# Analysis of the binary neutron star merger GW170817 using numerical-relativity calibrated waveforms

in prep.

credit: NASA/Goddard Space Flight Center

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**"Multi-Messenger Astrophysics in the GW Era," 10/3, 2019, YITP**

Gravitational Wave Physics and Astronomy Genesis

# Synergy between data analysis and theory researches

## A: BH binaries

- A-01 Testing gravity
- A-02 Gravity and Cosmology
- A-03 BH binary formation

## B: NS binaries

- B-01 Internal structure of NS
- B-02 Gamma-ray burst and BH
- B-03 *r*–process elements

## C: Supernovae

- C-01 SN explosion mechanism
- C-02 SN explosion mechanism via $\nu$ observation

# Physics and astronomy motivated by GW observations

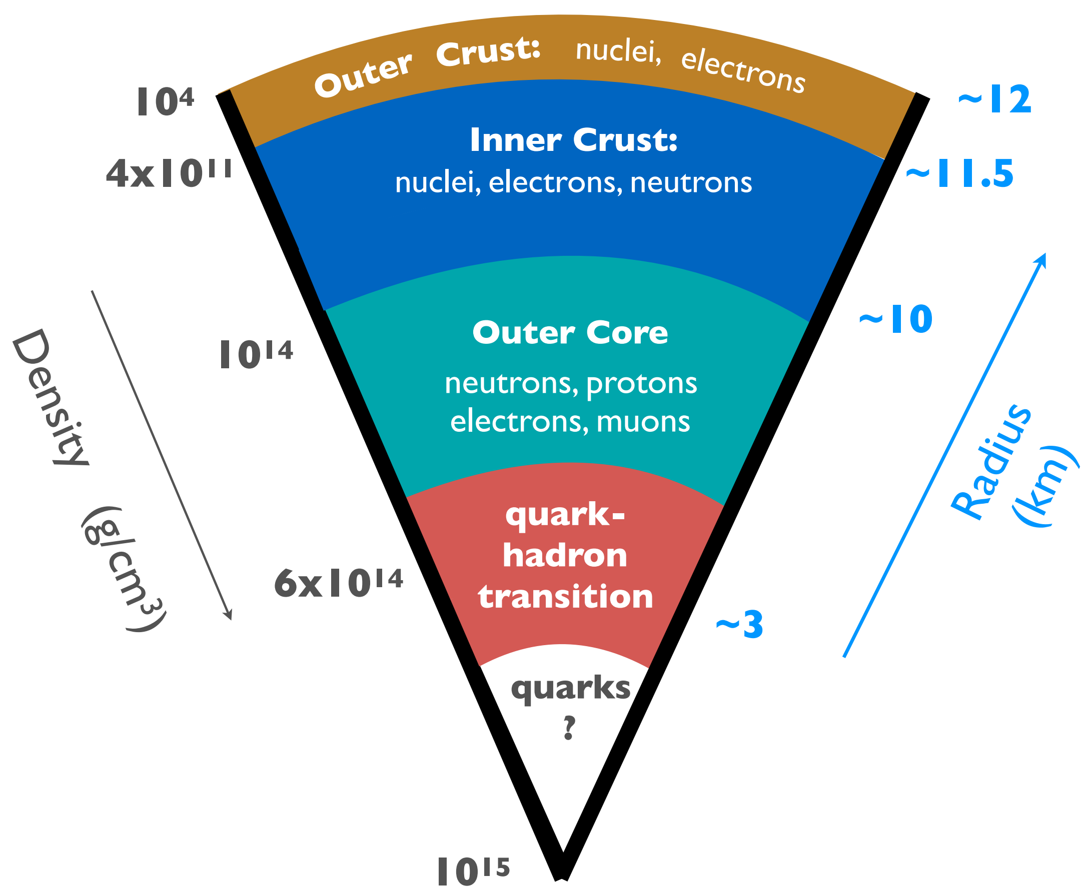
# Fundamental physics

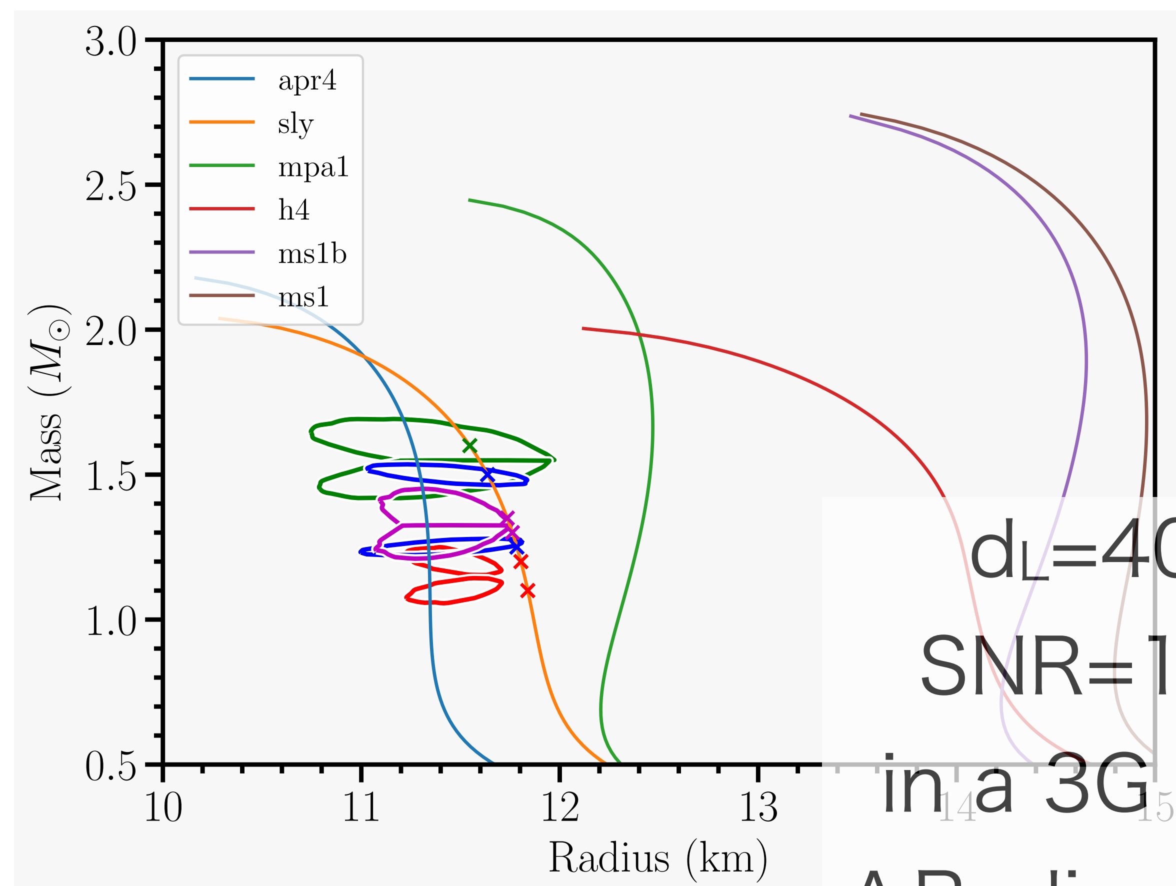
Structure of bulk matter at supranuclear density

