The p d -> $\eta\ ^3$He and $K^-\ ^3$He -> npΛ reactions and bound nuclear states

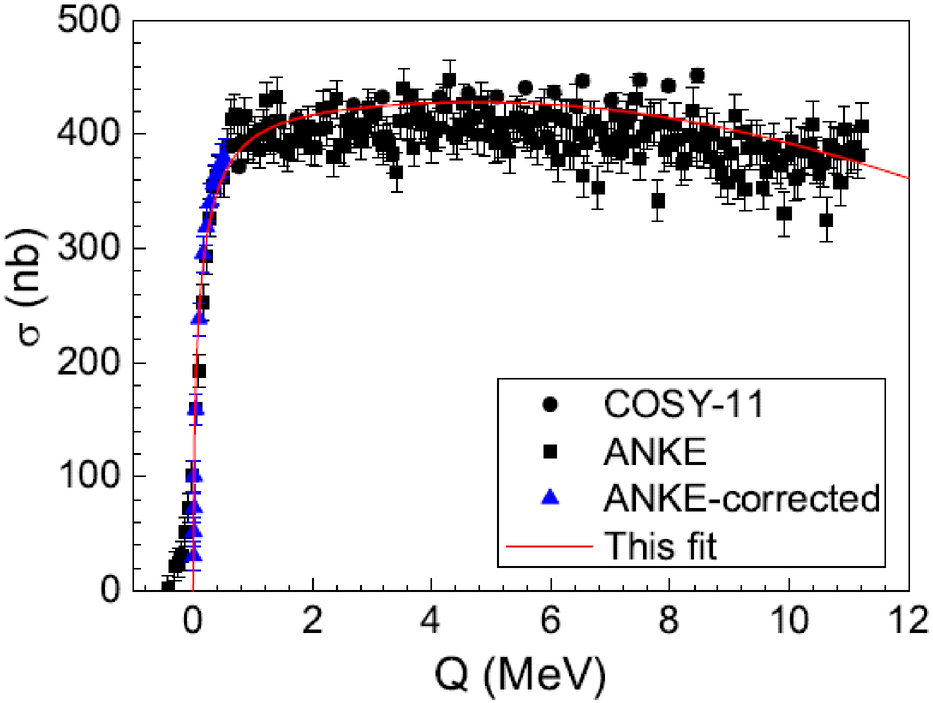
E. Oset, J.J. Xie, W. H. Liang, P. Moskal, M.Skurzok, C. Wilkin,
T. Sekihara, A. Ramos

The p d -> $\eta\ ^3$He reaction close to threshold
$\eta\ ^3$He bound state

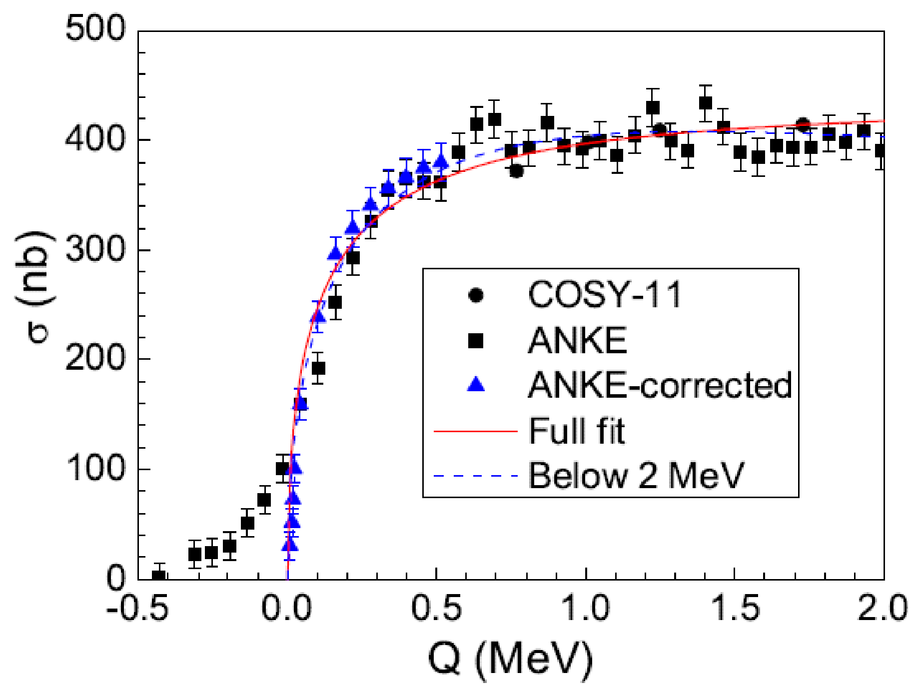
The K- ^{3}He -> np Λ reaction with in fligh kaons
A bound Kbar NN state

E. Oset , IFIC, CSIC-Universidad de Valencia

$pd \to \eta^3\text{He}$ total cross section



$$Q = \sqrt{s} - m_\eta - M_{^3\mathrm{He}}$$

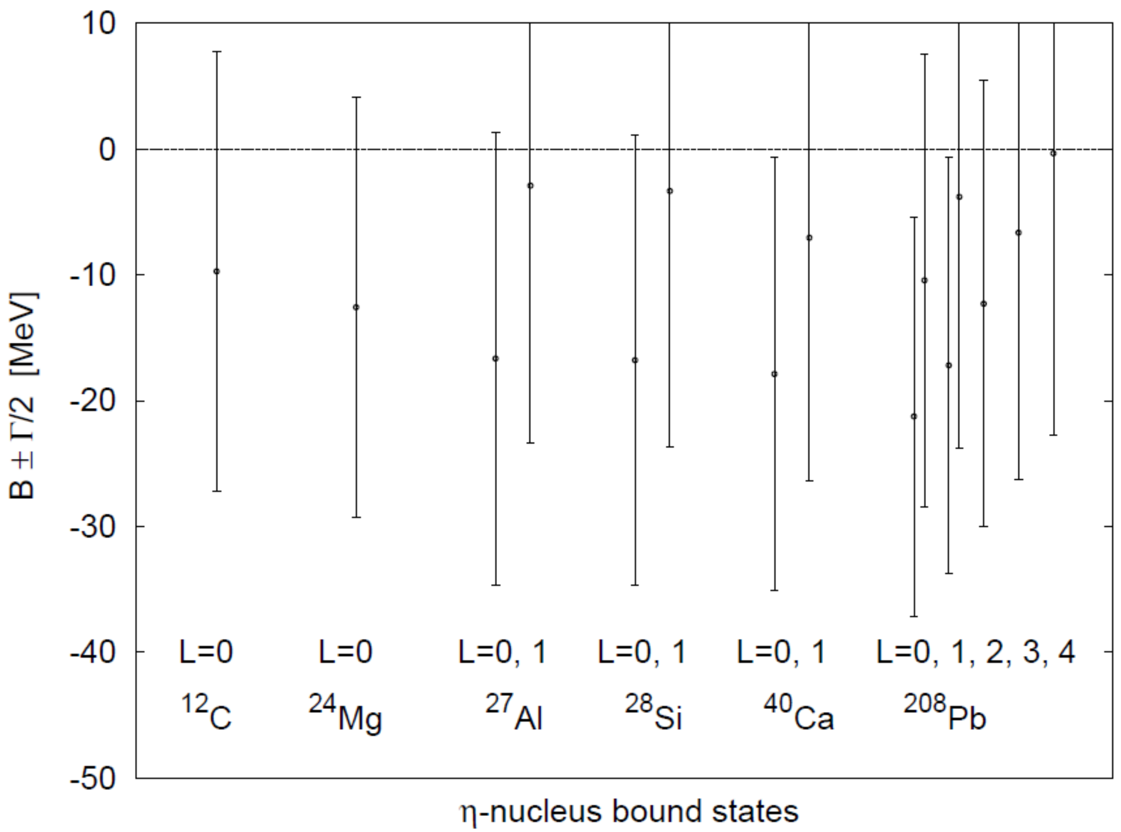


Fits to these data have been done before

T. Mersmann *et al.*, Phys. Rev. Lett. **98**, 242301 (2007)
C. Wilkin *et al.*, Phys. Lett. B **654**, 92 (2007)

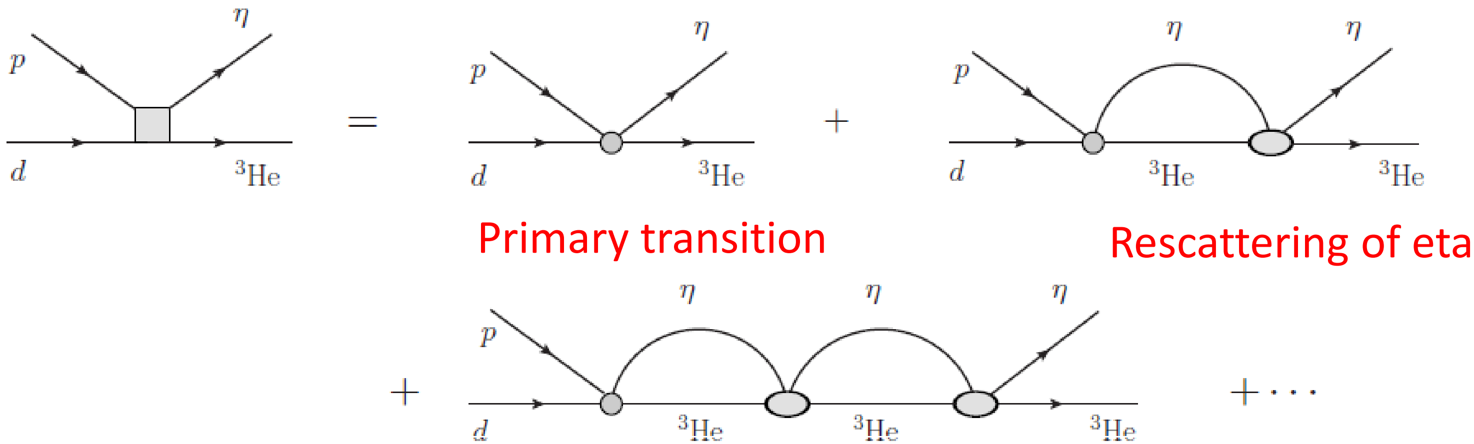
They get a small binding and very narrow width

This is in contrast with all calculations, that give Γ > B



Garcia-Recio, Nieves, Inoue, E. O
PLB (2002)

Theoretical approach



Primary transition

$$V_P = A\vec{\epsilon}\cdot\vec{p} + iB(\vec{\epsilon}\times\vec{\sigma})\cdot\vec{p},$$ S-wave in eta ^{3}He

$$V_{1P} = C\vec{\epsilon}\cdot\vec{p}_\eta + iD(\vec{\epsilon}\times\vec{\sigma})\cdot\vec{p}_\eta.$$ P-wave in eta ^{3}He

Eta rescattering $T = V + VGT$ V is an optical potential, complex

Full transition amplitude t= (VP + V1P) (1+GT)

Construction of T

In many body theory the low density theorem tells that for low densities,

$$V(\vec{r}) = t_{\eta N}\rho(\vec{r}) = 3t_{\eta N}\tilde{\rho}(\vec{r})$$

with $\tilde{\rho}(\vec{r})$ normalized to unity.

