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A machine learning approach to extract the EoS of dense matter from the neutron star properties

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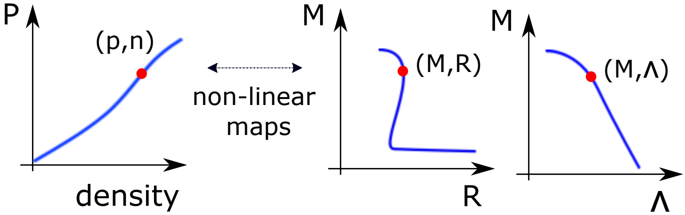
Introduction

The Neutron Star structure is defined by:

- TOV (General Relativity) + EoS (Nuclear Physics)
  - EoS of nuclear matter still unknown for $n > n_{sat}$
  - Big uncertainty on the NS structure
- Non-linear map between EoS and mass-radius (mass-Λ):
  - Standard NS structure problem



Motivation

Constraining nuclear matter parameters from astrophysical observations

- Parametrize the EOS as a Taylor expansion around n_{sat}
- Using supervised machine learning to learn the non-linear maps
- Impact/dependence of the nuclear matter parameters on Λ and R

Supervised Learning was already explored in the inverse stellar problem in *[Yuki Fujimoto, et. al., PRD 98, 023019 (2018))]*

- Used DNN to learn the non-linear map between mass-radius and EoS
- EoS inference from a set of mass-radius observational data

Generating EoS: parametrization

- Homogeneous nuclear matter in β-equilibrium: Taylor expansion (up to third order) around $x = (n - n_{sat})/(3n_{sat})$

$$\mathcal{E}(n_n, n_p) = e_{sat}(n) + e_{sym}(n)\delta^2$$

where the isoscalar/isovector parts

$$e_{sat}(x) = E_{sat} + \frac{1}{2}K_{sat}x^2 + \frac{1}{6}Q_{sat}x^3$$

$$e_{sym}(n) = E_{sym} + L_{sym}x + \frac{1}{2}K_{sym}x^2 + \frac{1}{6}Q_{sym}x^3$$

where $n = n_n + n_p$ is the baryonic density and $\delta = (n_n - n_p)/n$ is the asymmetry.

- Taylor expansions near the saturation density but parameterizations at supra-saturation densities.
  - *[N.B. Zhang, et. al., The Astrophysical Journal 859, 90 (2018)]*
  - *[Margueron, J.,et. al., PRC, 97, 025806 (2017)]*
  - ...

Generating EoS: physical constraints

- The EoS parameter space is 7-dimensional

$$\text{EoS}_i = (E_{sat}, K_{sat}, Q_{sat}, E_{sym}, L_{sym}, K_{sym}, Q_{sym})_i \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

- No a priori correlation between the empirical parameter ($\boldsymbol{\Sigma}$ diagonal)
- The correlations arise from the following constraints (filters):
  - Positive $P(n)$ and $\mathcal{E}(n)$ gradients (thermodynamic stability)
  - The speed of sound $\leq c$ (causality)
  - Supports $1.97 M_\odot$
  - Predicts $70 < \Lambda_{1.4M_\odot} < 580$
  - Positive symmetry energy e_{sym}
- We used the SLy4 EoS ($n < n_{sat}$) for the crust

Generating EoS: sample space

- We have fixed $E_{sat} = -15.8$ MeV
- Sampling from a 6-dimensional EOS parameter space

