Constraining neutron star equations of state using GW170817

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Talk Outline

- Background Information
- Methods
- Results
- Future work

GW170817

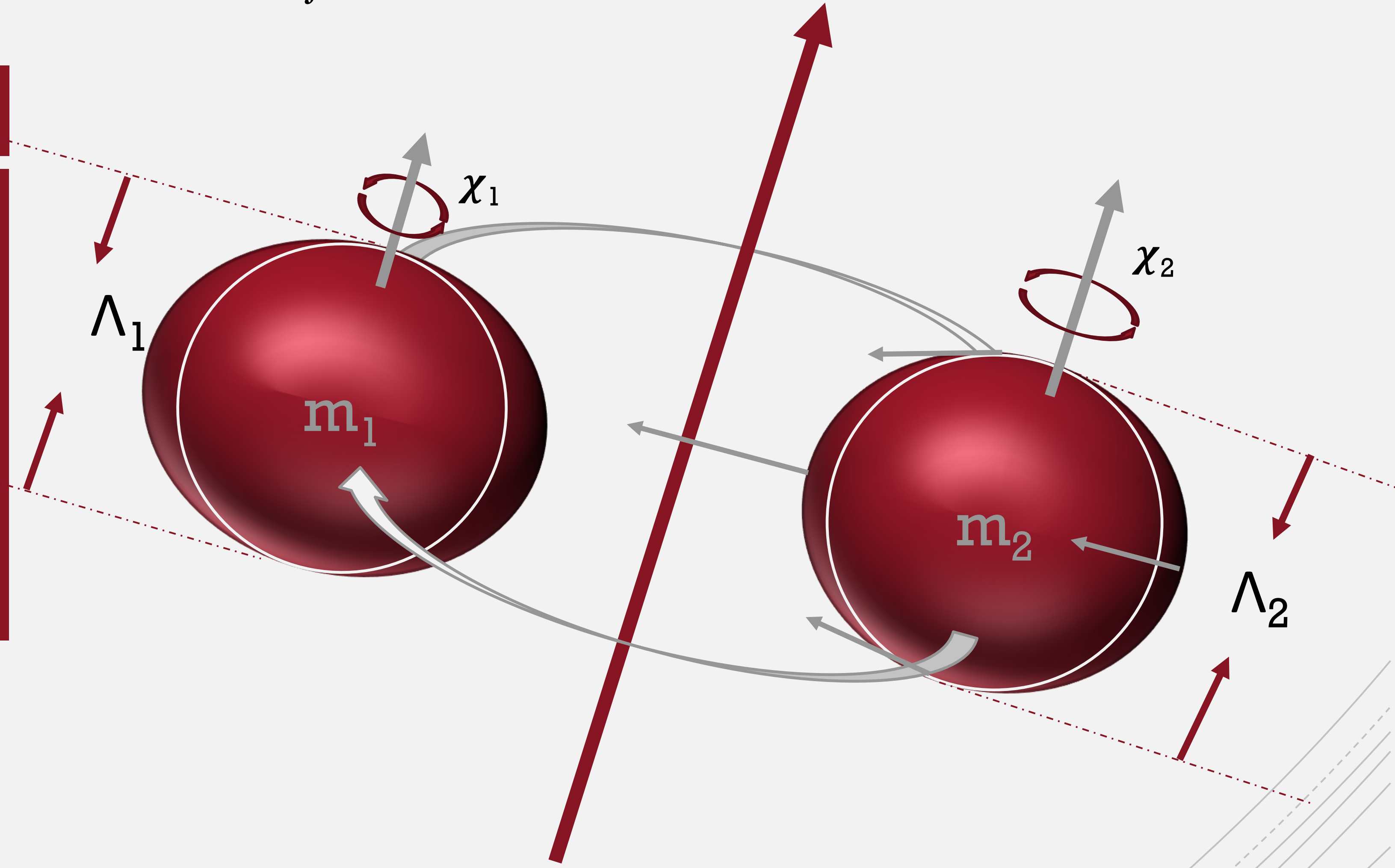
- 17 August 2017 12:41:01 UTC
- LIGO and Virgo
- First and so far only published Binary Neutron Star (BNS) detected with GW
- GRB Observed in Coincidence
- Follow up EM observations

Why are Neutron Stars Interesting?

- Matter inside a NS far more dense than any matter on earth, which makes them unique nuclear physics laboratories
- Neutron stars have matter effects that make them more complicated than black holes which can be described purely by mass, spin, and charge (no hair theorem).

NS Tidal Deformation

In a binary two neutron stars exert tidal forces on each other and both objects will deform



Tidal deformation and GW

- The deformability appears in the GW signal at 5PN in terms of the combined tidal deformability parameter

$$\Lambda_s = \frac{16}{13}\frac{(12q+1)\Lambda_1 + (12+q)q^4\Lambda_2}{(1+q)^5}$$

- The deformation is defined, at leading order, by the tidal deformability parameter

$$\Lambda = \frac{2}{3}k_2\left(\frac{c^2R}{Gm}\right)^5$$

- k_2 is the tidal love number: a proportionality constant between an external tidal field and the quadrupole deformation of a star
- k_2 comes from integrating a first order differential equation along with the TOV equations[1]

1. Zhao, T. and Lattimer. Phys. Rev. D 98, 063020 (2018)

Describing Neutron Star Matter

- Tolman-Oppenheimer-Volkov (TOV) Equations constrains spherically symmetric matter in general relativity. Combining TOV Equations with EOS allows one to completely describe the structure of a neutron star.
- The EOS describes the state of matter under a given set of physical conditions such as pressure and density.
- Hence, tidal deformability is strongly dependent on neutron star structure

Equations of State

- The exact EOS of ultra dense matter in a neutron star is unknown. Direct derivations from QCD are not yet possible.
- Currently cutting edge EOS solutions come from Quantum Monte Carlo simulations.
- EOS are constrained at low densities by nuclear physics experiments and at high densities by neutron star observations.

Our goal is to constrain the EOS of NSM using GW170817

Previous work

- Previous works have put constraints on Λ_1 and Λ_2 using PE of GW170817
- The original work by the LVC made no assumptions relating Λ_1 and Λ_2 [1]
- Later work by the LVC [2] and also by De et al [3] related Λ_1 and Λ_2 by studying common properties of EOS

1. B. P. Abbott, et al., Phys. Rev. Lett. 119, 161101 (2017)
2. B. P. Abbott, et al., Phys. Rev. Lett. 121, 161101 (2018)
3. S. De, et al., Phys. Rev. Lett 121, 091102 (2018).