## 2. Extreme Matter, Extreme Environs

**Science Target**

***Determine the properties of the hottest and densest matter in the Universe***




$d_L$=400Mpc

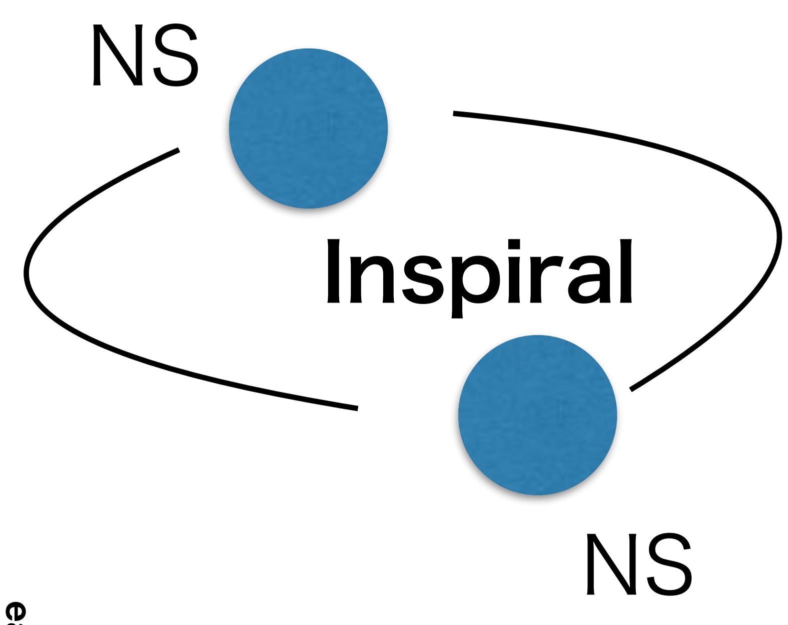
SNR=100-140

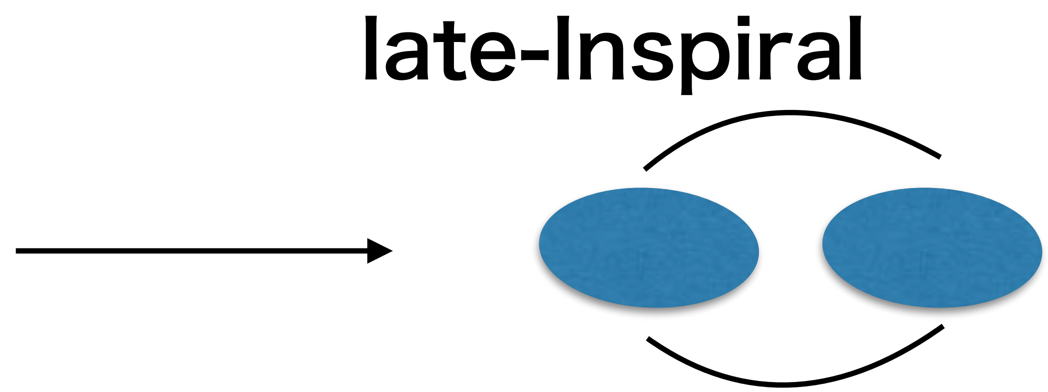
in a 3G detector

ΔRadius~0.5-1 km

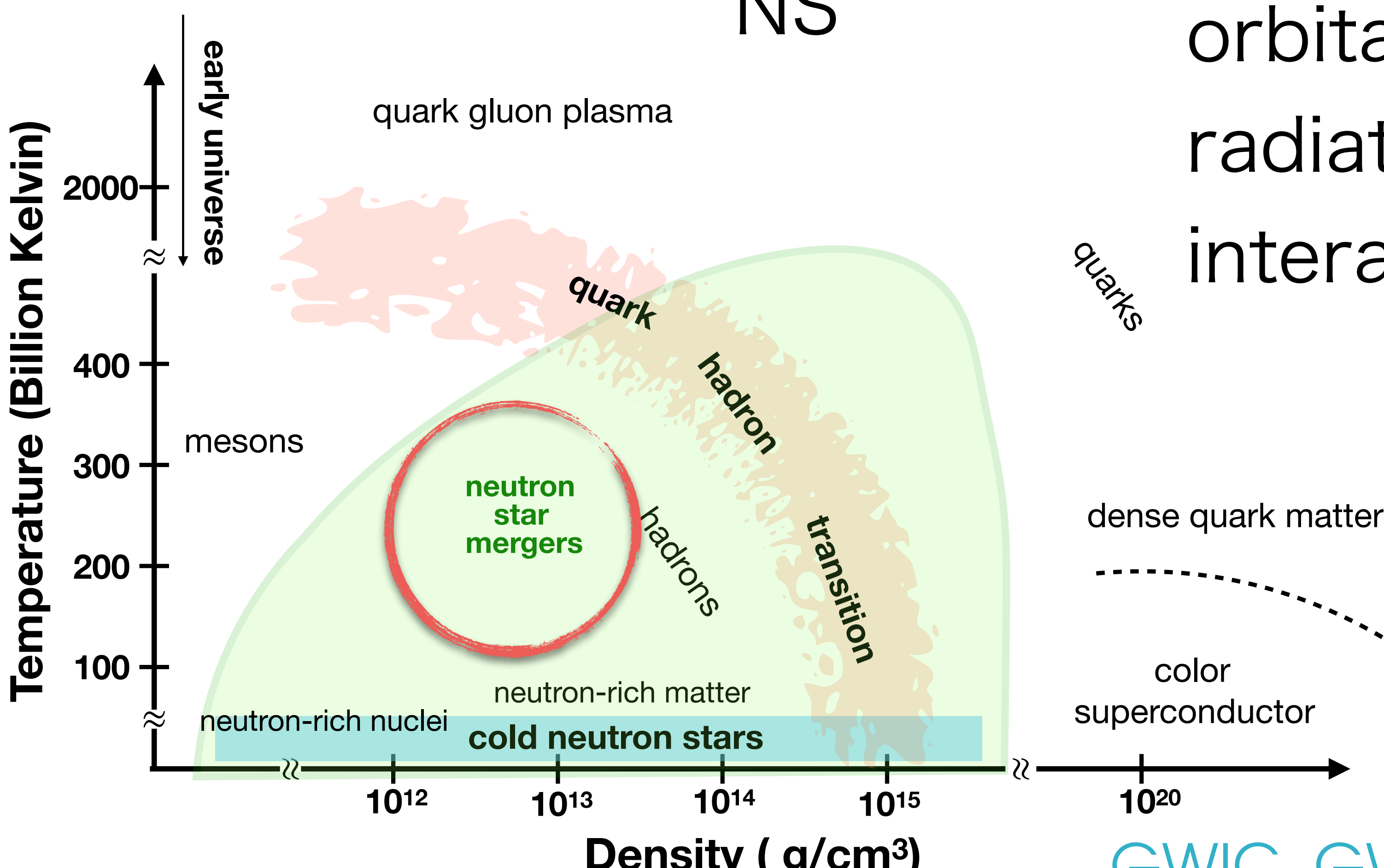
GWIC, GWIC-3G, GWIC-3G-SCT-Consortium

# Binary-neutron-star (BNS) merger



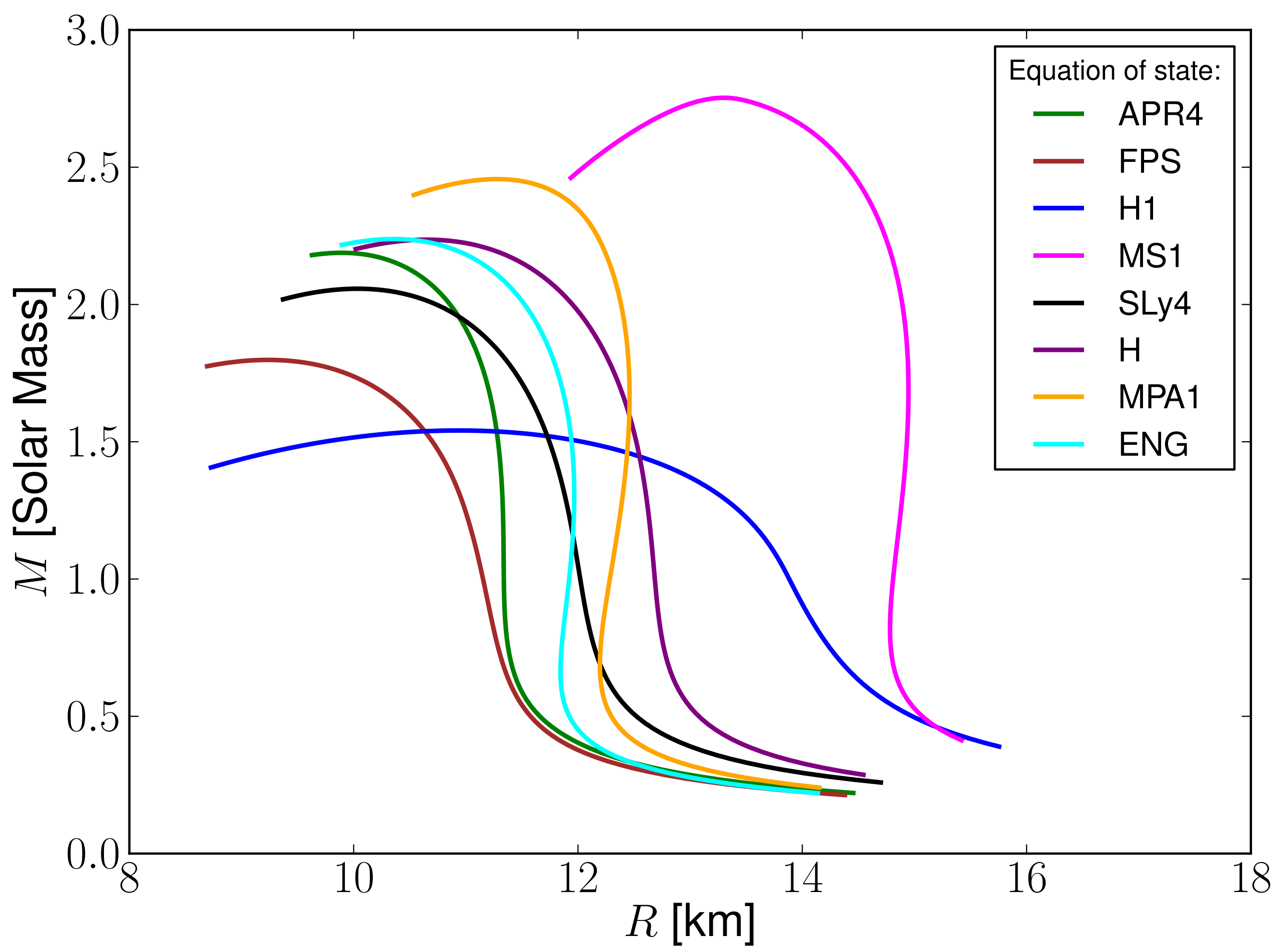

Matter effects influence the orbital evolution and gravitational radiation through the tidal interaction between the NSs.

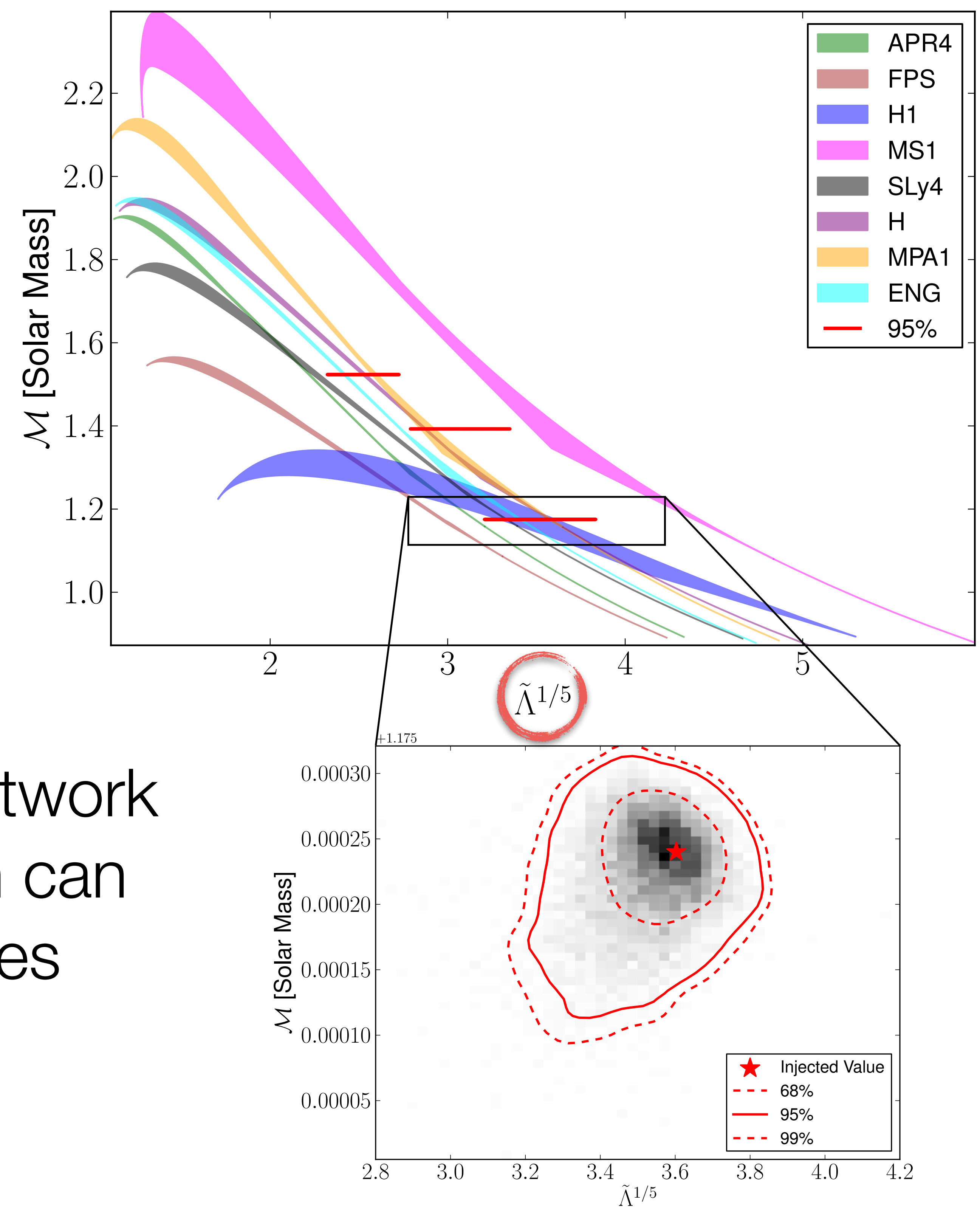



GWIC, GWIC-3G, GWIC-3G-SCT-Consortium

**BNS mergers are valuable laboratories for nuclear astrophysics.**

- **Can GWs help constrain the NS EOS?** [Wade+ 2013]
  - To visualize how a GW detection might constrain the NS EOS, we plot a 2D PDF from a single source on mass-radius-*like* curves