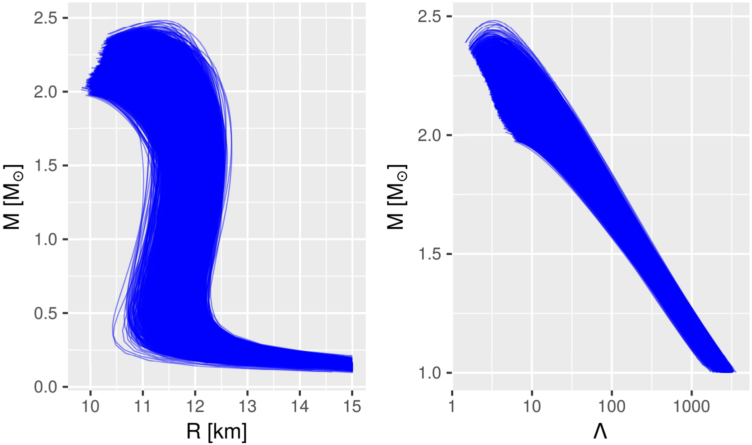
$$\text{EoS}_i = (K_{sat}, Q_{sat}, E_{sym}, L_{sym}, K_{sym}, Q_{sym})_i \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

- Mean values μ_i and variances σ_{ii}

	μ_i [MeV]	$\sqrt{\sigma_{ii}}$ [MeV]
K_{sat}	230	20
Q_{sat}	300	400
E_{sym}	32	2
L_{sym}	60.3	15
K_{sym}	-100	100
Q_{sym}	0	400

- From the 8×10^7 samped EoS, **only 13038 EoS have passed all filters.**

Diagrams $M - R$ and $M - \Lambda$ for the 13038 EoS



- $\Lambda_{1.4M_\odot} = 415.723 \pm 46.179$
- $R_{1.4M_\odot} = 12.055 \pm 0.194$ km

Supervised Machine Learning

- Our goal is to learn the following non-linear maps

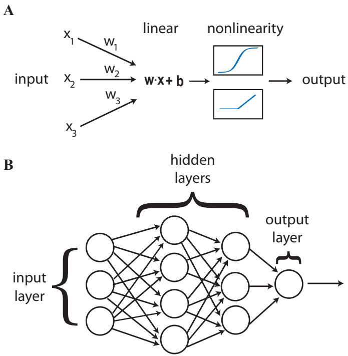
$$\Lambda_{M_i}(K_{sat}, Q_{sat}, E_{sym}, L_{sym}, K_{sym}, Q_{sym})$$

$$R_{M_i}(K_{sat}, Q_{sat}, E_{sym}, L_{sym}, K_{sym}, Q_{sym})$$

- One can then analyze the exact effect of each EoS parameter on Λ and R
- Two supervised machine learning methods
  - Deep Neural Networks (DNN)
  - Supportive Vector Machines Regresion (SVM-R)

Deep Neural Networks

- Hierarchical layers of neurons that perform a complex non-linear transformation of the inputs



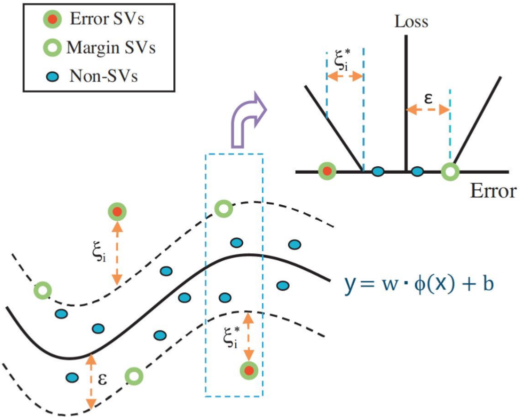
[Figure from Pankaj Mehta, et. al., Physics Reports 810 (2019) 1-124]

- Training a DNN consists in finding the optimal weights and biases by minimizing a loss function, e.g.,

$$L(\boldsymbol{w}, \boldsymbol{b}) = \frac{1}{N} \sum_i (\hat{y}_i(\boldsymbol{w}, \boldsymbol{b}) - y_i)^2 \quad \text{(MSE)}$$

Supportive Vector Machines Regression

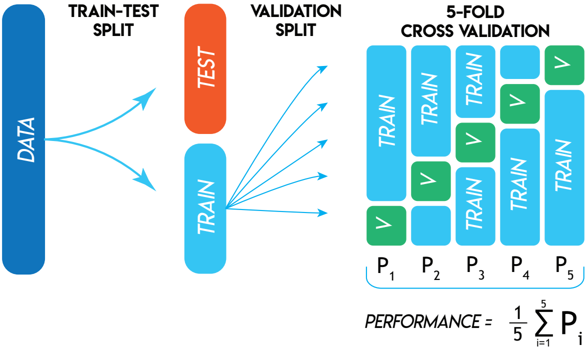
- Fits a hyper-tube of radius ϵ that encloses as many data points as possible with a minimal number of violations.