De et al

- Studies thousands of polytropic equations of state
- $\Lambda_1 / \Lambda_2 = q^6$ where $q = m_2 / m_1 \leq 1$
  - $\Lambda_1 = q^3 \Lambda_s$; $\Lambda_2 = q^{-3} \Lambda_s$
- Samples in Λ_s and uses the above relations to calculate Λ_1 and Λ_2

S. De, et al., Phys. Rev. Lett 121, 091102 (2018).

LVC

- EOS Insensitive (more constraining)
  - Studies a set of 12 EOS
  - Found a roughly EOS-insensitive relation between deformability and compactness
  - They report a maximum 6.5% error in the relation across a large sample of polytropic EOS
- Parameterized EOS
  - Build a family of EOS
  - SLy at low density
  - Above this they use a spectral parameterization
  - Sample in these indices

B. P. Abbott, et al., Phys. Rev. Lett. 121, 161101 (2018)

Our Work

- Nuclear physics improvement: We use a state-of-the-art nuclear physics model called chiral effective field theory
- Parameter estimation improvement: We avoid making any generalizations relating Λ_1 and Λ_2 by sampling EOS space directly in our parameter analysis

Chiral EFT

- Chiral-EFT has been show to be effective at low densities[1.2]
- It has well determined uncertainties at higher densities
- Our EOS are defined by Chiral EFT up to a transition density. Above that density the EOS are constrained only by the requirement that they are causal, stable, and able to support a neutron star of mass $1.9M_{\odot}$.

[1] D. Lonardoni, *et al.*, *Phys. Rev. Lett.* 120, 122502 (2018).
[2] I. Tews, J. Carlson, S. Gandolfi, S. Reddy, *Astrophys. J.* 860, 149 (2018).

EOS Models used

- At low densities, the EOS is solved using Quantum Monte Carlo methods to solve the Chiral EFT many-bodied Hamiltonian.
- These equations are designed to be as general as possible and include phase transitions
- The two families used have $n_{tr} = n_{sat}$ and $2n_{sat}$ respectively where $n_{sat} = 0.16\ fm^{-3}$

Parameter Estimation

- We use bayesian methods to perform parameter estimation and infer properties of the binary
- Given the data from the three interferometers $\boldsymbol{d} = \{\boldsymbol{d}_H, \boldsymbol{d}_L, \boldsymbol{d}_V\}$, we can define the probability that the binary has a set of parameters (ϑ) as

$$p(\vartheta|\boldsymbol{d}, h; I) = \frac{p(\boldsymbol{d}|\vartheta, h; I)p(\vartheta|\, h; I)}{p(\boldsymbol{d}|, h; I)}$$

  - h is the hypothesis or model of the gravitational-wave signal
  - I is additional information such as distribution of neutron star masses or nuclear physics of neutron stars
  - $p(\boldsymbol{d}|\vartheta, h; I)$ is the likelihood
  - $p(\vartheta|\, h; I)$ is the prior
  - $p(\boldsymbol{d}|, h; I)$ is the evidence

Biwer, C. M. et al. Publ. Astron. Soc. Pac. 131 **996** (2019)

Mass Priors

- For mass we used two priors
  - Uniform in the region [1,2)
  - Double Neutron Star or DNS, which is fit to observations of neutron stars in our galaxy[1] and is a truncated normal: $\mathcal{N}(\mu = 1.34, \sigma = 0.09, a = 1, b = 2)$

1. F. Ozel, P. Freire, Ann. Rev. Astron. Astrophys. 54, 401 (2016).

Generating the EOS Prior

- As I mentioned before, something unique about our approach is that we sample over the EOS directly
- EOS are generated and sorted into bins according to their radius at $1.4M_{\odot}$.
- 2000 EOS are selected such that the prior is uniform in R.
- This selection process is important as fewer EOS have very small or very large radii.

Sampling in the EOS

- Each EOS has a data file tabulating the radius, mass, and tidal deformability.
- PyCBC's Markov Chain Monte Carlo samples the EOS prior by drawing a number. The code then opens the data file associated with that EOS and the tidal deformability is taken from the table using the mass.
- The sampler draws two masses and an EOS and calculates Λ_1 and Λ_2 using the EOS, ensuring that both use the exact same EOS

Other priors

- Other priors are uniform
- Spin prior is $\chi_{1,2} \sim$ U(-0.05,0.05)
- Polarization: $\psi \in [0,2\pi)$
- Inclination: $\cos\iota \in [0,1)$
- Coalescence time: $t_c \in t_0 \pm 0.1$ s

EM Constraints

- Due to coincident GRB[1,2] detection we are able to fix the sky location and luminosity distance in our analysis.
- $\boldsymbol{\alpha} = 13^{h}09^{m}48.1^{s}$
- $\boldsymbol{\delta} = -23° 22' 53.4''$
- $d_L = 40.7$ Mpc

1. Soares-Santos, M. *et al.*, Astrophys. J. 848, L16 (2017).
2. Cantiello, M. *et al.*, Astrophys. J. 854, L31 (2018).

Additional Constraints

- We add a threshold mass constraint using the method discussed yesterday in Ben Margalit's presentation.

