The Astrophysical Journal, 577:904–908, 2002 October 1
© 2002. The American Astronomical Society. All rights reserved. Printed in U.S.A.

COLLAPSE OF A ROTATING SUPERMASSIVE STAR TO A SUPERMASSIVE BLACK HOLE: ANALYTIC DETERMINATION OF THE BLACK HOLE MASS AND SPIN

Stuart L. Shapiro[1,2] and Masaru Shibata[3]
Received 2002 May 13; accepted 2002 June 10

ABSTRACT

The collapse of a uniformly rotating, supermassive star (SMS) to a supermassive black hole (SMBH) has been followed recently by means of hydrodynamic simulations in full general relativity. The initial SMS of arbitrary mass M in these simulations rotates uniformly at the mass-shedding limit and is marginally unstable to radial collapse. The final black hole mass and spin have been determined to be $M_h/M \approx 0.9$ and $J_h/M_h^2 \approx 0.75$. The remaining mass goes into a disk of mass $M_{\rm disk}/M \approx 0.1$. Here we show that these black hole and disk parameters can be calculated *analytically* from the initial stellar density and angular momentum distribution. The analytic calculation thereby corroborates and provides a simple physical explanation for the computational discovery that SMS collapse inevitably terminates in the simultaneous formation of a SMBH *and* a rather substantial ambient disk. This disk arises even though the total spin of the progenitor star, $J/M^2 = 0.97$, is safely below the Kerr limit. The calculation performed here applies to *any* marginally unstable $n = 3$ polytrope uniformly rotating at the breakup speed, independent of stellar mass or the source of internal pressure. It illustrates how the black hole and disk parameters can be determined for the collapse of other types of stars with different initial density and rotation profiles.

Subject headings: black hole physics — hydrodynamics — relativity — stars: rotation

1. INTRODUCTION

Recent observations provide increasingly strong evidence that supermassive black holes (SMBHs) of mass $\sim 10^6$–$10^{10}\ M_\odot$ exist and that they are the central engines that power active galactic nuclei (AGNs) and quasars (Rees 1998, 2001). The dynamical formation of SMBHs, as well as the inspiral, collision, and merger of binary SMBHs, are promising sources of long-wavelength gravitational waves for the proposed *Laser Interferometer Space Antenna* (*LISA*; Thorne 1995; Schutz 2001). However, the scenario by which SMBHs form is still very uncertain (see, e.g., Rees 1984, for an overview). One promising route is the collapse of a supermassive star (SMS). Once they form out of primordial gas, sufficiently massive stars will evolve in a quasi-stationary manner via radiative cooling, slowly contracting until reaching the point of the onset of relativistic radial instability. At this point, such stars undergo catastrophic collapse on a dynamical timescale, leading to the formation of an SMBH (Bisnovatyi-Kogan, Zeldovich, & Novikov 1966; Zeldovich & Novikov 1971; Shapiro & Teukolsky 1983).

Because most objects formed in nature have some angular momentum, rotation is likely to play a significant role in the quasi-stationary evolution, as well as the final collapse of an SMS. The slow contraction of even a slowly rotating SMS will likely spin it up to the mass-shedding limit because such stars are so centrally condensed. At the mass-shedding limit, matter on the equator moves in a Keplerian orbit about the star, supported against gravity by centrifugal force and not by an outward pressure gradient. Baumgarte & Shapiro (1999) recently performed a detailed numerical analysis of the structure and stability of a rapidly rotating SMS in equilibrium. Assuming the viscous or magnetic braking timescale for angular momentum transfer is shorter than the evolution timescale of a typical SMS (Zeldovich & Novikov 1971; Shapiro 2000), the star will settle into rigid rotation and evolve to the mass-shedding limit following cooling and contraction. They found that all stars at the onset of quasi-radial collapse have an equatorial radius of $R \approx 640GM/c^2$ and a nondimensional spin parameter of $a/M \equiv cJ/GM^2 \approx 0.97$. Here J, M, c, and G are spin, gravitational mass, light velocity, and gravitational constant. (Hereafter we adopt gravitational units and set $c = G = 1$.) Because of the large value of a/M, it was uncertain whether the rotating SMS would collapse directly to a black hole and/or form a disk. Because of the growth of $T/|W| \propto 1/R$ during collapse (where T is the rotational kinetic energy and W is the gravitational potential energy), it was unclear whether the collapse would trigger the growth of bars or other nonaxisymmetric instabilities. We, therefore, investigated the collapse of a rotating SMS with a $3 + 1$ post-Newtonian (PN) hydrodynamic simulation (Saijo et al. 2002). We found that the collapse proceeds nearly homologously and axisymmetrically, and we inferred that an SMBH is likely to be formed, with some gas remaining outside the hole. Shibata & Shapiro (2002) then performed simulations of the collapse using an axisymmetric code in full general relativity. We showed conclusively that a black hole forms at the center and determined that the mass of the hole contains about 90% of the total mass of the system and has a spin parameter of $J/M^2 \sim 0.75$. The remaining gas forms a rotating disk about the nascent hole.

