Axial U(1) symmetry in 2-flavor QCD at finite temperature

Sinya AOKI for JLQCD Collaboration

Center for Gravitational Physics,
Yukawa Institute for Theoretical Physics, Kyoto University

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1. Introduction

Chiral symmetry of QCD

phase transition

low T $U(1)_B \otimes SU(N_f)_V$ → high T $U(1)_B \otimes SU(N_f)_L \otimes SU(N_f)_R$

restoration of chiral symmetry

Theoretical questions

1. Recovery of U(1)_A symmetry at high T ?

relation ?

2. Eigenvalue distribution of Dirac operator $\rho(\lambda)$ λ: eigenvalue of Dirac operator

Eigenvalue density

$$\rho(\lambda) = \sum_n \rho_n \frac{\lambda^n}{n!}$$

$$\lim_{m\to 0} \langle \bar\psi \psi \rangle = \pi \rho(0)$$

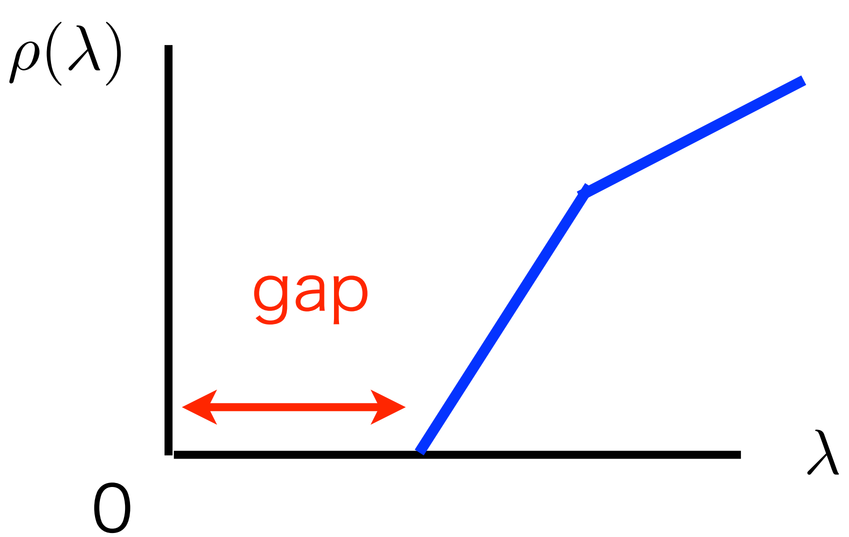
Banks-Casher relation

⇒ $\rho(0) = \rho_0 = 0$ if chiral symmetry is restored.

What are general consequences ? (This talk)

If $\rho(\lambda)$ has a gap ⇒ Anomalous $U(1)_A$ symmetry is fully restored.

(See later.)

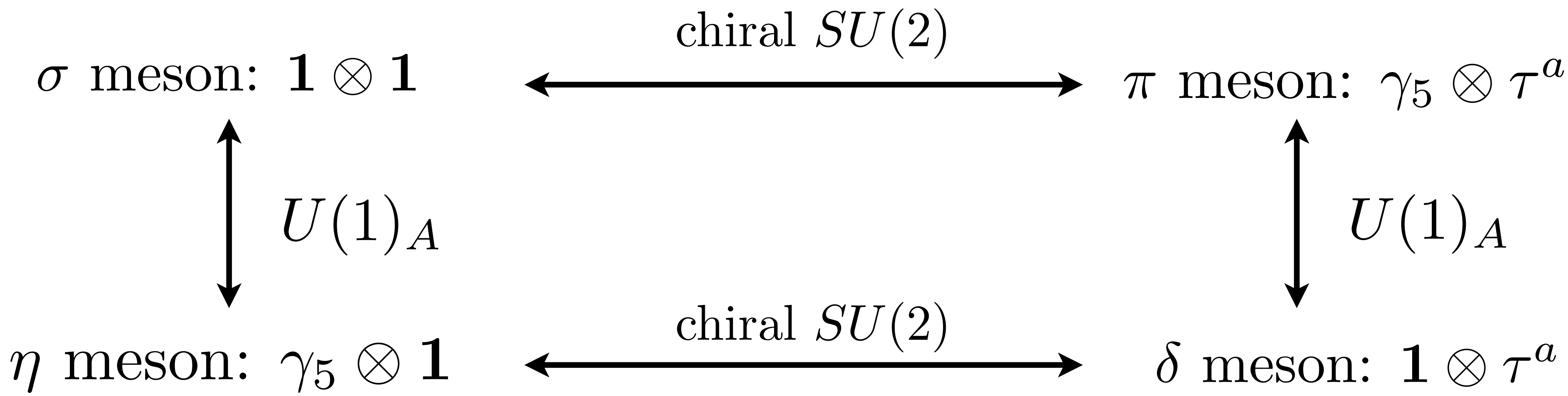


Susceptibility

$$\chi^A_\Gamma = \frac{1}{V}\int d^4x \langle M^A_\Gamma(x) M^A_\Gamma(0)\rangle$$

$N_f = 2$

$$M^A_\Gamma(x) = \bar{\psi}^a(x)^f_\alpha(\Gamma \otimes T^A)^{fg}_{\alpha\beta}\psi^a(x)^g_\beta$$



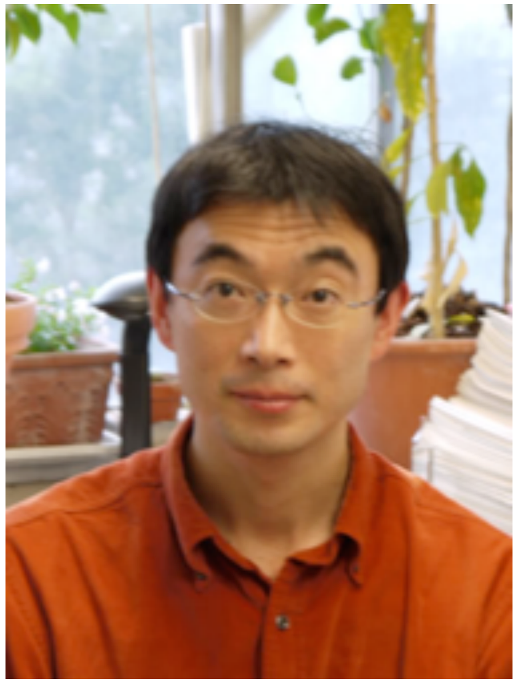
$U(1)_A$ susceptibilities

$$\chi^{\sigma-\eta} \equiv \chi^\sigma - \chi^\eta \qquad \chi^{\pi-\delta} \equiv \chi^\pi - \chi^\delta \qquad \chi^{\pi-\eta} \equiv \chi^\pi - \chi^\eta$$

If $U(1)_A$ is recovered, $\chi^{\sigma-\eta} = \chi^{\pi-\delta} = \chi^{\pi-\eta} = 0.$

2. Previous Theoretical Investigation

S.A, H. Fukaya, Y. Taniguchi,
“Chiral symmetry restoration, eigenvalue density of Dirac operator and axial U(1) anomaly at finite temperature”,
Phys. Rev D86(2012)114512.