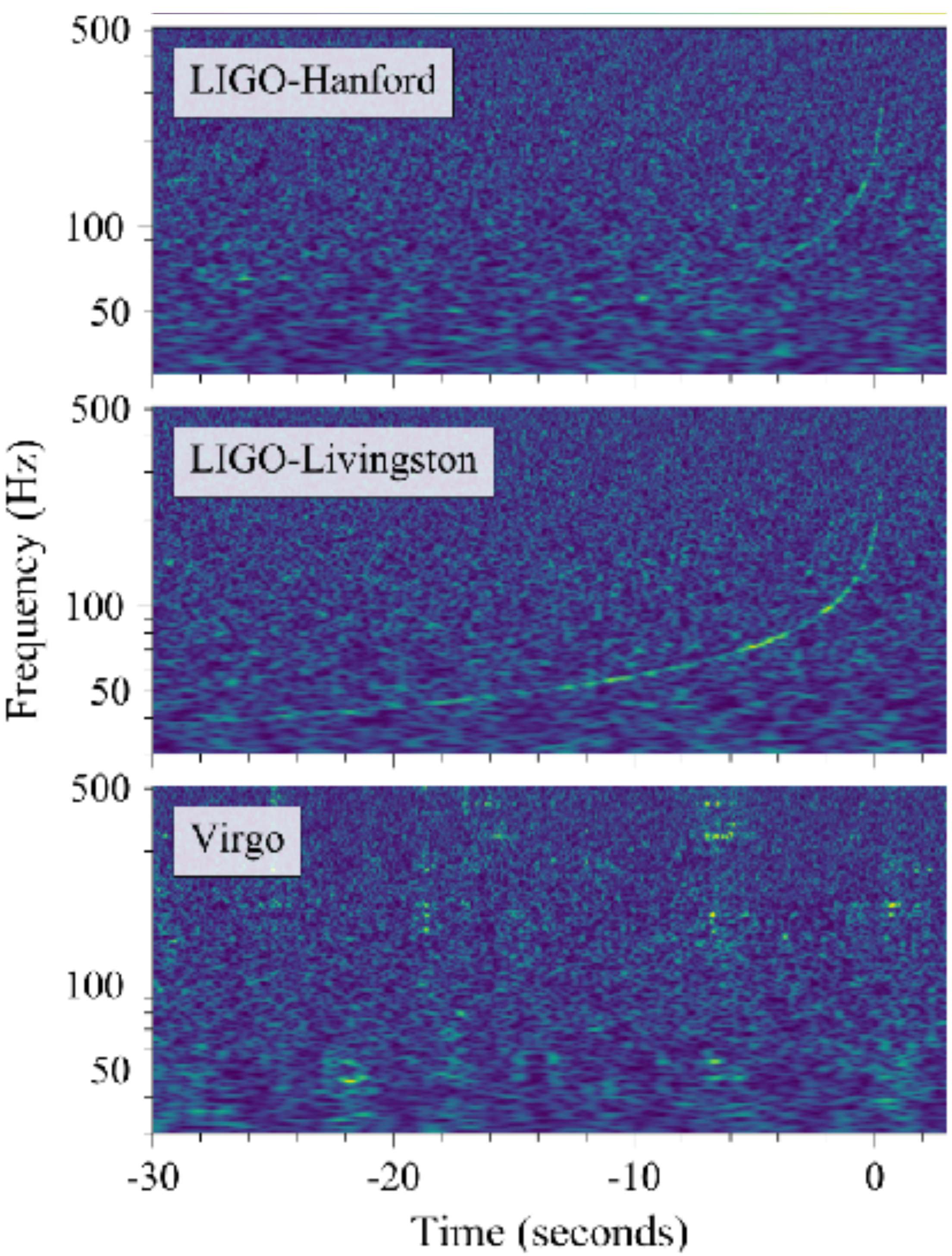




  - We find that a single, loud (Network SNR 32.4) BNS GW detection can rule out several NS EOS families
  - Multiple detections will highly constrain the NS EOS

# Discovery of GW170817

[LVC, 2017]

Spectrograms

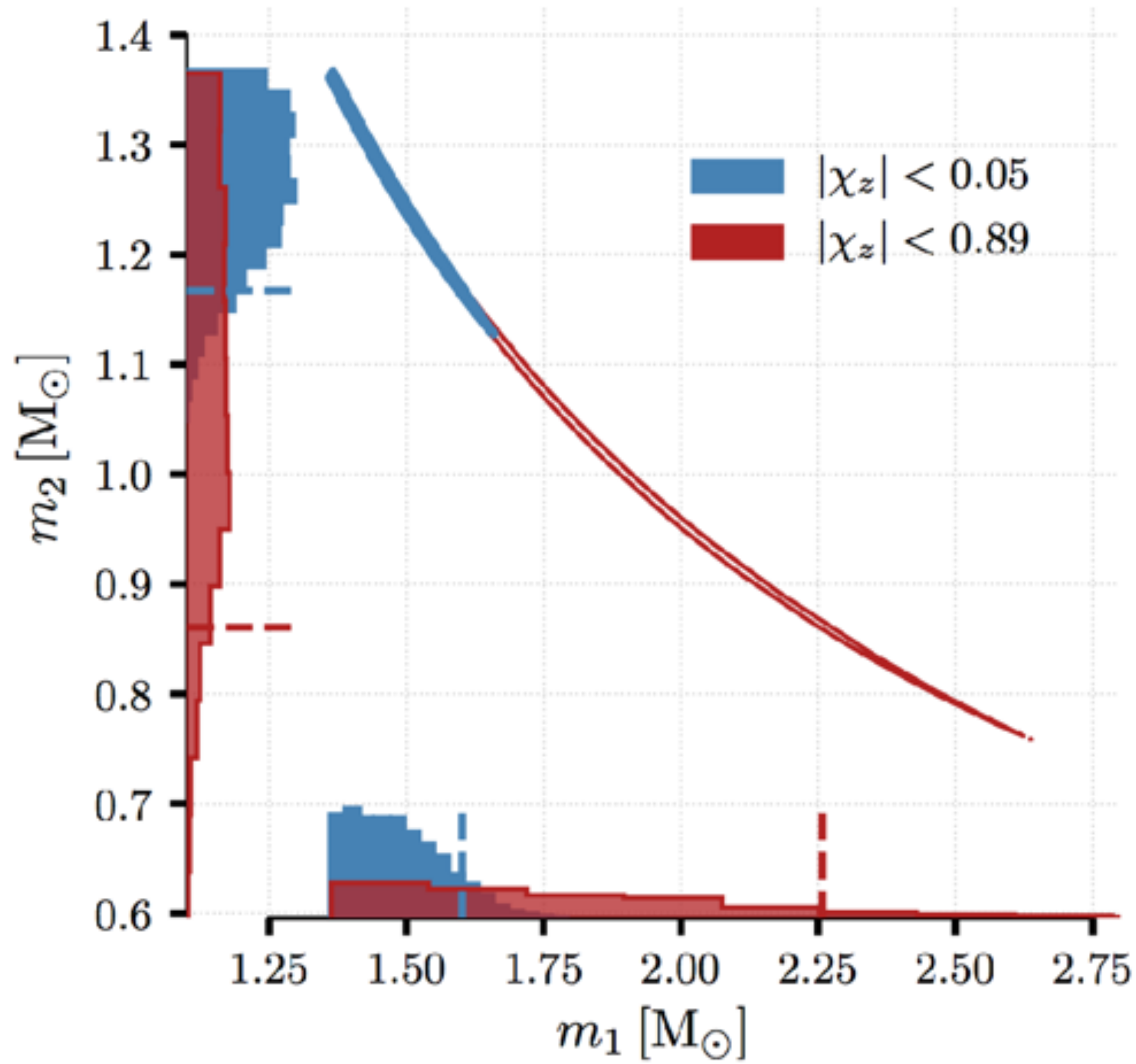



SNR

32.4 (Network)

18.8 (H), 26.4(L), 2.0 (V)

Estimates of the two components of the binary



# GW170817 enabled us to measure the tidal deformability for the first time

[LVC, 2017]

Λ constrains the nuclear EOS of NS matter.

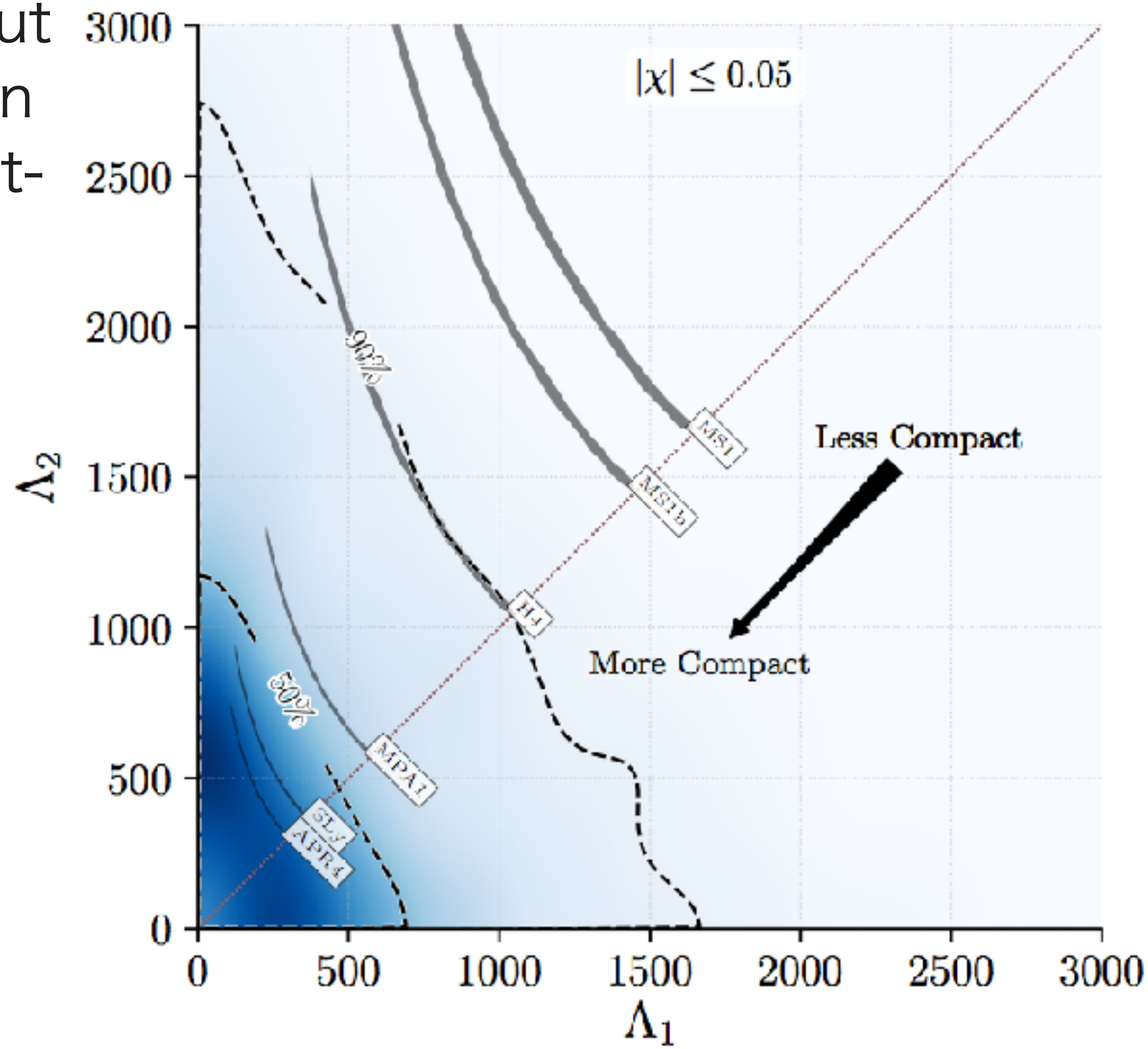
LIGO-Virgo Collaboration put conservative upper limits on tidal deformability with post-Newtonian waveform