Guided by our numerical simulations, we show here that the final black hole mass and spin can be calculated *analytically* (up to quadrature) from the stellar density and angular momentum profiles in the progenitor SMS. The calculation provides a simple physical explanation for the important

[1] Department of Physics, University of Illinois at Urbana-Champaign, Loomis Laboratory of Physics, 1110 West Green Street, Urbana, IL 61801-3080.

[2] Department of Astronomy and National Center for Supercomputing Applications, University of Illinois at Urbana-Champaign, 103 Astronomy Building, 1002 West Green Street, Urbana, IL 61801-3080.

[3] Graduate School of Arts and Sciences, University of Tokyo, Komaba, Meguro, Tokyo 153-8902, Japan.

finding that an SMBH formed from the collapse of a rotating SMS is always born with a "ready-made" disk that can provide fuel for accretion to power an energy source.

2. BASIC ASSUMPTIONS

Our analysis relies on several explicit assumptions, all of which we expect to hold to high accuracy in SMSs. In particular, we assume that the SMS has the following characteristics:

1. It is dominated by thermal radiation pressure.
2. It is fully convective.
3. It is uniformly rotating.
4. It is governed by nearly Newtonian gravitation.
5. It can be described by the Roche model in the outer envelope.

For large masses, the ratio between radiation pressure, P_r, and gas pressure, P_g, satisfies

$$\beta \equiv \frac{P_g}{P_r} = 8.49\left(\frac{M}{M_\odot}\right)^{-1/2} \quad (1)$$

(see, e.g., Shapiro & Teukolsky 1983, eqs. [17.2.8] and [17.3.5]); here the coefficient has been evaluated for a composition of pure ionized hydrogen. For SMSs with $M \gtrsim 10^6\ M_\odot$, we can therefore neglect the pressure contributions of the plasma in determining the equilibrium profile, even though the plasma can be important for determining the stability of the star when it is rotating sufficiently slowly (Zeldovich & Novikov 1971; Shapiro & Teukolsky 1983). A simple proof that SMSs are convective in this limit is given in Loeb & Rasio (1994). This result implies that the photon entropy per baryon,

$$s_r = \frac{4}{3}\frac{aT^3}{n}\ , \quad (2)$$

is constant throughout the star and so, therefore, is $\beta \approx 8(s_r/k)^{-1}$. Here a is the radiation density constant, n is the baryon density, and k is Boltzmann's constant. As a consequence, the equation of state of an SMS is that of an $n = 3$ polytrope:

$$P = K\rho^{4/3}, \quad K = \left[\left(\frac{k}{\mu m}\right)^4 \frac{3}{a}\frac{(1+\beta)^3}{\beta^4}\right]^{1/3} = \text{const}\ , \quad (3)$$

where m is the atomic mass unit and μ is the mean molecular weight [see Clayton 1983, eq. (2–289); note that Clayton adopts a different definition of β, which is related to ours by $\beta_{\rm Clayton} = \beta/(1+\beta)$]. In fact, our results depend only on the assumption that the star is an $n = 3$ polytrope and not on the origin of the internal pressure.

The third assumption, that the star is uniformly rotating, is probably the most uncertain of our assumptions. Nevertheless, it has been argued that convection and magnetic fields provide an effective turbulent viscosity that dampens differential rotation and brings the star into uniform rotation (Bisnovatyi-Kogan, Zeldovich, & Novikov 1967; Wagoner 1969; Shapiro 2000; for an alternative, see New & Shapiro 2001). Uniformly rotating configurations were adopted as initial configurations in Shibata & Shapiro (2002), whose numerical results we are attempting to corroborate.

