Two-solar-mass hybrid stars

a two model description with the Nambu−Jona-Lasinio quark model

Renan Câmara Pereira

in collaboration with: Pedro Costa & Constança Providência
Centre for Physics of the University of Coimbra

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Introduction

It is an **exciting era** to study **neutron stars**:

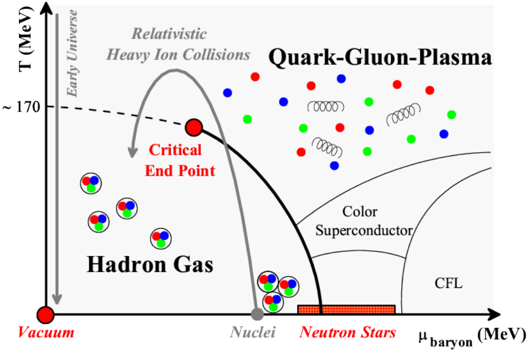
- The Equation of State (**EoS**) of dense cold nuclear matter is still unknown from first principles calculations;
- The discovery of Gravitational Waves opened the window for new experimental constraints;

Due to its large central densities, the core of a neutron star may be composed by pure quark matter.

Do massive neutron stars have a core composed by quark matter?

The QCD phase diagram

The different manifestations of QCD matter can be displayed in a $T - \mu_B$ phase diagram.



Neutron stars populate a very interesting regime in the **QCD** phase diagram.

In these conditions, to study QCD, we are limited in options: or

- **Lattice QCD**
  - first principle calculations;
  - currently only works on the finite temperature and zero/low density region due to the so called sign problem;

- **Dyson–Schwinger equations**
  - truncation required;

- **Effective models**
  - incorporate the most important features of QCD at a certain energy scale;
  - work on the entire range of the phase diagram;
  - coupling parameters need to be fixed to experimental data or first principle calculations;

In this work, to study the possibility of a quark core, we make a two-model approach to the description of the **EoS** of compact stars with two independent models.

- The hadronic phase will be described by the NL3$\omega\rho$ model;
- The quark phase by the 3 flavour **Nambu–Jona-Lasinio** model;

Following previous studies, to favour the appearance of quark matter in the core of neutron stars, we will consider:

- A low vacuum constituent quark mass;
- Different types of vector interactions, known to stiffen the **EoS**;
- That the deconfinement transition coincides with the partial restoration of chiral symmetry;

A phenomenological Bag constant will be used to make the transition between the hadronic and quark models coincide with the chiral symmetry restoration of the quark model, [Pagliara and Schaffner-Bielich, 2008].

B^* is fixed in such a way that: deconfinement phase transition and the chiral transition, coincide through:

$$P_{eff} = P^{\text{ quarks}} + B^*,$$
$$\epsilon_{eff} = \epsilon^{\text{ quarks}} - B^*.$$

To build the hybrid **EoS** we use the Maxwell construction:

$$\mu_B^H = \mu_B^Q \quad \text{chemical equilibrium,}$$
$$p_B^H = p_B^Q \quad \text{mechanical equilibrium,}$$
$$T_B^H = T_B^Q \quad \text{thermal equilibrium.}$$

The hadronic model

The NL3$\omega\rho$ model Lagrangian density is given by:

$$
\begin{aligned}
\mathscr{L}^{\mathrm{NL3}\omega\rho} = \overline{\psi}_N & \left[\gamma^\mu \left(i\partial_\mu - g_\omega \omega_\mu - \frac{1}{2} g_\rho \boldsymbol{\tau} \cdot \boldsymbol{\rho_\mu}\right) - (m - g_\sigma \sigma)\right] \psi_N \\
& + \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} bm (g_\sigma \sigma)^3 - \frac{1}{4} c (g_\sigma \sigma)^4 \\
& + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{4!} \xi g_\omega^4 \left(\omega_\mu \omega^\mu\right)^2 \\
& - \frac{1}{4} \boldsymbol{\rho}^{\mu\nu} \cdot \boldsymbol{\rho}_{\mu\nu} + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}^\mu \cdot \boldsymbol{\rho}_\mu \\
& + \Lambda_\omega \left(g_\omega^2 \omega_\mu \omega^\mu\right) \left(g_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu\right) .
\end{aligned}
$$

This model satisfies constraints imposed by observation ($M_{\max} > 2M_\odot$), CET (chiral effective field theory), and nuclear properties, [Fortin et al., 2016].