Set up

Lattice regularization with Overlap fermion, **2-flavors**

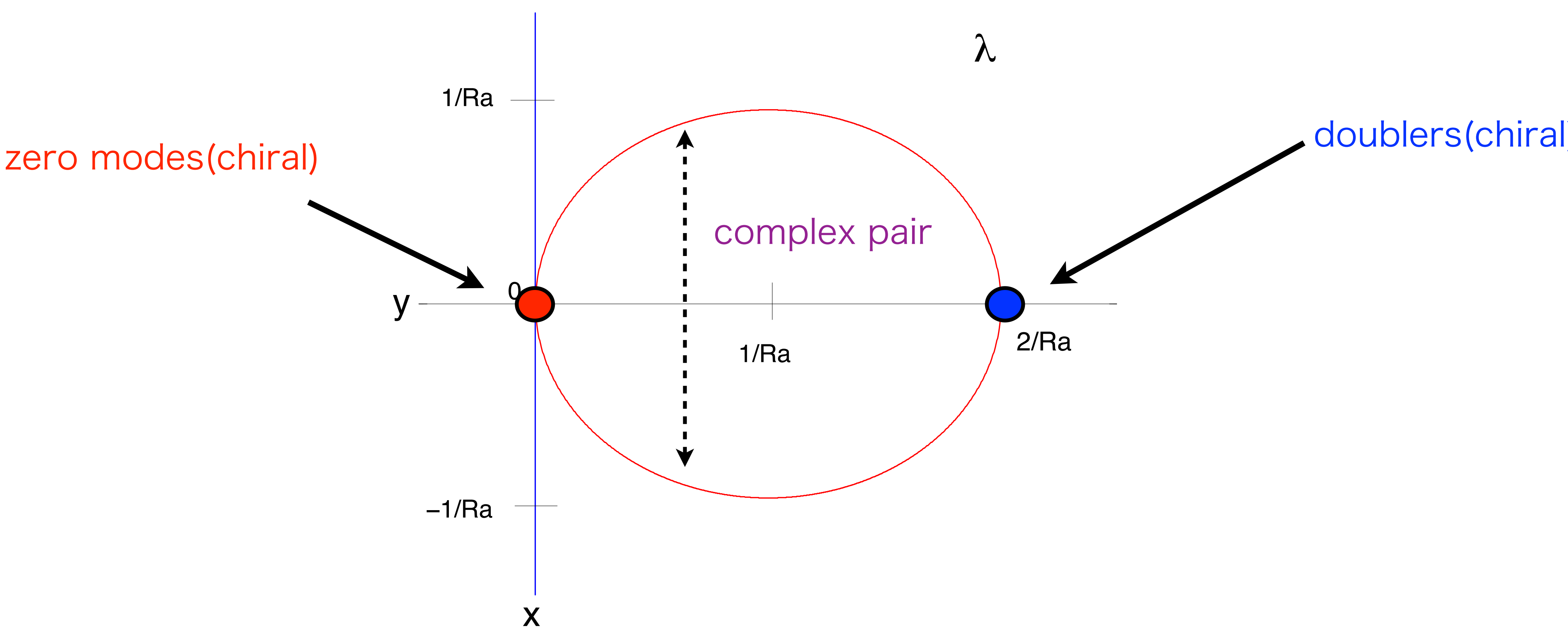
Exact “chiral” symmetry but explicit U(1)_A anomaly form Ginsparg-Wilson relation

$$D\gamma_5 + \gamma_5 D = aDR\gamma_5 D \qquad \gamma_5 D \gamma_5 = D^\dagger$$

Eigenvalue spectrum

$$\lambda_n^A + \bar{\lambda}_n^A = aR\bar{\lambda}_n^A \lambda_n^A$$

A: gauge configuration



$$D(A)\phi_n^A = \lambda_n^A \phi_n^A \quad \Rightarrow \quad D(A)\gamma_5\phi_n^A = \bar{\lambda}_n^A \gamma_5 \phi_n^A$$

Some assumptions

Assumption 1 non-singlet chiral symmetry is restored.

Assumption 2 if $\mathcal{O}(A)$ is m-independent A: gauge configuration

$$\langle \mathcal{O}(A) \rangle_m = f(m^2) \qquad f(x) \text{ is analytic at } x = 0$$

(Too strong. We should loosen this condition.)

Note that this does not hold if the chiral symmetry is spontaneously broken.

Ex.
$$\lim_{V \to \infty} \frac{1}{V} \langle Q(A)^2 \rangle_m = m \frac{\Sigma}{N_f} + O(m^2)$$

topological charge

Results

Non-singlet chiral Ward-Takahashi identities

$$\rho^A(\lambda) \equiv \lim_{V\to\infty} \frac{1}{V} \sum_n \delta\left(\lambda - \sqrt{\bar{\lambda}_n^A \lambda_n^A}\right) = \sum_{n=0}^{\infty} \rho_n^A \frac{\lambda^n}{n!}$$

eigenvalues density

$$\lim_{m\to 0} \langle \rho^A(\lambda) \rangle_m = \lim_{m\to 0} \langle \rho_3^A \rangle_m \frac{|\lambda|^3}{3!} + O(\lambda^4)$$

No constraints to higher $\langle \rho_n^A \rangle_m$

$\lim_{m\to 0} \langle \rho_3^A \rangle_m \neq 0$ even for "free" theory.