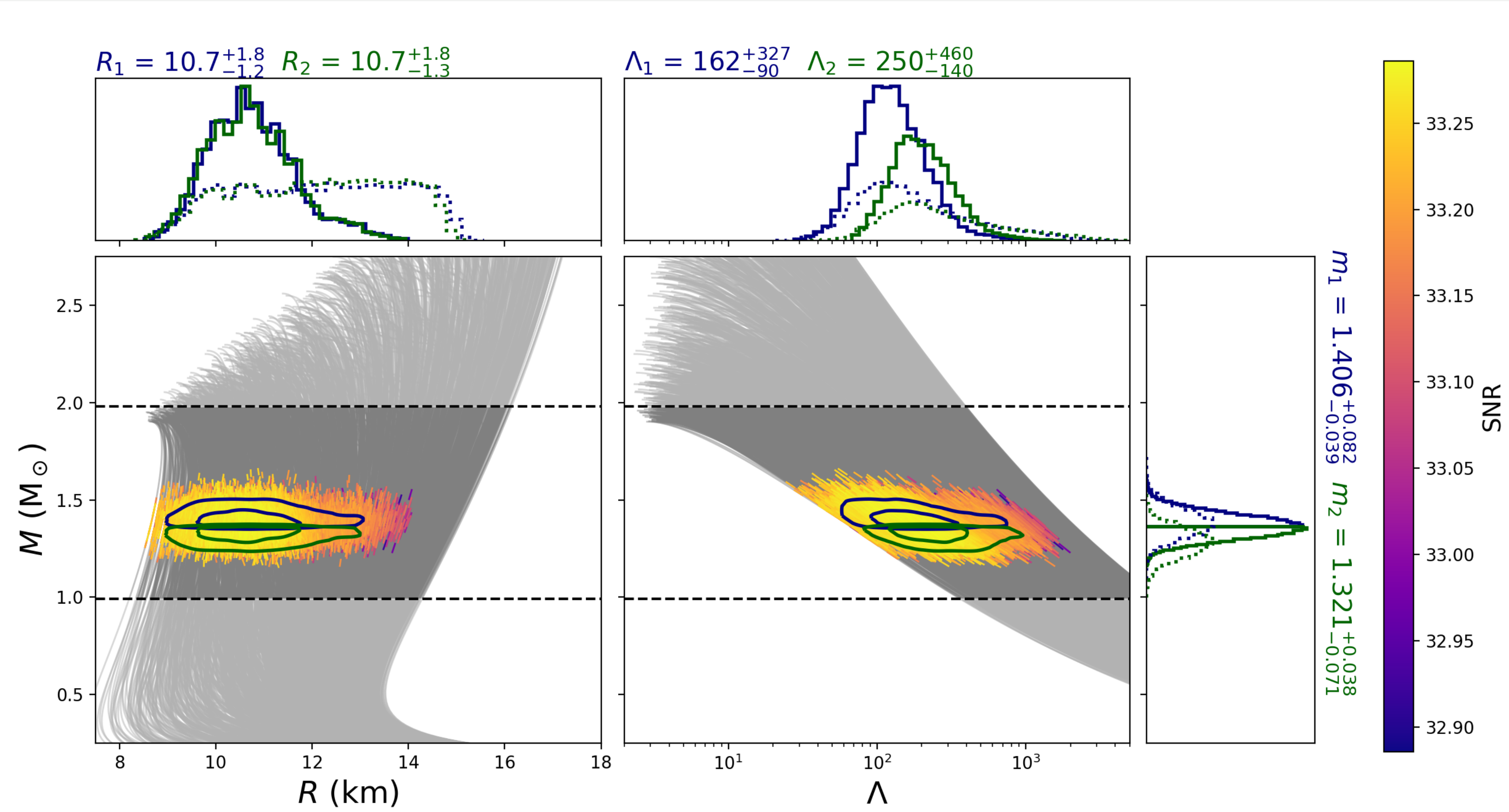
A. Bauswein, T. W. Baumgarte, and H.-T. Janka Phys. Rev. Lett. **111**, 131101 (2013)

Results

Interesting Results

- Several different results
  - We demonstrate the GW170817 favors soft equations of state and small radii: it can even rule out very stiff equations of state
  - We present evidence that suggests the DNS prior is favored over the uniform prior
  - We are able to place tighter constraints on the upper bound on the neutron star radius
  - Tightest constraints to date on tidal deformability of the objects in GW170817 .
  - We present limits on the pressure inside a NS
  - NSBH implications

EOS Parameter Analysis rules out some EOS entirely



Uniform nsat

Bayes Factors

	Uniform nsat	Uniform 2 nsat	DNS nsat
Uniform 2nsat	1.38 ±0.35		
DNS nsat	5 ± 1.3	3.6 ± 0.91	
DNS 2nsat	6.7 ±2.3	4.9 ±1.2	1.35 ±0.34

Robert E. Kass & Adrian E. Raftery (1995). Journal of the American Statistical Association. **90** (430): 791.

