Black hole dynamics in generic spacetimes

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Molecule workshop:
“Recent advances in numerical and analytical methods for black hole dynamics”
YITP, Kyoto, 28 March, 2012
1. Collisions of black holes in higher dimensional spacetimes

2. Black holes in a box

3. Conclusions and Outlook
Black hole collisions
in higher dimensional spacetimes
Consider particle collisions with $E = 2\gamma m_0 c^2 > M_{Pl}$

- Hoop - Conjecture (Thorne ’72)
  $\Rightarrow$ BH formation, if circumference of particle $< 2\pi r_S$

- Collisions of shock waves
  (Penrose ’74, Eardley & Giddings ’02)
  $\Rightarrow$ BH formation if $b \leq r_S$

- Numerical evidence in ultra relativistic collision of boson stars
  $\Rightarrow$ BH formation if boost $\gamma_c \geq 2.9$

$\Rightarrow$ black hole formation in high energy collisions of particles

- Higher dimensional theories of gravity $\Rightarrow$ TeV gravity scenarios
- Signatures of black hole production in high energy collision of particles
  - at the Large Hadron Collider
  - in ultra-high relativistic Cosmic rays interactions with the atmosphere
Life cycle of Mini Black Holes

1. Formation
   - lower bound on BH mass from area theorem (Yoshino & Nambu ’02)
2. Balding phase: end state is Myers-Perry black hole
3. Spindown phase: loss of angular momentum and mass
4. Schwarzschild phase: decay via Hawking radiation
5. Planck phase: $M \sim M_{Pl}$

Goal: more precise understanding of black hole formation
⇒ compute energy and angular momentum loss in gravitational radiation
Toy model: black hole collisions in higher dimensions
Numerical Relativity

in $D > 4$ Dimensions

- Yoshino & Shibata, Phys. Rev. **D80**, 2009,
Numerical Relativity in D Dimensions

Consider highly symmetric problems

Dimensional reduction by isometry on a (D-4)-sphere

General metric element

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \lambda(x^\mu) d\Omega_{D-4} \]
Numerical Relativity in D Dimensions

D dimensional vacuum Einstein equations \( G_{AB} = R_{AB} - \frac{1}{2} g_{AB} R = 0 \) imply

\[
(4) T_{\mu\nu} = \frac{D - 4}{16\pi\lambda} \left[ \nabla_\mu \nabla_\nu \lambda - \frac{1}{2\lambda} \partial_\mu \lambda \partial_\nu \lambda - (D - 5) g_{\mu\nu} + \frac{D - 5}{4\lambda} g_{\mu\nu} \nabla_\alpha \lambda \nabla^\alpha \lambda \right]
\]

\[
\nabla^\mu \nabla_\mu \lambda = 2(D - 5) - \frac{D - 6}{2\lambda} \nabla^\mu \lambda \nabla_\mu \lambda
\]

- 4D Einstein equations coupled to scalar field
Formulation of EEs as Cauchy Problem in $D > 4$

- 3+1 split of spacetime $\mathcal{M} = \mathbb{R}^4 + \Sigma$ (Arnowitt, Deser, Misner '62)
- $d\mathcal{s}^2 = g_{\mu\nu} \, dx^\mu \, dx^\nu = (-\alpha^2 + \beta_{k^l} \beta^k) \, dt^2 + 2\beta_i \, dx^i \, dt + \gamma_{ij} \, dx^i \, dx^j$
- 3+1 split of 4D Einstein equations with source terms
  $\implies$ Formulation as initial value problem with constraints (York 1979)

**Evolution Equations**

\[
\begin{align*}
\partial_t \gamma_{ij} &= -2\alpha K_{ij} + \mathcal{L}_\beta \gamma_{ij} \\
\partial_t K_{ij} &= -D_i D_j \alpha + \alpha \left( (3) R_{ij} - 2K_{il} K^l_j + KK_{ij} \right) + \mathcal{L}_\beta K_{ij} \\
&\quad - \alpha \frac{D - 4}{2\lambda} \left( D_i D_j \lambda - 2K_{ij} K\lambda - \frac{1}{2\lambda} \partial_i \lambda \partial_j \lambda \right) \\
\partial_t \lambda &= -2\alpha K_\lambda + \mathcal{L}_\beta \lambda \\
\partial_t K_\lambda &= -\frac{1}{2} \partial^l \alpha \partial_l \lambda + \alpha \left( (D - 5) + KK_\lambda + \frac{D - 6}{\lambda} K^2_\lambda \right) \\
&\quad - \frac{D - 6}{4\lambda} \partial^l \lambda \partial_l \lambda - \frac{1}{2} D^l D_l \lambda \right) + \mathcal{L}_\beta K_\lambda
\end{align*}
\]
Wave Extraction in $D > 4$

