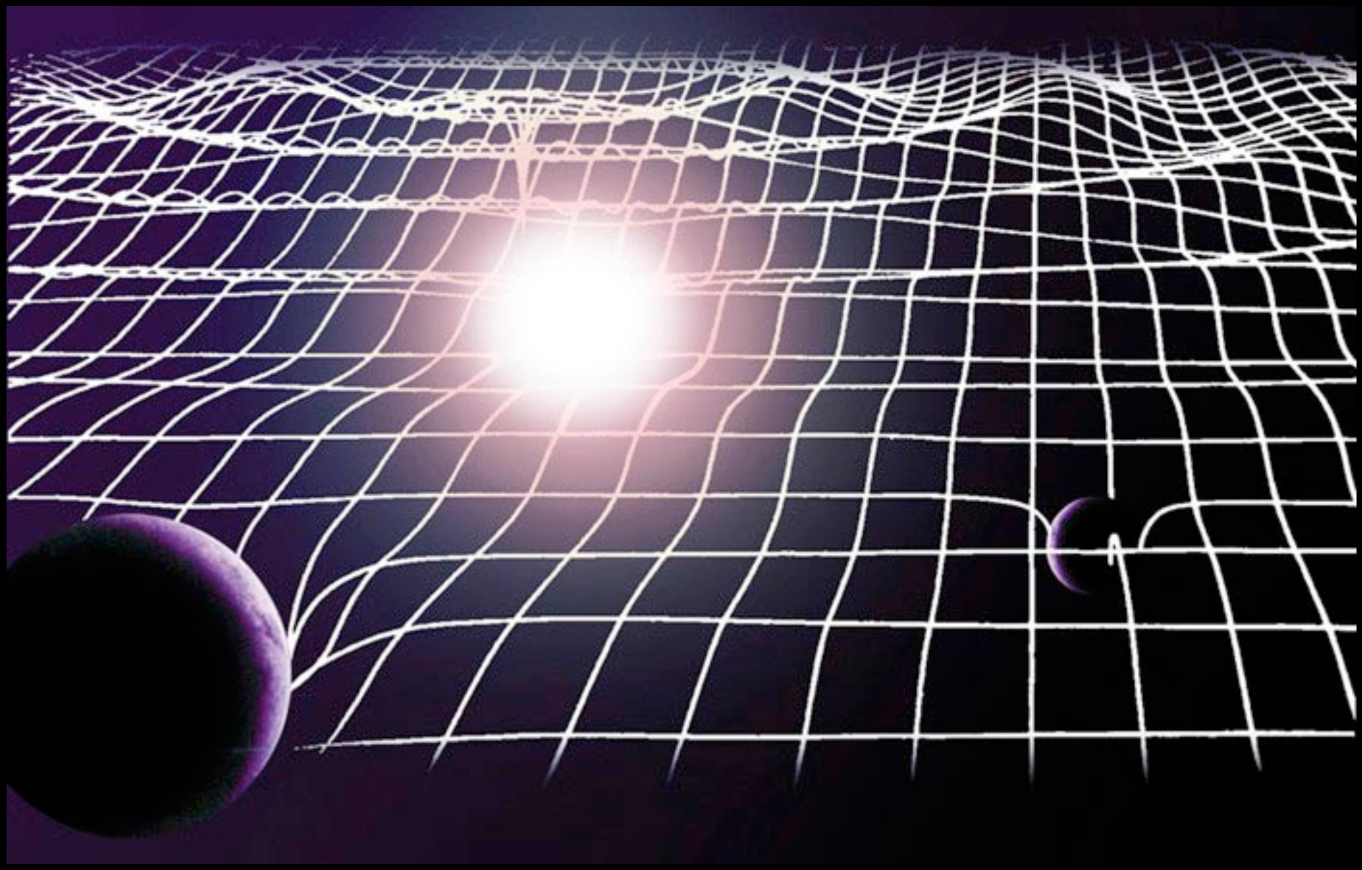
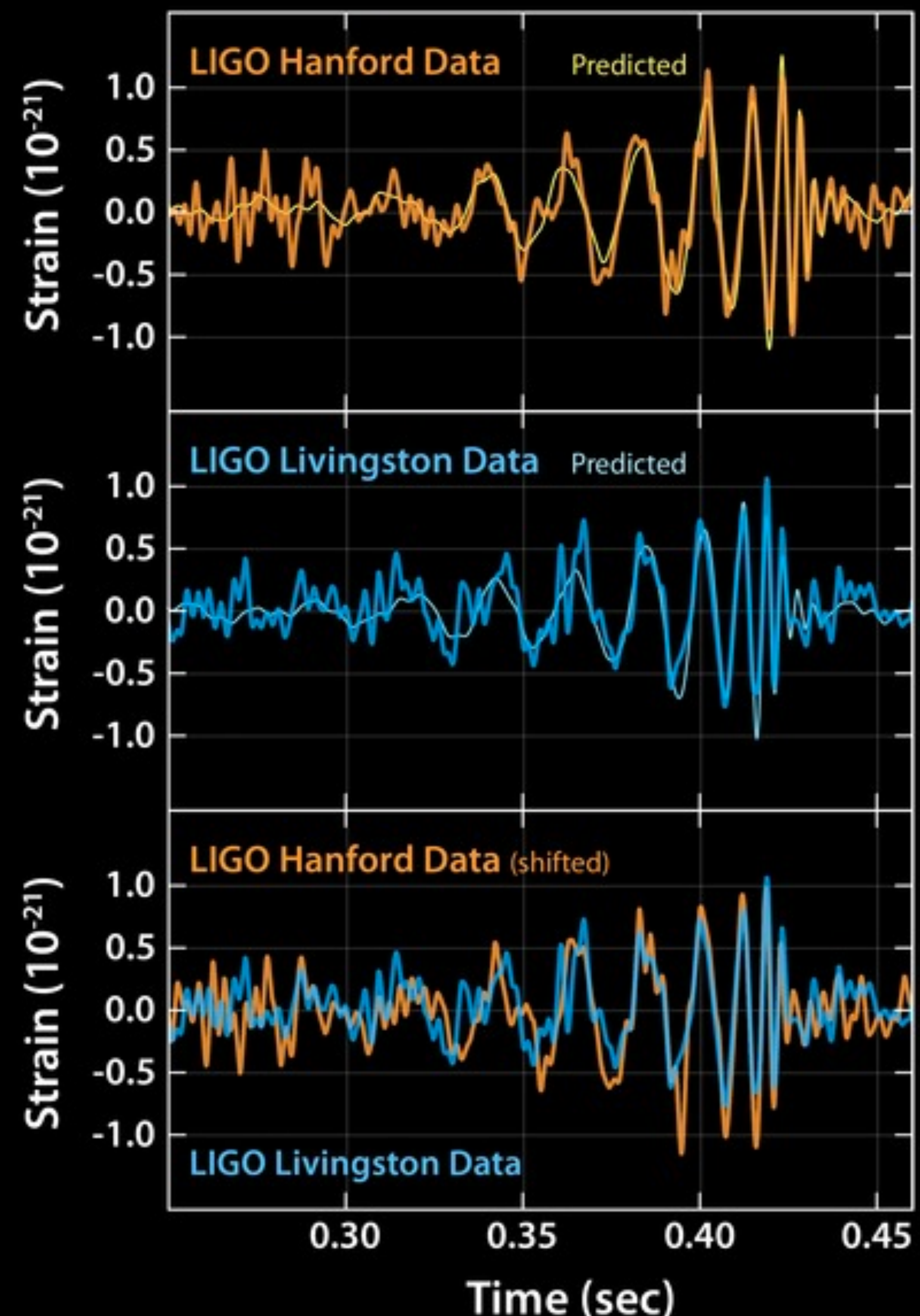
Formation of Eccentric BH Mergers

Johan Samsing
Princeton University



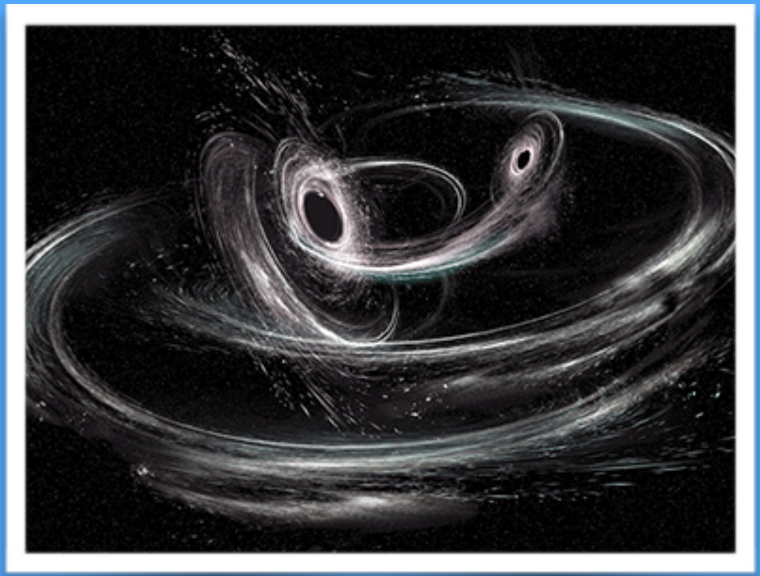
This is what we observe:



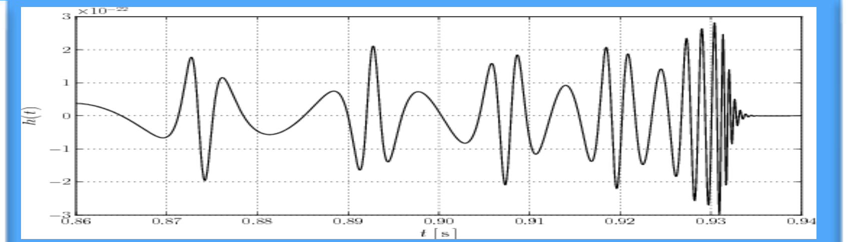
Nothing to test if you have no ‘useful’ predictions…

What is the origin of BBH mergers?

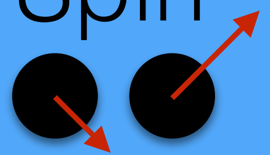
<u>Clusters</u>



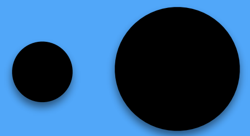
<u>parameters</u>



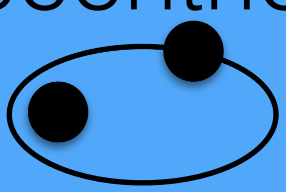
Spin



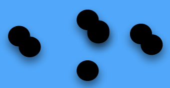
Mass



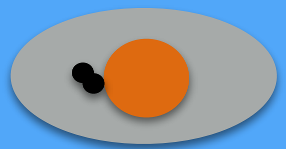
Eccentricity



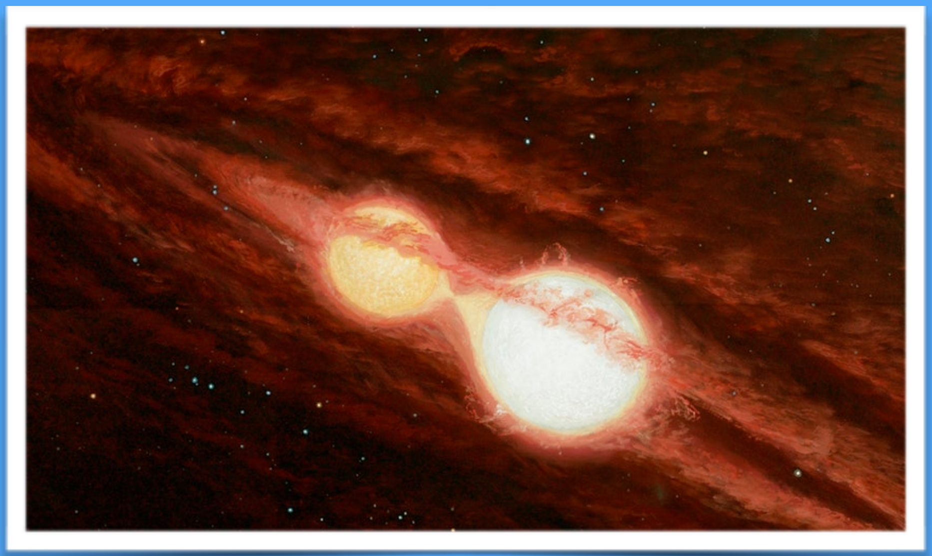
Rates

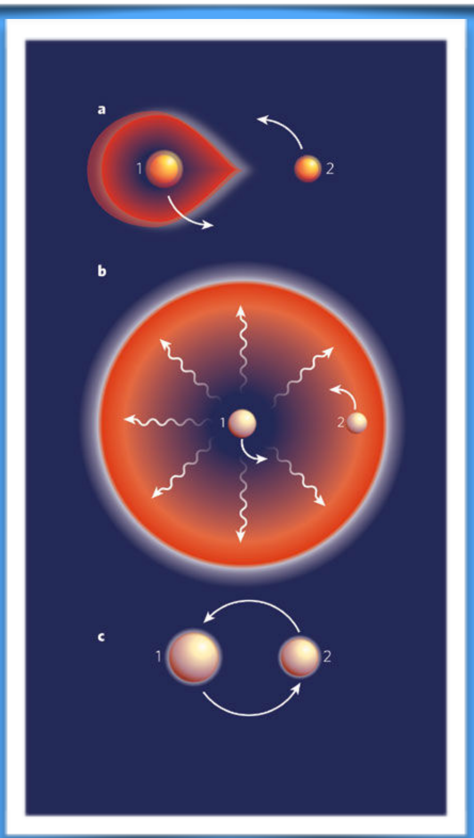


Location



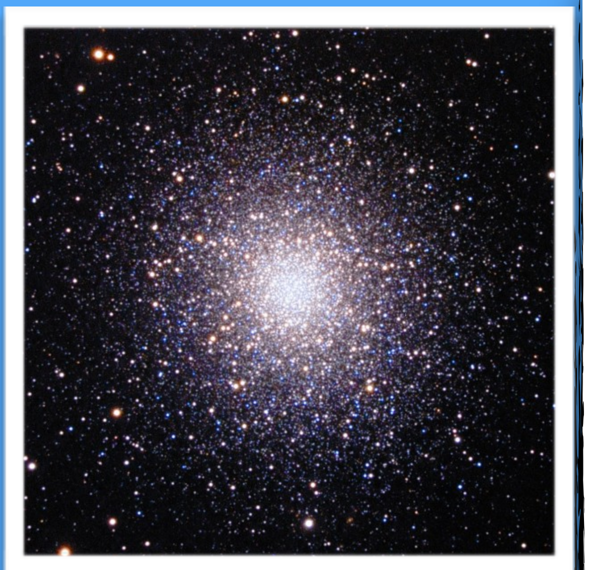
<u>Field</u>

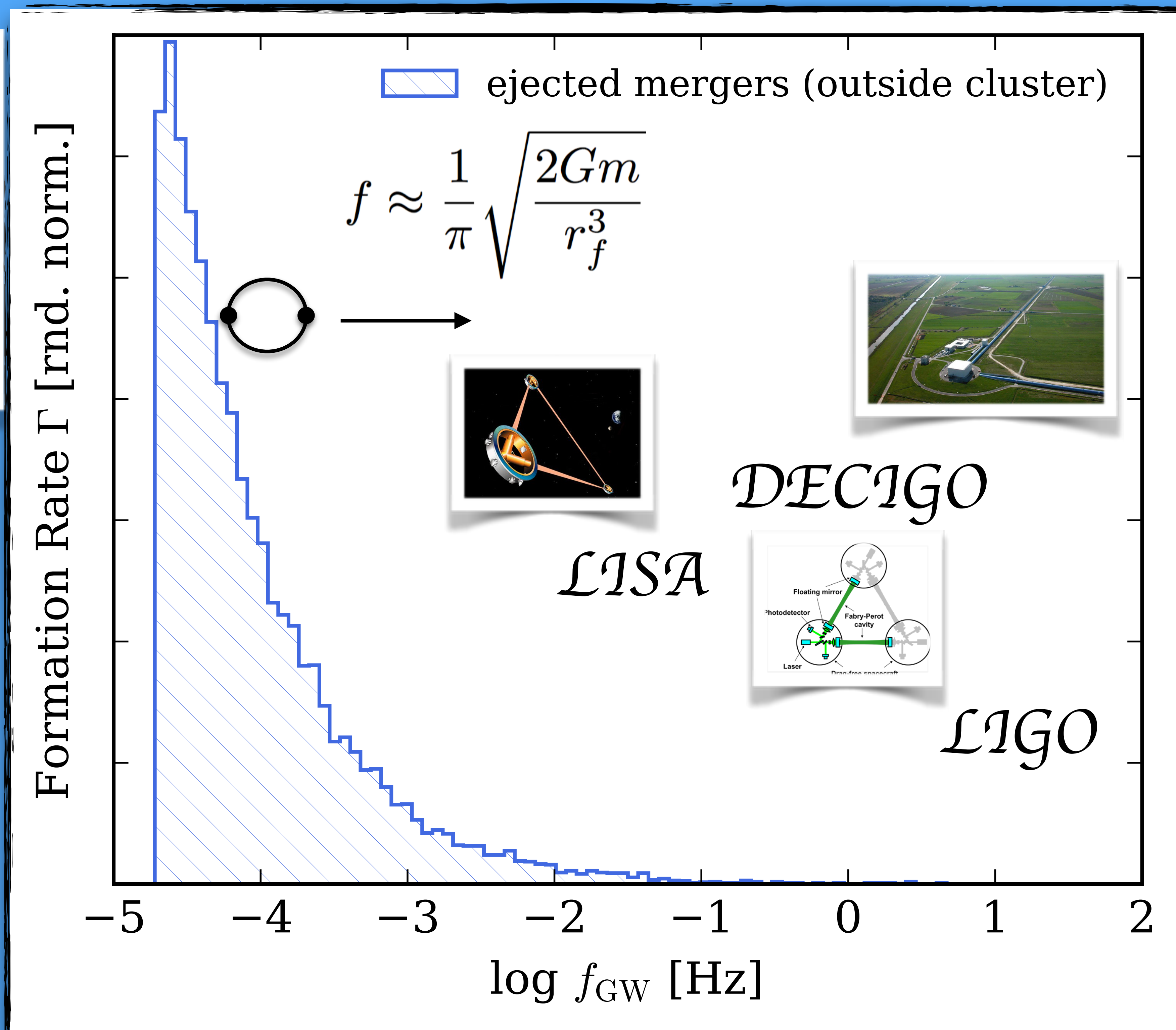




Old Newtonian Studies

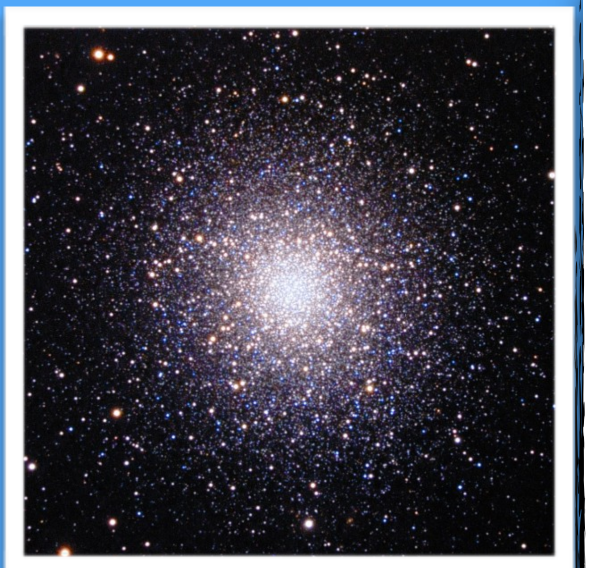
< 2017

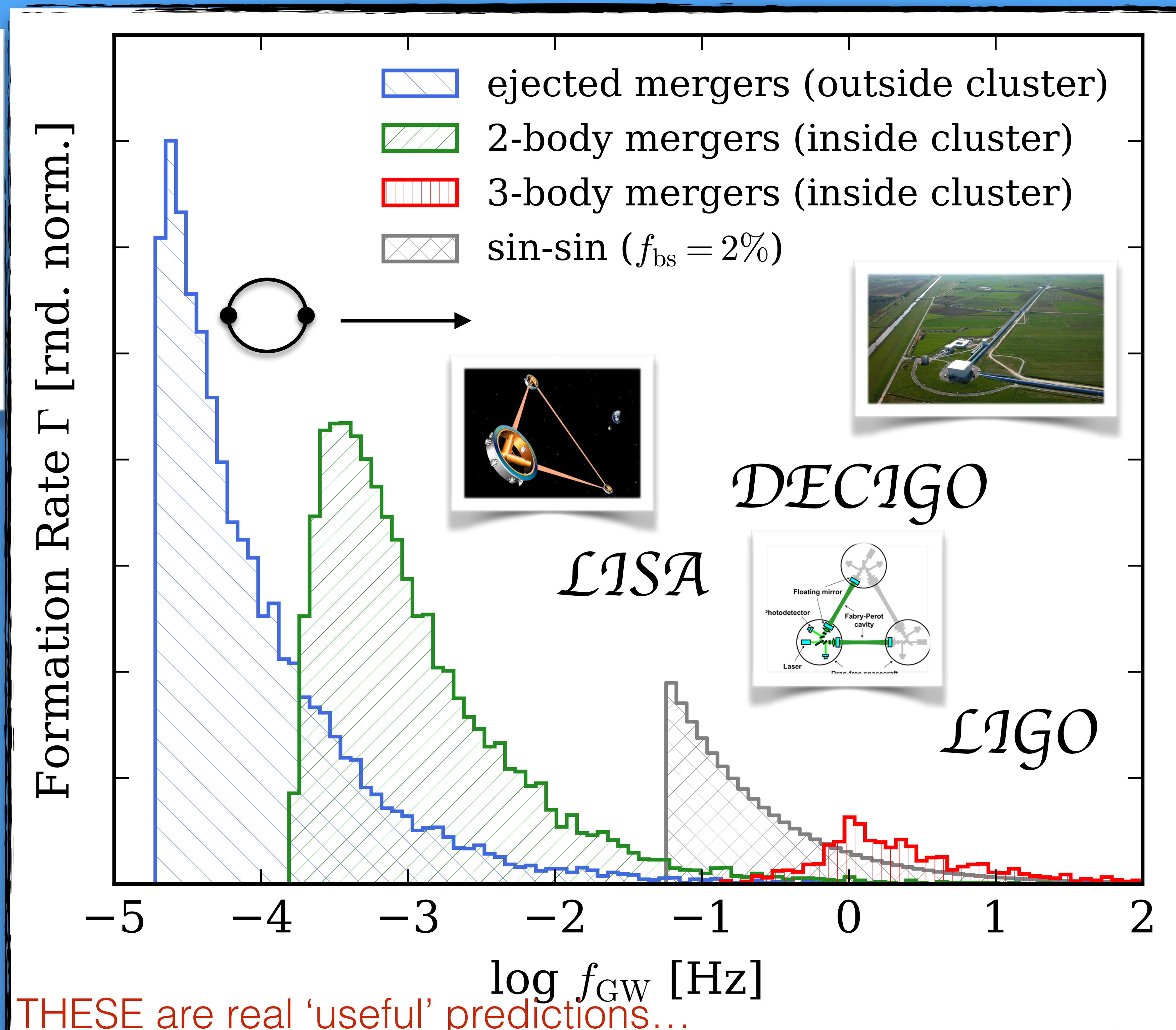




New Post-Newtonian Studies

> 2017





THESE are real ‘useful’ predictions…

Background

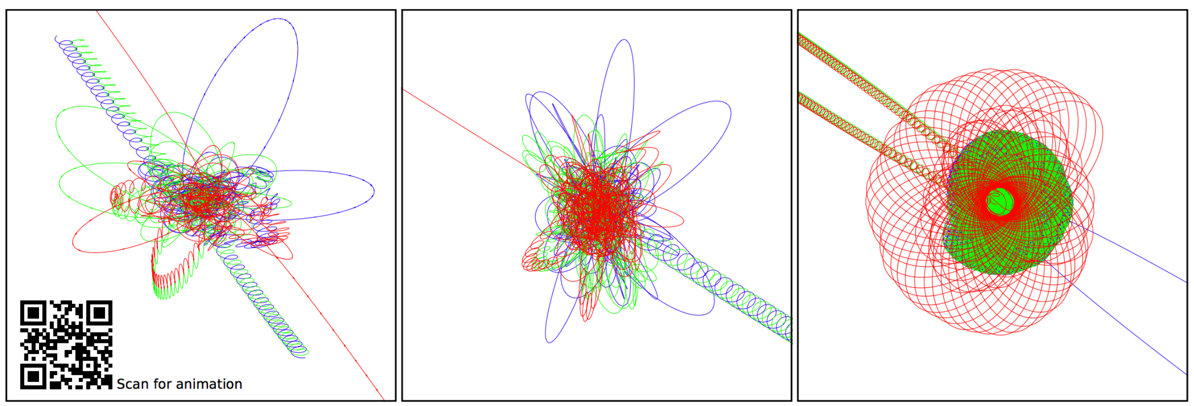
THE FORMATION OF ECCENTRIC COMPACT BINARY INSPIRALS AND THE ROLE OF GRAVITATIONAL WAVE EMISSION IN BINARY-SINGLE STELLAR ENCOUNTERS

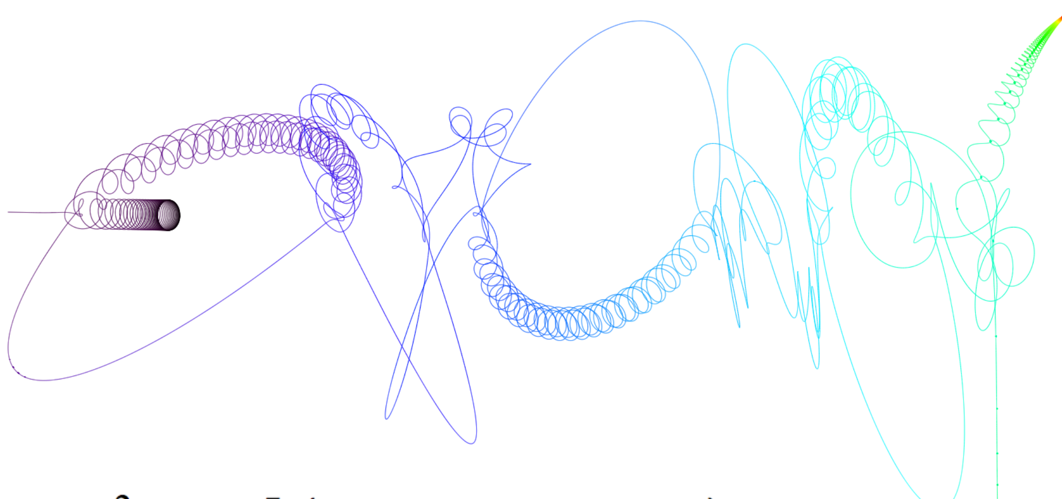
JOHAN SAMSING[1], MORGAN MACLEOD[2], ENRICO RAMIREZ-RUIZ[2]