(**restricted TF2**

**with 5+1PNTidal**).

symmetric contribution of tidal deformability

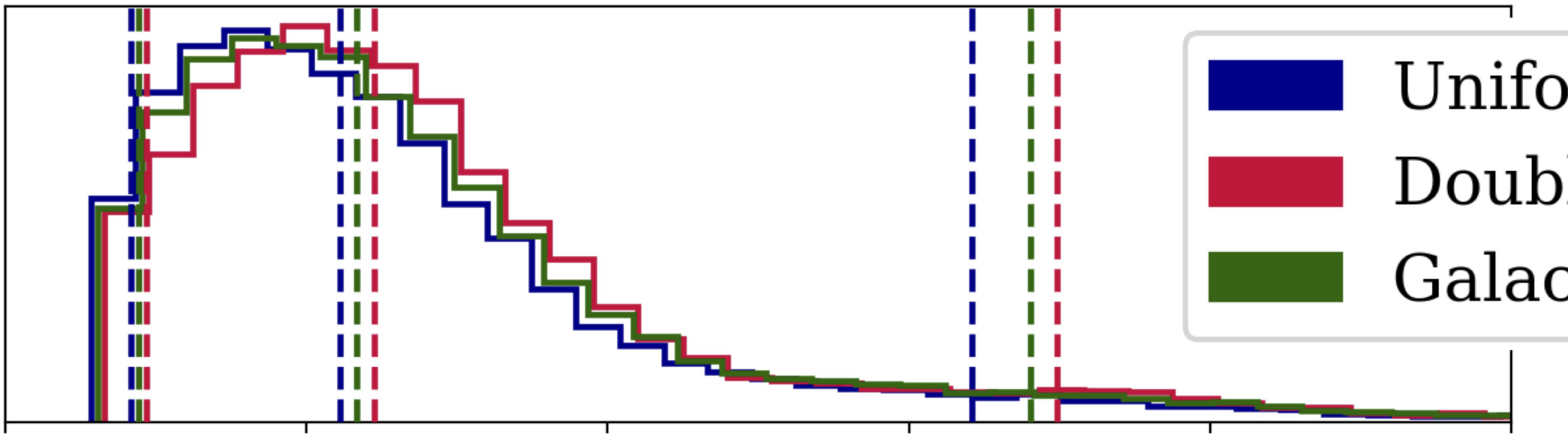
$$\tilde{\Lambda} \leq 900$$



# Independent analysis of GW170817

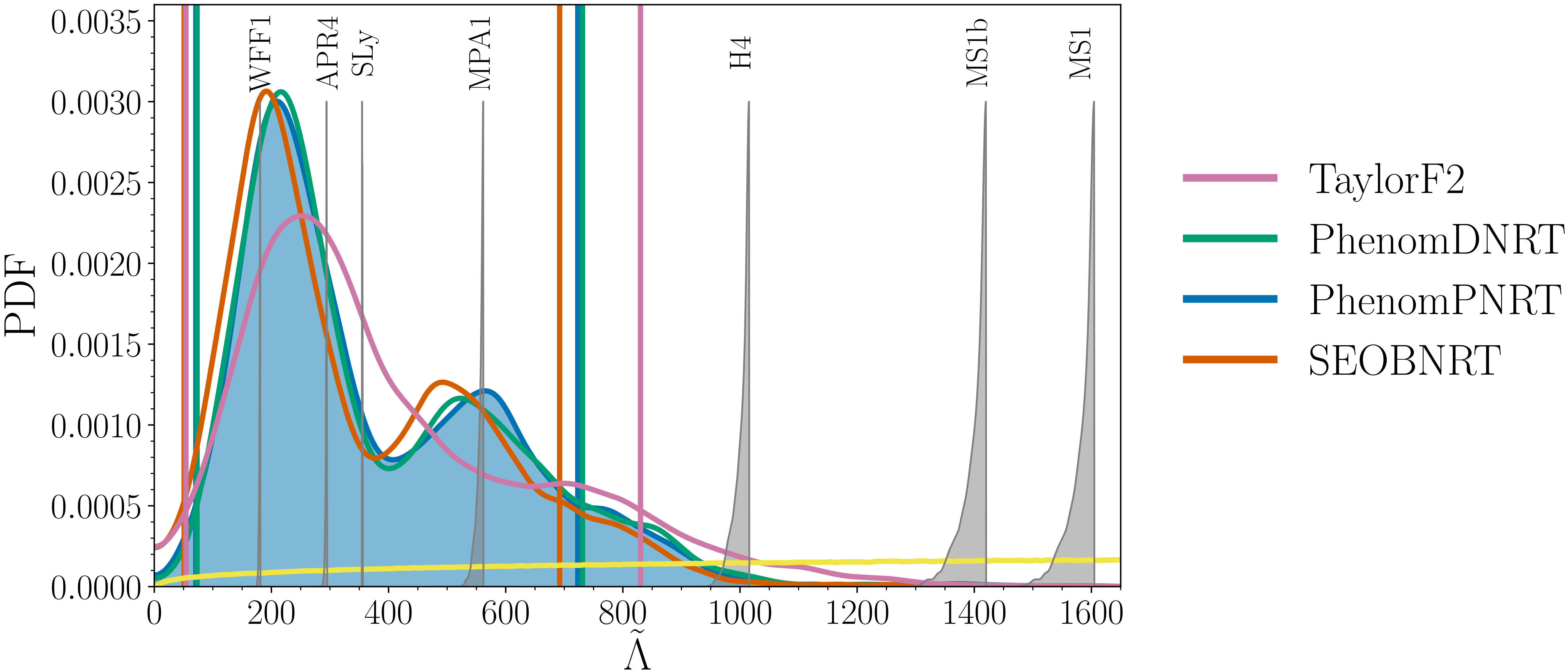
with **restricted TF2 with 5+1PNTidal** [De, et al., 2018]

$\tilde{\Lambda} = 222.29^{+419.83}_{-138.48}$ $245.39^{+453.12}_{-151.53}$ $233.39^{+447.55}_{-144.40}$




| Mass prior | $\tilde{\Lambda}$ | $\hat{R}$ (km) |
|---|---|---|
| Uniform | $310^{+679}_{-234}$ | $11.3^{+2.4}_{-2.4} \pm 0.2$ |
| Double neutron star | $354^{+691}_{-245}$ | $11.6^{+2.3}_{-2.1} \pm 0.2$ |
| Galactic neutron star | $334^{+669}_{-241}$ | $11.5^{+2.3}_{-2.2} \pm 0.2$ |

# Improved analysis of GW170817 [LVC,1805.11579]




Using sophisticated waveform models (**NRTidal**), an updated highest-posterior-density (HPD) interval, $\tilde{\Lambda} = 300^{+420}_{-230}$

If a common EOS is assumed, this is further restricted to $\tilde{\Lambda} = 190^{+390}_{-120}$

Parameter estimation methods

A. Data and Bayesian inference

# Our independent reanalysis of GW170817

- **NR calibrated tidal waveform models**
- flat prior on $\tilde{\Lambda}$, $\tilde{\Lambda}$~U[0, 3000]

basically following those adopted in the improved LVC analysis (e.g., arXiv:1805.11579) or our previous study (TN+, Phys. Rev. Res. 2019).

・Bayesian inference, Nested sampling implemented in LALInference.

・The sky position is fixed to the location determined by optical followup observations.

・BayesLine PSD

# Bayesian parameter estimation of GWs

Why Bayesian statistics and stochastic sampling

- A lot of parameters
- Parameter estimation (PE)
- Model selection

$$L(d|\vec{\theta}) \propto \exp\left(-2\int_0^\infty \frac{|\tilde{d}(f)-h(\vec{\theta},f)|^2}{S_n(f)}\mathrm{d}f\right)$$

Likelihood

## Bayes' theorem

Prior

Posterior

$$p(\vec{\theta}|\{d\},H) = \frac{p(\vec{\theta}|H)p(\{d\}|\vec{\theta},H)}{P(\{d\}|H)}$$

Evidence for the model H $P(\{d\}|H) \equiv Z_H$

H: hypothesis (signal embedded in data), {d}: data set, $\theta$: parameters

- 23 Hz≦f≦$f_{max}$, $f_{max}$=1000 Hz or 2048 Hz (min[$f_{ISCO}$, $f_s$/2]), $f_s$=4096 Hz.

We calculate posterior with Nested sampling
(LIGO Algorithm Library (LAL), LALInference)

Parameter estimation methods

B. Waveform models for inspiraling BNSs

**The gravitational waveform** $\tilde{h}(f) = A(f)e^{i\Psi(f)}$

where the amplitude and the phase

$$A(f) = A_{\rm point-particle}(f) + A_{\rm spin}(f) + A_{\rm tidal}(f)$$

$$\Psi(f) = \Psi_{\rm point-particle}(f) + \Psi_{\rm spin}(f) + \Psi_{\rm tidal}(f)$$

**BBH baseline: PP + Spin**

- **TF2**
- **TF2+**

**Tiadl contribution**

- **KyotoTidal**
- **NRTidal**
- **NRTidalv2**
- **PNTidal**

| Model name | Point-particle part | |
|---|---|---|
| | Amplitude | Phase |
| TF2 | 3PN | 3.5PN |
| **TF2+** | **6PN** | **6PN** |

| Model name | Tidal part | |
|---|---|---|
| | Amplitude | Phase |
| **KyotoTidal** | **Polynomial** | **Nonlinear** |
| NRTidal | - | Pade approx. |
| NRTidalv2 | Pade approx. | Pade approx. |
| PNTidal | 5+1PN | 5+2.5PN |

## BBH baseline: PP + Spin

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- **TF2+**

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| **TF2+** | **6PN** | **6PN** |