We assume that gravitational fields in the progenitor star are sufficiently weak so that we can apply Newtonian gravity in analyzing its envelope. SMSs at the onset of collapse have equatorial radii of $R/M \approx 640$ (Baumgarte & Shapiro 1999), so that this assumption certainly holds. Relativistic corrections are crucial for the stability of SMSs, but can be neglected in the analysis of the equilibrium state.

Finally, the Roche approximation provides a very accurate description of the envelope of a rotating stellar model with a soft equation of state, as in the case of an $n = 3$ polytrope (for numerical demonstrations, see, e.g., Papaloizou & Whelan 1973 and Baumgarte & Shapiro 1999). Since our analysis is based on this approximation, we will briefly review it together with some of its predictions in the following section.

We assume that the collapse proceeds

1. axisymmetrically;
2. adiabatically.

Our previous three-dimensional simulation in PN gravitation (Saijo et al. 2002) allowed for the growth of nonaxisymmetric perturbations, but none were observed. Hence, the assumption of axisymmetry, which was the basis for the fully relativistic simulation of Shibata & Shapiro (2002), appears to be justified. Cooling by photon and neutrino emission can be ignored on a dynamical timescale (Linke et al. 2001), hence the collapse is essentially adiabatic. The collapse will be accompanied by a burst of gravitational radiation that may be detectable by *LISA*. However, the fractional loss of mass-energy is sufficiently small that it can be ignored in determining the black hole mass (Shibata & Shapiro 2002); angular momentum is strictly conserved, as gravitational waves carry off no angular momentum in axisymmetry.

3. REVIEW OF THE ROCHE MODEL

Stars with soft equations of state are extremely centrally condensed: they have an extended, low-density envelope, while the bulk of the mass is concentrated in the core. For an $n = 3$ polytrope, for example, the ratio between central density to average density is $\rho_c/\bar{\rho} = 54.2$. The gravitational force in the envelope is, therefore, dominated by the massive core, and it is thus legitimate to neglect the self-gravity of the envelope. In the equation of hydrostatic equilibrium,

$$\frac{\nabla P}{\rho} = -\nabla(\Phi + \Phi_c)\ , \quad (4)$$

this neglect amounts to approximating the Newtonian potential Φ by

$$\Phi = -\frac{M}{r}\ . \quad (5)$$

In equation (4) we introduce the centrifugal potential Φ_c, which, for constant angular velocity Ω about the z-axis, can be written

$$\Phi_c = -\tfrac{1}{2}\Omega^2(x^2 + y^2) = -\tfrac{1}{2}\Omega^2 r^2 \sin^2\theta\ . \quad (6)$$

Integrating equation (4) yields the Bernoulli integral

$$h + \Phi + \Phi_c = H\ , \quad (7)$$

where H is a constant of integration and

$$h = \int \frac{dP}{\rho} = (n+1)\frac{P}{\rho} \tag{8}$$

is the enthalpy per unit mass. Evaluating equation (7) at the pole yields

$$H = -\frac{M}{R_p} , \tag{9}$$

since $h = 0$ on the surface of the star and $\Phi_c = 0$ along the axis of rotation. In the following we assume that the polar radius R_p of a rotating star is always the same as in the nonrotating case. This assumption has been shown numerically to be very accurate (e.g., Papaloizou & Whelan 1973). The mass of the star is hardly changed from its value in spherical equilibrium,

$$M = \frac{4}{\pi^{1/2}}(\xi_1^2|\theta'(\xi_1)|)K^{3/2} , \tag{10}$$

where the Lane-Emden factor for an $n = 3$ polytrope is given by $\xi_1^2|\theta'(\xi_1)| = 2.01824$ (Shapiro & Teukolsky 1983). A rotating star reaches mass shedding when the equator orbits with the Kepler frequency. Using equations (6) and (7), it is easy to show that at this point the ratio between equatorial and polar radius is

$$\left(\frac{R_e}{R_p}\right)_{\rm shedd} = \frac{3}{2} . \tag{11}$$

The corresponding maximum orbital velocity is

$$\Omega_{\rm shedd} = \left(\frac{2}{3}\right)^{3/2}\left(\frac{M}{R_p^3}\right)^{1/2} \tag{12}$$

(Zeldovich & Novikov 1971; Shapiro & Teukolsky 1983).