Draft version October 29, 2018

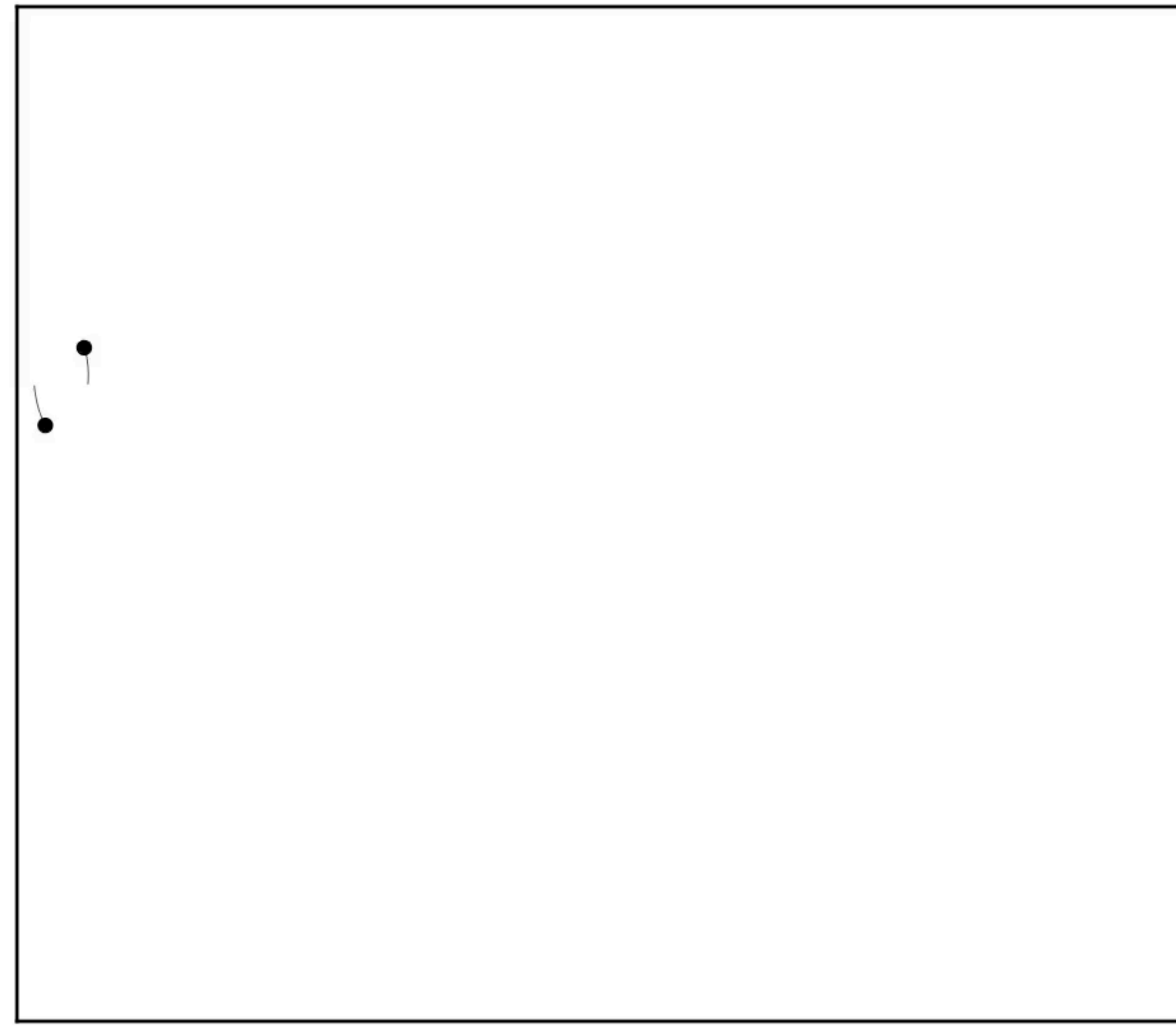
ABSTRACT

The inspiral and merger of eccentric binaries leads to gravitational waveforms distinct from those generated by circularly merging binaries. Dynamical environments can assemble binaries with high eccentricity and peak frequencies within the *LIGO* band. In this paper, we study binary-single stellar scatterings occurring in dense





$$\mathbf{a}_5 = \frac{4}{5}\frac{G^2 m_1 m_2}{r_{12}^3}\left[\left(\frac{2Gm_1}{r_{12}} - \frac{8Gm_2}{r_{12}} - v_{12}^2\right)\mathbf{v}_{12} + (\hat{\mathbf{r}}_{12}\cdot\mathbf{v}_{12})\left(\frac{52Gm_2}{3r_{12}} - \frac{6Gm_1}{r_{12}} + 3v_{12}^2\right)\hat{\mathbf{r}}_{12}\right]$$



eccentric black hole mergers forming in globular clusters Samsing, 18.

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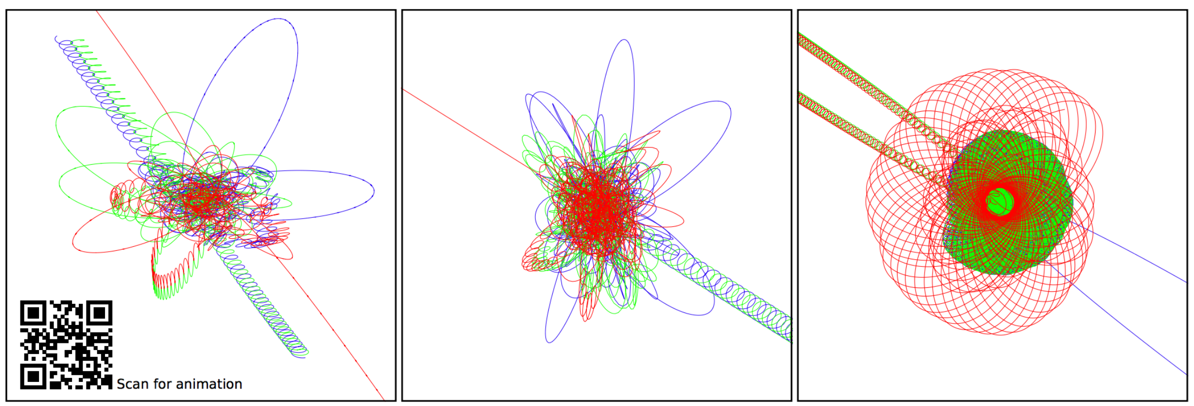
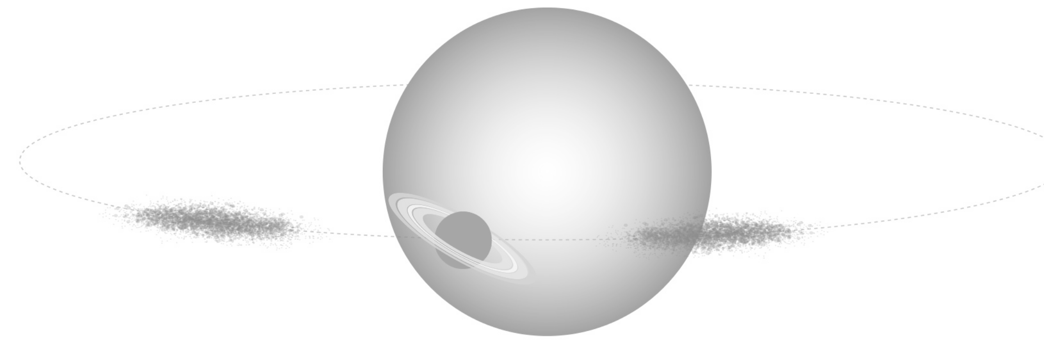
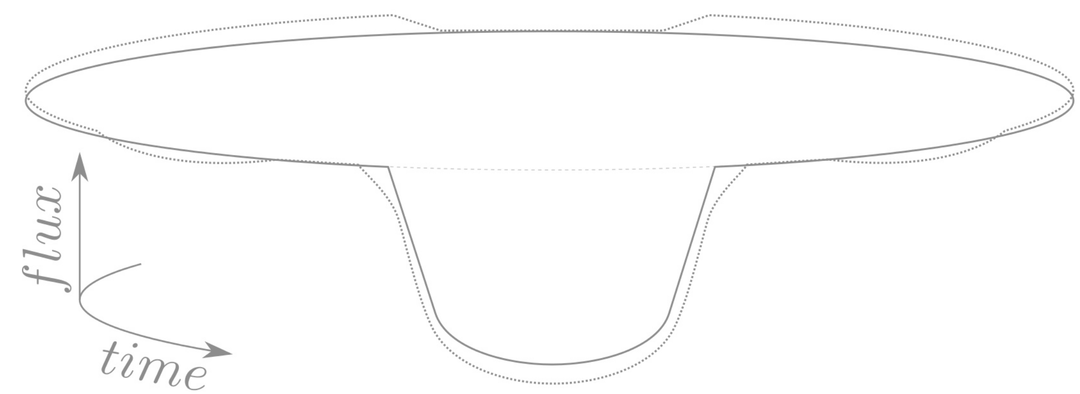
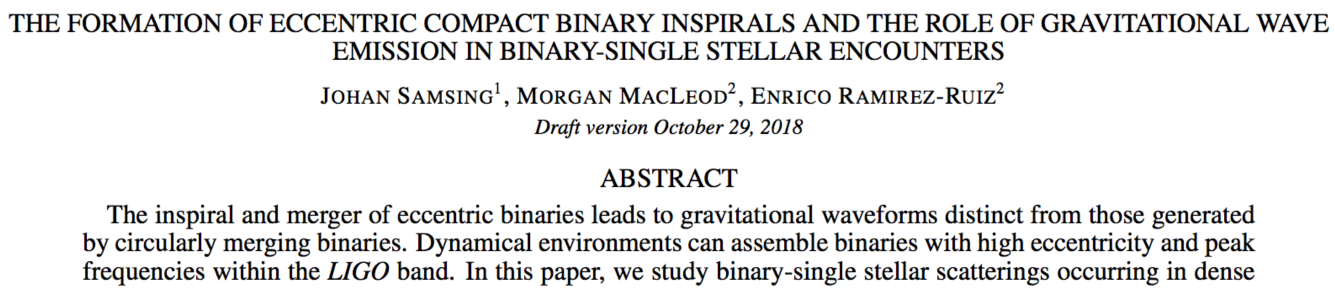


2015…





Background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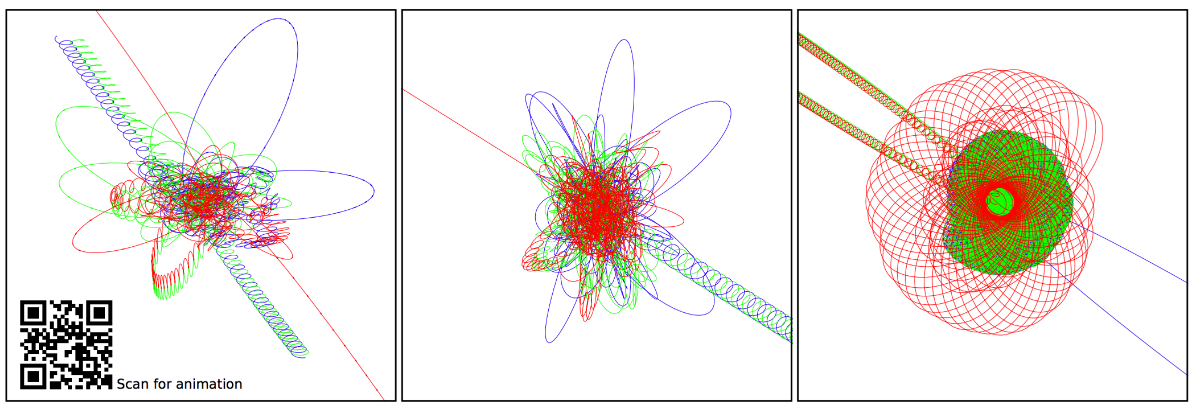

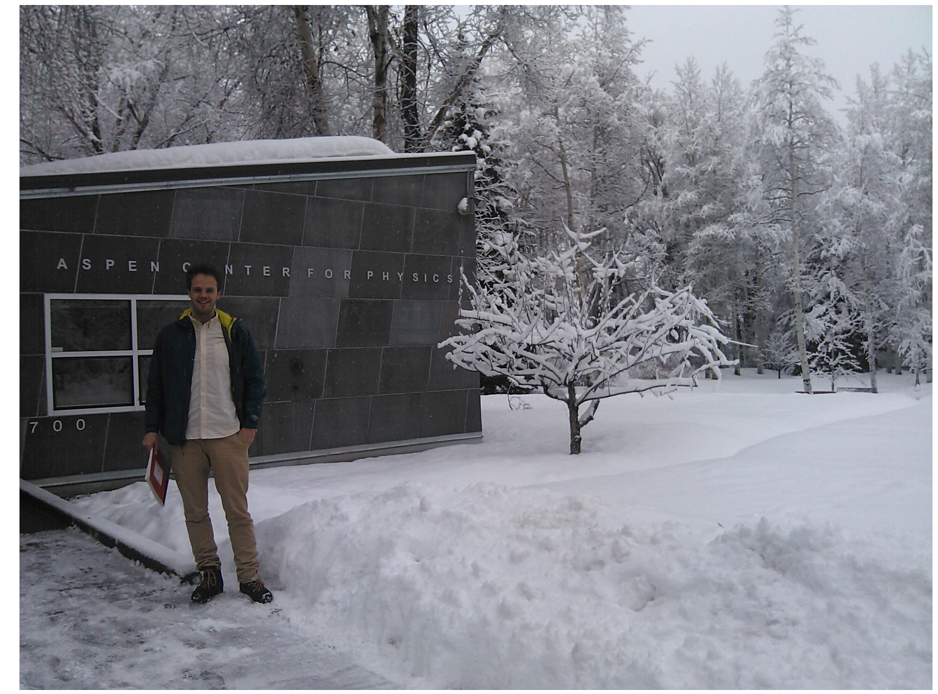

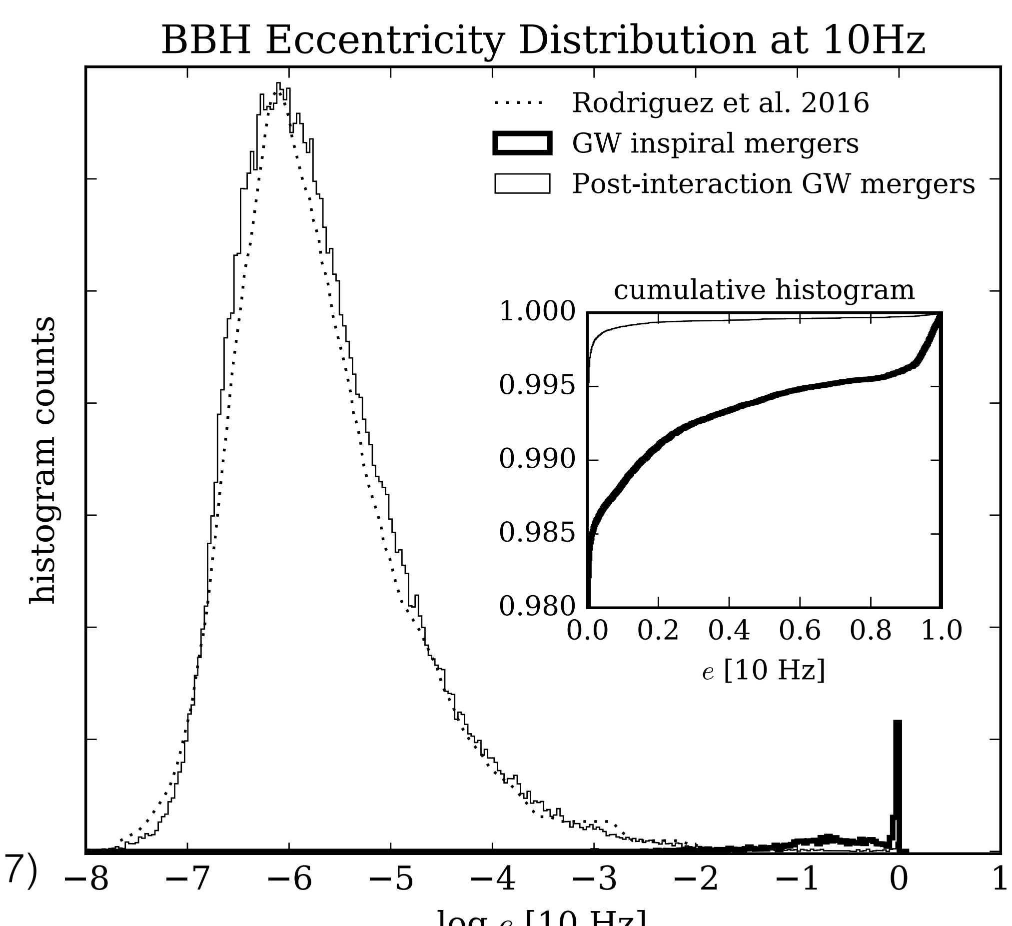

(Samsing, Ramirez-Ruiz, 17)

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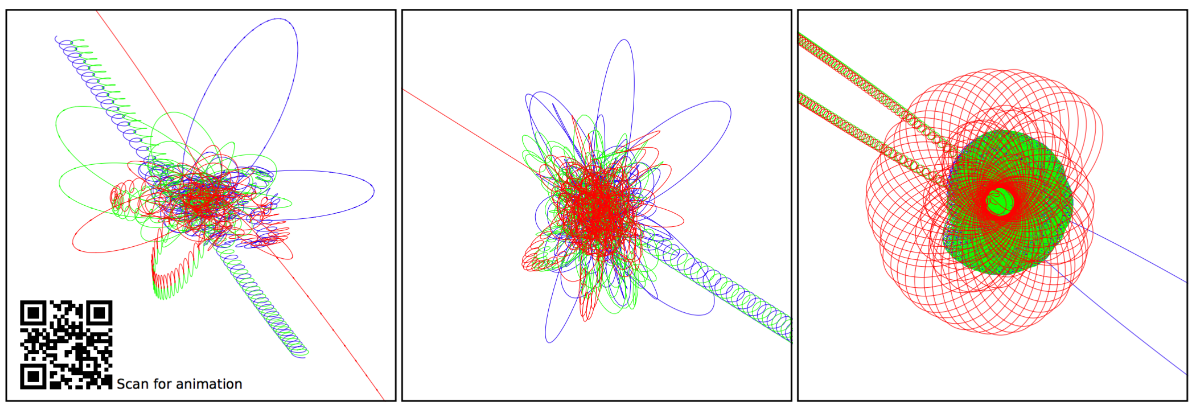

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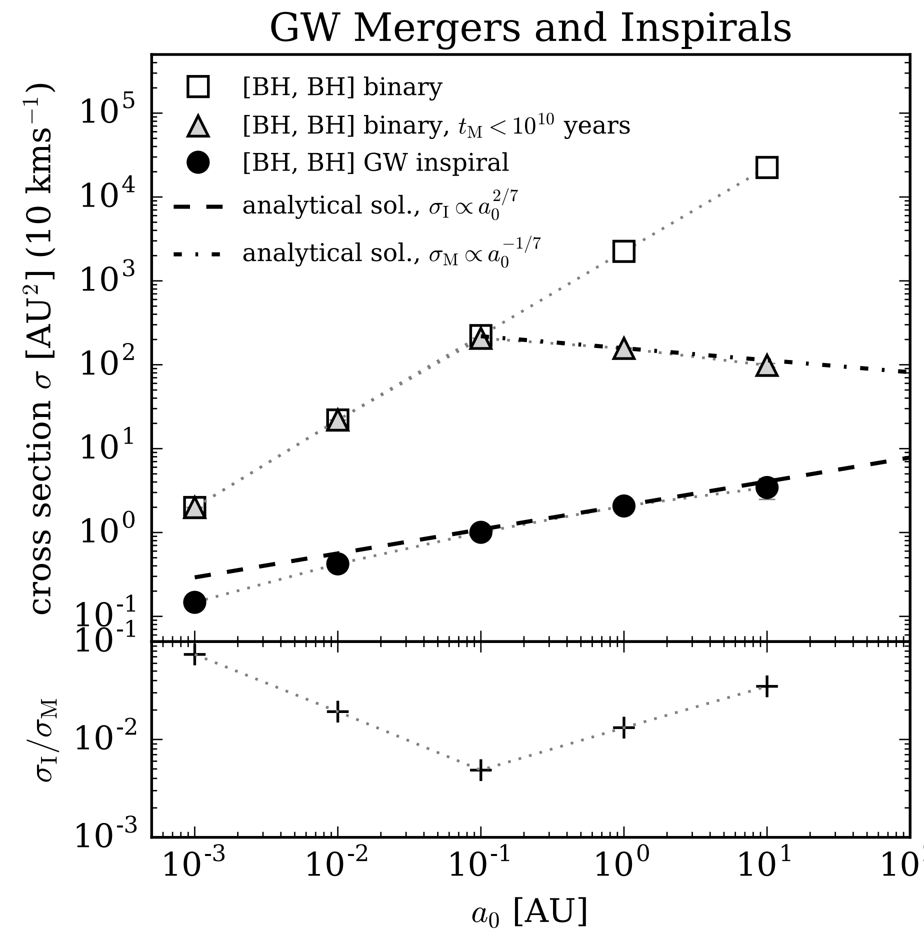
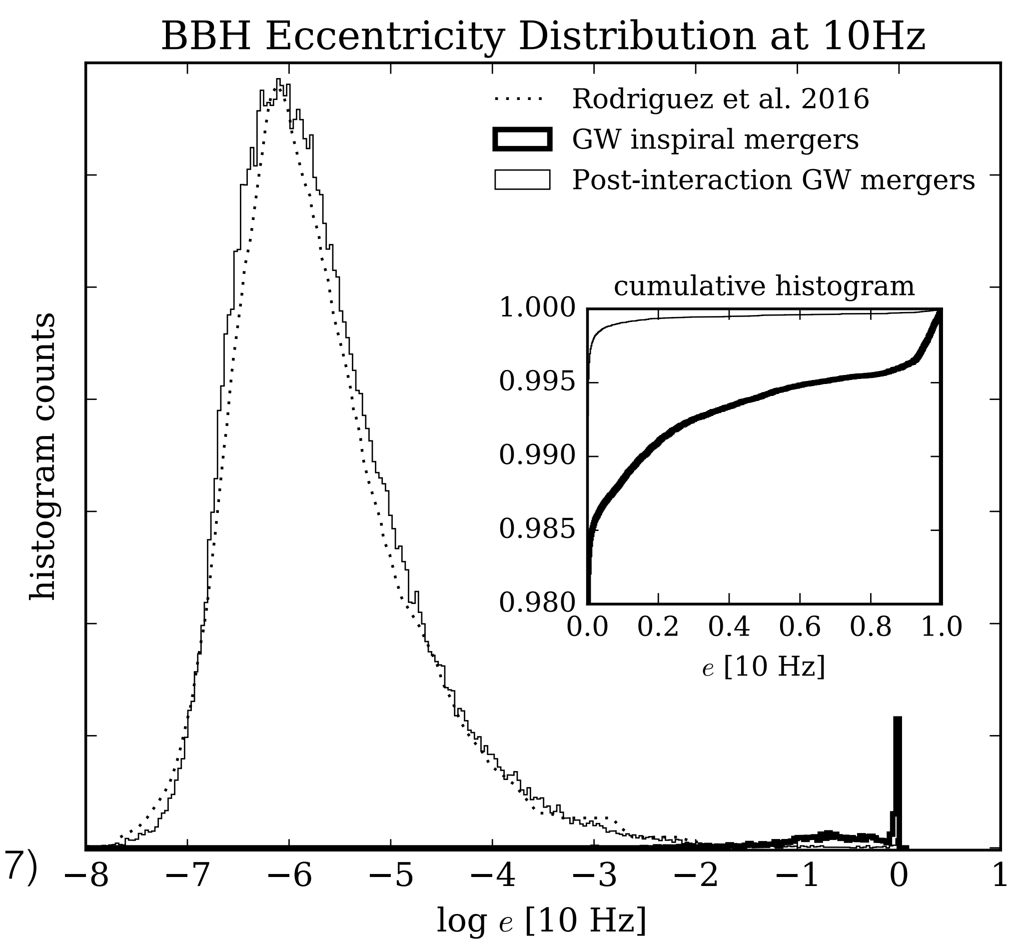


(Samsing, et a. 2017)



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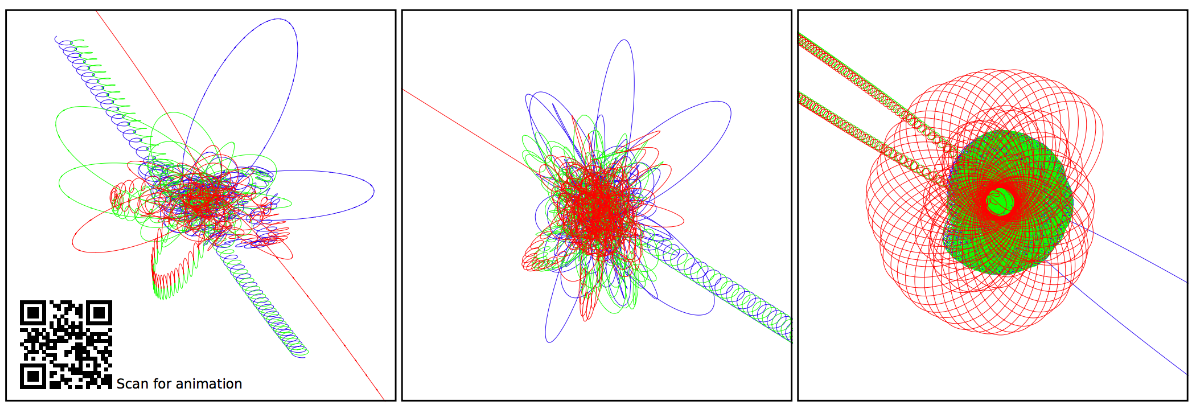


Clusters give rise to predictable outcomes.

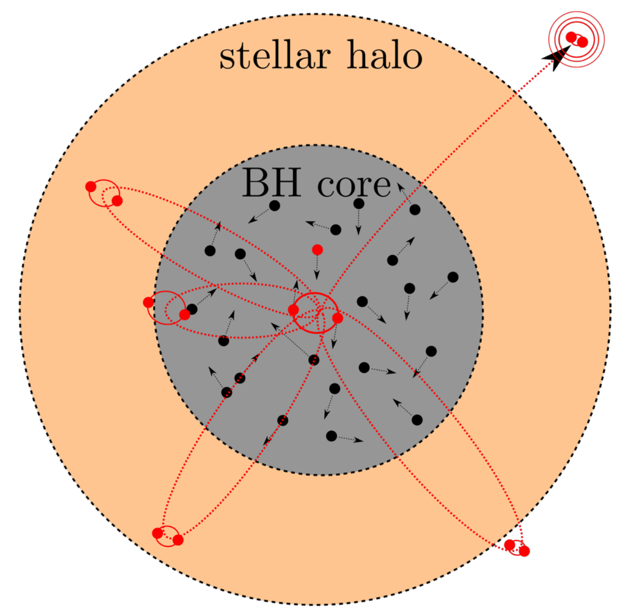
Pen and paper can reach percent precision!

