Simulations of a BH-axion system

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arXiv:1203.5070[gr-qc]

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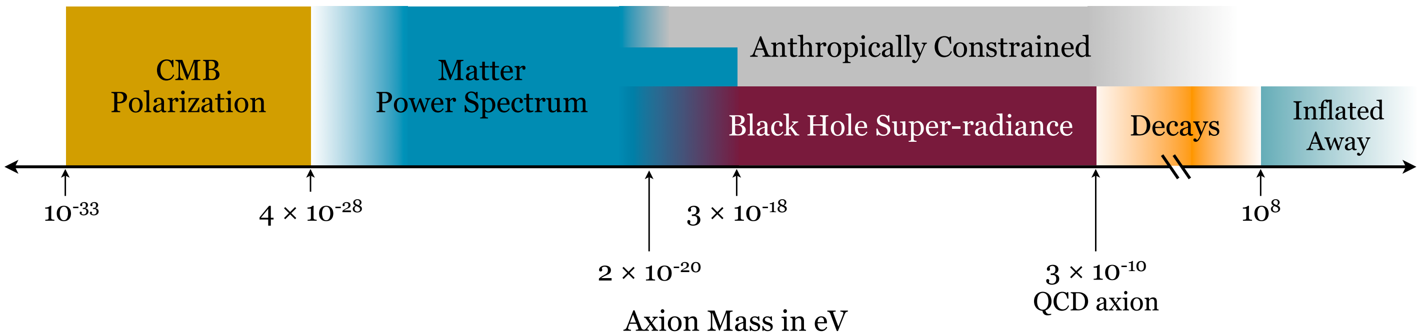
Axions

- Massive scalar particles (predicted theoretically).
  - QCD axion
    - QCD axion was introduced to solve the Strong CP problem.
    - It is one of the candidates of dark matter.
  - String axions

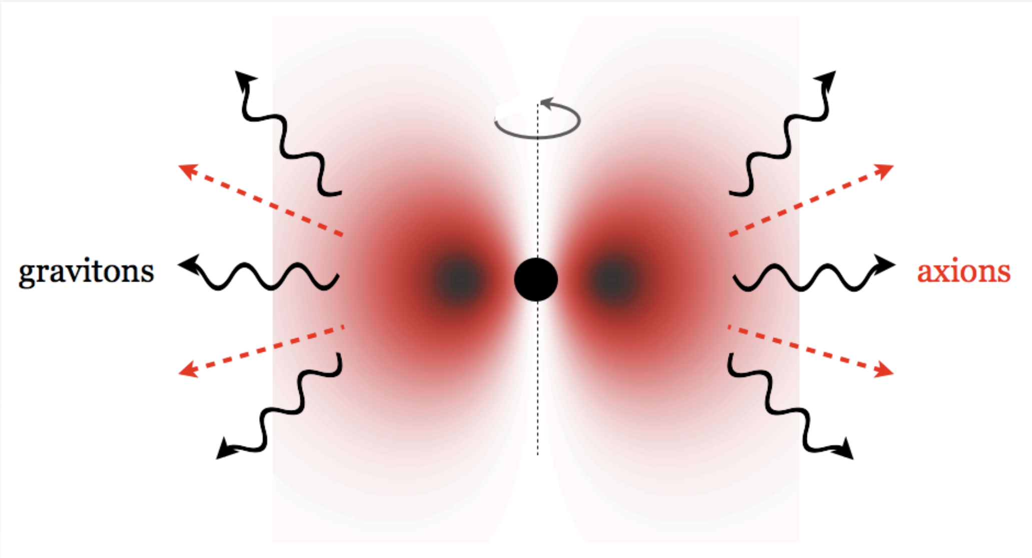
    Arvanitaki, Dimopoulos, Dubvosky, Kaloper, March-Russel, PRD81 (2010), 123530.

    - String theory predicts the existence of 10-100 axion-like massive scalar fields.
    - There are various expected phenomena of string axions.



Axion field around a rotating black hole

- Axion field makes a bound state and causes the superradiant instability

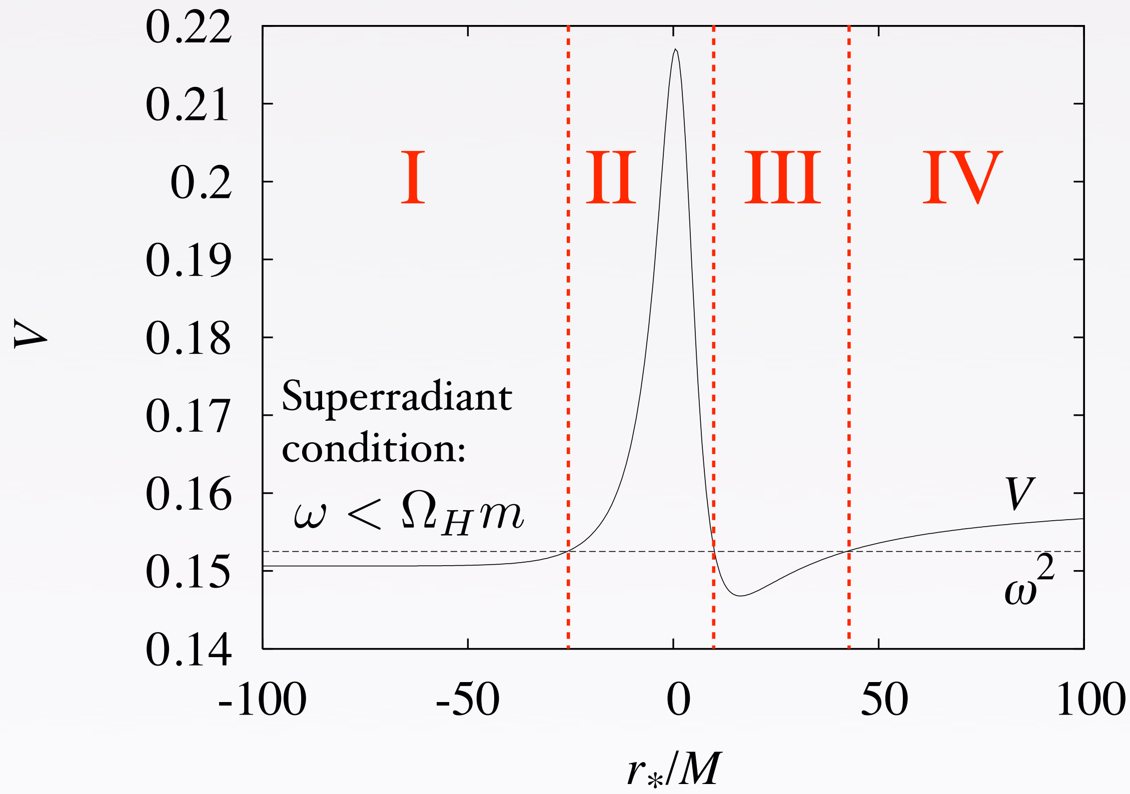


Detweiler, PRD22 (1980), 2323.

Zouros and Eardley, Ann. Phys. 118 (1979), 139.

Bound state

Zouros and Eardley, Ann. Phys. 118 (1979), 139.



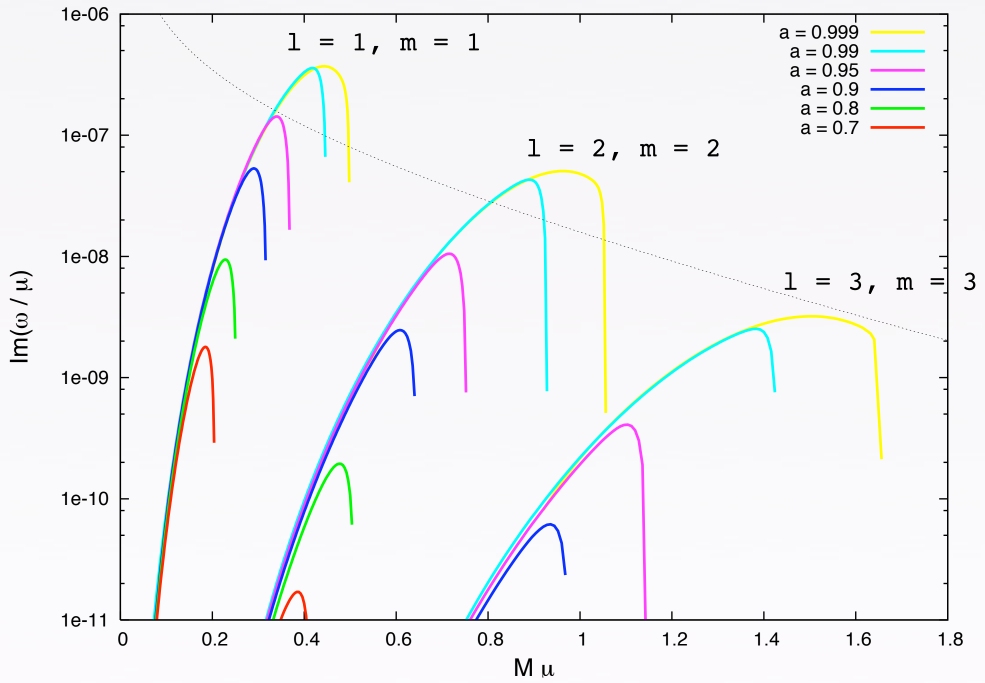
$$\Phi = \mathrm{Re}[e^{-i\omega t} R(r) S(\theta) e^{im\phi}]$$

$$R = \frac{u}{\sqrt{r^2 + a^2}} \quad \Rightarrow \quad \frac{d^2 u}{dr_*^2} + \left[\omega^2 - V(\omega)\right] u = 0$$

Growth rate

- Growth rate calculated by continued fraction method

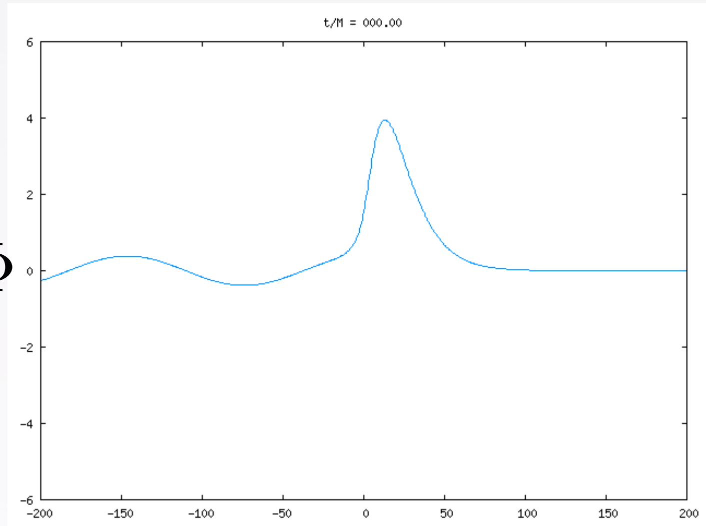
Dolan, PRD76 (2007), 084001.