This is only used to establish the range of the interaction

Momentum space
$$V(\vec{p}_\eta, \vec{p'}_\eta) = 3t_{\eta N}\int d^3\vec{r}\tilde{\rho}(\vec{r})e^{i(\vec{p}_\eta - \vec{p'}_\eta)\cdot\vec{r}} = 3t_{\eta N}F(\vec{p}_\eta - \vec{p'}_\eta)$$

$$F(\vec{q}) = \int d^3\vec{r}\tilde{\rho}(\vec{r})e^{i\vec{q}\cdot\vec{r}} \qquad F(\vec{q}) = e^{-\beta^2|\vec{q}|^2} \qquad \beta^2 = 13.7\ \mathrm{GeV}^{-2}.$$

s-wave projected
$$\begin{aligned} V(\vec{p}_\eta, \vec{p}_\eta\,') &= 3t_{\eta N}\frac{1}{2}\int_{-1}^{1} d\cos\theta\, e^{-\beta^2(|\vec{p}_\eta|^2 + |\vec{p'}_\eta|^2 - 2|\vec{p}_\eta||\vec{p'}_\eta|\cos\theta)} \\ &= 3t_{\eta N}e^{-\beta^2|\vec{p}_\eta|^2}e^{-\beta^2|\vec{p'}_\eta|^2}[1 + \frac{1}{6}(2\beta^2|\vec{p}_\eta||\vec{p'}_\eta|)^2 + ...]. \end{aligned}$$

The term [...] is essentially 1 in the range of study and, thus, the potential is separable

$$T(\vec{p}_\eta, \vec{p'}_\eta) = \tilde{V} e^{-\beta^2 |\vec{p}_\eta|^2} e^{-\beta^2 |\vec{p'}_\eta|^2} + \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{M_{^3\mathrm{He}}}{2\omega_\eta(\vec{q}) E_{^3\mathrm{He}}(\vec{q})} \frac{\tilde{V} e^{-\beta^2 |\vec{p}_\eta|^2} e^{-\beta^2 |\vec{q}|^2}}{\sqrt{s} - \omega_\eta(\vec{q}) - E_{^3\mathrm{He}}(\vec{q}) + i\epsilon} \tilde{T} e^{-\beta^2 |\vec{q}|^2} e^{-\beta^2 |\vec{p'}_\eta|^2}$$

But we do not take

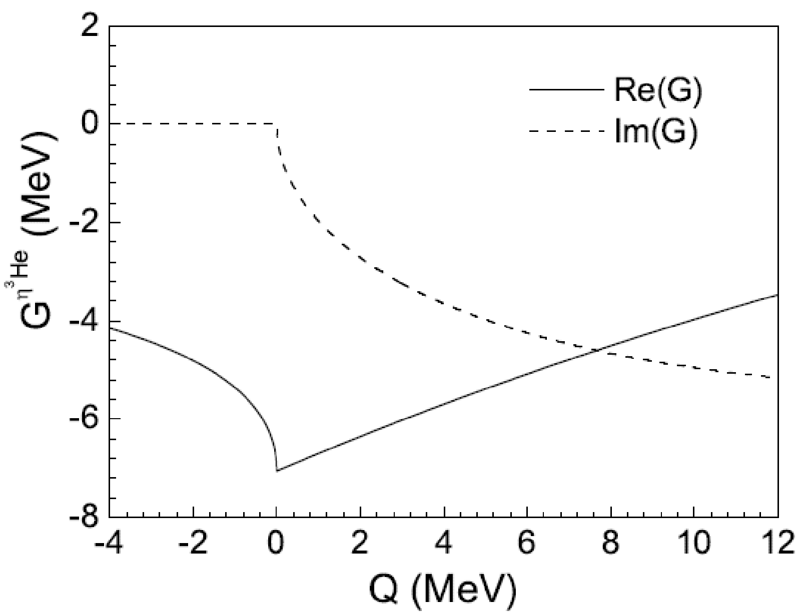
Take instead

$$3 t_{\eta N} e^{-\beta^2 |\vec{p}_\eta|^2} e^{-\beta^2 |\vec{p'}_\eta|^2} \longrightarrow \tilde{V} e^{-\beta^2 |\vec{p}_\eta|^2} e^{-\beta^2 |\vec{p'}_\eta|^2} \qquad T(\vec{p}_\eta, \vec{p'}_\eta) = \tilde{T} e^{-\beta^2 |\vec{p}_\eta|^2} e^{-\beta^2 |\vec{p'}_\eta|^2}$$

$$\tilde{T} = \tilde{V} + \tilde{V} G \tilde{T}$$

$\tilde{V}$ will be a fit parameter

$$G = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{2\omega_\eta(\vec{q})} \frac{M_{^3\mathrm{He}}}{E_{^3\mathrm{He}}(\vec{q})} \frac{e^{-2\beta^2 |\vec{q}|^2}}{\sqrt{s} - \omega_\eta(\vec{q}) - E_{^3\mathrm{He}}(\vec{q}) + i\epsilon};$$



$$a_{\eta N} = \frac{1}{4\pi}\frac{m_N}{\sqrt{s_{\eta N}}} t_{\eta N}|_{\sqrt{s_{\eta N}}=m_N+m_\eta}$$

$$a_{\eta^3\mathrm{He}} = \frac{1}{4\pi}\frac{M_{^3\mathrm{He}}}{\sqrt{s}} T|_{\sqrt{s}=M_{^3\mathrm{He}}+m_\eta}$$

$$a'_{\eta N} = \frac{1}{4\pi}\frac{m_N}{\sqrt{s_{\eta N}}}\frac{\tilde{V}}{3}|_{\sqrt{s_{\eta N}}=m_N+m_\eta}$$

S-wave

$$t_{dp\to\eta^3\mathrm{He}} = V_P e^{-\beta^2|\vec{p}_\eta|^2} + V_P G\tilde{T} e^{-\beta^2|\vec{p}_\eta|^2} = V_P e^{-\beta^2|\vec{p}_\eta|^2}(1+G\tilde{T}) = \frac{V_P e^{-\beta^2|\vec{p}_\eta|^2}}{1-\tilde{V}G},$$

$$\sigma = \frac{m_p M_{^3\mathrm{He}}}{12\pi s}(|A'|^2 + 2|B'|^2)|\vec{p}_\eta||\vec{p}|e^{-2\beta^2|\vec{p}_\eta|^2}$$

$$A' = \frac{A}{1-\tilde{V}G}; \qquad B' = \frac{B}{1-\tilde{V}G}.$$

INCLUSION OF p-WAVE

$$V_{1P} = C\vec{\epsilon}\cdot\vec{p}_\eta + iD(\vec{\epsilon}\times\vec{\sigma})\cdot\vec{p}_\eta$$

$$\frac{d\sigma}{d\Omega} = \frac{m_p M_{^3\mathrm{He}}}{48\pi^2 s}\frac{|\vec{p}_\eta|}{|\vec{p}|}\Bigg((|A'|^2 + 2|B'|^2)|\vec{p}|^2 e^{-\beta^2|\vec{p}_\eta|^2} + (|C|^2 + 2|D|^2)|\vec{p}_\eta|^2$$
$$+2\mathrm{Re}(A'C^* + 2B'D^*)|\vec{p}||\vec{p}_\eta|\cos(\theta_\eta)\Bigg),$$

Asymmetry

$$\alpha = \frac{d}{d(\cos\theta_\eta)}\ln\left(\frac{d\sigma}{d\Omega}\right)\Big|_{\cos(\theta_\eta)=0}.$$

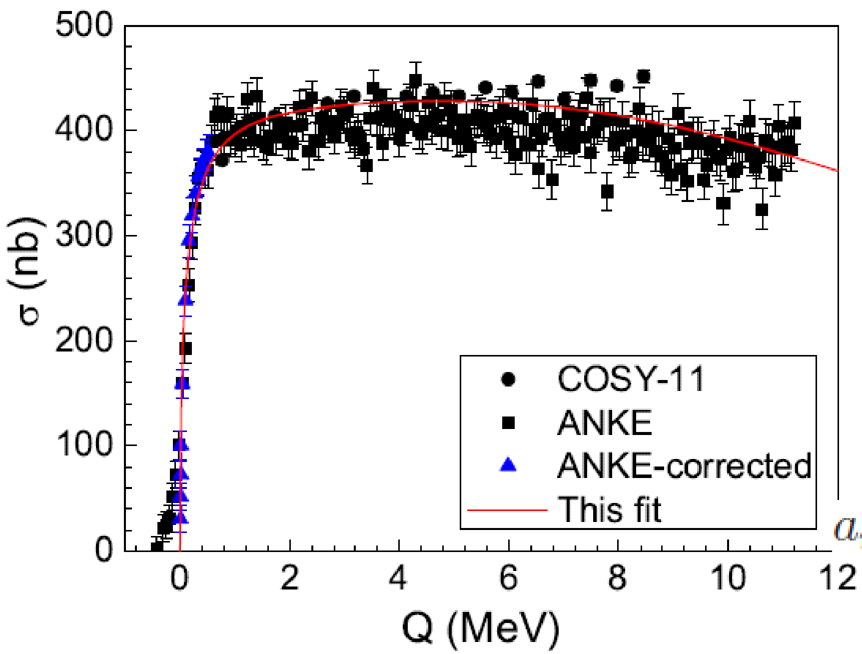
$$\alpha = \frac{2\mathrm{Re}(A'C^* + 2B'D^*)|\vec{p}||\vec{p}_\eta|}{(|A'|^2 + 2|B'|^2)|\vec{p}|^2 e^{-2\beta^2|\vec{p}_\eta|^2} + (|C|^2 + 2|D|^2)|\vec{p}_\eta|^2}.$$

$$\sigma = \frac{m_p M_{^3\mathrm{He}}}{12\pi s}\frac{|\vec{p}_\eta|}{|\vec{p}|}\left((|A'|^2 + 2|B'|^2)|\vec{p}|^2 e^{-2\beta^2|\vec{p}_\eta|^2} + (|C|^2 + 2|D|^2)|\vec{p}_\eta|^2\right)$$