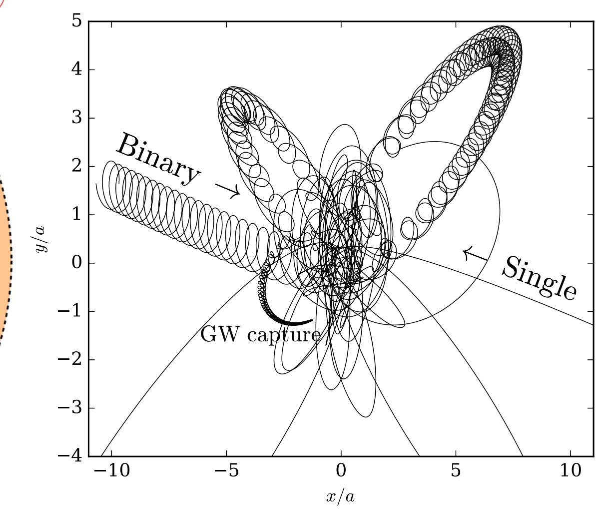
Our two methods greatly complement each other!

Eccentric Black Hole Mergers Forming in Globular Clusters

Johan Samsing*
Department of Astrophysical Sciences, Princeton University, Peyton Hall, 4 Ivy Lane, Princeton, NJ 08544, USA.

We derive the probability for a newly formed binary black hole (BBH) to undergo an eccentric gravitational wave (GW) merger during binary-single interactions inside a stellar cluster. By integrating over the hardening interactions such a BBH must undergo before ejection, we find that the observable rate of BBH mergers with eccentricity > 0.1 at 10 Hz relative to the rate of circular mergers can be as high as $\sim 5\%$ for a typical globular cluster (GC). This further suggests that BBH mergers forming through GW captures in binary-single interactions, eccentric or not, are likely to constitute $\sim 10\%$ of the total BBH merger rate from GCs. Such GW capture mergers can only

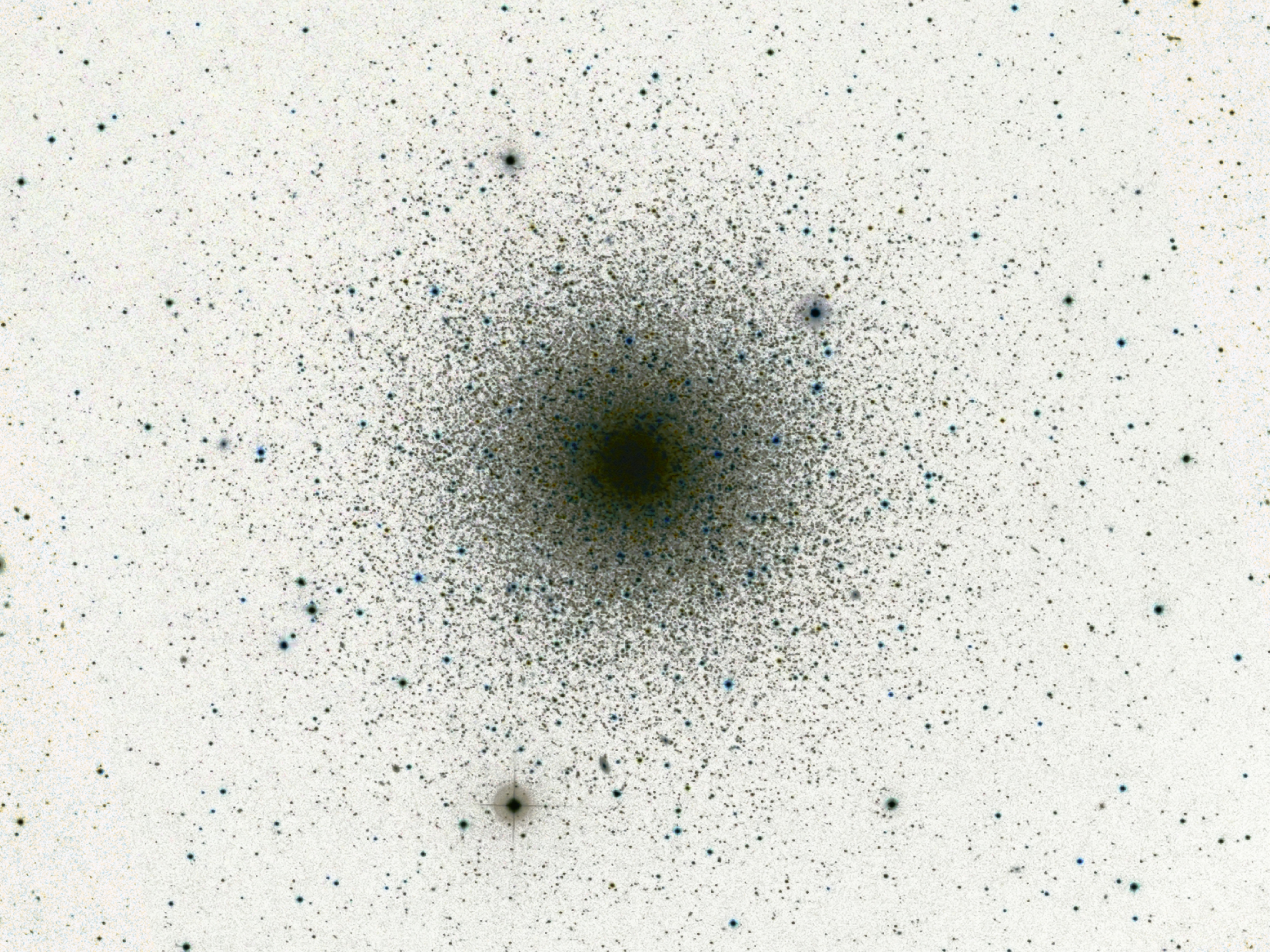


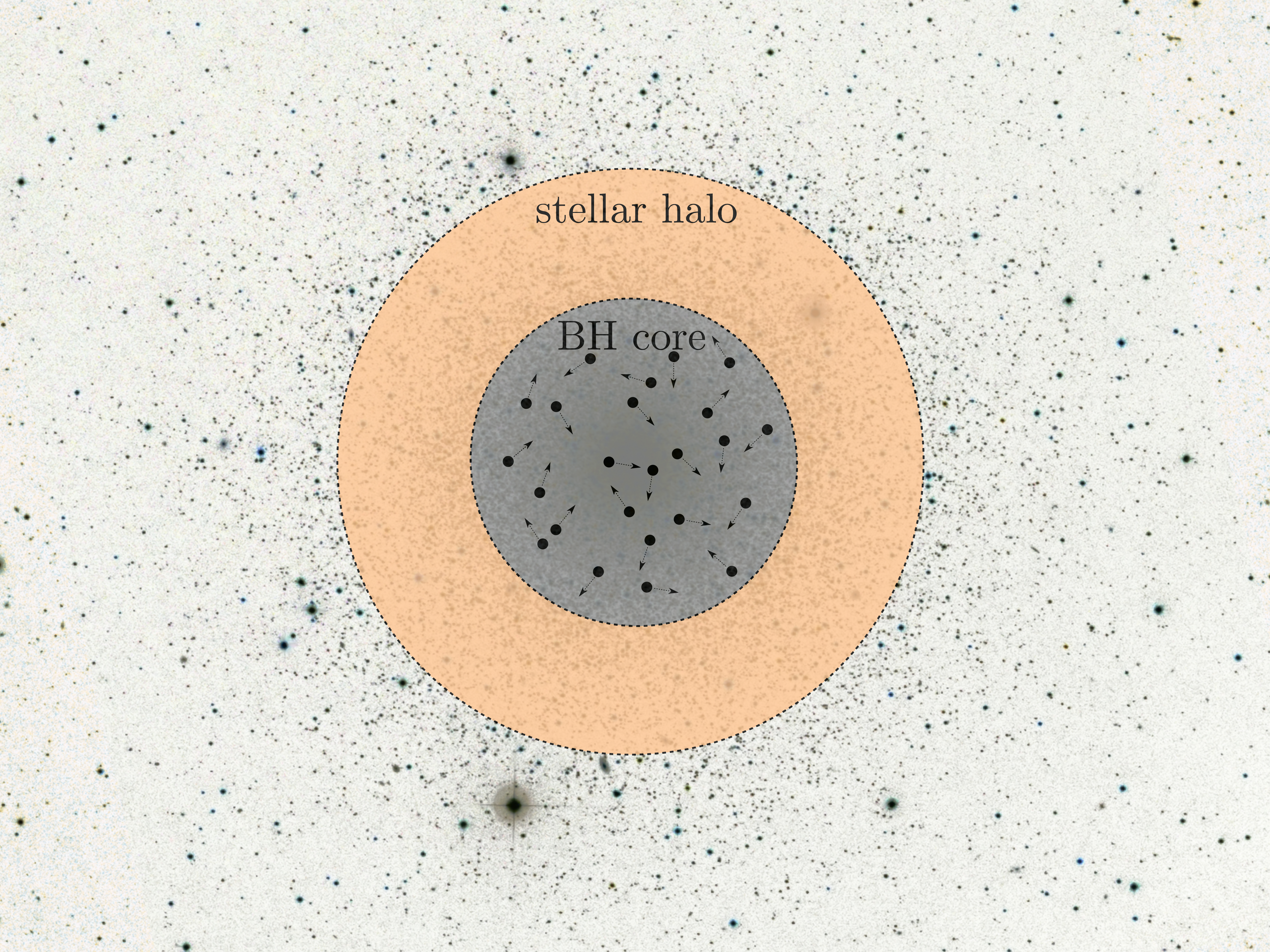


Post-Newtonian Dynamics in Dense Star Clusters: Formation, Masses, and Merger Rates of Highly-Eccentric Black Hole Binaries

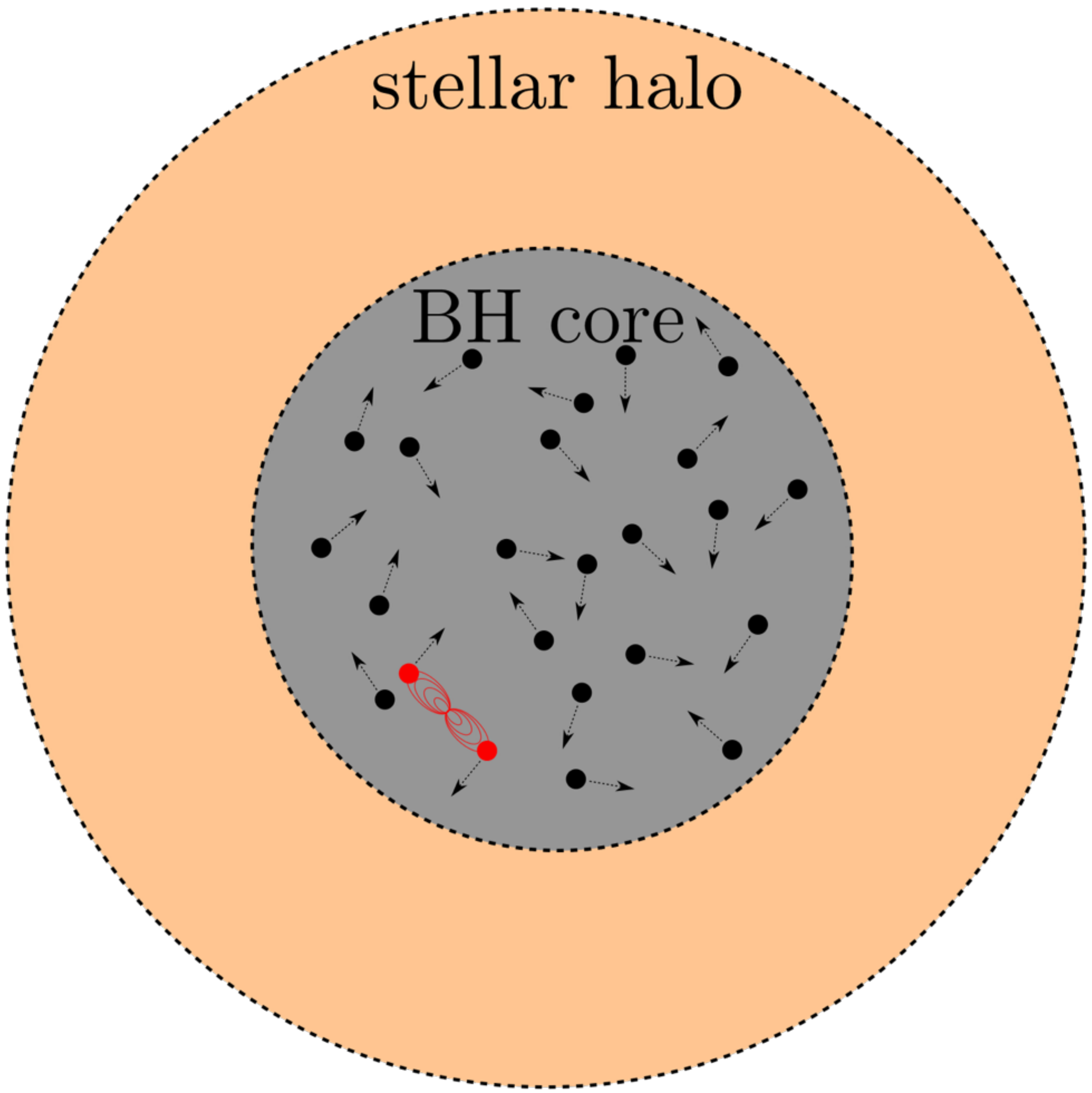
Carl L. Rodriguez,[1] Pau Amaro-Seoane,[2] Sourav Chatterjee,[3] Kyle Kremer,[4] Frederic A. Rasio,[4] Johan Samsing,[5] Claire S. Ye,[4] and Michael Zevin[4]

Using state-of-the-art dynamical simulations of globular clusters, including radiation reaction during black hole encounters and a cosmological model of star cluster formation, we create a realistic population of dynamically-formed binary black hole mergers across cosmic space and time. We show that in the local universe, 10% of these binaries form as the result of gravitational-wave emission between unbound black holes during chaotic resonant encounters, with roughly half of those events having eccentricities detectable by current ground-based gravitational-wave detectors. The mergers

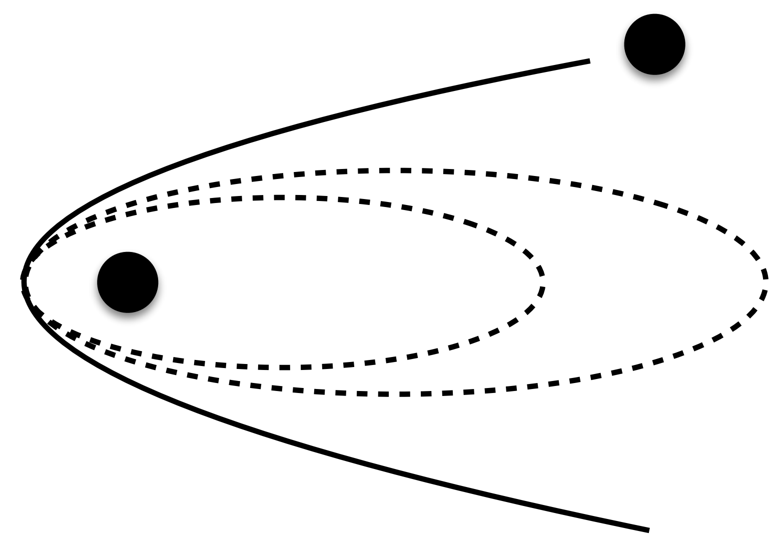



stellar halo
BH core

Merger Type: Single-Single



Capture:

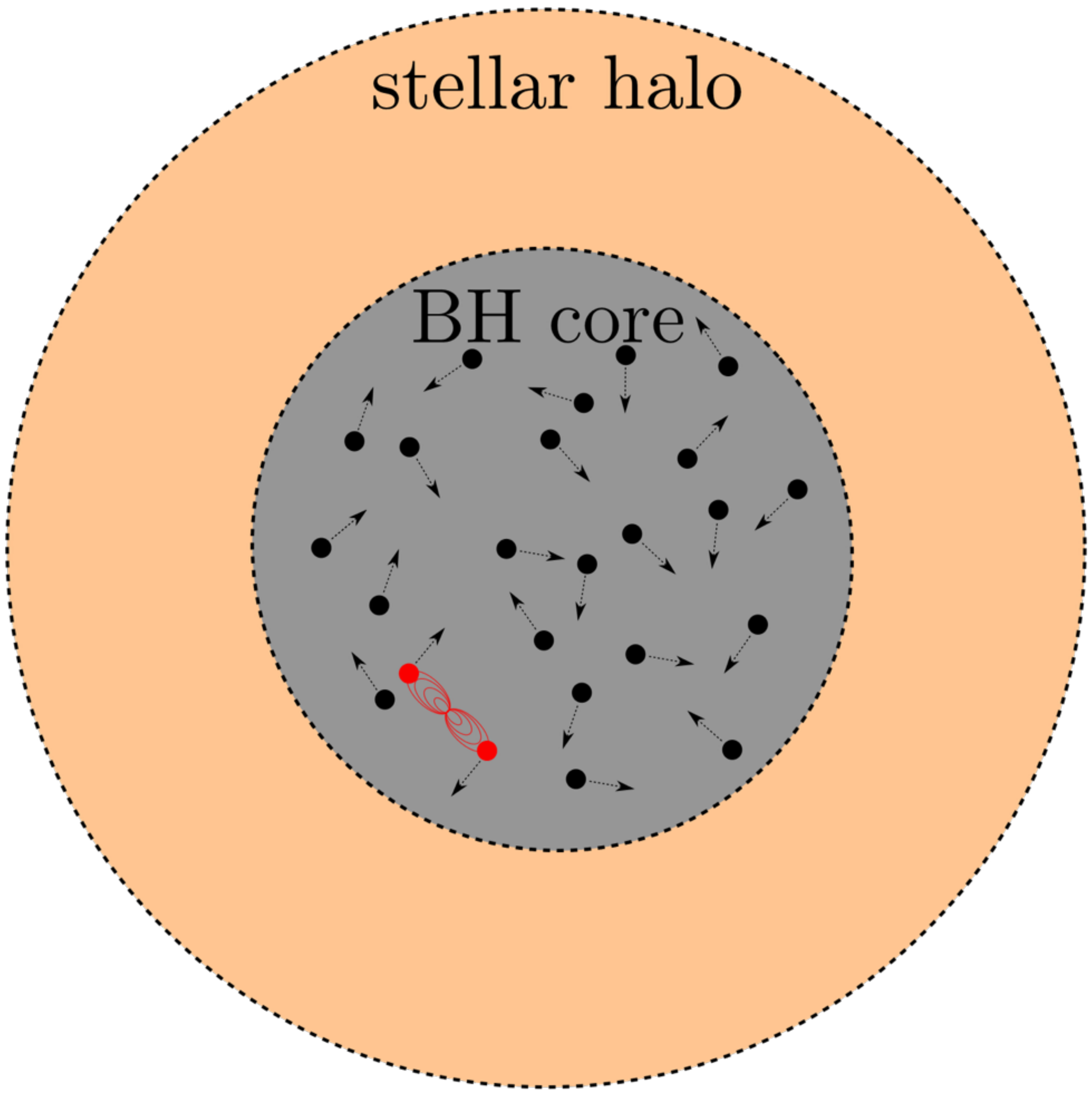


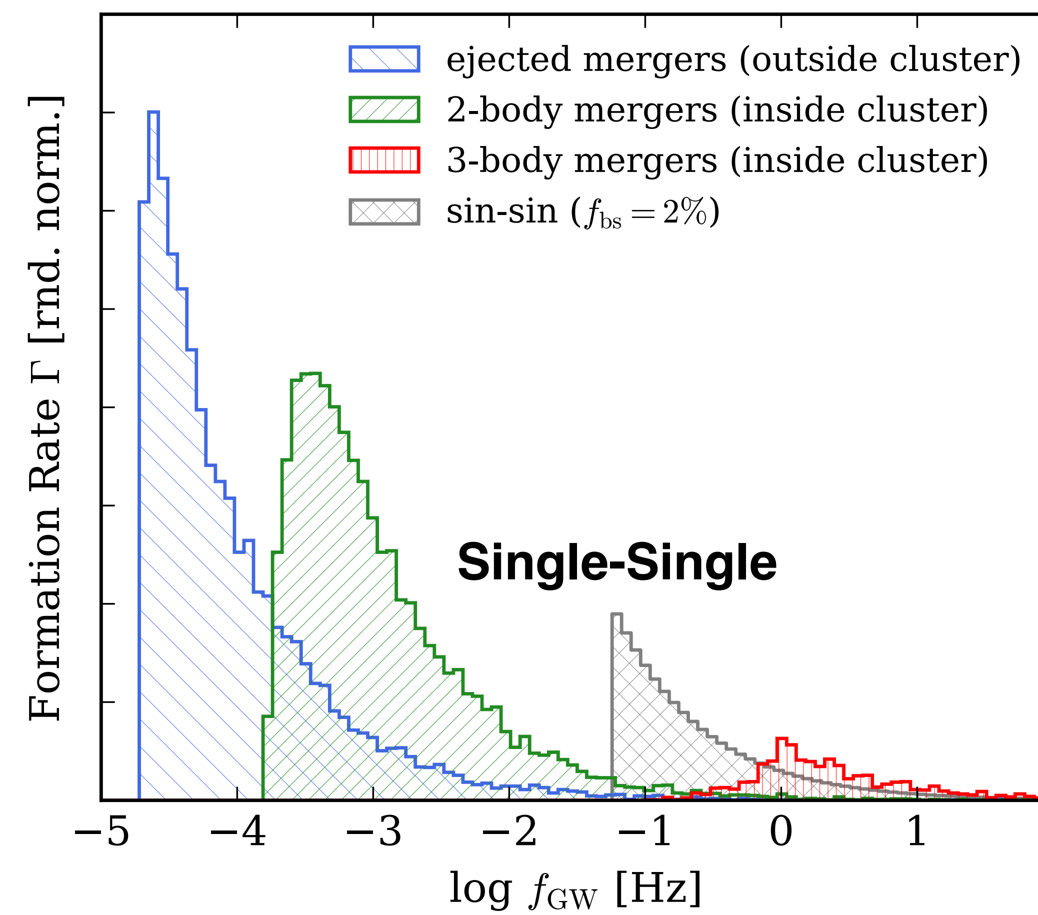
$$\Delta E_{\rm p} \approx (85\pi/12) G^{7/2} c^{-5} m^{9/2} r_{\rm p}^{-7/2}$$

$$E_{\rm ss} \approx \mu v^2/2$$

$$\mathcal{R}_{\rm ss} = \left(\frac{85\pi}{24\sqrt{2}}\right)^{2/7} \times \mathscr{R}_{\rm m} \left(\frac{c^2}{v^2}\right)^{2/7}$$

Merger Type: Single-Single





$$\mathcal{R}_{ss} = \left(\frac{85\pi}{24\sqrt{2}}\right)^{2/7} \times \mathscr{R}_m \left(\frac{c^2}{v^2}\right)^{2/7} \qquad f \approx \frac{1}{\pi}\sqrt{\frac{2Gm}{r_f^3}}$$

S-S captures do not only operate in Galactic Nuclei!

Distribution depends on:

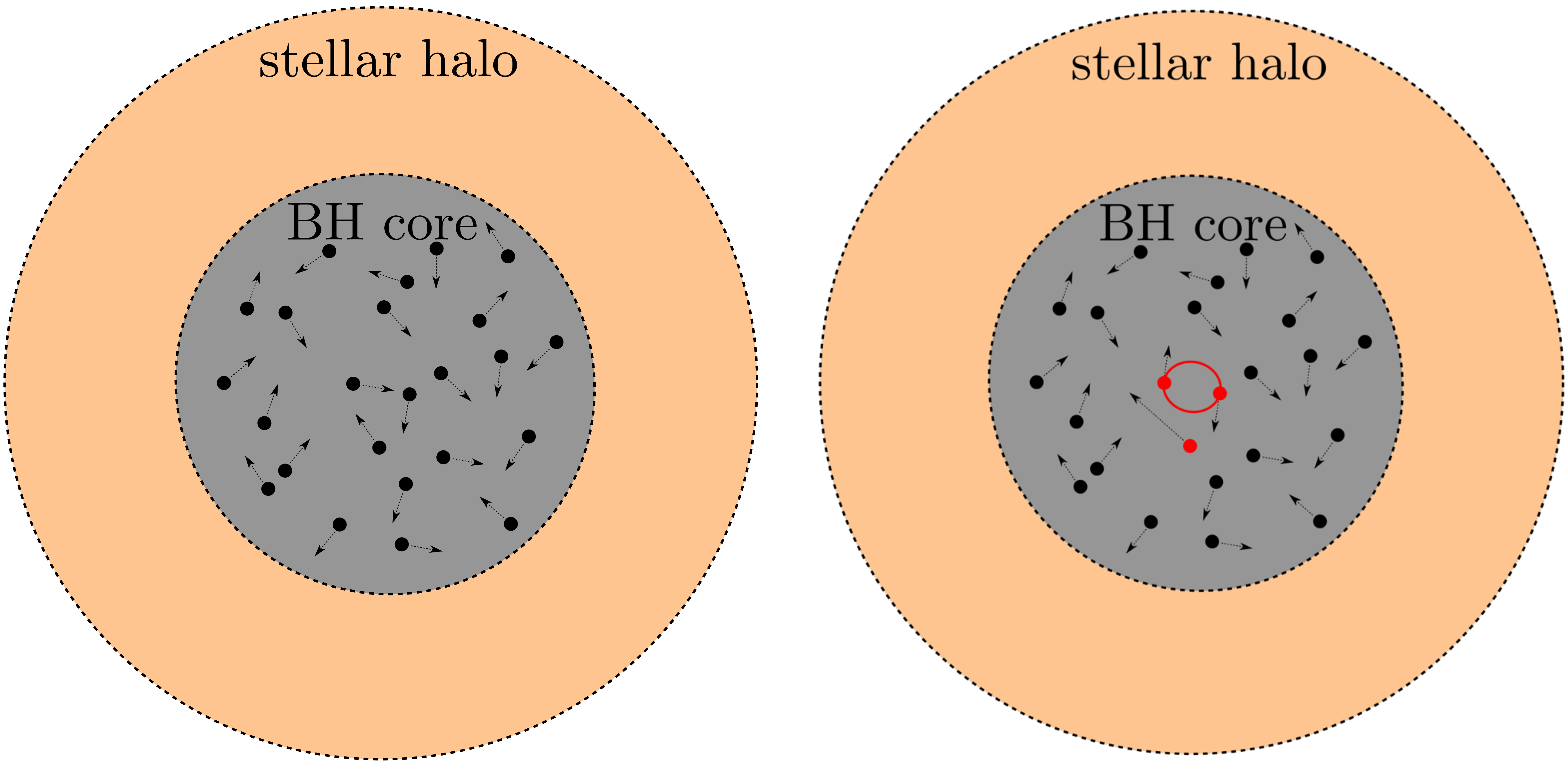
BH distribution (Plummer/Uniform..)