- Time evolution

$$l = m = 1 \qquad M\mu = 0.4$$

Φ

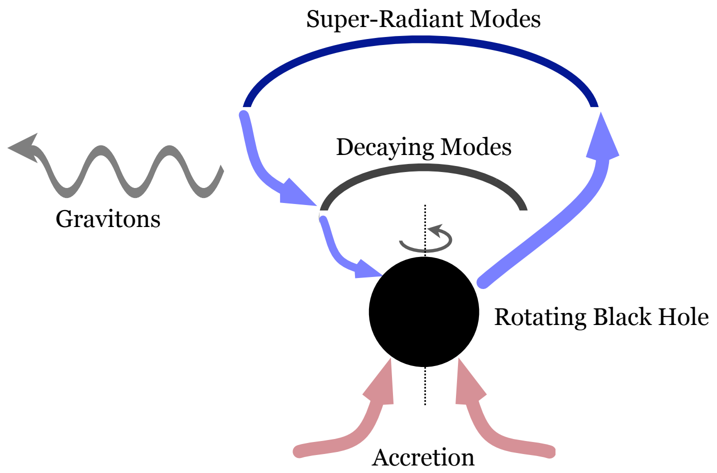


r_*

(Near the horizon)

$$\Phi \sim e^{-i\omega t} e^{-i\tilde{\omega} r_*}$$
$$\tilde{\omega} = \omega - m\Omega_H$$

BH-axion system



Arvanitaki and Dubovsky, PRD83 (2011), 044026.

- Superradiant instability
  - Emission of gravitational waves
  - Pair annihilation of axions
- Effects of nonlinear self-interaction
  - Bosenova
  - Mode mixing

Nonlinear effect

- Typically, the potential of axion field becomes periodic

$$V = f_a^2 \mu^2 [1 - \cos(\Phi / f_a)]$$

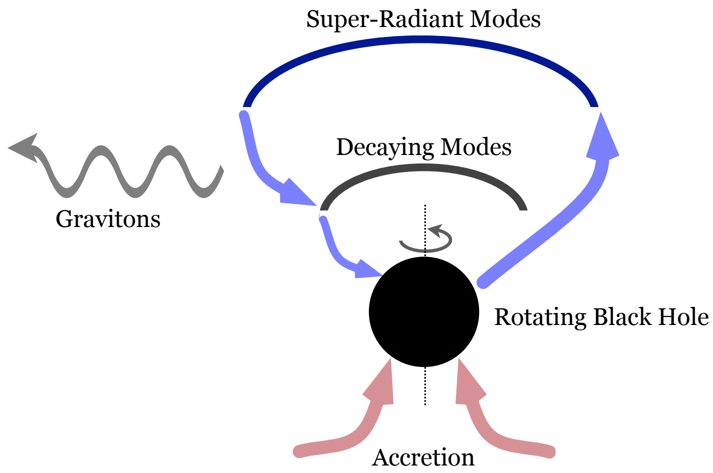
$$\Rightarrow \quad \boxed{\nabla^2 \varphi - \mu^2 \sin \varphi = 0} \qquad \varphi \equiv \frac{\Phi}{f_a}$$

- c.f., QCD axion

PQ phase transition　　　　QCD phase transition

U(1)PQ symmetry ⇒ potential becomes like a wine bottle ⇒ Z(N) symmetry

BH-axion system



Arvanitaki and Dubovsky, PRD83 (2011), 044026.

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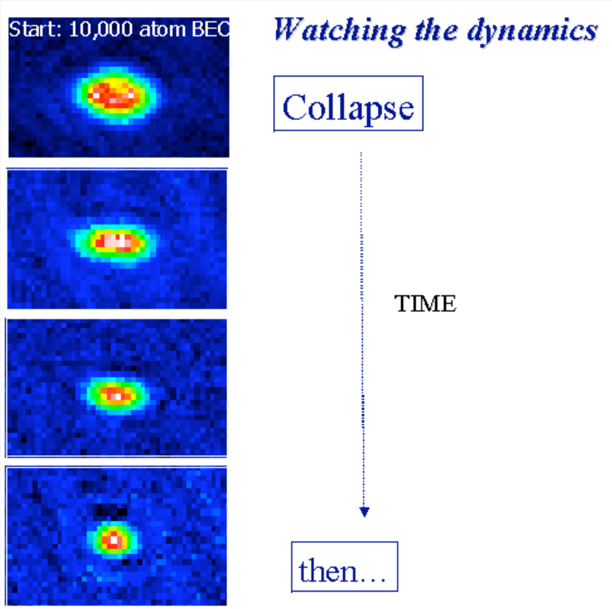
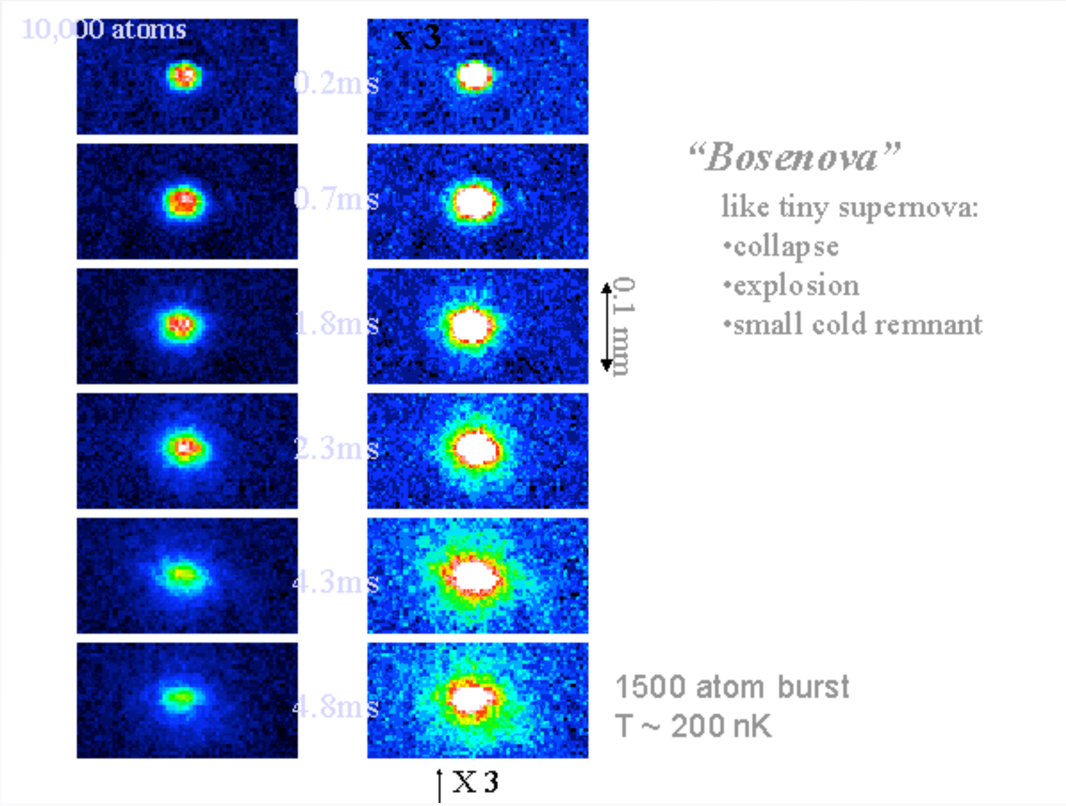
Bosenova in condensed matter physics

http://spot.colorado.edu/~cwieman/Bosenova.html





BEC state of Rb85 (interaction can be controlled)

Switch from repulsive interaction to attractive interaction

Wieman et al., Nature 412 (2001), 295

What we would like to do

- We would like to study the phenomena caused by axion cloud generated by the superradiant instability around a rotating black hole.

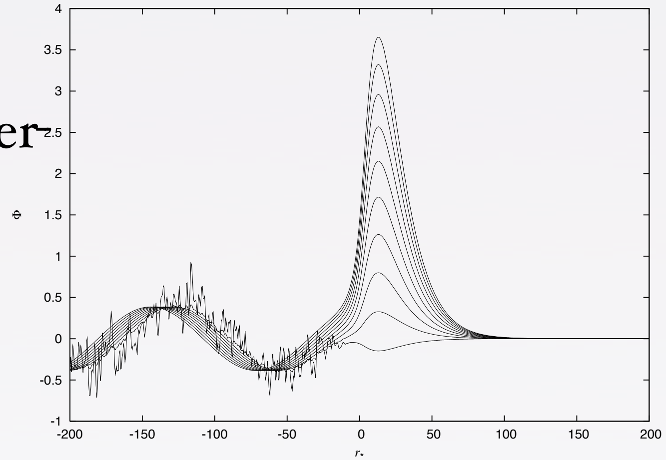
- In particular, we study numerically whether “Bosenova” happens when the nonlinear interaction becomes important.

- We adopt the background spacetime as the Kerr spacetime, and solve the axion field as a test field.

Contents

First difficulty

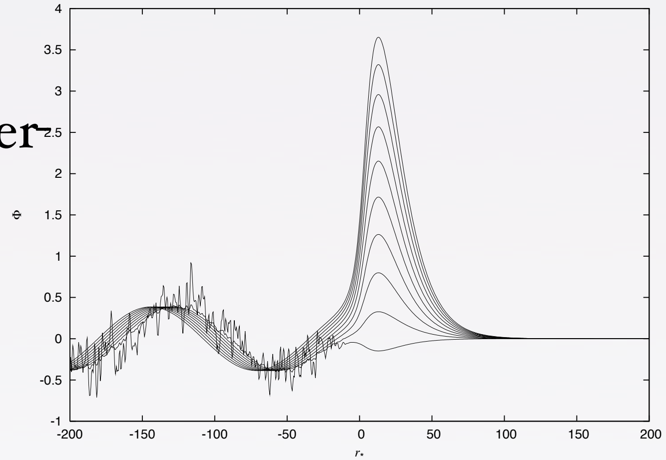
- Stable simulation cannot be realized in Boyer-Lindquist coordinates.

