Status of Numerical Relativity

--From my personal point of view--

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1 Introduction

2 Key implementations in numerical relativity

3 Current status of implementation

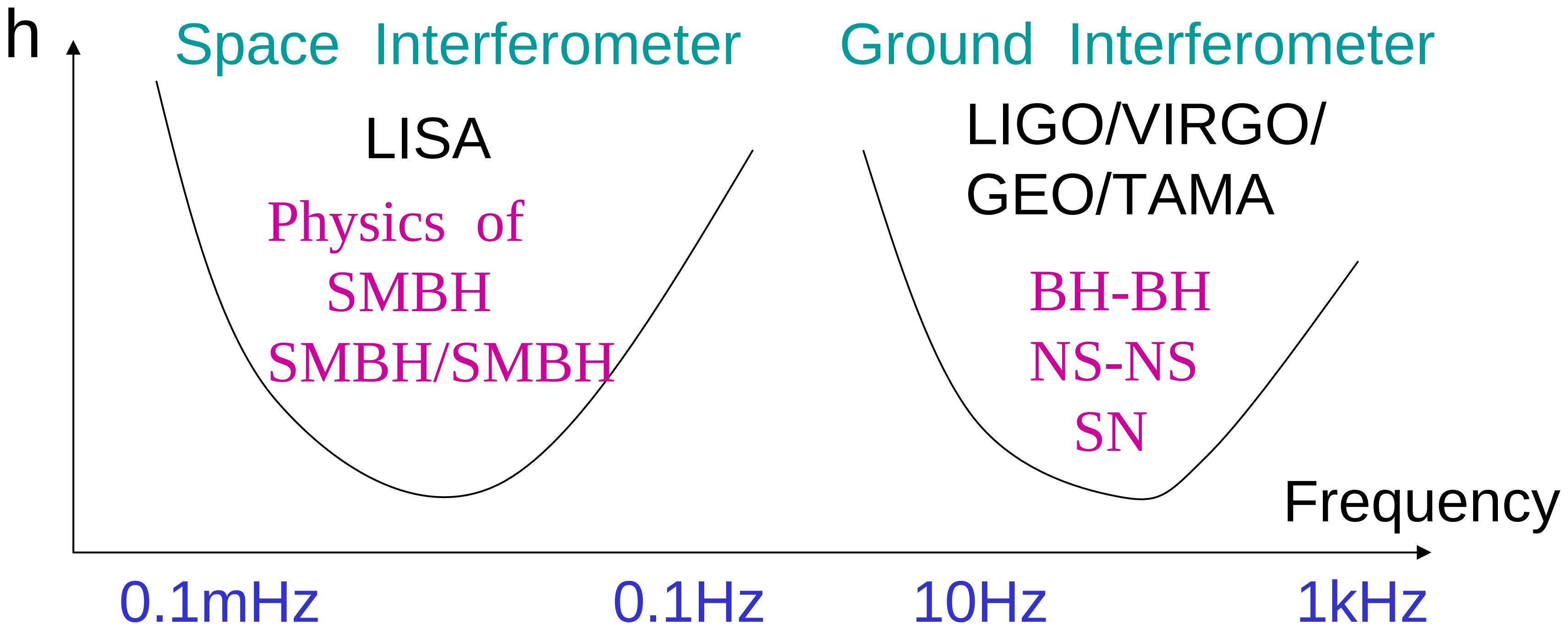
4 Some of our latest numerical results:
NS-NS merger

5 Summary & perspective

1: Introduction: Roles of NR

A To predict gravitational waveforms:

Two types of gravitational-wave detectors are working or will work in near future.

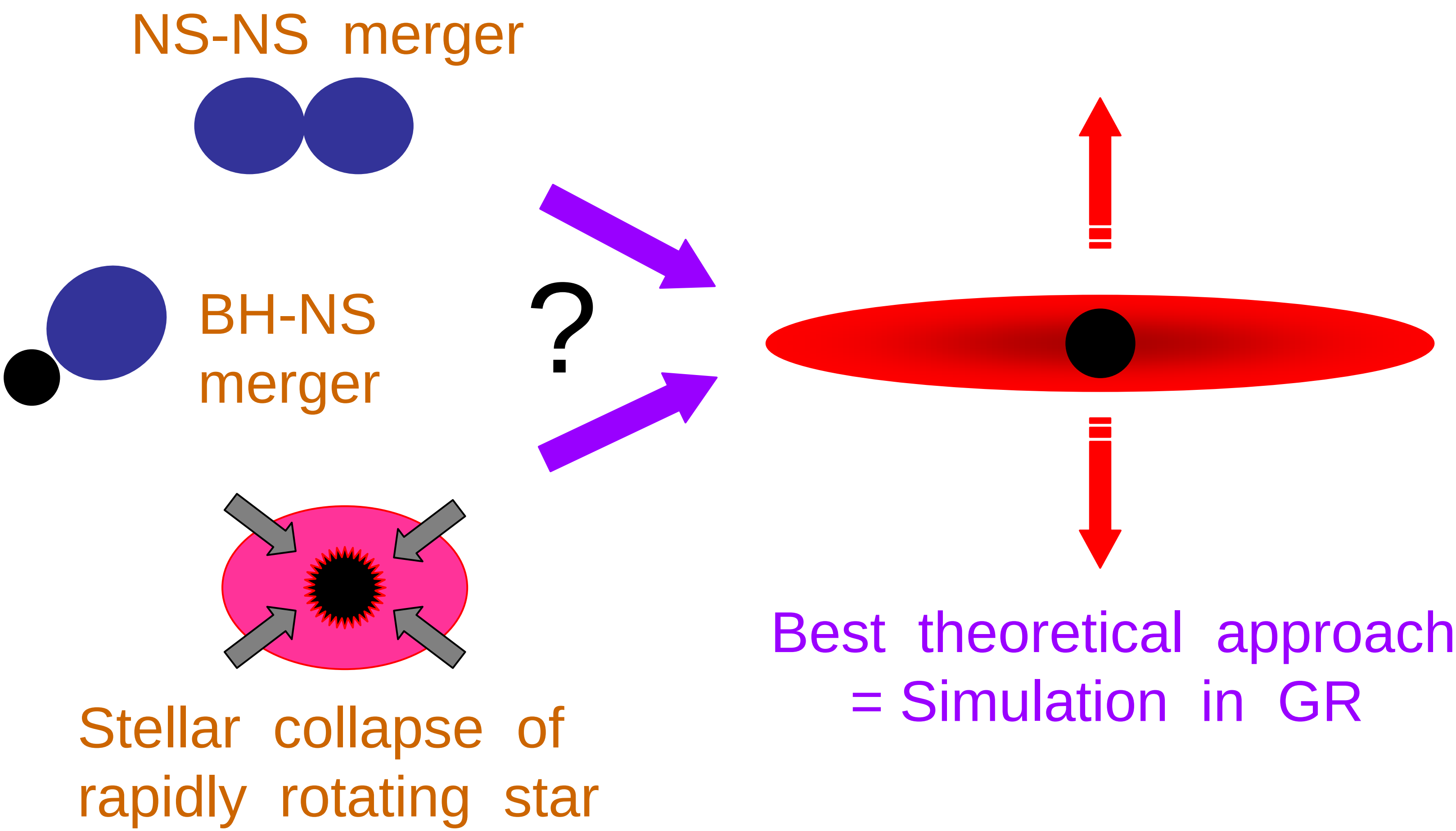


Templates (for compact binaries, core collapse, etc) should be prepared

B To simulate astrophysical phenomena

e.g. Central engine of γ-ray bursts

= Stellar-mass black hole + disks (Probably)



C To discover new phenomena in GR

In the past 20 years, community has discovered e.g.,

1: Critical phenomena (Choptuik)

2: Toroidal black hole (Shapiro-Teukolsky)

3: Naked singularity formation (Nakamura, S-T)

There may be many others.

2 Necessary elements for GR simulations

- Einstein's evolution equations solver
- GR Hydrodynamic equations solver
- Appropriate gauge conditions (coordinate conditions)
- Realistic initial conditions in GR
- Gravitational wave extraction techniques
- Apparent horizon (hopefully Event horizon) finder
- Special techniques for handling BHs / BH excision
- Micro physics (EOS, neutrino processes, B-field …)
- Powerful supercomputers

RED = Indispensable elements

3: Current Status: Achievements in the past decade

Here, focus on progress in main elements:

- Einstein evolution equation solver in 3D
- GR Hydro equation solver
- Appropriate gauge conditions in 3D
- Supercomputers

Progress I

- Formulations for Einstein's evolution equation

Many people 10 yrs ago believed the standard ADM formalism works well.

BUT:

Unconstrained free evolution

Standard ADM

Variables in standard ADM formalism:

$\gamma_{ij}, \quad K_{ij}$

12 components

Numerical simulation becomes unstable even in the evolution of linear GW

(Nakamura 87, Shibata 95, Baumgarte-Shapiro 99)

Due to constraint violation instabilities

- New formulations for Einstein's evolution eqs :

(i) BSSN formalism

Nakamura (87), Shibata-Nakamura (95), Baumgarte-Shapiro (99)…..

Choose variables:

$$\phi \equiv \frac{1}{12}\ln\left(\det(\gamma)\right)$$

$$\tilde{\gamma}_{ij} \equiv e^{-4\phi}\gamma_{ij}$$

$$K \equiv K^k_k$$

$$\tilde{A}_{ij} \equiv e^{-4\phi}\left(K_{ij} - \frac{1}{3}\gamma_{ij}K\right)$$

$$F_i \equiv \delta^{jk}\partial_j\tilde{\gamma}_{ik}$$

17 components

→

An Important step

Rewrite ADM equations using

$$\left\{\begin{array}{l} constraint \ \ equations \\ \det\left(\tilde{\gamma}_{ij}\right) = 1 \end{array}\right\}$$

↓ Unconstrained free evolution

Stable numerical simulation

(So far no problem in the absence of black holes)

- New formulations for Einstein's evolution eqs. :

(ii) Hyperbolic formulations

Bona-Masso (92) ……… many references ……..

Kidder-Scheel-Teukolsky (KST) (01)

$$\partial_t g^{ij} + \partial_k Q^{kij} = F^{ij}(g, Q, \ldots)$$

No derivatives

30~40 variables are defined.
Direction of characteristic is clear.

Advantage for imposing boundary conds. at BH
→ Perhaps, robust for BH spacetimes
But, so far, no success in 2BH merger.
(Something is short of. Need additional ideas.)