Generalization of Regge-Wheeler-Zerilli formalism by Kodama & Ishibashi '03

Master function

$$\Phi, t = (D - 2)r^{(D-4)/2} \frac{2rF, t - F_r^r}{k^2 - D + 2 + \frac{(D-2)(D-1)}{2} \frac{r_s D^3}{r D^3}}, \quad k = l(l + D - 3)$$

Energy flux & radiated energy

$$\frac{dE_l}{dt} = \frac{(D - 3)k^2(k^2 - D + 2)}{32\pi(D - 2)}(\Phi', t)^2, \quad E = \sum_{l=2}^{\infty} \int_{-\infty}^{\infty} dt \frac{dE_l}{dt}$$

Momentum flux & recoil velocity

$$\frac{dP^i}{dt} = \int_{S_\infty} d\Omega \frac{d^2E}{dtd\Omega} n^i, \quad v_{\text{recoil}} = \left| \int_{-\infty}^{\infty} dt \frac{dP}{dt} \right|$$
Numerical Setup

- use Sperhake's extended Lean code (Sperhake '07, Zilhão et al '10)
  - 3+1 Einstein equations with scalar field
  - Baumgarte-Shapiro-Shibata-Nakamura formulation with moving punctures
  - dynamical variables: $\chi$, $\tilde{\gamma}_{ij}$, $K$, $\tilde{A}_{ij}$, $\tilde{\Gamma}^i$, $\zeta$, $K_\zeta$
- modified puncture gauge

\[
\partial_t \alpha = \beta^k \partial_k \alpha - 2\alpha (K + (D - 4)K_\zeta)
\]
\[
\partial_t \beta^i = \beta^k \partial_k \beta^i - \eta_\beta \beta^i + \eta_i \tilde{\Gamma}^i + \eta_\lambda \frac{D - 4}{2\zeta} \tilde{\gamma}^{ij} \partial_j \zeta
\]

- measure lengths in terms of $r_S$ with

\[
r_S^{D-3} = \frac{16\pi}{(D - 2)A^{S_{D-2}}} M
\]
Equal mass head-on in $D = 4, 5, 6$

Brill-Lindquist type initial data

$$\psi = 1 + r_{S,1}^{D-3}/4r_1^{D-3} + r_{S,2}^{D-3}/4r_2^{D-3}$$

Key (technical) issues:
- modification of gauge conditions
- modification of formulation
- increase in $E/M$ with $D$  
  $\Rightarrow$ qualitative agreement with PP calculations (Berti et al, 2010)

<table>
<thead>
<tr>
<th>$D$</th>
<th>$r_S \omega(l = 2)$</th>
<th>$E/M$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.7473 $- i0.1779$</td>
<td>0.055</td>
</tr>
<tr>
<td>5</td>
<td>0.9477 $- i0.2561$</td>
<td>0.089</td>
</tr>
<tr>
<td>6</td>
<td>1.140 $- i0.304$</td>
<td>0.104</td>
</tr>
</tbody>
</table>
Unequal mass head-on in $D = 5$

- consider mass ratios $q = r_{S,1}^{D-3}/r_{S,2}^{D-3} = 1, 1/2, 1/3, 1/4$

$$E/M \sim \eta^2 \ (\text{M.Lemos '10, MSc thesis})$$

**Modes of $\Phi, t$**

- fitting function $\frac{E}{M \eta^2} = 0.0164 - 0.0336\eta^2$
- within $< 1\%$ agreement with point particle calculation (Berti et al, 2010)
Initial data for boosted BHs in $D > 4$

- construct initial data by solving the constraints
- assumption: $\bar{\gamma}_{ab} = \psi^{\frac{4}{D-3}} \delta_{ab}$, $\bar{K} = 0$, $\bar{K}_{ab} = \psi^{-2} \hat{A}_{ab}$
- constraint equations

$$\partial_a \hat{A}^{ab} = 0, \quad \hat{\Delta} \psi + \frac{D - 3}{4(D - 2)} \psi^{-\frac{3D - 5}{D - 3}} \hat{A}^{ab} \hat{A}_{ab} = 0, \quad \text{with} \quad \hat{\Delta} \equiv \partial_a \partial^a$$

- analytic ansatz for $\hat{A}_{ab} \rightarrow$ generalization of Bowen-York type initial data
- elliptic equation for $\psi \rightarrow$ puncture method (Brandt & Brügmann '97)

$$\psi = 1 + \sum_i r_{S(i)}^{D-3} / 4r_{(i)}^{D-3} + u$$

$\Rightarrow$ Hamiltonian constraint becomes

$$\hat{\Delta} u + \frac{D - 3}{4(D - 2)} \hat{A}^{ab} \hat{A}_{ab} \psi^{-\frac{3D - 5}{D - 3}} = 0$$
Head-on of boosted BHs (preliminary results)