**TaylorF2** waveform (**TF2**) [Blanchet+2006, Buonanno+2009]
with point-particle phase up to 3.5PN-order + spin

$$\Psi_{\rm point-particle}(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128\eta} x^{-5/2} \left\{ 1 + \left( \frac{3715}{756} + \frac{55}{9}\eta \right) x - 16\pi x^{3/2} + \ldots \right\}$$

$$\eta = m_1 m_2/(m_1+m_2)^2$$

0PN 1PN 1.5PN

where the dimensionless post-Newtonian (PN) parameter

$$x = (\pi M_{\rm tot} f)^{2/3}$$

The best estimated mass parameter is the chirp mass

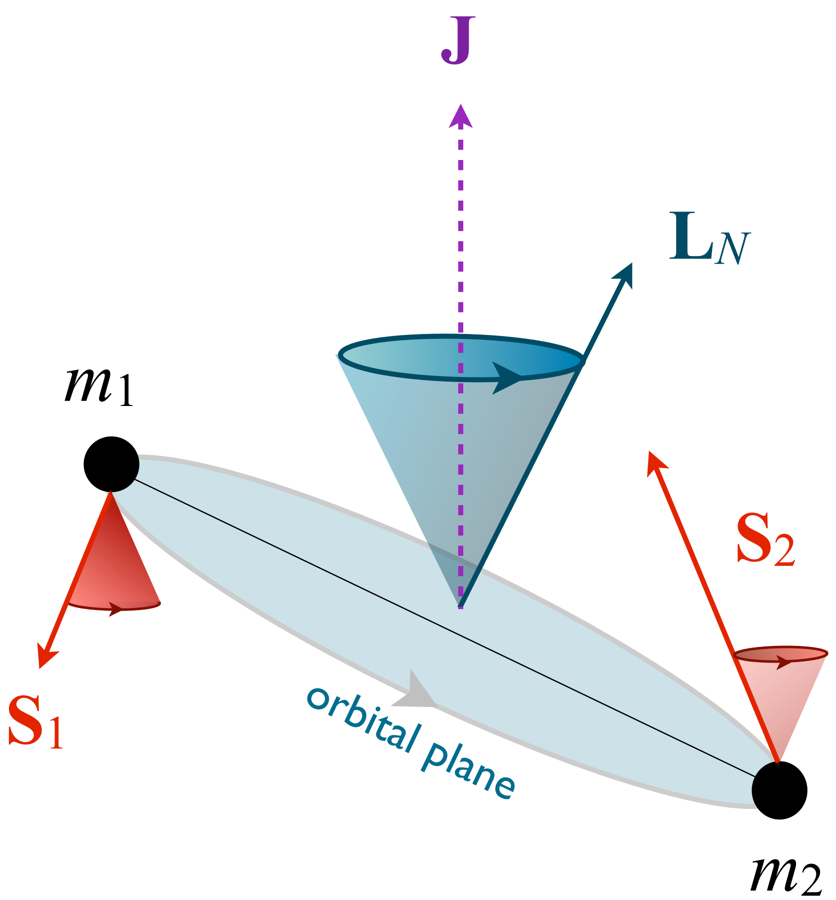
$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}} \simeq \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f^{-11/3}\dot{f}\right]^{3/5}$$

and only place weak constraints on the mass ratio q=m2/m1≦1 and the spin.

**TF2+** adds high-PN order terms to TF2. [Kawaguchi+, 2018]

# Spin contribution

J=L+S1+S2




If $S_{1,2}$ misaligned with respect to L, they cause the binary's orbital plane to precess around the almost-constant direction of the total angular momentum of the binary, J.

**We assume that the spins are aligned with L.**

dimensionless spin magnitude $\chi = c\,|\mathbf{S}|/(Gm^2) \leq 1$

a simple mass-weighted linear combination of the spins

$$\chi_{\rm eff} = \frac{c}{G}\left(\frac{\boldsymbol{S}_1}{m_1} + \frac{\boldsymbol{S}_2}{m_2}\right) \cdot \frac{\hat{\boldsymbol{L}}}{M},$$

which takes values between −1 (both objects have maximal spins antialigned with respect to L) and +1 (maximal aligned spins).

# Tidal effects

When binary orbital separations are small, each star is tidally distorted by its companion.

**tidal deformability: $\lambda = -\dfrac{\text{Q: (tidal induced) quadrupole moment}}{\varepsilon\text{: companion's tidal field}}$**

The information about the NS EOS can be quantified by $\lambda$.

The leading order tidal contribution to GW phase

[Vines, Flanagan & Hinderer 2011]

$$\Psi_{\rm tidal} = \frac{3}{128\eta}\left[-\frac{39}{2}\tilde{\Lambda}\, x^{5/2}\left(1+\frac{3115}{1248}x \qquad\qquad\qquad \right)\right]$$

5PN 5+1PN

**binary tidal deformability**, mass-weighted combination of $\Lambda_{1,2}$

$$\tilde{\Lambda} = \frac{16}{13}\frac{(m_1+12m_2)m_1^4\Lambda_1 + (m_2+12m_1)m_2^4\Lambda_2}{(m_1+m_2)^5}$$

$\Lambda = \lambda/m^5$ : dimensionless

**PNTidal**

(Post-Newtonian)

5+2.5PN

$$\Psi_{\rm tidal}^{\rm PNTidal} = \frac{3}{128\eta}\left[-\frac{39}{2}\tilde{\Lambda}\, x^{5/2} \times \left(1 + \frac{3115}{1248}x - \pi x^{3/2} + \frac{28024205}{3302208}x^2 - \frac{4283}{1092}\pi x^{5/2}\right)\right]$$

5PN 5+1PN 5+1.5PN 5+2PN 5+2.5PN

5+1PN [Vines, Flanagan & Hinderer, (2011)]

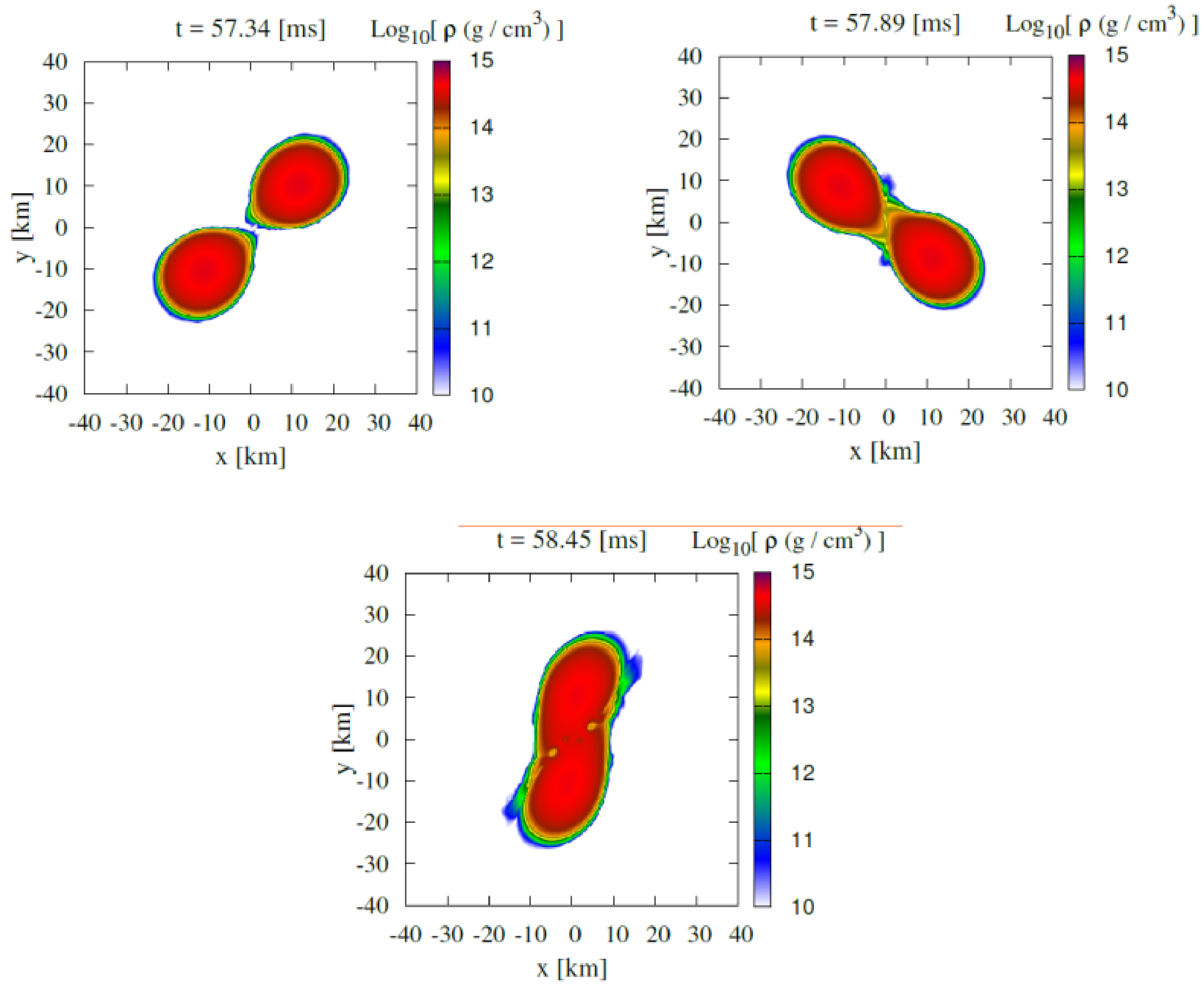
5+2.5PN [Damour, Nagar & Villain, (2012)]

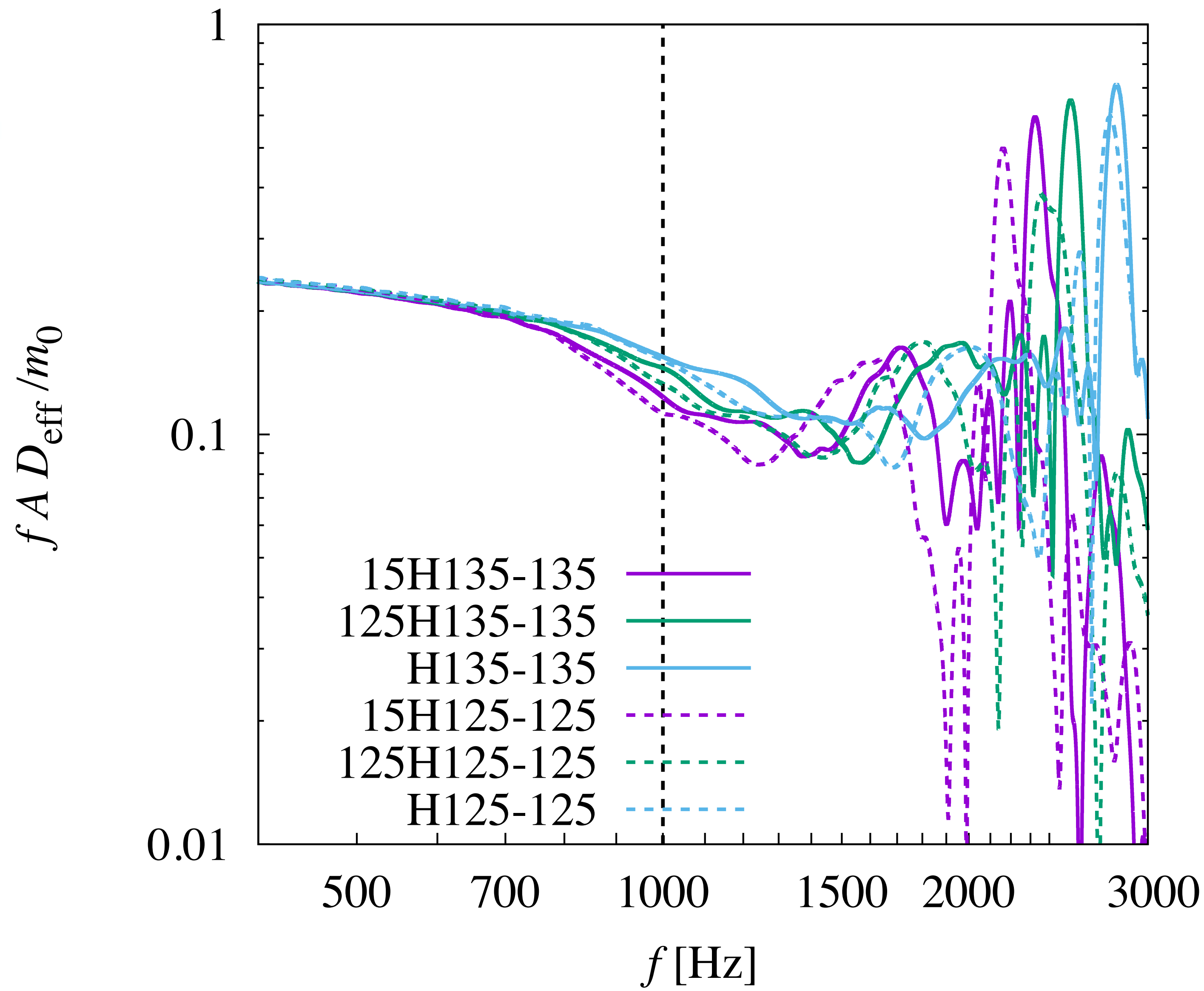
$$A_{\rm tidal}^{\rm PNTidal} = \sqrt{\frac{5\pi\eta}{24}}\frac{M_{\rm tot}^2}{d_L}\tilde{\Lambda} x^{-7/4} \times \left(-\frac{27}{16}x^5 - \frac{449}{64}x^6\right)$$