Progress II

- GR Hydro scheme

Trend until the middle of 1990

$\Rightarrow$ Add artificial viscosity to capture shocks

(Wilson 1980, Centrella 1983, Hawley et al. 1984, Stark-Piran 1985, Evans 1986, Nakamura 1993, Shibata 1999)

Schematically,

$$\frac{\partial \rho v_i}{\partial t} + \frac{\partial (\rho v_i v^j + P\gamma_i^{\ j})}{\partial x^j} = \underline{[Viscous \ \ term]_i} + ...$$

Very phenomenological;
Not very physical

Drawback : Strong shocks cannot be captured accurately.
Concern : We do not know if it always gives the correct answer for any problems ???

- Hydro scheme: Current trend

High-resolution shock-capturing scheme
= Solve equations using characteristics
(+ Piecewise-Parabolic interpolation
+ Approximate Riemann solver) : very physical !

No artificial viscosity

Developed by Valencia (Ibanez, Marti, Font, …)
& Munich (Mueller …) groups in 1990s.

Now used by many groups (including myself)

- Strong shocks & oscillations of stars are computed accurately
- Physical Scheme → No concern on the outputs

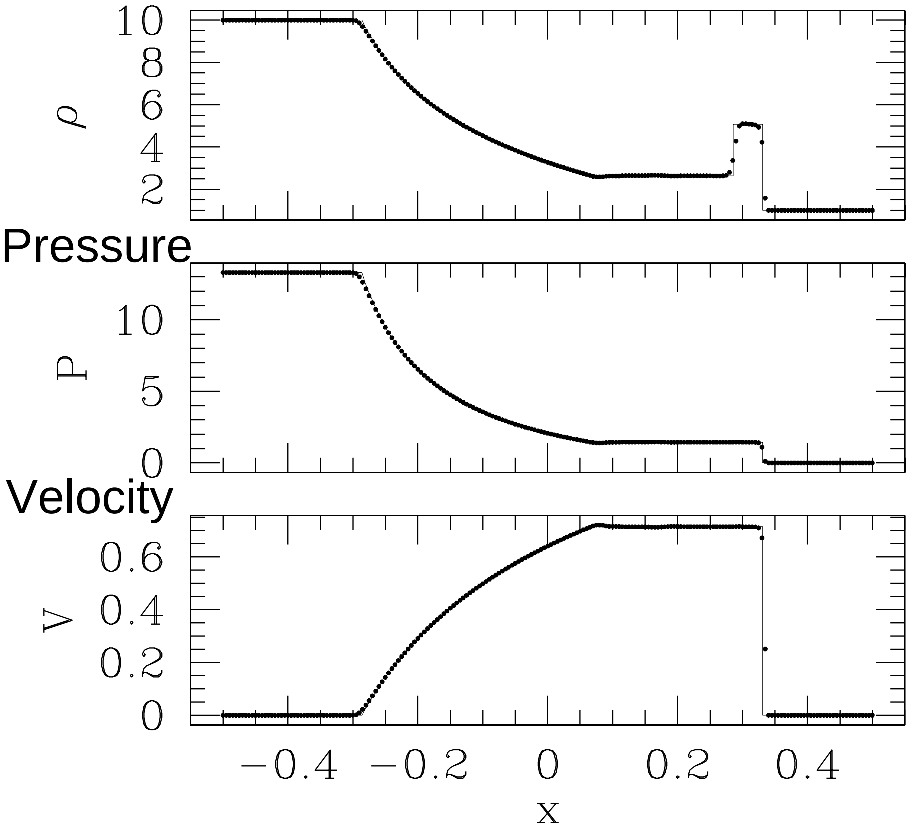
⇒ This is currently the best choice for simulations of
-- Stellar core collapse
-- NS-NS merger

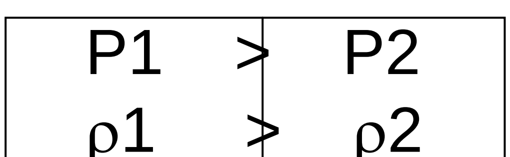
Standard tests for hydro code in special relativity

Riemann Shock Tube

N = 400, $\Gamma = 5/3$

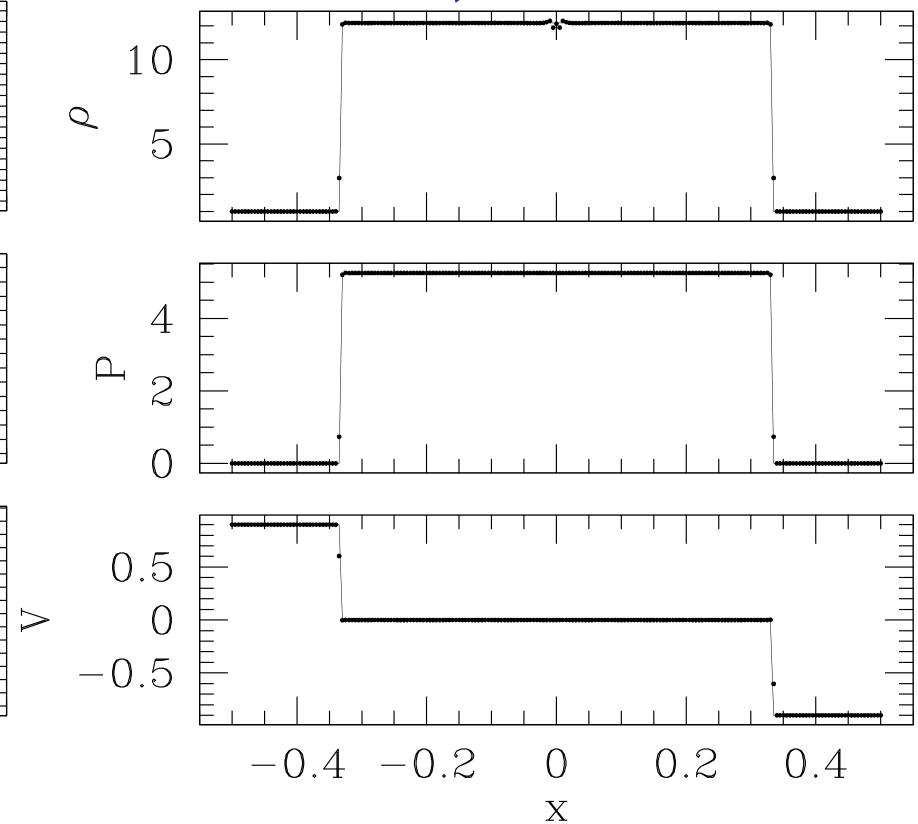
Density

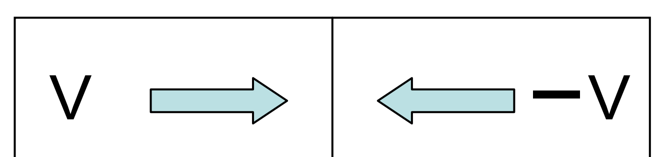




V = 0.9c. Wall Shock

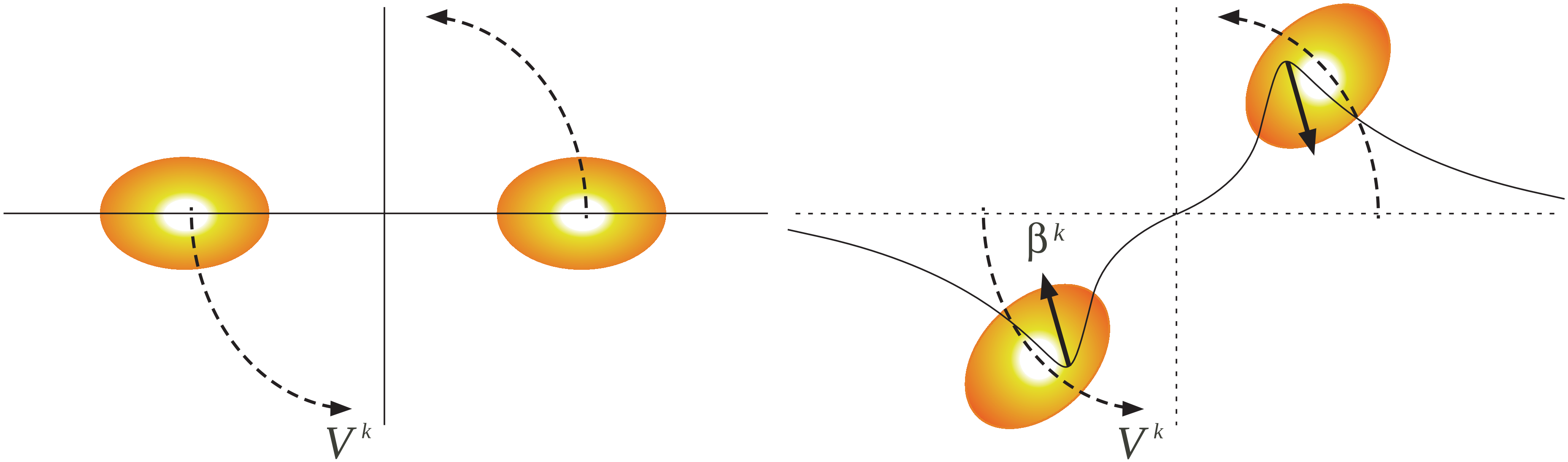
N = 400, $\Gamma = 4/3$





Progress III

- Choice of appropriate spatial gauge condition :