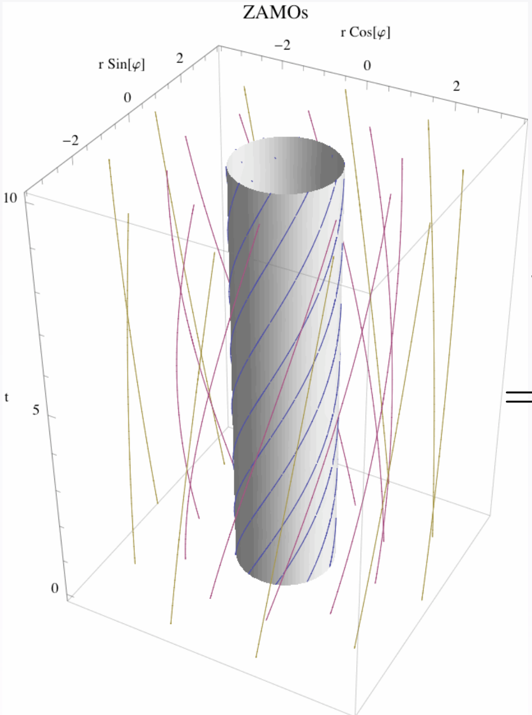


First difficulty

- Stable simulation cannot be realized in Boyer-Lindquist coordinates.

➡ We use ZAMO coordinates.

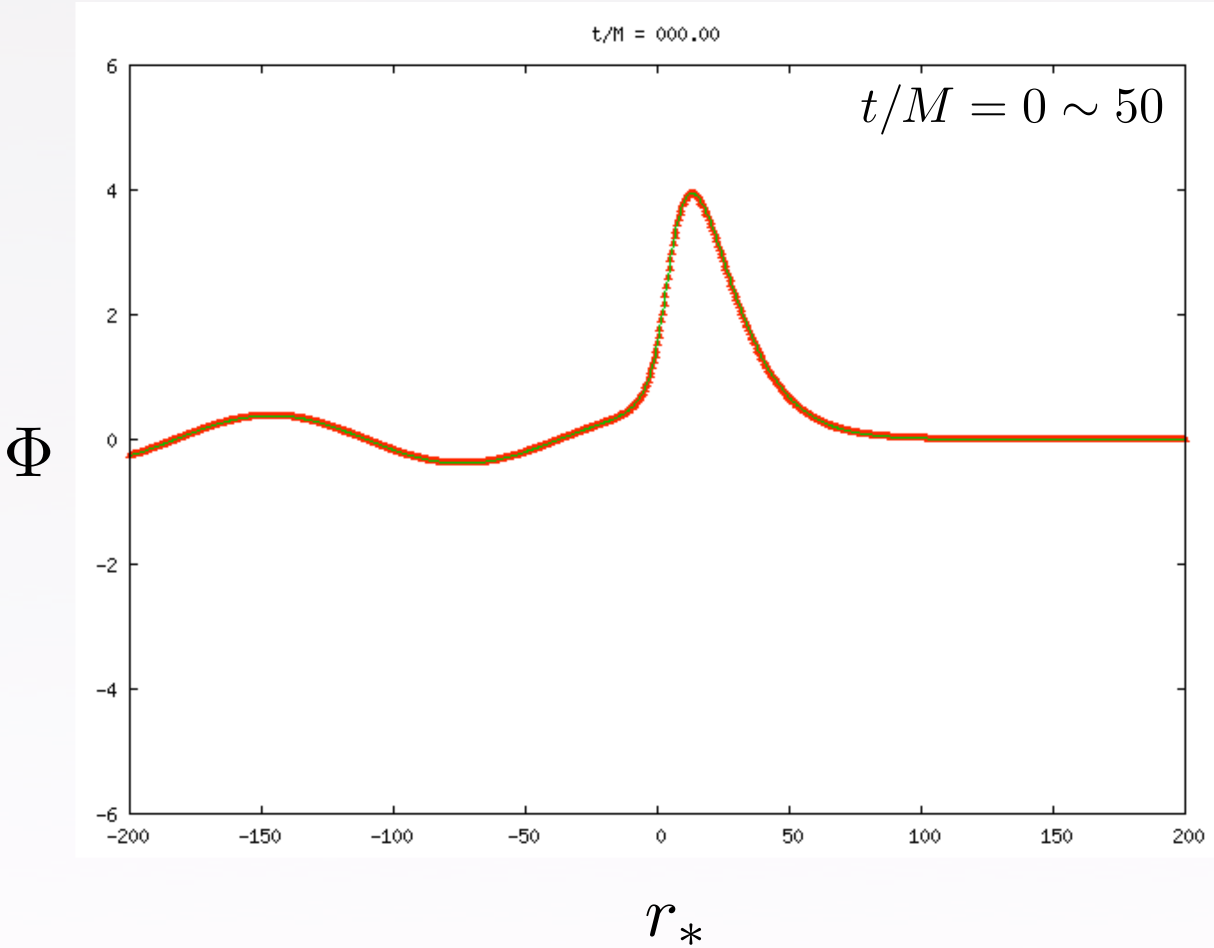




$$\Omega = \frac{d\phi}{dt} = \frac{u^{\phi}}{u^{t}} = -\frac{g_{t\phi}}{g_{\phi\phi}}$$
$$= \frac{2Mar}{(r^2+a^2)^2 - \Delta a^2 \sin^2\theta}$$

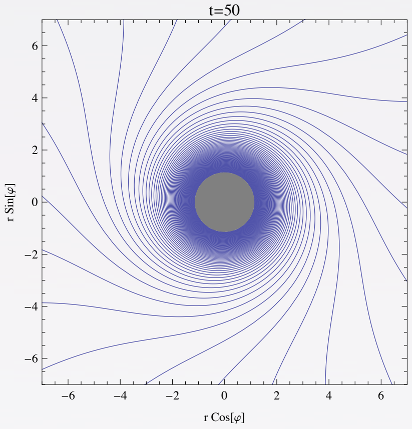
$$\begin{aligned}\tilde{t} &= t,\\ \tilde{\phi} &= \phi - \Omega(r,\theta)t,\\ \tilde{r} &= r,\\ \tilde{\theta} &= \theta,\end{aligned}$$

Numerical solution in the ZAMO coordinates

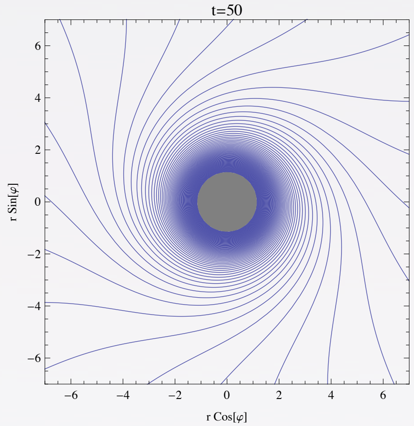


Second difficulty

- ZAMO coordinates become more and more distorted in the time evolution



Second difficulty

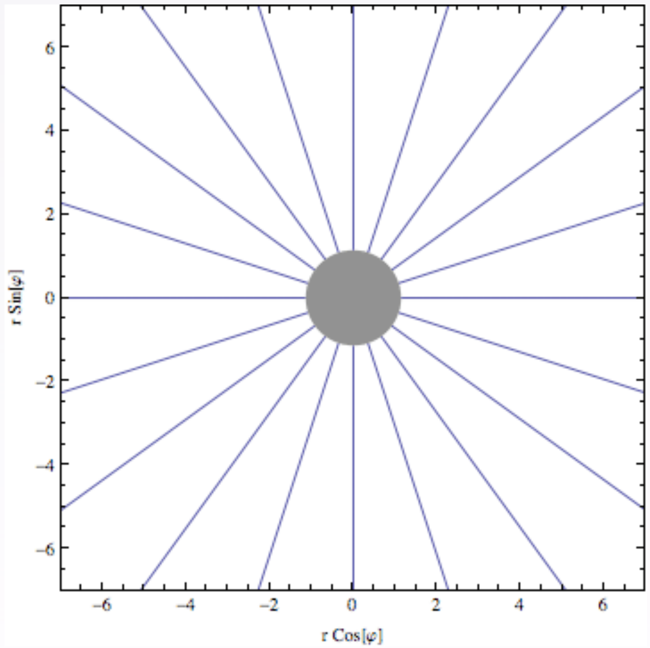


- ZAMO coordinates become more and more distorted in the time evolution

➡ We "pull back" the coordinates

$$nT_P \leq t \leq (n+1)T_P :$$

$$\begin{aligned} t^{(n)} &= t, \\ \phi^{(n)} &= \phi - \Omega(r,\theta)(t - nT_P), \\ r^{(n)} &= r, \\ \theta^{(n)} &= \theta. \end{aligned}$$



Our 3D code

- Space direction : 6th-order finite discretization

- Time direction : 4th-order Runge-Kutta

- Grid size: $$\Delta r_* = 0.5 \quad (M = 1)$$ $$\Delta\theta = \Delta\phi = \pi/30$$

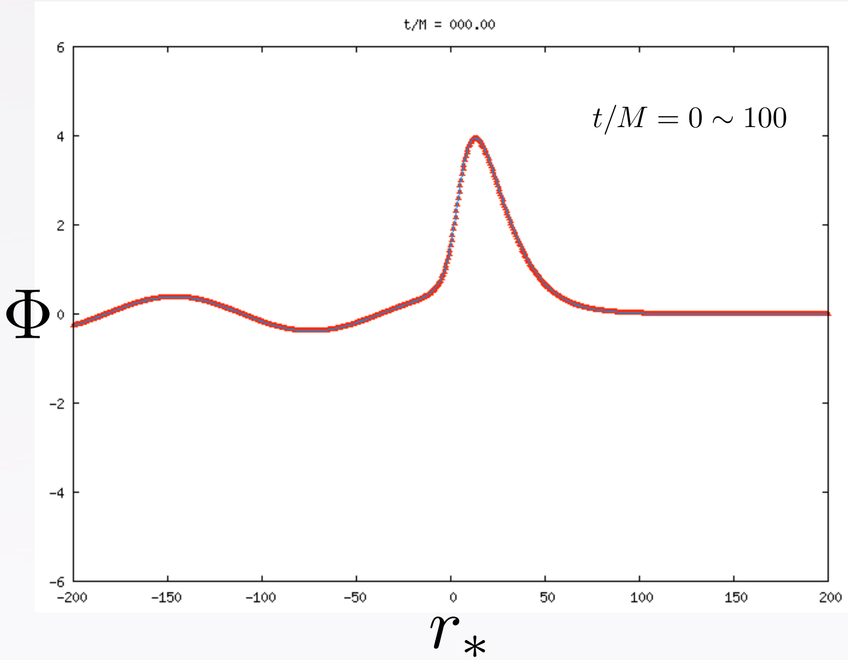
- Courant number: $$C = \frac{\Delta t}{\Delta r_*} = \frac{1}{20}$$

