Black holes in modified gravity

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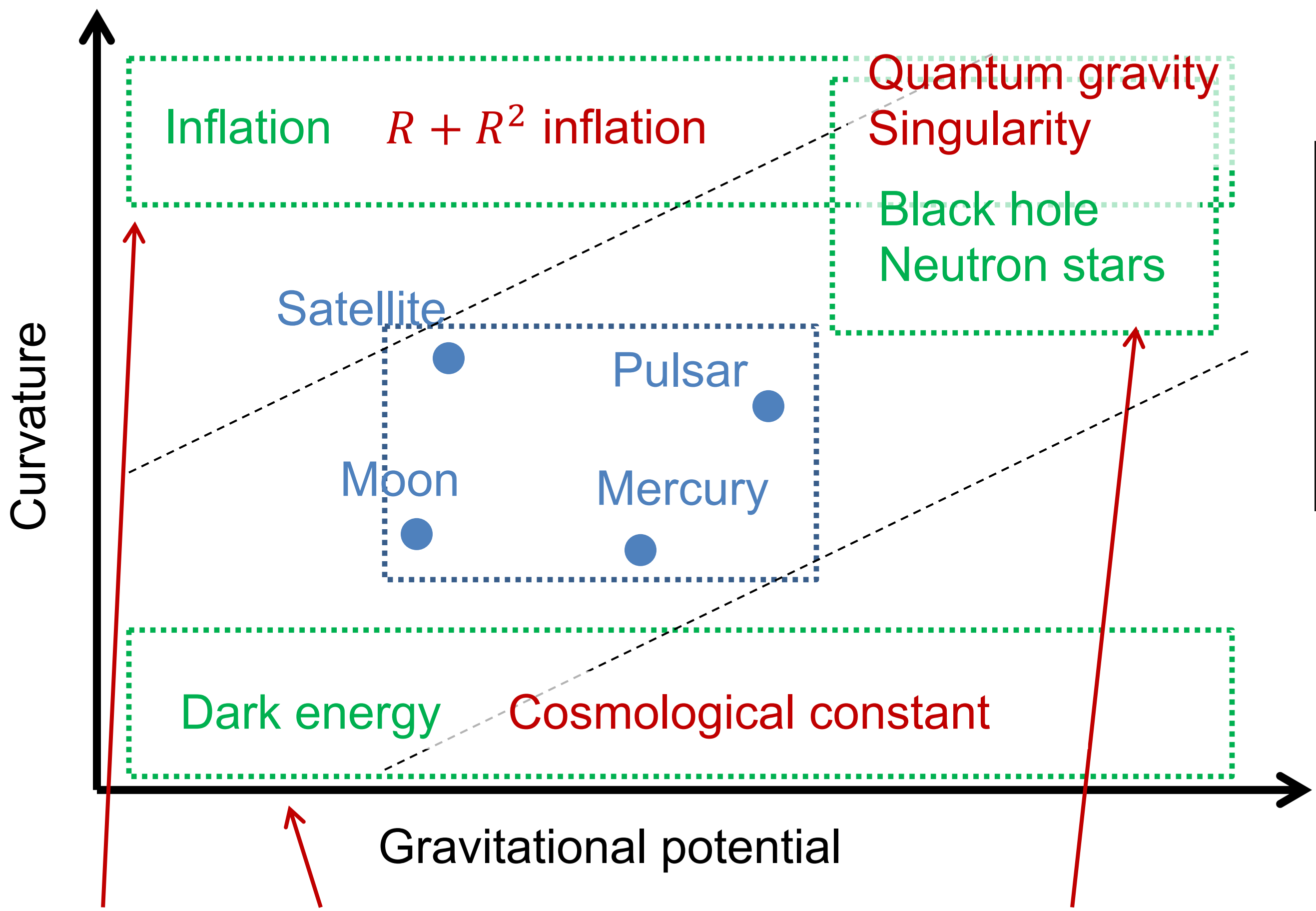
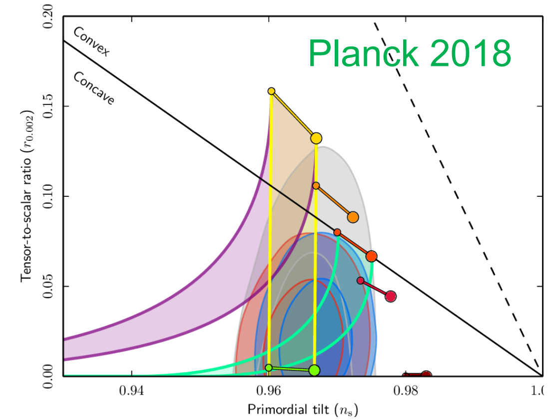
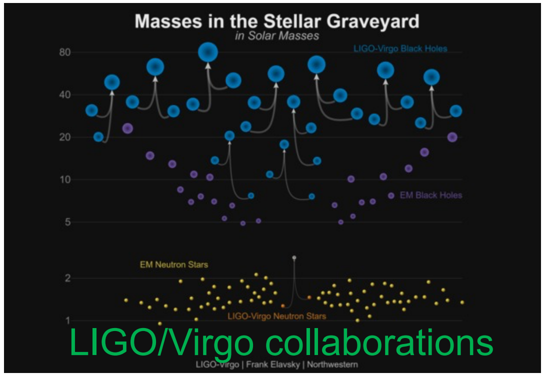
2019.10.3 YITP long-term workshop
Multi-Messenger Astrophysics in the Gravitational Wave Era