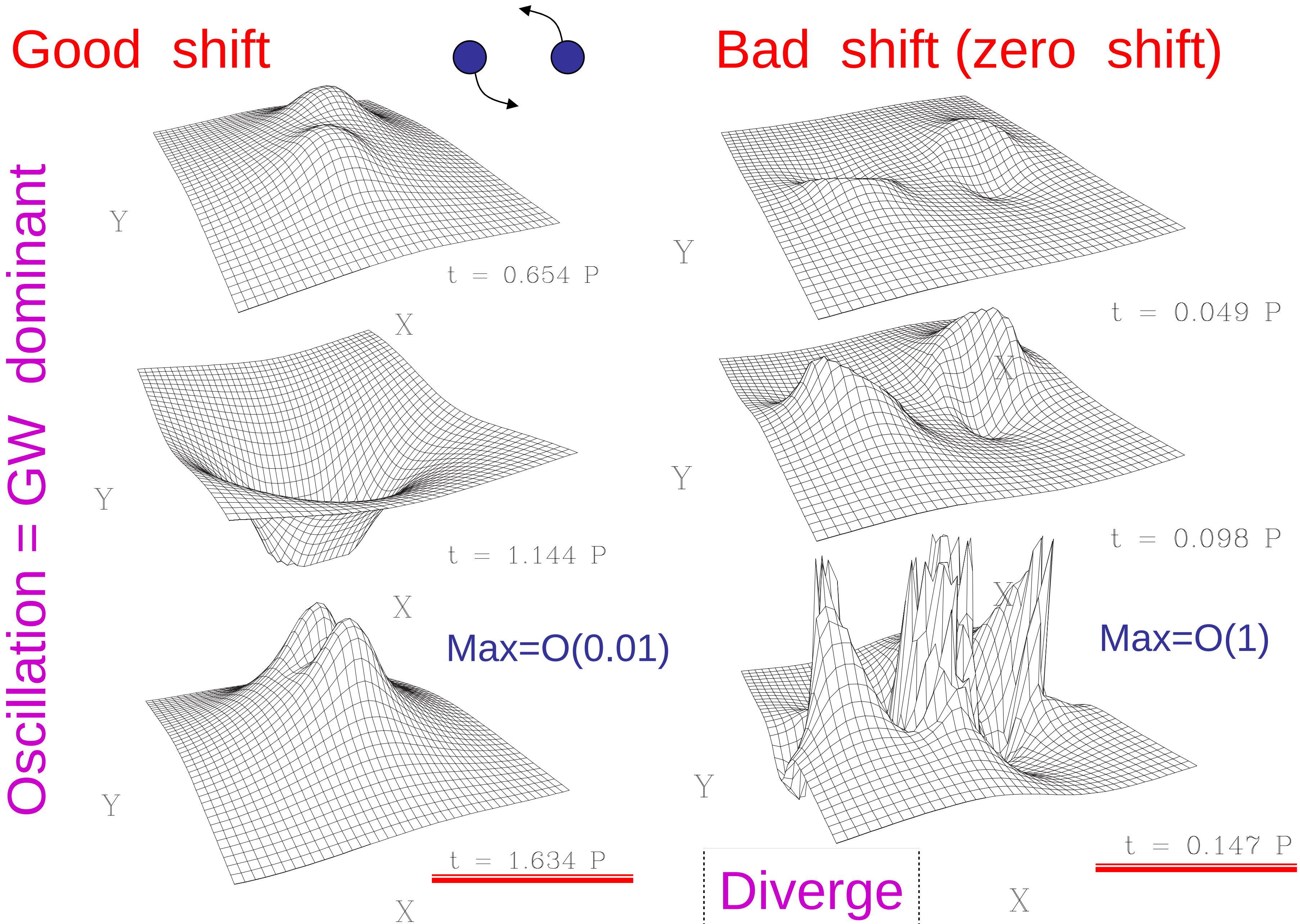


Frame dragging → Coordinate distortion

Could increase the magnitude of unphysical parts of metric

We need to suppress it for a long-term evolution.

γ_{xx}-1 on the equatorial plane



Previous belief: Minimal distortion gauge

(Smarr & York 1978)

Require that an action which denotes the global magnitude of the coordinate distortion is minimized.

$$\text{MD gauge}: \Delta\beta^k + \frac{1}{3}D^k D_j \beta^j = S^k$$

Physically good.
But, computationally time-consuming

New Trend: Dynamical gauge (Alcubierre et al 2000, Lindblom & Scheel 2003, Shibata 2003 …..)

Schematic form:

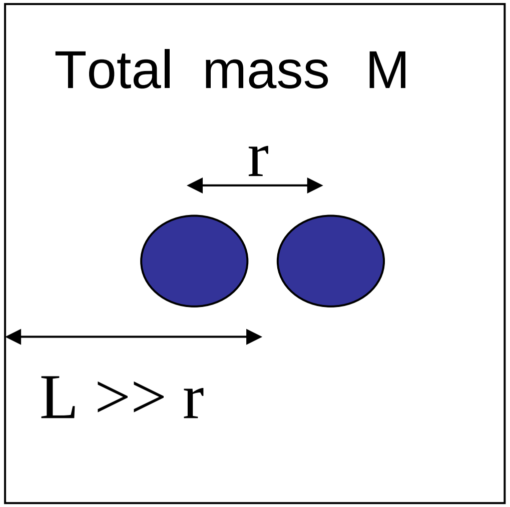
$$\ddot{\beta}^l \approx \Delta\beta^l + \frac{1}{3}D^l D_j \beta^j - S^l$$

Save CPU time significantly !!
Recent numerical experiments show it works well !!

Progress IV

Computational resources

Minimum required grid number for extraction of gravitational waveforms



$$\lambda_{GW} \le \lambda_{ISCO} \approx 58\left(\frac{GM}{c^2}\right)\left(\frac{rc^2}{7GM}\right)^{3/2}$$

$$\text{Require} \quad \mathrm{L} \ge \lambda_{GW} \quad \& \quad \Delta\mathrm{x} \le 0.2\left(\frac{GM}{c^2}\right)$$

$$\Rightarrow \quad \frac{\mathrm{L}}{\Delta\mathrm{x}} \ge 290\left(\frac{rc^2}{7GM}\right)^{3/2} \quad \& \quad \mathrm{N} \ge 580\left(\frac{rc^2}{7GM}\right)^{3/2} = 1000\left(\frac{rc^2}{10GM}\right)^{3/2}$$

Minimum grid number required (in uniform grid):
~ 600 * 600 * 300 (equatorial symmetry is assumed)
$\Rightarrow$ Memory required ~ 200 GBytes (~200 variables)

An example of current supercomputer

FUJITSU FACOM VPP5000 at NAOJ

- Vector-Parallel Machine (60 vector PEs)
- Maximum memory → 0.96TBytes
- Maximum speed → 0.58TFlops
- Our typical run with 32PEs

Typical current memory & speed

633 * 633 * 317 grid points = 240 Gbytes memory (in my code)

About 20,000 time steps ~ 100 CPU hours /model

Minimum grid number can be taken

But, hopefully, we could use hypercomputers for better-resolved simulations in near future.
(e.g. Earth simulator ~ 10TBytes, ~ 40TFlops)

Summary of current status

- Einstein evolution equations solver OK
- Gauge conditions (coordinate conditions) OK
- GR Hydrodynamic equations solver OK
- Powerful supercomputer ~OK

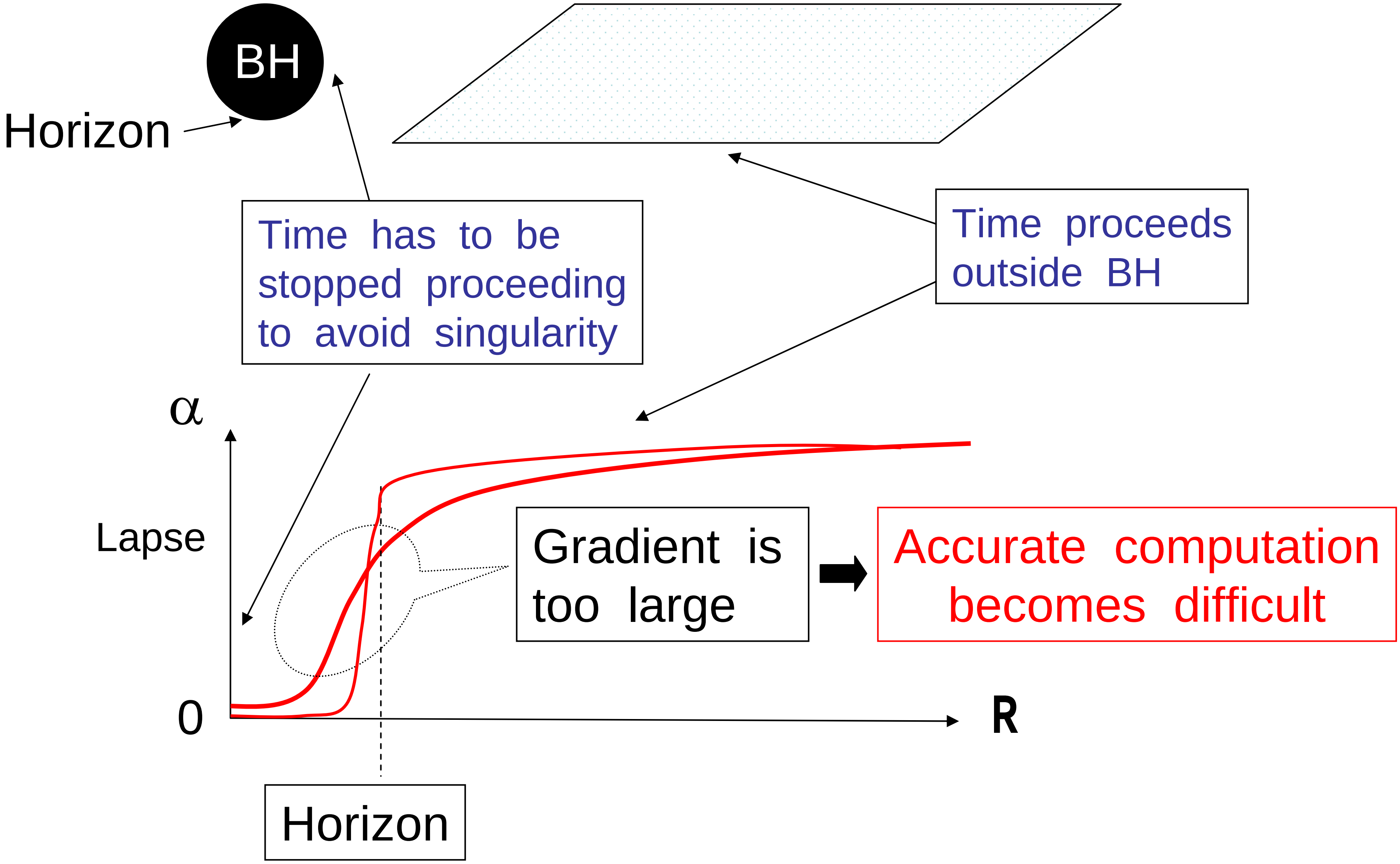
but hopefully need hypercomputers

Long-term scientific GR simulations are feasible (in the absence of BHs)