- Pure ingoing BC at the inner boundary,
  Fixed BC at the outer boundary

- Pullback: 7th-order Lagrange interpolation

Code check (I)

- Comparison with semianalytic solution of the Klein-Gordon case



- Growth rate

$$\omega_I = \frac{\dot{E}}{2E} \simeq \frac{E(100M) - E(0)}{200ME(0)}$$

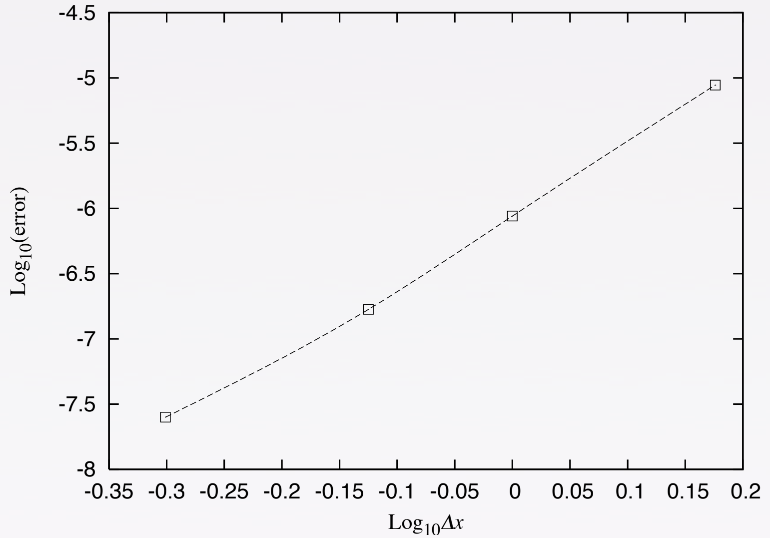
$$\omega_I^{(\mathrm{CF})}/\mu = 3.31 \times 10^{-7}$$

$$\omega_I^{(\mathrm{Numerical})}/\mu = 3.26 \times 10^{-7}$$

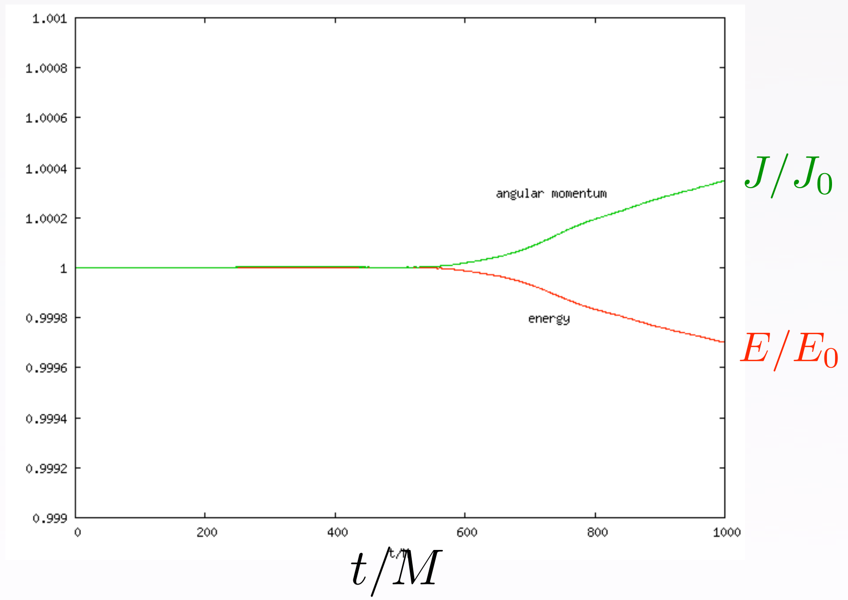
Code check (2)

- Convergence

$(t = 12.5M)$



- Conserved quantities



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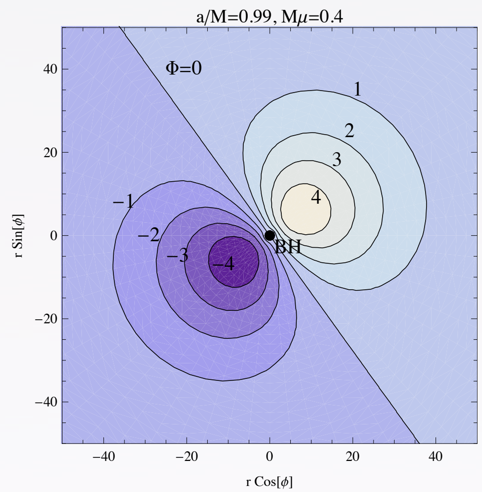
Numerical simulation

- Sine-Gordon equation

$$\nabla^2 \varphi - \mu^2 \sin \varphi = 0$$

- Setup $\quad a/M = 0.99, \quad M\mu = 0.4$

As the initial condition, we choose the bound state of the Klein-Gordon field of the $l = m = 1$ mode.

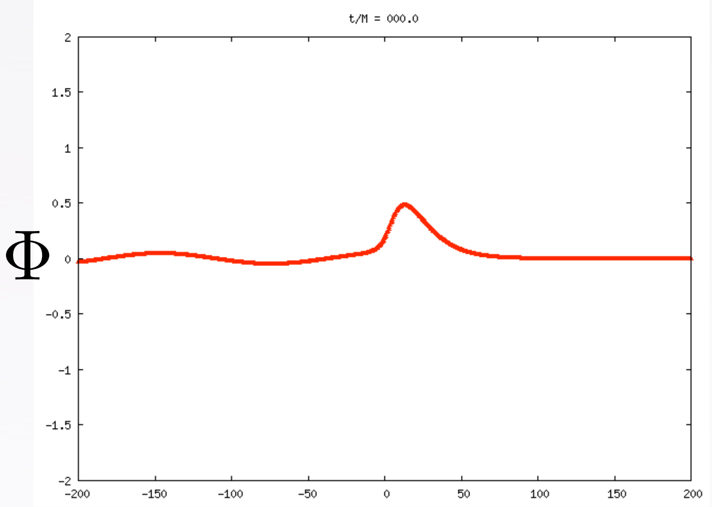


	Initial peak value	$E/[(f_a/M_p)^2 M]$
(A)	0.6	1370
(B)	0.7	1862

Simulation (A) $\varphi_{\text{peak}}(0) = 0.6$

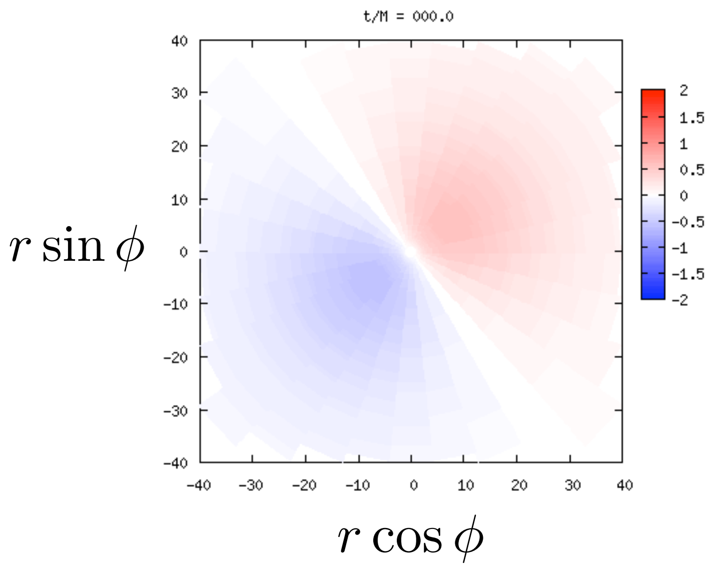
- Axion field on the equatorial plane $(\theta = \pi/2)$

$(\phi = 0)$



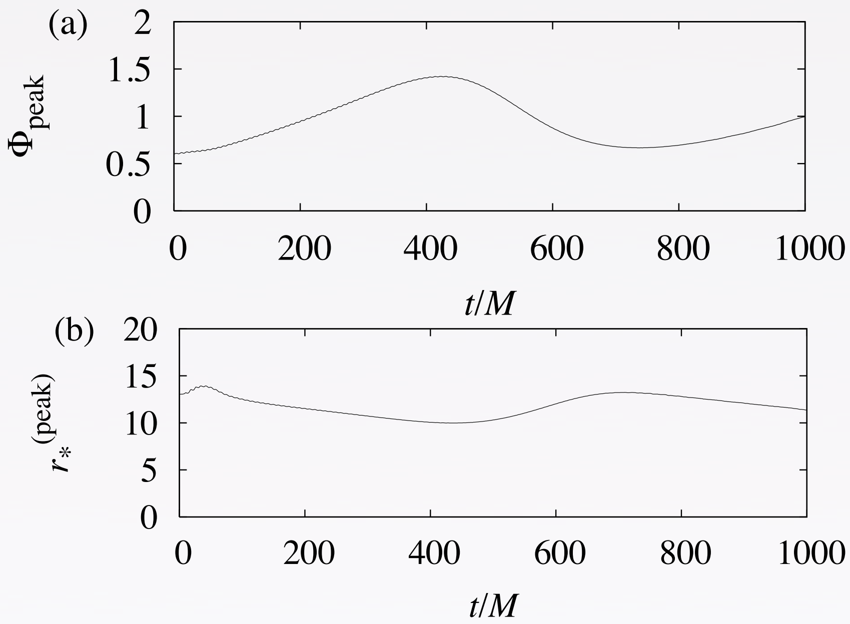
$-200 \leq r_*/M \leq 200$

density plot

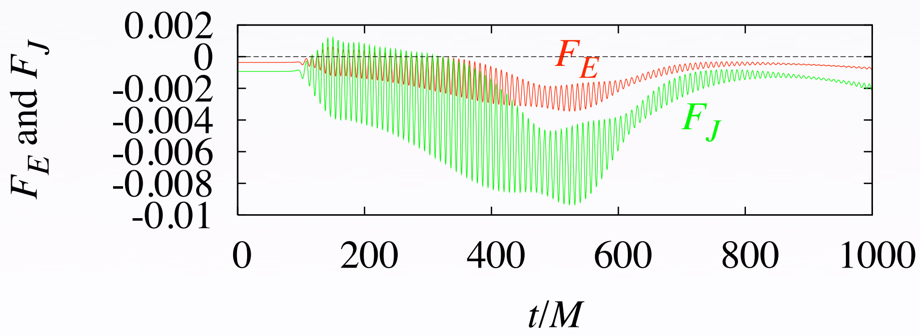


Simulation (A)