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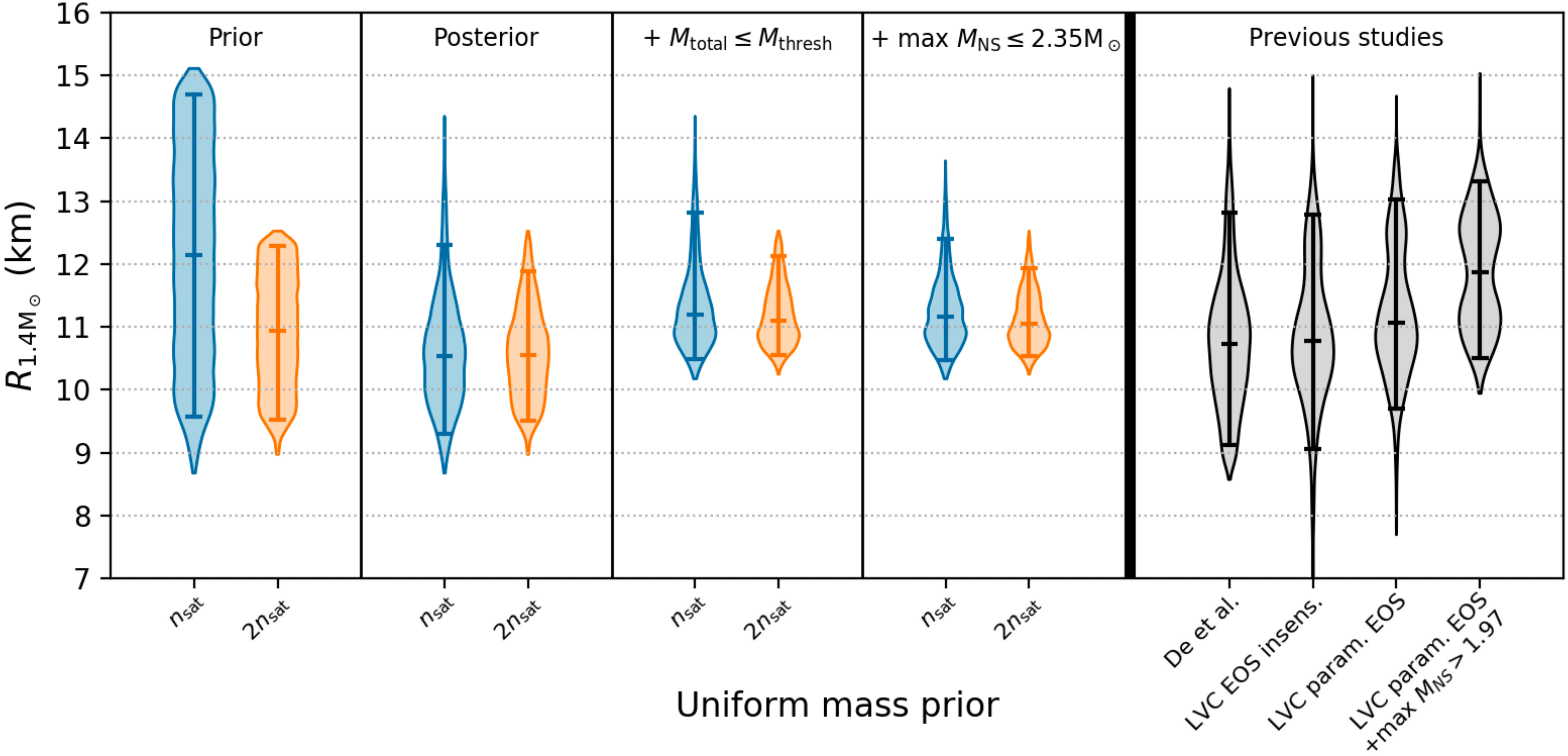
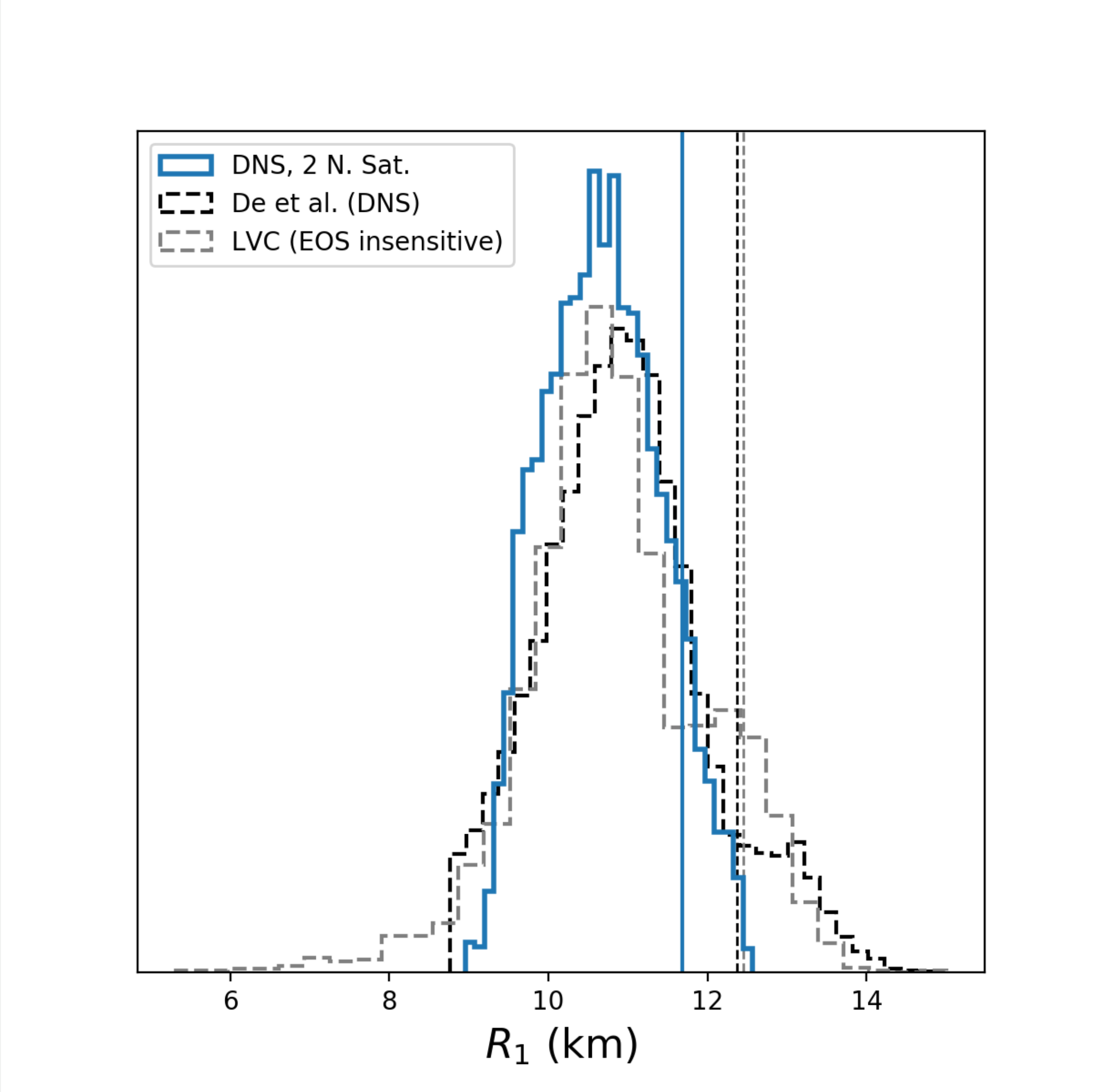


Prior
Posterior
+ $M_{\rm total} \leq M_{\rm thresh}$
+ max $M_{\rm NS} \leq 2.35 M_\odot$
Previous studies
$R_{1.4 M_\odot}$ (km)
16
15
14
13
12
11
10
9
8
7
$n_{\rm sat}$
$2n_{\rm sat}$
De et al.
LVC EOS insens.
LVC param. EOS
LVC param. EOS +max $M_{NS} > 1.97$
Uniform mass prior