The quark model

The general Lagrangian density of the **NJL** model for N_f flavours of quarks interacting through a local scalar and pseudoscalar four point interaction, is given by:

$$\mathscr{L}^{\text{NJL}} = \overline{\psi}\left(i\partial\!\!\!/ - \hat{m} + \hat{\mu}\gamma^0\right)\psi + G_S \sum_{a=0}^{N_f^2-1}\left[\left(\overline{\psi}\Gamma^a\psi\right)^2 + \left(\overline{\psi}i\gamma_5\Gamma^a\psi\right)^2\right] + \mathscr{L}^{det} - \mathscr{L}^{vec}.$$

We consider a 3-momentum cut off as a regularization scheme.

$\mathscr{L}^{det}$ explicitly break the $U_A(1)$ symmetry:

$$\mathscr{L}^{det} = G_D\left(\det_f\left[\overline{\psi}\left(1+\gamma_5\right)\psi\right] + \det_f\left[\overline{\psi}\left(1-\gamma_5\right)\psi\right]\right).$$

$\mathscr{L}^{vec}$ is a vector interaction:

$$\mathscr{L}^{vec} = G_{vec}\sum_{a=0}^{N_f^2-1}\left[(\overline{\psi}\gamma^\mu\Gamma^a\psi)^2 + (\overline{\psi}\gamma^\mu\gamma_5\Gamma^a\psi)^2\right].$$

The operators Γ^a, are N_f^2 matrix operators that form a $U\left(N_f\right)$ algebra.

We will consider 3 types of vector interactions

$$\text{NJL(V+P+VI+PI)}: \ \mathscr{L}_I^{vec} = G_\omega \left[(\overline{\psi}\gamma^\mu \Gamma^0 \psi)^2 + (\overline{\psi}\gamma^\mu \gamma_5 \Gamma^0 \psi)^2 \right] + G_\rho \sum_{a=1}^{N_f^2-1} \left[(\overline{\psi}\gamma^\mu \Gamma^a \psi)^2 + (\overline{\psi}\gamma^\mu \gamma_5 \Gamma^a \psi)^2 \right],$$

$$\text{NJL(V+P)}: \ \mathscr{L}_{II}^{vec} = G_\omega \left[(\overline{\psi}\gamma^\mu \Gamma^0 \psi)^2 + (\overline{\psi}\gamma^\mu \gamma_5 \Gamma^0 \psi)^2 \right],$$

$$\text{NJL(VI+PI)}: \ \mathscr{L}_{III}^{vec} = G_\rho \sum_{a=1}^{N_f^2-1} \left[(\overline{\psi}\gamma^\mu \Gamma^a \psi)^2 + (\overline{\psi}\gamma^\mu \gamma_5 \Gamma^a \psi)^2 \right].$$

Neutron star matter

To study matter inside neutron stars, we impose:

- Zero electrical charge density, $\rho_Q = 0$;
- $\beta-$equilibrium (the neutron decay and electron capture happens at the same rate);
- Neutrinos escape because they interact very poorly with the rest of matter;
- We use the $T = 0$ limit of the **EoS**;

Due to $\beta-$equilibrium, a free gas of electrons must be added to the pressure.