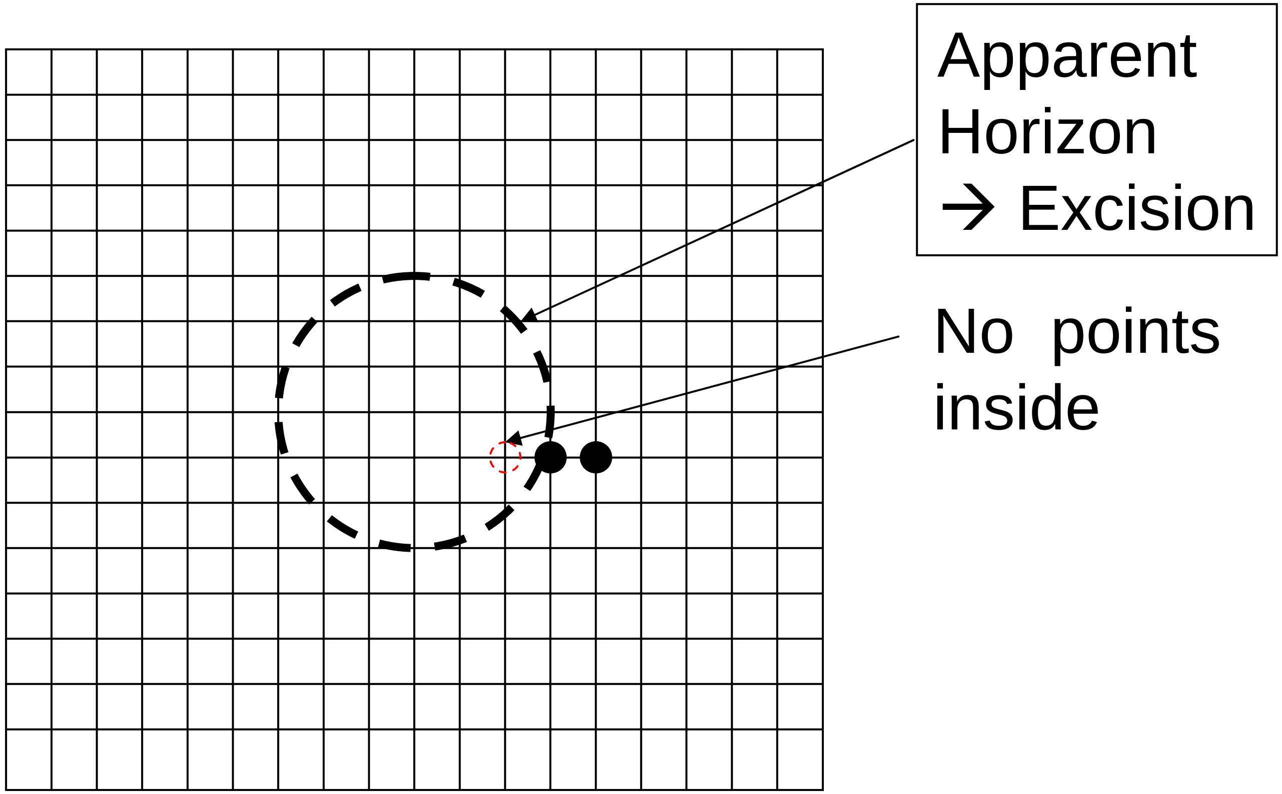
In the past 5 yrs, scientific runs have been done for

- NS-NS merger (Shibata-Uryu, Miller, …)
- Stellar core collapse (Font, Papadopoulas, Mueller, Shibata)
- Collapse of supermassive star (Shibata-Shapiro)
- Bar-instabilities of NSs (Shibata-Baumgarte-Shapiro) etc

Unsolved Issue : Handling BHs



A solution = Excision (Unruh 84)



What are appropriate formulation, gauge,
boundary conditions …. ?

-- 1BH → OK (Cornell, Potsdam, Illinois…)

-- 2BH → No success for a longterm simulation

(But see PRL**92**, 211101, Bruegmann et al. for one orbit)

4. Our latest numerical results:

Current implementation in our group

1. GR : BSSN (or Nakamura-Shibata). But modified year by year; latest version = Shibata et al. PRD **68**, 084020, 2003

2. Gauge : Maximal slicing (K=0) + Dynamical gauge

3. Hydro : High-resolution shock-capturing scheme (Roe-type method with 3rd-order PPM interpolation)

Latest results for merger of 2NS:

Last ~10 msec of BNS

EOS: Hybrid equations of state

$$P = P_{\text{Cold}} + P_{\text{Thermal}}$$

$$P_{\text{Thermal}} = \left(\Gamma_{\text{Thermal}} - 1\right)\rho(\varepsilon - \varepsilon_{\text{Cold}})$$

P_{Cold}, $\varepsilon_{\text{Cold}}$: A realistic EOS by Haensel et al. (SLy)

We set $\Gamma_{\text{Thermal}} = 2$

I here show animations for

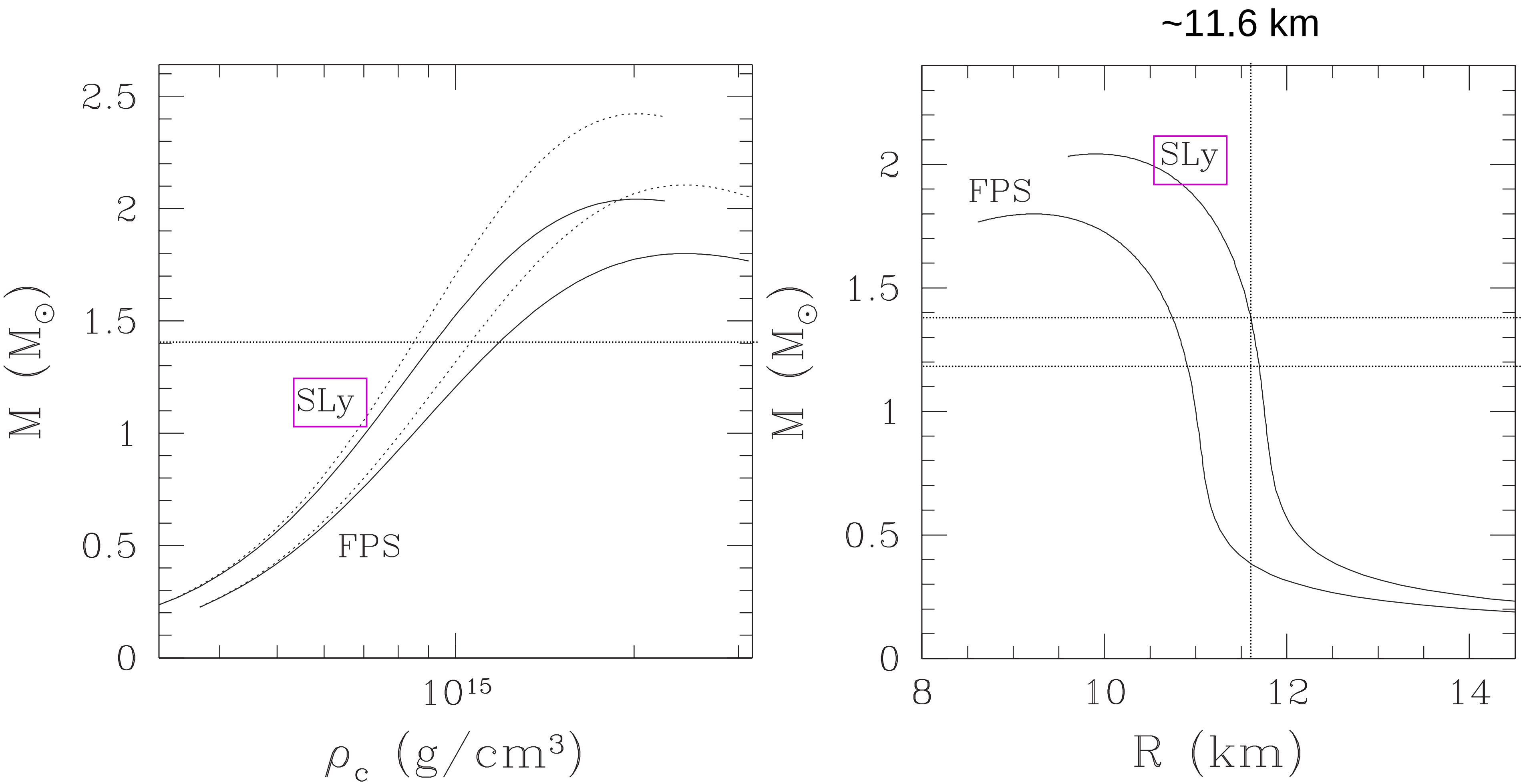
(a) 1.30 – 1.30 M_sun,

(b) 1.40 – 1.40 M_sun,

(c) 1.25 – 1.35 M_sun,

Typical grid size : 633 * 633 * 317

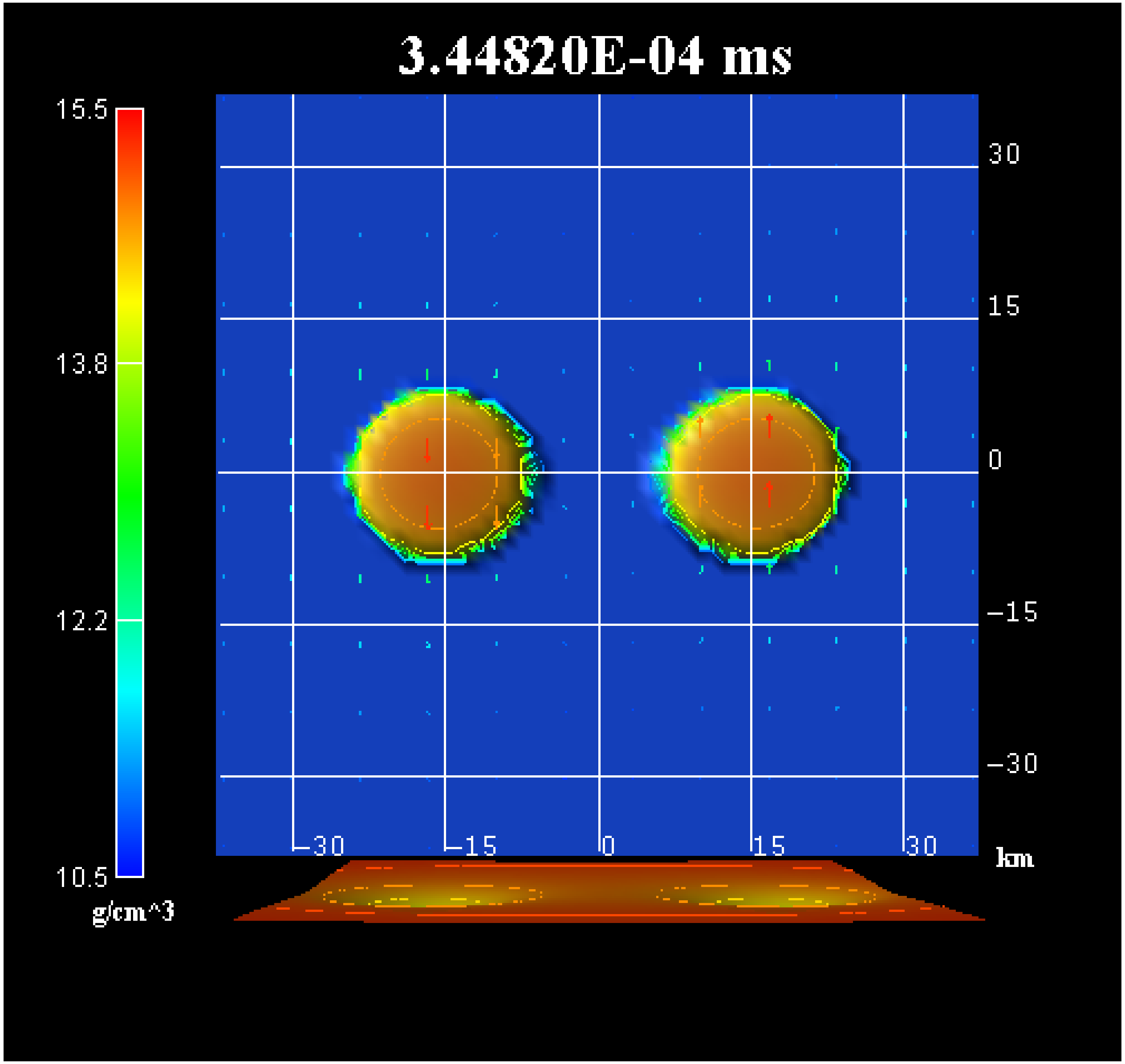
Relations for spherical stars



Sly: Maximum mass ~ 2.04 M_sun;