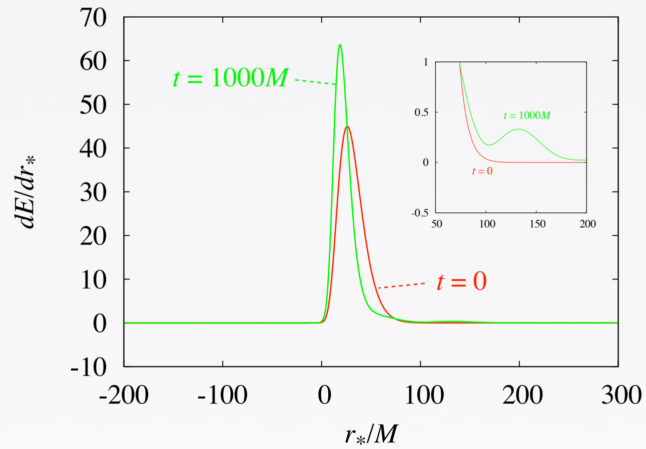
- Peak value and peak location

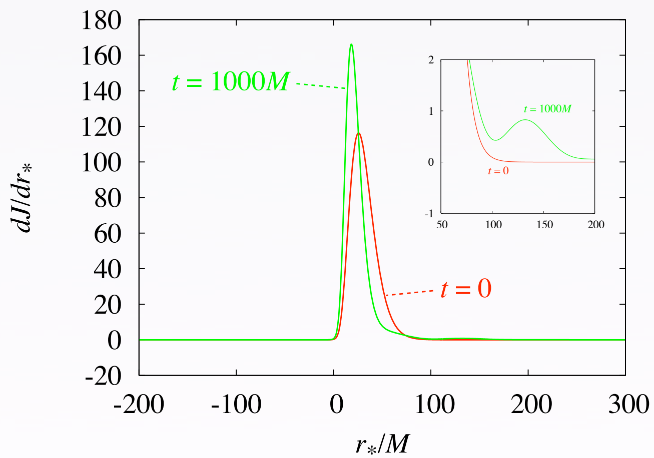


- Fluxes toward the horizon



- Energy and angular momentum distribution



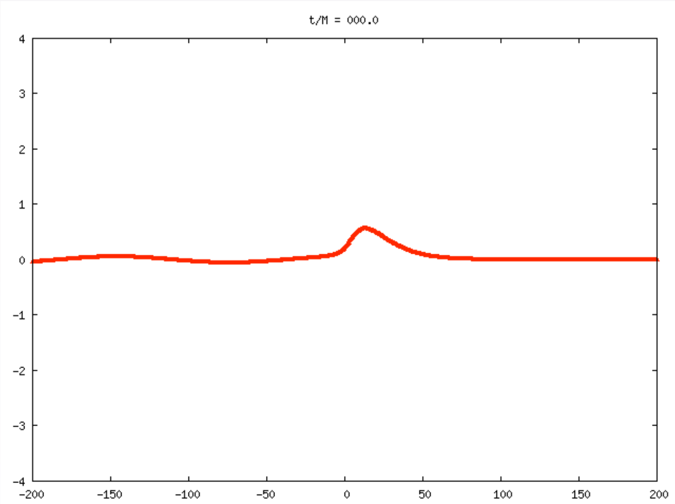


Simulation (B) $\varphi_{\rm peak}(0) = 0.7$

- Axion field on the equatorial plane $(\theta = \pi/2)$

$(\phi = 0)$

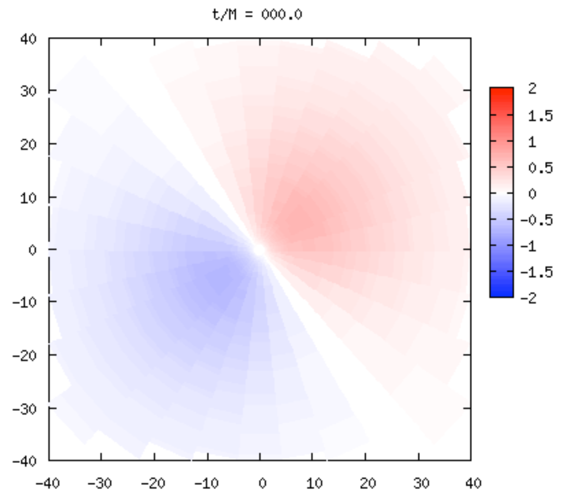
Φ



$-200 \le r_*/M \le 200$

density plot

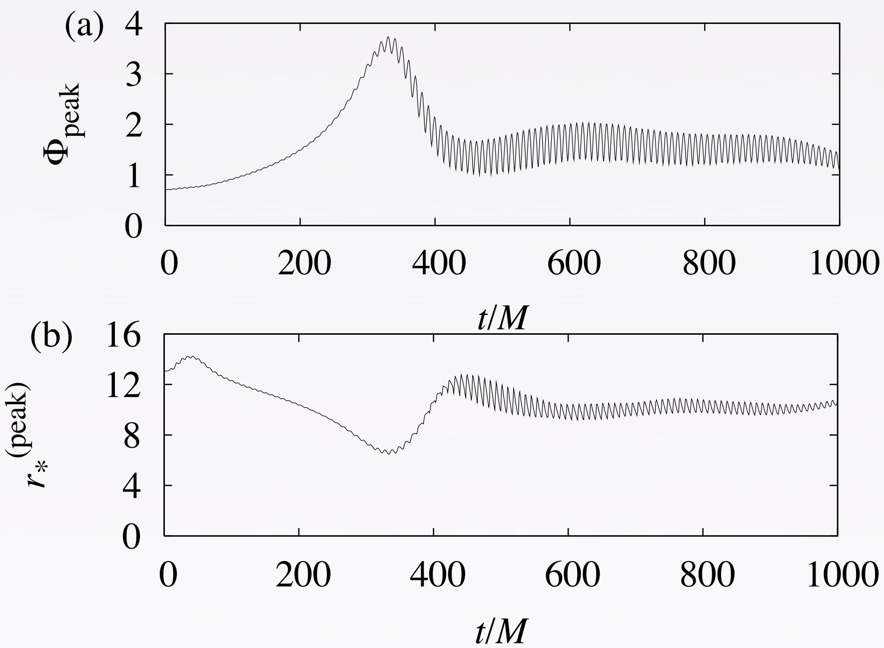
$r \sin\phi$



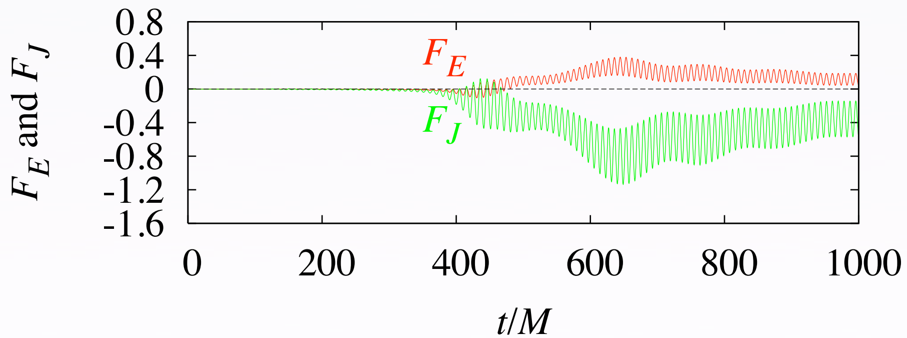
$r \cos\phi$

Simulation (B)