- evolution of puncture with $z/r_S = \pm 30.185$ with $P/r_S^{D-3} = 0.4$

$l = 2$ mode of $\Phi, t$

PP calculations (Berti et al, 2010)

Issues:

- dependence of radiated energy on $D$ and boost
- long-term stable evolutions for larger boosts
- adjustment of (numerical) gauge
- requirement of very high resolution in wavezone for reasonable accuracy
Black hole binaries in a box
AdS / CFT correspondence (Maldacena '97)

- Anti-de Sitter spacetime
  ⇒ spacetime with negative cosmological constant
- duality between
  theory with gravity on $\text{AdS}_d \times X$
  \[\updownarrow\]
  conformal field theory on conformal boundary

- consider scalar field propagation in SAdS background
  \[\frac{d^2}{dr^2_r} \psi + (\omega^2 - V)\psi = 0\]
  potential $V \to \infty$ as $r \to \infty$
  ⇒ view AdS as boxed spacetime

⇒ toy model: BH evolution in a box
Black Hole stability

- superradiant scattering of rotating BH (Penrose '69, Christodoulou '70, Misner '72)
  - impinging wave amplified as it scatters off a BH if $\omega < m\Omega_H$
  - extraction of energy and angular momentum of the BH by superradiant modes
- “black hole bomb” (Press & Teukolsky '72)
  - consider Kerr BH surrounded by a mirror
  - subsequent amplification of superradiant modes
- stability of Kerr-AdS BHs (Hawking & Reall '99, Cardoso et al. '04)
  - AdS infinity behaves as box $\Rightarrow$ amplification of superradiant instabilities?
  - large Kerr-AdS BH: stable
  - small Kerr-AdS BH: unstable
mimic AdS-BH spacetime by BHs in a box

impose reflecting b.c. via
\[ \partial_t K_{ij} = 0 \] and \[ \partial_t \gamma_{ij} = 0 \] at boundary

wave extraction: Newman Penrose scalars \( \Psi_4 \) (outgoing) and \( \Psi_0 \) (ingoing)

initial data: non-spinning, equal mass BHs

head-on collision \( \Rightarrow \) non-spinning final BH quasi-circular inspiral \( \Rightarrow \) spinning final BH with \( a/M = 0.69 \)

Movie
- spectrum of $\Psi_{22}^4$ after merger and before interaction with boundary
- assume quasi Kerr BH
  - critical frequency for superradiance $M\omega_C = m\Omega = 0.4$
- signal contains frequencies within and above superradiance regime
BBHs in a Box - Convergence Test

\[ \Re(\psi^4_{22}) \]

\[ \Re(\psi^0_{22}) \]

- simulations at 3 different resolutions:
  \[ h_c = 1/48M, \quad h_m = 1/52M, \quad h_f = 1/56M \]
- 4\(^{th}\) order accurate in merger signal
- 2\(^{nd}\) order convergence in 1\(^{st}\) & 2\(^{nd}\) reflection
- loosing convergence afterwards \(\Rightarrow\) consider first 3 cycles
successive increase in horizon area, mass and angular momentum during interaction between BH and gravitational radiation

absorption of $\sim 15\%$ of GW energy per cycle

increase of $\sim 5\%$ in angular momentum in *first* cycle

*no* indication of superradiant amplification
BBHs in a Box

- IDEA:
  - complementary phenomena
    - high frequency modes:
      absorption of mass and angular momentum from radiation
    - low frequency modes:
      amplification of superradiant modes
      with $\omega < m\Omega_H$
  - frequencies in superradiant and absorption regime
    $\Rightarrow$ at transition point between stable and superradiant regime?

ToDo:
- improvement of bcs $\Rightarrow$ long-term evolution
- model highly spinning BHs

Press & Teukolsky '74
Conclusions I

Evolutions of BH head-on collisions in $D = 5, 6$ dimensions

- good agreement between PP and numerical results for unequal mass binaries
- collisions from rest: increase in radiated energy with increasing dimension
- setup of initial data for boosted BH solving the constraints
- evolutions of collisions with small boost parameter

Issues & ToDo list:

- dependence of radiated energy on $D$ and boost
- go beyond $D = 6$
- long-term stable evolutions for large boosts
- adjustment of gauge conditions
- modification of formulation
Conclusions II

mimic AdS-BH spacetimes by BHs in a box

- “BH bomb” like setup
- monitor interaction between Kerr BH and gravitational radiation
- evidence for absorption of radiation by the BH

Issues & ToDo list:

- no clear evidence for superradiant amplification
- systems becomes numerically unstable after few reflections
- improvement of boundary conditions
  ⇒ long-term modelling of system
- increasing amplification rate with increasing spin
  ⇒ evolve highly spinning BHs
Arigato!

http://blackholes.ist.utl.pt