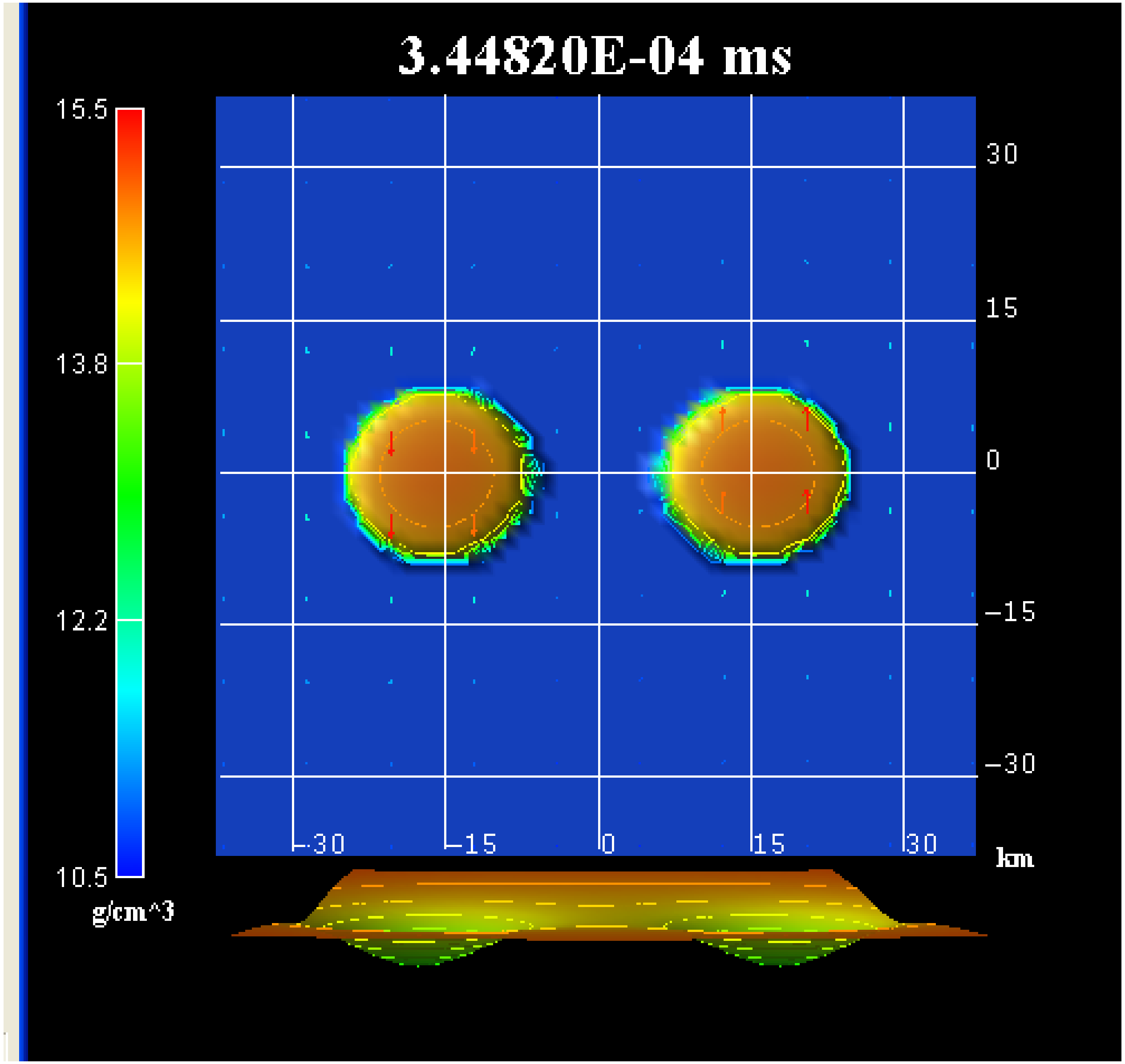
M=1.4M_sun ←→R=11.6 km

1.4-1.4M_sun : Density in the z=0



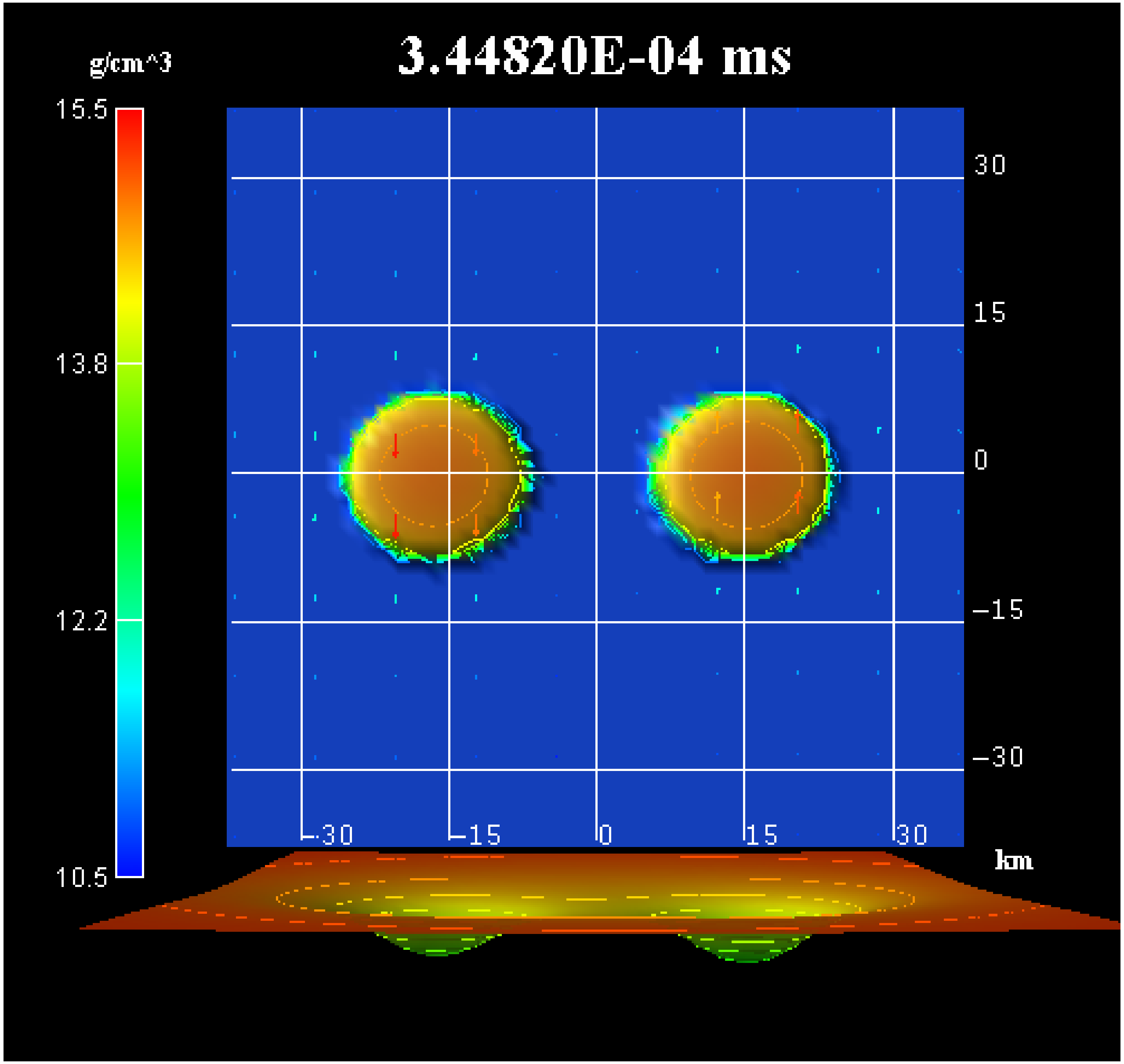
Lapse

1.3-1.3M_sun : Density in the z=0



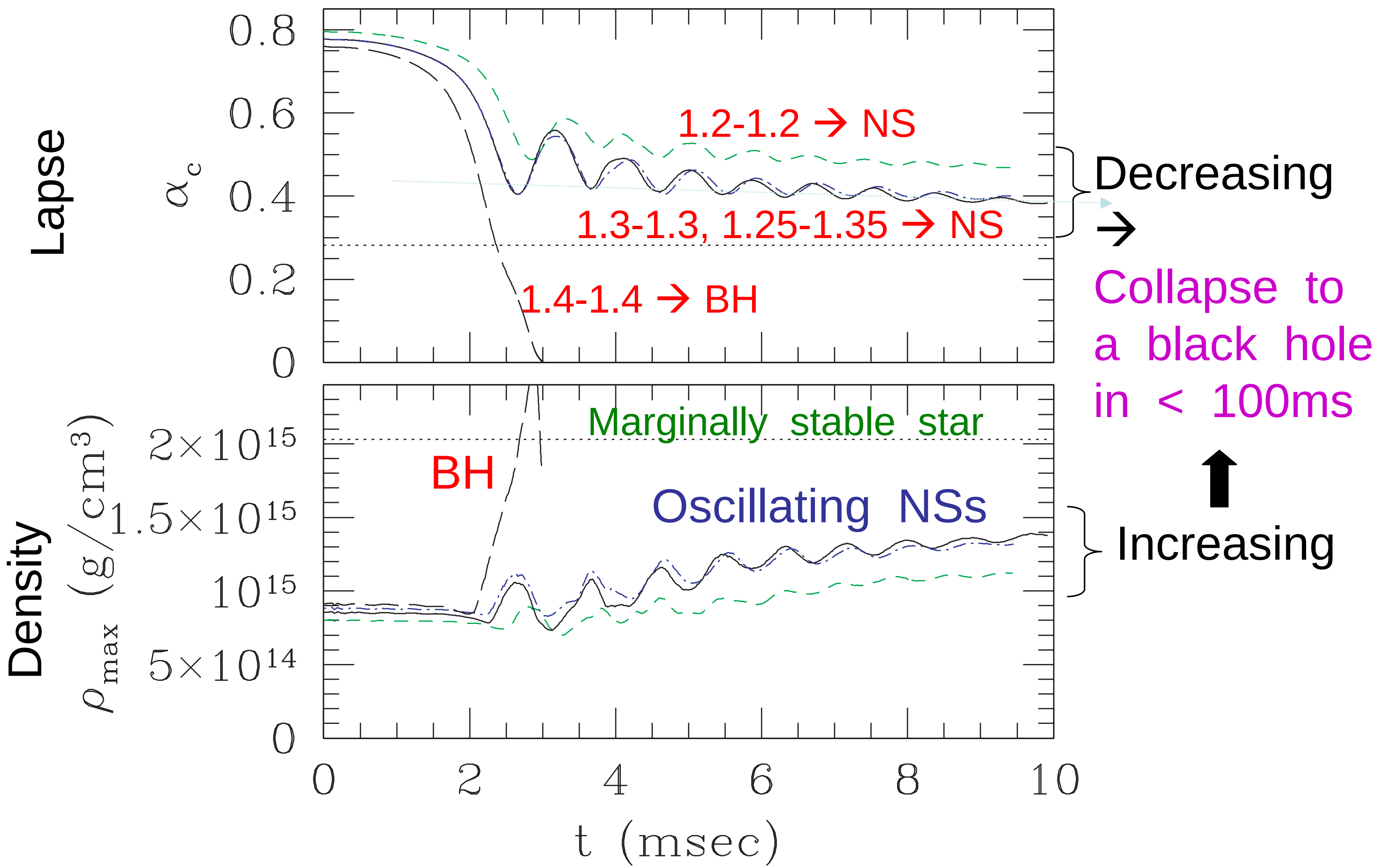
Lapse

1.25-1.35M_sun

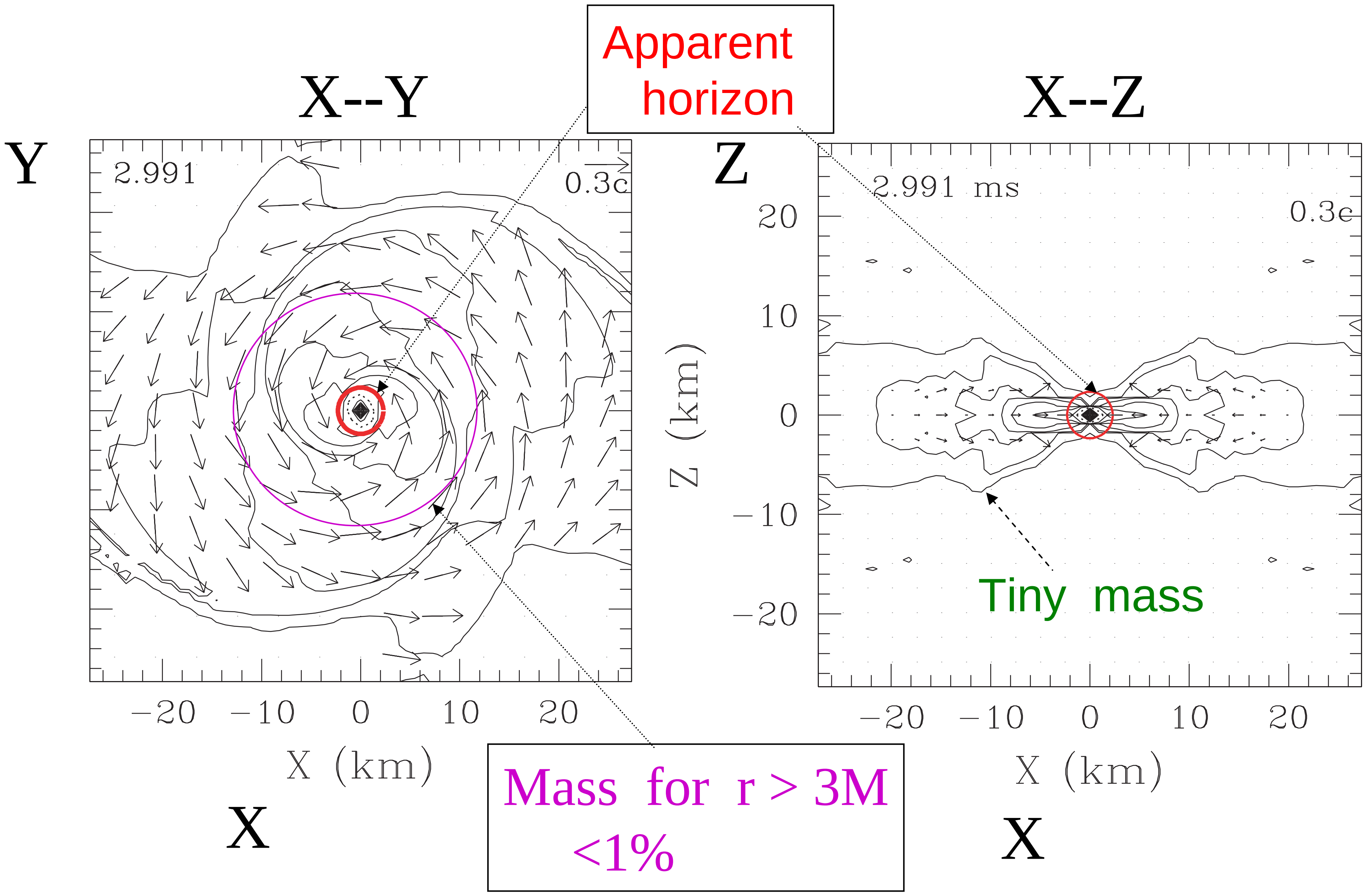


Lapse

Evolution of maximum density & central lapse

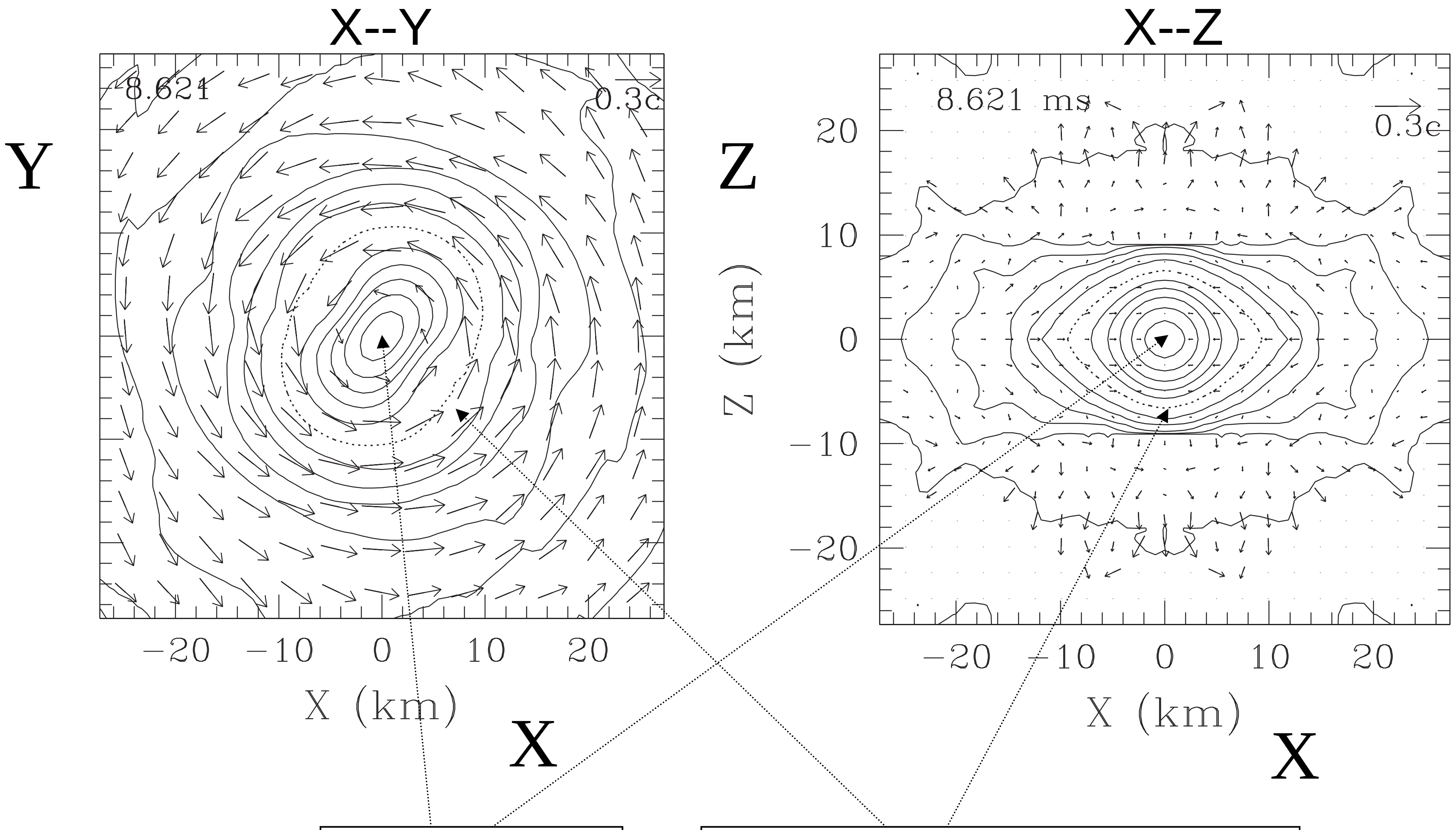