5PN 5+1PN

# Numerical Relativity calibrated waveforms for inspiraling BNS

Kiuchi+, "Sub-radian-accuracy gravitational waveforms of coalescing BNSs in NR", 2017.




NR calibrated waveform model, **TF2+_KyotoTidal**

point-particle: **TF2+**, calibrated by SEOBNRv2

tidal: **KyotoTidal**, calibrated by hybrid waveform (SEOBNRv2T+NR)

Kawaguchi+, 2018

# KyotoTidal

(NR calibrated)

[Kawaguchi, et al., 2018]

Nonlinear correction

$$\Psi_{\rm tidal}^{\rm KyotoTidal} = \frac{3}{128\eta}\left[-\frac{39}{2}\tilde{\Lambda}\left(1 + a\tilde{\Lambda}^{2/3}x^{p}\right)\right]x^{5/2}$$

$$\times\left(1 + \frac{3115}{1248}x - \pi x^{3/2} + \frac{28024205}{3302208}x^2 - \frac{4283}{1092}\pi x^{5/2}\right)$$

5PN 5+1PN 5+1.5PN 5+2PN 5+2.5PN

$$A_{\rm tidal} = \sqrt{\frac{5\pi\eta}{24}}\frac{m_0^2}{D_{\rm eff}}\tilde{\Lambda}x^{-7/4}$$

$$\times\left(-\frac{27}{16}x^5 - \frac{449}{64}x^6 - 4251\,x^{7.890}\right).$$

5PN 5+1PN higher-order tidal effects

**NRTidal**

(NR calibrated)

[Dietrich, et al., 2017]

$$\Psi_{\rm tidal}^{\rm NRTidal} = \frac{3}{128\eta}\left[-\frac{39}{2}\tilde{\Lambda}x^{5/2} \times \frac{1+\tilde{n}_1 x+\tilde{n}_{3/2}x^{3/2}+\tilde{n}_2 x^2+\tilde{n}_{5/2}x^{5/2}}{1+\tilde{d}_1 x+\tilde{d}_{3/2}x^{3/2}}\right]$$

Pade approx. (Linear)

**NRTidalv2**

(NR calibrated)

[Dietrich, et al., 2019]

$$\Psi_{\rm tidal}^{\rm NRTidalv2} = \frac{3}{128\eta}\left[-\frac{39}{2}\tilde{\Lambda}x^{5/2} \times \frac{1+\tilde{n}'_1 x+\tilde{n}'_{3/2}x^{3/2}+\tilde{n}'_2 x^2+\tilde{n}'_{5/2}x^{5/2}+\tilde{n}'_3 x^3}{1+\tilde{d}'_1 x+\tilde{d}'_{3/2}x^{3/2}+\tilde{d}'_2 x^2}\right]$$

Pade approx. (Linear)

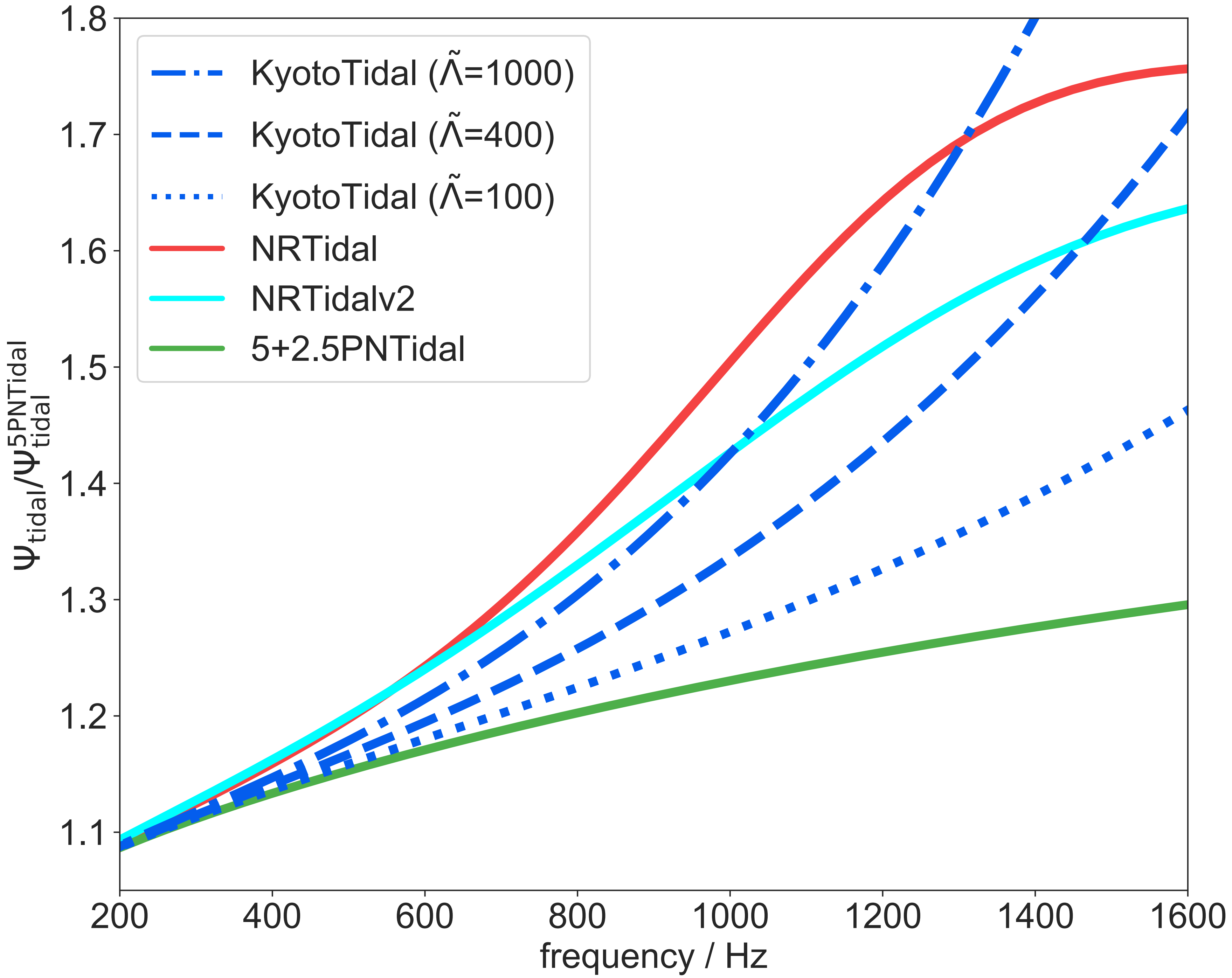
$$A_{\rm tidal}^{\rm NRTidalv2} = \sqrt{\frac{5\pi\eta}{24}}\frac{M_{\rm tot}^2}{d_L}\tilde{\Lambda}x^{-7/4} \times \left(-\frac{27}{16}x^5\right)\frac{1+\frac{449}{108}x+\frac{22672}{9}x^{2.89}}{1+dx^4}$$

Pade approx. (Linear)

# Tidal waveform models

divided by the leading tidal formula.

phase shift:
1. NRTidal
2. NRTidalv2
3. KyotoTidal (Lamtilde=400)
4. PNTidal




| Model name | Point-particle part | | Tidal part | |
|---|---|---|---|---|
| | Amplitude | Phase | Amplitude | Phase |
| TF2_PNTidal | 3PN | 3.5PN | 5+1PN | 5+2.5PN |
| TF2+_PNTidal | 6PN | 6PN | 5+1PN | 5+2.5PN |
| TF2+_KyotoTidal | 6PN | 6PN | Polynomial | Non-linear |
| TF2+_NRTidal | 6PN | 6PN | - | Padé approximation |
| TF2+_NRTidalv2 | 6PN | 6PN | Padé approximation | Padé approximation |

# Source parameters

- parameters for BNS $\{m_{1,2}, \chi_{1,2}, \tilde{\Lambda}\}$
  - $m_{1,2}$~U[0.83, 7.7]$M_\odot$, $M_c^{det}$~U[1.184, 2.168]$M_\odot$
  - $\chi_{1,2}$~U[-0.05, 0.05]
  - $\tilde{\Lambda}$~U[0, 3000]