RESULTS

Next, we perform six-parameter ($A = B = r_A$, $C = D = r_C e^{i\theta}(1 + \beta Q)$, and $\tilde{V} = \mathrm{Re}(V) + i\mathrm{Im}(V)$) χ^2 fits to the experimental data on the total cross sections and asymmetry

Parameters	Fitted values	parameters	Fitted values
$r_A[\mathrm{MeV}^{-2}]$	$(9.44 \pm 2.85) \times 10^{-7}$	$\beta[\mathrm{MeV}^{-1}]$	$(-5.25 \pm 2.47) \times 10^{-2}$
$r_C[\mathrm{MeV}^{-2}]$	$(6.85 \pm 4.79) \times 10^{-6}$	$\mathrm{Re}(V)[\mathrm{MeV}^{-1}]$	$(-14.58 \pm 6.04) \times 10^{-2}$
θ[degree]	347 ± 29	$\mathrm{Im}(V)[\mathrm{MeV}^{-1}]$	$(-5.37 \pm 2.31) \times 10^{-2}$



$$a'_{\eta N} = -(0.48 \pm 0.20) - i(0.18 \pm 0.08) \ \ \mathrm{fm}.$$

$a_{\eta N} = (-0.264 - i0.245)$ fm in Ref. [14]

$a_{\eta N} = (-0.20 - i0.26)$ fm in Ref. [5].

$a_{\eta N} = (-0.87 - i0.27)$ fm [22]

$a_{\eta N} = (-0.691 - i0.174)$ fm [23]

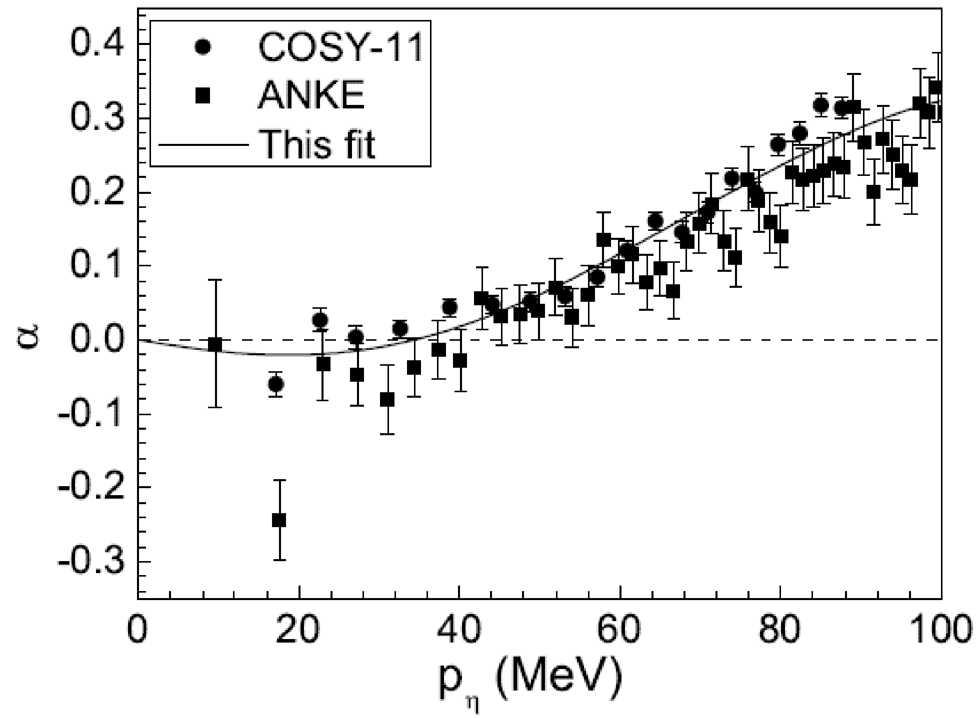
$a_{\eta N} = (-0.968 - i0.281)$ fm

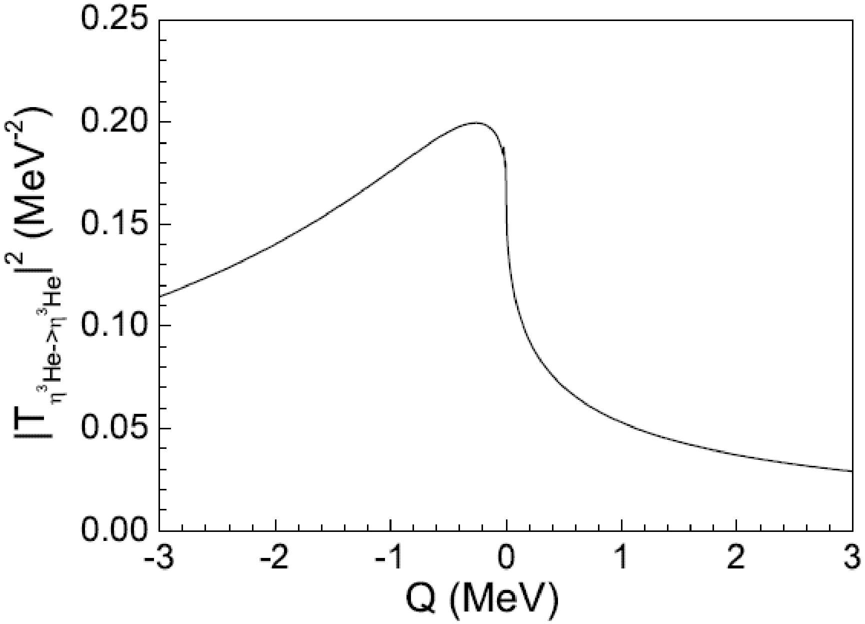
$a_{\eta N} = (-0.910 \pm 0.050 + i(0.290 \pm 0.04))$ fm [24]

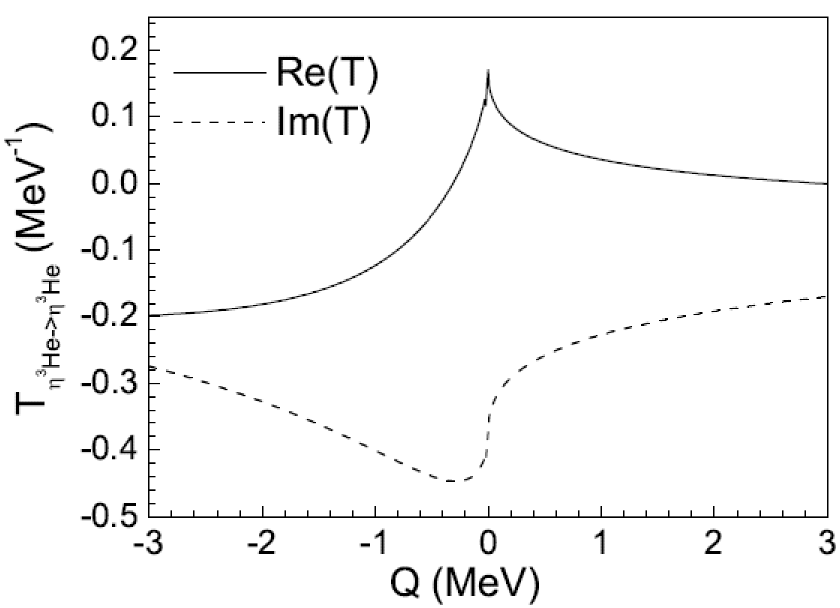
[5] T. Waas, N. Kaiser and W. Weise, Phys. Lett. B **379**, 34 (1996).

[14] T. Inoue and E. Oset, Nucl. Phys. A **710**, 354 (2002)

[22] A. M. Green and S. Wycech, Phys. Rev. C **60**, 035208 (1999)
[23] M. Batinic, I. Slaus and A. Svarc, Phys. Rev. C **52**, 2188 (1995).
[24] M. Batinic, I. Slaus, A. Svarc and B. M. K. Nefkens, Phys. Rev. C **51**, 2310 (1995) Erratum: [Phys. Rev. C **57**, 1004 (1998)].


COSY-11
ANKE
This fit
α
p_η (MeV)


$|T_{\eta^3He\to\eta^3He}|^2$ (MeV^{-2})
Q (MeV)