$$\langle \rho_0^A \rangle_m = 0$$

$$\lim_{V\to\infty} \frac{1}{V^k} \langle (N_{R+L}^A)^k \rangle_m = 0, \quad \lim_{V\to\infty} \frac{1}{V^k} \langle Q(A)^{2k} \rangle_m = 0$$

total number of zero modes $N_{R+L}^A = N_R^A + N_L^A$

topological charge $Q(A) = N_R^A - N_L^A$

N_R^A a number of right-handed zero modes

N_L^A a number of left-handed zero modes

Consequences

Singlet susceptibility at high T

$$\lim_{V\to 0} \chi^{\pi-\eta} = \lim_{m\to 0} \lim_{V\to\infty} \frac{N_f^2}{m^2 V} \langle Q(A)^2 \rangle_m = 0$$

This, however, does not mean U(1)_A symmetry is recovered at high T.

$$\lim_{m\to 0} \chi^{\pi-\eta} = 0 \qquad \text{"} m_\pi = m_\eta \text{"}$$

is necessary but NOT “sufficient” for the recovery of U(1)_A .

Effective symmetry at hight T

full $U(1)_A$ is not recovered.

$SU(2)_L \otimes SU(2)_R \otimes Z_4$ not $SU(2)_L \otimes SU(2)_R \otimes U(1)_A$

What is the order of chiral phase transition in 2-flavor QCD ?
1st or 2nd ?

Order of phase transition at N_f=2

U(1)$_A$ is still broken at T > T$_c$

$$SU(2)_L \otimes SU(2)_R$$

2nd order

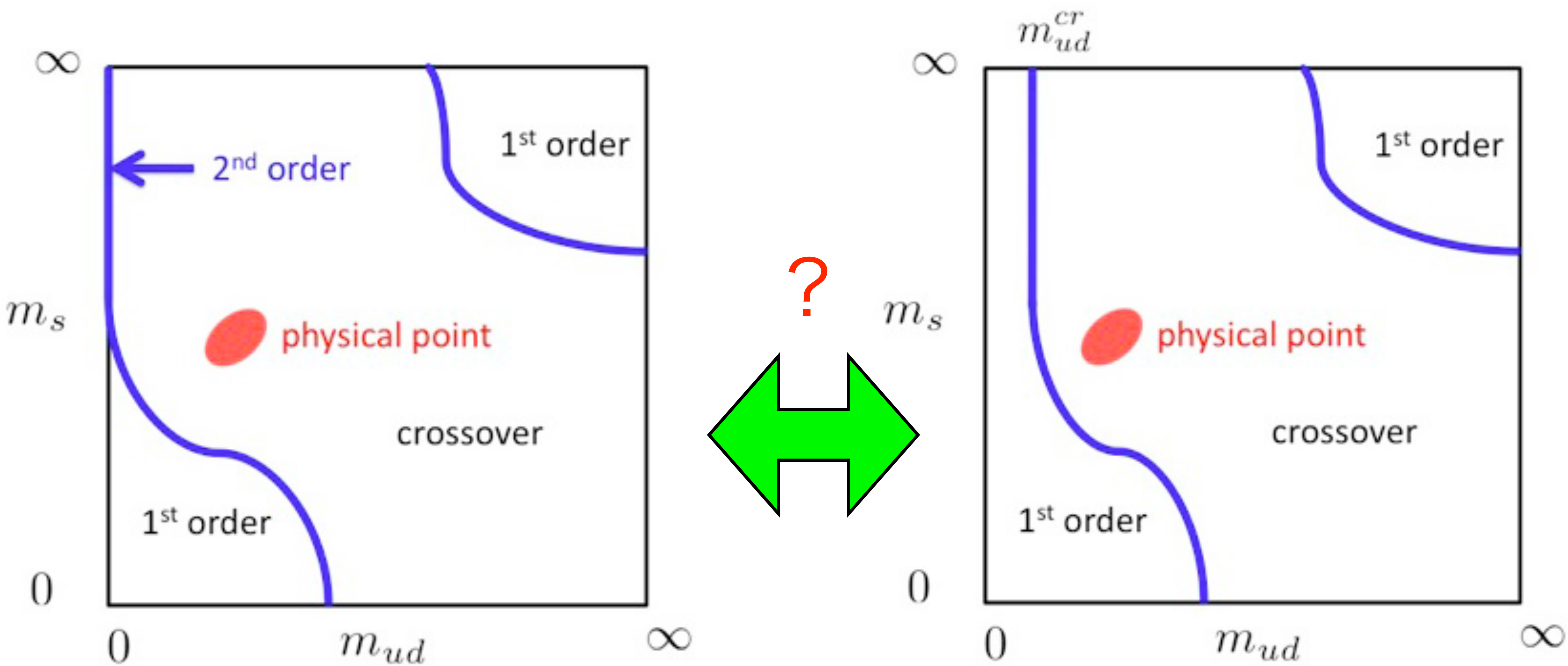
$$SU(2)_L \otimes SU(2)_R \otimes Z_4$$

U(1)$_A$ is restored at T > T$_c$

$$SU(2)_L \otimes SU(2)_R \otimes U(1)$$

1st order ?

phase diagram of 2+1 flavor QCD



Remarks

Important conditions

Large volume limit $V \to \infty$

chiral limit $m \to 0$

lattice chiral symmetry Ginsparg-Wilson relation $D\gamma_5 + \gamma_5 D = aDR\gamma_5 D$

Fractional power for the eigenvalue density

$\rho^A(\lambda) \simeq c_A \lambda^\gamma, \ \gamma > 0$ non-singlet chiral symmetry is recovered.