Inserting equation (3), (5), (6), (8), (9), and (12) into equation (7) yields the density throughout the extended envelope,

$$\rho = \frac{(\xi_1^2|\theta'(\xi_1)|)^2}{4\pi}\frac{M}{R_p^3}\left(\frac{R_p}{r} - 1 + \frac{4}{27}\frac{r^2}{R_p^2}\sin^2\theta\right)^3 . \tag{13}$$

The scale of the density is set by the value of the polar radius at the point of the onset of radial collapse,

$$R_p/M = 427 \tag{14}$$

(Baumgarte & Shapiro 1999). The stellar surface is the boundary along which $\rho = 0$ and is thus defined by the curve $r(\theta)$ given by

$$\frac{4}{27}\frac{r^3}{R_p^3}\sin^2\theta - \frac{r}{R_p} + 1 = 0 . \tag{15}$$

The solution to this cubic equation is given by

$$\frac{r(\theta)}{R_p} = \frac{3\sin(\theta/3)}{\sin\theta} \tag{16}$$

(R. Cooper 2002, private communication).

4. CALCULATING THE BLACK HOLE MASS AND SPIN

Consider the implosion of matter from the envelope onto the central black hole formed from the collapse of the centrally condensed interior region. If the specific angular momentum of the imploding envelope matter is below $j_{\rm ISCO}$, the specific angular momentum of a particle at the innermost stable circular orbit (ISCO) about the hole, then the matter will be captured. If the angular momentum of the infalling matter exceeds $j_{\rm ISCO}$, the matter will escape capture and continue to orbit outside the hole, forming a disk. This capture criterion is well supported by the numerical simulation of Shibata & Shapiro (2002), where the capture of fluid in noncircular orbits with $j > j_{\rm ISCO}$ was demonstrated to be negligible. This criterion suggests a simple iterative scheme for calculating the final mass and spin of the hole and disk from the initial stellar density and angular momentum profile. First, guess the mass and spin of the hole, M_h and J_h. For our initial guess, we shall take a black hole that has consumed all the mass and angular momentum of the star, so that $M_h/M = 1$ and $J_h/M_h^2 = 0.97$. Next, use the initial stellar profile to "correct" this guess by calculating the escaping mass and angular momentum of the outermost envelope with specific angular momentum exceeding $j_{\rm ISCO}$ (see Fig. 1). We note that the value $j_{\rm ISCO}$ depends on M_h and J_h. We then "correct" the black hole mass and spin by deducting the values of the escaping mass and angular momentum of the envelope material from the guessed values of M_h and J_h. We recompute $j_{\rm ISCO}$ for the "corrected" black hole mass and spin and repeat the calculation of the escaping envelope mass and angular momentum until convergence is achieved. The calculation described above relies on the theorem that for an axisymmetric dynamical system, the specific angular momentum spectrum, i.e., the integrated baryon rest mass of all fluid elements with specific angular momentum j less than a given value (e.g., $j_{\rm ISCO}$), is strictly conserved in the absence of viscosity (Stark & Piran 1987). Any viscosity, if present, is expected to be unimportant on dynamical timescales, as required by the theorem.

For a Kerr black hole of mass M_h and spin parameter $a = J_h/M_h$, the value of $j_{\rm ISCO}$ is given by

$$j_{\rm ISCO} = \frac{\sqrt{M_h r_{\rm ms}}(r_{\rm ms}^2 - 2a\sqrt{M_h r_{\rm ms}} + a^2)}{r_{\rm ms}(r_{\rm ms}^2 - 3M_h r_{\rm ms} + 2a\sqrt{M_h r_{\rm ms}})^{1/2}} , \tag{17}$$

