Role of tensor force in light nuclei with tensor-optimized antisymmetrized molecular dynamics (TOAMD)

TOAMD ... New variational model for nuclei to treat V_{NN} directly

Takayuki MYO

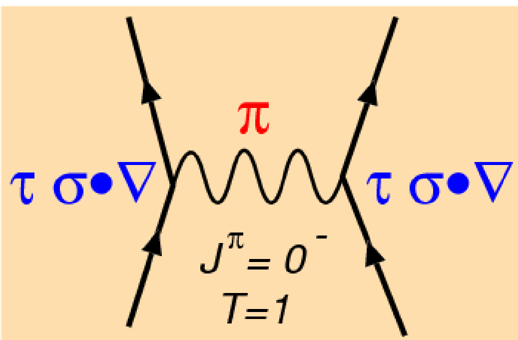
OSAKA INSTITUTE OF TECHNOLOGY

Hiroshi TOKI (RCNP)
Kiyomi IKEDA (RIKEN)
Hisashi HORIUCHI (RCNP)
Tadahiro SUHARA (Matsue Coll. of Tech.)

Pion exchange interaction & V_{tensor}

$$3(\vec{\sigma}_1 \cdot \hat{q})(\vec{\sigma}_2 \cdot \hat{q})\frac{q^2}{m^2+q^2} = (\vec{\sigma}_1 \cdot \vec{\sigma}_2)\frac{q^2}{m^2+q^2} + S_{12}\frac{q^2}{m^2+q^2}$$

$$= (\vec{\sigma}_1 \cdot \vec{\sigma}_2)\left[\frac{m^2+q^2}{m^2+q^2} - \frac{m^2}{m^2+q^2}\right] + \boxed{S_{12}\frac{q^2}{m^2+q^2}}$$



δ interaction

Yukawa interaction

involve large momentum

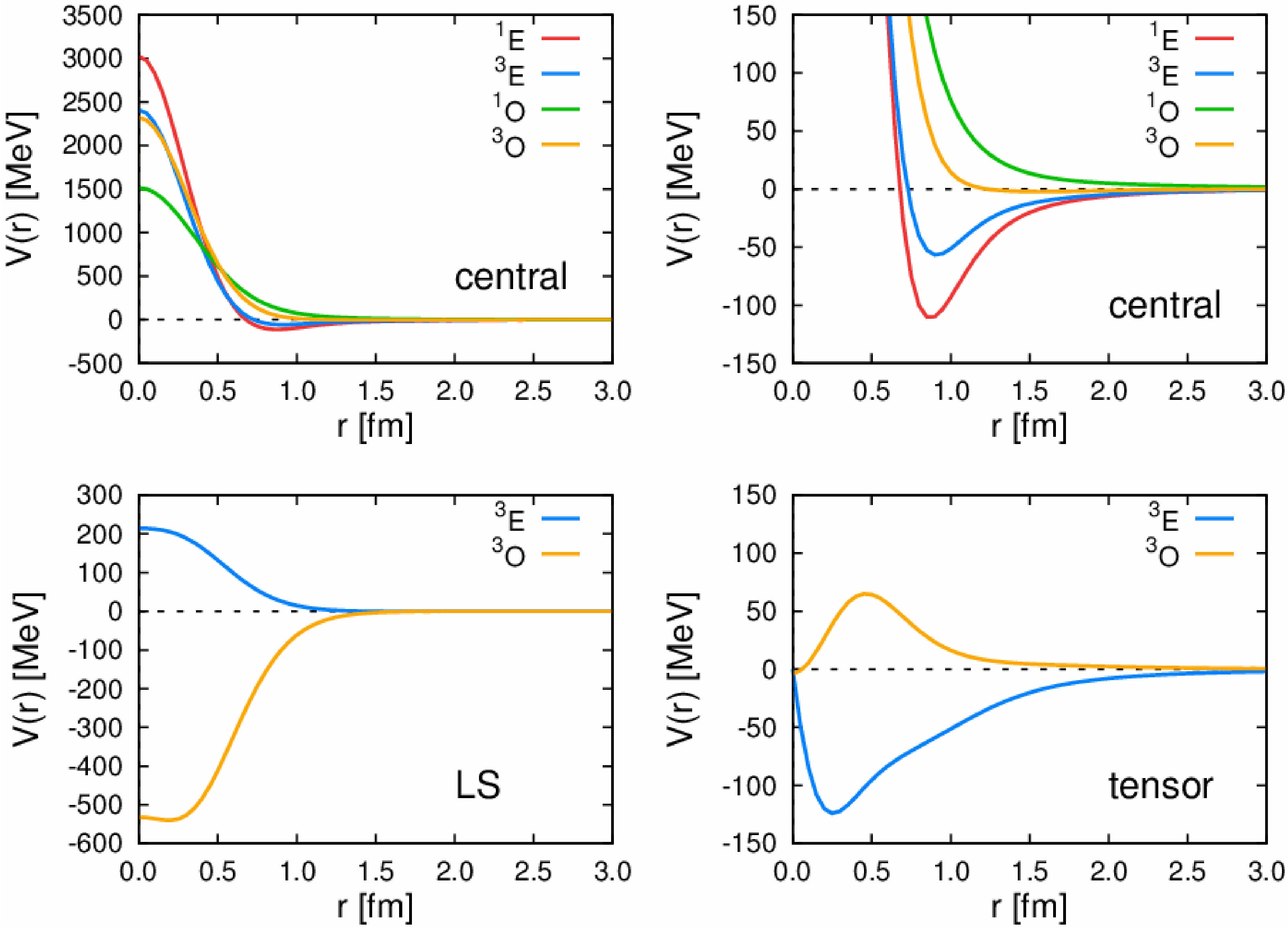
Tensor operator

$$S_{12} = 3(\vec{\sigma}_1 \cdot \hat{q})(\vec{\sigma}_2 \cdot \hat{q}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

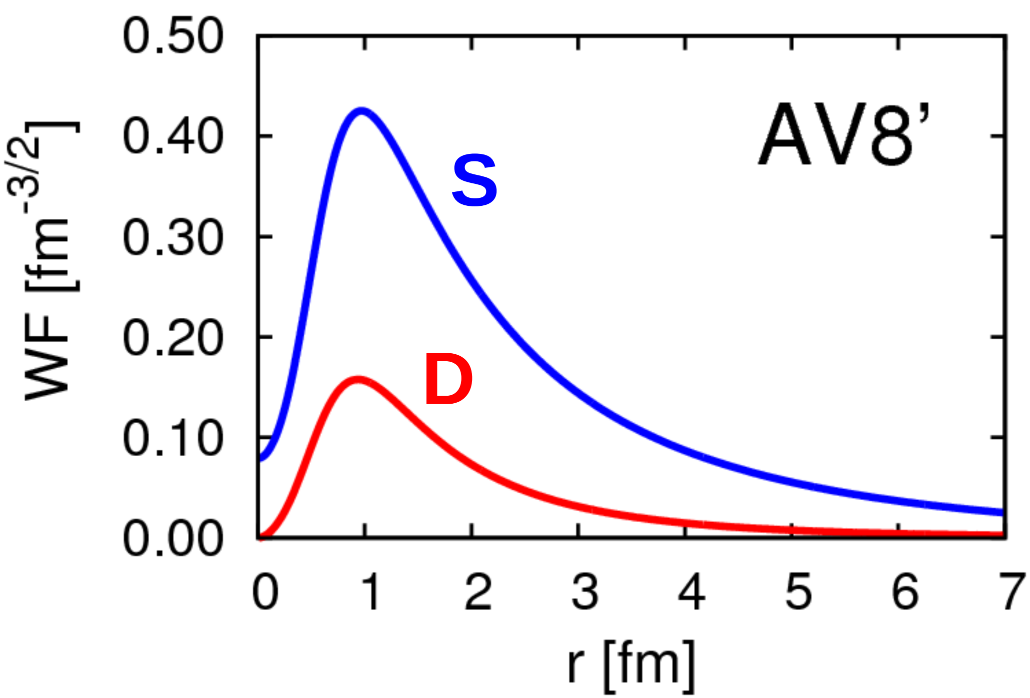
- V_{tensor} produces the high momentum component.

AV8' potential

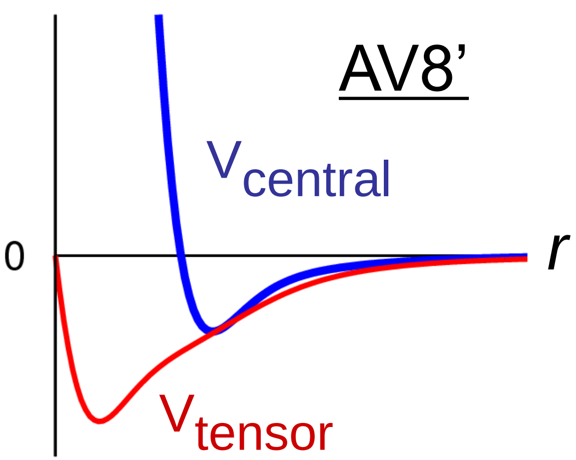
R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C**51**, 38 (1995).