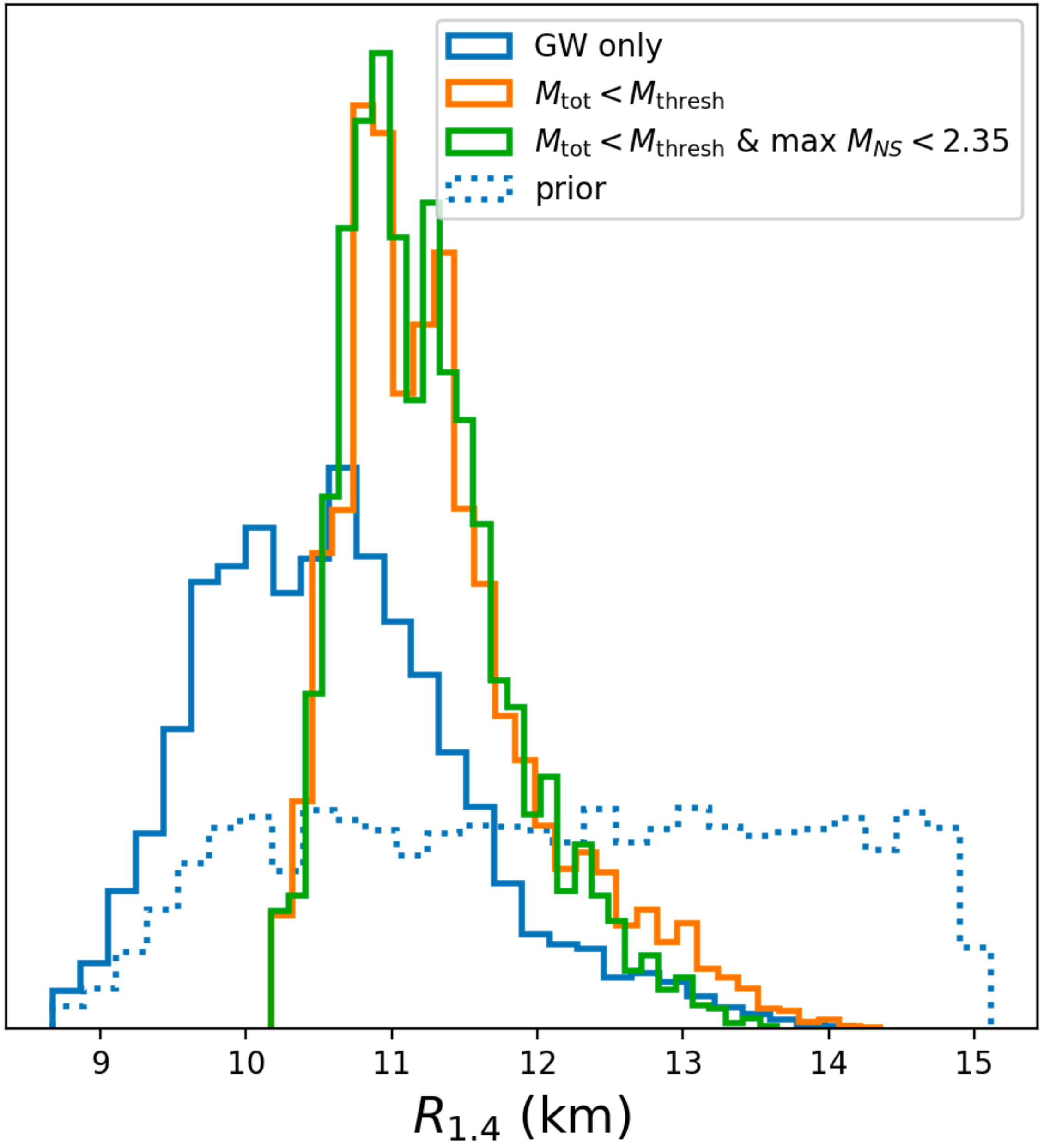


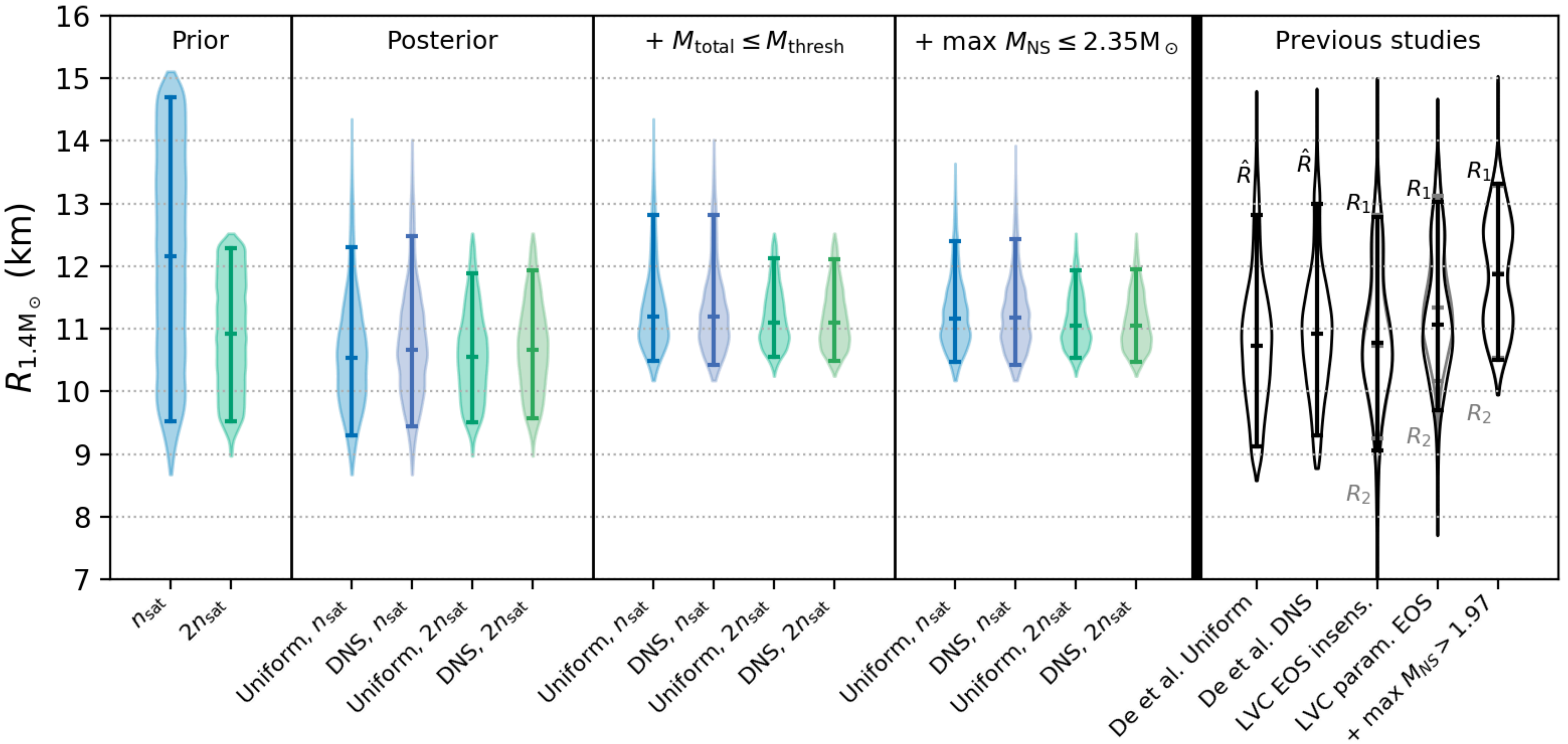
Using just GW wave data and no EM constraints, we still have the most constraining upper limit on radius.