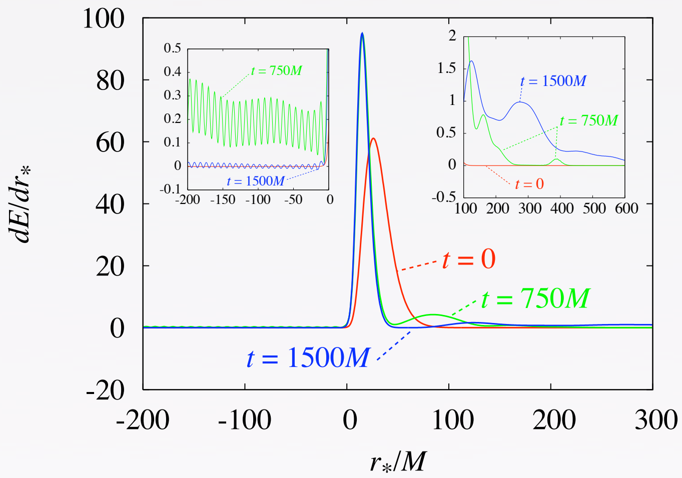
- Peak value and peak location

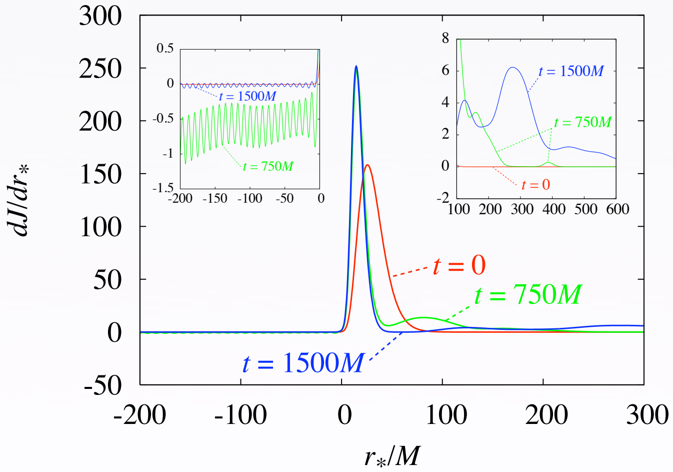


- Fluxes toward the horizon



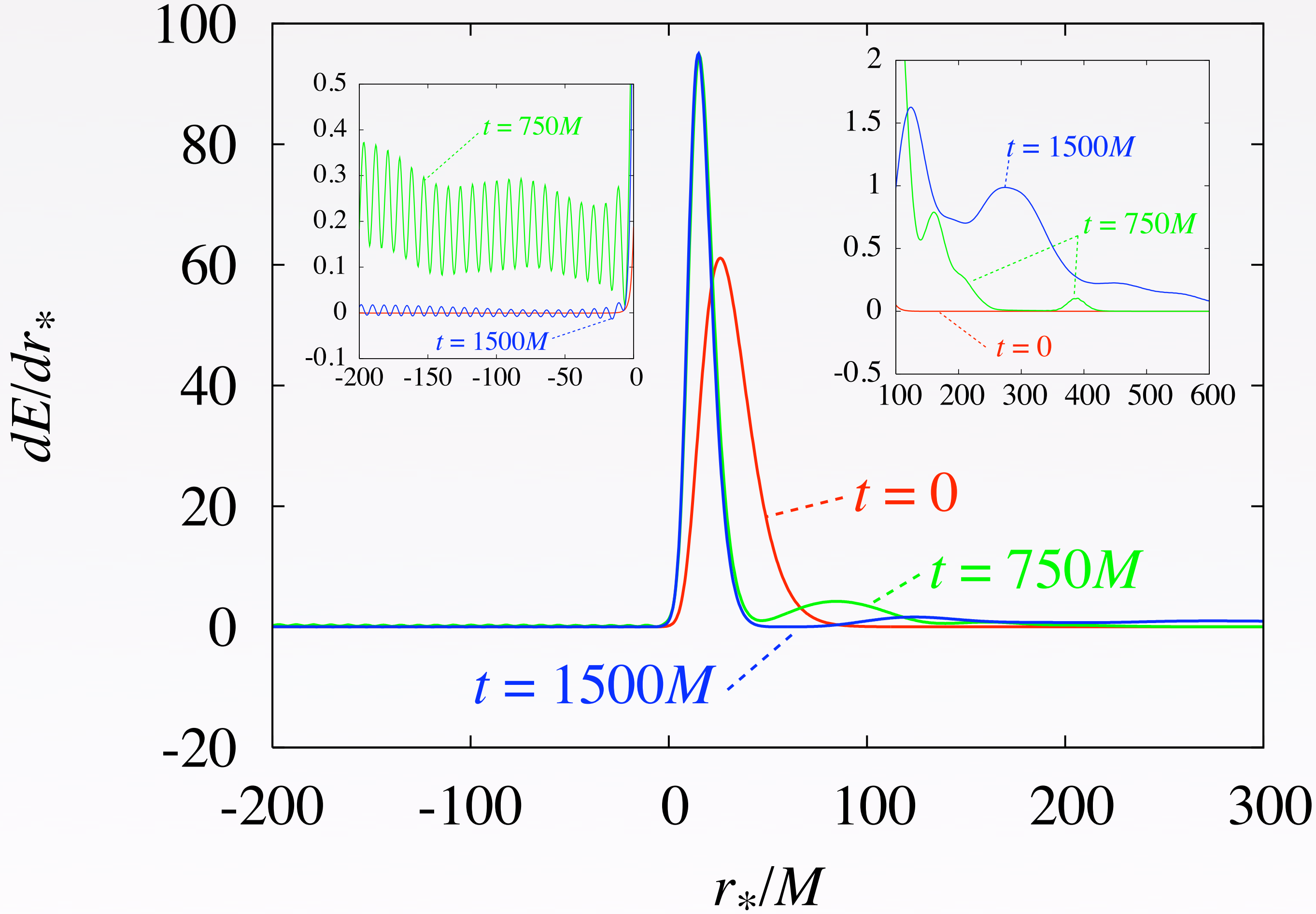
- Energy and angular momentum distribution





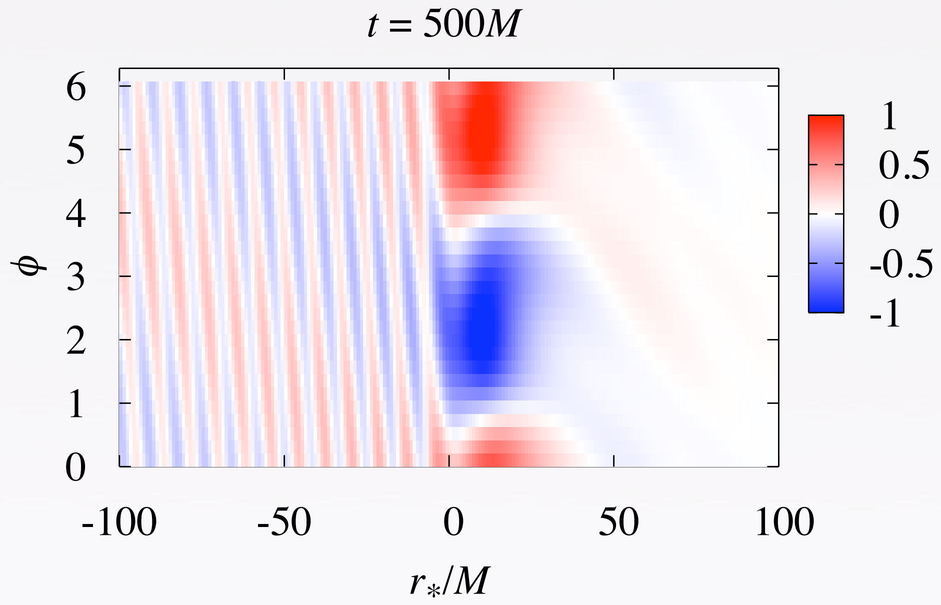
Simulation (B)

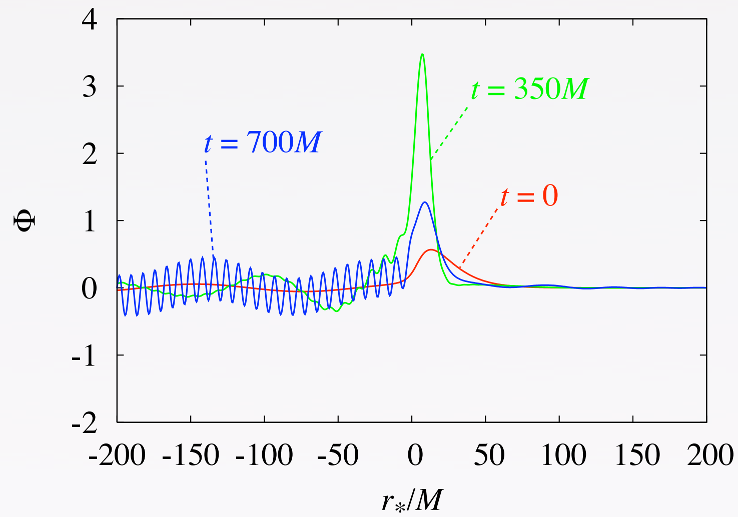
- Energy distribution



Simulation (B)

- Snapshots





m=-1 mode is generated!

(Near the horizon) $$\Phi \sim e^{-i\omega t} e^{-i\tilde{\omega} r_*}$$
$$\tilde{\omega} = \omega - m\Omega_H$$

$M\omega_{\rm KG} = 0.39$	$M\omega_{\rm NL} = 0.35$
$M\tilde{\omega}_{\rm KG} = -0.04$	$M\tilde{\omega}_{\rm NL} = 0.87$

Summary of the simulations (A) and (B)

- (A)
  - When the peak value is not very large, the nonlinear term enhances the rate of superradiant instability.
  - The nonlinear effect makes energy distribute in the neighborhood of the black hole.
- (B)
  - When the peak value is sufficiently large, the bosenova collapse happens.
  - Once the bosenova happens, positive energy falls into the black hole, while the angular momentum continues to be extracted.

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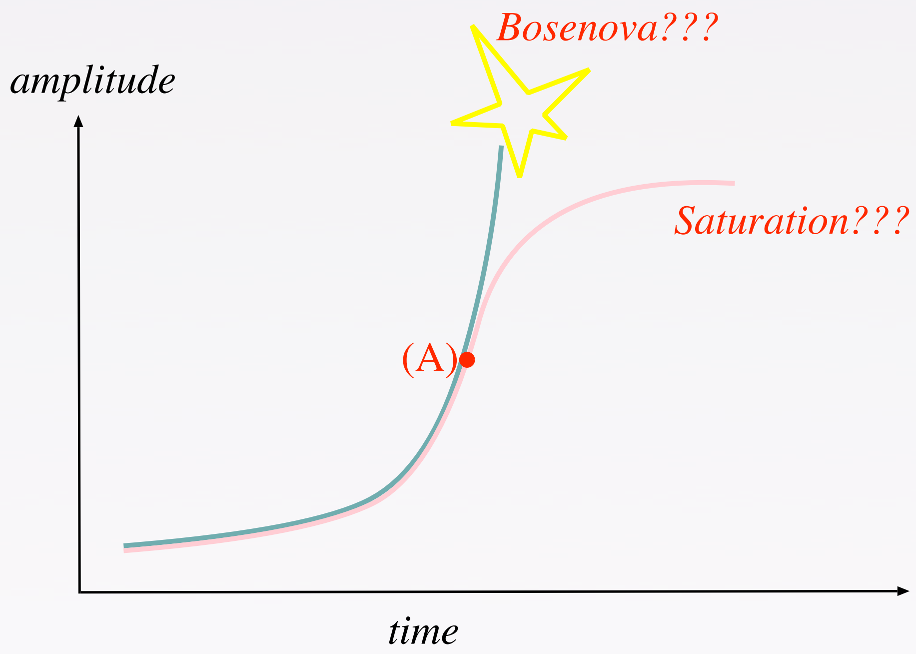
Does bosenova really happen?