Deuteron properties & tensor force



Energy	-2.24 MeV	
Kinetic	19.88	S 11.31 D 8.57
Central	-4.46	
Tensor	**-16.64**	SD -18.93 DD 2.29
LS	-1.02	
P(L=2)	5.77%	
Radius	1.96 fm	



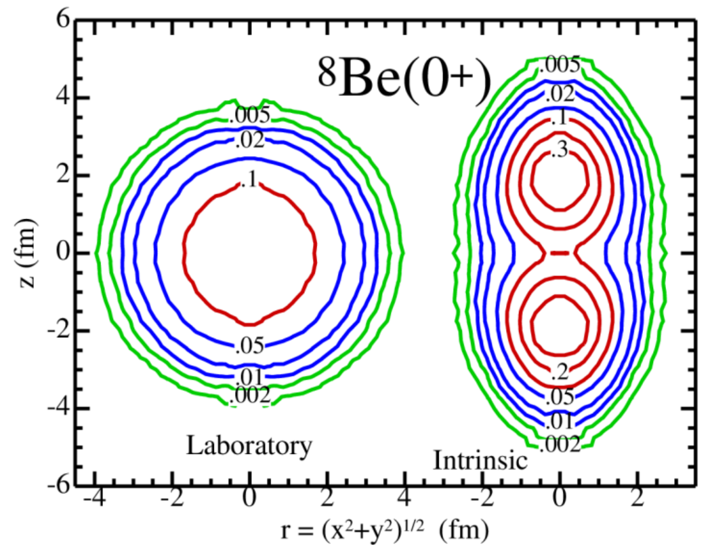
$R_m(s)$=2.00 fm

$R_m(d)$=1.22 fm

d-wave is
"**spatially compact**"
(high momentum)

Nuclear clustering & tensor force

- Argonne Group
  - Green's function Monte Carlo
    C.Pieper, R.B.Wiringa,
    Annu.Rev.Nucl.Part.Sci.51 (2001)



α-α structure

- Unitary Correlation Operator Method (**UCOM**)
  - Neff, Feldmeier, NPA 713 (2004) 311.
  - Unitary transformation of V_{NN} into V_{eff} within 2-body approximation.
  - Fermionic Molecular Dynamics (**FMD**) for nuclear w.f.
- Antisymmetrized Molecular Dynamics (**AMD**)
  - V_{eff} with central+LS, Kyoto Group (Horiuchi, En'yo, Kimura, Suhara...)
- Role of tensor force on the mechanism of nuclear clustering is still ***unclear***, such as Hoyle (triple-α) state in ^{12}C.

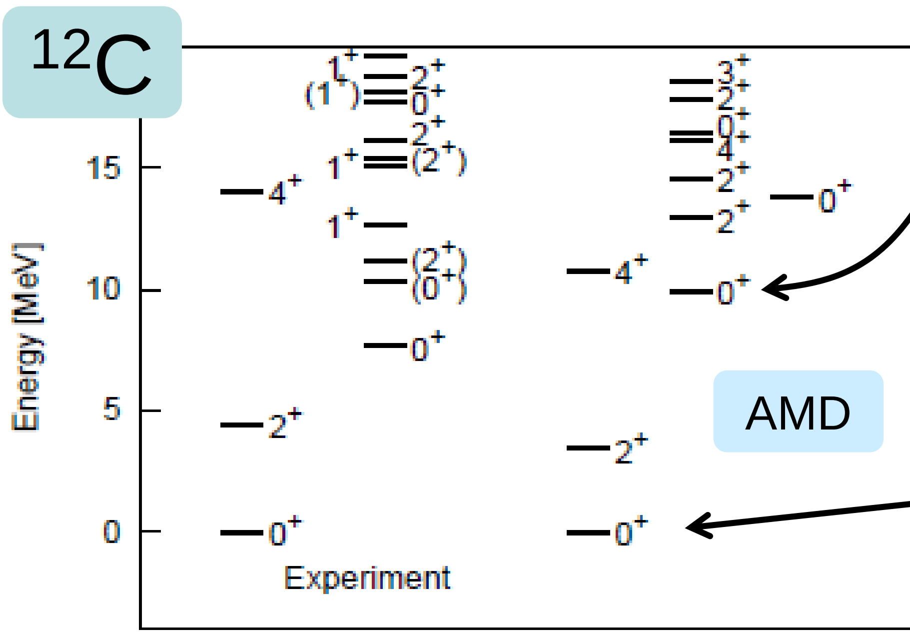
^{12}C with AMD

$$\left|\Phi_{AMD}\right\rangle = \det\left\{\varphi_1 \cdots \varphi_A\right\}$$

nucleon w.f. $\varphi \propto e^{-\nu(\vec{r}-\vec{Z})^2}\chi_\sigma\chi_\tau$

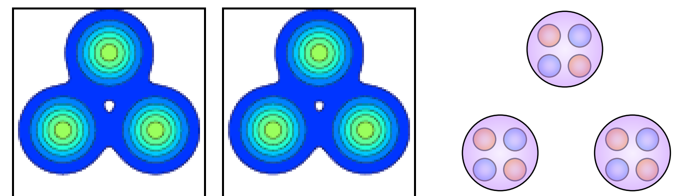
Gaussian wave packet

V_{eff} : Effective central force
+ *LS* force
(NO tensor force)



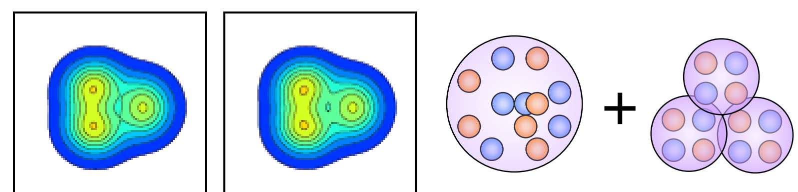
T. **Suhara**. and Y. Kanada-En'yo, Phys. Rev. C **82,** 044301 (2010).

0^+(Hoyle) triple-α config.



α-particle ... s-wave state, $(0s)^4$

0^+(GS) shell-model-like



proton neutron

PTEP Prog. Theor. Exp. Phys. **2015**, 073D02 (38 pages)
DOI: 10.1093/ptep/ptv087

Tensor-**O**ptimized **A**ntisymmetrized **M**olecular **D**ynamics (**TOAMD**)

TM, Hiroshi Toki, Kiyomi Ikeda, Hisashi Horiuchi, and Tadahiro Suhara

- Toward clustering description of nuclei from V_{NN} .
- Multiply pair-type correlation function $\boldsymbol{F}$ to AMD w.f.
  - ✓ Idea from S. Nagata, T. Sasakawa, T. Sawada, R. Tamagaki, PTP22 (1959) 274.
- Correlated Hamiltonian, $\boldsymbol{F}^{\dagger}\boldsymbol{H}\boldsymbol{F}$ generates **many-body operators** using the cluster expansion

Formulation of TOAMD

- Deuteron wave function

TM, K. Kato, K. Ikeda
PTP113 (2005) 763

$$|\text{Deuteron}\rangle = |s\text{-wave}\rangle + |\underline{d\text{-wave}}\rangle$$

$R_{d-\text{wave}} \sim 0.6 \times R_{s-\text{wave}}$

Involve high-k component induced by V_{tensor}

spatially compact

- Tensor optimized AMD (**TOAMD**)

cf. S. Nagata et al.
PTP22 (1959) 274

$$|\Phi_{\text{TOAMD}}\rangle = |\Phi_{\text{AMD}}\rangle + \underline{F_D}|\Phi_{\text{AMD}}\rangle$$

$$F_D = \sum_{t=0}^{1}\sum_{i<j}^{A} f_D^t(\vec{r}_i - \vec{r}_j)\ , \quad f_D(\vec{r}\,) = \underline{S_{12}}\sum_{n}^{N_G} C_n e^{-a_n r^2}$$

- Pair excitation via tensor operator with d-wave transition
- Optimize relative motion with Gaussian expansion

General formulation of TOAMD

tensor

short-range (central type)

$$\left|\Phi_{\mathrm{TOAMD}}\right\rangle = \left|\Phi_{\mathrm{AMD}}\right\rangle + F_D\left|\Phi_{\mathrm{AMD}}\right\rangle + F_S\left|\Phi_{\mathrm{AMD}}\right\rangle$$
$$+ F_D F_S\left|\Phi_{\mathrm{AMD}}\right\rangle + F_D F_D\left|\Phi_{\mathrm{AMD}}\right\rangle + F_S F_S\left|\Phi_{\mathrm{AMD}}\right\rangle + \cdots$$

tensor × short-range

F_D, F_S : Gaussian expansion

- Variational principle
  - $\delta E_{\mathrm{TOAMD}} = 0$ for $E_{\mathrm{TOAMD}} = \dfrac{\langle \Phi_{\mathrm{TOAMD}} | H | \Phi_{\mathrm{TOAMD}} \rangle}{\langle \Phi_{\mathrm{TOAMD}} | \Phi_{\mathrm{TOAMD}} \rangle}$
- Variational parameters
  - AMD : $\nu,\ \mathbf{Z}_i\ (i=1,\ldots,A)$
  - F_D : $S_{12} \sum_{n=1}^{N_G} C_n e^{-a_n r^2}$
  - F_S : $\sum_{n=1}^{N_G} C'_n e^{-a'_n r^2}$

$$\left|\Phi_{\mathrm{AMD}}\right\rangle = \det\left\{\varphi_1 \cdots \varphi_A\right\}$$

nucleon w.f. $\quad \varphi \propto e^{-\nu(\vec{r}-\vec{Z})^2} \chi_\sigma \chi_\tau$