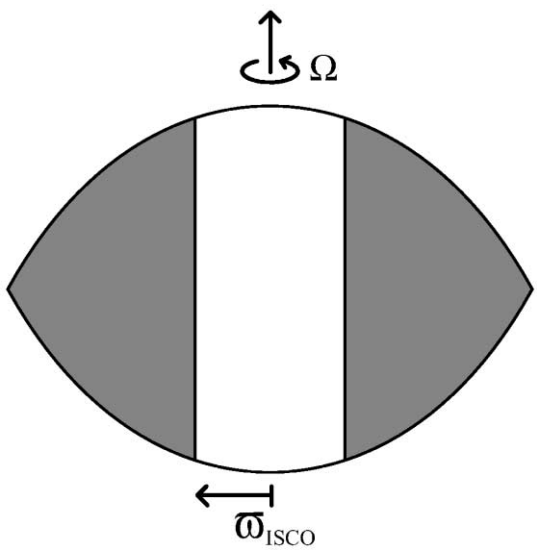

FIG. 1.—SMS rotating uniformly at the mass-shedding limit. The shaded region shows the portion of the Roche envelope $\varpi > \varpi_{\rm ISCO}$ in which the matter has specific angular momentum $j > j_{\rm ISCO}$. Consequently, this gas will escape capture by the black hole that forms at the center following collapse of the interior.

where $r_{\rm ms}$ is the ISCO given by

$$r_{\rm ms} = M_h\{3 + Z_2 - [(3 - Z_1)(3 + Z_1 + 2Z_2)]^{1/2}\} , \quad (18)$$

where

$$Z_1 \equiv 1 + \left(1 - \frac{a^2}{M_h^2}\right)^{1/3}\left[\left(1 + \frac{a}{M_h}\right)^{1/3} + \left(1 - \frac{a}{M_h}\right)^{1/3}\right] , \quad (19)$$

and

$$Z_2 \equiv \left(3\frac{a^2}{M_h^2} + Z_1^2\right)^{1/2} \quad (20)$$

(see, e.g., Shapiro & Teukolsky 1983). Clearly, the infalling gas corotates with the black hole.

The mass of the escaping matter in the envelope with $j > j_{\rm ISCO}$ is given by

$$\Delta M = \int\int 2\pi\varpi d\varpi\rho , \quad (21)$$

where the density is given by equation (13) and the quadrature is performed over all cylindrical shells in the star with cylindrical radii $\varpi > \varpi_{\rm ISCO} = (j_{\rm ISCO}/\Omega)^{1/2}$ (see Fig. 1). Here we set $\Omega = \Omega_{\rm shedd}$ given by equation (12). Defining $\bar{\varpi} = \varpi/R_p$ and $\bar{z} = z/R_p$ gives the nondimensional integral

$$\Delta M/M = \left(\xi_1^2|\theta'(\xi_1)|\right)^2 \times \int\int \bar{\varpi}d\bar{\varpi}d\bar{z}\left[\frac{1}{(\bar{\varpi}^2 + \bar{z}^2)^{1/2}} - 1 + \frac{4}{27}\bar{\varpi}^2\right]^3 . \quad (22)$$

Similarly, the angular momentum carried off by the escaping matter in the envelope is given by

$$\Delta J = \int\int 2\pi\varpi d\varpi\rho\Omega\varpi^2 \quad (23)$$

or

$$\frac{\Delta J}{M^2} = \left(\xi_1^2|\theta'(\xi_1)|\right)^2\left(\frac{2}{3}\right)^{3/2}\left(\frac{R_p}{M}\right)^{1/2} \times \int\int \bar{\varpi}^3 d\bar{\varpi}d\bar{z}\left[\frac{1}{(\bar{\varpi}^2 + \bar{z}^2)^{1/2}} - 1 + \frac{4}{27}\bar{\varpi}^2\right]^3 , \quad (24)$$

where R_p/M is given by equation (14) and where, once again, the integral is performed over all cylindrical shells in the star with $\varpi > \varpi_{\rm ISCO} = (j_{\rm ISCO}/\Omega)^{1/2}$.

The mass and angular momentum of the black hole can then be determined from equations (22) and (24) according to

$$\frac{M_h}{M} = 1 - \frac{\Delta M}{M} \quad (25)$$

and

$$\frac{J_h}{M_h^2} = \frac{\left(J/M^2 - \Delta J/M^2\right)}{(1 - \Delta M/M)^2} . \quad (26)$$

Once the iteration of equations (17)–(20) with equations (22), (25), and (26) converge, the mass of the ambient disk can be found from

$$\frac{M_{\rm disk}}{M} = \frac{\Delta M}{M} . \quad (27)$$

Convergence to better than 1% is achieved after only four iterations. We find that $\varpi_{\rm ISCO}/R_p = 0.43$. We obtain a black hole mass $M_h/M = 0.87$ and black hole spin $J_h/M_h^2 = 0.71$, and a disk mass $M_{\rm disk}/M = 0.13$. These values are consistent with those inferred from the numerical simulation of Shibata & Shapiro (2002): $M_h/M \approx 0.9$, $J_h/M_h^2 \approx 0.75$, and $M_{\rm disk}/M \approx 0.1$. The differences are well within the numerical accuracy of the dynamical integrations in the simulation.