⇒ $\gamma \leq 2$ is excluded. ⇒ $\gamma > 2$

consistent with the integer case (n > 2)

Universal treatment ? (future investigations)

3. Recent Numerical Results

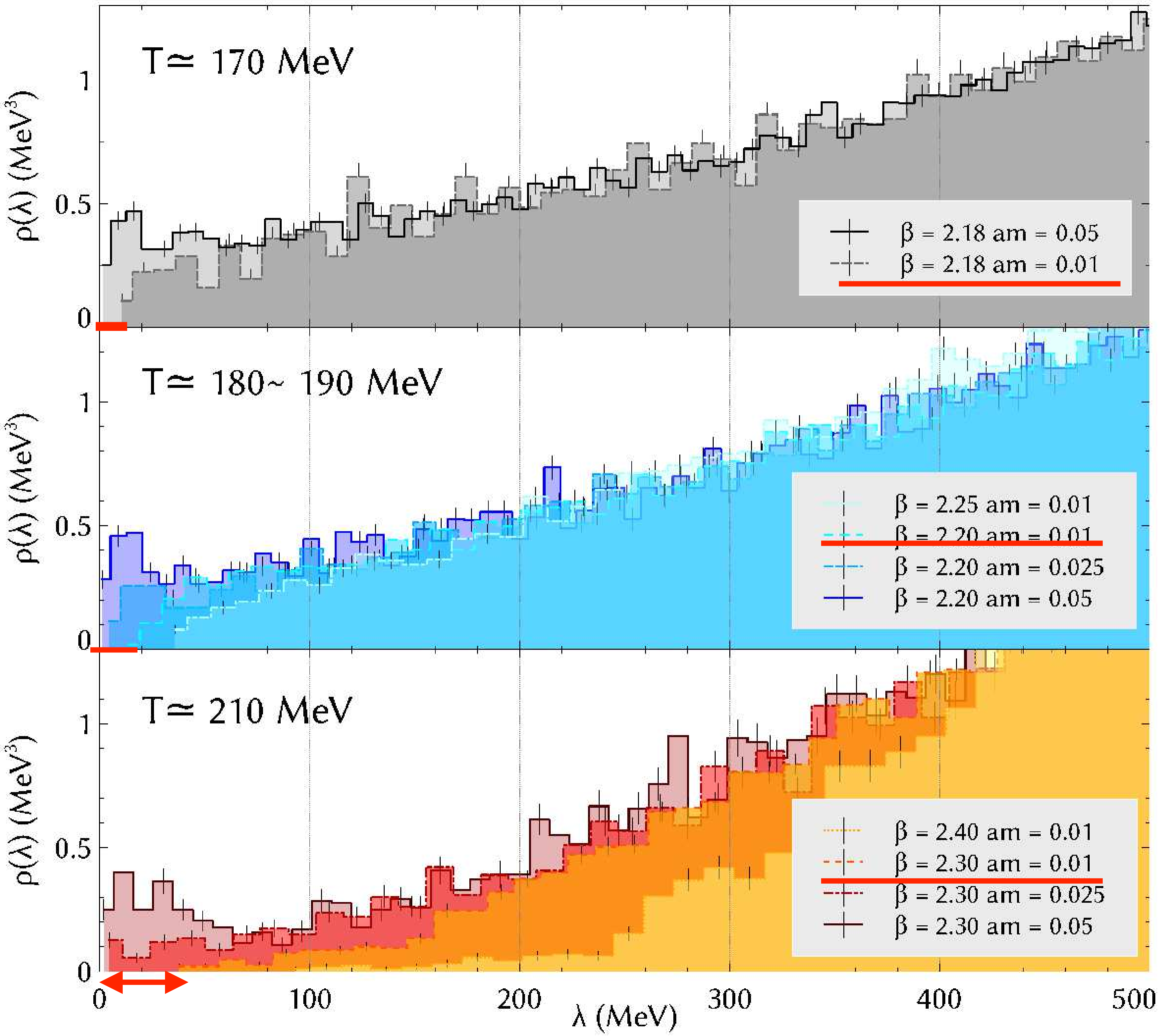
A. Tomiya et al. (JLQCD), Lat2015

G. Cossu et al. (JLQCD), Lat2015

Eigenvalue densities

$$\rho(\lambda) = \lim_{V \to \infty} \frac{1}{V} \sum_n \delta(\lambda - \lambda_n)$$

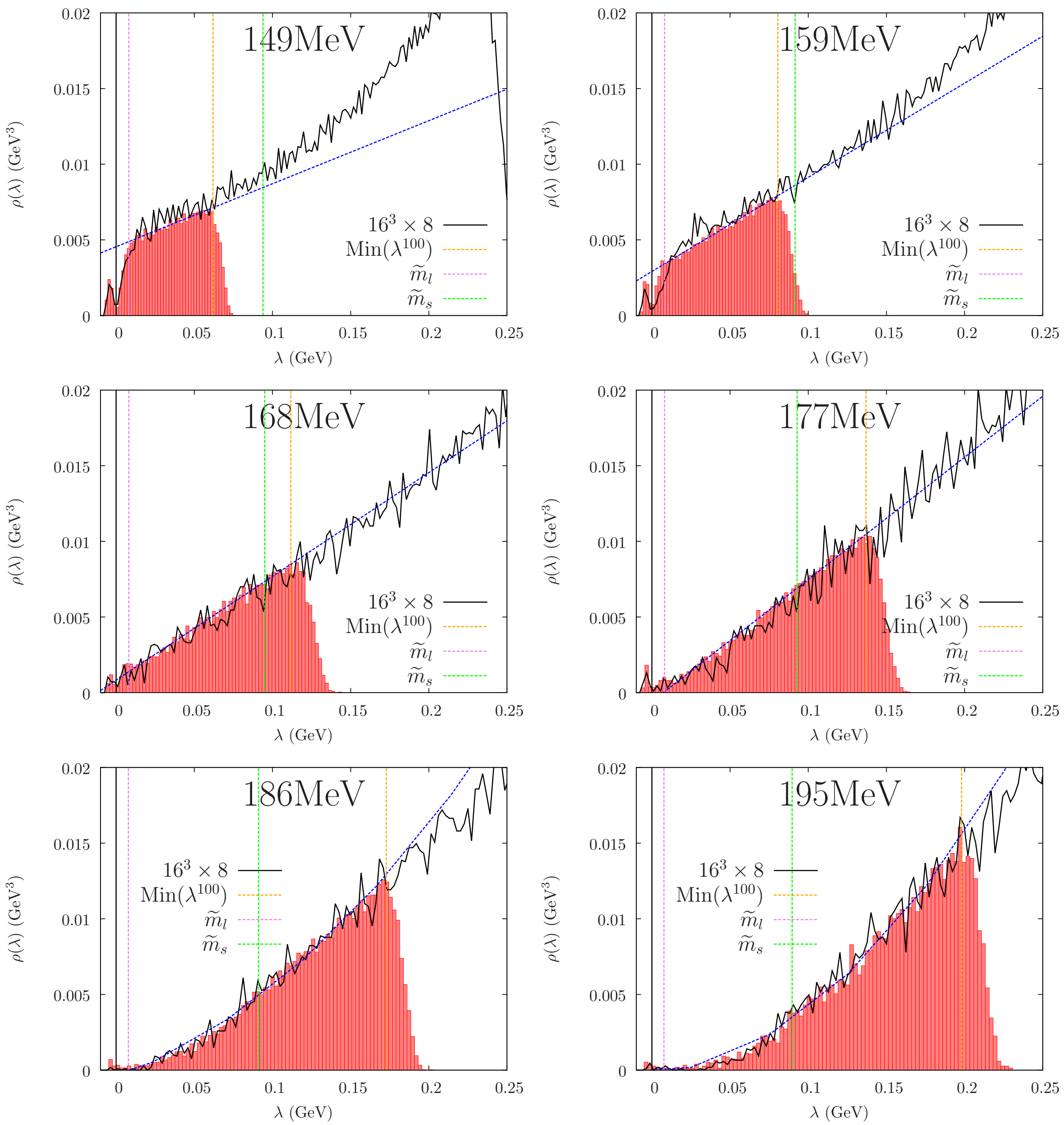
Cossu *et al.* (JLQCD), Overlap
Phys. Rev. D87 (2013) 114514