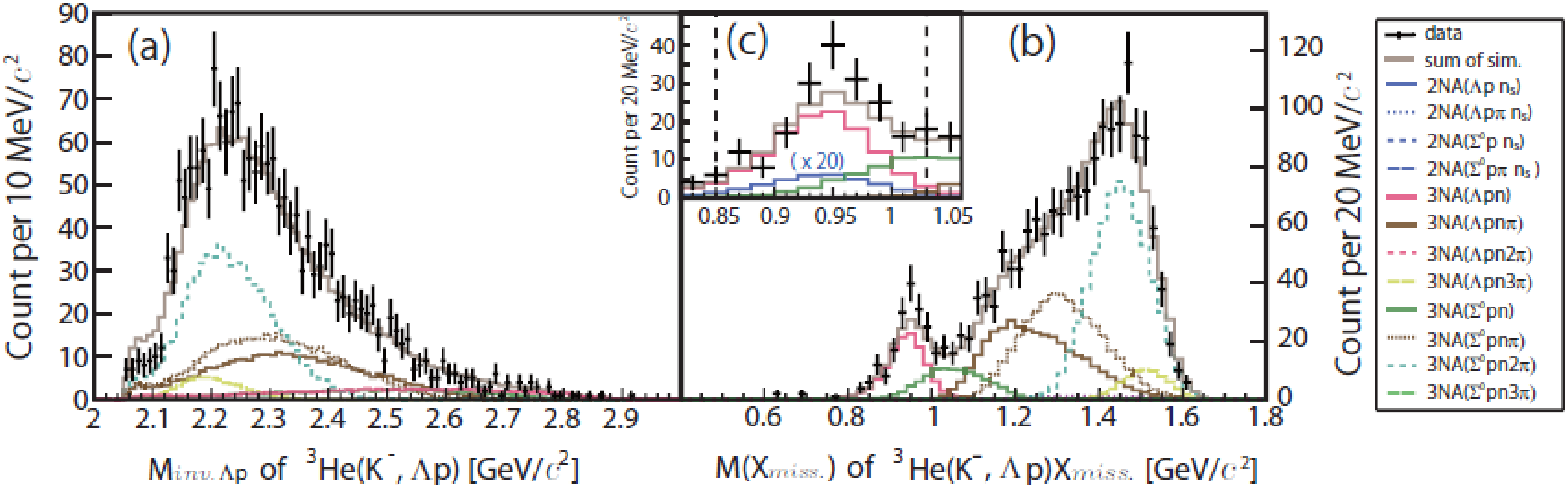

Re(T)
Im(T)
$T_{\eta^3He\to\eta^3He}$ (MeV^{-1})
Q (MeV)

$$T = \frac{g^2}{\sqrt{s} - M_R + i\Gamma/2} = \frac{g^2(\sqrt{s} - M_R)}{(\sqrt{s} - M_R)^2 + \Gamma^2/4} - i\frac{g^2\Gamma/2}{(\sqrt{s} - M_R)^2 + \Gamma^2/4}$$

$B = 0.3$ MeV with $\Gamma = 3$ MeV

On the structure observed in the in-flight ^{3}He(K^-, Λp)n reaction at J-PARC

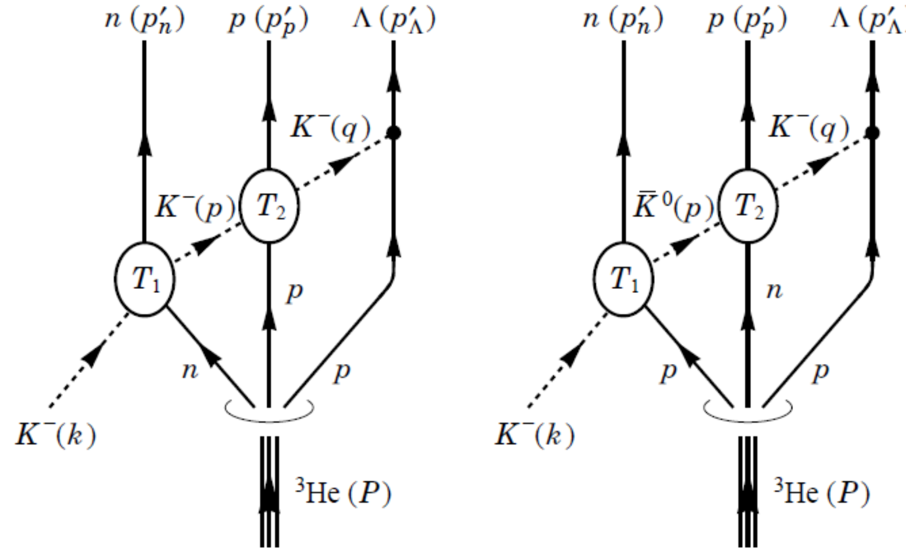
Takayasu Sekihara[1,*] Eulogio Oset[2], and Angels Ramos[3]



Structure near $K^- + p + p$ threshold in the in-flight ^{3}He(K^-,Λp)n reaction

JPARC E15, PTEP 2016, 051D01

Theoretical interpretation



If we want to produce a Kbar NN system, we must get the first rescattered Kbar as much at rest as possible . This happens at 1Gev/c and backward scattering in CM. The n goes forward in the lab system.

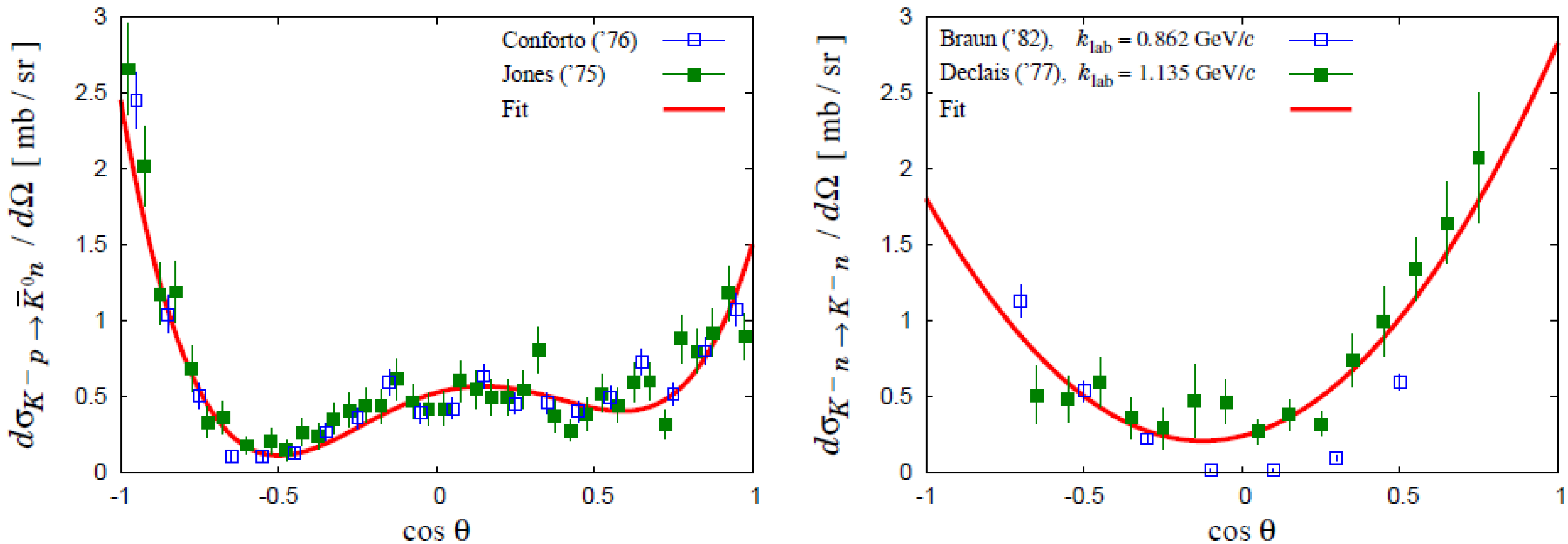


Fig. B1 Differential cross sections of the $K^-p \to \bar{K}^0 n$ (left) and $K^-n \to K^-n$ (right)

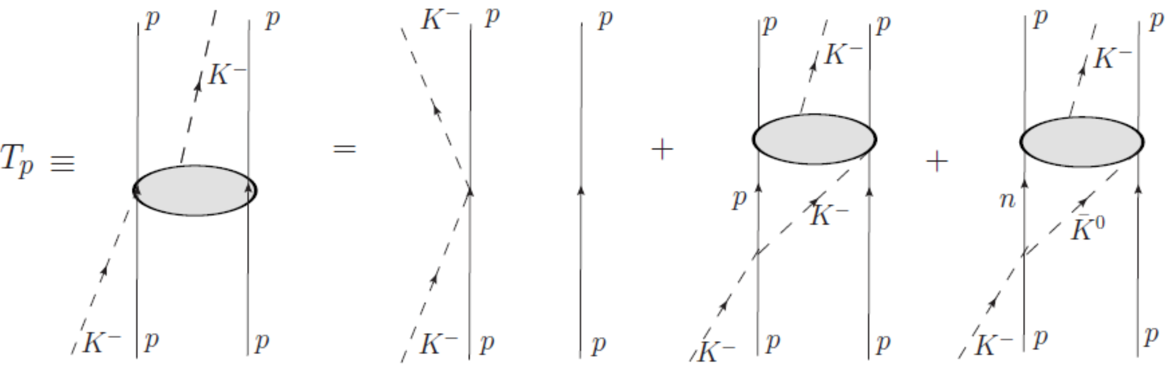
$$
\left|{}^3\mathrm{He}(\chi)\right\rangle = \frac{1}{\sqrt{6}}\tilde{\Psi}(p_\lambda, p_\rho)\big[|n(p_1,\chi)p(p_2,\chi_\uparrow)p(p_3,\chi_\downarrow)\rangle - |n(p_1,\chi)p(p_3,\chi_\downarrow)p(p_2,\chi_\uparrow)\rangle
- |p(p_2,\chi_\uparrow)n(p_1,\chi)p(p_3,\chi_\downarrow)\rangle + |p(p_3,\chi_\downarrow)n(p_1,\chi)p(p_2,\chi_\uparrow)\rangle
+ |p(p_2,\chi_\uparrow)p(p_3,\chi_\downarrow)n(p_1,\chi)\rangle - |p(p_3,\chi_\downarrow)p(p_2,\chi_\uparrow)n(p_1,\chi)\rangle\big]
$$

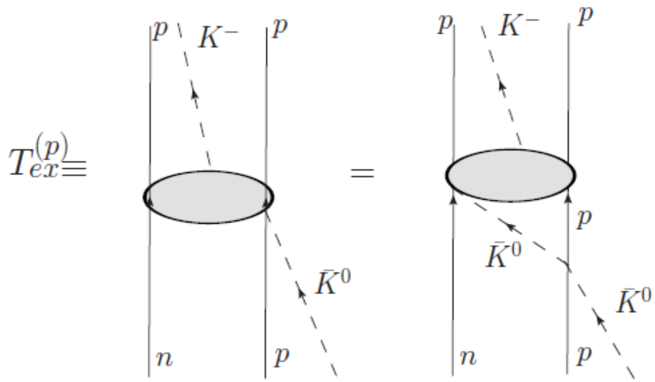
Different orders of the interactions amount to a factor 6 in the cross section when this wave function is explicitly considered. Jacobi coordinates are used for the 3He.