Binary Fraction

see our new paper:

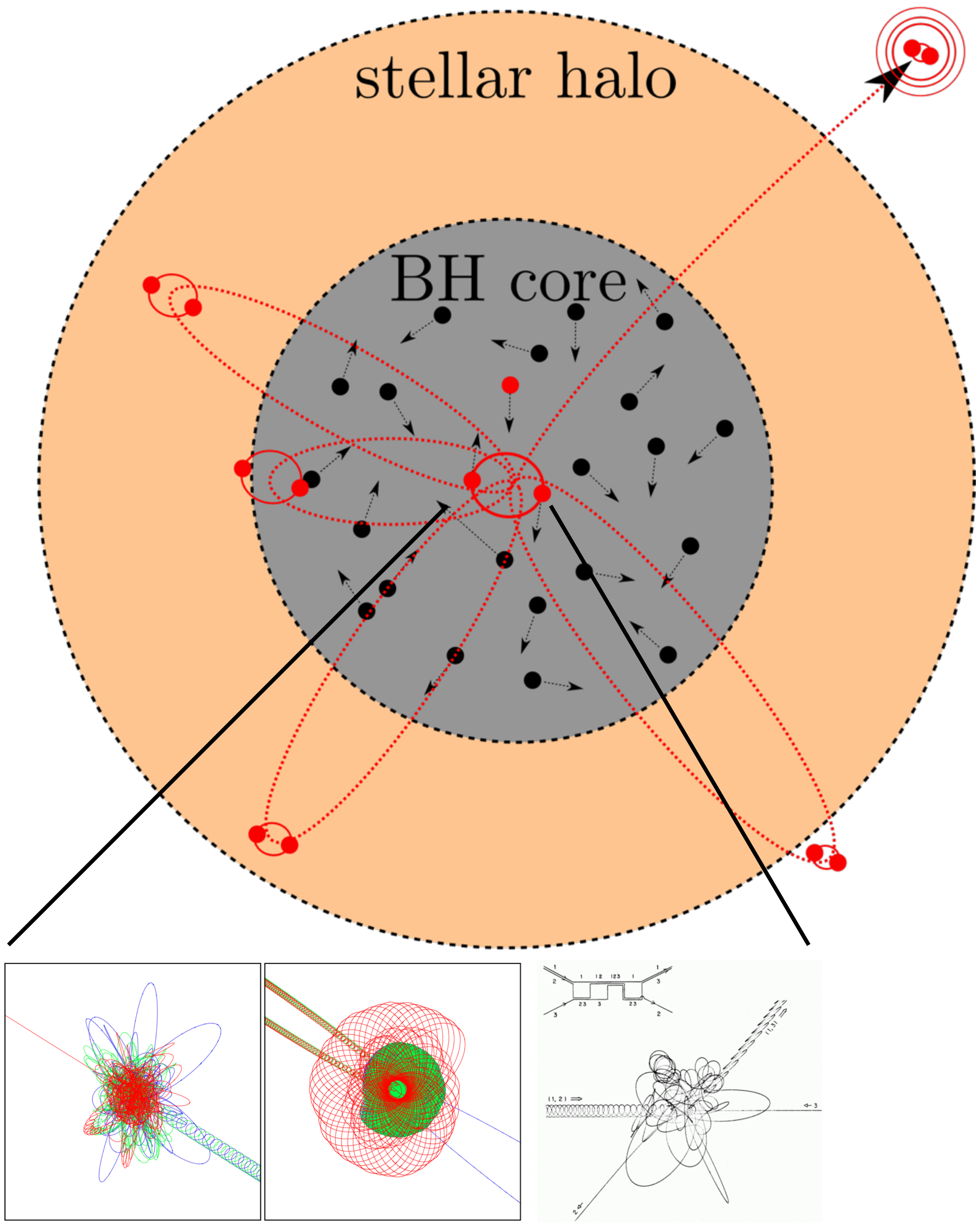
'GW captures of single BHs in GCs' (Samsing, et al. 2019)

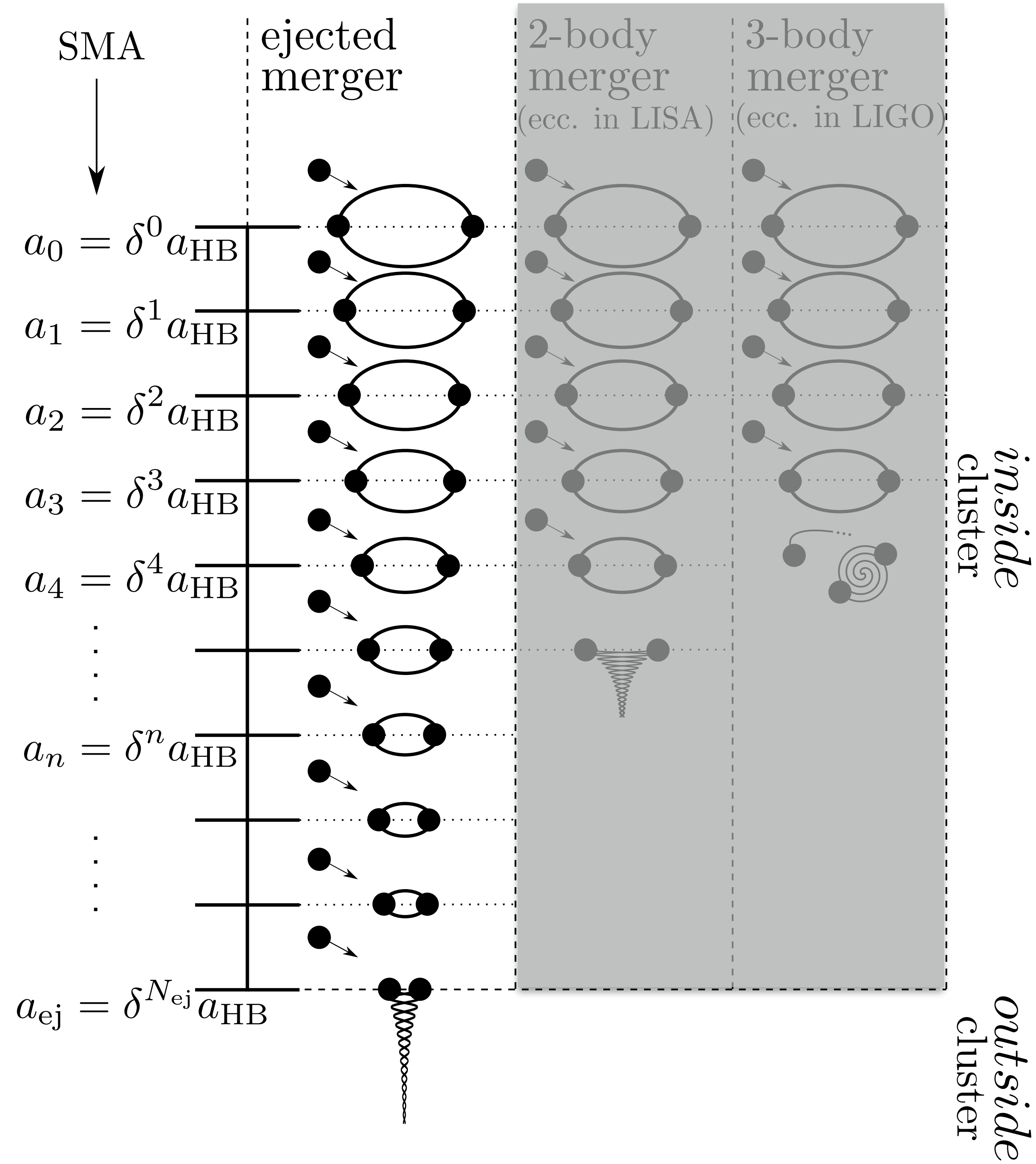
Few-body BBH mergers: Formation of a BBH



$$a_{\rm HB} = \frac{3}{2}\frac{Gm_{\rm BH}}{v_{\rm dis}^2}$$

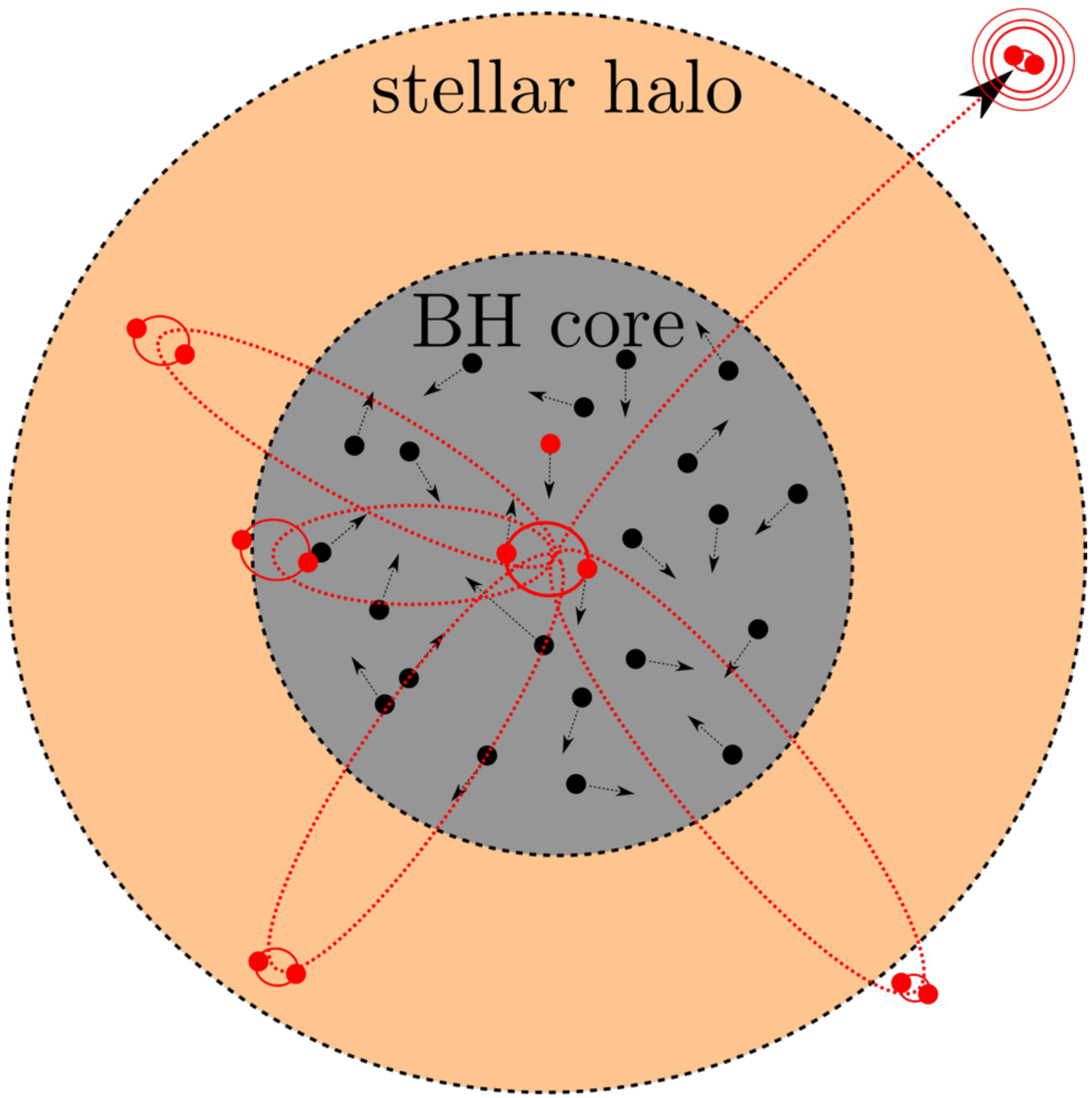
Merger Type: Ejected Merger





Samsing, D'Orazio. 18.

Merger Type: Ejected Merger



Classical way of forming BBH mergers
Suggested by: P. Zwart, S. McMillan

$$\Delta E_{\rm bs} = (1/\delta - 1) \times E_{\rm B}(a)$$

$$a_{\rm ej} \approx \frac{1}{6}\left(\frac{1}{\delta} - 1\right)\frac{Gm}{v_{\rm esc}^2}$$

$$a_{\rm HB} = \frac{3}{2}\frac{Gm_{\rm BH}}{v_{\rm dis}^2}$$

$$a(k) = a_{\rm HB}\delta^k$$

$$da = -a(1-\delta)dk$$

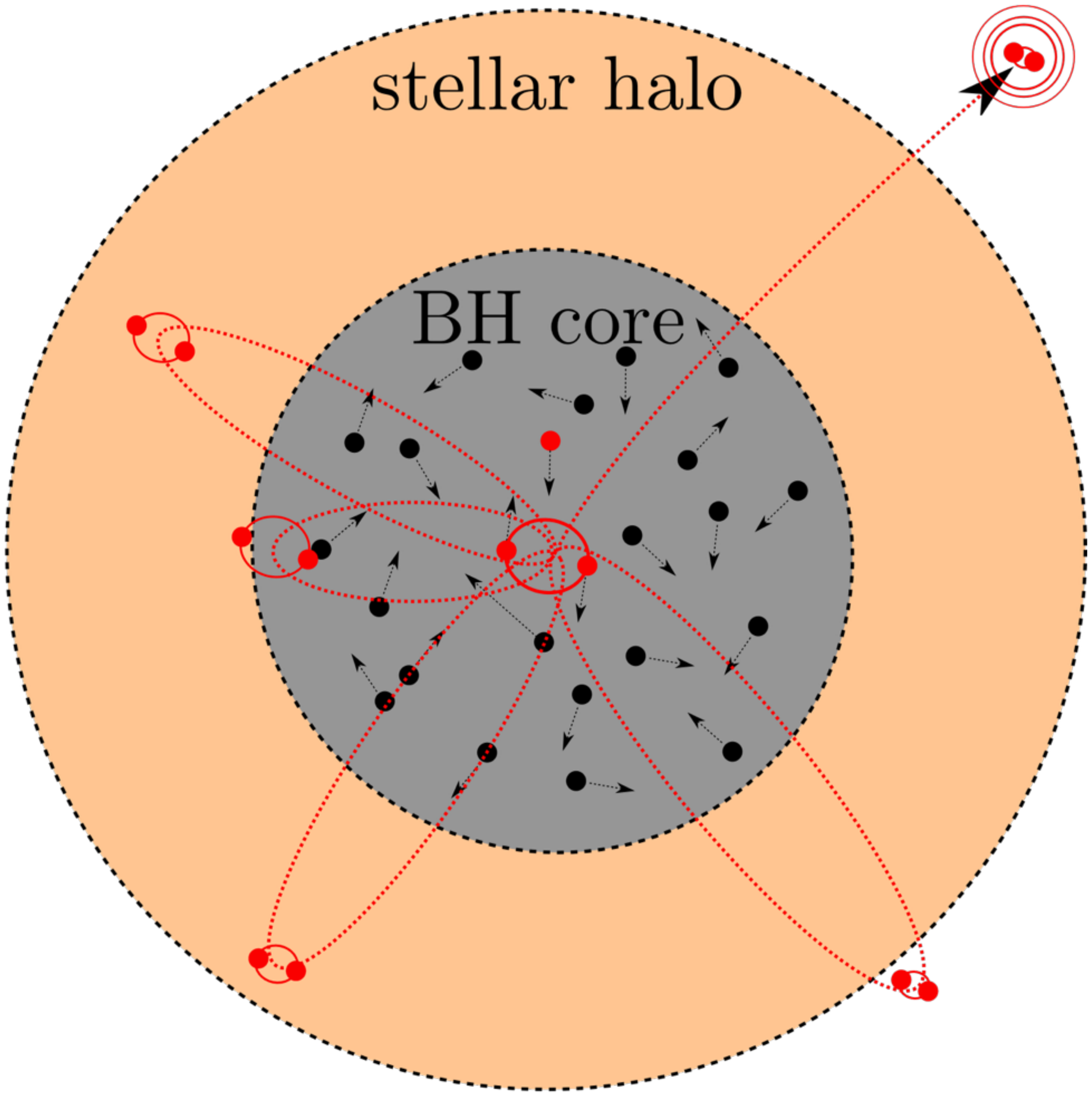
$$T_{\rm ej} = \int_{a_{\rm ej}}^{a_{\rm HB}} \frac{1}{n_0\sigma_{\rm bs}v_0}\frac{da}{a(1-\delta)}, \approx \frac{(6\pi G)^{-1}}{(1-\delta)}\frac{v_0}{n_0}\frac{m^{-1}}{a_{\rm ej}},$$

Newtonian outcome.

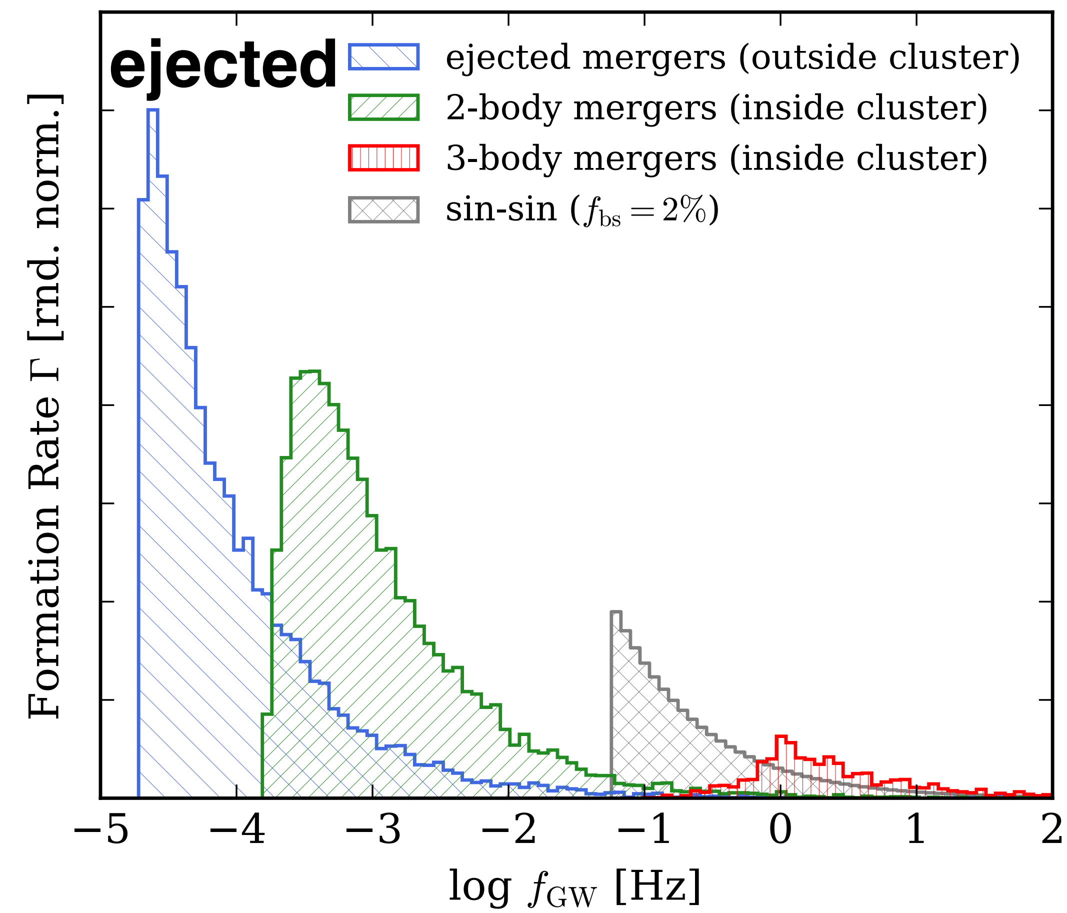
Leads to interesting rate,
but standard circular mergers.

$$N_{\rm bs}(a_{\rm in}, a_{\rm ej}) = \int_{a_{\rm ej}}^{a_{\rm in}} \frac{1}{1-\delta}\frac{1}{a}da = \frac{1}{1-\delta}\ln\left(\frac{a_{\rm in}}{a_{\rm ej}}\right)$$

Merger Type: Ejected Merger



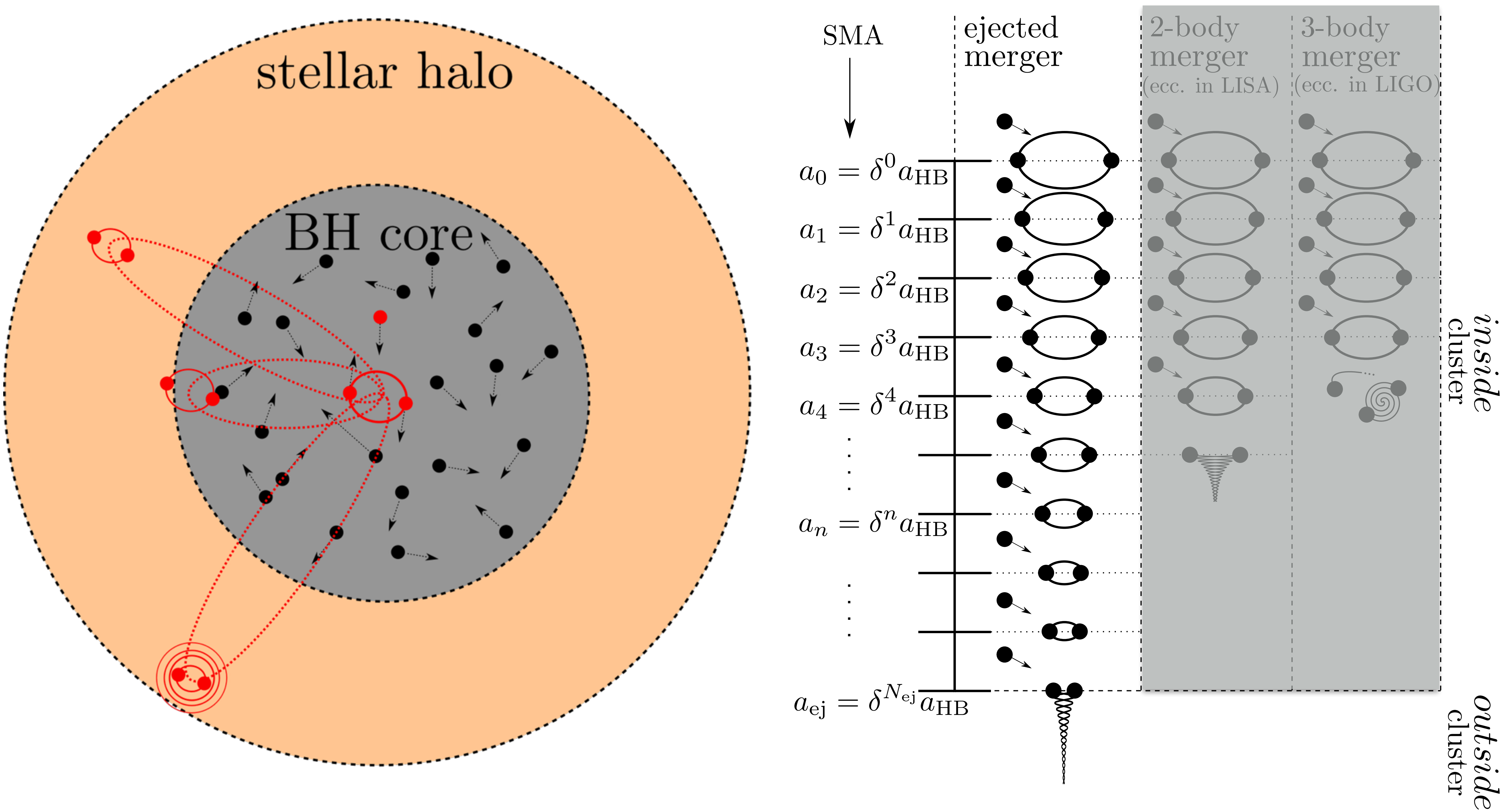
Circular GW sources



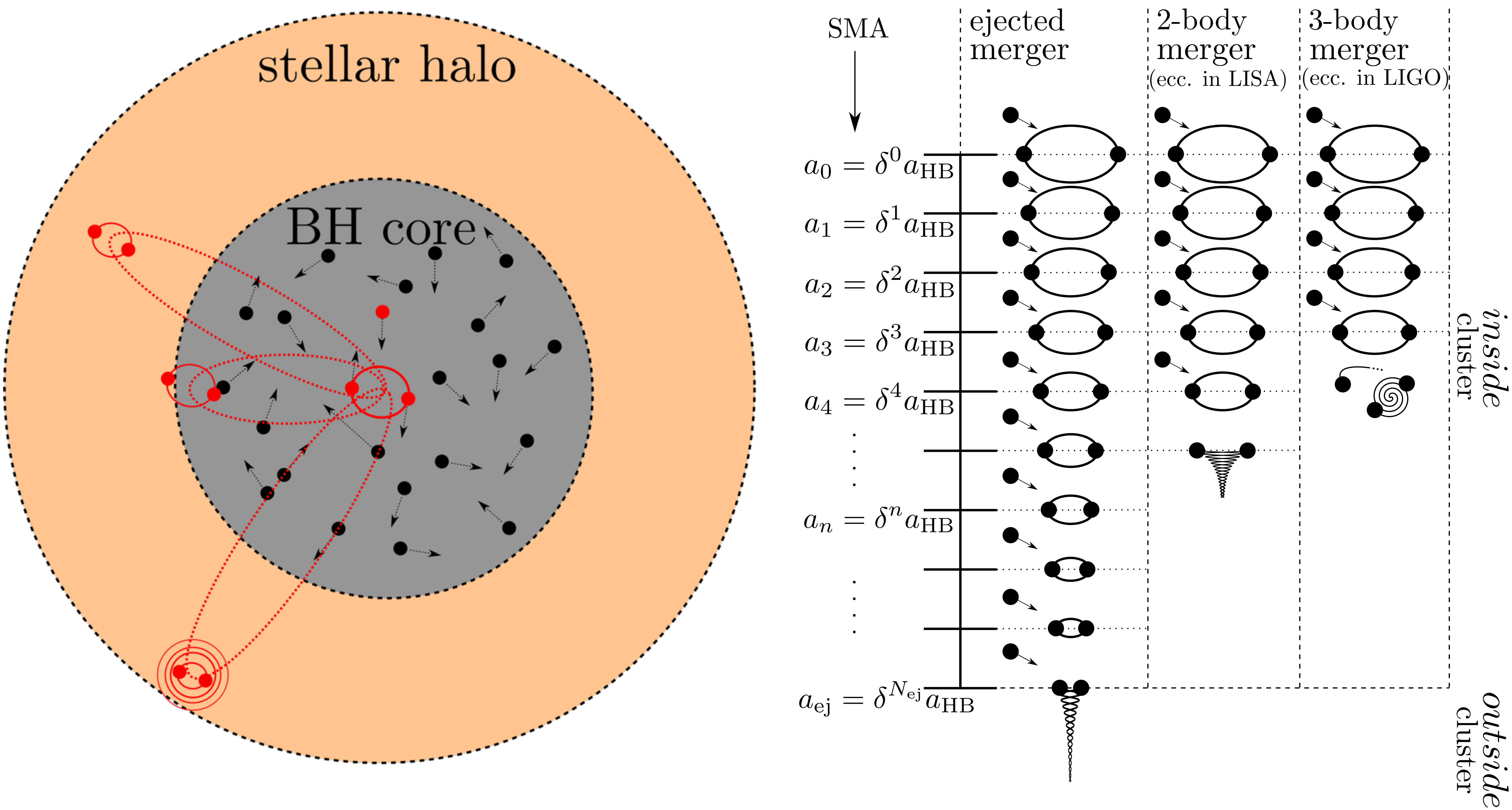
Important point: It is all about characteristic time scales!