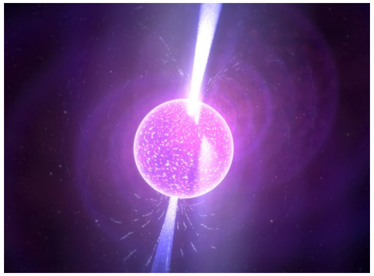
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS
CGP
Center for Gravitational Physics
Yukawa Institute for Theoretical Physics
Gravitational Wave Physics and Astronomy Genesis
科研費 KAKENHI

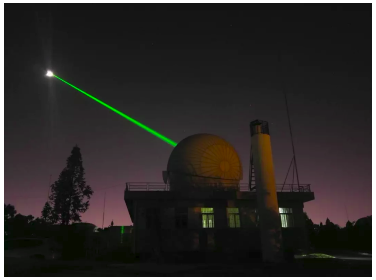
Testing gravity











Not clear if GR is valid for these regions.

Sensible theory

No ghost, no tachyon instability, …

⬇

BH solution same as in GR

Condition for existence, stability, …

This talk

⬇

BH solution different from GR

⬇

Parametrized deformation from BH in GR

⬇

Prediction of waveform, QNM, BH shadow…

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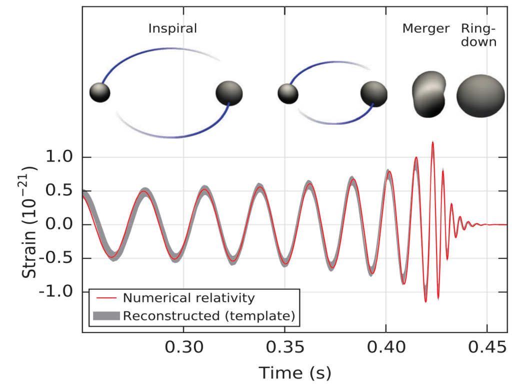
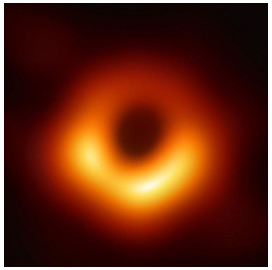
Test with observational data



LIGO collaboration



EHT collaboration

1850

Ostrogradsky theorem

Nondegenerate higher-order Lagrangian → Ghost DOF

1971

Lovelock theory

- 4D diffeo. inv.
- Metric only
- 2nd order EL eqs

1974

Horndeski theory

- 4D diffeo. inv.
- Metric + scalar field
- 2nd order EL eqs

2014

Beyond Horndeski (GLPV)

Higher-order EL eqs but no ghost DOF

Gleyzes et al, 1404.6495

2014 – 2018

Degenerate higher-order theories

HM, Suyama, 1411.3721

Langlois, Noui, 1510.06930

HM, Suyama, Yamaguchi, Langlois, Noui, 1603.09355

HM, Suyama, Yamaguchi, 1711.08125, 1804.07990

2011

Generalized Galileon

Deffayet et al, 1103.3260

Rediscovery of Horndeski theory

Kobayashi et al, 1105.5723

Degenerate theories

- $L = L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a) \qquad \phi^a = \phi^a(t)$
- $K_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \ddot{\phi}^b}, \quad M_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \dot{\phi}^b} - \frac{\partial^2 L}{\partial \ddot{\phi}^b \partial \dot{\phi}^a}$


Sena (1969)

Ostrogradsky theorem (1850)

$\det K \neq 0 \Longrightarrow H \supset PQ$ is unbounded.

