、高次相関で増大

従来補正法は、正しい測定値を再現できない

Summary

Correct Efficiency Correction Formulas

$$\left\langle \frac{Q}{N} \right\rangle = \left\langle\!\!\left\langle \sum_{i=1}^{n} \xi_i k_i \right\rangle\!\!\right\rangle_{n\neq 0} \qquad \left\langle \frac{\{Q_1 Q_2\}}{N(N-1)} \right\rangle = \left\langle\!\!\left\langle \sum_{i\neq j} q_{1,i} q_{2,j} k_{2;i,j} \right\rangle\!\!\right\rangle_{n\neq 0,1},$$

$$k_i = \frac{1}{r_i} \int_0^1 d\sigma \prod_{j\neq i} \frac{\sigma + r_j \alpha_j}{r_j}, \qquad k_{2;i,j} = \frac{1}{r_i r_j} \int_0^1 d\sigma' \int_0^{\sigma'} d\sigma \prod_{l\neq i, l\neq j} \left(\frac{\sigma}{r_l} + \alpha_l \right),$$

They will play crucial roles in experimental studies in HIC.
e.g. flow (dv_1/dy, $v_n\{m\}$, v_2-p_T correlation, etc.)

Further Development

General "unbiased estimator" that is applicable for any observables

Murase, MK, in prep.

Should Self-correlations be Eliminated?

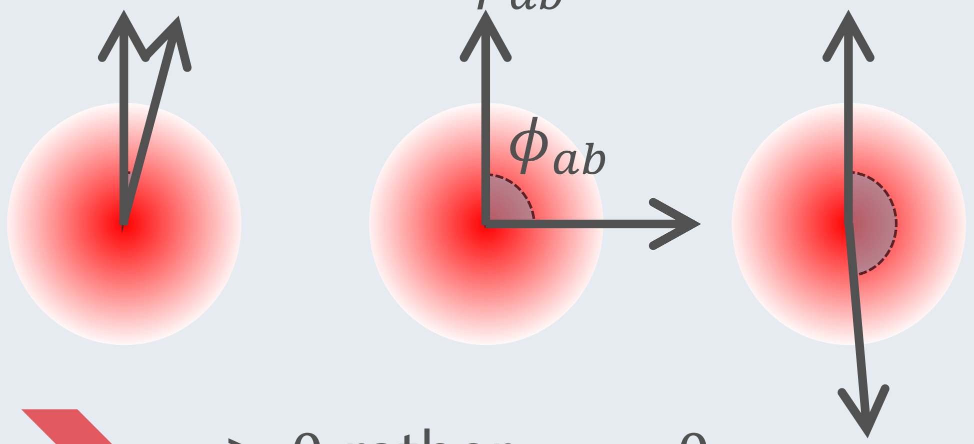
Flow correlations: $v_2^2 = \left\langle \frac{\sum_{i \neq j} e^{i(\phi_i - \phi_j)}}{N(N-1)} \right\rangle$

The "self correlation" terms are usually neglected. **Why?**

Argument 1:

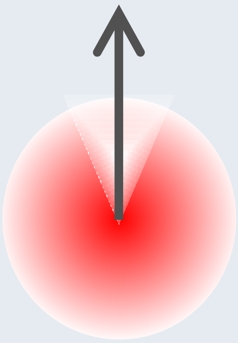
Emission of 2 independent particles

ϕ_{ab}: random



$v_2 > 0$ rather $v_2 = 0$

Argument 2:



Emission of a particle take away density

suppress probability to emit particles to same direction

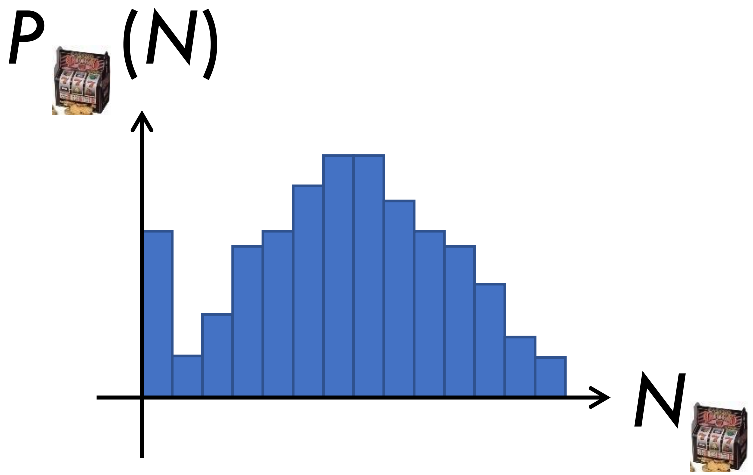
Simpler Example: Particle Number Fluc.

How can we obtain the cumulants of the true distribution only from observed information on $\tilde{P}(n)$?