$$f_{r,0}^{\rm peak}(\mathcal{T}) \approx 2{\cdot}10^{-5}\,{\rm Hz}\left(\frac{\mathcal{T}}{10^{10}{\rm yrs}}\right)^{-3/7}\left(\frac{a}{0.5{\rm au}}\right)^{3/14}\left(\frac{m}{30M_{\odot}}\right)^{-11/14}$$

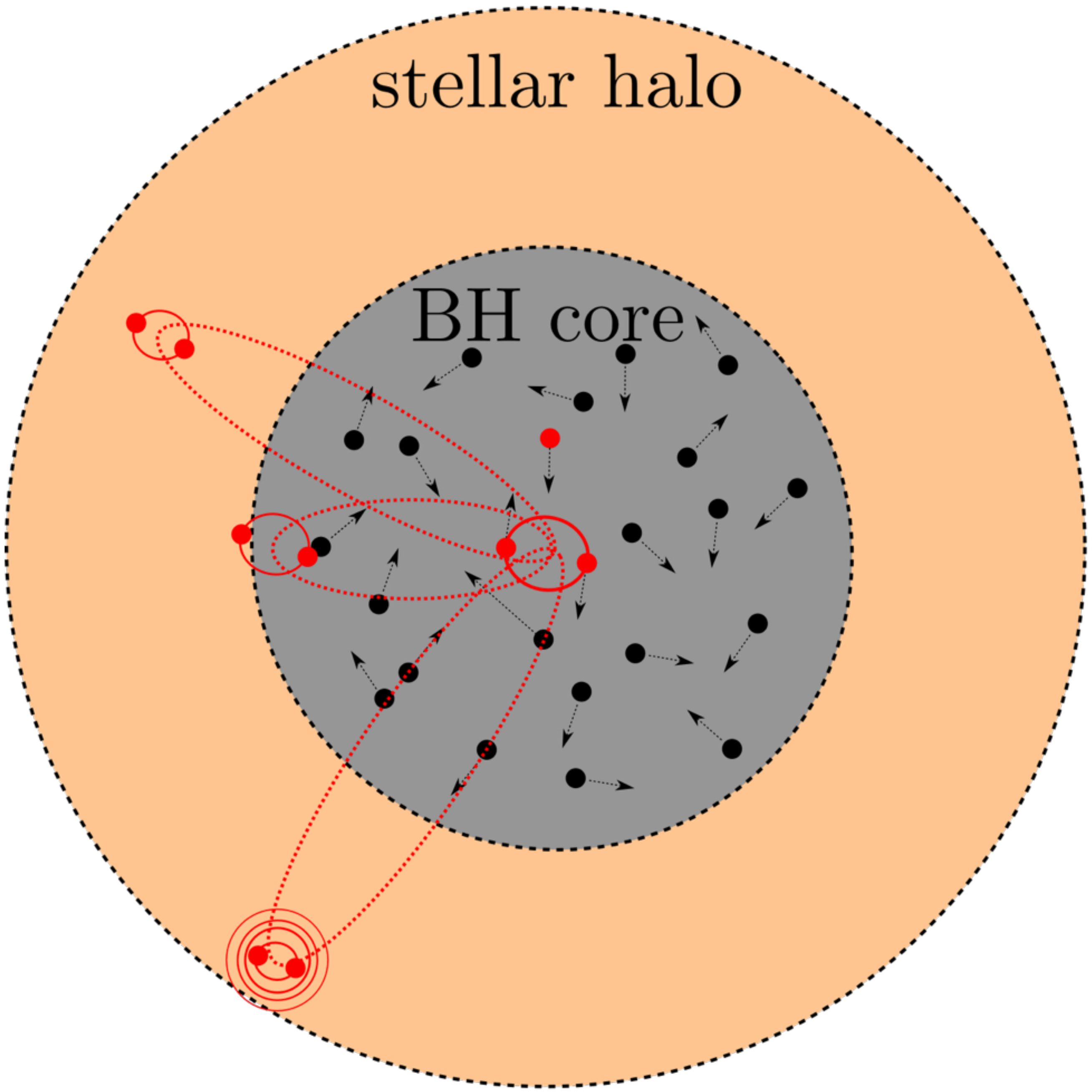
Merger Type: 2-body Merger



Merger Type: 2-body Merger



Merger Type: 2-body Merger



Time scales:

$$t_{\rm life}(a,e) \approx t_{\rm life}(a)(1-e^2)^{7/2}$$

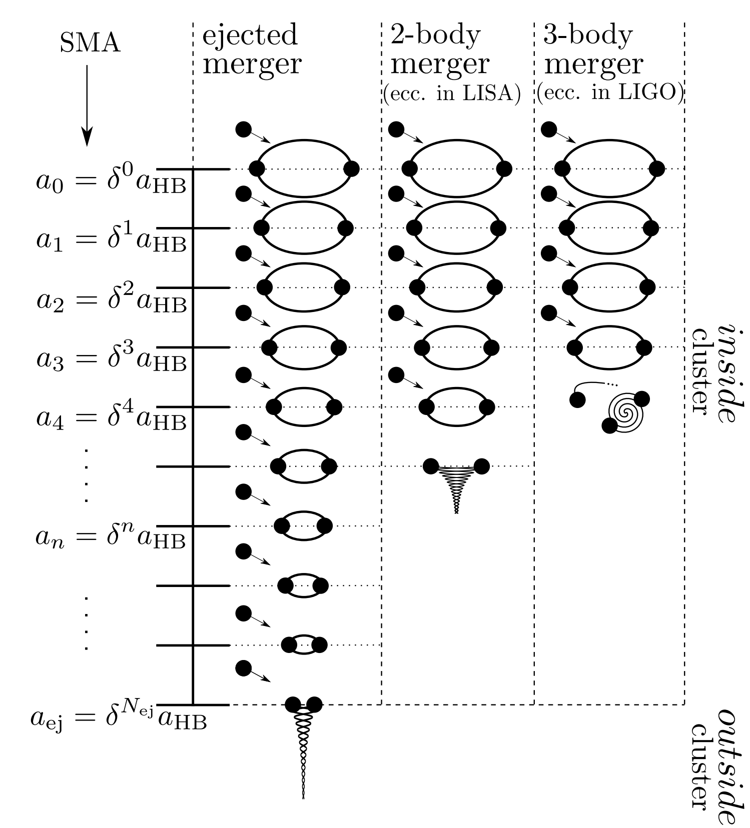
$$t_{\rm bs}(a) \approx 1/\Gamma_{\rm bs} \approx (n_{\rm s}\sigma_{\rm bs}v_{\rm disp})^{-1}$$

Eccentricity:

$$P(e) = 2e$$

Probability:

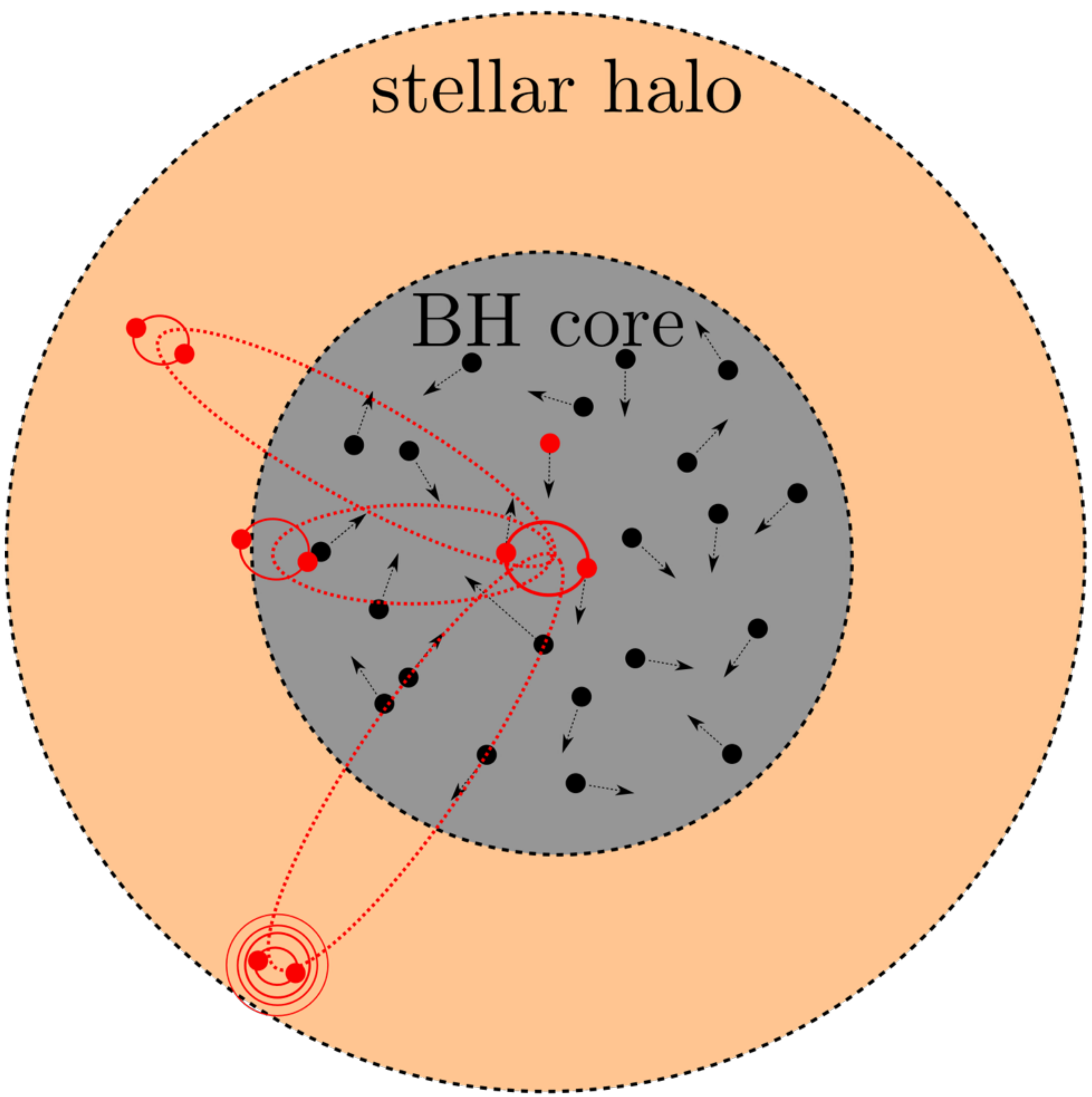
$$P_{\rm IM}(a) \approx (t_{\rm bs}(a)/t_{\rm life}(a))^{2/7}$$

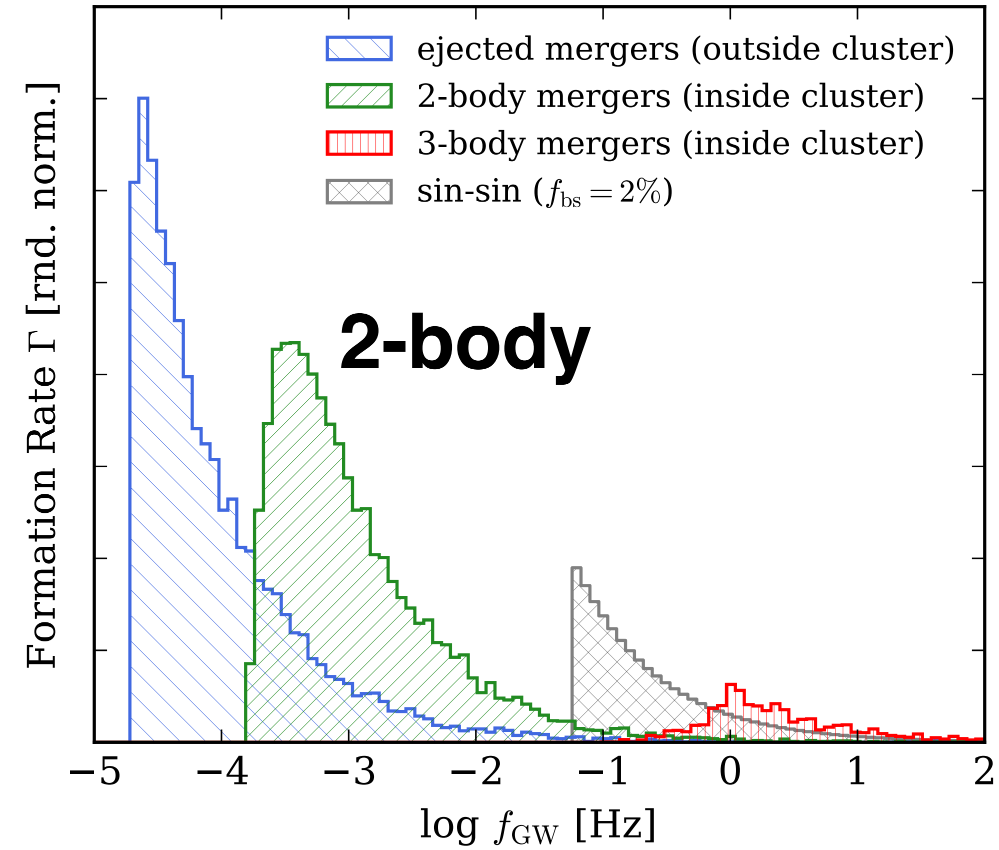


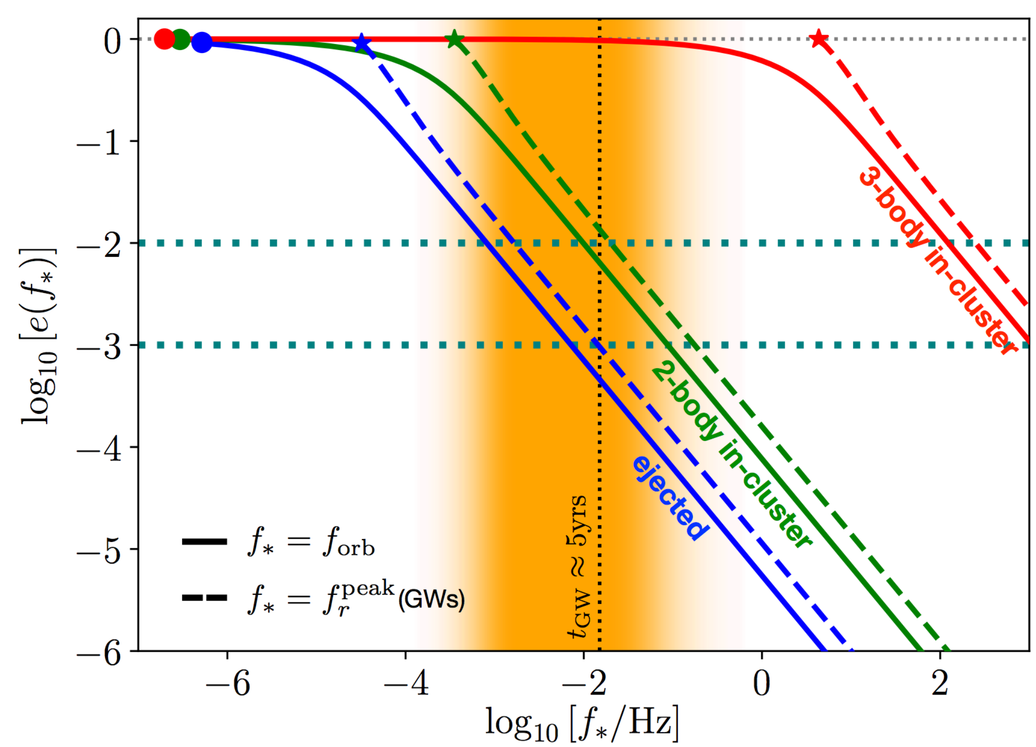
$$P_{\rm IM}(a_{\rm in}, a_{\rm ej}) \approx \frac{1}{1-\delta}\int_{a_{\rm ej}}^{a_{\rm in}} \frac{P_{\rm IM}(a)}{a}da \approx \frac{7}{10}\frac{P_{\rm IM}(a_{\rm ej})}{1-\delta}$$

Merger Type: 2-body Merger

Eccentric LISA sources





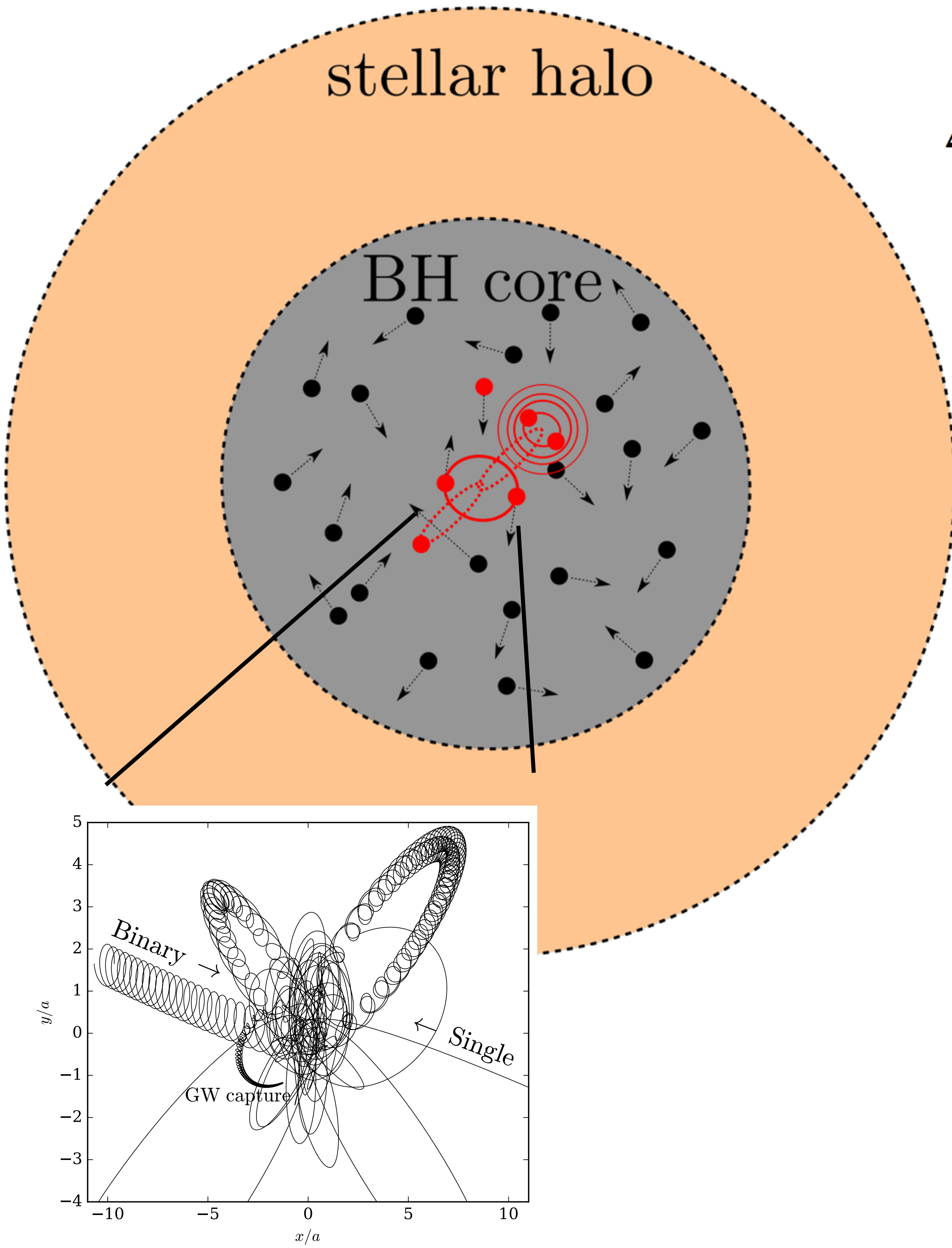


Consider again the characteristic time-scale argument:

$$f_{r,0}^{\rm peak}(\mathcal{T}) \approx 2 \cdot 10^{-5}\,{\rm Hz} \left(\frac{\mathcal{T}}{10^{10}{\rm yrs}}\right)^{-3/7} \left(\frac{a}{0.5{\rm au}}\right)^{3/14} \left(\frac{m}{30 M_\odot}\right)^{-11/14}$$

series of papers: Samsing/D'Orazio

Merger Type: 3-body Merger

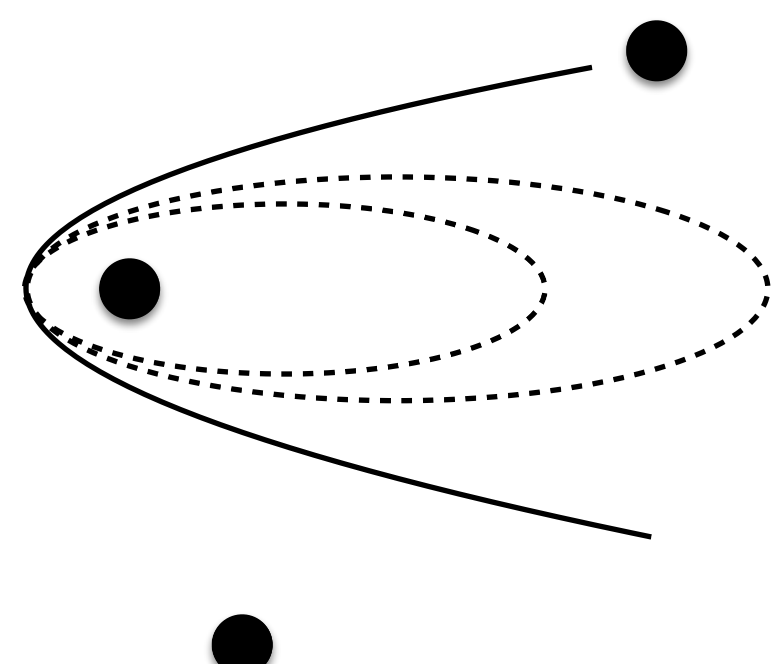


3-body GW capture:

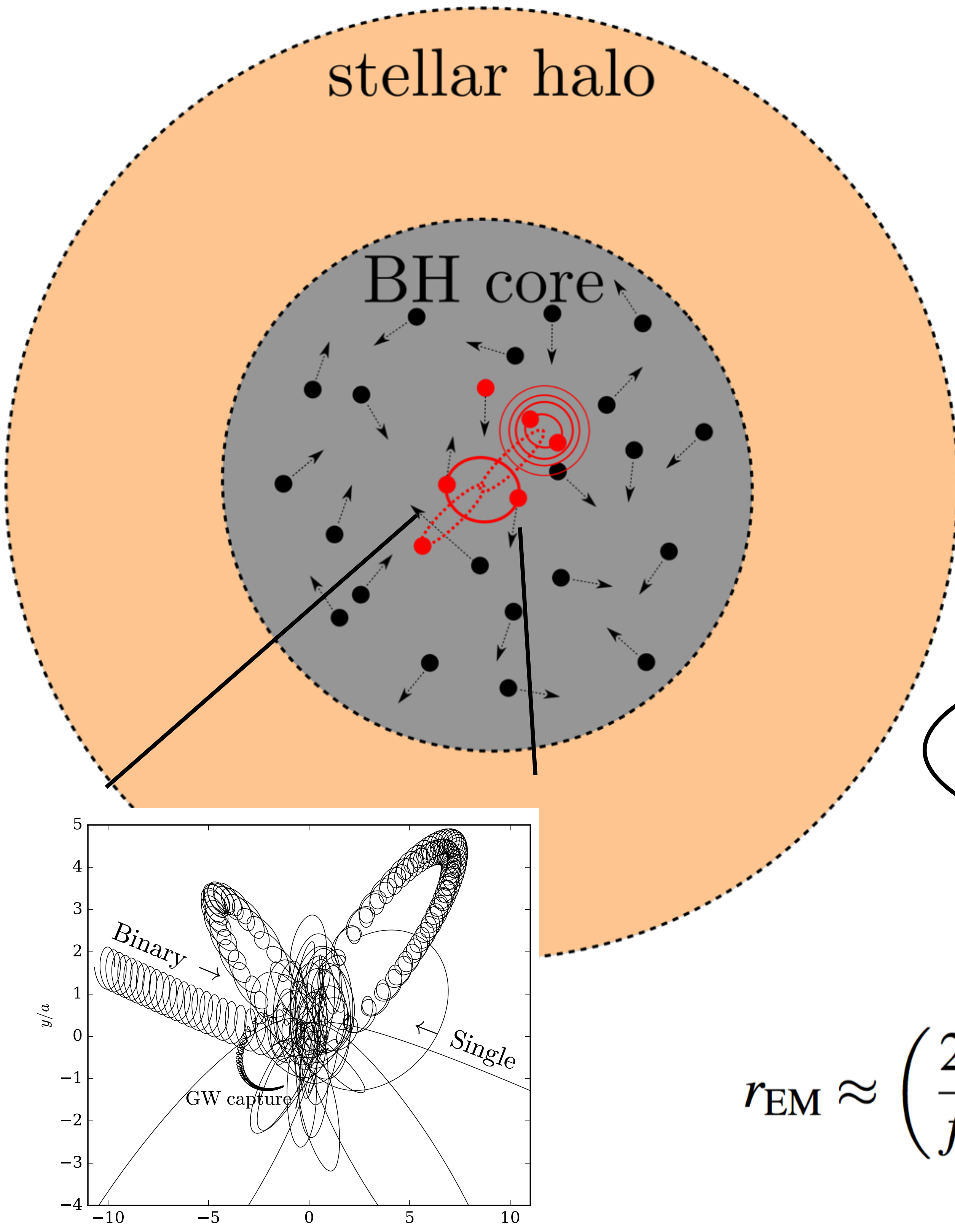
$$E_{\mathrm{B}} \approx Gm^2/(2a)$$

$$\Delta E_{\mathrm{p}} \approx (85\pi/12)G^{7/2}c^{-5}m^{9/2}r_{\mathrm{p}}^{-7/2}$$

$$\mathcal{R}_{\mathrm{bs}} \approx \left(\frac{85\pi}{24\sqrt{2}}\right)^{2/7} \times \mathscr{R}_{\mathrm{m}} \left(\frac{a}{\mathscr{R}_{\mathrm{m}}}\right)^{2/7}$$