GW Only: $10.5^{+1.8}_{-1.2}$

$M_{tot} < M_{thresh}$: $11.2^{+1.6}_{-0.7}$

$M_{tot} < M_{thresh}$ **& max** $M_{NS} <$ **2.35:** $11.2^{+1.2}_{-0.7}$



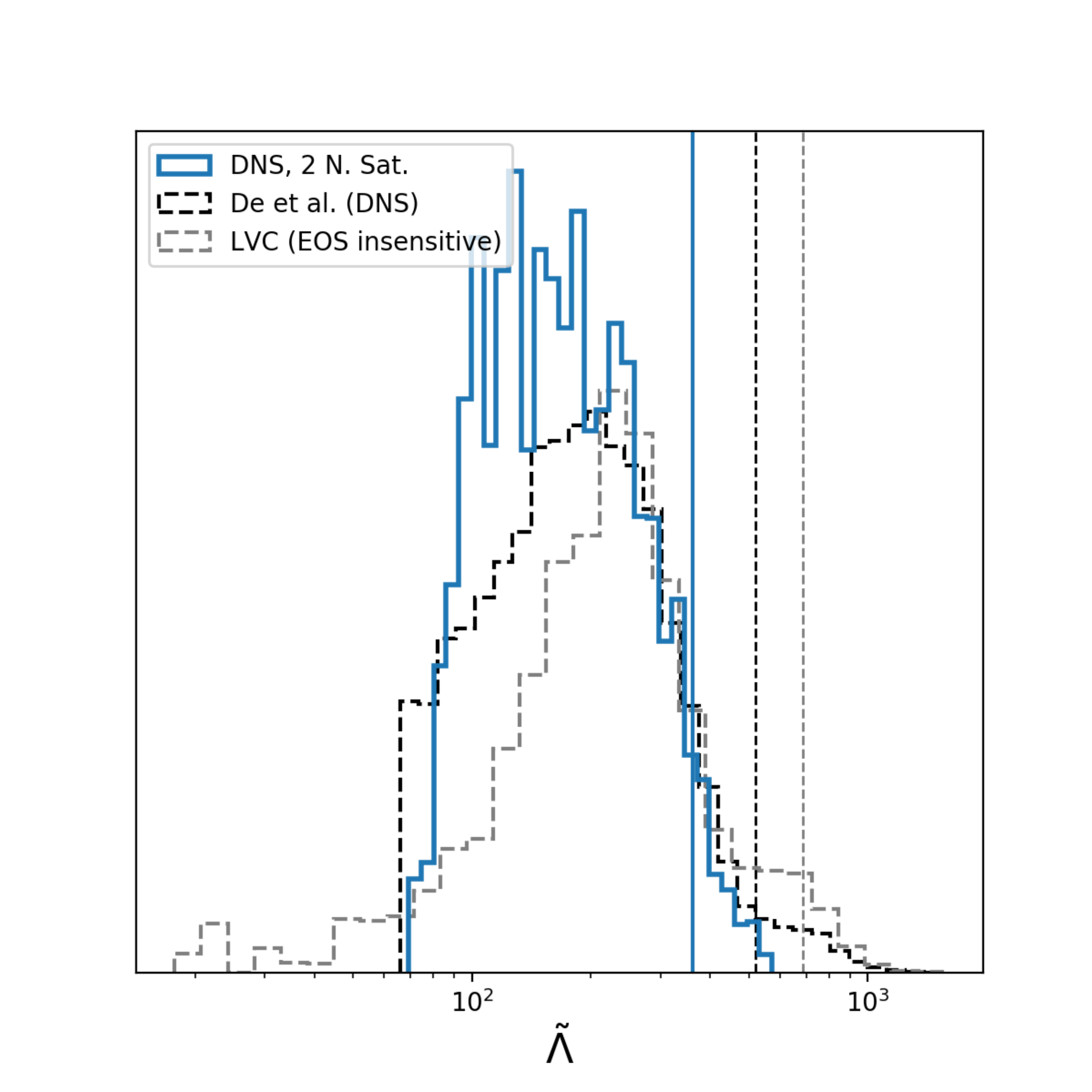


Prior
Posterior
+ $M_{total} \leq M_{thresh}$
+ max $M_{NS} \leq 2.35M_\odot$
Previous studies
$R_{1.4M_\odot}$ (km)
16
15
14
13
12
11
10
9
8
7
$\hat{R}$
$\hat{R}$
R_1
R_1
R_1
R_2
R_2
R_2
n_{sat}
$2n_{sat}$
Uniform, n_{sat}
DNS, n_{sat}
Uniform, $2n_{sat}$
DNS, $2n_{sat}$
Uniform, n_{sat}
DNS, n_{sat}
Uniform, $2n_{sat}$
DNS, $2n_{sat}$
Uniform, n_{sat}
DNS, n_{sat}
Uniform, $2n_{sat}$
DNS, $2n_{sat}$
De et al. Uniform
De et al. DNS
LVC EOS insens.
LVC param. EOS
+ max $M_{NS} > 1.97$