Gap seems to open at smaller quark mass.

$T_c \simeq 180$ MeV

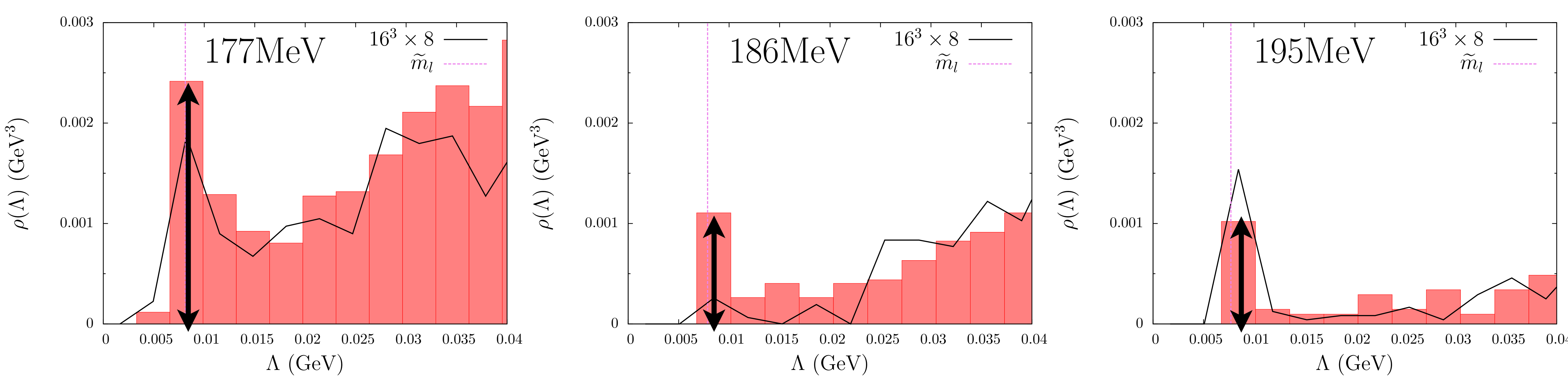
Buchoff *et al.* (LLNL/RBC), DomainWall, Phys. Rev. D89 (2014) 5,054514



Small eigenvalues appear.

Gap seems to close at or above critical temperature

$T_c \simeq 180$ MeV



Summary of recent results from chiral fermions.

Group	Fermion	Size	Gap in the spectrum	$U_A(1)$ Correlator	$U(1)_A$ $T \gtrsim Tc$
JLQCD (2013)	**Overlap (Top. fixed)**	2 fm	**Gap**	Degenerate	**Restored**
TWQCD (2013)	Optimal domain-wall	3 fm	**No gap**	Degenerate	**Restored**
LLNL/RBC, Hot QCD (2013,2014)	**(Möbius)-Domain-wall (W/ ov)**	2, 4, 11 fm	**No gap**	No degeneracy	**Violated**
Viktor Dick et al (2015)	**OV on HISQ sea**	3, 4 fm	**No gap**	No degeneracy	**Violated**

What causes this difference ?

volume ? quark mass ? lattice chiral symmetry ?