Gaussian wave packet

Matrix elements of the correlated operator

$$\langle \Phi_{\mathrm{TOAMD}} | H | \Phi_{\mathrm{TOAMD}} \rangle = \left\langle \Phi_{\mathrm{AMD}} \middle| F^{\dagger} H F \middle| \Phi_{\mathrm{AMD}} \right\rangle$$
$$\propto \left\langle \varphi_i \varphi_j \ldots \middle| \tilde{H} \middle| \det\{\varphi_{i'} \varphi_{j'} \ldots\} \right\rangle$$

Slater determinant

$$\left| \Phi_{\mathrm{AMD}} \right\rangle = \det\left\{ \varphi_1 \cdots \varphi_A \right\}$$

Correlated Hamiltonian, Norm

$$\tilde{H} = F^{\dagger} H F \qquad \tilde{N} = F^{\dagger} F$$

Classify the connections between F & H into **many-body operators** using cluster expansion method.

$$\tilde{N} = F^{\dagger} F = \tilde{N}^{[2]} + \tilde{N}^{[3]} + \tilde{N}^{[4]}$$
$$\tilde{H} = F^{\dagger} H F = \tilde{H}^{[2]} + \cdots + \tilde{H}^{[6]}$$

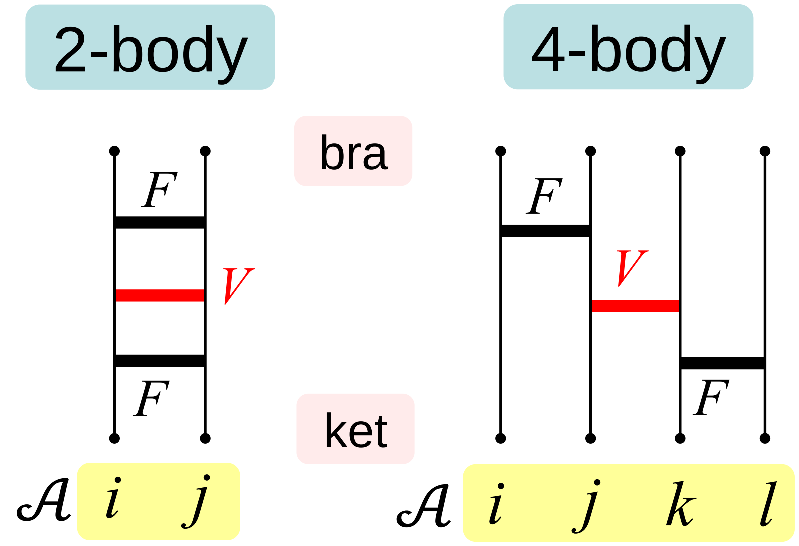
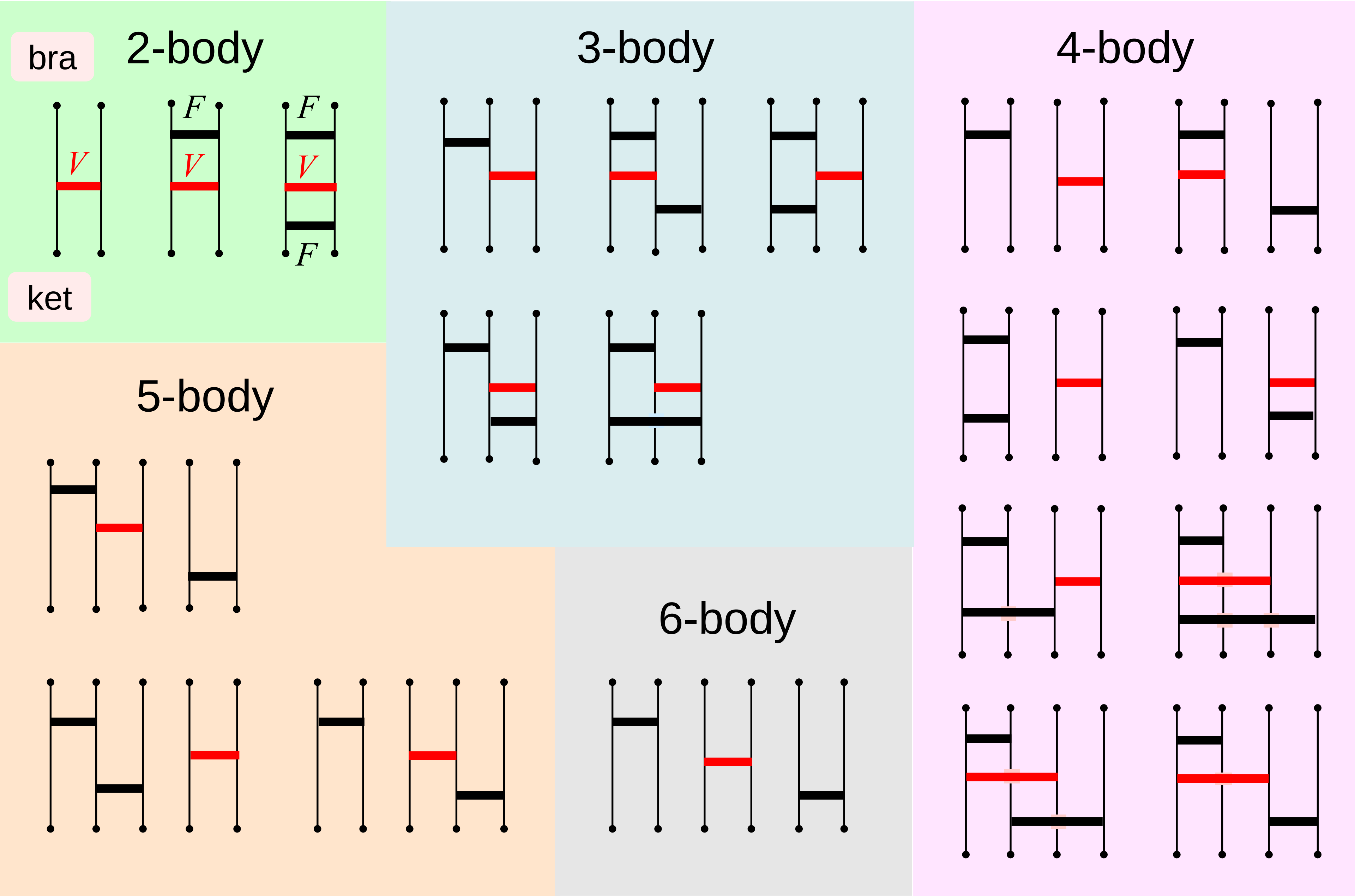
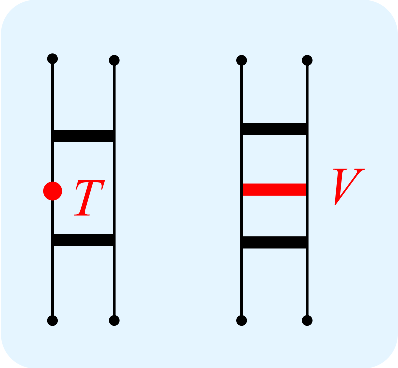


Diagram - 2-body Interaction -

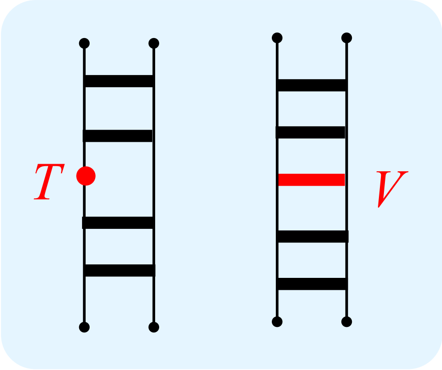


Multiple correlation functions in TOAMD

- **Signle *F***
  - $|\Phi_{\mathrm{TOAMD}}\rangle = (1 + F_D + F_S)|\Phi_{\mathrm{AMD}}\rangle$
  - Correlated Hamiltonian $\widetilde{H} = F^\dagger H F$