$$
-i\mathcal{T}_1 = \int \frac{d^3q}{(2\pi)^3}\frac{i}{(q^0)^2-\omega_{K^-}(\boldsymbol{q})^2}\int\frac{d^3p}{(2\pi)^3}\frac{i}{(p^0)^2-\omega_{K^-}(\boldsymbol{p})^2+im_{K^-}\Gamma_K}\,\tilde{\Psi}(p_\lambda,p_\rho)
\times\left[-i\chi_p^\dagger T_2^{(K^-p\to K^-p)}(w_2)\chi_\uparrow\right]\left[-i\chi_n^\dagger T_1^{(K^-n\to K^-n)}(w_1,\cos\theta_1)\chi\right]
\times\left[\tilde{V}\mathcal{F}(\boldsymbol{q})\boldsymbol{q}\left(\chi_\Lambda^\dagger\boldsymbol{\sigma}\chi_\downarrow\right)\right],
$$

We introduce a witdth for the Kbar to account for Kbar aborption by two nucleons based on the work of Bayar, Oset PRC 88 , 044003 (2013)

$\bar{K}NN$ Absorption within the Framework of the Fixed Center Approximation to Faddeev equations

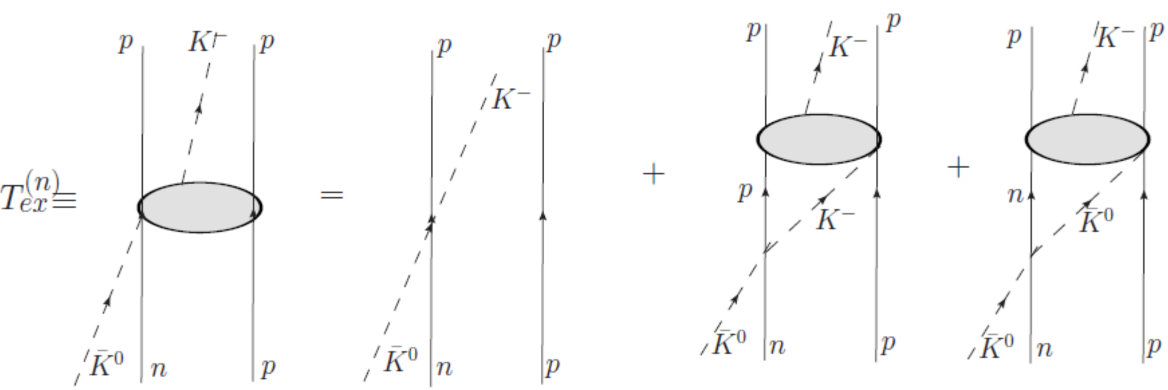




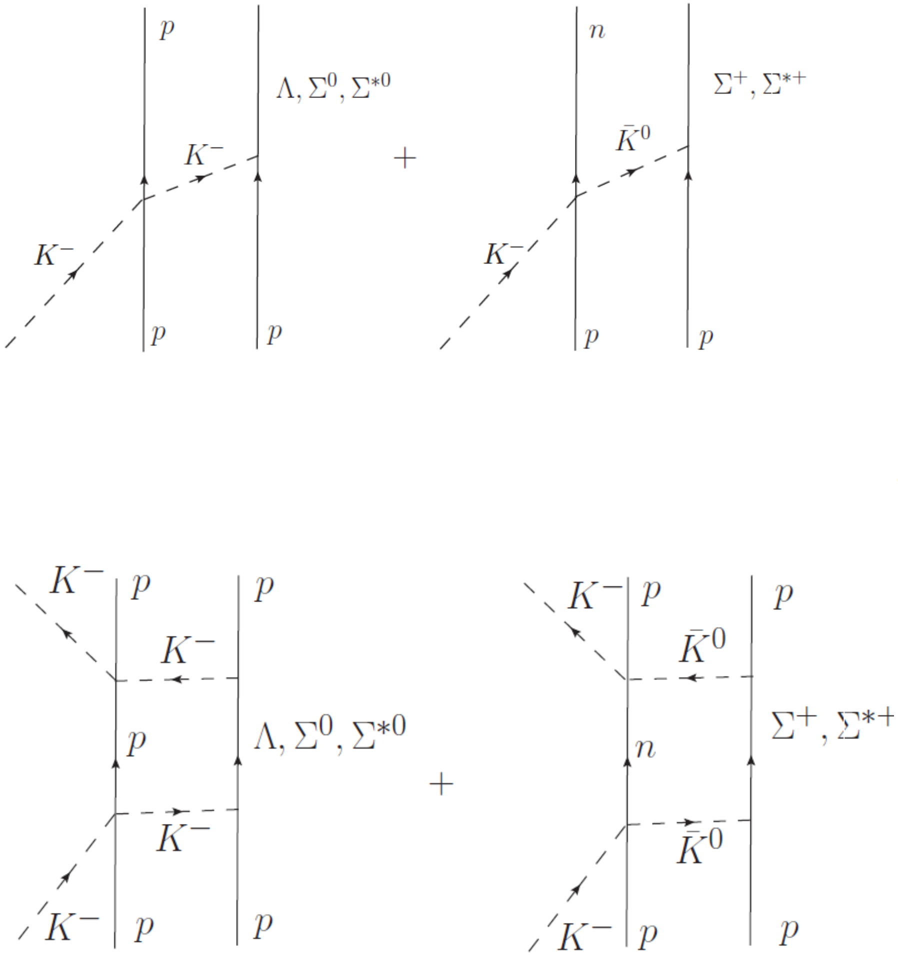
$$T_p = t_p + t_p G_0 T_p + t_{ex} G_0 T_{ex}^{(p)}$$

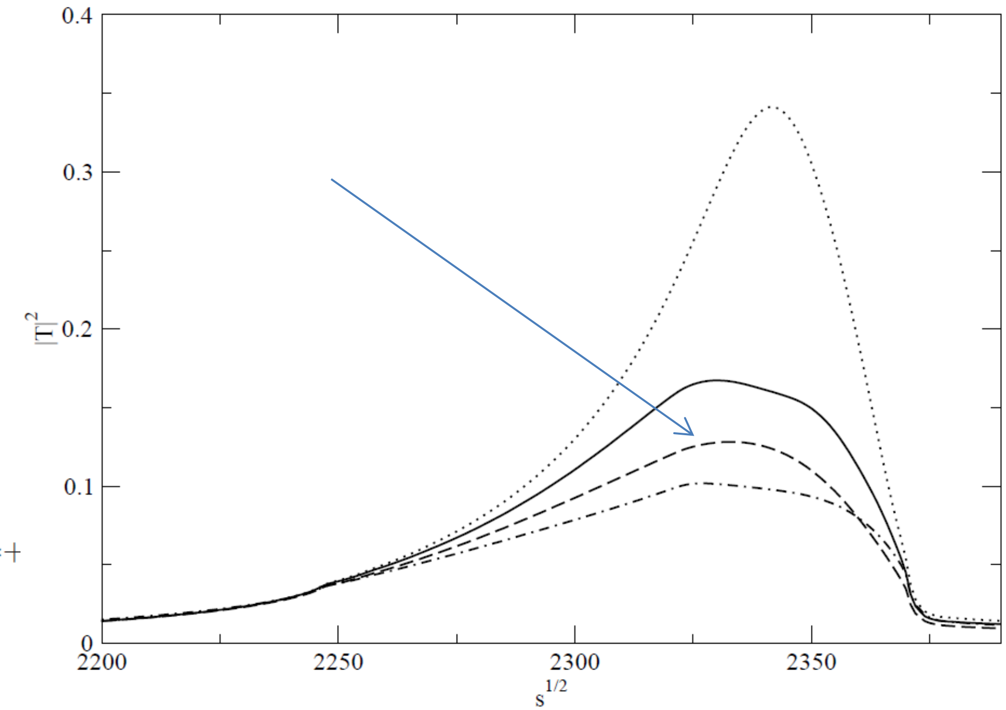
$$T_{ex}^{(p)} = t_0^{(p)} G_0 T_{ex}^{(n)}$$

$$T_{ex}^{(n)} = t_{ex} + t_{ex} G_0 T_p + t_0^{(n)} G_0 T_{ex}^{(p)}$$



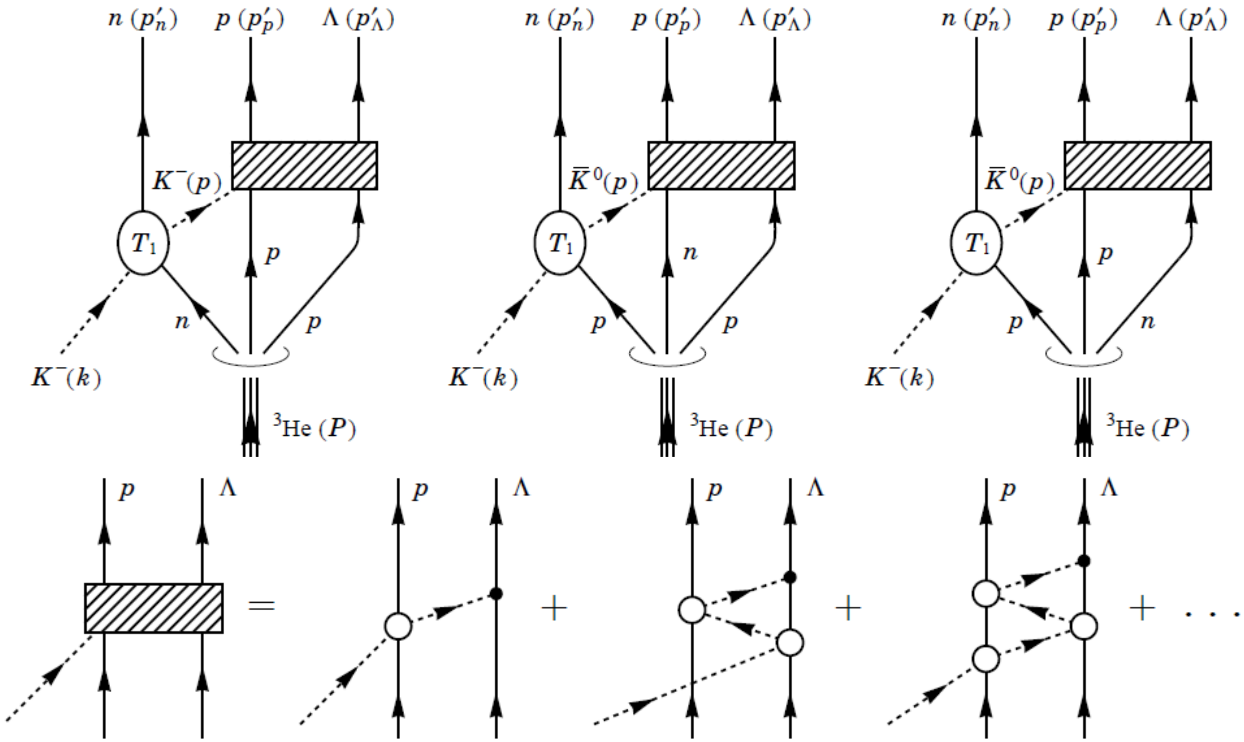
$$G_0 = \int \frac{d^3q}{(2\pi)^3} F_{NN}(q) \frac{1}{q^{0\,2} - \vec{q}^{\,2} - m_{\bar{K}}^2 + i\epsilon}$$





B= 20 MeV, Γ=75-80 MeV

In Bayar's paper it is shown that this can be taken into account taking a Kbar width of about 15 MeV. This is the only work in which the Kbar absorption by two nucleons is evaluated. It provides about 30 MeV more to the width of the Kbar NN state.