5. DISCUSSION

We have considered an SMBH formed by the collapse of a radially unstable, uniformly rotating SMS of mass M, spinning at the mass-shedding limit. We calculated that the black hole has a mass of $M_h/M \approx 0.87$ and a spin of $J_h/M_h^2 \approx 0.71$. Most significantly, we found that at birth such a black hole will be automatically embedded in a substantial disk of mass $M_{\rm disk}/M \approx 0.13$. A disk forms even though the total angular momentum of the star at the onset of collapse is $J/M^2 \approx 0.97$, below the Kerr limit.

We have also calculated the black hole and disk parameters for unstable SMSs that, at the onset of collapse, are spinning more slowly than the angular frequency at breakup (i.e., mass shedding). We find that the parameters are very insensitive to the ratio $\alpha = \Omega/\Omega_{\rm shedd}$, where $\Omega_{\rm shedd}$ is given by equation (12), over the range $0.1 \le \alpha \le 1$. The reason is that for a small Ω, a rotating SMS destabilizes at large R_p/M ($R_p/M \approx 427/\alpha^2$), with a value of $J/M^2 \approx 0.97$, nearly independent of α. The corresponding value of $\varpi_{\rm ISCO}/R_p \approx 0.43$ is nearly unchanged, and as a result, the final black hole and disk parameters do not vary significantly as α decreases. (For sufficiently low spin rates, the effects of thermal gas pressure compete with rotation to determine the onset of stability and must be included.) Nevertheless, we believe, on evolutionary grounds, that it is most likely that an SMS will be spinning near the breakup value at the onset of collapse (Baumgarte & Shapiro 1999).

The nascent disk is fairly massive. Such a massive, rotating disk may be dynamically unstable to nonaxisymmetric instabilities. If the disk deforms, it could emit long-wavelength, quasi-periodic gravitational waves with a frequency of about $10^{-3}(M/10^6\ M_\odot)^{-1}$ Hz, which might be detectable by *LISA*.

At birth, the disk will be hot and thick, radiation-dominated, and will extend from a radius of $\sim 640\ M$ at the outer edge in the equatorial plane all the way down to the horizon. Subsequently, the matter in the disk will likely cool by both photon and neutrino emission and flatten. Under the influence of viscosity, the gas will diffuse outward in the outermost region, transporting angular momentum away from the inner region. At the same time, gas in the inner region will accrete onto the central black hole, increasing its mass and powering a source of luminous energy. The lifetime of the disk $t_{\rm disk}$ may be estimated crudely by speculating that the accretion eventually will settle into a steady

state, generating photons at the Eddington luminosity at $\sim$10% efficiency: $L = L_{\rm Edd} = 1.3 \times 10^{38}(M_h/M_\odot)$ and $\dot{M} = 10L/c^2$. Accordingly, we have $t_{\rm disk} = M_{\rm disk}/\dot{M}$ or

$$t_{\rm disk} = \frac{M_{\rm disk}c^2}{10L_{\rm Edd}} \approx 5 \times 10^6 \text{ yr} . \quad (28)$$

The above estimate for $t_{\rm disk}$ is independent of M. However, it may be off by a considerable factor: a lower luminosity will increase the disk lifetime, while a lower efficiency (e.g., an efficiency characterizing an advection-dominated accretion flow disk[ADAF]; Narayan & Yi 1994) or an enhancement in the total luminosity via the addition of neutrino cooling, could decrease the lifetime. Moreover, the disk at black hole birth may comprise far more material than the gas that escapes capture during the final implosion. Prior to undergoing collapse, a rotating SMS will likely evolve along a mass-shedding sequence as it cools and contracts, provided the stellar viscosity and/or magnetic braking proves sufficient to enforce uniform rotation (Baumgarte & Shapiro 1999). In this case, the secular cooling and contraction phase will be accompanied by mass loss, and the gas that is ejected might then already form an extensive, ambient disk prior to collapse. Finally, we note the possibility that such a massive disk around the hole may suffer a global "runaway" instability that may cause a more rapid destruction of the disk than implied by steady state accretion (Abramowicz, Calvani, & Nobili 1983; Nishida et al. 1996; Nishida & Eriguchi 1996). These are important issues that are ripe for further analysis.