1.4 – 1.4 M_sun case : final snapshot



1.3 – 1.3 M_sun case : final snapshot

Massive ellipsoidal neutron star is formed



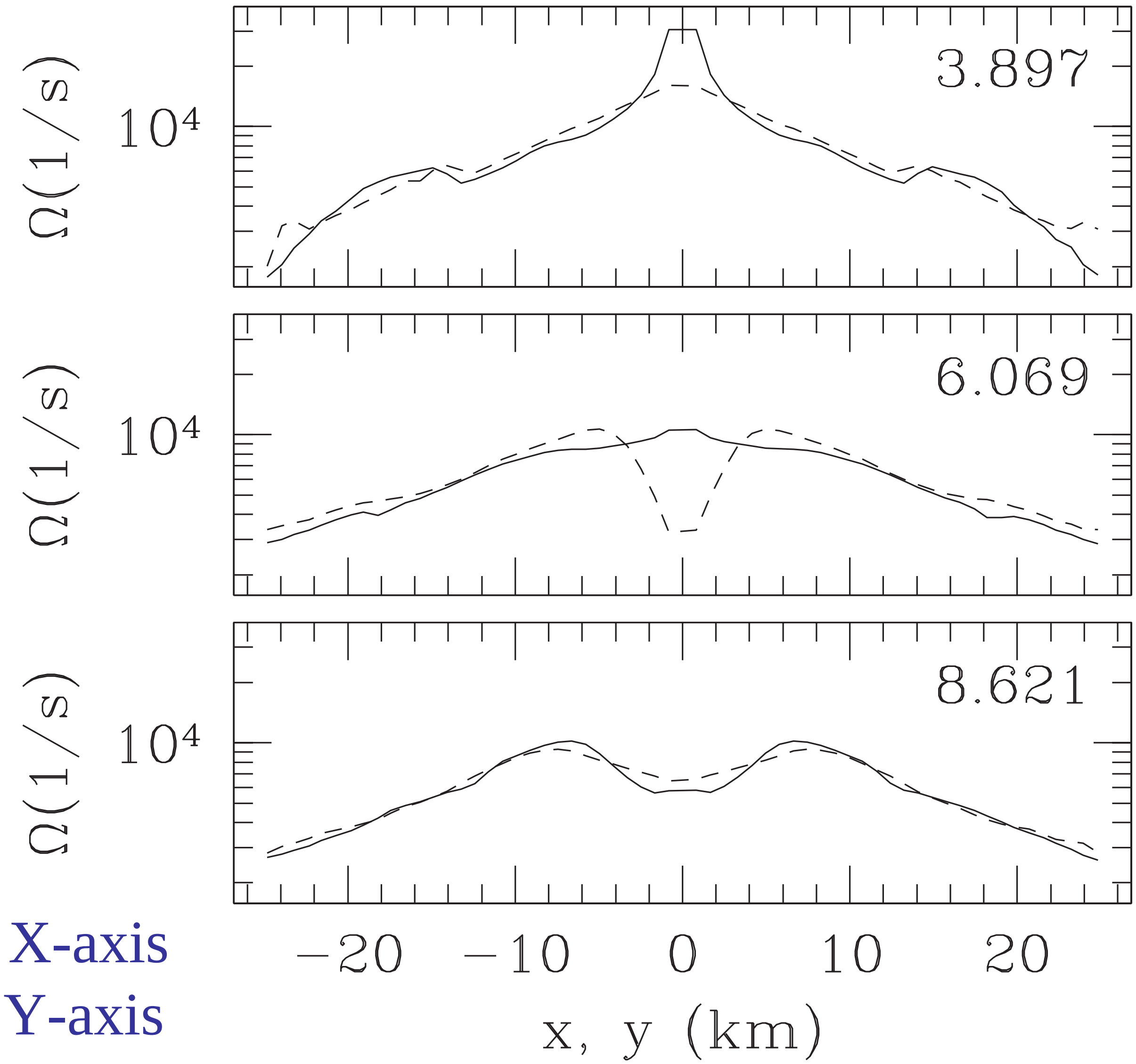
1.3e15 g/cc

Dotted curve=2e14 g/cc

Formed Massive toroidal NS is differentially and rapidly rotating

Angular velocity

Rotation Period ~ 0.6 ms



Solid curve : X-axis
Dashed : Y-axis

Products of mergers

Equal-mass or nearly equal-mass cases