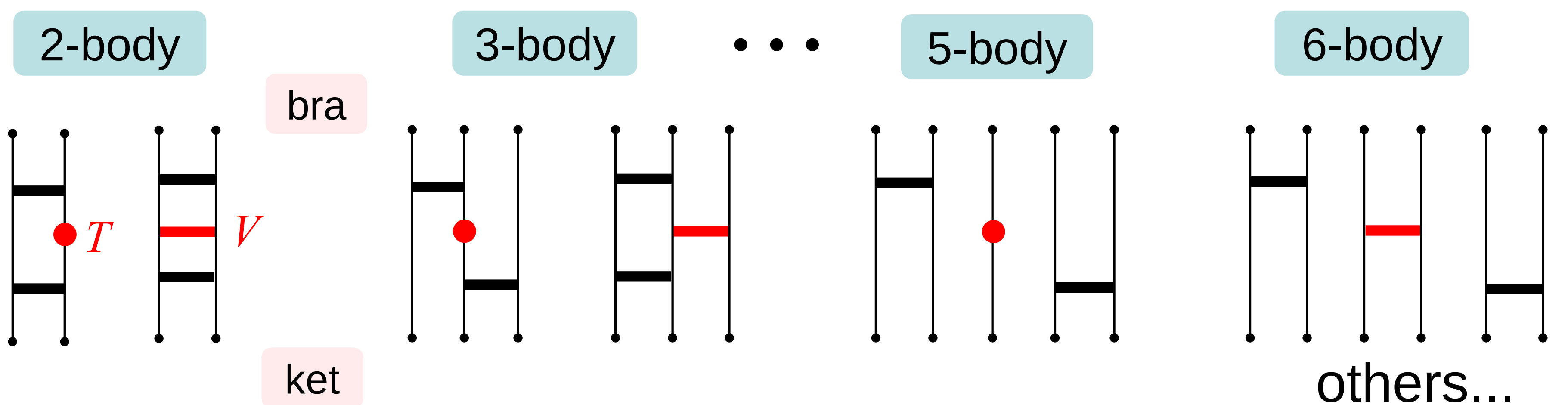
- **Double *F***
  - $|\Phi_{\mathrm{TOAMD}}\rangle = \underline{(1 + F_D + F_S)}^2|\Phi_{\mathrm{AMD}}\rangle$
    $= 1 + 2F_D + 2F_S + F_S F_S + F_S F_D + F_D F_S + F_D F_D$
  - $\widetilde{H} = F^\dagger F^\dagger H F F$
  - Each F is determined independently.



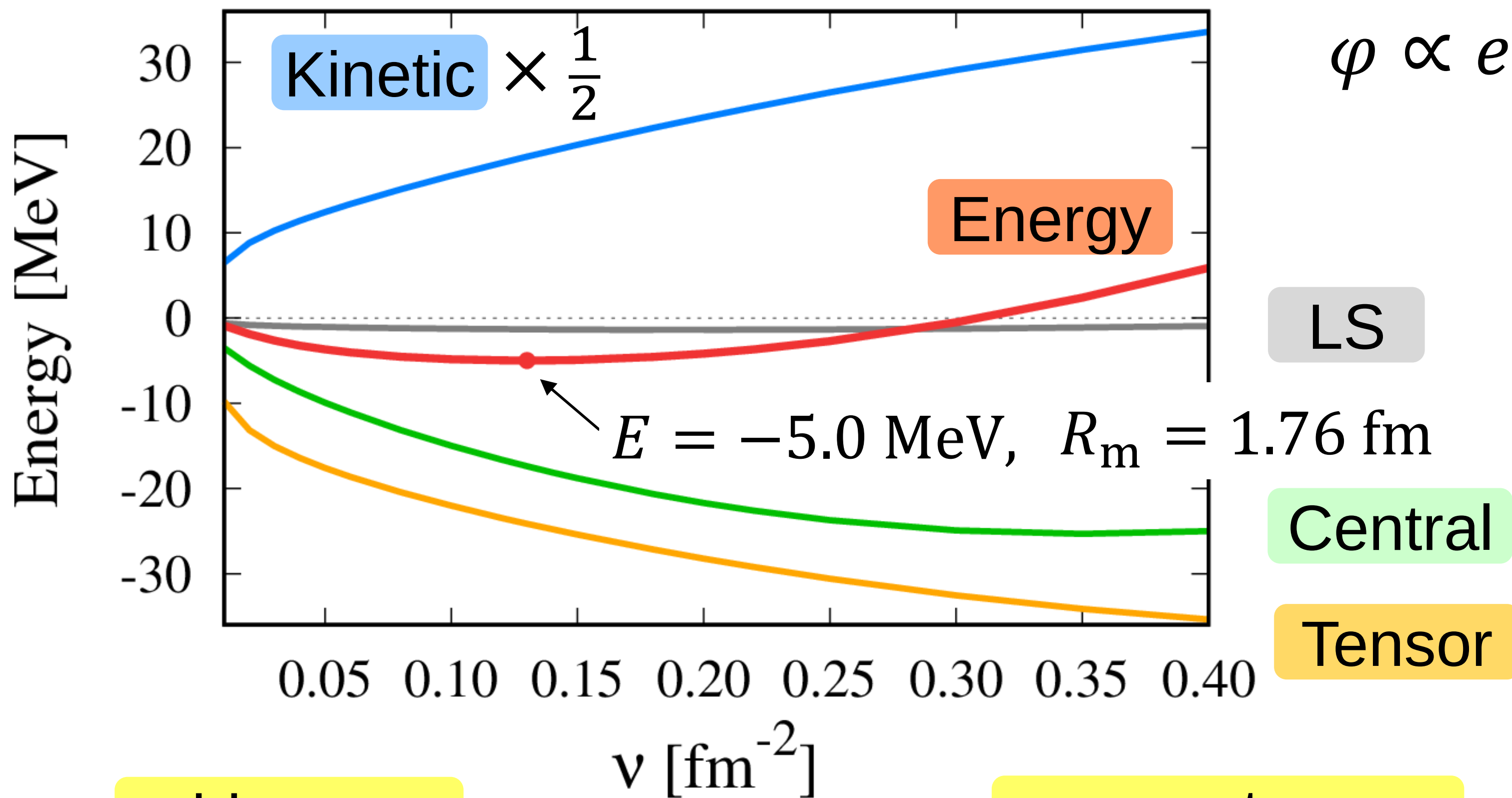
- **Triple *F***
  - $|\Phi_{\mathrm{TOAMD}}\rangle = (1 + F_D + F_S)^3\, |\Phi_{\mathrm{AMD}}\rangle$
  - $\widetilde{H} = F^\dagger F^\dagger F^\dagger H F F F$

Results

- ^{3}H, ^{4}He, (^{6}He, ^{6}Li)
- V_{NN} : AV8' (central+LS+tensor)
- TOAMD with 1) single F 2) double F
- 7 Gaussians for F_D, F_S to converge the solution.
- Full treatment of many-body operators (all diagrams) to retain the variational principle.



^{3}H in TOAMD with single F



$$\varphi \propto e^{-\nu(\vec{r}-\vec{Z})^2}$$

$$\nu = \frac{1}{2b^2}$$

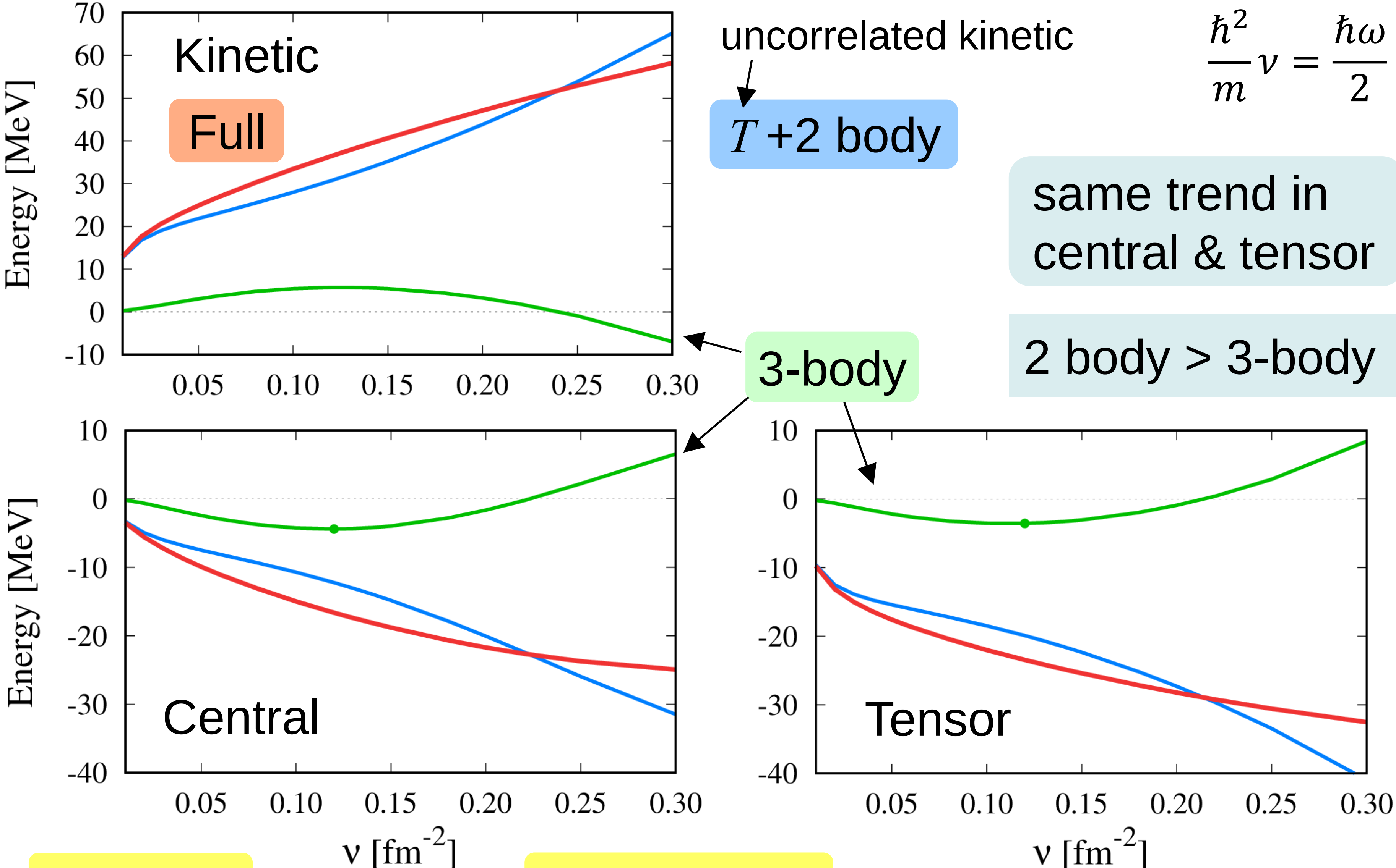
$$\frac{\hbar^2}{m}\nu = \frac{\hbar\omega}{2}$$