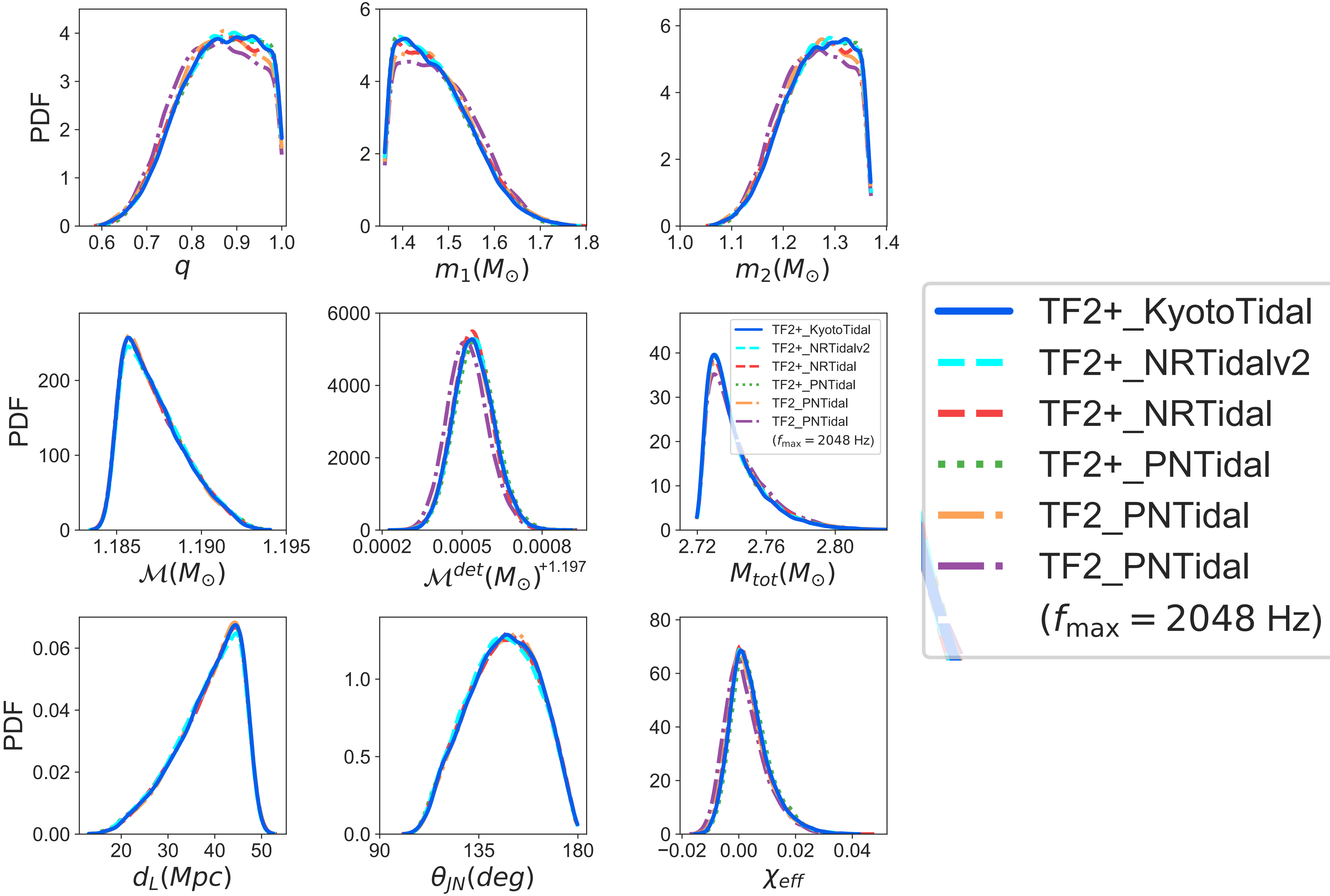
- additional parameters for fully describe the binary:

$\{D_L, \theta_{JN}, \psi, t_c, \phi_c,$ ~~$\alpha, \delta$~~$\}$

- a uniform prior in [0,2π] for $\phi_c$.
- sources uniformly distributed in volume

Results

A. Source properties other than the tidal deformability




**Almost no systematic bias associated with a difference in the estimates of the parameters among waveform models.**

Results

B. Posterior of binary tidal deformability

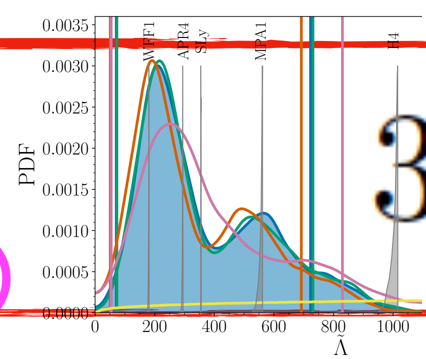
# **Comparison with LIGO-Virgo analysis** as a sanity check

**restricted TF2 with 5+1PN-order tidal phase**

**uniform prior on $\Lambda_{1,2}$**

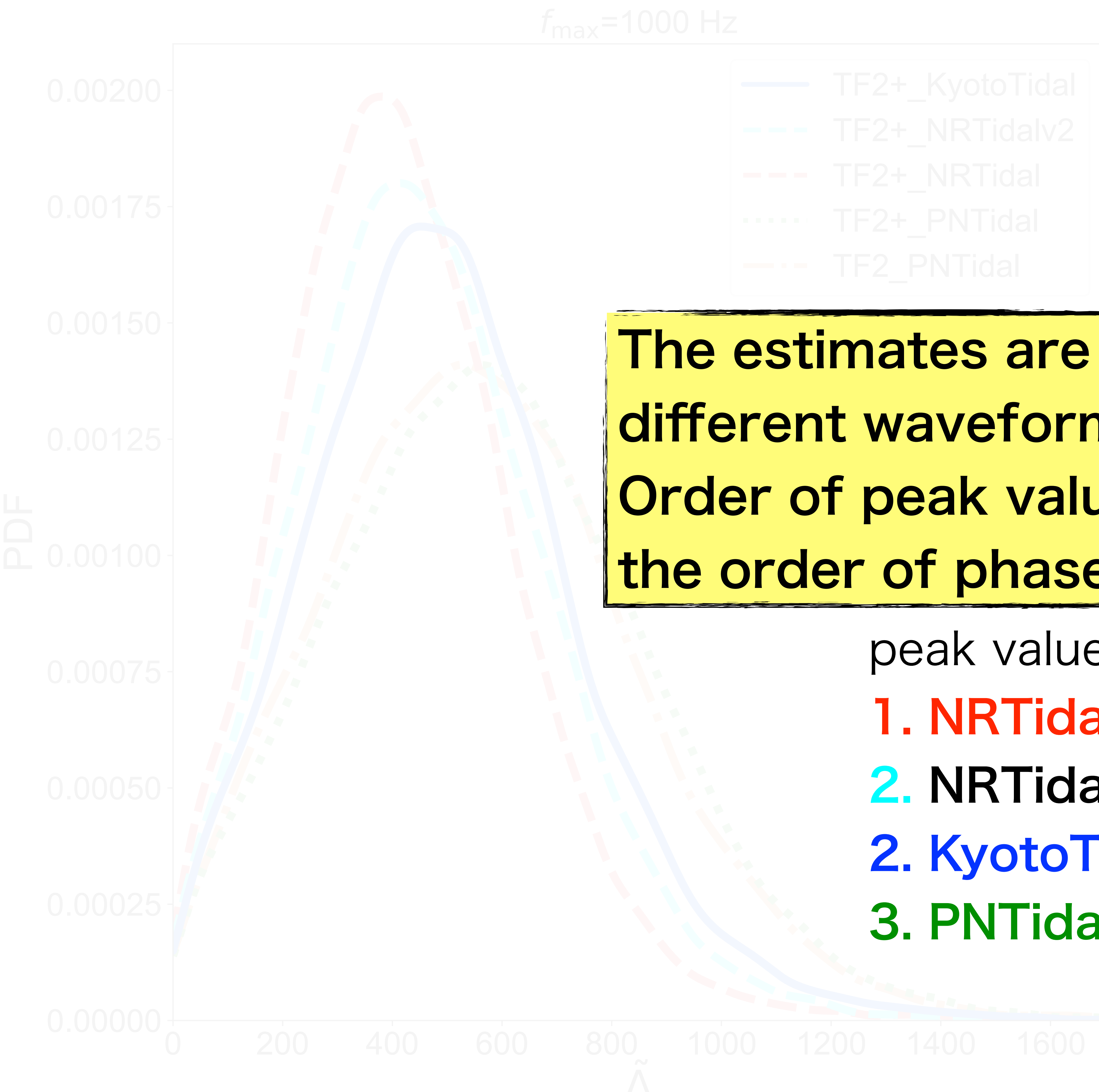
**$f_{max}$=2048 Hz**

| | Symmetric interval | HPD (Highest probability distribution) |
|---|---|---|
| HLV **(our results)** | $347^{+564}_{-243}$ | $347^{+453}_{-295}$ |
| HLV (LVC report, arXiv:1805.11579) | $340^{+580}_{-240}$ | $(340^{+490}_{-290})$ |
| HLV (our results, flat prior on Lamtilde) | $316^{+504}_{-224}$ | $(316^{+367}_{-291})$ |




