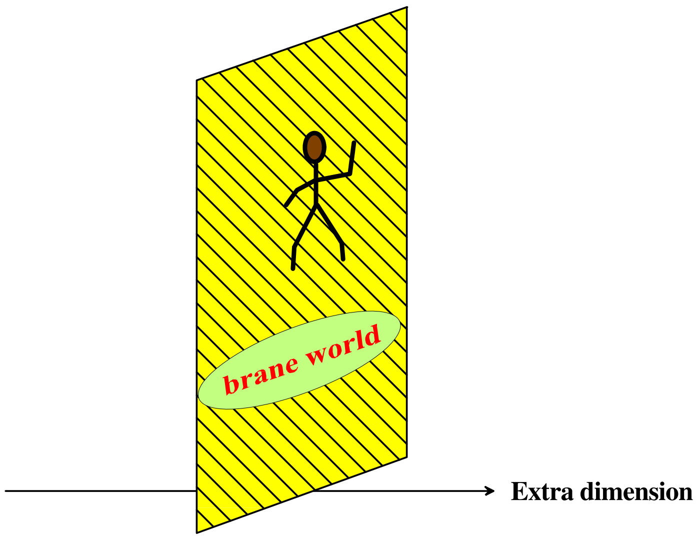
Theory of Cosmological Perturbations

Addendum

— a touch on brane-cosmological perturbations —

§ 1. Braneworld scenario

· Our universe is a (timelike) singular surface (brane) in a higher dimensional spacetime.



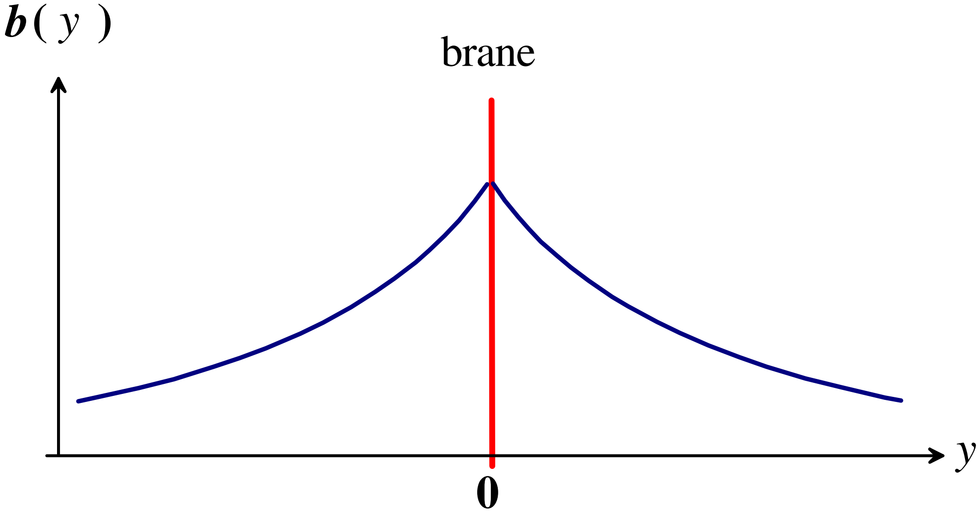
· Standard matter fields are confined on the brane, while gravity propagates in the whole spacetime (called the bulk).

· One of the most studied models is the Randall-Sundrum model in which the bulk is 5D anti-de Sitter (AdS) space.

· Cosmology on the brane = brane dynamics in the bulk.

- Randall-Sundrum braneworld metric (AdS_5 + Minkowski brane):

$$ds^2 = dy^2 + b^2(y)\eta_{\mu\nu}\, dx^\mu dx^\nu \,; \quad b(y) = e^{-|y|/\ell}$$



- Generalization to cosmology:

Brane fixed at $y = 0$ ($\sim$ view from the brane)

$$ds^2 = dy^2 + \left(-n^2(y,t)dt^2 + a^2(y,t)d\sigma^2_{(3)}\right)$$

Or, a brane moving in the AdS bulk ($\sim$ view from the bulk)

$$ds^2 = -A(R)dT^2 + \frac{dR^2}{A(R)} + R^2 d\Omega_K^2 \quad A(R) = K + \frac{R^2}{\ell^2} \quad (K = \pm 1, 0)$$

with brane trajectory: $(T, R) = (T(\tau), R(\tau))$

§ 2. Large-scale cosmological perturbations on the brane

· General formalism for braneworld perturbations

Kodama, Ishibashi & Seto ('00), Mukohyama ('00),
Langlois ('01), Koyama & Soda ('01), $\cdots$.