Merger Type: 3-body Merger

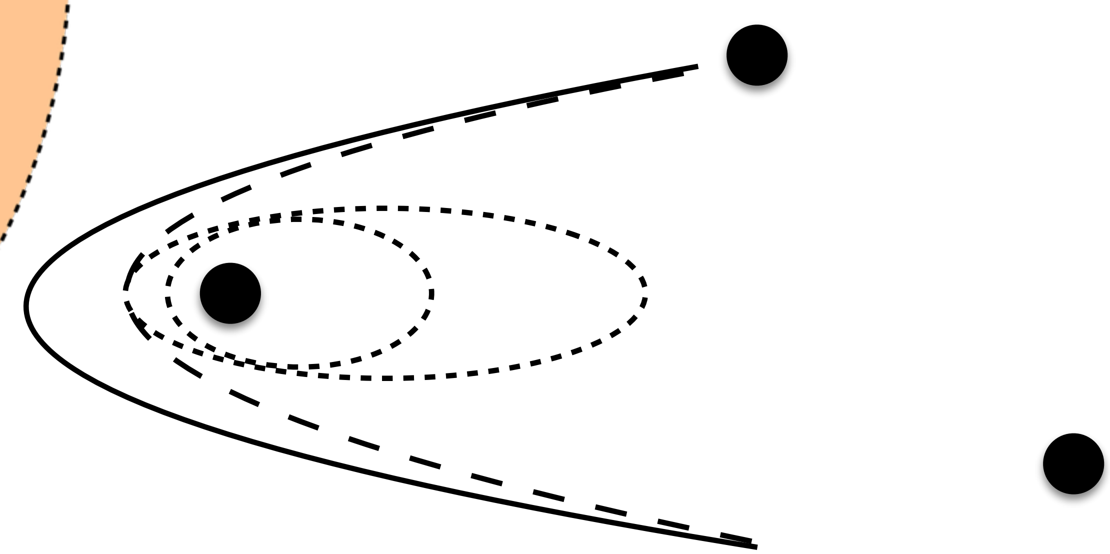


3-body Ecc. encounter:

$$r_f \approx \left(\frac{2Gm}{f^2\pi^2}\right)^{1/3}$$

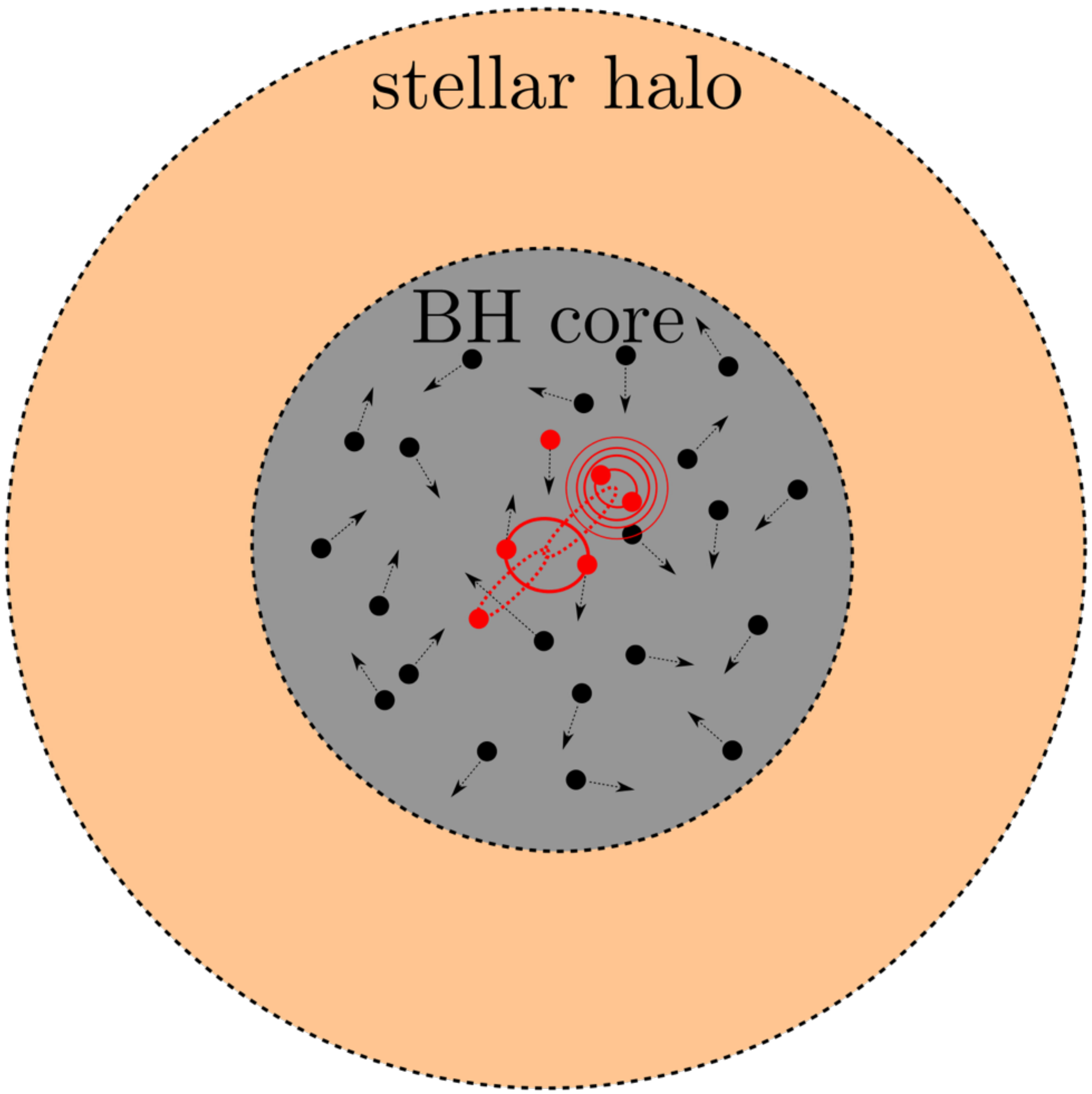
$$r_{\rm EM} \approx r_f \times \frac{1}{2F(e_f)} \left(\frac{425}{304}\right)^{870/2299}$$

$$F(e) = \frac{e^{12/19}}{1+e}\left(1+\frac{121}{304}e^2\right)^{870/2299}$$

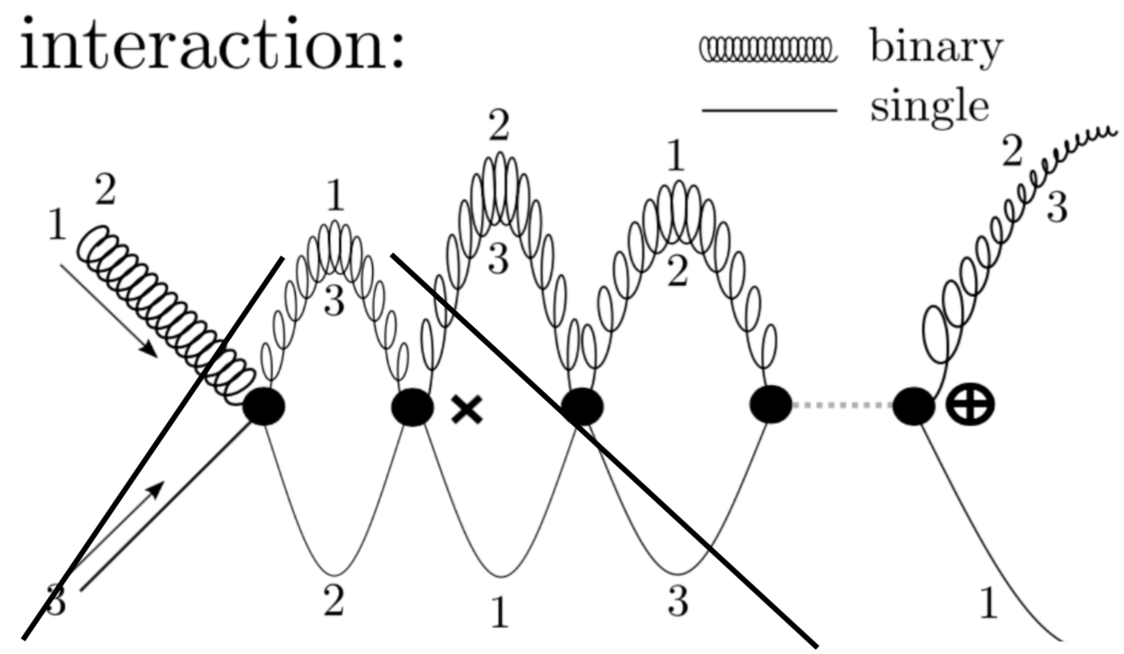


$$r_{\rm EM} \approx \left(\frac{2Gm}{f^2\pi^2}\right)^{1/3} \frac{1}{2}\frac{1+e_f}{e_f^{12/19}}\left[\frac{425}{304}\left(1+\frac{121}{304}e_f^2\right)^{-1}\right]^{870/2299}$$

Merger Type: 3-body Merger



3-body Ecc. Probability:



$$r_{\mathrm{p}} = a(1-e) \quad P(e) = 2e$$

$$P_{\mathrm{EM}}(a) \approx \frac{2r_{\mathrm{EM}}}{a} \times N_{\mathrm{IMS}}$$

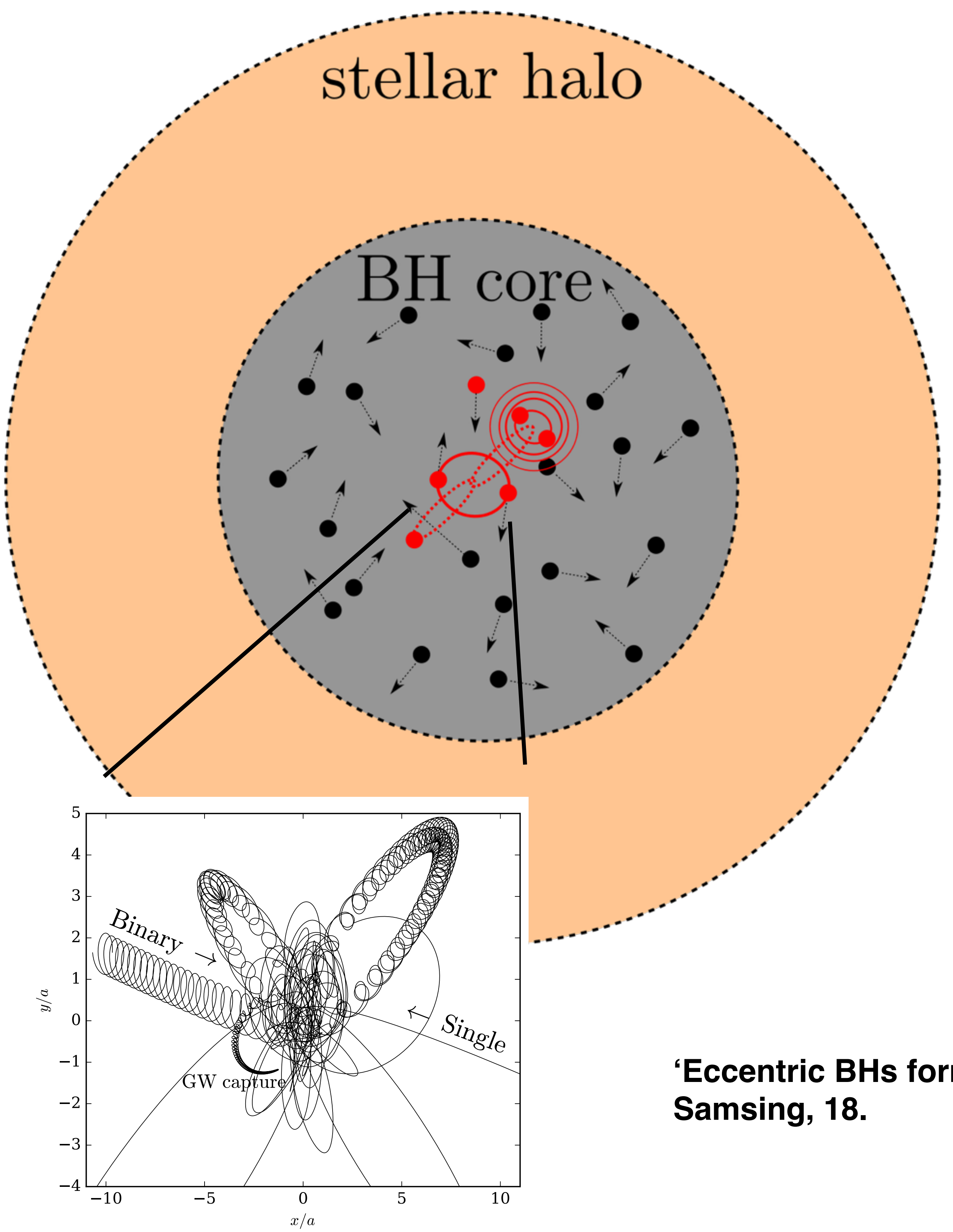
$$r_{\mathrm{EM}} \approx \left(\frac{2Gm}{f^2\pi^2}\right)^{1/3} \frac{1}{2}\frac{1+e_f}{e_f^{12/19}}\left[\frac{425}{304}\left(1+\frac{121}{304}e_f^2\right)^{-1}\right]^{870/2299}$$

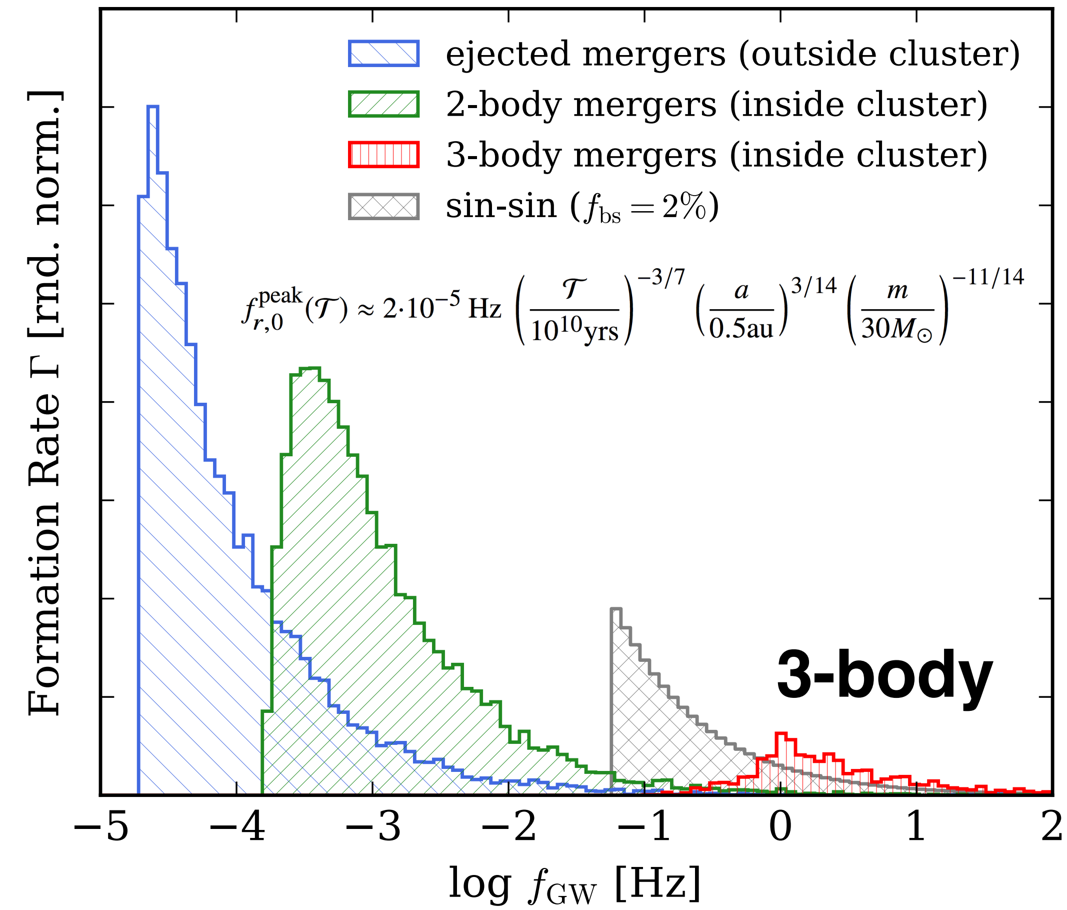
$$P_{\mathrm{EM}}(a_{\mathrm{in}}, a_{\mathrm{ej}}) = \frac{1}{1-\delta}\int_{a_{\mathrm{ej}}}^{a_{\mathrm{in}}} \frac{P_{\mathrm{EM}}(a)}{a} da \approx \frac{P_{\mathrm{EM}}(a_{\mathrm{ej}})}{1-\delta}$$

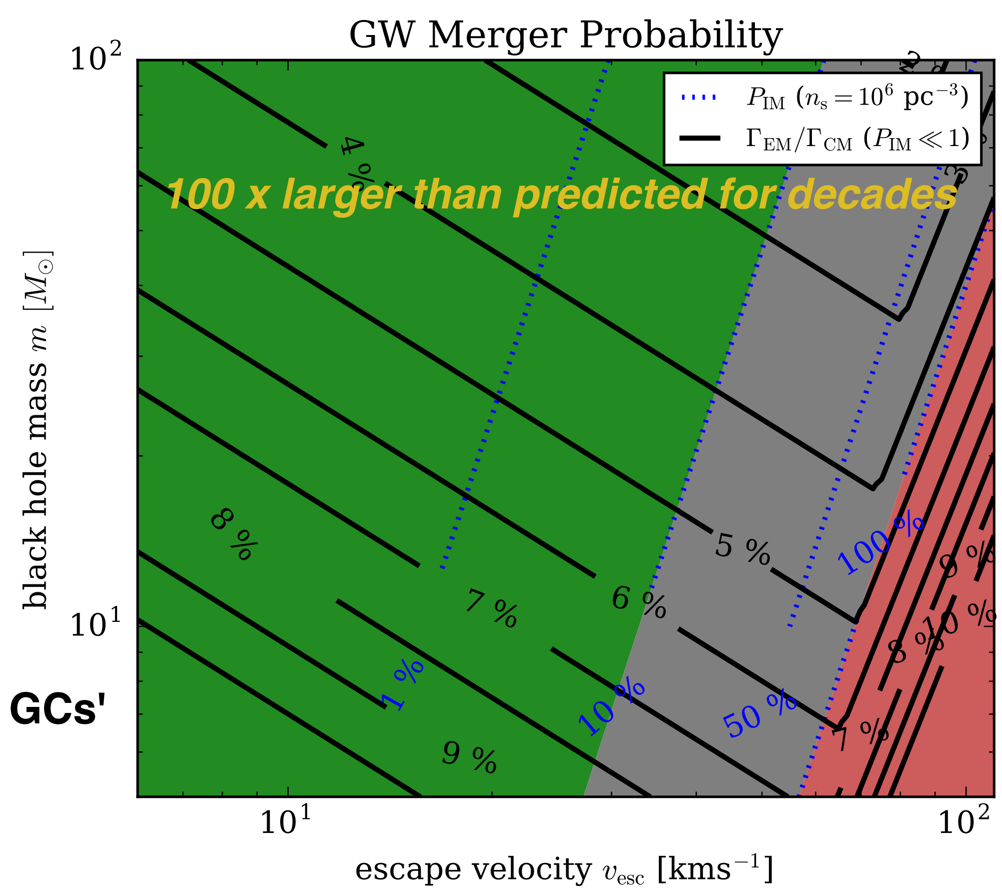
'Eccentric BHs forming in GCs'
Samsing, 18.

Merger Type: 3-body Merger

Eccentric LIGO sources

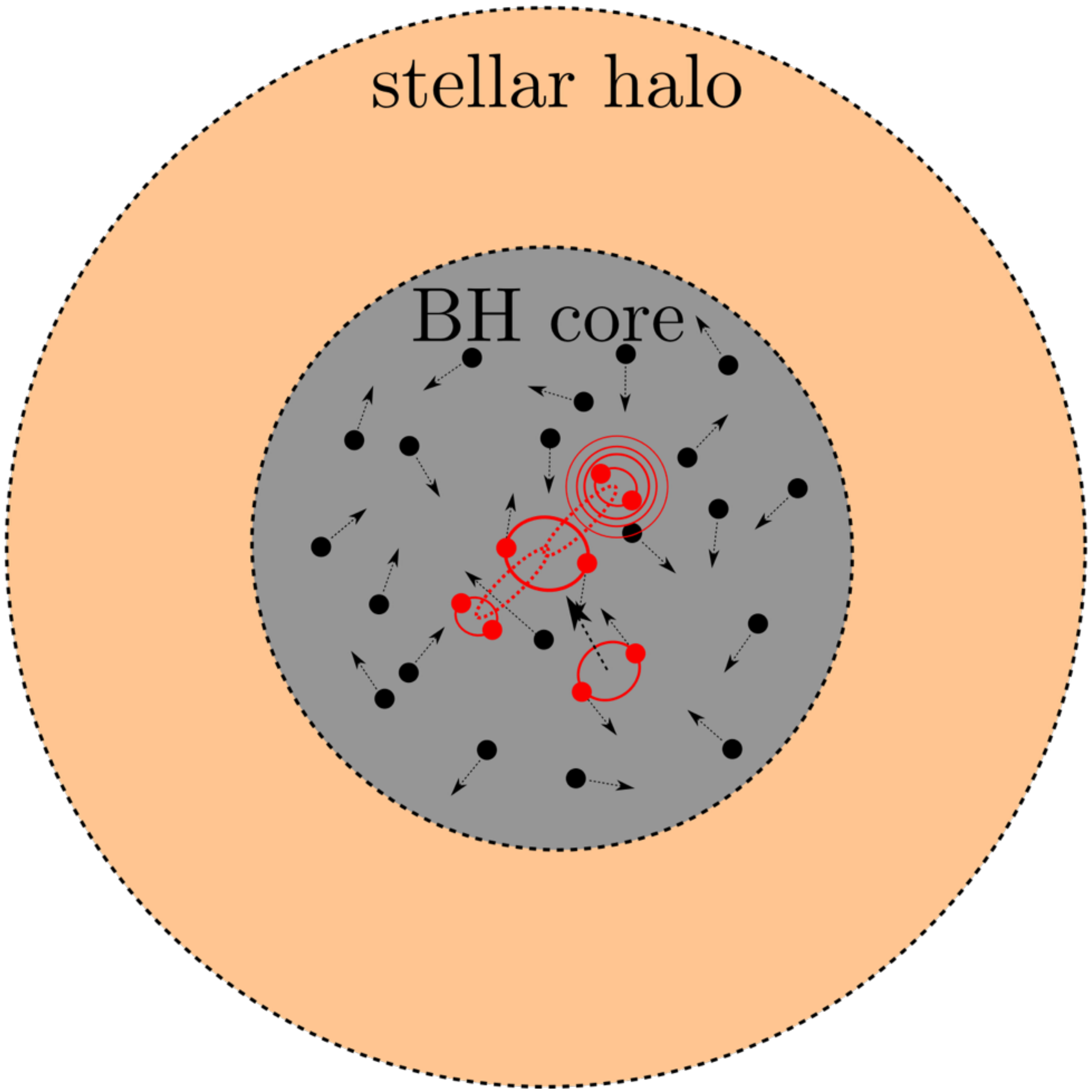


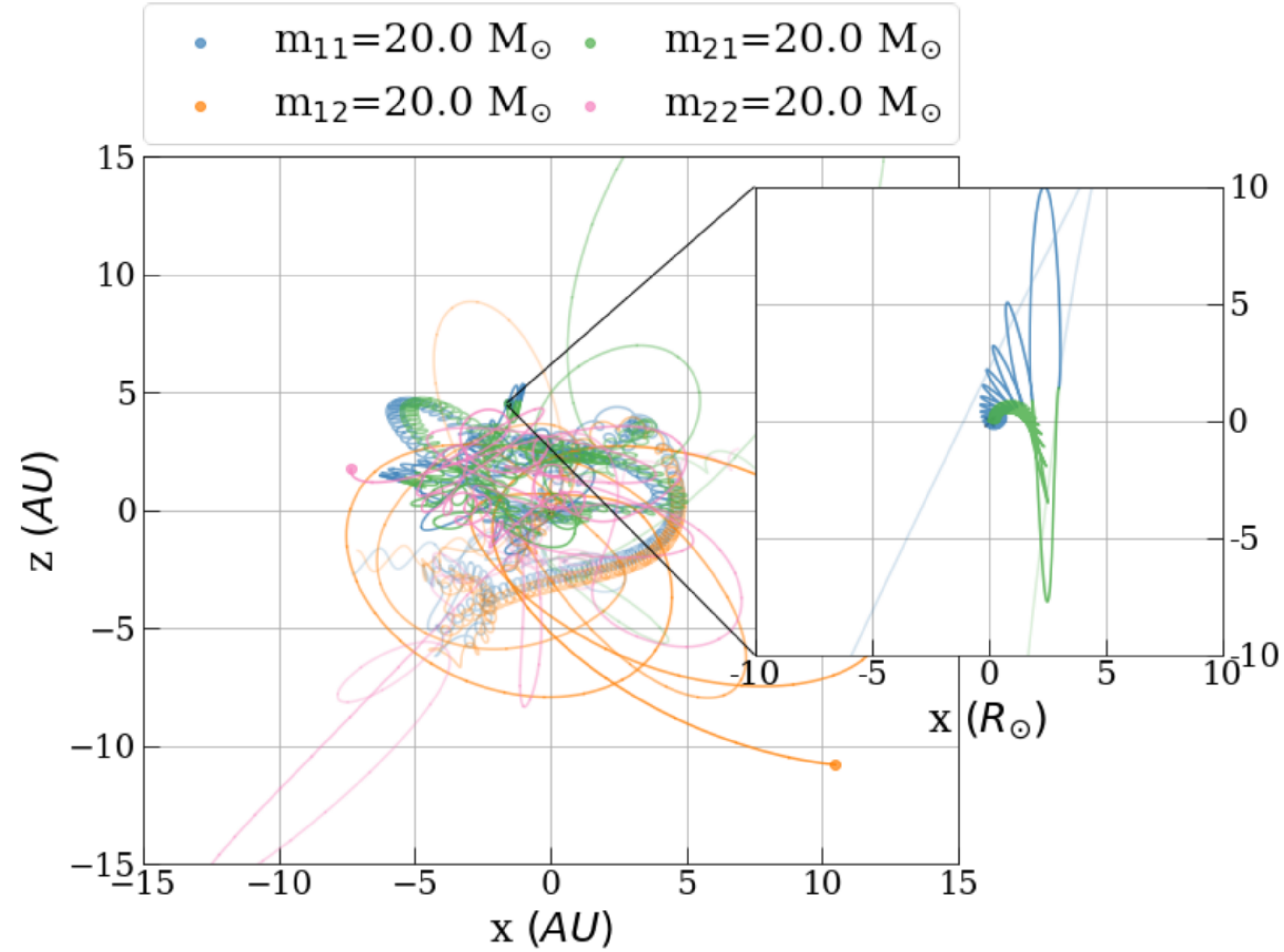




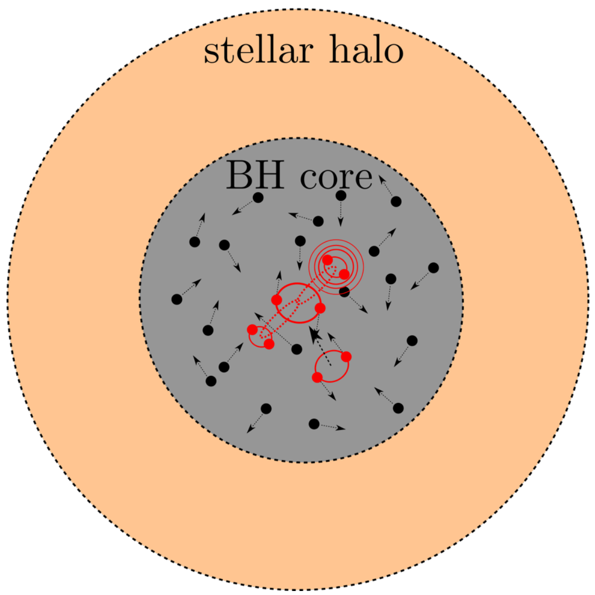
'Eccentric BHs forming in GCs'
Samsing, 18.