**very close!**

# Posterior of binary tidal deformability

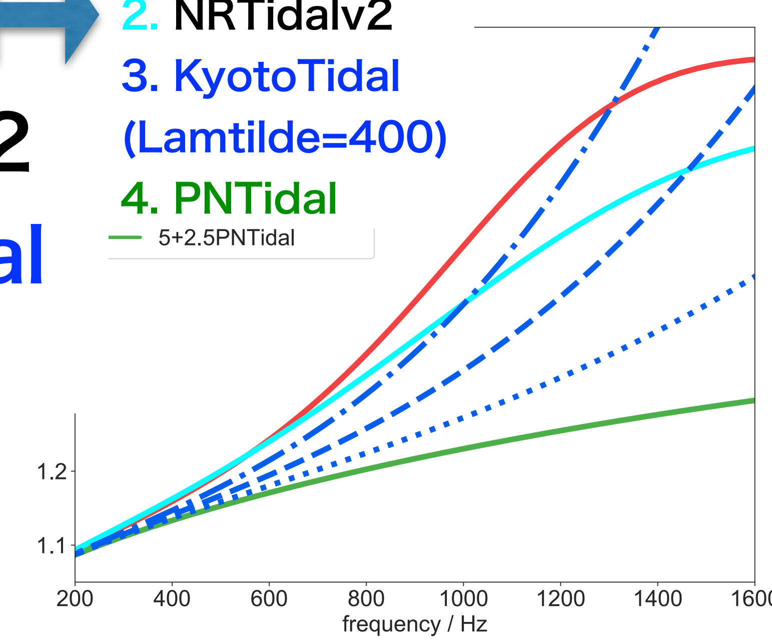



likely ~400,
not above 1000

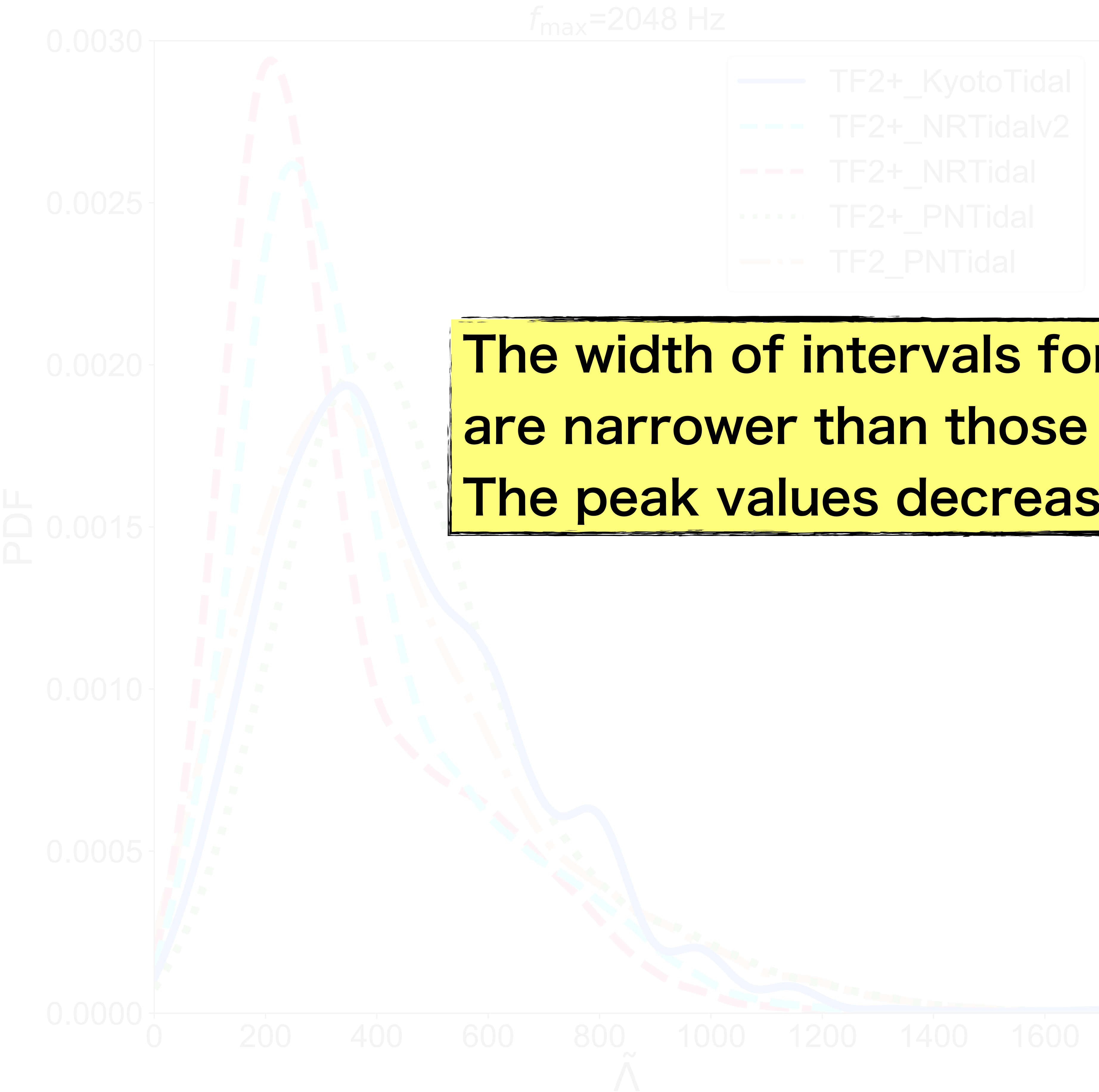
**The estimates are biased by using different waveform models.**
**Order of peak value is consistent with the order of phase shift.**

peak value:
1. NRTidal
2. NRTidalv2
2. KyotoTidal
3. PNTidal

phase shift:
1. NRTidal
2. NRTidalv2
3. KyotoTidal
(Lamtilde=400)
4. PNTidal



# For $f_{max}$=2048 Hz (only a reference)




**The width of intervals for $f_{max}$=2048 Hz are narrower than those for $f_{max}$=1000 Hz. The peak values decrease as $f_{max}$ increases.**

peak value:

1. NRTidal
2. NRTidalv2
2. KyotoTidal
3. PNTidal

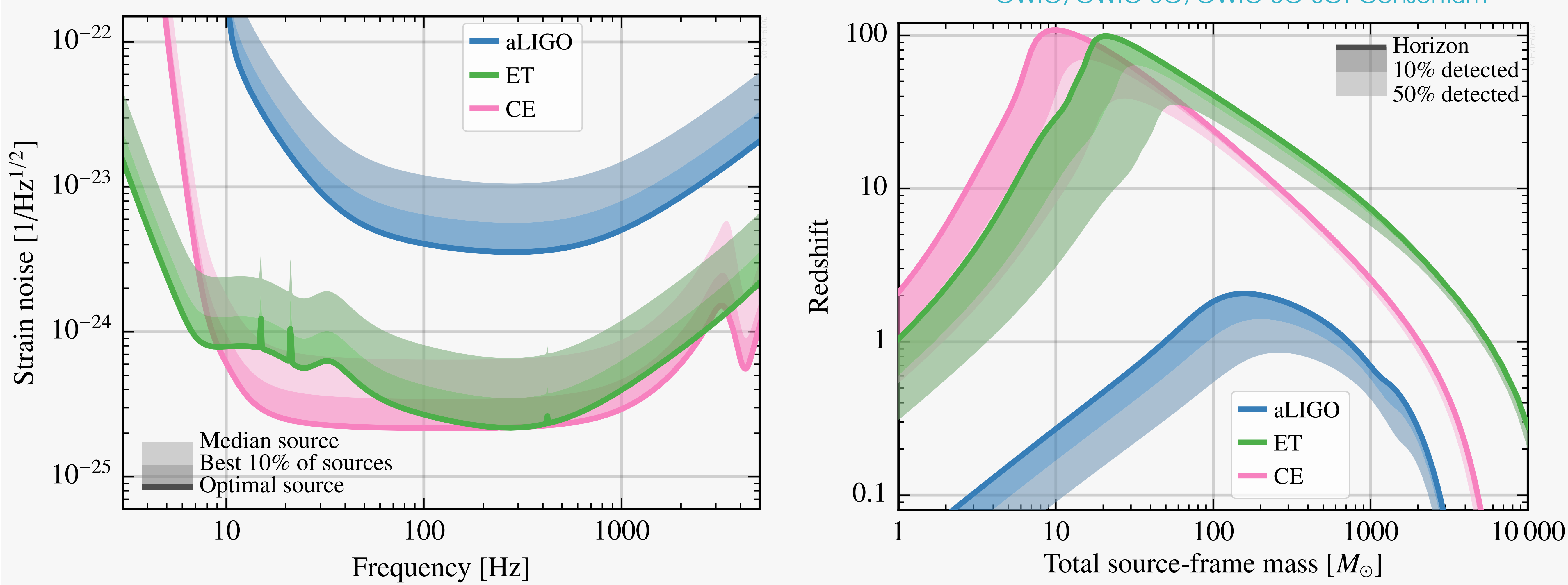
Discussion

need to improve the current waveform model

# Systematic error for $f_{max}$=1000 Hz

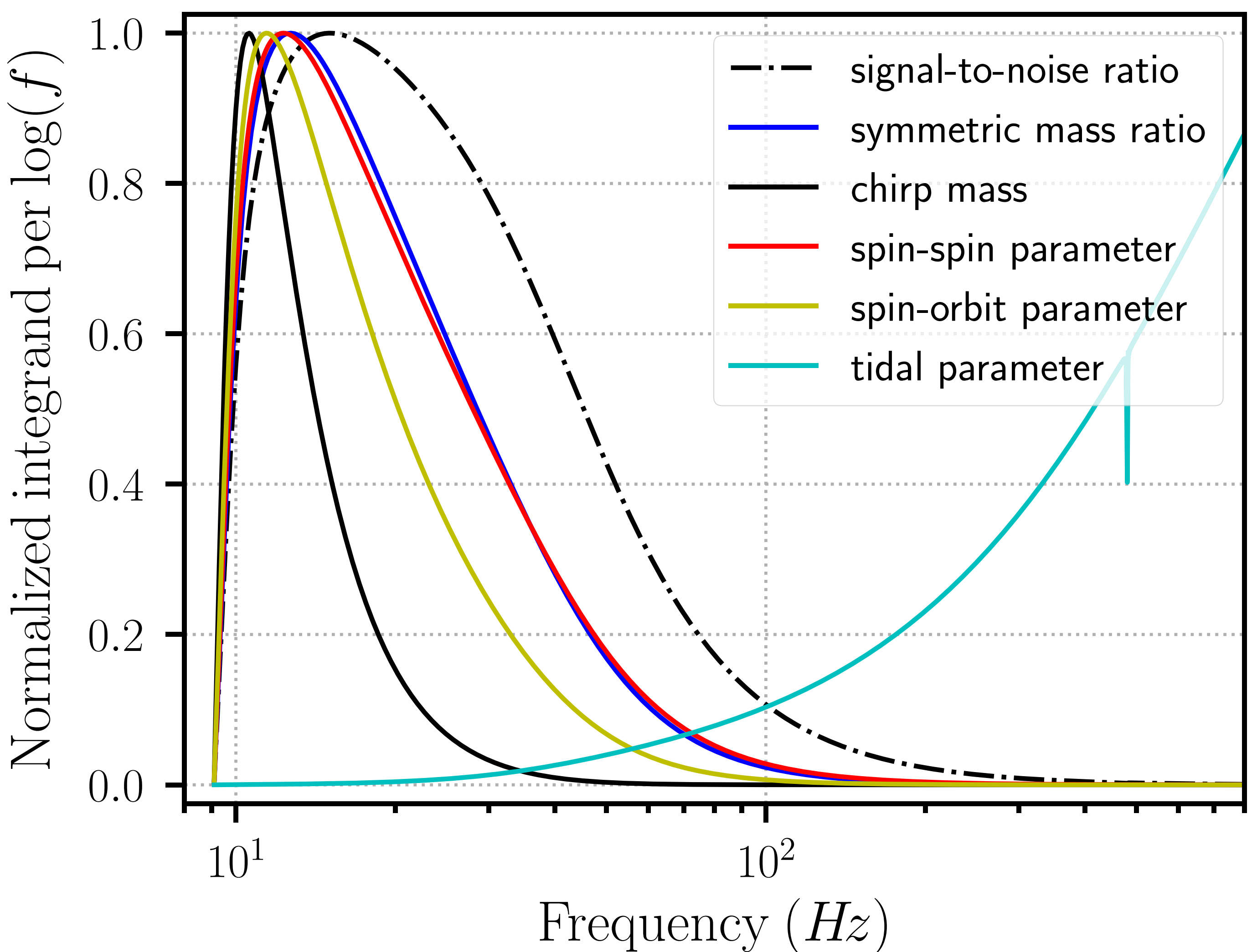
For GW170817, systematic error is dominant over statistical error. For 10 times louder SNR event than GW170817, systematic error between KyotoTidal and NRTidal is comparable with the statistical error. 3G detector's sensitivity is 10 times better than current detectors. Toward 3G detector era, it is needed to improve current waveform models.

GWIC, GWIC-3G, GWIC-3G-SCT-Consortium



# Need to improve at frequency higher than 1000 Hz

High frequency data are generally more informative to measure tidal deformability. Our results indicate that $\tilde{\Lambda}$ is indeed constrained tighter determined for $f_{max}$=2048 Hz than for $f_{max}$=1000 Hz. However, since the TF2+ KyotoTidal model is calibrated by NR waveforms only up to 1000 Hz, toward 3G detector era, it is needed to further improve the model in the frequency higher than 1000 Hz.




[Harry, Hinderer, 2018] (c.f. [Damour, Nagar, Villain, 2012])

# Summary

・LIGO-Virgo Collaboration put conservative upper limits on tidal deformability with post-Newtonian waveform (**PNTidal**) and measure it with **NRTidal**.

・We reanalyze GW170817 with a new NR calibrated waveform model, **KyotoTidal**.

・We compare our results with another NR calibrated waveform, **NRTidal** and its upgraded model, **NRTIdalv2**.

**The estimates are biased by using different waveform models. Order of peak value is consistent with the order of phase shift.**

**The width of intervals for $f_{max}$=2048 Hz are narrower than those for $f_{max}$=1000 Hz. The peak values decrease as $f_{max}$ increases.**