wide, ρ_{low}

compact, ρ_{high}

- Large cancelation of *T* & *V* makes the small total energy.
- $\vec{Z}_1 = \vec{Z}_2 = \vec{Z}_3 = 0$, s-wave configuration of AMD w.f.

Many-body terms of $F^\dagger HF$ in ^{3}H



$$\frac{\hbar^2}{m}\nu = \frac{\hbar\omega}{2}$$

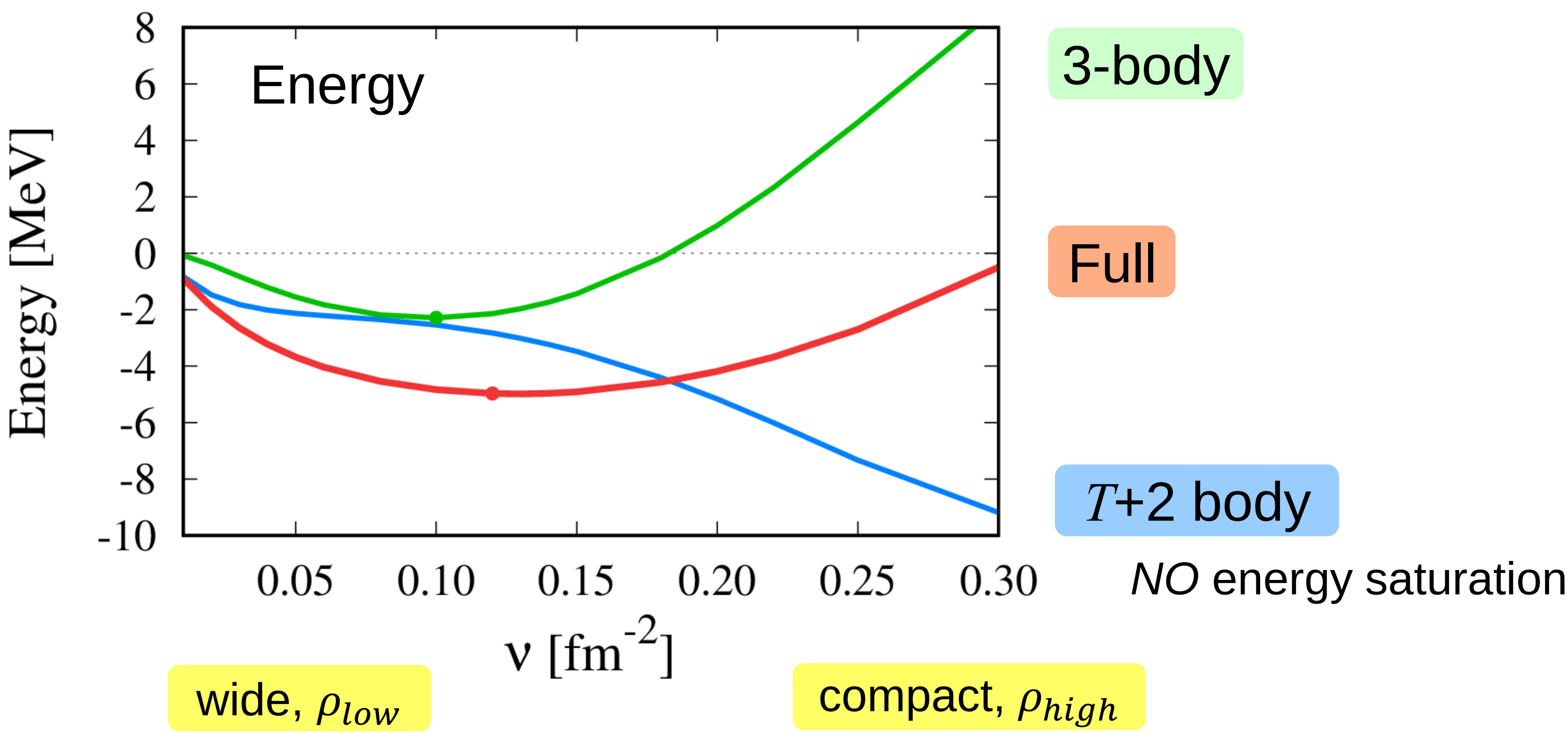
same trend in central & tensor

2 body > 3-body

wide, ρ_{low}

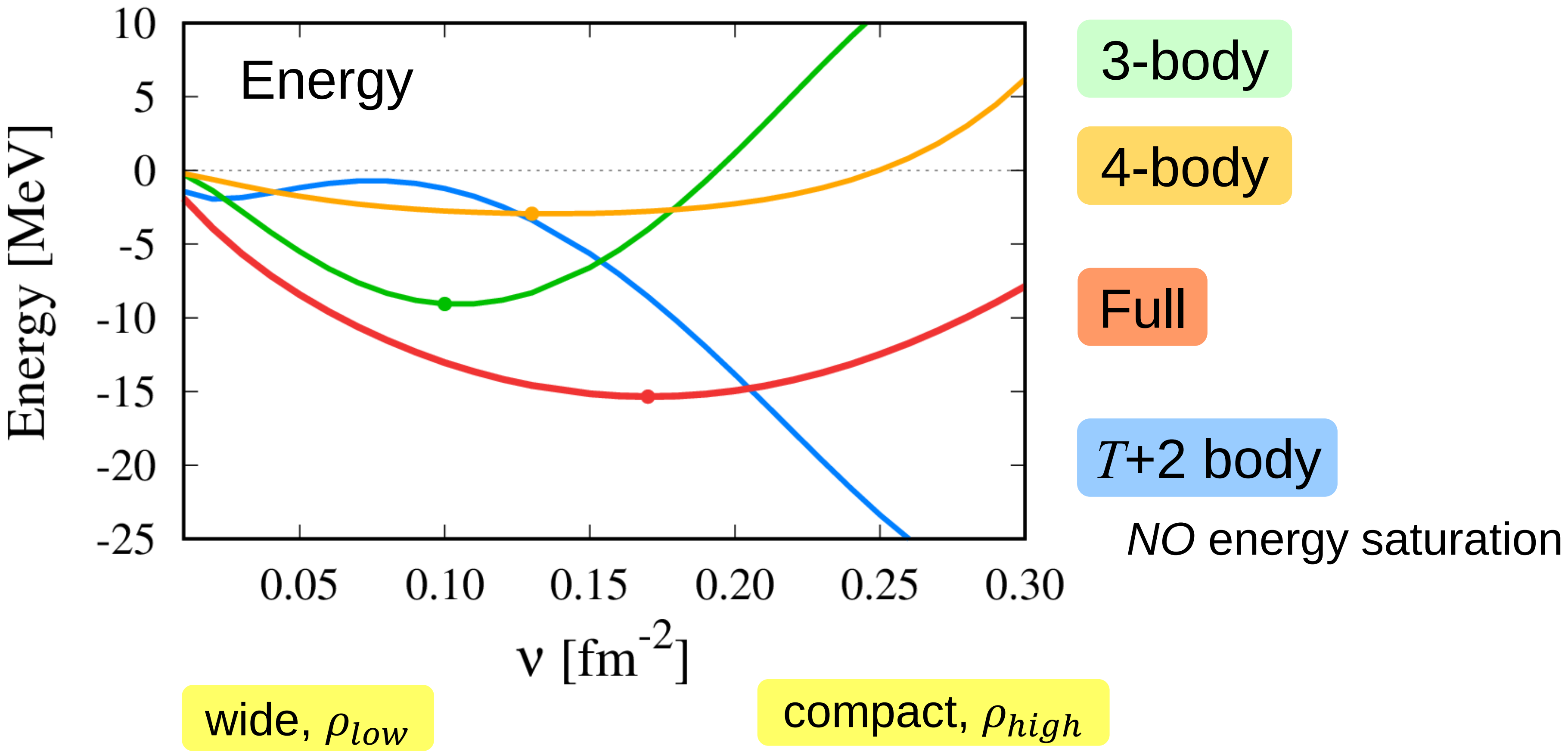
compact, ρ_{high}

Many-body terms of $F^\dagger H F$ in ^{3}H



- Large cancelation of T & V in each many-body term
- 3-body term has a saturation behavior.
  - Similar to Bethe-Brueckner-Goldstone approach with G-matrix (Baldo)