No-(Ostrogradsky-)ghost condition HM, Suyama, 1411.3721

a.k.a. Degeneracy condition

$K_{ab} = 0$ & $M_{ab} = 0 \Longrightarrow H \not\ni PQ$

Degenerate theories

- $L = a\ddot{\phi}^2 + 2b_i\ddot{\phi}\dot{q}^i + k_{ij}\dot{q}^i\dot{q}^j + \cdots = k^{-1}\left(b\ddot{\phi} + k\dot{q}\right)^2 + \cdots$

  $K = 0$ ➡ quadratic DHOST Langlois, Noui, 1510.06930

- $L = L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a, \dot{q}^i, q^i)$

  $K_{ab} = 0$ & $M_{ab} = 0$

  HM, Suyama, Yamaguchi, Langlois, Noui, 1603.09355

- $L = L(\phi^{i_d(d+1)}, \ldots; \phi^{i_{d-1}(d)}, \ldots; \ldots; \ddot{\phi}^{i_1}, \dot{\phi}^{i_1}, \phi^{i_1}, \dot{\phi}^{i_0}, \phi^{i_0})$

  HM, Suyama, Yamaguchi, 1711.08125, 1804.07990

All the constraints

$$
\begin{array}{cccccc|cccccc}
 & & & & & \Psi^{i_1}_{11} & \Upsilon^{i_1}_{10} & & & & & \\
 & & & & \Psi^{i_2}_{22} & \Upsilon^{i_2}_{21} & \Upsilon^{i_2}_{20} & \Omega^{i_2}_{20} & & & & \\
 & & & \Psi^{i_3}_{33} & \Upsilon^{i_3}_{32} & \Upsilon^{i_3}_{31} & \Upsilon^{i_3}_{30} & \Omega^{i_3}_{30} & \Omega^{i_3}_{31} & & & \\
 & & \Psi^{i_4}_{44} & \Upsilon^{i_4}_{43} & \Upsilon^{i_4}_{42} & \Upsilon^{i_4}_{41} & \Upsilon^{i_4}_{40} & \Omega^{i_4}_{40} & \Omega^{i_4}_{41} & \Omega^{i_4}_{42} & & \\
 & \cdot^{\cdot^{\cdot}} & \cdot^{\cdot^{\cdot}} & & & \vdots & \vdots & \vdots & & & \ddots & \\
\Psi^{i_d}_{dd} & \Upsilon^{i_d}_{d,d-1} & & \cdots & & \Upsilon^{i_d}_{d1} & \Upsilon^{i_d}_{d0} & \Omega^{i_d}_{d0} & & \cdots & & \Omega^{i_d}_{d,d-2}
\end{array}
= 0
$$

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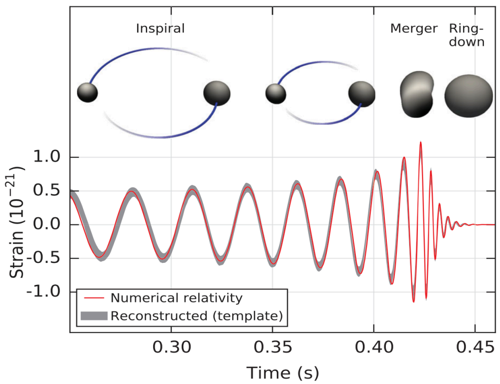
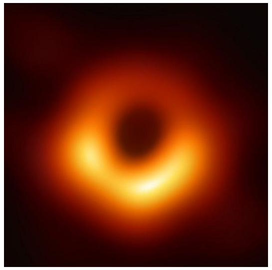


LIGO collaboration



EHT collaboration

Theories allowing GR solution

Suppose: No deviation from GR solution

What kind of modified gravity allow GR solution?