JLQCD collaboration

Overlap: exact GW relation

LLNL/RBC collaborations

DomainWall: approximated GW relation

Recent study by A. Tomiya et al. for JLQCD collaboration Preliminary

generate gauge configurations with an improved DomainWall quarks

very small violation of GW relation

(0)calculate eigenvalue distribution of DW operator on these configurations

original

(1)calculate eigenvalue distribution of overlap operator on these configurations

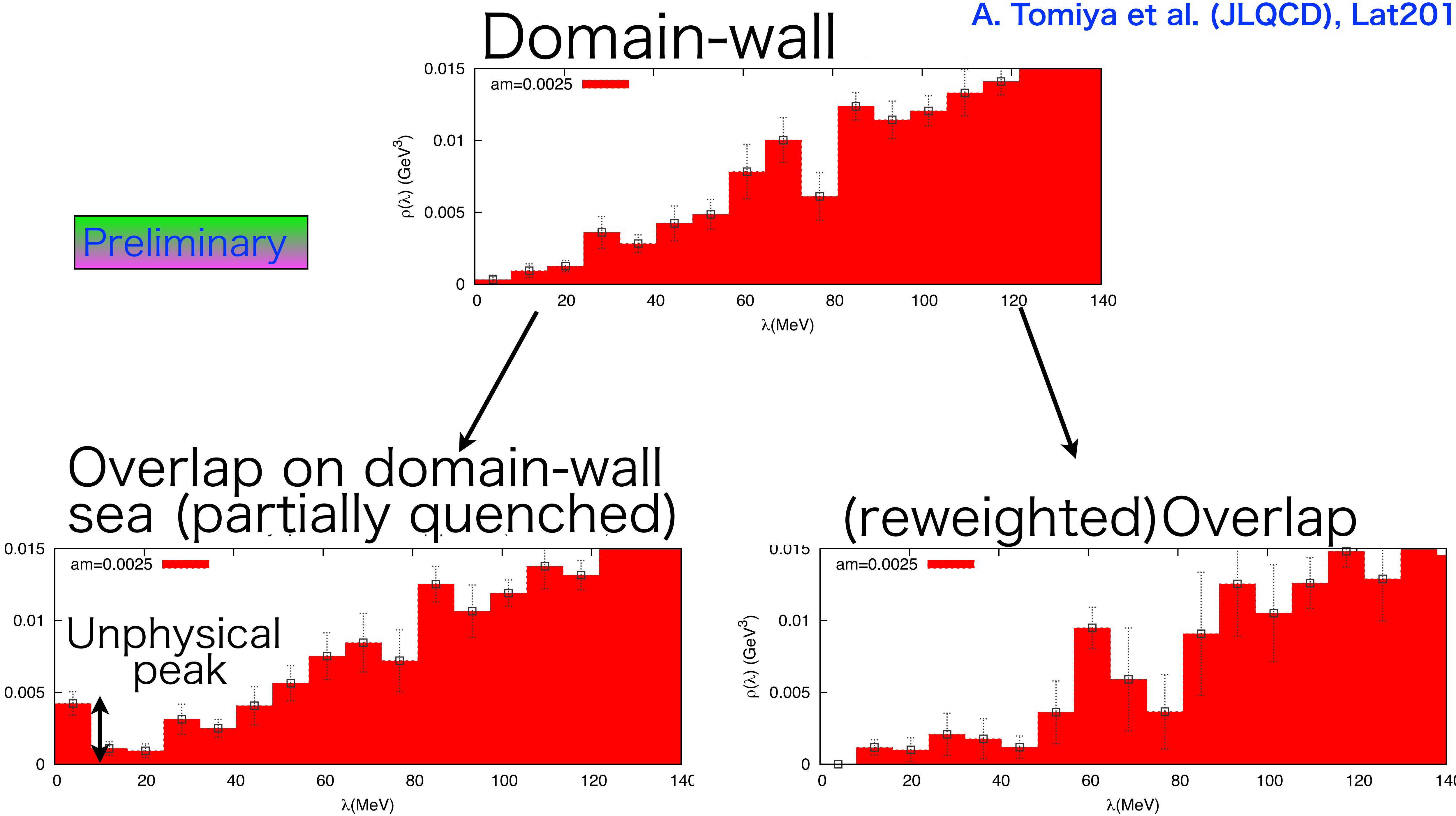
partially quenched

(2)reweighting factor from the improved DW to Overlap is introduced to obtain the full eigenvalue distribution

full Overlap

T=190 MeV for L=3 fm, T=1.05 T_c

A. Tomiya et al. (JLQCD), Lat2015



After the reweighting, small eigenvalues in PQ disappear, and the gap seems to open in full Overlap.

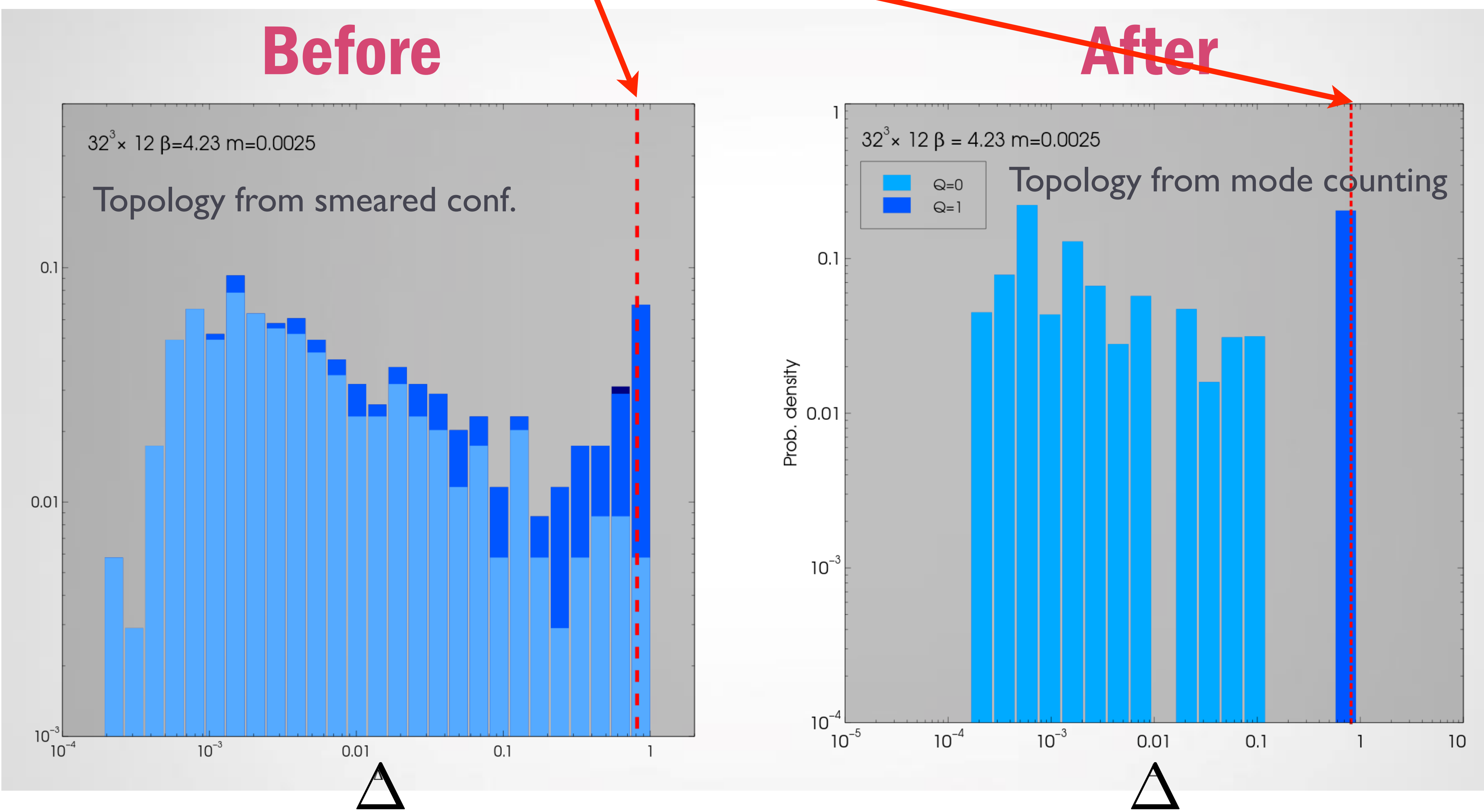
An exact lattice chiral symmetry is essential. A tiny violation of the chiral symmetry may destroy the theoretically expected relation.