Many-body terms of $F^{\dagger}HF$ in ^{4}He



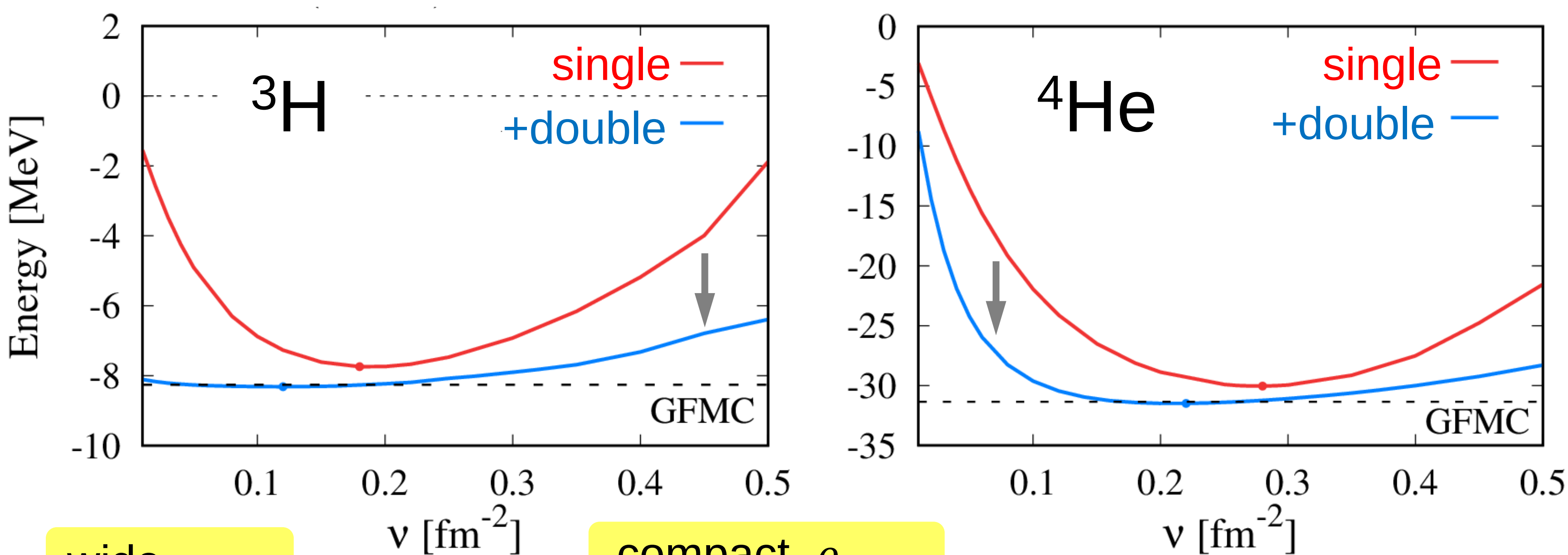
- Large cancelation of *T* & *V* in each many-body term
- Many-body terms are necessary to obtain E_{mininum}.

Double F_s effect in TOAMD

Malfliet-Tjon V
(central short-range)

$$(1 + \underbrace{F_S}_{\text{single}} + \underbrace{F_S F_S}_{\text{double}})|\Phi_{\text{AMD}}\rangle \qquad \frac{\hbar^2}{m}\nu = \frac{\hbar\omega}{2}$$

F are independent



wide, ρ_{low}

compact, ρ_{high}

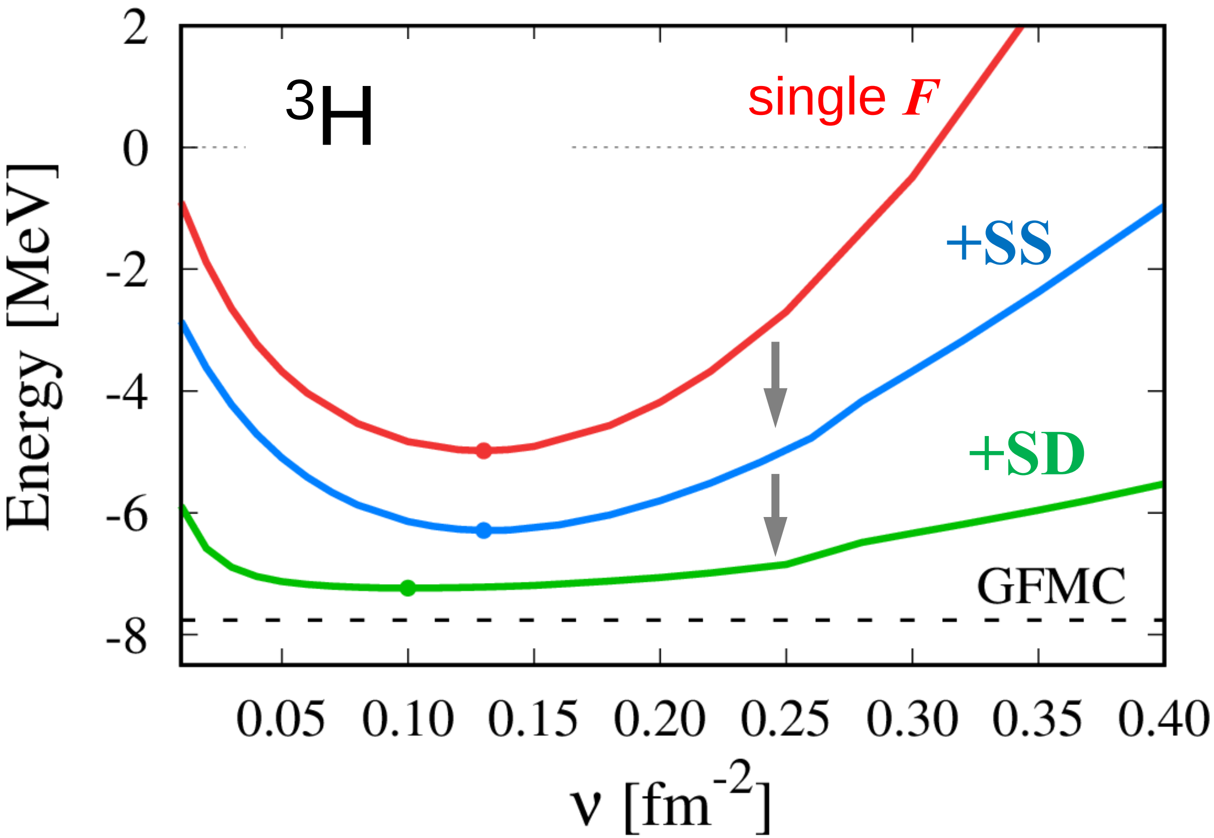
- Double F_S reproduces the GFMC energy.
- Small ν-dependence indicates the flexibility of F_S.

Double F effect in TOAMD

AV8'

$$(1 + \underbrace{F_D + F_S}_{\text{single}} + \underbrace{F_S F_S}_{\text{SS}} + \underbrace{F_S F_D}_{\text{SD}})|\Phi_{\text{AMD}}\rangle$$

$$\frac{\hbar^2}{m}\nu = \frac{\hbar\omega}{2}$$



F are independent

- Get close to the GFMC energy.
- ν-dependence is small due to the flexibility of F.