The calculation performed here applies to *any* marginally unstable $n = 3$ polytrope rotating at the mass-shedding limit. The results are independent of stellar mass or the source of internal pressure. The method should be applicable to estimating the black hole and disk parameters for the collapse of other types of stars with different equations of state, initial density, and rotation profiles. For example, a preliminary calculation indicates that for a marginally unstable star uniformly rotating at breakup speed, the mass fraction that avoids collapse and forms a disk falls sharply as the polytropic index of the star drops below $n = 3$. The reason is that such stars are more compact at the onset of collapse, with smaller values of R_p/M and, hence, less extended envelopes. This may provide a qualitative explanation for the results of previous simulations in full general relativity for the collapse of rotating neutron stars modeled as $n = 1$ polytropes (Shibata, Baumgarte, & Shapiro 2000; Shibata 2000). We hope to provide further examples in the future.

We are grateful to the Illinois Undergraduate Research Team (H. Agarwal, R. Cooper, B. Hagan, and D. Webber) for assistance with visualization. This work was supported in part by NSF grant PHY 00-90310 and NASA grants NAG 5-8418 and NAG 5-10781 at the University of Illinois at Urbana-Champaign and Monbu-Kagaku-sho grants 13740143 and 14047207 at the University of Tokyo.

REFERENCES

Abramowicz, M. A., Calvani, M., & Nobili, L. 1983, Nature, 302, 597
Baumgarte, T. W., & Shapiro, S. L. 1999, ApJ, 526, 941
Bisnovatyi-Kogan, G. S., Zeldovich, Ya. B., & Novikov, I. D. 1967, Soviet Astron., 11, 419
Clayton, D. 1983, Principles of Stellar Evolution and Nucleosynthesis (Chicago: Univ. Chicago Press)
Linke, F., Font, J. A., Janka, H.-T., Müller, E., & Papadopoulas, P. 2001, A&A, 376, 568
Loeb, A., & Rasio, F. A. 1994, ApJ, 432, 52
Narayan, R., & Yi, I. 1994, ApJ, 428, L13
New, K. C. B., & Shapiro, S. L. 2001, ApJ, 548, 439
Nishida, S., & Eriguchi, Y. 1996, ApJ, 461, 320
Nishida, S., Lanza, A., Eriguchi, Y., & Abramowicz, M. A. 1996, MNRAS, 278, L41
Papaloizou, J. C. B., & Whelan, F. A. J. 1973, MNRAS, 164, 1
Rees, M. J. 1984, ARA&A, 22, 471
———. 1998, in Black Holes and Relativistic Stars, ed. R. M. Wald (Chicago: Univ. Chicago Press), 79
———. 2001, in Black Holes in Binaries and Galactic Nuclei, ed. L. Kaper, E. P. J. van den Heuvel, & P. A. Woudt (New York: Springer), 351
Saijo, M, Baumgarte, T. W., Shapiro, S. L., & Shibata, M. 2002, ApJ, 569, 349
Schutz, B. F. 2001, preprint (gr-qc/0111095)
Shapiro, S. L. 2000, ApJ, 544, 397
Shapiro, S. L., & Teukolsky, S. A. 1983, Black Holes, White Dwarfs, and Neutron Stars (New York: Wi1ey)
Shibata, M. 2000, Prog. Theor. Phys., 104, 325
Shibata, M., Baumgarte, T. W., & Shapiro, S. L. 2000, Phys. Rev. D, 61, 044012
Shibata, M., & Shapiro, S. L. 2002, ApJ, 572, L39
Stark, R. F., & Piran, T. 1987, Comput. Phys. Rep., 5, 221
Thorne, K. S. 1995, in Proc. of Snowmass 95 Summer Study on Particle and Nuclear Astrophysics and Cosmology, ed. E. W. Kolb & R. Peccei (Singapore: World Scientific), 398
Wagoner, R. V. 1969, ARA&A, 7, 553
Zeldovich, Ya. B., & Novikov, I. D. 1971, Relativistic Astrophysics, Vol. 1 (Chicago: Univ. Chicago Press)