[Figure from Shi, et. al., Flood Prediction Using Support Vector Machines (SVM)]

Learning procedure

- Data set: 13038 EoS
- Split the data set into training (80%) and test (20%) sets
- 5-fold validation



- Grid search to find the best SVMR and DNN model
- Finally, the performance (RMSE) is measured on the test set

Best DNN and SVM-R models: performance

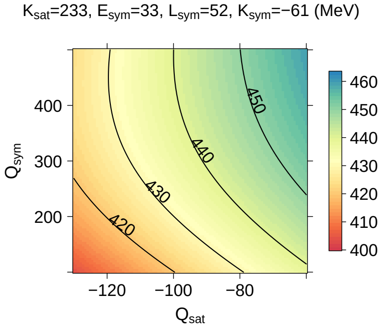
- Evaluation of RMSE $= \sqrt{\frac{1}{N}\sum_i(\hat{y}_i - y_i)^2}$ on test set (2608 EoS)

	RMSE	
	DNN	SVM-R
$\Lambda_{1.0M_\odot}$	16.646	23.547
$\Lambda_{1.4M_\odot}$	1.932	2.236
$\Lambda_{1.9M_\odot}$	0.227	0.556
$R_{1.0M_\odot}$ [km]	0.007	0.012
$R_{1.4M_\odot}$ [km]	0.006	0.010
$R_{1.9M_\odot}$ [km]	0.007	0.019

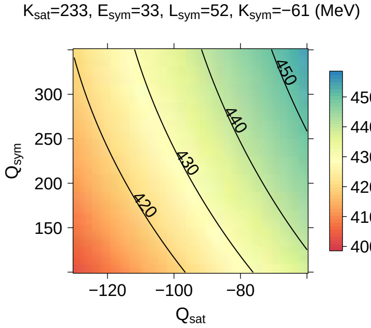
Results: SVMR and DNN comparison

- Dependence of $\Lambda_{1.4M_\odot}$ on (Q_{sym}, Q_{sat})

SVMR



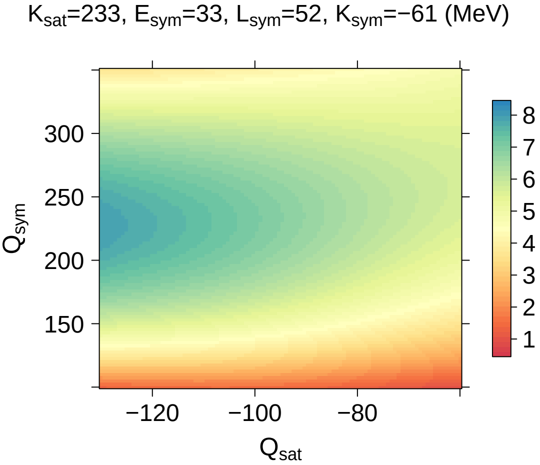
DNN



- DNN predicts almost a linear correlation $\Lambda_{1.4M_\odot} \sim aQ_{sym} + bQ_{sat}$
- There is a region where $\Lambda_{1.4M_\odot}$ is insensitive to Q_{sym} for SVMR

Results: SVMR and DNN comparison

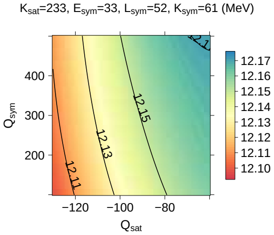
- Diference in prediction: SVMR($\Lambda_{1.4M_\odot}$) - DNN($\Lambda_{1.4M_\odot}$)