Dns,2nsat 210^{+200}_{-110}

De et al.: 245^{+453}_{-151}

LVC: 290^{+510}_{-160}



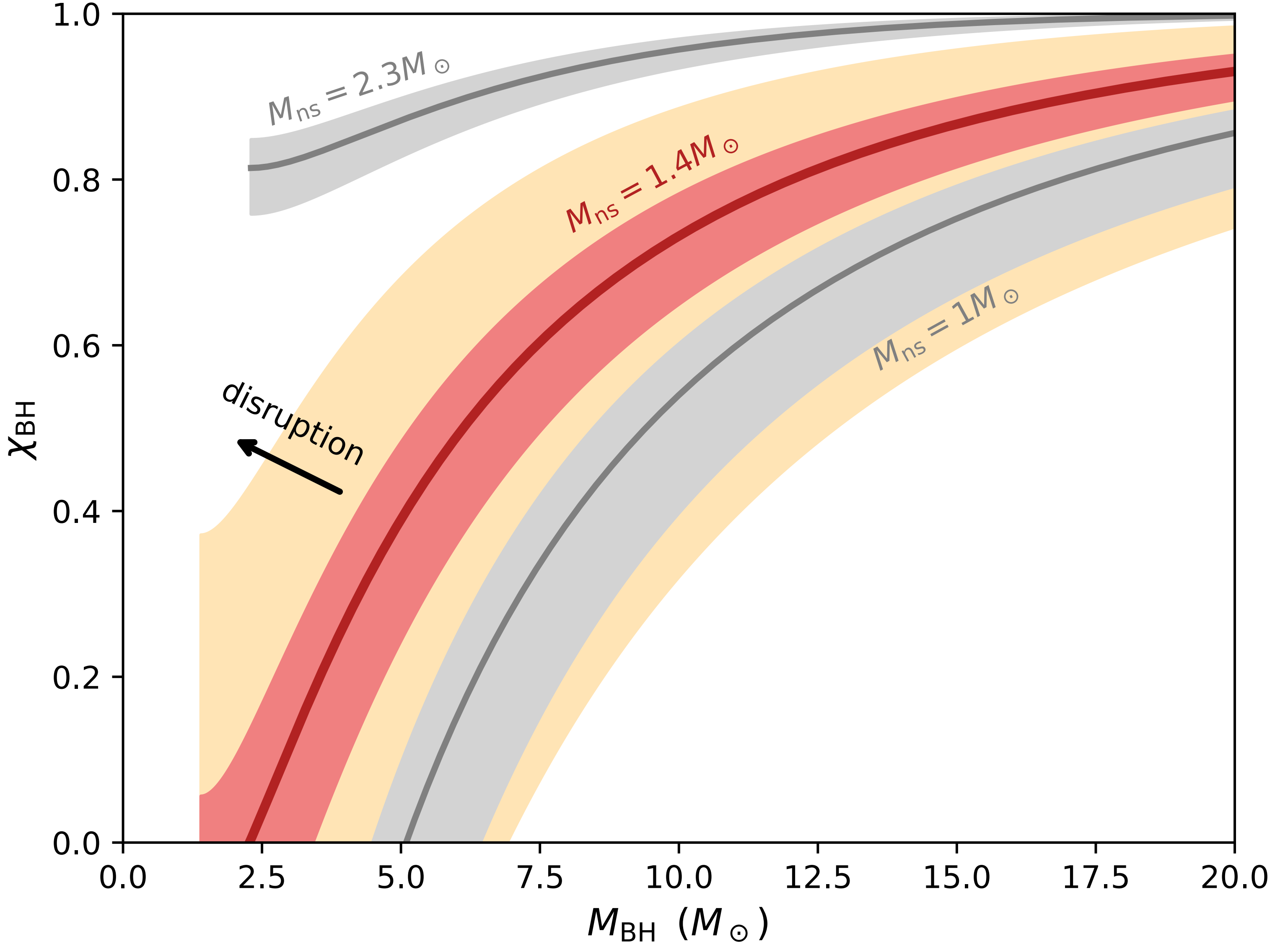
Observable	pr./po.	Uni n_{sat}	Uni $2n_{sat}$	DNS n_{sat}	DNS $2n_{sat}$
$P_{max}[\mathrm{MeV/fm^3}]$	prior	517^{+512}_{-371}	644^{+437}_{-394}	517^{+512}_{-371}	644^{+437}_{-394}
	+GW	11.54^{+350}_{-380}	11.54^{+350}_{-440}	11.07^{+330}_{-360}	11.07^{+380}_{-410}
	+EM	600^{+380}_{-330}	570^{+320}_{-320}	600^{+330}_{-330}	570^{+360}_{-320}
$P_{max}[\mathrm{atm}]$	prior	$8.17^{+8.10}_{-5.87} \times 10^{29}$	$10.02^{+6.91}_{-6.23} \times 10^{29}$	$8.17^{+8.10}_{-5.87} \times 10^{29}$	$10.02^{+6.91}_{-6.48} \times 10^{29}$
	+GW	$13.01^{+5.53}_{-6.00} \times 10^{29}$	$13.01^{+5.53}_{-6.96} \times 10^{29}$	$12.48^{+5.22}_{-5.69} \times 10^{29}$	$12.48^{+6.00}_{-7.30} \times 10^{29}$
	+EM	$9.49^{+6.00}_{-5.22} \times 10^{29}$	$9.01^{+5.06}_{-5.06} \times 10^{29}$	$9.49^{+5.22}_{-5.22} \times 10^{29}$	$9.02^{+5.69}_{-5.06} \times 10^{29}$

- Pressure inside a neutron star is thought to be maximal pressure observed in the universe

Strasbourg astronomical Data Center

NSBH Implications

- EM counter parts are critical indicators of the existence of a NS in a merger
- Mass ejection only occurs when the neutron star is tidally disrupted before the merger
- Whether or not a NS is disrupted depends on the radius of the NS
- Our radius constraint allows us to predict what NSBH events will have an EM counterpart.


$M_{ns} = 2.3M_\odot$
$M_{ns} = 1.4M_\odot$
$M_{ns} = 1M_\odot$
disruption
χ_{BH}
M_{BH} $(M_\odot)$
1.0
0.8
0.6
0.4
0.2
0.0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

Future Work

- The lower radius constraint was calculated using numerical relativity simulations with nucleonic EOS, not with chiral eft. We would like to redo the calculation of the lower limit with chiral-eft for consistency
- More detections of BNS by LIGO/Virgo and future missions will provide more data to constrain the EOS.
- Exploring chiral eft at higher densities
- Exploring how gravitational waves may be able to distinguish between NSBH, BBH, and BNS

Questions?