Six configurations: K^-pp, $\bar{K}^0np$, $\bar{K}^0pn$, ppK^-, $np\bar{K}^0$, and $pn\bar{K}^0$

$$T_{ij}^{\mathrm{FCA}} = V_{ij}^{\mathrm{FCA}} + \sum_{k=1}^{6} \tilde{V}_{ik}^{\mathrm{FCA}} G_0 T_{kj}^{\mathrm{FCA}} = \sum_{k=1}^{6} \left[1 - \tilde{V}^{\mathrm{FCA}} G_0 \right]^{-1}_{ik} V_{kj}^{\mathrm{FCA}}$$

with

$$V^{\mathrm{FCA}} = \begin{pmatrix} t_1 & t_2 & 0 & 0 & 0 & 0 \\ t_2 & t_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & t_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_1 & 0 & t_2 \\ 0 & 0 & 0 & 0 & t_4 & 0 \\ 0 & 0 & 0 & t_2 & 0 & t_3 \end{pmatrix}, \quad \tilde{V}^{\mathrm{FCA}} = \begin{pmatrix} 0 & 0 & 0 & t_1 & t_2 & 0 \\ 0 & 0 & 0 & t_2 & t_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & t_4 \\ t_1 & 0 & t_2 & 0 & 0 & 0 \\ 0 & t_4 & 0 & 0 & 0 & 0 \\ t_2 & 0 & t_3 & 0 & 0 & 0 \end{pmatrix}.$$

$$t_1(M_{\Lambda p}) = T^{\mathrm{ChUA}}_{K^- p \to K^- p}$$

$$t_2(M_{\Lambda p}) = T^{\mathrm{ChUA}}_{K^- p \to \bar{K}^0 n}$$

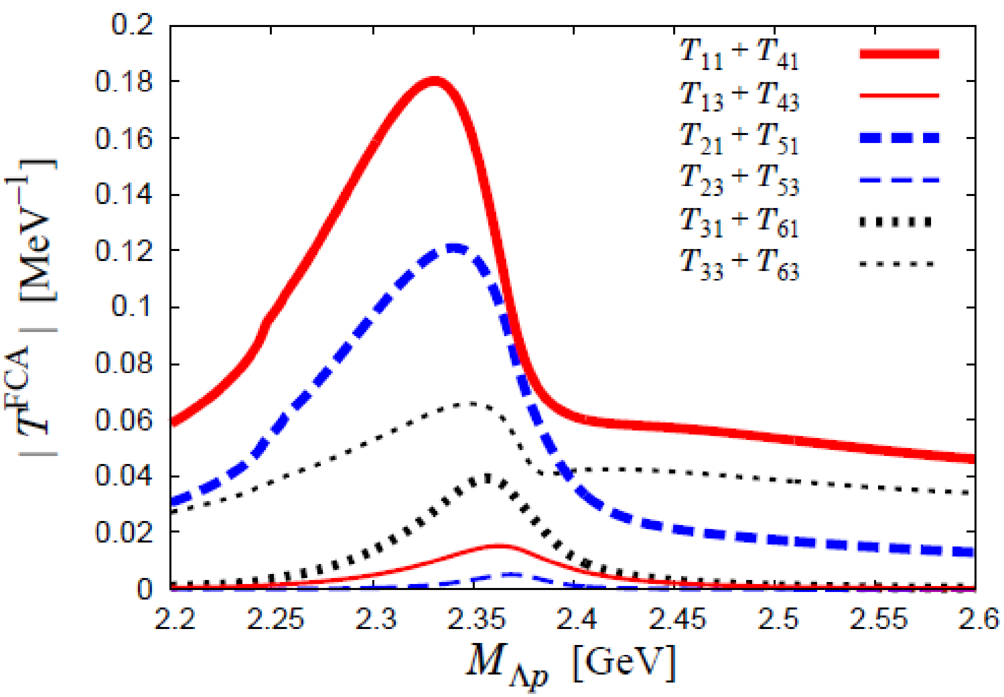
$$t_3(M_{\Lambda p}) = T^{\mathrm{ChUA}}_{\bar{K}^0 n \to \bar{K}^0 n}$$

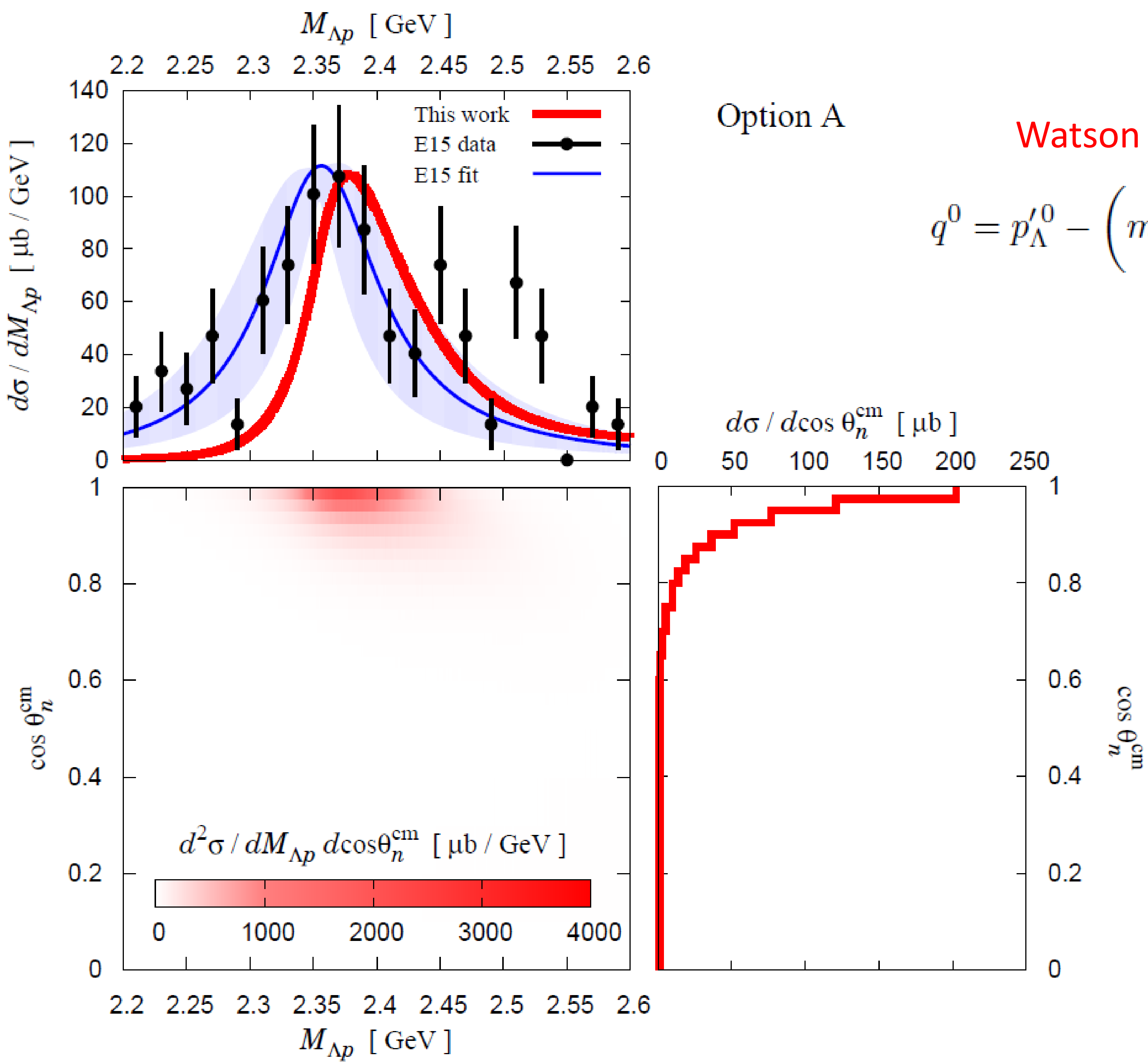
$$t_4(M_{\Lambda p}) = T^{\mathrm{ChUA}}_{\bar{K}^0 p \to \bar{K}^0 p}$$

$$\mathcal{T}_1 = i\left(\chi_n^\dagger \chi\right)\left(\chi_p^\dagger \chi_\uparrow\right) T_1^{(K^- n \to K^- n)}(w_1,\, \cos\theta_1)\tilde{V} \int \frac{d^3 q}{(2\pi)^3} \mathcal{F}(\boldsymbol{q})\boldsymbol{q}\left(\chi_\Lambda^\dagger \boldsymbol{\sigma} \chi_\downarrow\right)$$
$$\times \left[\frac{T_{11}^{\rm FCA} + T_{41}^{\rm FCA}}{(q^0)^2 - \omega_{K^-}(\boldsymbol{q})^2} + \frac{T_{13}^{\rm FCA} + T_{43}^{\rm FCA}}{(q^0)^2 - \omega_{\bar{K}^0}(\boldsymbol{q})^2}\right] \int \frac{d^3 p}{(2\pi)^3} \frac{\tilde{\Psi}(p_\lambda,\, p_\rho)}{(p^0)^2 - \omega_{K^-}(\boldsymbol{p})^2 + i m_{K^-}\Gamma_K};$$

....

