

## Erratum and Addendum: Gravitational waves induced by a spinning particle falling into a rotating black hole

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A coding error was discovered in the computer program written to generate the numerical results presented in Phys. Rev. D **53**, 622 (1996). The corrected results, as well as some additional results, are presented here. [S0556-2821(99)03502-X]

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In Ref. [1] we presented the total radiated energy and wave form of gravitational waves from a spinning particle of mass  $\mu$  and spin angular momentum  $\mu S$  falling along the  $z$  axis into a rotating black hole of mass  $M \gg \mu$  and spin angular momentum  $Ma$ , using the Teukolsky–Sasaki–Nakamura formalism for black hole perturbations. We have recently found a coding error in our computer program for the numerical calculation. Since the error was on an  $Sa$  coupling term in the source term of the Teukolsky equation, the results were correct only for  $a=0$  or  $S=0$ ; otherwise they were incorrect. Although the qualitative feature and main conclusion do not change at all, the quantitative results change somewhat. In particular, for the case where both  $|a|$  and  $|S|$  are as large as  $\sim M$ , the magnitude of the error was about 20%. For these reasons, we correct here the previous results.

We recalculated the total radiated energy of gravitational waves  $\Delta E$  for a wide variety of parameter space of  $a$  and  $S$ . In Fig. 1 we show  $\Delta E$  in units of  $(\mu/M)^2$  as a function of  $a$  and  $S$  for  $0 \leq a/M \leq 0.9$  and  $-1 \leq S/M \leq 1$ . We note that for  $aS < 0$  ( $aS > 0$ ), the value of  $\Delta E$  is larger (smaller) than the previous incorrect results. For example, for  $a=S=0.99M$ , the previous incorrect  $\Delta E$  was  $0.0106\mu^2/M$ , while the correct one is  $0.00853\mu^2/M$ ; also for  $a=-S=0.99M$ , the previous incorrect value was  $0.0298\mu^2/M$ , while the correct one is  $0.0356\mu^2/M$ .

In Fig. 2 we also show  $\Delta E$  in units of  $(\mu/M)^2$  as a function of  $S$  for  $a=0, 0.9$ , and  $0.999$ .  $|a|$  is formally restricted to be less than  $M$ , but  $|S|$  is not. Hence numerical calculations

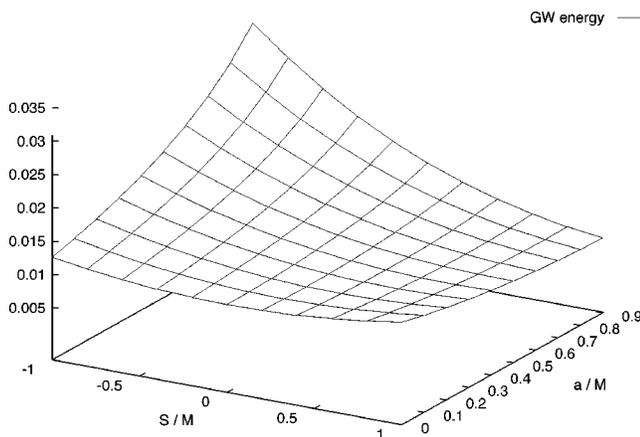


FIG. 1.  $\Delta E$  in units of  $(\mu/M)^2$  as a function of  $a$  and  $S$ .

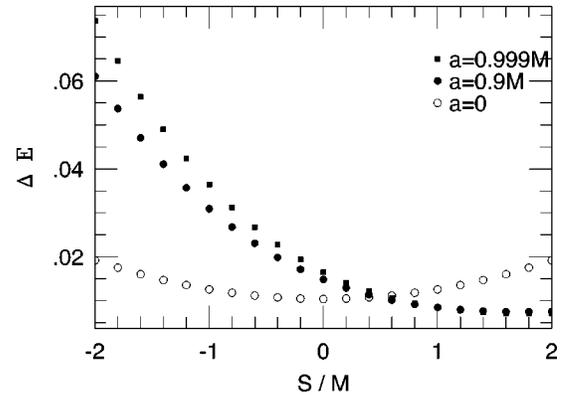


FIG. 2.  $\Delta E$  in units of  $(\mu/M)^2$  as a function of  $S$  for  $a=0, 0.9$ , and  $0.999$ .

were performed not only for small  $S$ , but for large  $|S|$  ( $\leq 2M$ ). Although in a realistic astrophysical situation there may be no compact objects of such large  $S$ , the calculation will be useful to see the effect of large  $S$ . From Fig. 2 it is found that even when  $a$  approaches  $M$ ,  $\Delta E$  does not increase drastically. This is because the quasinormal mode frequency that mainly determines the energy spectrum and  $\Delta E$  depends only weakly on  $a$  for the axially symmetric case compared to the nonaxisymmetric case [2]. As for the coupling effect between  $a$  and  $S$ , we found the following feature already pointed out in Ref. [1]:  $aS$  coupling tends to decrease the total radiated energy, i.e., with increasing (decreasing)  $aS$ ,  $\Delta E$  decreases (increases). Even for the case of large  $|S|$