- Low mass cases

  Hypermassive neutron stars of ellipsoidal shape

  → Collapse to a black hole in ~ 100 ms

- High mass cases

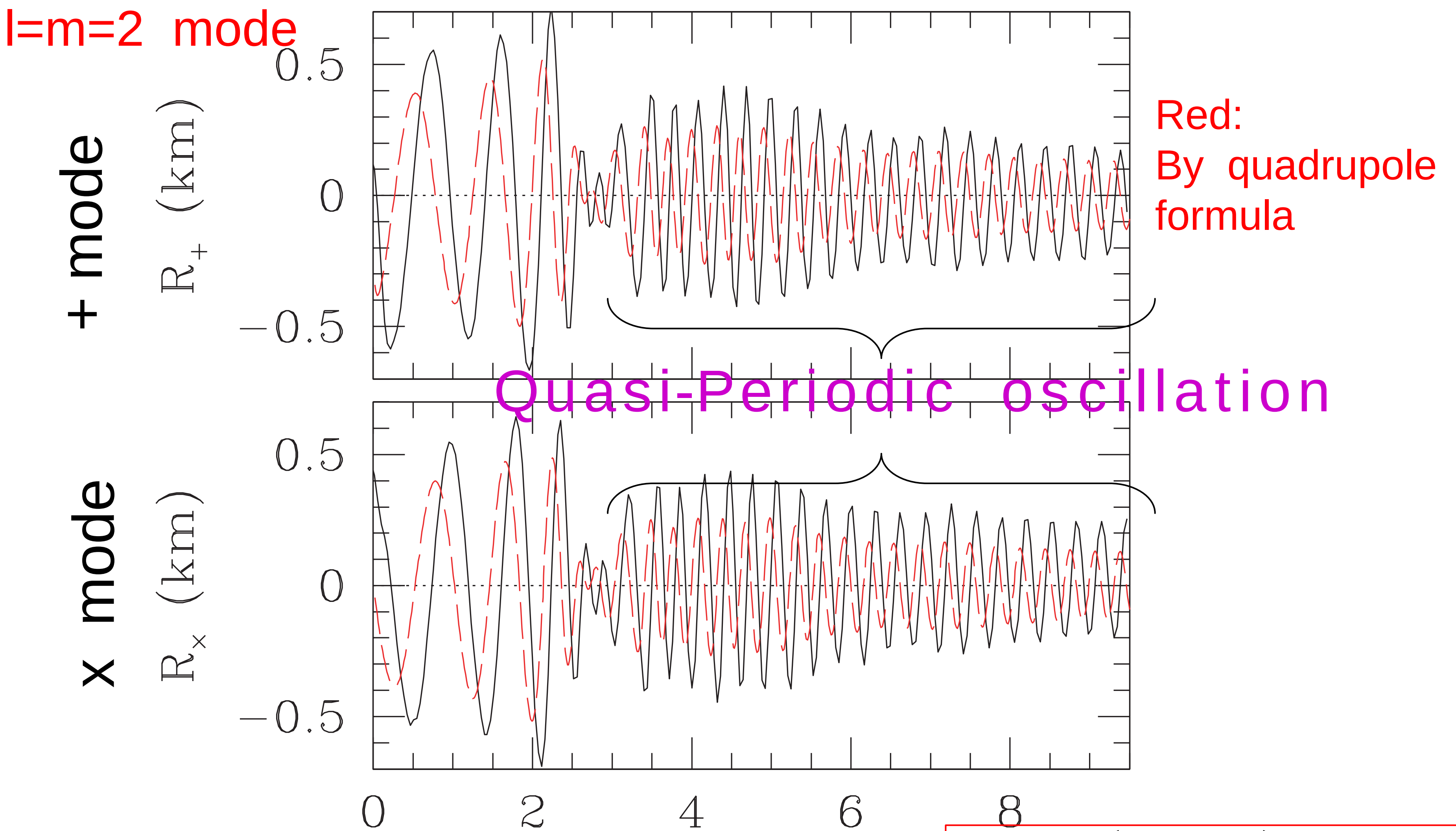
  Direct formation of Black holes

  with very small disk mass

Unequal-mass cases with large mass difference ?

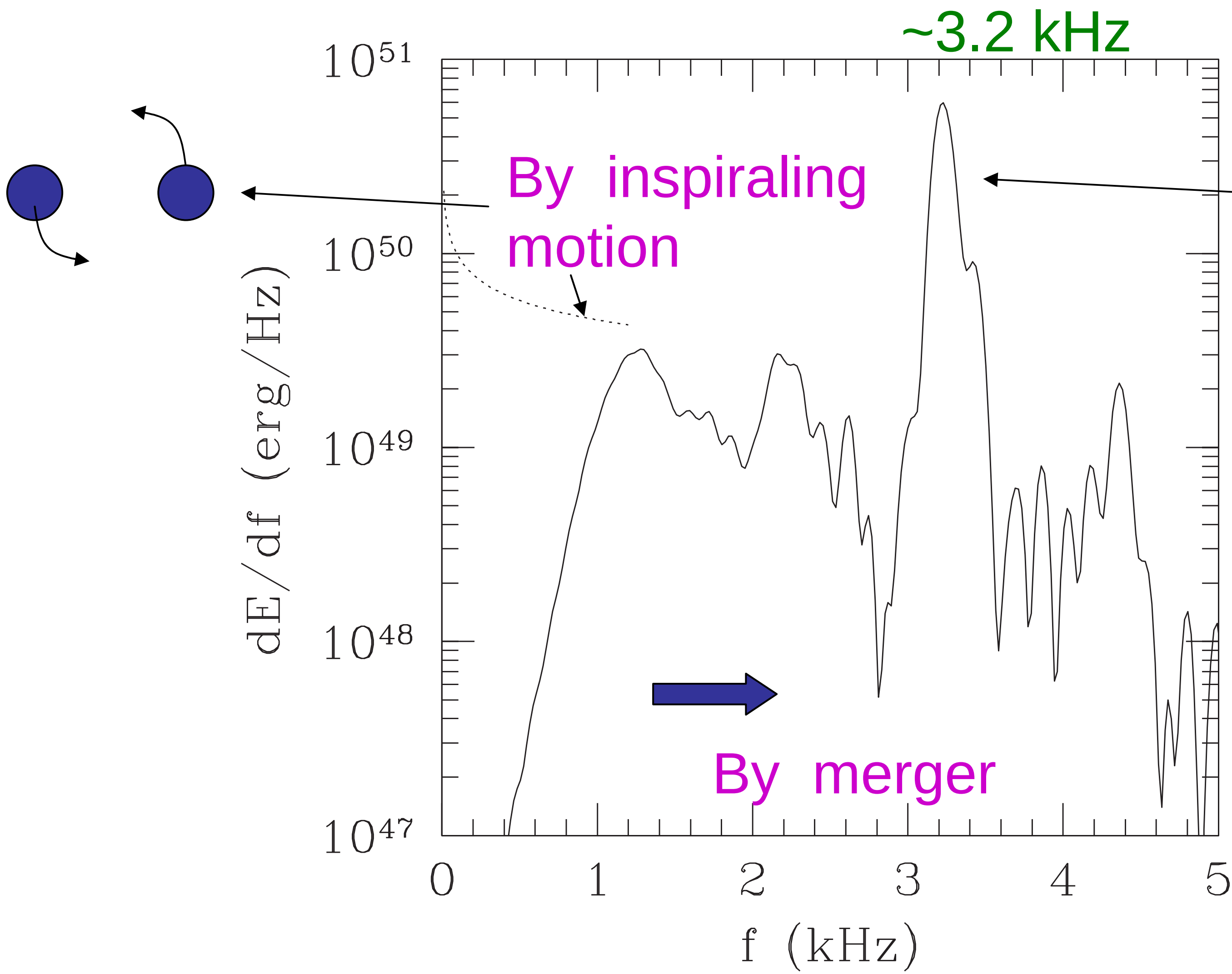
- Disk mass may be larger but need smaller mass ratio $M2 / M1 < 0.9$