Let us clarify condition for ∃ GR solution.

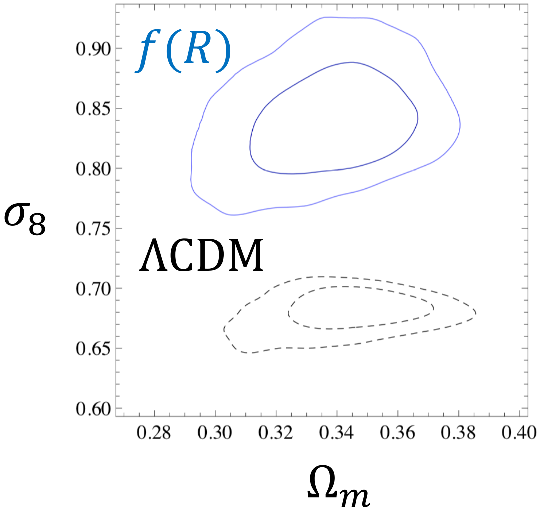
(If GR solution is unique $\Longrightarrow$ No hair theorem)

c.f. Cosmology

ΛCDM expansion history

$\Longleftarrow$ ΛCDM, quintessence, $f(R)$

$\rightarrow$ Observational tests



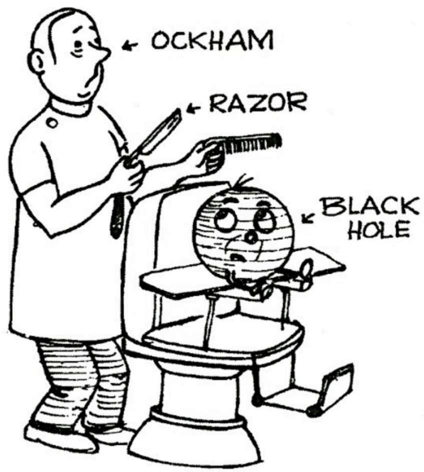
HM, Starobinsky, Yokoyama, 1203.6828

	$g_{\mu\nu}$	ϕ	L
(1)	Any GR solution $G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$	ϕ =const.	Theories with multiple scalars and arbitrary higher-order derivs.
(2)	Vacuum GR solution $R_{\mu\nu} = 0$ → Strong coupling	$\phi(r)$	Horndeski subclass: $c_t = c$, no shift sym.
(3)	Schwarzschild & Schwarzschild-(A)dS	$\phi =$ $qt + \psi(r)$ X =const.	Shift-sym. quadratic DHOST theories

HM, Minamitsuji, 1804.01731, 1809.06611, 1901.04658

Takahashi, HM, Minamitsuji, 1904.03554

$\phi(t,r) = qt + \psi(r)$ in shift-sym. theories



Vishveshwara (1980)

Why?

- Simplicity:
  Compatible with static spacetime.

- Nontrivial:
  Circumvent static scalar assumption of no-hair theorem.

Hui, Nicolis, 1202.1296

Babichev, Charmousis, Lehebel, 1702.01938

- Interesting:
  "Stealth" Schwarzschild-de Sitter solution.

Babichev, Charmousis, 1312.3204

Kobayashi, Tanahashi, 1403.4364

Stealth BH solutions in quadratic DHOST

HM, Minamitsuji, 1901.04658

We found novel exact BH solutions.

- Theory = Shift-sym. quadratic DHOST theories.
- $g_{\mu\nu}$ = Schwarzschild, Schwarzschild-de Sitter (SdS)
- $\phi(t,r) = qt + \psi(r)$ with $X \equiv \partial_\mu \phi \partial^\mu \phi$ = const.