$$\mathcal{T}_6 = -i\left(\chi_n^\dagger \chi_\downarrow\right)\left(\chi_p^\dagger \chi_\uparrow\right) T_1^{(K^- p \to \bar{K}^0 n)}(w_1',\, \cos\theta_1')\tilde{V} \int \frac{d^3 q}{(2\pi)^3} \mathcal{F}(\boldsymbol{q})\boldsymbol{q}\left(\chi_\Lambda^\dagger \boldsymbol{\sigma} \chi\right)$$
$$\times \left[\frac{T_{31}^{\rm FCA} + T_{61}^{\rm FCA}}{(q^0)^2 - \omega_{K^-}(\boldsymbol{q})^2} + \frac{T_{33}^{\rm FCA} + T_{63}^{\rm FCA}}{(q^0)^2 - \omega_{\bar{K}^0}(\boldsymbol{q})^2}\right] \int \frac{d^3 p}{(2\pi)^3} \frac{\tilde{\Psi}(p_\lambda,\, p_\rho)}{(p'^{\,0})^2 - \omega_{\bar{K}^0}(\boldsymbol{p})^2 + i m_{\bar{K}^0}\Gamma_K}$$





Option A

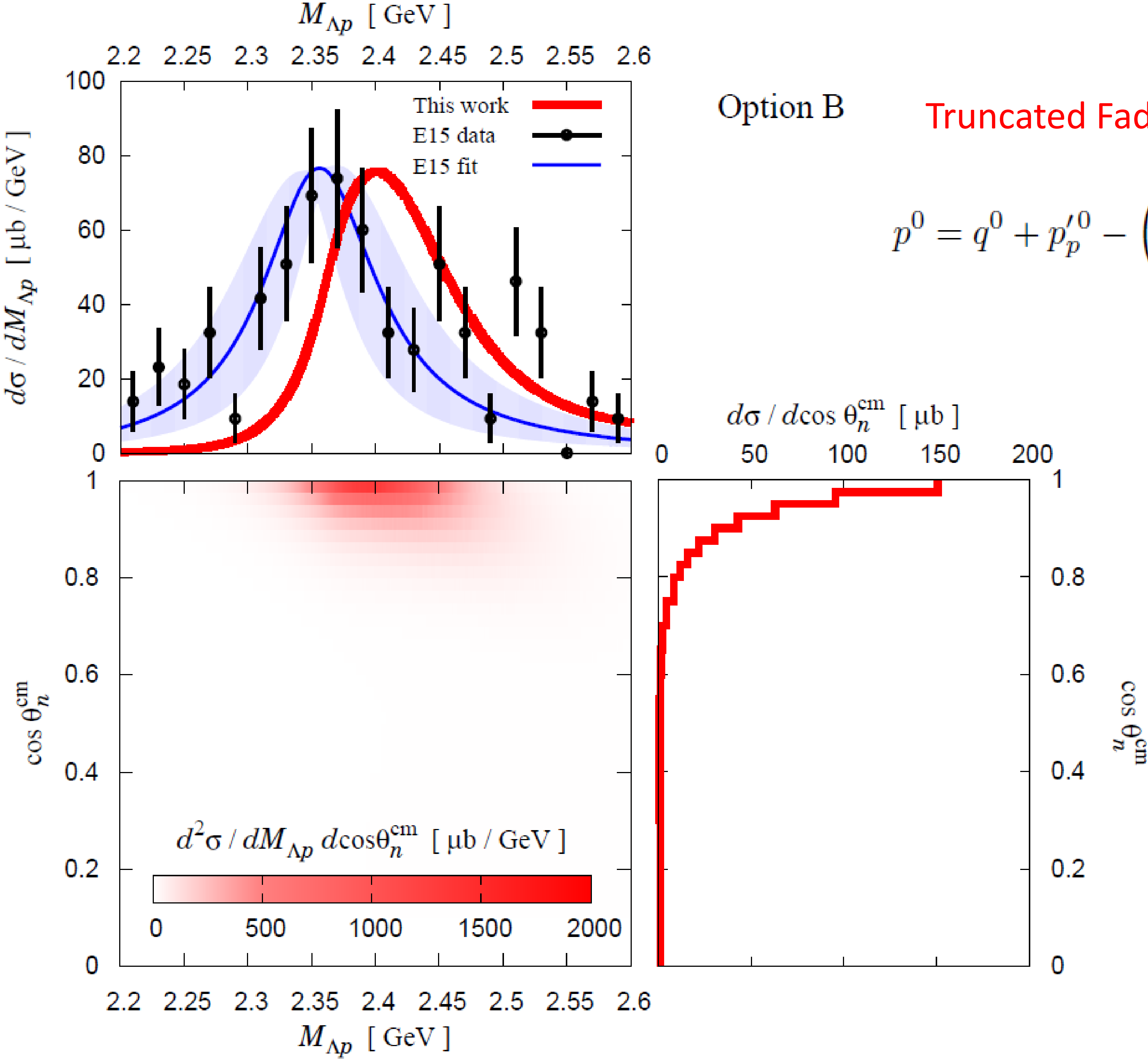
Watson approach

$$q^0 = p_\Lambda'^0 - \left(m_p - \frac{B_{^3\mathrm{He}}}{3} \right)$$

Truncated Faddeev approach

Option B

$$p^0 = q^0 + p_p'^0 - \left(m_p - \frac{B_{^3\mathrm{He}}}{3} \right)$$



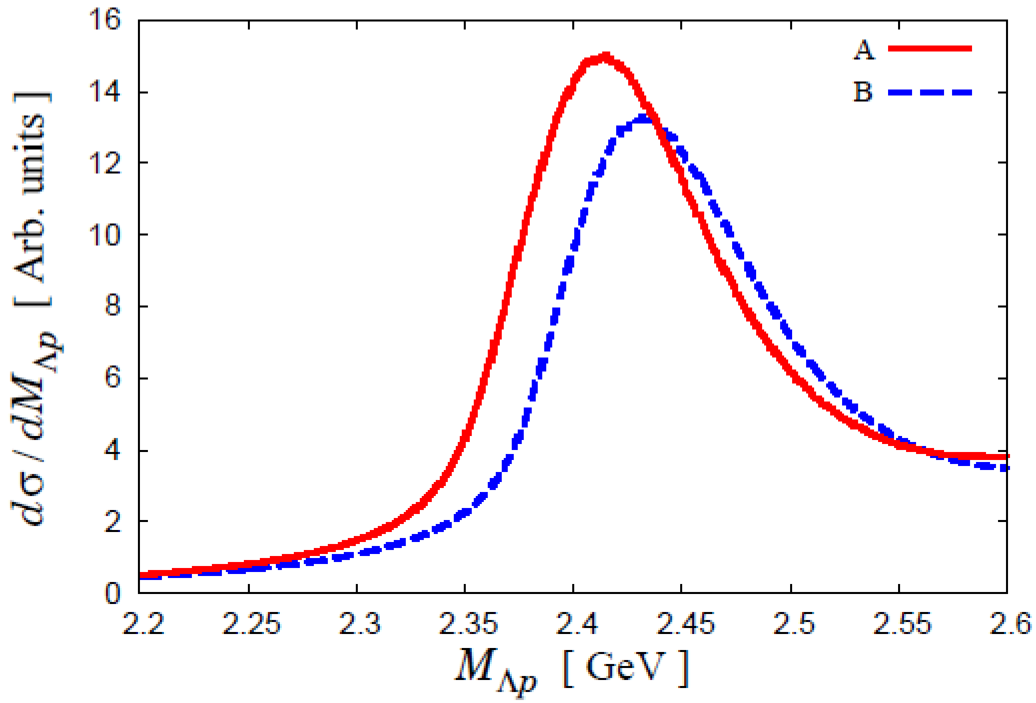
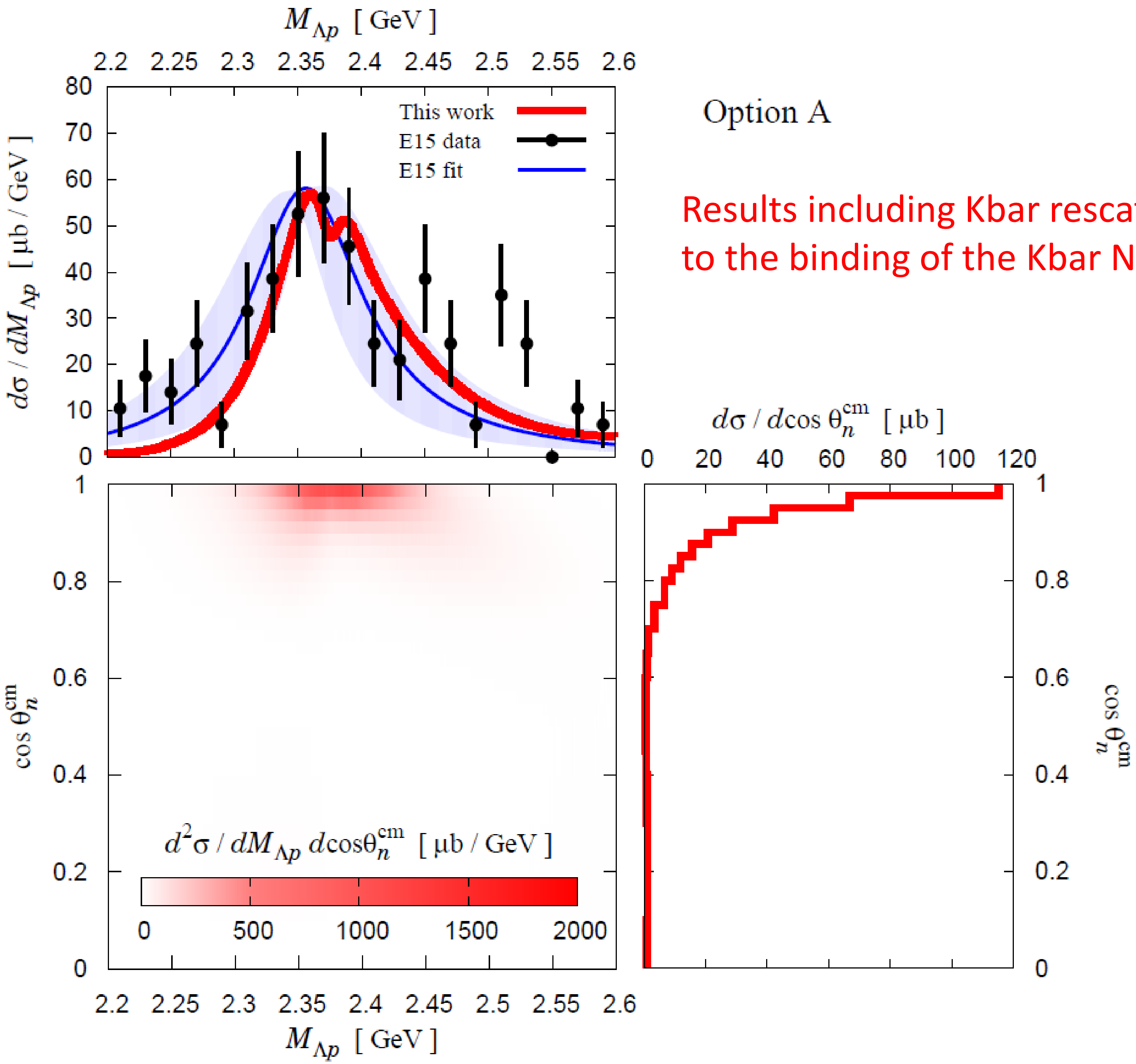