U(1)_A susceptibility $\Delta := \chi^{\pi} - \chi^{\delta}$

G. Cossu et al. (JLQCD), Lat2015

$$\Delta = \boxed{\frac{2N_{R+L}}{Vm^2}} + \sum_{\lambda \neq 0} \frac{2m^2}{V(\lambda^2 + m^2)^2}$$

zero-modes

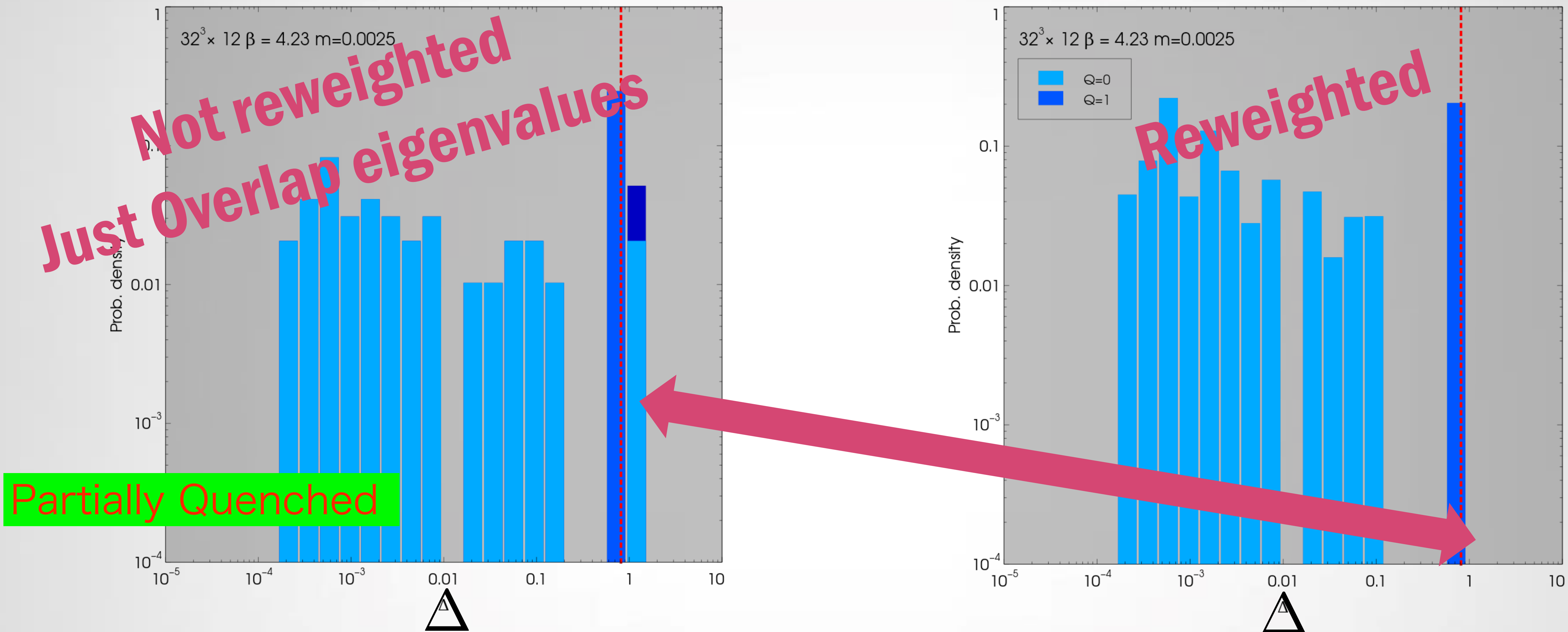


DomainWall

reweighted Overlap

REWEIGHTING IS CRUCIAL

G. Cossu et al. (JLQCD), Lat2015



Point: Reweighting is crucial

Partially quenched results show accumulation of unphysical near zero modes

If the gap opens, the effective symmetry is

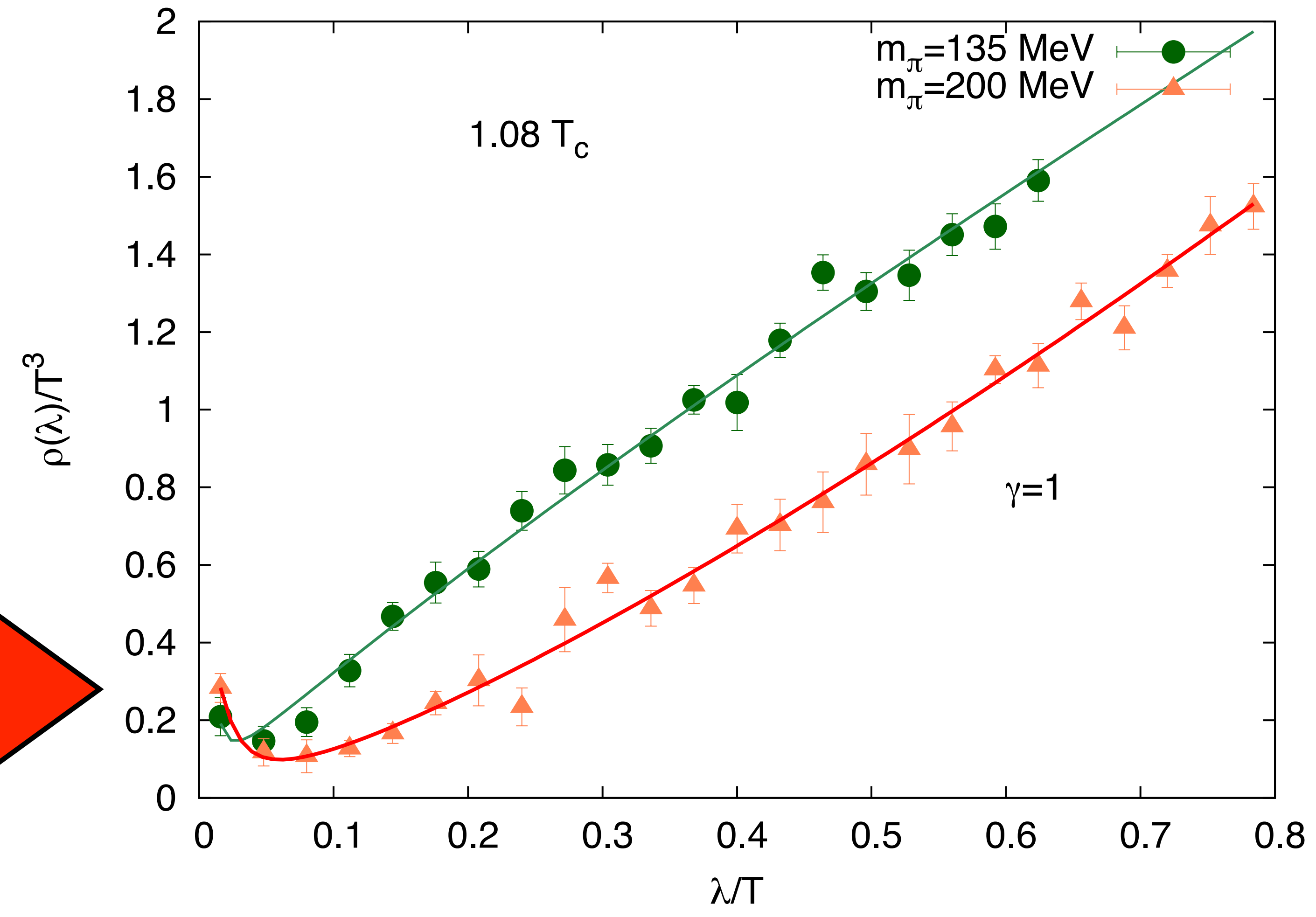
$$\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_A$$

S. Sharma, V. Dick, F. Karsch, E. Laermann, S. Mukherjee, Lattice2015
Eigenvalues density of Overlap on DomainWall (partially quenched !)