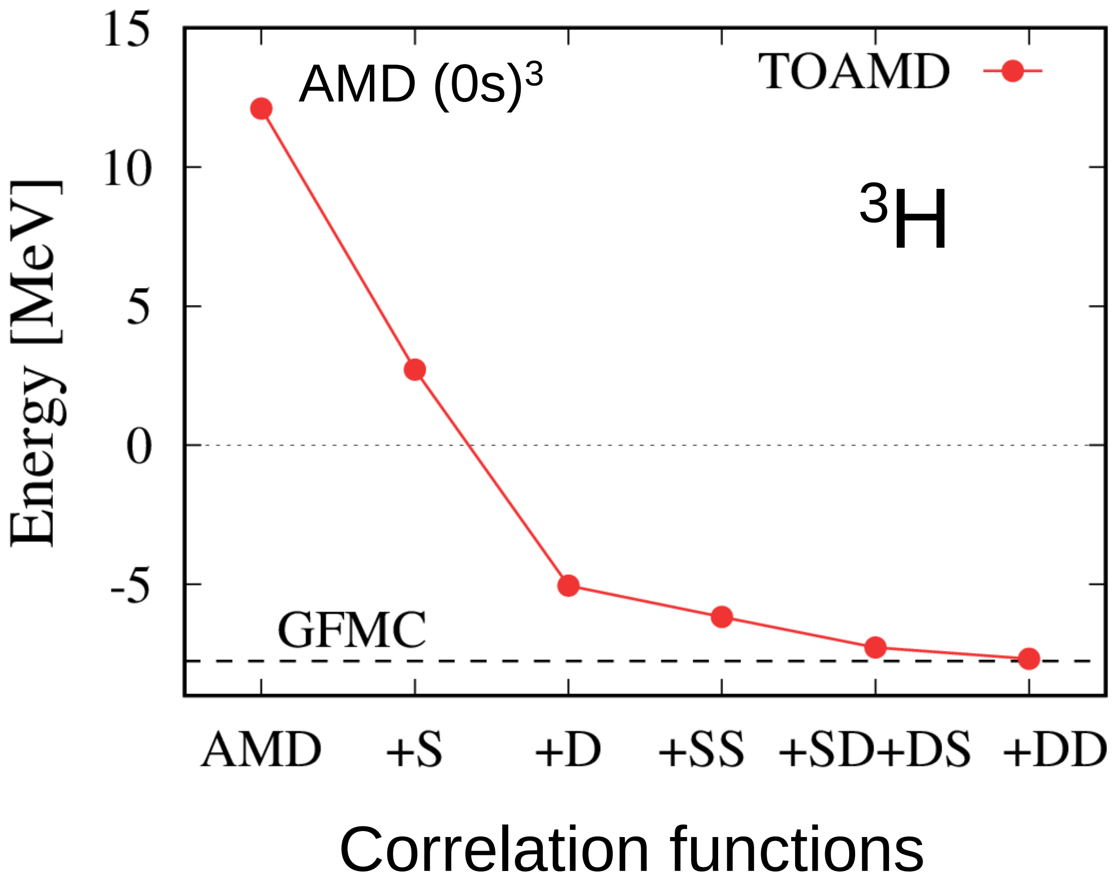
Double F effect in TOAMD

AV8'

$$(1 + \underline{F_D} + \underline{F_S} + \underline{F_S F_S} + \underline{F_S F_D} + \underline{F_D F_S} + \underline{F_D F_D})|\Phi_{\rm AMD}\rangle$$

D **S** **SS** **SD** **DS** **DD**

$$\frac{\hbar^2}{m}\nu = \frac{\hbar\omega}{2}$$



F are independent

- Reproduce the GFMC energy

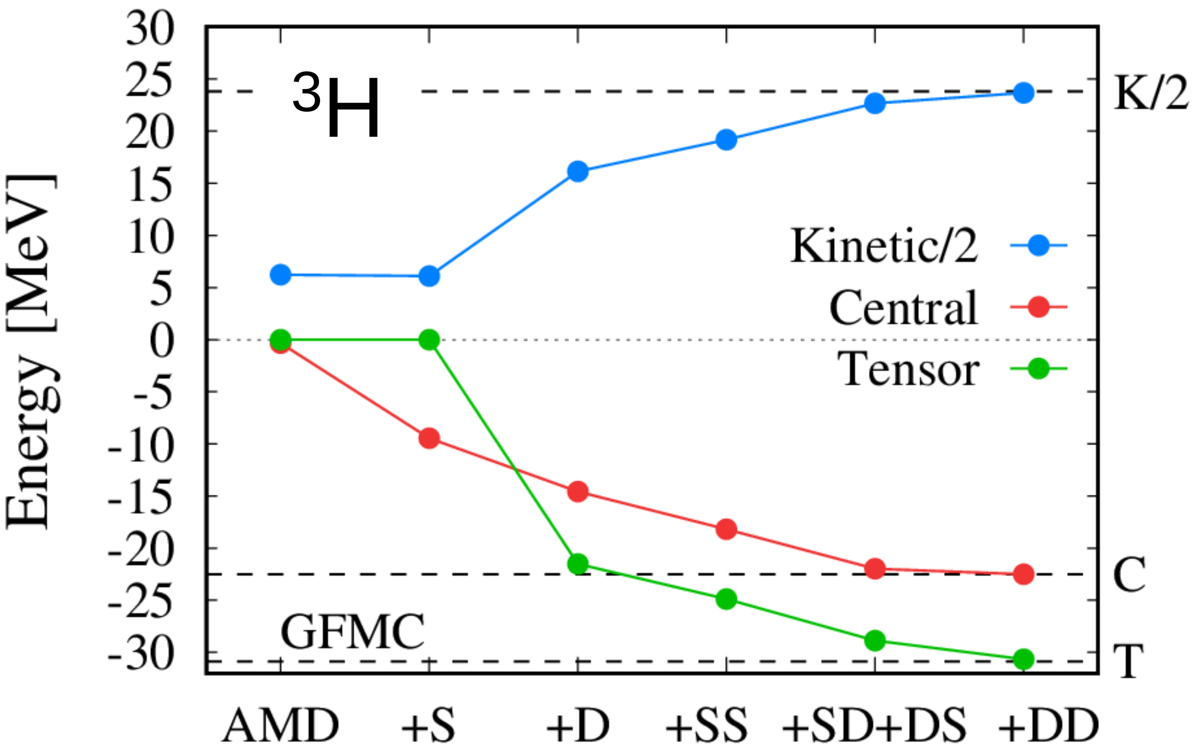
Double F effect in TOAMD

AV8'

$$(1 + \underline{F_D} + \underline{F_S} + \underline{F_S F_S} + \underline{F_S F_D} + \underline{F_D F_S} + \underline{F_D F_D})|\Phi_{\mathrm{AMD}}\rangle$$

D **S** **SS** **SD** **DS** **DD**

$$\frac{\hbar^2}{m}\nu = \frac{\hbar\omega}{2}$$



Correlation functions

F are independent

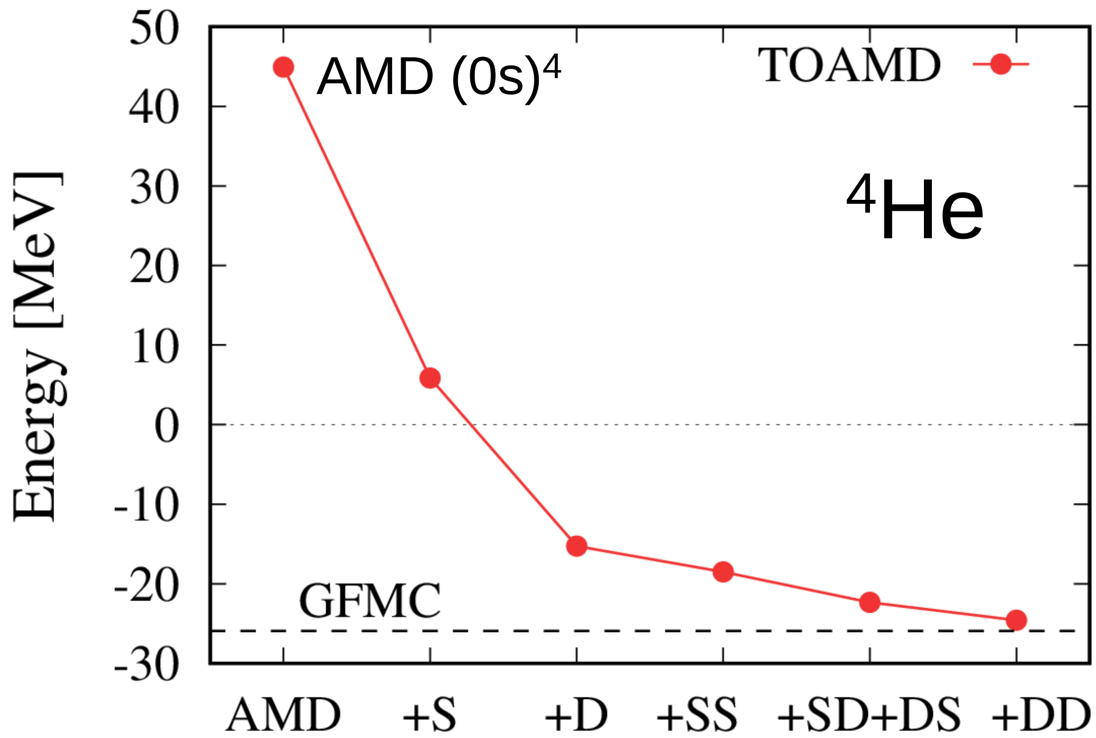
- Reproduce the Hamiltonian components of ^{3}H

Double F effect in TOAMD

AV8'

$$(1 + \underbrace{F_D}_{\mathbf{D}} + \underbrace{F_S}_{\mathbf{S}} + \underbrace{F_S F_S}_{\mathbf{SS}} + \underbrace{F_S F_D}_{\mathbf{SD}} + \underbrace{F_D F_S}_{\mathbf{DS}} + \underbrace{F_D F_D}_{\mathbf{DD}})|\Phi_{\mathrm{AMD}}\rangle$$

$$\frac{\hbar^2}{m}\nu = \frac{\hbar\omega}{2}$$



Correlation functions

F are independent

- Good energy with F^2
- Next order is triple-F such as $F_S F_D F_D$.