Supplemental Material

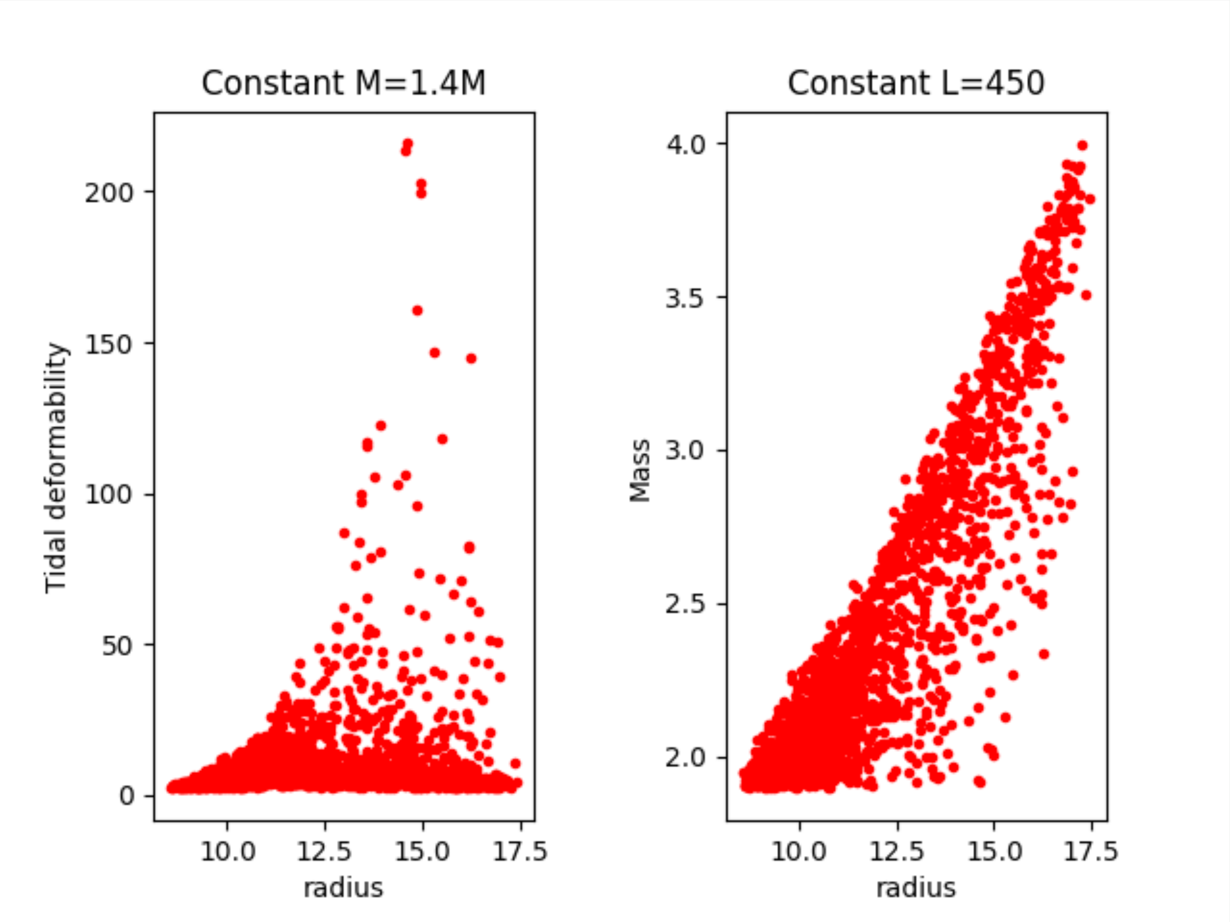
Chiral Effective Field Theory

- Chiral-EFT is a method that incorporates the symmetries of strong interactions, quantum chromodynamics, and low-energy observations into extrapolations to regimes where experimental data is not sufficient.

Model Cross Checks

- Using IMRPhenomD_NRTidal we confirmed
  - Parameter Estimation
  - Evidence Calculations

How do mass and tidal deformability affect radius results



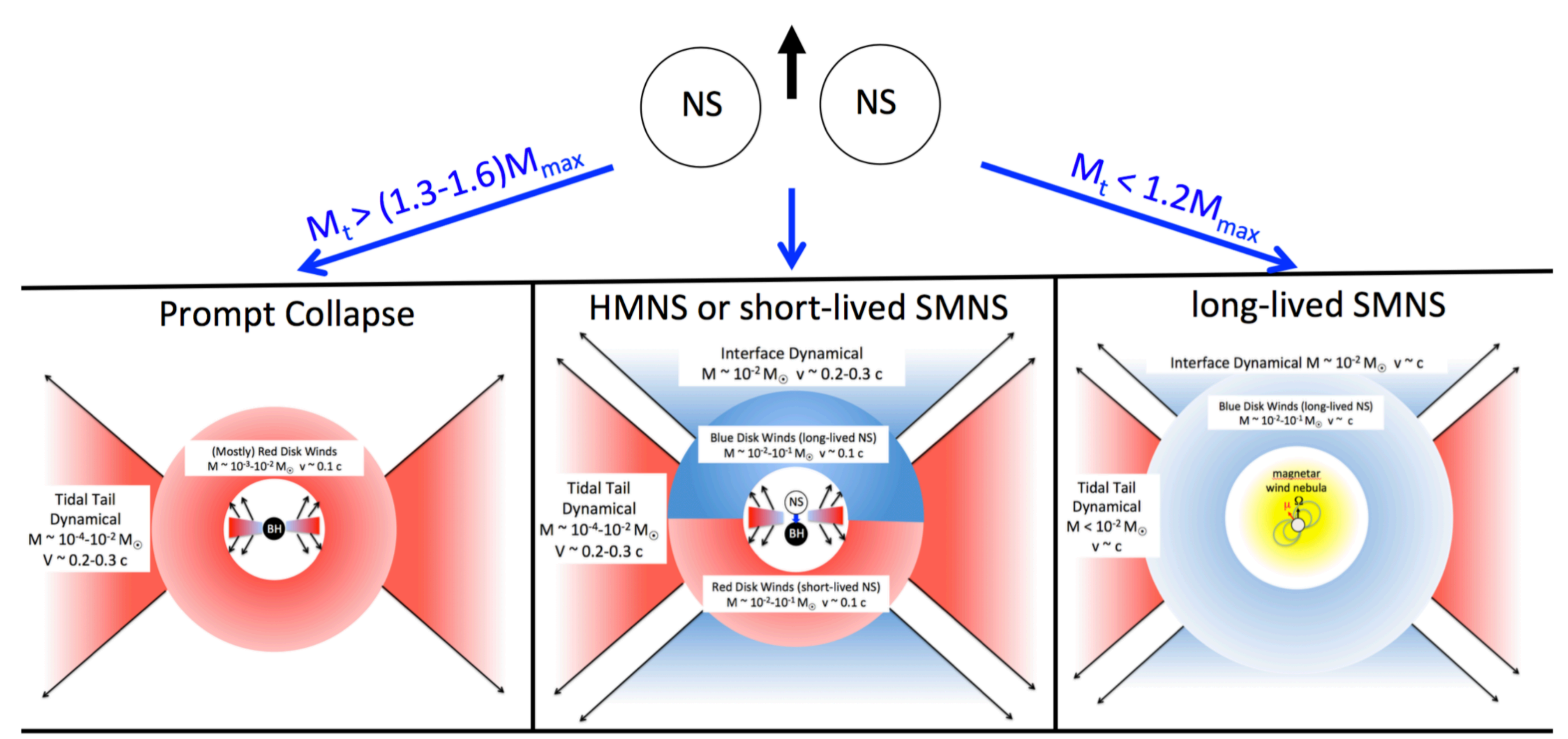


Fig. 1 B. Margalit and B. D. Metzger, Astrophys. J. 850, L19 (2017).

Additional Constraints

- If the remnant did not collapse into a BH immediately after the merger.

$$M_{Thresh} > M_{tot}^{GW170817}$$

- There is an empirical relation for the prompt collapse threshold mass

$$M_{thresh} = \left(-3.38\,\frac{G\,M_{max}}{c^2\,R_{max}} + 2.43\right) M_{max} > 2.74\,M_{\odot}$$

- Relation was calculated in Bauswein et al using numerical simulations of neutron star mergers. Explored 12 EOS spanning a wide range of stiffnesses.
- These EOS are not chiral EFT and we consider this an approximate limit.

A. Bauswein, T. W. Baumgarte, and H.-T. Janka Phys. Rev. Lett. **111**, 131101 (2013)

Cont

- We enforce Causality $(v_s \leq c)$

$$M_{max} \leq \frac{1}{2.82}\frac{c^2 R_{max}}{G}$$

- Combining these conditions yields

$$M_{tot}^{GW170817} \leq \left(-3.38\,\frac{G\,M_{max}}{c^2\,R_{max}} + 2.43\right) M_{max}$$

$$\leq \left(-\frac{3.38}{2.82} + 2.43\right)\frac{1}{2.82}\frac{c^2 R_{max}}{G}$$