- Additional simulation:

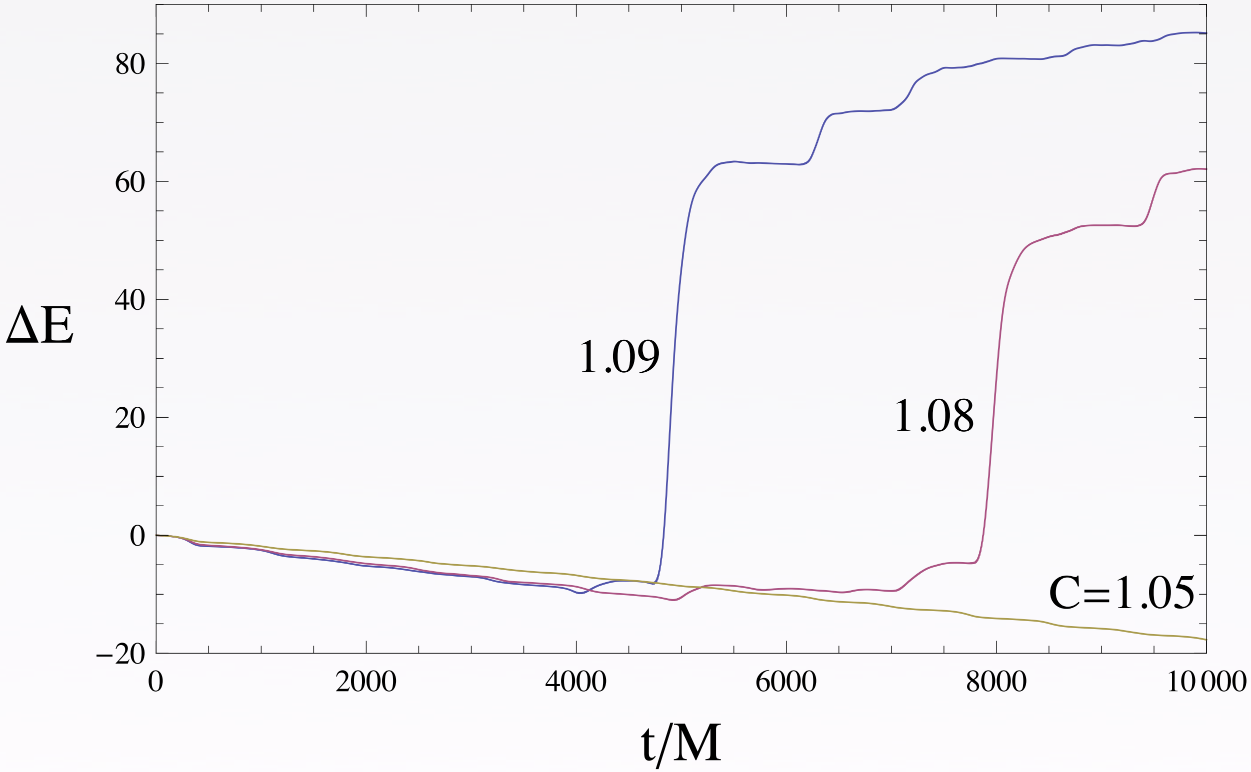
$$\varphi(0) = C\varphi^{(\mathrm{A})}(1000M)$$
$$\dot{\varphi}(0) = C\dot{\varphi}^{(\mathrm{A})}(1000M)$$

$$C = \begin{cases} 1.05 \\ 1.08 \\ 1.09 \end{cases}$$

Supplementary simulation

$$\varphi(0) = C\varphi^{(\mathrm{A})}(1000M)$$
$$\dot{\varphi}(0) = C\dot{\varphi}^{(\mathrm{A})}(1000M)$$

- Energy absorbed by the black hole $\Delta E := \int_0^t F_E dt$ $\quad C = \begin{cases} 1.05 \\ 1.08 \\ 1.09 \end{cases}$



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Green's function approach (I)

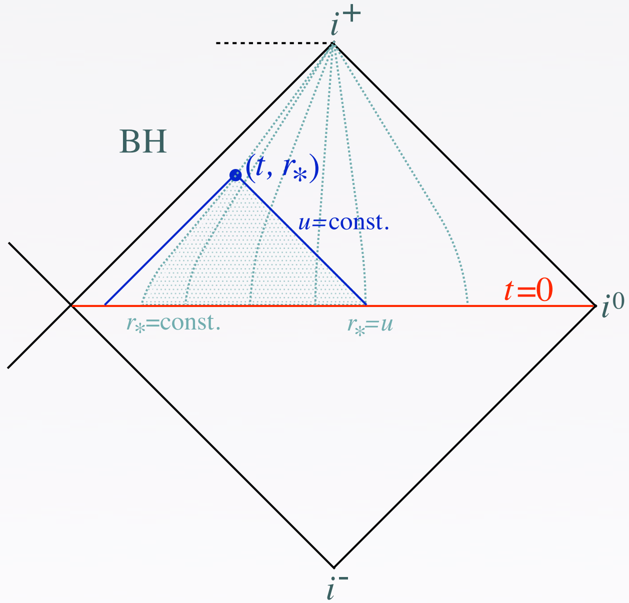
- Approximation

$$\varphi(x) = \varphi_0(x) + \Delta\varphi, \qquad \varphi_0 = 2\mathrm{Re}\left[e^{(\gamma - i\omega_0)t} P(r) S_1^1(\cos\theta) e^{i\phi}\right],$$

$O(\varphi_0^4)$ is ignored

- Equation

$$(\nabla^2 - \mu^2)\Delta\varphi = J(\varphi_0) := -\frac{\mu^2}{6}\varphi_0^3$$



- Green's function

$$(\nabla^2 - \mu^2)_{x'} G(x, x') = \delta^4(x, x')$$

- Formal solution

$$\Delta\varphi(x) = \int_{D'} d^4x' \sqrt{g(x')}\, G(x, x') J(\varphi_0(x'))$$

Green's function approach (2)

- Constructing the Green's function

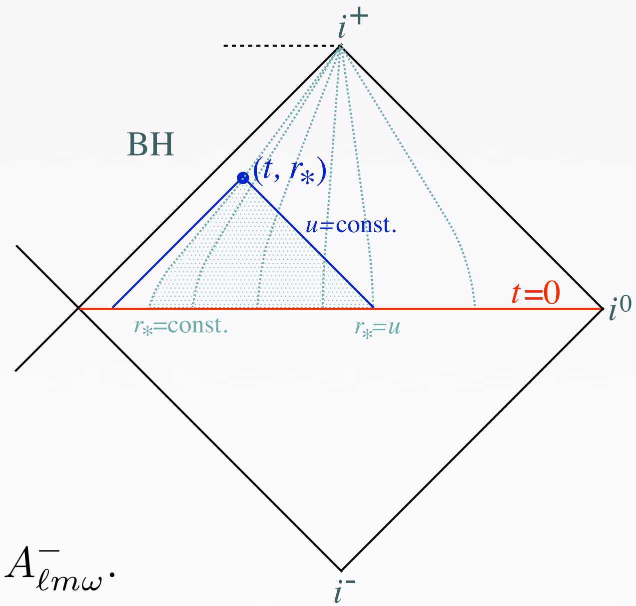
$$G(x,x') = \frac{1}{(2\pi)^2}\sum_{\ell,m}\int_{-\infty}^{\infty} d\omega G^{\omega}_{\ell m}(r,r') e^{-i\omega(t-t')+im(\phi-\phi')} S^m_\ell(\cos\theta)\bar{S}^m_\ell(\cos\theta'),$$

$$G^{\omega}_{\ell m}(r,r') = \frac{1}{W_{\ell m\omega}}\left[\theta(r-r')R^+_{\ell m\omega}(r)R^-_{\ell m\omega}(r') + \theta(r'-r)R^-_{\ell m\omega}(r)R^+_{\ell m\omega}(r')\right],$$

- Radial function $k = \sqrt{\omega^2-\mu^2}, \mathrm{Im}[k] \geq 0$

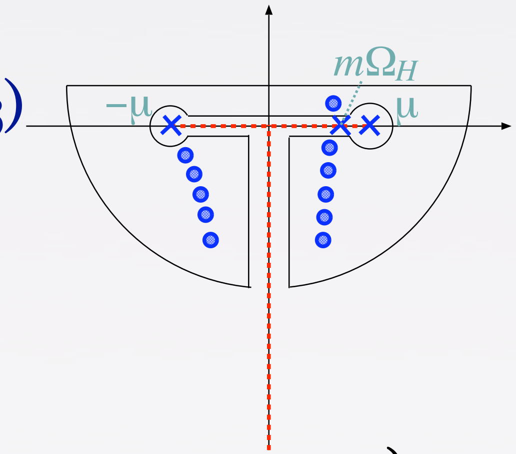
$$R^+_{\ell m\omega} \simeq \begin{cases} C^+_{\ell m\omega} e^{ikr}/r, & r\to\infty; \\ A^+_{\ell m\omega} e^{i\tilde{\omega} r_*} + B^+_{\ell m\omega} e^{-i\tilde{\omega} r_*}, & r \simeq r_+, \end{cases}$$

$$R^-_{\ell m\omega} \simeq \begin{cases} A^-_{\ell m\omega} e^{-ikr}/r + B^-_{\ell m\omega} e^{ikr}/r, & r\to\infty; \\ C^-_{\ell m\omega} e^{-i\tilde{\omega} r_*}, & r \simeq r_+, \end{cases}$$



$$W(R^-,R^+) = 2i\tilde{\omega}(r_+^2+a^2)C^-_{\ell m\omega}A^+_{\ell m\omega} = 2ikC^+_{\ell m\omega}A^-_{\ell m\omega}.$$

Green's function approach (3)



- Near-horizon solution

$$\Delta\varphi = \sum_{\ell,m} e^{im\phi} S_\ell^m(\cos\theta) \frac{e^{im\Omega_H r_*}}{2i(r_+^2 + a^2)}$$
$$\times \left\{ e^{(3\gamma - im\omega_0)u} D_{\ell m}(u, r_*) - \int_{-\infty}^{\infty} d\omega \frac{e^{-i\omega u}}{\tilde{\omega} A_{\ell m}^{+(\omega)} [3\gamma + i(\omega - m\omega_0)]} E_{\ell m}^{(\omega)}(u, r_*) \right\},$$

  - First term $\quad \sim e^{-i(m\omega_0 + 3i\gamma)t}$
  - Second term
    - Pole $\quad \omega = m\omega_0 + 3i\gamma \quad \Rightarrow \quad \sim e^{-i(m\omega_0+3i\gamma)t}$
    - Pole $\quad A_{\ell m}^{+(\omega_{\rm BS}^{(\ell m n)})} = 0 \quad \Rightarrow \quad \sim \sum_n (\cdots) e^{-i\omega_{\rm BS}^{(\ell m n)} t}$

$$\omega_{\rm BS}^{(n)} \simeq \pm\mu \simeq \pm\omega_0$$

Nonlinear term makes transfer from growing bound state to decaying bound state with negative frequency.