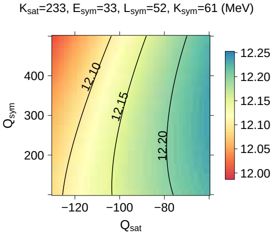
- The discrepancy is of $\Delta\Lambda_{1.4M_\odot} < 9$

Results: dependence of $R(Q_{sym}, Q_{sat})$ on M

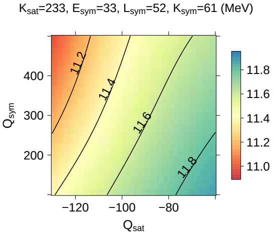
$R_{1.0M_\odot}$



$R_{1.4M_\odot}$



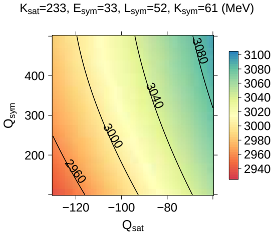
$R_{1.9M_\odot}$



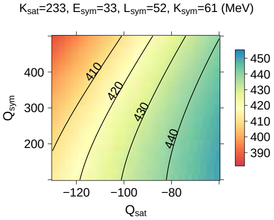
- The dependence changes with increasing M
- $R_{1.0M_\odot}$ and $R_{1.4M_\odot}$ are quite insensitive to Q_{sym} and Q_{sat}

Results: dependence of $\Lambda(Q_{sym}, Q_{sat})$ on M

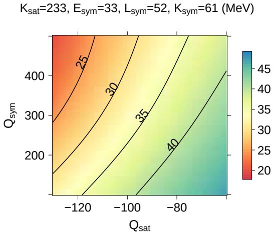
$\Lambda_{1.0M_\odot}$



$\Lambda_{1.4M_\odot}$

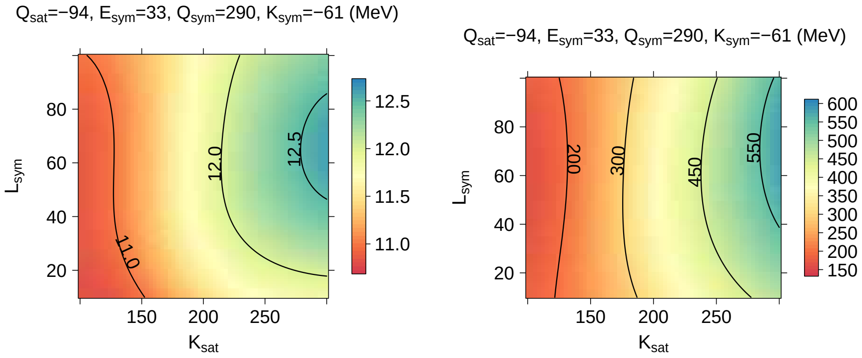


$\Lambda_{1.9M_\odot}$



- The dependence of Λ with increasing M is similar to R_M

Results: effect of $\{L_{sym}, K_{sat}\}$ on $R_{1.4M_\odot}$ and $\Lambda_{1.4M_\odot}$



- L_{sym} shows almost no effect $\Lambda_{1.4M_\odot}$.
- These same happens with $R_{1.4M_\odot}$ for $L_{sym} > 40$.

Conclusions

- The supervised ML methods are able to accurately learn the non-linear maps between the EoS empirical parameters and both R and Λ.
- They allow to study the exact dependence of each empirical parameter on astrophysical observables.
- These non-linear maps allow to constraining nuclear matter properties from astrophysical quantities.
- NS observations are presently the only way of accessing the cold high density QCD phase diagram

Acknowledgments

G2NET
COST ACTION CA17137
A NETWORK FOR GRAVITATIONAL WAVES, GEOPHYSICS AND MACHINE LEARNING

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