To calculate the mass-radius and mass-central density diagrams we use the Tolman–Oppenheimer–Volkoff equations (**TOV**) for static and spherically symmetric stars:

$$\frac{dP(r)}{dr} = -\frac{G}{r^2}\left[\rho(r) + \frac{P(r)}{c^2}\right]\left[M(r) + 4\pi r^3 \frac{P(r)}{c^2}\right]\left[1 - \frac{2GM(r)}{c^2 r}\right]^{-1},$$

$$M(r) = 4\pi \int_0^r dr' \, r'^2 \epsilon\left(r'\right).$$

The tidal deformability can be calculated solving:

$$\frac{\mathrm{d}^2 H(r)}{\mathrm{d}r^2} + \left\{\frac{2}{r} + \left[\frac{2M(r)}{4\pi r^2} + r(P(r) - \epsilon(r))\right]\frac{4\pi}{1 - \frac{2M(r)}{r}}\right\}\frac{\mathrm{d}H(r)}{\mathrm{d}r}$$
$$+ \left\{\left[5\epsilon(r) + 9P(r) + (P(r) + \epsilon(r))\frac{\mathrm{d}\epsilon}{\mathrm{d}P} - \frac{3}{2\pi r^2}\right]\frac{4\pi}{1 - \frac{2M(r)}{r}}\right\}H(r)$$
$$- 4\left\{\frac{\frac{\mathrm{d}P(r)}{\mathrm{d}r}}{P(r) + \epsilon(r)}\right\}^2 H(r) = 0.$$

The initial conditions for the differential equations are: $P(r = R) = 0$, $M(r = 0) = 0$ and $H'(r = 0) = H(r = 0) = 0$.

Results

We use a parameter set in which the quark constituent mass in the vacuum, is approximately one third of the baryonic mass of the nucleon.

Parameter set	Λ [MeV]	$m_{u,d}$ [MeV]	m_s [MeV]	$G_S\Lambda^2$	$G_D\Lambda^5$	$M_{u,d}$ [MeV]	M_s [MeV]
$SU_f(3)$−I	630.0	5.5	135.7	1.781	9.29	312	508

Table: Λ is the model cut-off, $m_{u,d}$ and m_s are the quark current masses, G_S and G_D are coupling constants. $M_{u,d,s}$ are the quark constituent masses in the vacuum.

We have considered the BPS **EoS** for the outer crust [Baym, Pethick, and Sutherland, 1971] and the inner crust was calculated within a Thomas-Fermi calculation of the pasta phases.

Equations of state

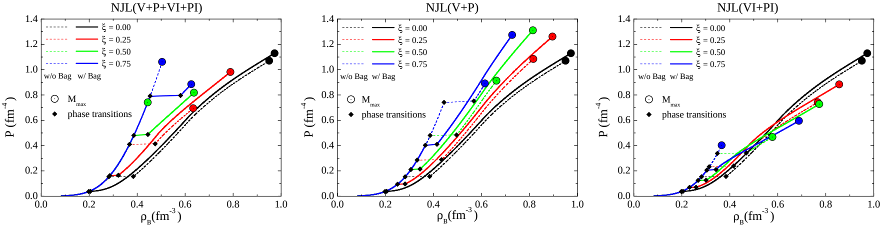


Table: **EoS** as a function of the baryonic density (ρ_B), with $B^* \neq 0$ and $B^* = 0$ for different values of $\xi = G_V/G_S$.

- The inclusion of a $B^* \neq 0$ makes the transition form the hadronic **EoS** to the quark **EoS** occur earlier;
- Increase G_V makes the **EoS** harder in models with vector-isoscalar interaction (NJL(V+P));

M-R relations

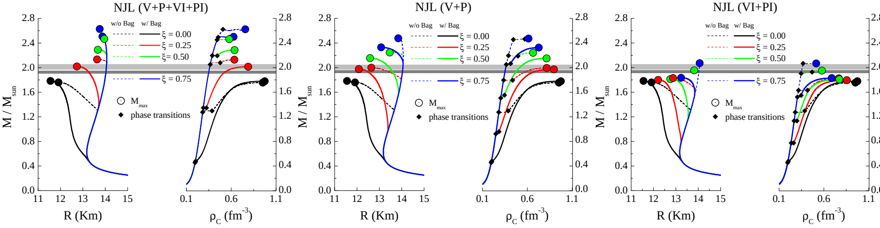


Table: mass-radius and mass-central density diagrams with $B^* \neq 0$ and $B^* = 0$ for different values of $\xi = G_V/G_S$.