Contents

Action

Saito and Ueda, PRA63 (2001), 043601

- BEC
  - Action

$$S = N\hbar \int d^3x dt \left[i\psi^* \dot{\psi} + \frac{1}{2}\psi^* \nabla^2 \psi - \frac{r^2}{2}\psi^*\psi - \frac{g}{2}(\psi^*\psi)^2 \right]$$

$$i\dot{\psi} = -\frac{1}{2}\nabla^2\psi + \frac{r^2}{2}\psi + g|\psi|^2\psi$$

Gross-Pitaevskii equation

- BH-axion
  - Action

$$\hat{S} = \int d^4x \sqrt{-g} \left[-\frac{1}{2}(\nabla\varphi)^2 - \mu^2 \left(\frac{\varphi^2}{2} + \hat{U}_{\rm NL}(\varphi) \right) \right],$$

  - Non-relativistic approximation

$$\varphi = \frac{1}{\sqrt{2\mu}} \left(e^{-i\mu t}\psi + e^{i\mu t}\psi^* \right)$$

$$\hat{S}_{\rm NR} = \int d^4x \left[\frac{i}{2} \left(\psi^* \dot{\psi} - \psi \dot{\psi}^* \right) - \frac{1}{2\mu} \partial_i \psi \partial_i \psi^* + \frac{\alpha_g}{r}\psi^*\psi - \mu^2 \tilde{U}_{\rm NL}(|\psi|^2/\mu) \right]$$

$$\tilde{U}_{\rm NL}(x) = -\sum_{n=2}^{\infty} \frac{(-1/2)^n}{(n!)^2} x^n .$$

Effective theory

Saito and Ueda, PRA63 (2001), 043601

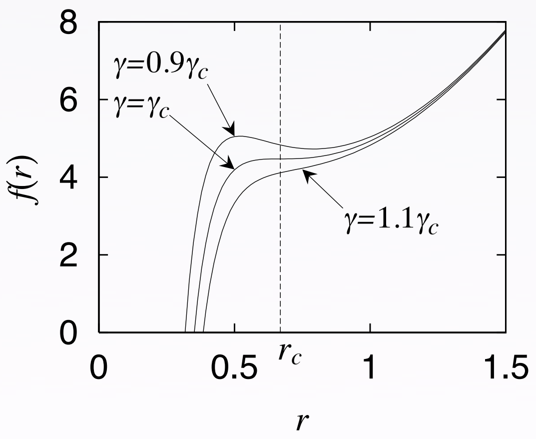
- BEC $\psi = A(x,y,z,t)e^{i\phi(x,y,z,t)}$

$$A = \frac{\exp\left[-\left(\frac{x^2}{2d_x^2(t)} + \frac{y^2}{2d_y^2(t)} + \frac{z^2}{2d_z^2(t)}\right)\right]}{\sqrt{\pi^{3/2} d_x(t) d_y(t) d_z(t)}}$$

$$\phi = \frac{\dot{d}_x(t)}{2d_x(t)}x^2 + \frac{\dot{d}_y(t)}{2d_y(t)}y^2 + \frac{\dot{d}_z(t)}{2d_z(t)}z^2$$

- Spherical case $d_x = d_y = d_z = r(t)$

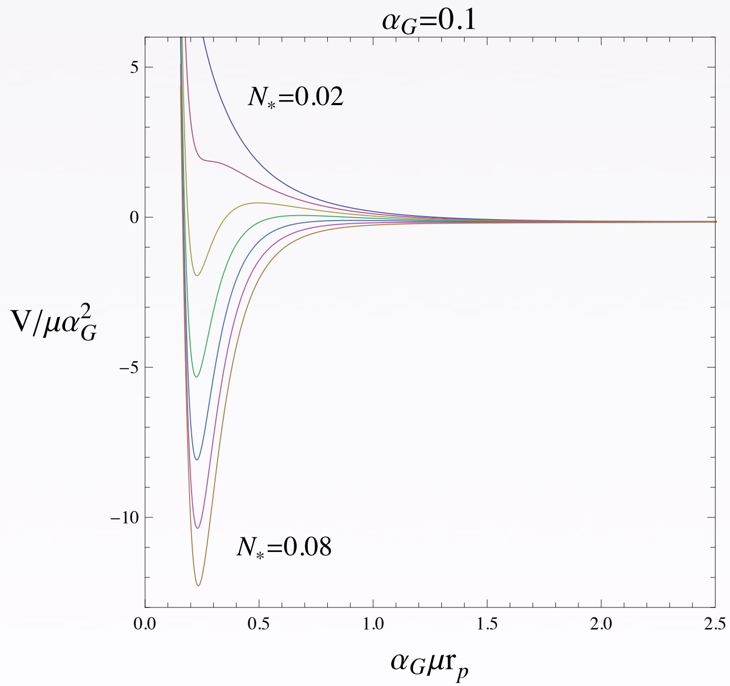
$$S = \frac{N\hbar}{4}\int dt\,\left[3\dot{r}^2 + 3\dot{r} - f(r)\right]$$



- BH-axion $\psi = A(t,r,\nu)e^{iS(t,r,\nu)+im\phi}$

$$A(t,r,\nu) \approx A_0 \exp\left[-\frac{(r-r_p)^2}{4\delta_r r_p^2} - \frac{(\nu-\nu_p)^2}{4\delta_\nu}\right],$$

$$S(t,r,\nu) \approx S_0(t) + p(t)(r-r_p) + P(t)(r-r_p)^2 + \pi_\nu(t)(\nu-\nu_p)^2 + \cdots,$$

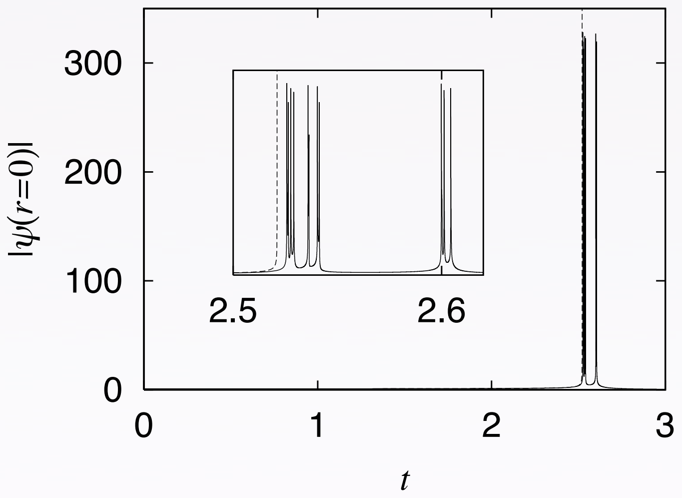


Simulation results

Saito and Ueda, PRA63 (2001), 043601

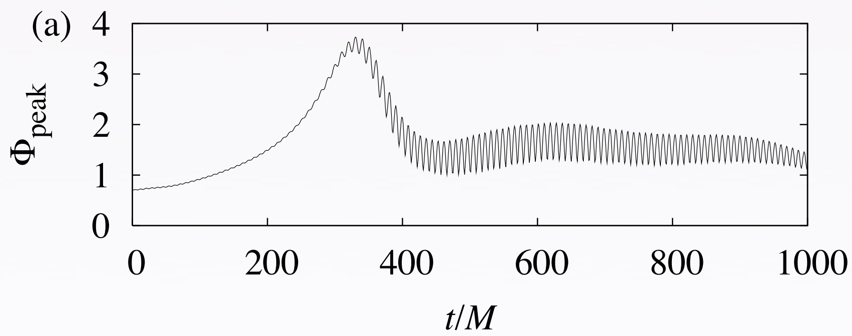
- BEC

$$i\dot{\psi} = -\frac{1}{2}\nabla^2\psi + \frac{r^2}{2}\psi + g|\psi|^2\psi - \frac{i}{2}\left(\frac{L_2}{2}|\psi|^2 + \frac{L_3}{6}|\psi|^4\right)\psi$$



- BH-axion

- Our simulation results

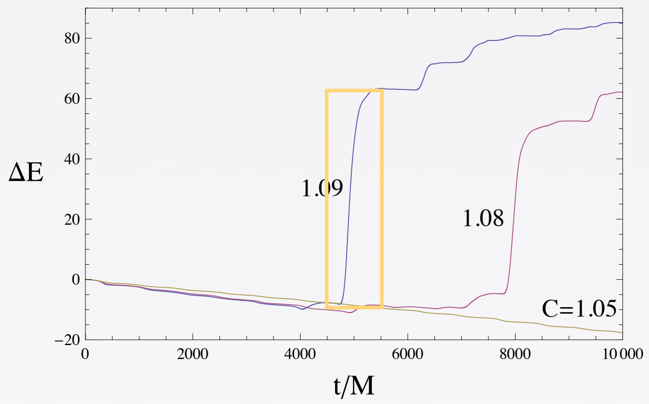


Contents

GWs emitted in the bosenova (rough estimate)

- Quadrupole moment
  - $Q_{ij} \sim r_p^2 E$

    $r_p \sim 10M \qquad E_0 \sim 10^{-3} M$



- About 5% of energy falls into the BH
  - $E = E_0 + (\Delta E/2)\left[\cos(\pi t/\Delta t) - 1\right]$

    $\Delta E \sim 0.05 E_0 \qquad \Delta t \sim 500M$

- Amplitude of generated GWs

$$h \sim \frac{\ddot{Q}_{ij}}{r_{\rm obs}} \sim 10^{-7} \frac{M}{r_{\rm obs}}$$

Detectability

$$\boxed{h \sim \frac{\ddot{Q}_{ij}}{r_{\rm obs}} \sim 10^{-7}\frac{M}{r_{\rm obs}}}$$

- Supermassive BH of our galaxy (Sagittarius A*)

$$h_{\rm rss} := \left[\int |h|^2 dt\right]^{1/2} \sim 10^{-16}(\mathrm{Hz})^{-1/2}$$

➡ Detectable by the LISA

$(10^{-4}\ \mathrm{Hz})$

Angular frequency of GW

$(10^{2}\ \mathrm{Hz})$

- Solar-mass BH (e.g., Cygnus X-1)

$$h_{\rm rss} \sim 10^{-24}(\mathrm{Hz})^{-1/2}$$

➡ below the sensitivity of the Advanced LIGO, KAGRA (LCGT), etc.

Contents

Summary

- We developed a reliable code and numerically studied the behaviour of axion field around a rotating black hole.
- The nonlinear effect enhances the rate of superradiant instability when the amplitude is not very large.
- The bosenova collapse would happen as a result of superradiant instability.

Issues for future

- Calculation of the gravitational waves emitted in bosenova.
- The case where axions couple to magnetic fields.