· Essentially a 5-dimensional, PDE problem:

$Q(y, x^\mu) = Q_p(y,t)Y_p(x^i) \cdots$ $Q_p(y,t)$ is *not* separable.

· Nevertheless, some simplifications on super-horizon scales.

Langlois, Maartens, MS & Wands ('01)

- Effective gravitational equations on the brane (in AdS_5 bulk background)

$$\begin{aligned} G_{\mu\nu} + \Lambda_4\, q_{\mu\nu} &= 8\pi G_4\, T_{\mu\nu} + (8\pi G_5)^2\, \Pi_{\mu\nu} - E_{\mu\nu} \\ &\equiv 8\pi G_4\, T^{\mathrm{tot}}_{\mu\nu} \qquad (\Pi_{\mu\nu} \sim \text{ quadratic in } T_{\mu\nu}) \end{aligned}$$

$$E_{\mu\nu} \equiv \overset{(5)}{C}{}_{a\mu b\nu} n^a n^b\,; \qquad \overset{(5)}{C}{}_{acbd} \cdots \text{ 5D Weyl tensor}, \qquad n^a \cdots \text{ unit normal to the brane}$$

· $E_{\mu\nu}$ contains all the dynamics of the bulk

· By definition, $E^\mu{}_\mu = 0$. In addition, $D^\mu E_{\mu\nu} = 0$ on FLRW background.

"$-E_{\mu\nu}$" : **Weyl fluid** (or "dark radiation")

The energy momentum tensor of the brane matter:

$$T_{\mu\nu} = \rho\, u_\mu u_\nu + p\,(u_\mu u_\nu + g_{\mu\nu}) + \pi_{\mu\nu}$$

For FLRW background, with perturbations of $O(\epsilon)$,

$$\pi_{\mu\nu}\ :\ \text{anisotropic stress} = O(\epsilon) \quad \Rightarrow \quad u^\mu D^\nu E_{\mu\nu} = O(\epsilon^2)$$

On superhorizon scales, only the energy conservation law is important.
($\because$ momentum cannot be transfered over super-Hubble scales)

Weyl fluid decouples from the brane matter on superhorizon scales.

$\Downarrow$

Standard 4D theory is applicable with slight modifications due to $\Pi_{\mu\nu}$ (quadratic in $T_{\mu\nu}$).

$$\rho_{\rm eff} = \rho\left(1 + \frac{\rho}{2\sigma}\right) \quad (\sigma = \text{brane tension})\,, \quad \rho_{\mathcal{E}} = \frac{E^0{}_0}{8\pi G_4}\,, \quad \text{etc.}$$

What is missing is an equation for anisotropic stress of the Weyl fluid:

$$\pi^{ij}_{\mathcal{E}} = -\frac{E^{ij}_T}{8\pi\, G_4}\,; \quad E^i_{Tj} \equiv E^i{}_j - \frac{1}{3}\delta^i_j\, E^k{}_k$$

This must be determined by solving the bulk perturbation equations.

$\Rightarrow$ back to 5D problem

- Some recent ($\gtrsim$ 2003) efforts to solve the bulk perturbation equations:
  (in the context of inflation in RS-type brane cosmology)

  · Expanding equations around the brane ($\sim$ gradient expansion).

  Koyama ('03), Battye, Van de Bruck & Mennim ('03), $\cdots$

  · Approximating the brane by a de Sitter space.

  Minamitsuji, Himemoto & MS ('03), Du, Wang, Abdalla & Su ('04), $\cdots$

  · Analyzing exactly soluble models.

  Koyama & Takahashi ('03), Yoshiguchi & Koyama ('04), Kobayashi & Tanaka ('04), $\cdots$

  · Focusing on tensor-type perturbations

  Easther, Langlois, Maartens & Wands ('03), Hiramatsu, Koyama & Taruya ('03), $\cdots$

  · Elaborating the perturbation theory

  Deffayet ('03), Malik, Rodriguez-Martinez & Langlois ('03), $\cdots$

But no rigorous treatment of scalar-type perturbations so far