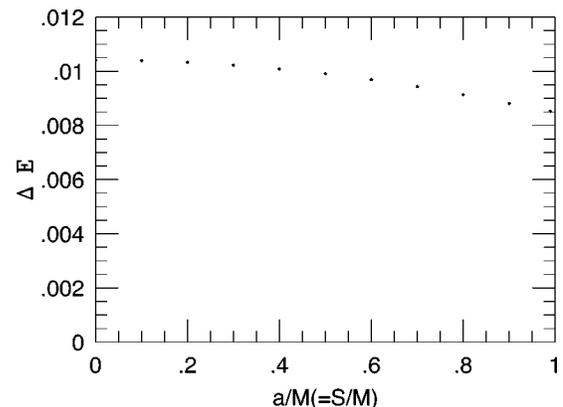


FIG. 3.  $\Delta E$  for the case  $S=a$ .

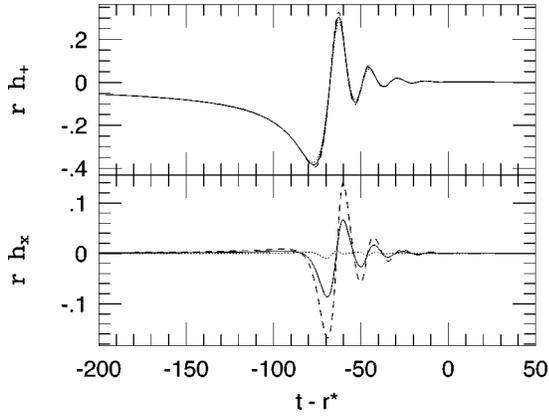


FIG. 4. Gravitational wave forms for  $a/M=0.6$  and  $S/M=0$  (solid line),  $-0.6$  (dashed line), and  $0.6$  (dotted line).

$>a$ , this feature does not change for  $aS<0$ . However, for  $aS>0$ , this is not the case: The coupling effect between  $a$  and  $S$  becomes less remarkable for  $S\geq a$ .

In Fig. 3 we show  $\Delta E$  for the case  $S=a$ . We can see that  $\Delta E$  does not depend on  $a$  ( $=S$ ) so much, but the trend is not so remarkable as reported previously.

In Figs. 4 and 5 we exhibit wave forms of gravitational waves. Figure 4 is for  $a/M=0.6$ . We plot for  $S/M=0$  (solid line),  $-0.6$  (dashed line), and  $0.6$  (dotted line). Figure 5 is for  $a/M=0.9$ . We plot for  $S/M=0$  (solid line),  $-0.9$  (dashed line), and  $0.9$  (dotted line). In our previous paper [1] we pointed out that (i) for the case  $a=S$ , the amplitude of  $\times$  modes of gravitational waves is much smaller than that of  $+$

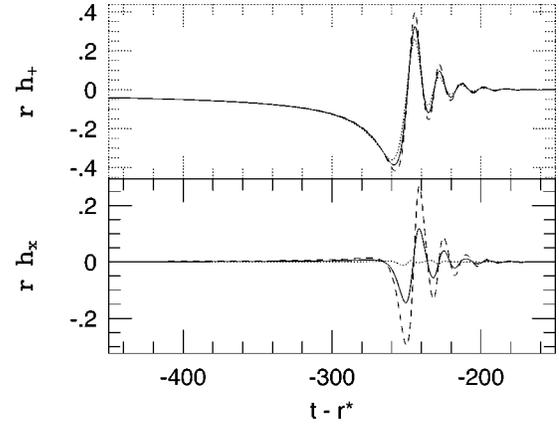


FIG. 5. Gravitational wave forms for  $a/M=0.9$  and  $S/M=0$  (solid line),  $-0.9$  (dashed line), and  $0.9$  (dotted line).

modes and (ii) the amplitude of  $+$  modes is weakly dependent on  $S$  for  $a=S$ . These features can be seen also in the present correct results. However, in the correct results, we find that feature (i) is more significant than that of the incorrect results in our previous paper and also that the dependence of the  $+$  mode amplitude on  $S$  is not so weak compared to the previous incorrect results.

Previous as well as present numerical calculations were performed by M.S. He wishes to apologize for any inconvenience caused by his negligence. We thank M. Saijo for checking the present numerical results independently.

[1] Y. Mino, M. Shibata, and T. Tanaka, Phys. Rev. D **53**, 622 (1996). Note that Figs. 3, 4, 7, and 9 in this reference are correct, but the other figures are incorrect. Figures 1, 2, 5, 6,

and 8 in this reference have to be replaced with Figs. 1, 2, 4, 5, and 3 in the present paper, respectively.

[2] E. W. Leaver, Proc. R. Soc. London **A402**, 285 (1985).