- Stars with a pure quark core are predicted both with $B^* \neq 0$ and if $B^* = 0$, except in some cases if $\xi = G_V/G_S = 0.75$ is too large;
- The vector-isovector interaction (NJL(VI+PI)) has a much smaller effect than the vector-isoscalar (NJL(V+P)) interaction;

Quark fractions

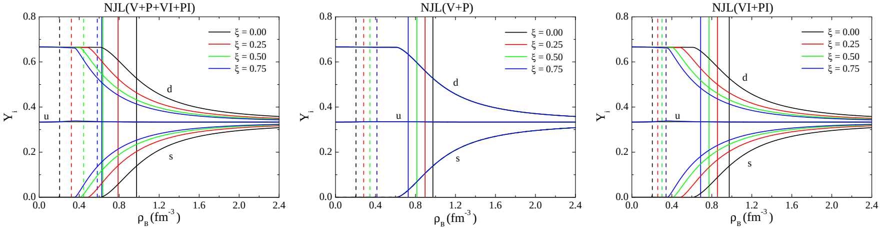


Table: Fractions of each flavour of quark (Y_i) in function of the baryonic density (ρ_B). The central density (ρ_c, full line) and initial quark phase density (ρ_c, dashed line) are shown.

- The different interactions influence the chemical constitution of the quark phase, e.g. the strangeness content;
- As soon as the *strange* quark appear, the *down* quark suffers a strong reduction;
- The onset of *strangeness* in the NJL(V+P) model is G_V independent;

Tidal deformability

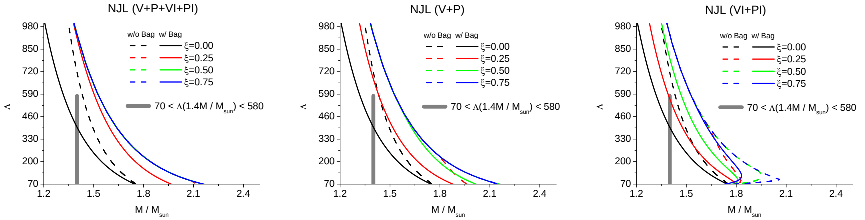


Table: Tidal deformability with $B^* \neq 0$ and $B^* = 0$ for different values of $\xi = G_V/G_S$. The constrain for $\Lambda(1.4)$ represents a 90% confidence interval from [Abbott, 2018].

- Some models with quark core are not ruled out by the tidal deformability constraint;
- The tidal deformability for 1.4 $M_{\rm sun}$ is very constrained by the hadronic model;

Conclusions

How to fix the parameters of the NJL EoS for neutron star quark matter?

- Large versus low vacuum quark constituent mass: a low mass is necessary for a quark core;
- Using a bag constant to coincide the hadron-quark transition and restoration of chiral symmetry favours a quark core;
- Vector-isoscalar interaction: larger masses are attained;
- Vector-isovector interaction: larger strangeness fractions and larger radii;

Further Work

- Due to the tidal deformability constraint, different nuclear models for the hadronic part of the hybrid **EoS** should be considered;
- Consider different quark models like the Quark-Meson model;
- Improve the quark model description by going beyond the usual mean field approximation and include quantum fluctuations (e.g. using the Functional Renormalization Group)

Thank you for your attention!

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Thermodynamics

After calculating Ω (the grand canonical potential), several thermodynamic quantities of interest, can be derived:

$$
\begin{aligned}
P(T,\mu) - P_0 &= -\Omega(T,\mu), \\
\rho_i(T,\mu) &= -\left(\frac{\partial \Omega(T,\mu)}{\partial \mu_i}\right)_T, \\
s(T,\mu) &= -\left(\frac{\partial \Omega(T,\mu)}{\partial T}\right)_\mu, \\
\epsilon(T,\mu) &= -P(T,\mu) + Ts(T,\mu) + \sum_i \mu_i \rho_i(T,\mu).
\end{aligned}
$$

The constants P_0 and ϵ_0 are the pressure and energy density in the vacuum, respectively.