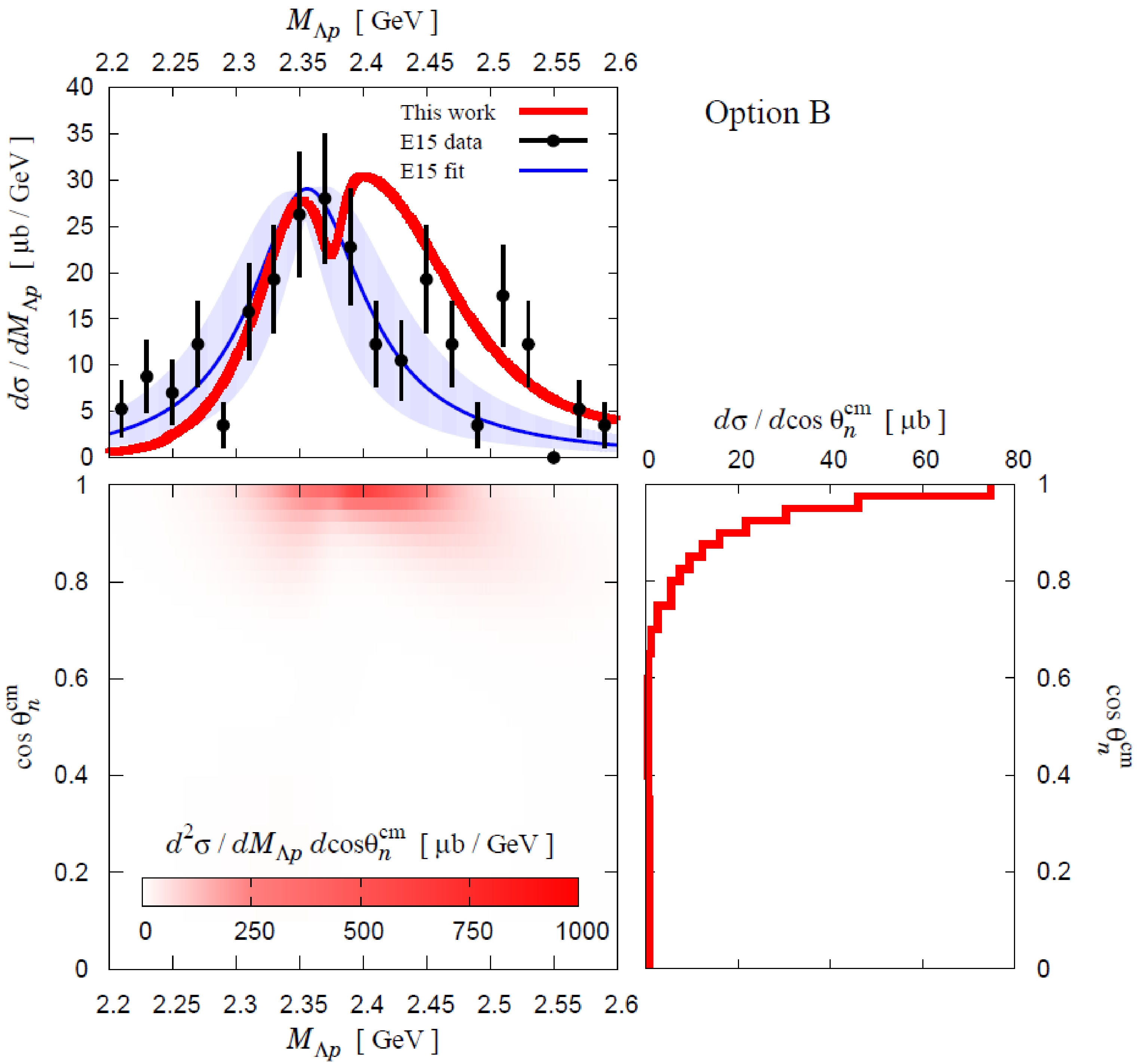
Fig. 7 Mass spectrum for the Λp invariant mass of the in-flight $^3\mathrm{He}(K^-, \Lambda p)n$ reaction with a constant T_2.

This figure shows that the shape is mostly induced by setting the Kbar propagator on shell after the first rescattering. Not due to the Λ(1405), which is not here.



Option A

Results including Kbar rescattering that leads to the binding of the Kbar NN system

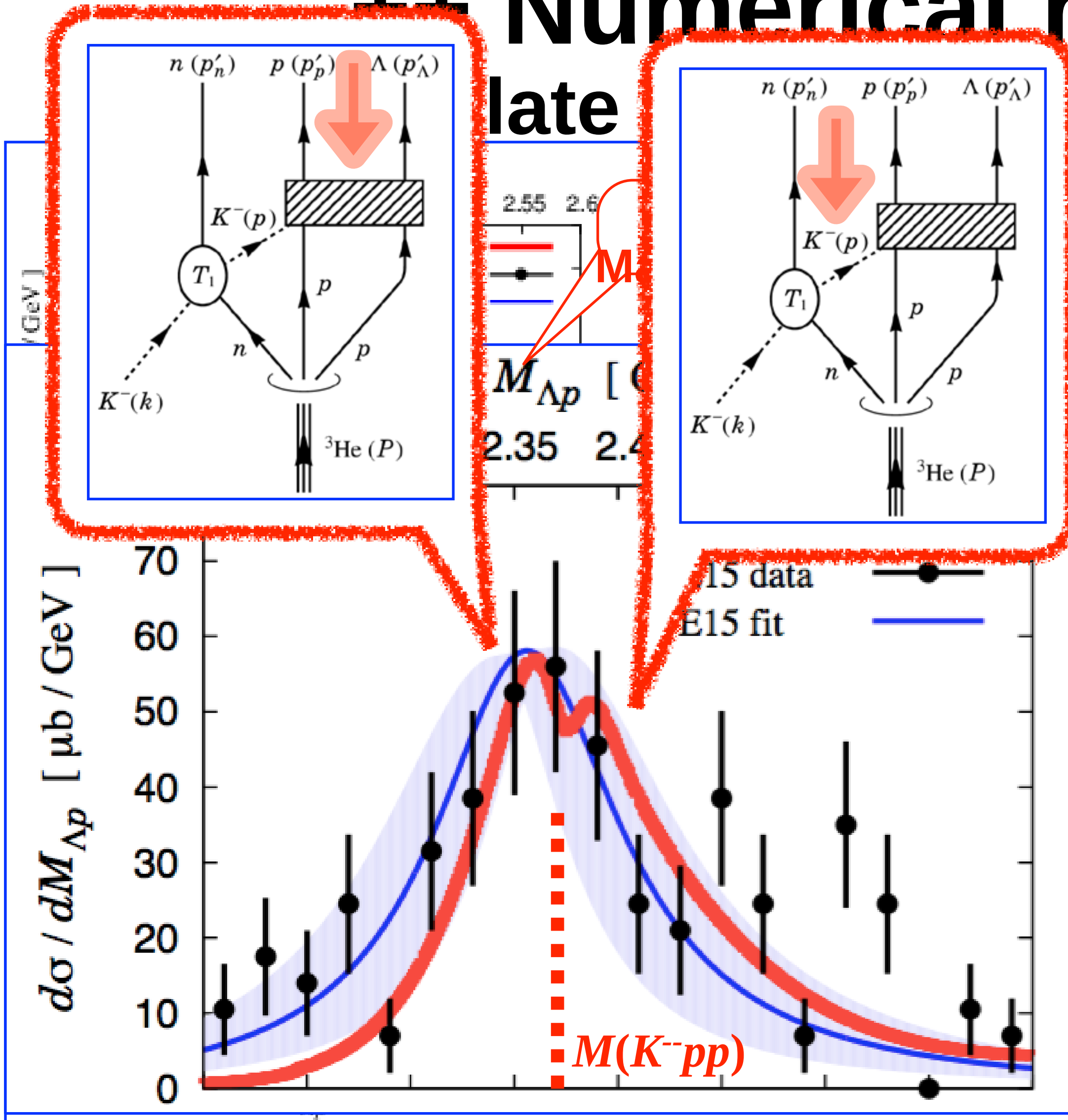


$M_{\Lambda p}$ [GeV]
Option B
This work
E15 data
E15 fit
$d\sigma / dM_{\Lambda p}$ [μb / GeV]
$d\sigma / d\cos\theta_n^{cm}$ [μb]
$\cos\theta_n^{cm}$
$d^2\sigma / dM_{\Lambda p}\, d\cos\theta_n^{cm}$ [μb / GeV]
0
250
500
750
1000
$M_{\Lambda p}$ [GeV]

3. $K\bar{N}N$ bound state

++ Numerical results ++

late pectrum and cross



□ One

Our

a

--- The

lower peak is

the

The integrated strength is σ= 7 μb , in also good agreement with experiment

Our conclusion would be that the peak observed gives support to the existence of the so much searched Kbar NN state.

The agreement of our results with experiment would say that

B ~ 20 MeV and Γ~ 80 MeV

Similar to Dote, Hyodo, Weise (include K^- absorption perturbatively)
Ikeda, Sato with energy dependent potential
Barnea, Gal, Liverts

But the width is bigger because of the accurate evaluation of K^- absorption