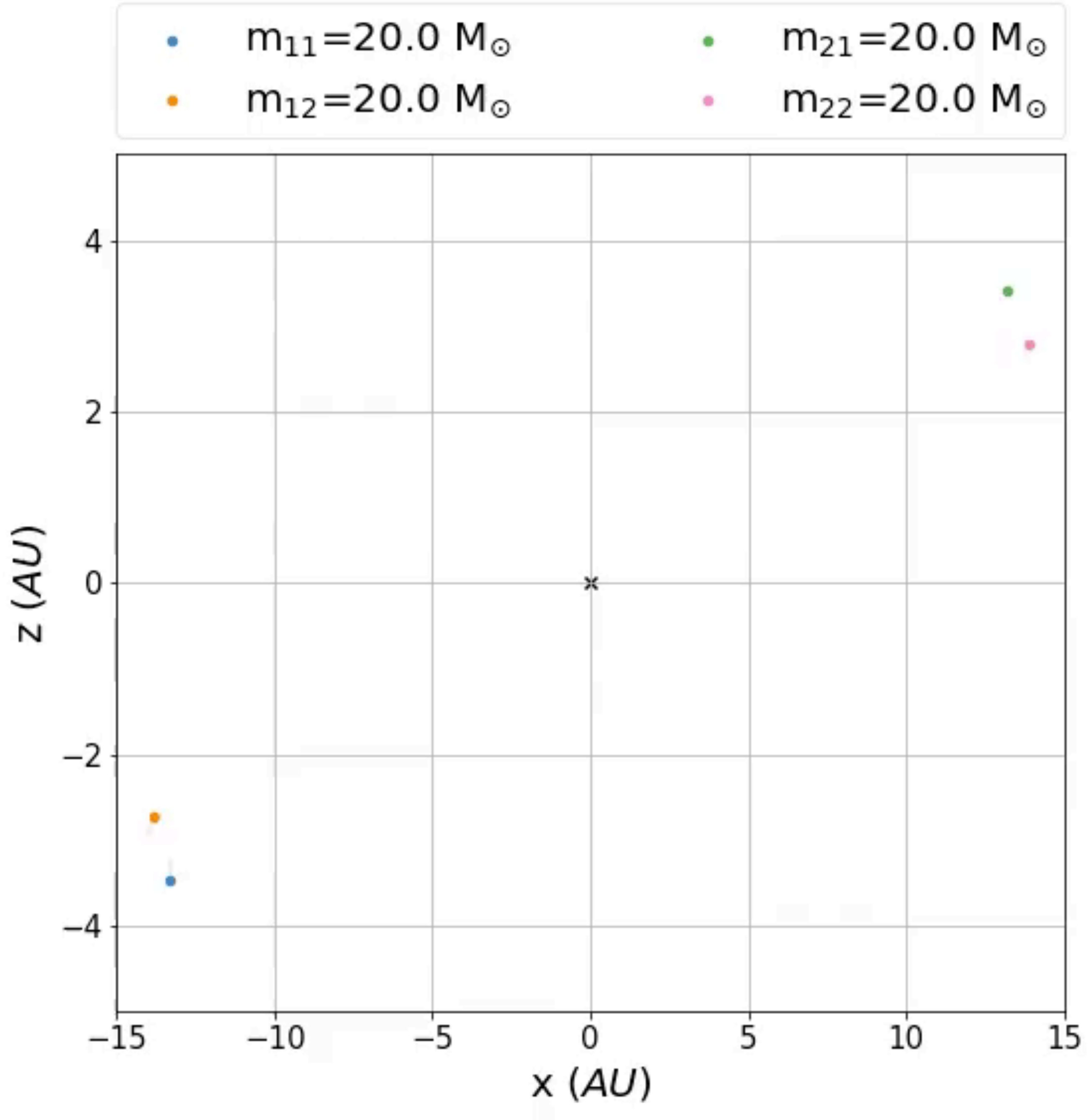
Merger Type: 4-body Merger

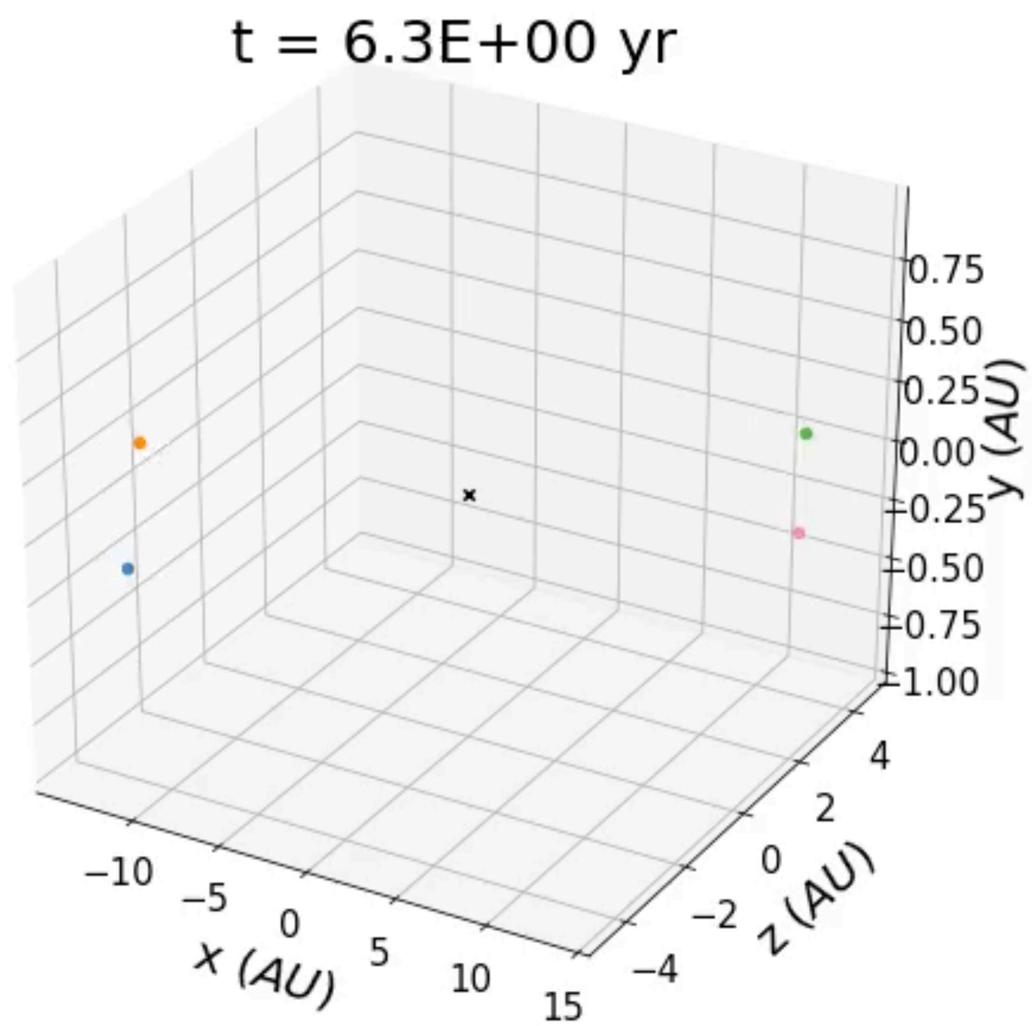




Merger Type: 4-body Merger

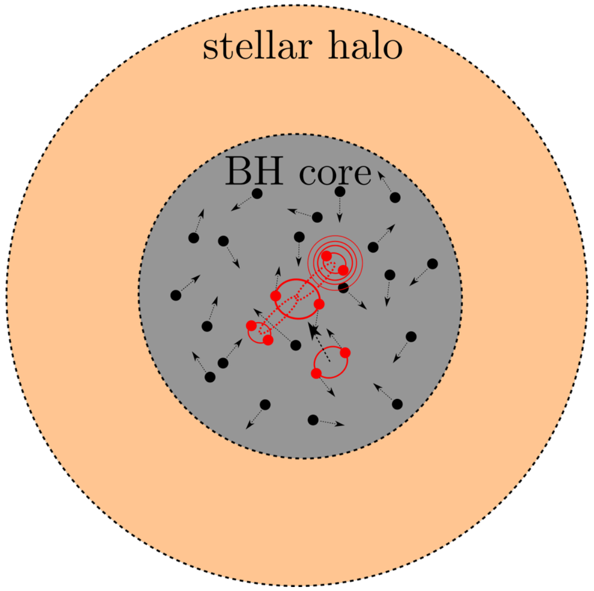


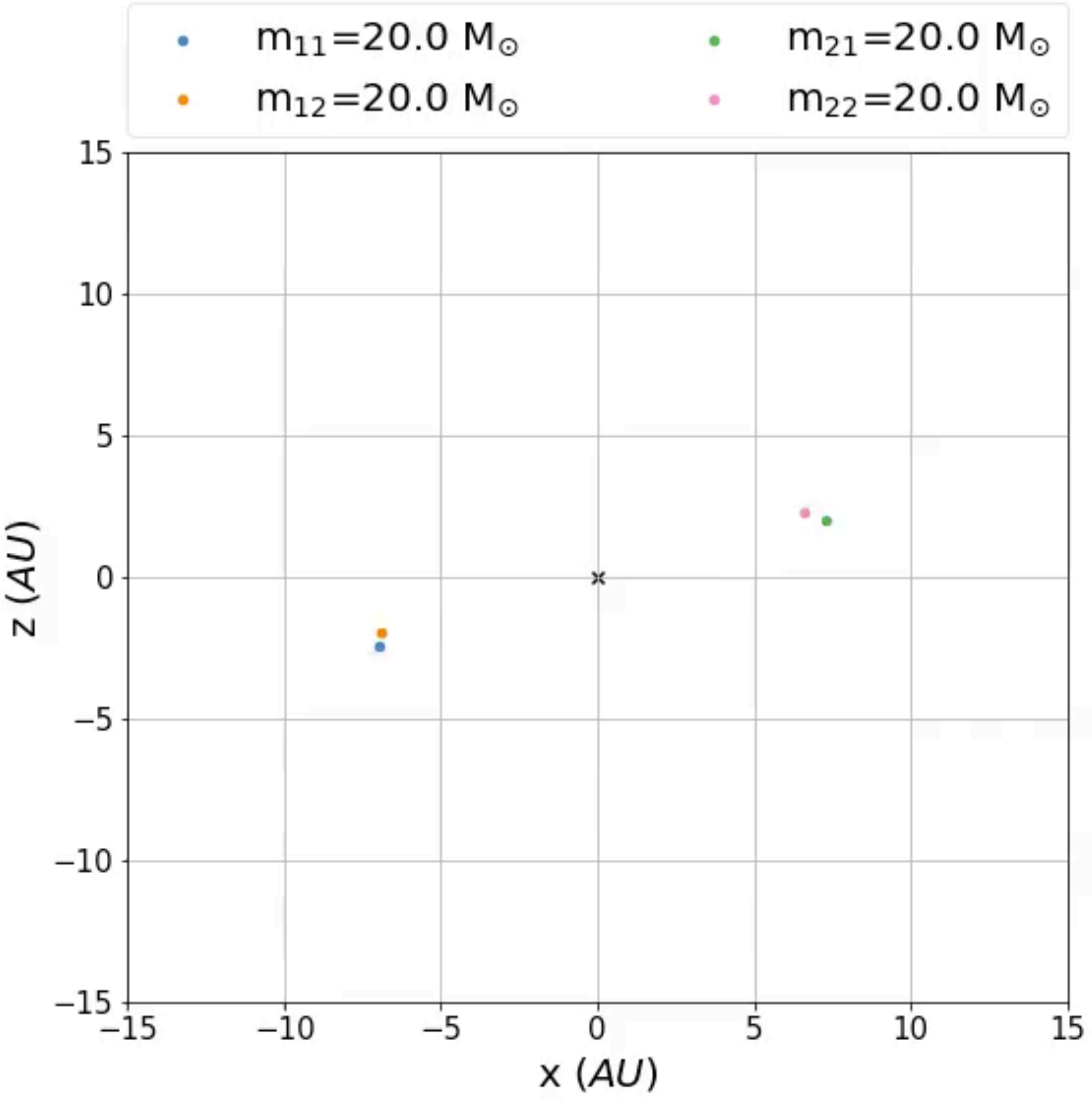


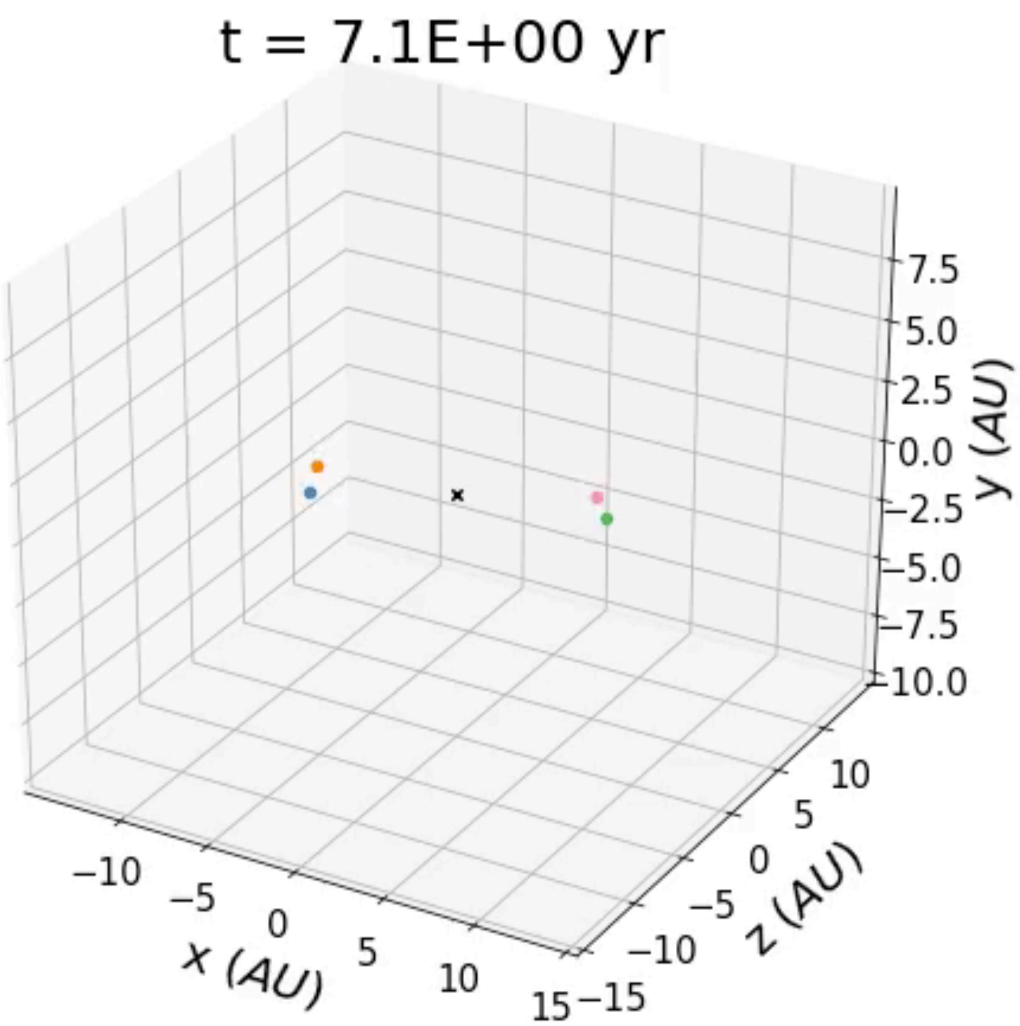


Merger Type: 4-body Merger

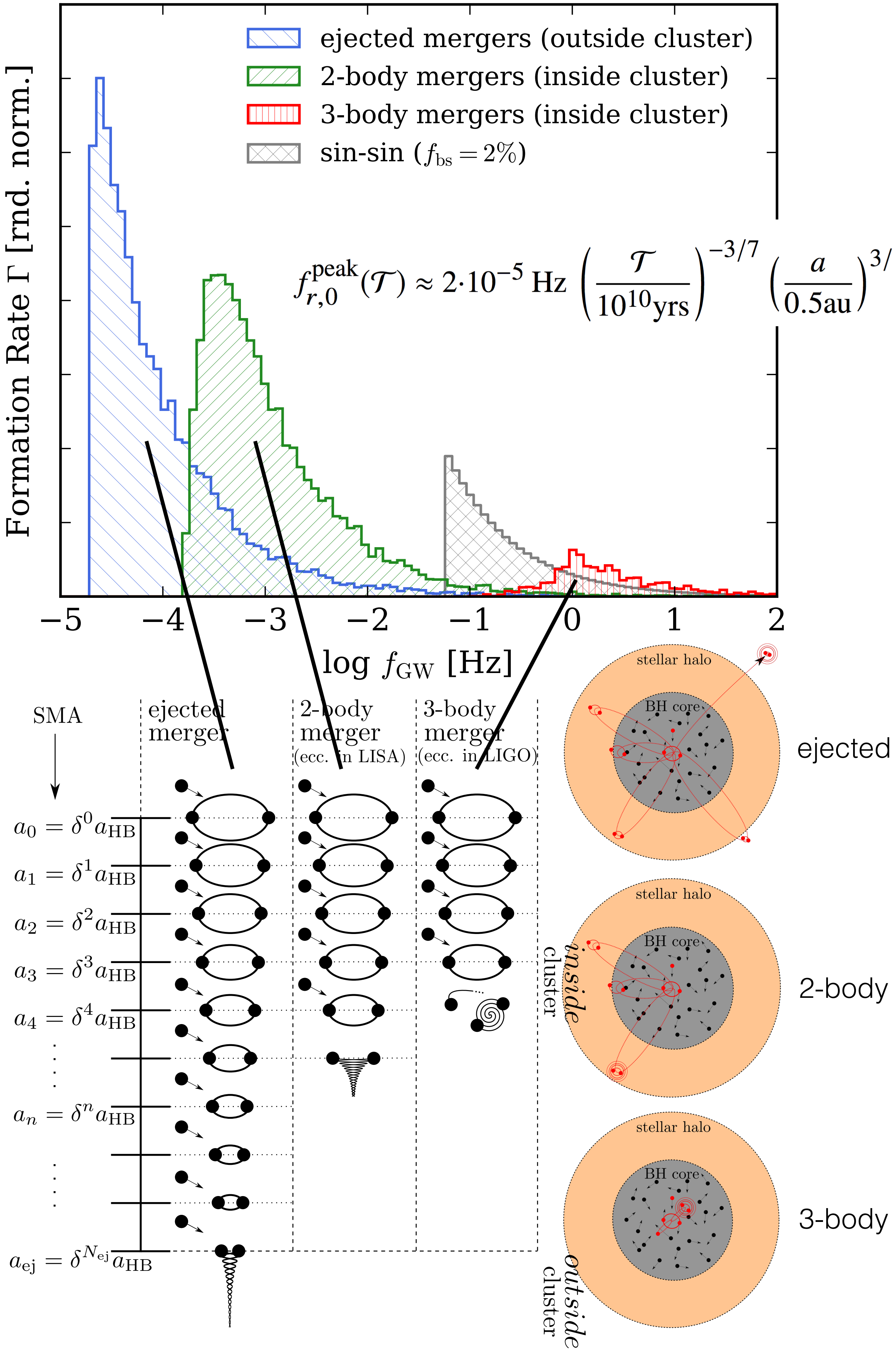
Zevin et al. 2019 (highlighted in AAS NOVA)







Peak Normalizations:



$$f_{r,0}^{\mathrm{peak}}(\mathcal{T}) \approx 2{\cdot}10^{-5}\ \mathrm{Hz} \left(\frac{\mathcal{T}}{10^{10}\mathrm{yrs}}\right)^{-3/7} \left(\frac{a}{0.5\mathrm{au}}\right)^{3/14} \left(\frac{m}{30M_{\odot}}\right)^{-11/14}$$

$$P_i \approx F_i \times \left(\frac{\tau_i(a_{ej})}{t_{GW}^{e=0}(a_{ej})}\right)^{2/7}$$

$$F_{in} \approx (7/10)/(1-\delta) \approx 3$$

$$F_{GW} \approx (7/5)/(1-\delta) \times N_{MS} \approx 120$$

$a_{ej} \sim 0.5$ AU $\quad M \sim 30M_{\odot}$

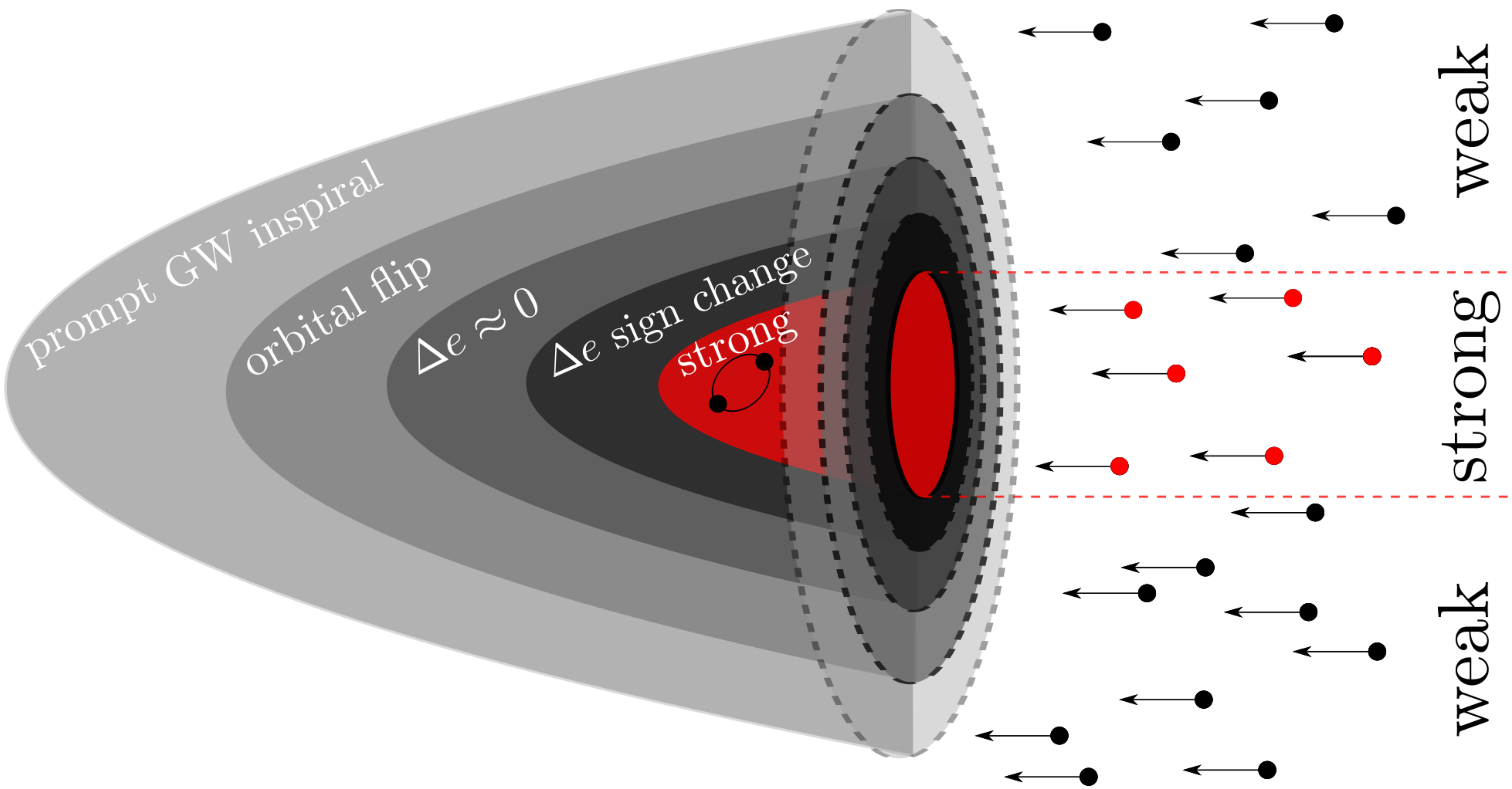
$\tau_{in} \sim 10^7$ years $\quad \tau_{GW} \sim 0.1$ year

$$P_{in} \approx 0.15 \quad P_{GW} \approx 0.03$$
$$P(t_{GW}(a_{ej}) < T_H) \approx 0.35$$
$$0.82 \times 0.35 \approx 0.3$$

start to reach 10% high ecc. LIGO mergers

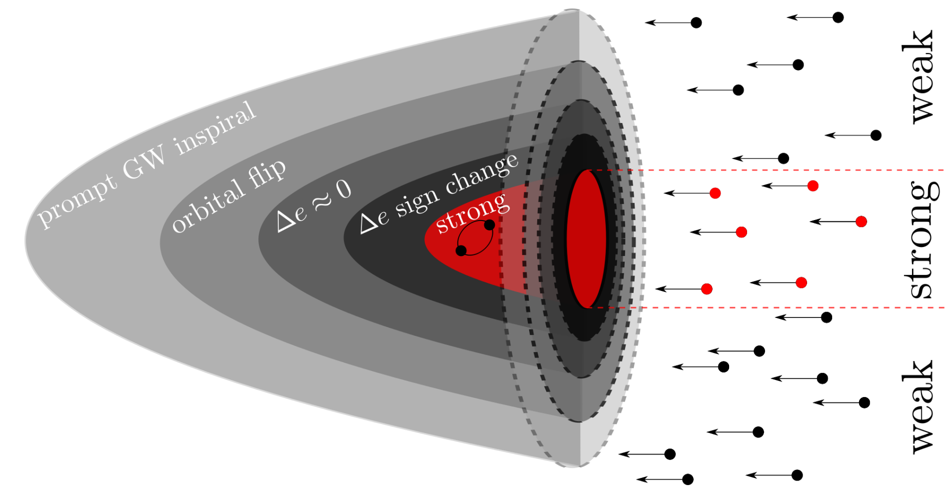
Merger Type: Secular-processes

- work done with Adrian Hamers (IAS)



Merger Type: Secular-processes

Why is this important?