NL3$\omega\rho$ model properties

This model has the following saturation properties [Fortin et al., 2016]:

$$
\begin{aligned}
\text{saturation density:} \quad & \rho_0 = 0.148\mathrm{fm}^{-3} \\
\text{binding energy:} \quad & E/A = -16.30\mathrm{MeV} \\
\text{incompressibility:} \quad & K = 271.76\mathrm{MeV} \\
\text{symmetry energy:} \quad & J = 31.7\mathrm{MeV} \\
\text{symmetry energy slope:} \quad & L = 55.5\mathrm{MeV} \\
\text{effective mass:} \quad & M^*/M = 0.60;
\end{aligned}
$$

$SU(3)_f$ NJL model (MFA)

Using the mean field approximation, the following grand canonical potential for the $SU_f(3)$ NJL model is found:

$$\begin{aligned}\Omega_{\text{MFA}} - \Omega_0 &= 2G_S\left(\sigma_u^2+\sigma_d^2+\sigma_s^2\right) - 4G_D\sigma_u\sigma_d\sigma_s \\ &- \frac{2}{3}G_\omega\left(\rho_u+\rho_d+\rho_s\right)^2 - G_\rho\left(\rho_u-\rho_d\right)^2 - \frac{1}{3}G_\rho\left(\rho_u+\rho_d-2\rho_s\right)^2 \\ &- 2T\,N_c\sum_{f=u,d,s}\int\frac{d^3p}{(2\pi)^3}\left[\beta E_f + \ln\left(1+e^{-\beta(E_f+\tilde{\mu}_f)}\right) + \ln\left(1+e^{-\beta(E_f-\tilde{\mu}_f)}\right)\right].\end{aligned}$$

The effective chemical potential for each flavour of quark is:

$$\begin{aligned}\tilde{\mu}_u &= \mu_u - \frac{4}{3}\left(G_\omega+2G_\rho\right)\rho_u - \frac{4}{3}\left(G_\omega-G_\rho\right)\rho_d - \frac{4}{3}\left(G_\omega-G_\rho\right)\rho_s, \\ \tilde{\mu}_d &= \mu_d - \frac{4}{3}\left(G_\omega+2G_\rho\right)\rho_d - \frac{4}{3}\left(G_\omega-G_\rho\right)\rho_s - \frac{4}{3}\left(G_\omega-G_\rho\right)\rho_u, \\ \tilde{\mu}_s &= \mu_s - \frac{4}{3}\left(G_\omega+2G_\rho\right)\rho_s - \frac{4}{3}\left(G_\omega-G_\rho\right)\rho_u - \frac{4}{3}\left(G_\omega-G_\rho\right)\rho_d.\end{aligned}$$

The values of condensates σ_u, σ_d and σ_s are determined by minimizing the grand canonical potential:

$$\frac{\partial \Omega_{\mathrm{MFA}}}{\partial \sigma_u} = \frac{\partial \Omega_{\mathrm{MFA}}}{\partial \sigma_d} = \frac{\partial \Omega_{\mathrm{MFA}}}{\partial \sigma_s} = 0.$$

The *gap* equations for three flavours are :

$$M_i = m_i - 4G_S\sigma_i + 2G_D\sigma_j\sigma_k \quad i \neq j \neq k \in \{u, d, s\},$$

here the quark condensate for each flavour is given by:

$$\sigma_i = \langle \overline{\psi}_i \psi_i | \overline{\psi}_i \psi_i \rangle = -2N_c \int \frac{d^3p}{(2\pi)^3} \frac{M_i}{E_i} \left(1 - n_i - \overline{n}_i\right)$$