Stealth SdS solution:

BH (& cosmological) solution with nontrivial scalar hair.

cf. BH solutions in non-shift-sym. DHOST

for $c_t = c$ and $X = -q^2$

Ben Achour, Liu, 1811.05369

Stealth BH solutions in quadratic DHOST

Action (+ degeneracy condition)

$$S = \int d^4x\sqrt{-g}\left[F_0(X) + F_1(X)\Box\phi + F_2(X)R + \sum_{I=1}^{5} A_I(X)L_I^{(2)}\right]$$

$$L_1^{(2)} = \phi^{;\mu\nu}\phi_{;\mu\nu},\ \ L_2^{(2)} = (\Box\phi)^2,\ \ L_3^{(2)} = (\Box\phi)\phi^{;\mu}\phi_{;\mu\nu}\phi^{;\nu},$$

$$L_4^{(2)} = \phi^{;\mu}\phi_{;\mu\nu}\phi^{;\nu\rho}\phi_{;\rho},\ \ L_5^{(2)} = \left(\phi^{;\mu}\phi_{;\mu\nu}\phi^{;\nu}\right)^2.$$

Metric

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + 2C(r)dtdr + D(r)r^2d\Omega^2$$

Scalar field profile

$\phi(t,r) = qt + \psi(r)$ with $X \equiv \partial_\mu\phi\partial^\mu\phi = \text{const.}$

Gauge fixing with $g_{\mu\nu}(r)$ and $\phi(t, r)$

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + 2C(r)dtdr + D(r)r^2 d\Omega^2$$

$\phi(t, r) = qt + \psi(r)$ with $X \equiv \partial_\mu \phi \partial^\mu \phi =$ const.

From the fundamental theorem on gauge fixing at the action level,

HM, Suyama, Takahashi, 1608.00071

$D(r) = 1$: OK

$C(r) = 0$: leads to a loss of independent EL eq.

so it should be substituted after deriving EL eq.

The argument is independent of the form of the action.

Example

HM, Suyama, Takahashi, 1608.00071

Simple example $L = \frac{1}{2}(\dot{x} - \ddot{y})^2 \quad \leftrightarrow \quad \frac{1}{2}\dot{X}^2$

which is invariant under a gauge transformation

$$x \to x + \dot{\xi}, \qquad y \to y + \xi$$

Euler-Lagrange eqs

$$E_x = -\ddot{x} + \dddot{y} = 0, \qquad E_y = -\dddot{x} + \ddddot{y} = 0$$

Off-shell identity (a.k.a. Noether identity)

$$-\dot{E}_x + E_y = 0$$

$\Rightarrow E_x$: independent / E_y: redundant

Gauge fixing at action level:

1) $x = 0$: ~~E_x~~, E_y Independent EOM is lost

2) $y = 0$: E_x, ~~E_y~~ OK

Schwarzschild solution

HM, Minamitsuji, 1901.04658

$$\mathcal{E}_A = \frac{X_0}{Q} Q_0 A_1 - \frac{q^2}{Q}\mathcal{E}_B + \frac{q}{\sqrt{Q} f}\mathcal{E}_C + \frac{X_0}{2Q}\mathcal{E}_D, \tag{15}$$

$$\mathcal{E}_B = \frac{1}{f}\left(\frac{9Q^2+Q_0^2}{2Q} - Q_0\right)(A_1+A_2) - \frac{1}{f}\left(Q_0 A_1 + \frac{1}{2}\mathcal{E}_D\right) + \frac{Q}{2f^2}\left[2r^2 F_{0X} + \frac{r(3Q+Q_0)}{Q^{1/2}} F_{1X} + \frac{(3Q+Q_0)^2}{2Q}(A_{1X}+A_{2X}) - 2Q_0(2A_{1X}+A_3)\right], \tag{16}$$

$$\mathcal{E}_C = \frac{q}{\sqrt{Q}}\left[2Q_0 A_1 - \left(\frac{9Q^2+Q_0^2}{2Q} - Q_0\right)(A_1+A_2) + 2f\mathcal{E}_B + \mathcal{E}_D\right], \tag{17}$$

$$\mathcal{E}_D = r^2 F_0 + \frac{(9Q-Q_0)(Q-Q_0)}{4Q}(A_1+A_2), \tag{18}$$

$$\mathcal{E}_\psi = -\frac{r(3Q+Q_0)}{Q^{1/2}} F_{0X} - 4Q_0 F_{1X} + \frac{(Q-Q_0)[27Q^3 - (11q^2+2X_0)Q^2 - (3q^2+X_0)Q_0 Q + 3q^2 Q_0^2]}{4rX_0 Q^{5/2}}(A_1+A_2) + \frac{(9Q-Q_0)(3Q+Q_0)(Q-Q_0)}{4rQ^{3/2}}(A_{1X}+A_{2X}) - \frac{Q-Q_0}{rQ^{1/2}} Q_0(2A_{1X}+A_3), \tag{19}$$