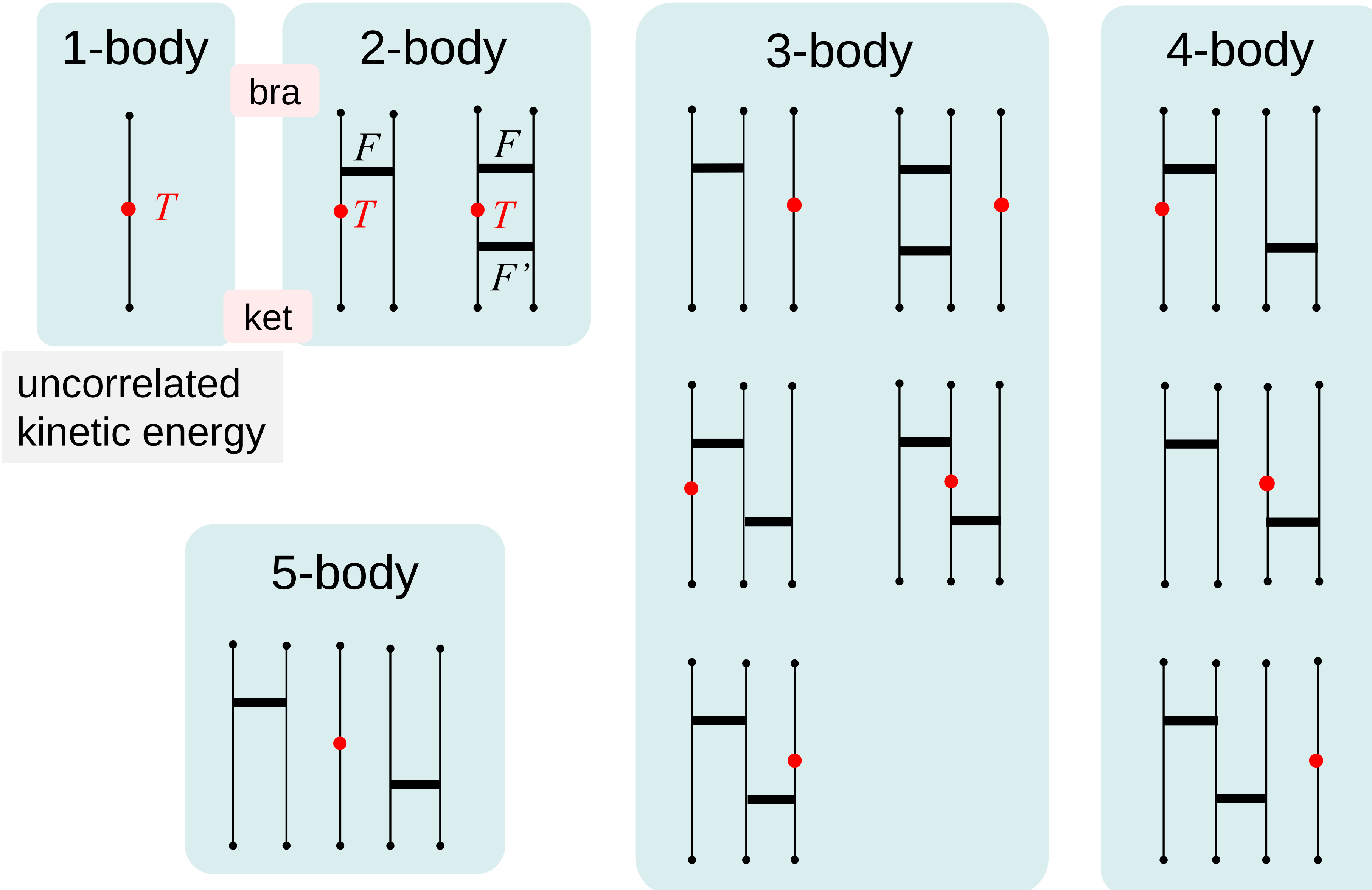
Summary

- **Tensor-Optimized AMD (TOAMD)**.
  - Variational model for nuclei to treat V_{NN} directly.
  - Correlation functions, F_D (tensor) , F_S (short-range).
  - Full treatment of many-body operators.
  - At the F^2 level, good reproduction of *s*-shell nuclei.
  - We can increase the multiple correlation functions systematically.
  - We can include V_{NNN} in the same manner.
  - We can apply TOAMD to hyper nuclei with ΛN-ΣN coupling.

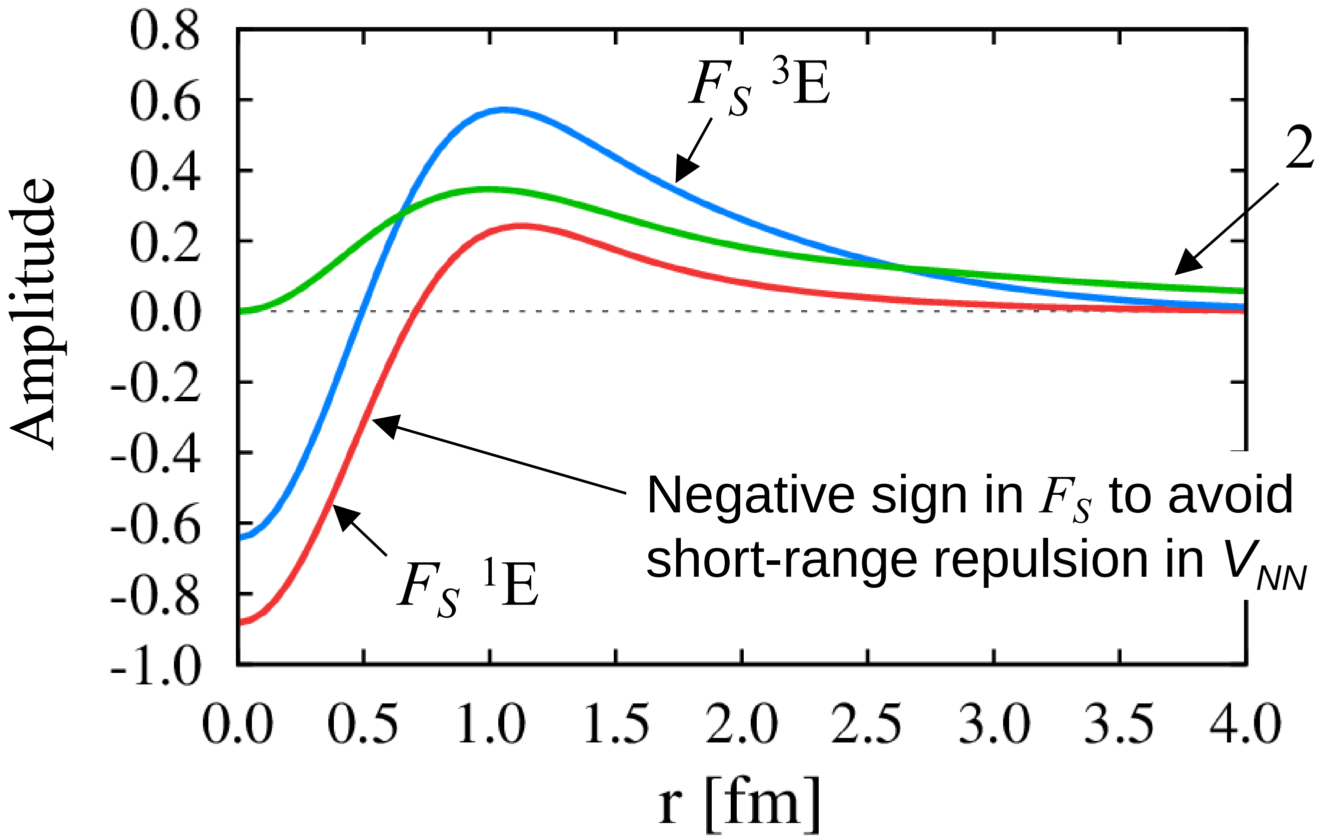
- Collaborators
  - Hiroshi TOKI (RCNP) Kiyomi IKEDA (RIKEN)
    Hisashi HORIUCHI (RCNP) Tadahiro SUHARA (Matsue College of Tech.)

Backup

Diagram -Kinetic energy -



Correlation functions F_D, F_S in ^{3}H



tensor

similar to TOSM

- Ranges of F_D, F_S are not short.
- Range b of $F_D \Phi_{\mathrm{AMD}}$ ~ 0.6 b_{AMD} → spatially compact, high-k in relative motion
  $(= 1/\sqrt{2a+\nu})$ $(1/\sqrt{\nu})$