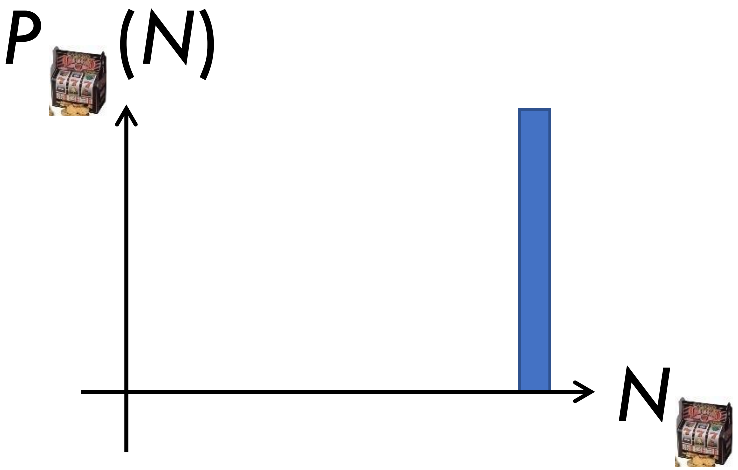
MK, Asakawa, 2012

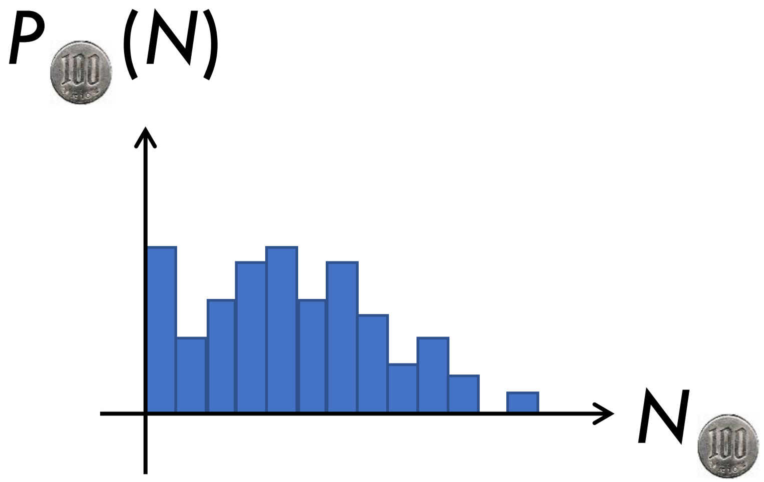
MK, Asakawa, 2012;2012

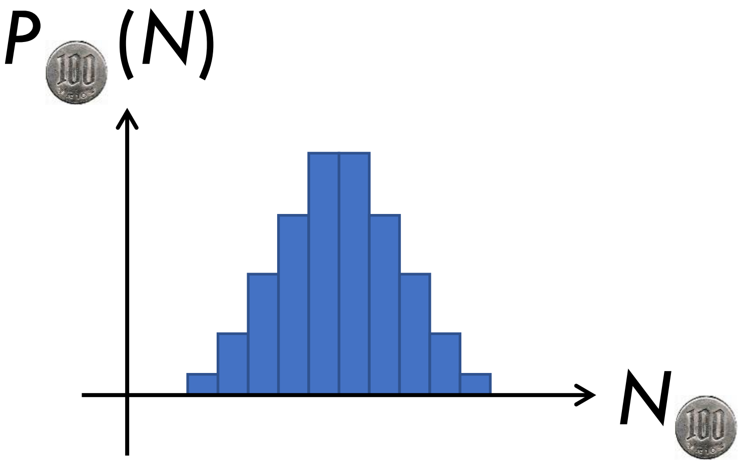
Slot Machine Analogy



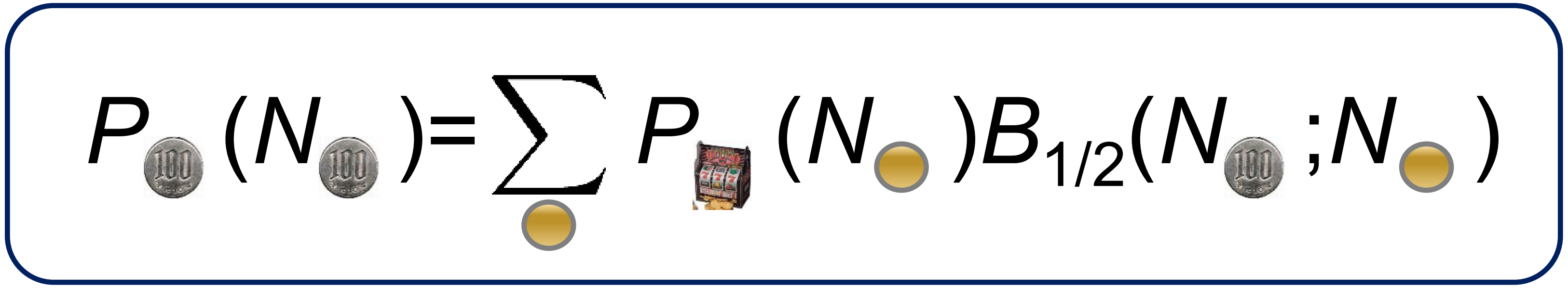
Fixed # of coins







Reconstructing Total Coin



- Example

$$
\begin{cases}
2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \cdots
\end{cases}
$$

genuine info. Poisson noise

Note: Higher order cumulants are more fragile.

MK, Asakawa, 2012;2012