From Sharma's talk@Lat2015,

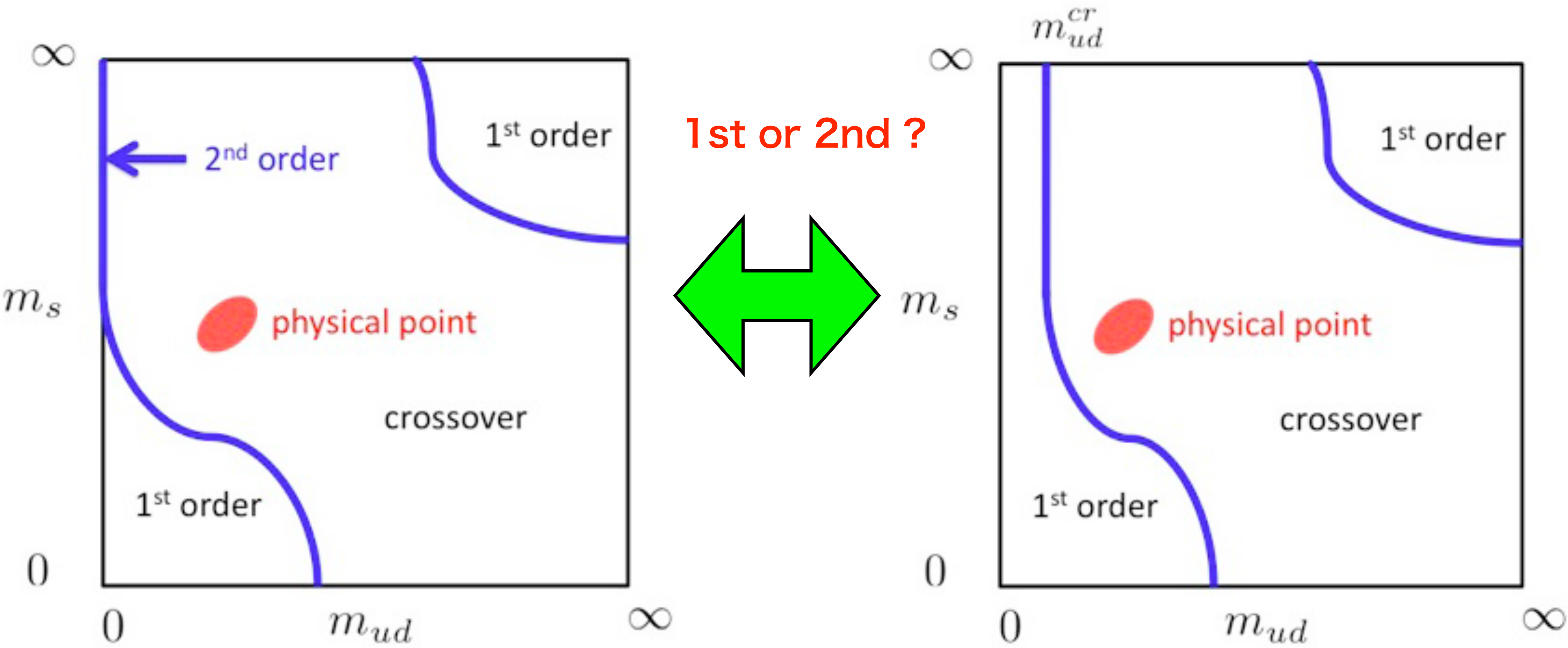
- General features: Near zero mode peak +bulk
- We fit to the ansatz: $\rho(\lambda) = \frac{A\epsilon}{\lambda^2+A} + B\lambda^\gamma$
- Bulk rises linearly as λ,no gap seen.
- No gap even when quark mass reduced!

This is an artifact due to PQ !



4. Conclusion

Order of phase transition in 2-flavor QCD



gapless EV density $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{Z}_4$

gapped EV density $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_A$

Conformal bootstrap method predicts IR fixed point for these cases.

Even if the phase transition is of 2nd order, its universality class should be different from O(4).