Gravitational waves for 1.3-1.3M_sun to NS



$$h = 10^{-21}\left(\frac{R_{+,\times}}{0.31\mathrm{km}}\right)\left(\frac{10\mathrm{Mpc}}{r}\right)$$

Fourier power spectrum



By quasi periodic rotation of formed NS

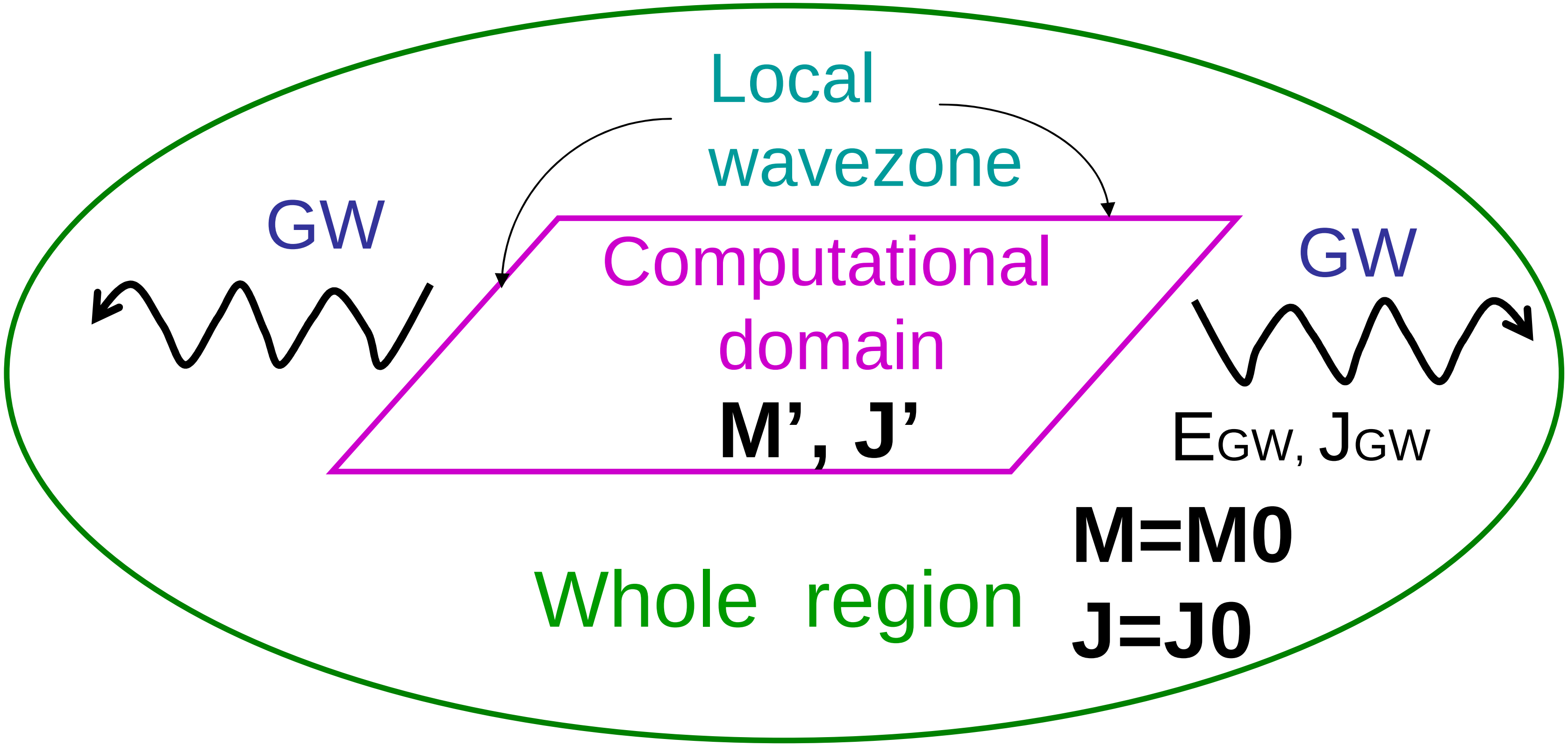
Large peak!

Maybe detectable by Advanced detectors

$$h_{\mathrm{eff}} = 1.8\times 10^{-21}\left(\frac{dE/df}{10^{51}\,\mathrm{erg/Hz}}\right)^{1/2}\left(\frac{100\mathrm{Mpc}}{r}\right)$$

Conservation of mass and angular momentum

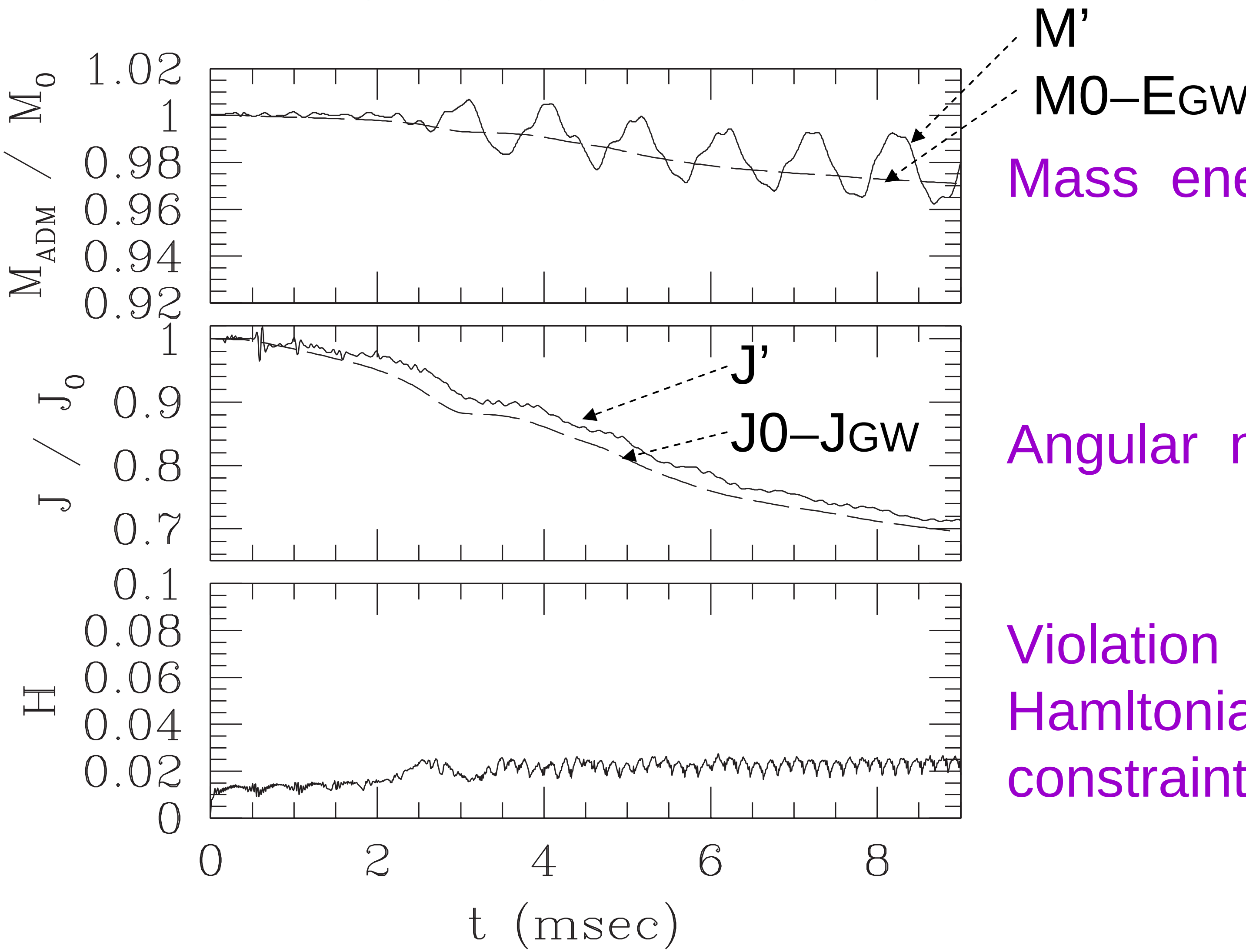
-- Important Check ! --



$M0 - E_{GW} = M'$ & $J0 - J_{GW} = J'$
should be satisfied

Radiation reaction : OK within ~ 1--2%

NS formation: 1.3—1.3



Mass energy

Angular mom.

Violation of Hamltonian constraint

Good accuracy

5 Summary

1 Rapid progress in particular in the past 5 yrs

2 Scientific (quantitative) runs are feasible now.

3 (Astrophysically) Accurate and longterm simulations are feasible for many phenomena in the absence of BHs : NS-NS merger, Stellar collapse, Bar-instabilities of NSs ....

4 Numerical implementations for fundamental parts have been almost established (for the BH-absent spacetimes)

Issues for the near future

1 Several (technical) Issues still remain :

- Grid numbers are still not large enough in 3D

  → We would need hypercomputer (~10TBytes, ~10TFlops)

  → Probably becomes available in a couple of yrs.

- Computation crashed due to grid stretching around BH horizon

  → We need to develop excision techniques.

2 Incorporate more realistic physics in hydro simulation

More realistic EOS, Neutrino cooling, Magnetic fields

Where are we ?

- 1: Make a code which runs anyhow stably
  (do not care accuracy)
- 2: Improve the code which can provide a qualitatively correct result; care accuracy somewhat (say we admit an error of ~10%)
- 3: Improve the code gradually getting qualitatively new results which can be obtained only by an improved code
  (error ~ 1%)

★ 4: Goal: Make a code which provides a quantitatively accurate result. ← Necessary for making GW templates (error << 1%)

We are here.

Similar to construction of detectors in some sense