where

$$Q(r) := q^2 + X_0 f(r), \quad Q_0 := q^2 + X_0, \tag{20}$$

Euler-Lagrange equations are satisfied if

$$F_0 = F_{0X} = F_{1X} = Q_0 A_1 = A_1 + A_2 = A_{1X} + A_{2X} = Q_0(2A_{1X} + A_3) = 0$$

at $X = X_0$. ⟹ Two branches: Cases 1, 2

Schwarzschild-de Sitter solution

HM, Minamitsuji, 1901.04658

Sufficient conditions

Raw Euler-Lagrange equations

Case 1-Λ

cf. Takahashi, HM, Minamitsuji, 1904.03554

Reduced Euler-Lagrange equations

At $X_0 = -q^2$,

$$F_0 = -2\Lambda(F_2 + q^2 A_1)$$

$$2F_{0X} = -\Lambda[8F_{2X} - 2A_1 - q^2(4A_{1X} + 3A_3)]$$

$$F_{1X} = A_1 + A_2 = A_{1X} + A_{2X} = 0$$

Case 2-Λ

At $X_0 = \text{const} \neq -q^2$,

$$F_0 = -2\Lambda F_2$$

$$F_{0X} = -\Lambda(4F_{2X} - X_0 A_{1X})$$

$$F_{1X} = A_1 = A_2 = A_{1X} + A_{2X} = 0$$

Compatible with DHOST Classes I, III

Example theories

HM, Minamitsuji, 1901.04658

Simple examples in DHOST subclass where $c_t = c$:

- Stealth Schwarzschild solution

$$F_0 = M^4 a(X),$$

$$F_2 = \frac{M_{\rm Pl}^2}{2} + M^2 b(X),$$

$$A_3 = \frac{c(X)}{M^6}$$

- Self-tuned S(A)dS solution

$$F_0 = -M_{\rm Pl}^2 \Lambda_{\rm b} + M^4 h(X),$$

$$F_2 = \frac{M_{\rm Pl}^2}{2} + \frac{\alpha}{2} M^2 h(X),$$

$$A_3 = -8\beta M^2 \frac{h\prime(X)}{X}$$

Stability analysis

Takahashi, HM, Minamitsuji, 1904.03554

We found novel solution ($\notin$ Cases 1-Λ, 2-Λ) and clarified stability conditions.

- $g_{\mu\nu}$ = Static, spherically sym.
- $\phi(t,r) = qt + \psi(r)$ with X =const.
- Reduced 2nd order EOMs
- $\delta g_{\mu\nu}$ = Odd parity pert. (1DOF ↔ Regge-Wheeler)

$$F_2 > 0, \qquad F_2 - XA_1 > 0, \qquad F_2 - \left(\frac{q^2}{A} + X\right)A_1 > 0$$

Future work: Stability for even parity perturbations.
(2DOFs ↔ Zerilli + $\delta\phi$)

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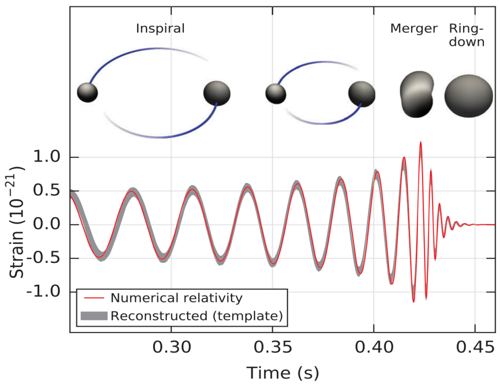

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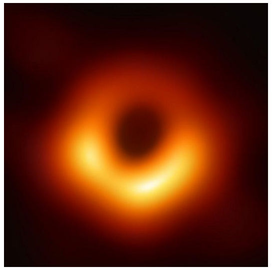

EHT collaboration

Summary

Degenerate theories

- Allow systematic construction of nontrivial higher-derivative theories without Ostrogradsky ghosts.

Stealth solution in shift sym. q-DHOST theories

- Derived condition for EL eqs to allow Schwarzschild-de Sitter solution with deficit solid angle.
- Derived condition for stability of odd parity perturbation (Regge-Wheeler analysis)