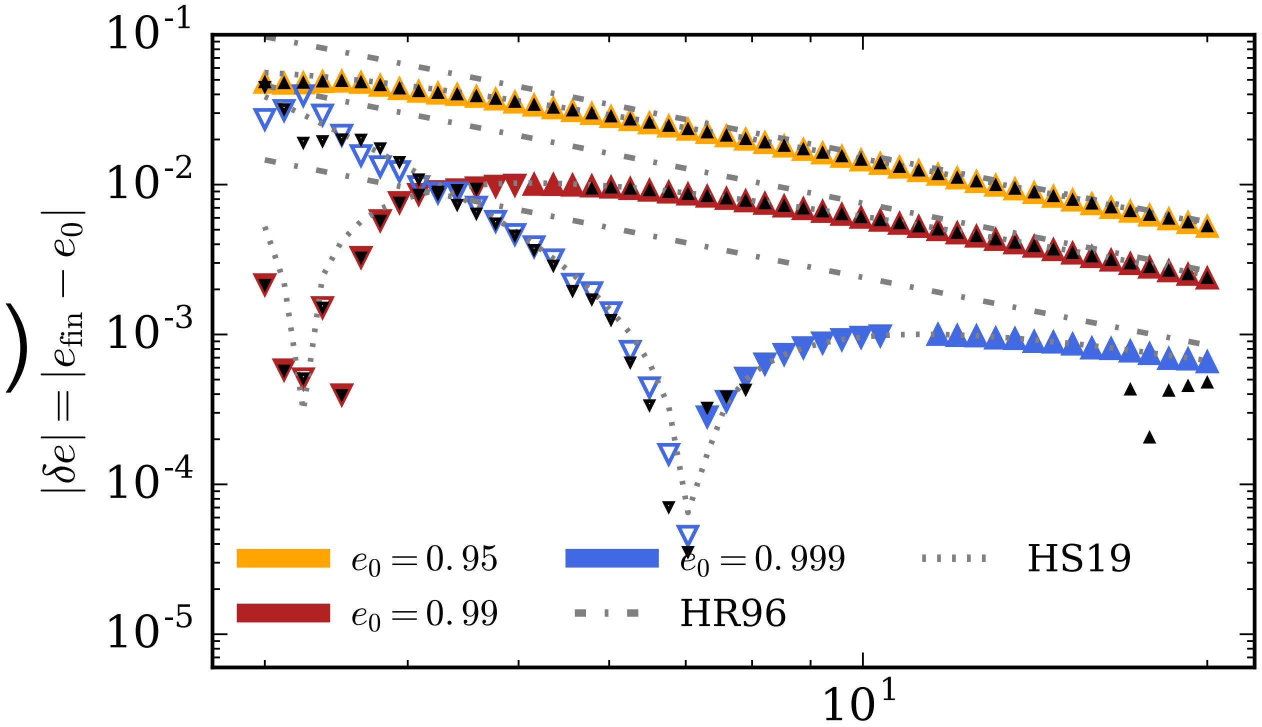


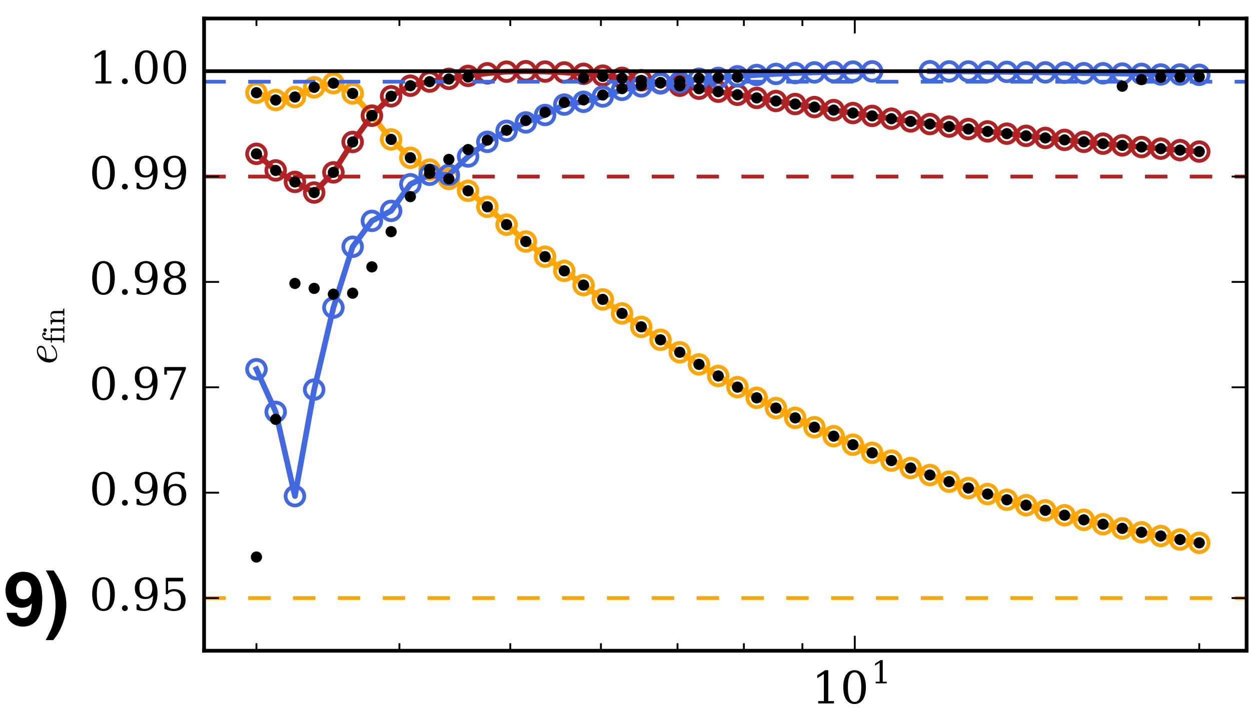
1.order (Heggie, Rasio 96)

$$\delta e = -\frac{15\pi}{16}\left(\frac{2m_3^2 a^3}{M_{123}M_{12}r_{\rm p}^3}\right)^{1/2} e\sqrt{1-e^2}\sin 2\Omega \sin^2 i$$

2.order 2x(Hamers, Samsing 19)

$$\begin{aligned}\Delta e_{\rm SO} =& \Delta e_{\rm FO} + \epsilon^2 \frac{3}{512}\pi e_0 \Big[-100\left(1-e_0^2\right)\sin 2\Omega \\ & \left\{(5\cos i + 3\cos 3i)\cos 2\omega + 6\sin i \sin 2i\right\} \\ & + 4\cos 2i \Big\{3\pi\left(81e_0^2-56\right) - 200\left(1-e_0^2\right) \\ & \cos 2\Omega \sin 2\omega\Big\} + 3\pi\Big\{200 e_0^2 \sin^4 i \cos 4\Omega \\ & + 8\left(16e_0^2+9\right)\sin^2 2i \cos 2\Omega \\ & + \left(39e_0^2+36\right)\cos 4i - 299e_0^2 + 124\Big\}\Big],\end{aligned}$$





and 1x(Samsing, Hamers, Types 19)

non-parabolic limit…

$$\begin{aligned}\boldsymbol{g}_e^{(1)} = \Big\{ & -\frac{15}{512}\pi \Big(2541 e_x^3 e_y + e_x^2 \big(36\pi (e_y^2 - 163 e_z^2 + 55 j_y^2 - 5 j_z^2) - 847 j_x j_y\big) + e_x \Big(2037 e_y^3 - e_y \big(294 e_z^2 + 637 j_x^2 + 2592\pi j_x j_y + 3969 j_y^2 + 4074 j_z^2 - 420\big) + 24 e_z j_z (76\pi j_x - 77 j_y)\Big) \\ & + 60\pi j_x^2 (3e_y^2 + 15 e_z^2 - 3 j_y^2 + j_z^2) + 7 j_x \big(j_y (163 e_y^2 - 482 e_z^2 - 62 j_z^2 + 20) - 264 e_y e_z j_z + j_y^3\big) \\ & + 12\pi \big(3e_y^4 + e_y^2(-91 e_z^2 - 36 j_y^2 + 3 j_z^2 + 4) + 72 e_y e_z j_y j_z + 96 e_z^4 + e_z^2(81 j_y^2 + 32 j_z^2 - 12) - 15 j_y^4 + 15 j_y^2 j_z^2 + 12 j_y^2 - 4 j_z^2\big) - 49 j_x^3 j_y\Big), \\ & \frac{15}{512}\pi \Big(2541 e_x^4 + 36\pi e_x^3 e_y + e_x^2 (2037 e_y^2 - 2513 e_z^2 - 1484 j_x^2 + 1380\pi j_x j_y - 903 j_y^2 - 7623 j_z^2 + 420) \\ & + 4 e_x \big(9\pi e_y^3 + e_y (3\pi(398 e_z^2 - 35 j_y^2 + 18 j_z^2 + 4) - 453\pi j_x^2 - 658 j_x j_y) - 2 e_z j_z (469 j_x + 36\pi j_y)\big) - e_z^2 \big(707 e_y^2 + 3213 j_x^2 - 72\pi j_x j_y + 7(j_y^2 - 480 j_z^2 + 20)\big) + 707 e_y^2 j_x^2 \\ & - 12\pi e_y^2 j_x j_y - 2037 e_y^2 j_z^2 + 56 e_y e_z j_z (47 j_y - 12\pi j_x) + 1120 e_z^4 - 49 j_x^4 - 180\pi j_x^3 j_y + 7 j_x^2 j_y^2 + 637 j_x^2 j_z^2 + 140 j_x^2 - 180\pi j_x j_y^3 + 120\pi j_x j_y j_z^2 + 144\pi j_x j_y + 903 j_y^2 j_z^2 - 420 j_z^2\Big), \\ & \frac{15}{512}\pi \Big(2892\pi e_x^3 e_z - e_y \big(e_z (-2219 e_x^2 + 1561 j_x^2 + 264\pi j_x j_y + 3059 j_y^2 - 140) + 2 e_x j_z (2891 j_x + 780\pi j_y) + 1120 e_z^3\big) + 3 e_x^2 j_z (3143 j_y - 492\pi j_x) + e_y^2 (2892\pi e_x e_z + 84\pi j_x j_z + 2471 j_y j_z) \\ & - 2 e_x e_z \big(6\pi(224 e_z^2 + 179 j_y^2 - 28) + 1206\pi j_x^2 - 679 j_x j_y\big) + 707 e_y^3 e_z + 3 j_z \big(-7 j_y (160 e_z^2 + 51 j_x^2 - 20) + 4\pi j_x (96 e_z^2 + 35 j_x^2 - 12) + 140\pi j_x j_y^2 - 301 j_y^3\big)\Big)\Big\};\end{aligned}$$

$$\begin{aligned}\boldsymbol{g}_e^{(2)} = \Big\{ & \frac{225}{32768}\pi \Big(13041 e_y^5 + 288 e_x \pi e_y^4 + (31626 e_x^2 + 36092 e_z^2 + 2458 j_x^2 - 23654 j_y^2 - 52164 j_z^2 + 640 j_x j_y \pi - 672) e_y^3 \\ & + 4\big(72\pi e_x^3 + (920\pi j_x^2 + 1583 j_y j_x + 8(-527 e_z^2 - 234 j_y^2 + 9 j_z^2 + 12)\pi) e_x + 2 e_z j_z (200\pi j_x + 11097 j_y)\big) e_y^2 \\ & + \big(20097 e_x^4 + 2(12502 e_z^2 + 507 j_x^2 - 32777 j_y^2 - 31626 j_z^2 - 13472 j_x j_y \pi + 2016) e_x^2 + 48 e_z j_z (53 j_x + 216 j_y \pi) e_x + 68096 e_z^4 + 3825 j_x^4 + 1925 j_y^4 - 3136 j_x^2 - 1570 j_x^2 j_y^2 + 1568 j_y^2 - 4916 j_x^2 j_z^2 - 14124 j_y^2 j_z^2 + 1344 j_z^2\big) e_y \\ & + \big(4 e_z^2 (8067 j_x^2 + 400 j_y \pi j_x - 19099 j_y^2 - 2688 j_z^2 - 3248) + 3200 j_x j_y^3 \pi - 960 j_x j_y j_z^2 \pi + 1600 j_x^3 j_y \pi - 1280 j_x j_y \pi + 560\big) e_y + 8 e_x^2 e_z j_z (4472\pi j_x + 1167 j_y) \\ & - 8 e_z j_z \big(-3883 j_y^3 + 1080 j_x \pi j_y^2 + (-17024 e_z^2 - 5767 j_x^2 + 2128) j_y + 40 j_x (32 e_z^2 + 15 j_x^2 - 4)\pi\big) - 4 e_x^3 \big(7343 j_x j_y + 216(73 e_z^2 - 19 j_y^2 + j_z^2)\pi\big) \\ & + 4 e_x \big(281 j_y j_x^3 + 40(115 e_z^2 - 17 j_y^2 - 21 j_z^2)\pi j_x^2 + j_y (-20602 e_z^2 + 2237 j_y^2 - 3802 j_z^2 + 672) j_x + 8(864 e_z^4 + 3(243 j_y^2 + 32 j_z^2 - 36) e_z^2 - 135 j_y^4 - 12 j_z^2 + j_y^2 (65 j_z^2 + 108))\pi\big)\Big), \\ & -\frac{225}{32768}\pi \Big(20097 e_x^5 + 432 e_y \pi e_x^4 + (31626 e_y^2 - 2884 e_z^2 - 28358 j_x^2 - 8294 j_y^2 - 80388 j_z^2 + 8896 j_x j_y \pi + 4032) e_x^3 \\ & + 4\big(144\pi e_y^3 + (-3896\pi j_x^2 - 10513 j_y j_x + 8(1363 e_z^2 - 170 j_y^2 + 27 j_z^2 + 24)\pi) e_y + 2 e_z j_z (2697 j_x + 472 j_y \pi)\big) e_x^2 \\ & + \big(13041 e_y^4 + 2(7126 e_z^2 - 3209 j_x^2 + 3531 j_y^2 - 31626 j_z^2 - 64 j_x j_y \pi - 336) e_y^2 + 16 e_z j_z (6207 j_y - 1256 j_x \pi) e_y + 57344 e_z^4 + 4949 j_x^4 + 2145 j_y^4 - 448 j_x^2\big) e_x \\ & + \big(14558 j_x^2 j_y^2 - 2464 j_y^2 - 2028 j_x^2 j_z^2 + 16588 j_y^2 j_z^2 - 8064 j_z^2 - 4 e_z^2 (10699 j_x^2 - 4272 j_y \pi j_x - 2691 j_y^2 - 16128 j_z^2 + 2912) - 1920 j_x j_y^3 \pi - 2880 j_x j_y j_z^2 \pi - 320 j_x^3 j_y \pi + 1536 j_x j_y \pi + 560\big) e_x \\ & + 4\big(36\pi e_y^5 - (240\pi j_x^2 + 7679 j_y j_x + 8(81 e_z^2 - 5 j_y^2 + 9 j_z^2 - 12)\pi) e_y^3 + 6 e_z j_z (613 j_x + 312 j_y \pi) e_y^2\big) \\ & + 4\Big(\big(300\pi j_x^4 - 1795 j_y j_x^3 + 40(15 e_z^2 + 30 j_y^2 + 39 j_z^2 - 8)\pi j_x^2 - j_y (20602 e_z^2 + 55 j_y^2 + 3802 j_z^2 - 1008) j_x + 4(704 e_z^4 + 6(135 j_y^2 + 32 j_z^2 - 36) e_z^2 + 125 j_y^4 - 24 j_z^2 - 10 j_y^2 (7 j_z^2 + 12) + 16)\pi\big) e_y\Big) \\ & + 4\Big(2 e_z j_z \big(4219 j_x^3 - 40 j_y \pi j_x^2 + (14336 e_z^2 + 2407 j_y^2 - 1792) j_x + 88 j_y (32 e_z^2 + 5 j_y^2 - 4)\pi\big)\Big)\Big), \\ & -\frac{225}{2048}\pi \Big(1024\pi e_z^5 + 32\big(20\pi j_x^2 + 21 j_y j_x + 4(e_y^2 - 11 j_y^2 - 2)\pi\big) e_z^3 + 32 e_y j_z (77 j_x + 24 j_y \pi) e_z^2 + \big(75\pi j_x^4 + 519 j_y j_x^3 - 10(17 e_y^2 + 17 j_y^2 + 8)\pi j_x^2 + j_y (4307 e_y^2 + 573 j_y^2 - 84)\big) e_z\Big) \\ & -\frac{225}{2048\pi}\Big(\big(j_x - (21 e_y^4 + 2(159 j_y^2 + 8) e_y^2 + 245 j_y^4 - 176 j_y^2 - 16)\pi\big) e_z + 1387 e_x^4 \pi e_z + e_y j_z \big(649 j_x^3 - 100 j_y \pi j_x^2 + (-2953 e_y^2 + 769 j_y^2 - 308) j_x + 4 j_y (7 e_y^2 + 55 j_y^2 - 24)\pi\big) + e_x^3 (1743 e_y e_z + 6061 j_y j_z - 804 j_x j_z \pi) \\ & + e_x \big(1365 e_z e_y^3 + j_z (28\pi j_x + 4021 j_y) e_y^2 + e_z (672 e_z^2 + 1769 j_x^2 - 6557 j_y^2 + 344 j_x j_y \pi - 84) e_y\big) + e_x \Big(j_z \big(-1573 j_y^3 + 860 j_x \pi j_y^2 + (-6496 e_z^2 - 1525 j_x^2 + 812) j_y + 4 j_x (448 e_z^2 + 135 j_x^2 - 56)\pi\big)\Big) \\ & - e_x^2 \big(2944\pi e_z^3 + (1534\pi j_x^2 + 3047 j_y j_x - 2(683 e_y^2 - 1013 j_y^2 + 184)\pi) e_z + e_y j_z (4777 j_x + 804 j_y \pi)\big)\Big)\Big\};\end{aligned}$$

$$\begin{aligned}\boldsymbol{h}_e^{(0)} = \Big\{ & \frac{3}{512}\pi \Big(-1200\pi e_x^3 j_z + 15 e_x^2 \big(1361 e_y j_z + 80\pi e_z j_x + (369 + 384\pi^2) e_z j_y\big) - 6 e_x \Big(300\pi e_y^2 j_z + 15 e_y e_z \big((64\pi^2 - 109) j_x + 160\pi j_y\big) + j_z \big(120\pi j_x^2 + (581 + 192\pi^2) j_x j_y + 20\pi j_y^2\big)\Big) - 7575 e_y^3 j_z \\ & + 15 e_y^2 e_z (440\pi j_x + 681 j_y) + e_y j_z \big((1152\pi^2 - 3353) j_x^2 + 960\pi j_x j_y + 399 j_y^2\big) + e_z (720\pi j_x^3 - 313 j_x^2 j_y + 1080\pi j_x j_y^2 - 927 j_y^3)\Big), \frac{9}{512}\pi \Big(-4475 e_x^3 j_z - 5 e_x^2 (80\pi e_y j_z + 177 e_z j_x - 480\pi e_z j_y) \\ & + e_x \Big(5655 e_y^2 j_z + 10 e_y e_z \big((65 + 192\pi^2) j_y - 200\pi j_x\big) + j_z \big(1043 j_x^2 - 80\pi j_x j_y - (1001 + 384\pi^2) j_y^2\big)\Big) - 600\pi e_y^3 j_z - 5 e_y^2 e_z \big((384\pi^2 - 387) j_x + 520\pi j_y\big) \\ & + 2 e_y j_z \big(-80\pi j_x^2 + (192\pi^2 - 371) j_x j_y + 140\pi j_y^2\big) + e_z (29 j_x^3 + 240\pi j_x^2 j_y + 95 j_x j_y^2 + 360\pi j_y^3)\Big), -\frac{3}{64}\pi \Big(2400\pi e_x^2 e_z j_z \\ & + e_y \big(-420 e_x e_z j_z + 45(21 + 16\pi^2) e_z^2 j_x + j_z^2 (48\pi^2 j_x + 7 j_x - 240\pi j_y)\big) + 15 e_x e_z^2 \big(80\pi j_x + (133 - 48\pi^2) j_y\big) + e_x j_z^2 \big((77 - 48\pi^2) j_y - 480\pi j_x\big) + 1200\pi e_y^2 e_z j_z - 12 e_z j_x j_z (20\pi j_x + 49 j_y)\Big)\Big\};\end{aligned}$$

The functions associated with the vector angular-momentum changes are given by

$$\boldsymbol{f}_j^{(0)} = \Big\{-\frac{3}{2}\pi(5 e_y e_z - j_y j_z), \frac{3}{2}\pi(5 e_x e_z - j_x j_z), 0\Big\};$$

$$\boldsymbol{f}_j^{(1)} = \Big\{-\frac{75}{16}\pi(-7 e_x e_y e_z + e_x j_y j_z + e_y j_x j_z + e_z j_x j_y), \frac{15}{32}\pi \big(e_z (-73 e_x^2 - 3 e_y^2 + 15 j_x^2 + 5 j_y^2 - 4) + 10 j_z (3 e_x j_x + e_y j_y) + 32 e_z^3\big), \frac{15}{32}\pi \big(e_y (3 e_x^2 - 32 e_z^2 - 5 j_x^2 - 15 j_y^2 + 4) - 10 e_x j_x j_y + 3 e_y^3\big)\Big\};$$

$$\begin{aligned}\boldsymbol{g}_j^{(0)} = \Big\{ & \frac{3}{16}\pi \Big(75 e_x^2 j_y + 60\pi e_x e_y j_y + j_x \big(5 j_x j_y - 6\pi (10 e_y^2 + j_z^2)\big) - 50 e_y e_z j_z + 10 e_z^2 (5 j_y - 9\pi j_x)\Big), \\ & -\frac{3}{16}\pi \Big(15 e_x^2 (5 j_x + 4\pi j_y) - 10 e_x (6\pi e_y j_x + 5 e_y j_y + 15 e_z j_z) + 50 e_y^2 j_x + 90\pi e_z^2 j_y + 5 j_x^3 - 10 j_x j_z^2 + 6\pi j_y j_z^2\Big), \\ & -\frac{15}{8}\pi(5 e_x e_y j_z + 5 e_x e_z j_y + 5 e_y e_z j_x + j_x j_y j_z)\Big\};\end{aligned}$$

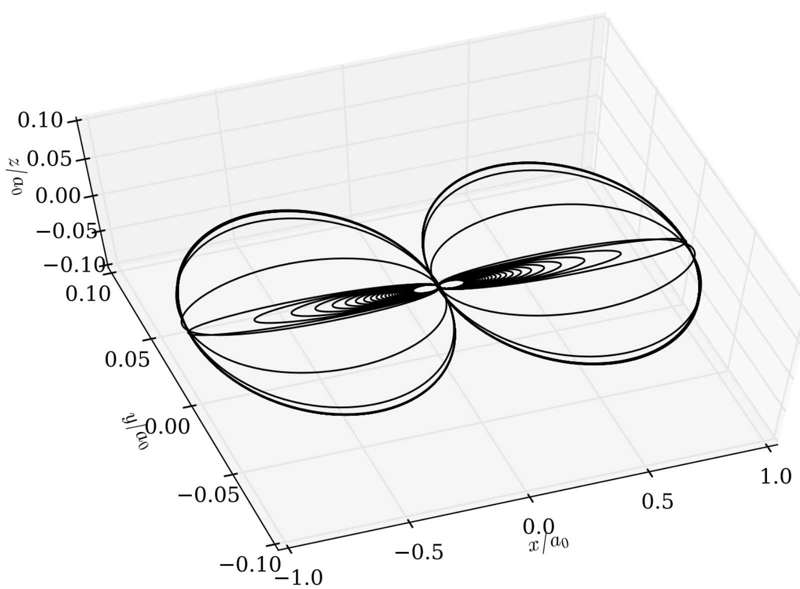
$$\begin{aligned}\boldsymbol{g}_j^{(1)} = \Big\{ & \frac{15}{512}\pi \Big(-2541 e_x^3 j_y + e_y \Big(7 j_x (121 e_x^2 + 682 e_z^2 - 63 j_y^2 + 62 j_z^2 - 20) - 24 \big(116\pi e_x^2 j_y - 322 e_x e_z j_z + \pi j_y (41 e_z^2 - 5 j_y^2 - 5 j_z^2 + 8)\big) + 49 j_x^3\Big) - 96\pi e_x^2 e_z j_z \\ & + 3 e_y^2 (1080\pi e_x j_x + 343 e_x j_y - 32\pi e_z j_z) + e_x \big(7 j_y (42 e_z^2 + 91 j_x^2 - 258 j_z^2 - 60) + 360\pi j_x (11 e_z^2 + j_z^2) + 120\pi j_x j_y^2 + 903 j_y^3\big) + e_y^3 (456\pi j_y - 707 j_x) \\ & - 64 e_z j_z \big(3\pi (8 e_z^2 - 1) + 15\pi j_x^2 - 7 j_x j_y\big)\Big), \\ & \frac{15}{256}\pi \Big(e_x^3 (847 j_x + 1446\pi j_y) - e_x^2 \big(3 e_y (558\pi j_x + 511 j_y) + 6748 e_z j_z\big) + e_x \big(7 j_x (196 e_y^2 - 359 e_z^2 - 64 j_y^2 + 91 j_z^2 + 40) - 6\pi j_y (29 e_y^2 - 406 e_z^2 + 35 j_y^2 - 50 j_z^2 - 28) - 343 j_x^3 - 270\pi j_x^2 j_y\big) \\ & - 54\pi e_y^3 j_x - 1372 e_y^2 e_z j_z + e_y \big(3 e_z^2 (12\pi j_x + 511 j_y) + 210\pi j_x^3 + 217 j_x^2 j_y + 6\pi j_x (25 j_y^2 - 30 j_z^2 - 12) - 217 j_y j_z^2\big) + 8 e_z j_z (280 e_z^2 + 49 j_x^2 - 60\pi j_x j_y + 56 j_y^2 - 35)\Big), \\ & \frac{15}{512}\pi \Big(36\pi e_x^3 j_z + e_x^2 (3549 e_y j_z - 492\pi e_z j_x + 2219 e_z j_y) + 2 e_x \Big(18\pi e_y^2 j_z + e_y e_z (60\pi j_y - 21 j_x) + j_z \big(-6\pi (32 e_z^2 + 25 j_y^2 - 4) + 30\pi j_x^2 + 49 j_x j_y\big)\Big) \\ & + e_z \big(-7 j_y (337 e_y^2 + 23 j_x^2 - 20) - 36\pi j_x (17 e_y^2 + 5 j_x^2 - 4) - 180\pi j_x j_y^2 + 7 j_y^3\big) + e_y j_z (2037 e_y^2 - 1071 j_x^2 + 360\pi j_x j_y - 469 j_y^2 + 420) - 3360 e_y e_z^2 j_z - 32 e_z^3 (36\pi j_x + 35 j_y)\Big)\Big\};\end{aligned}$$

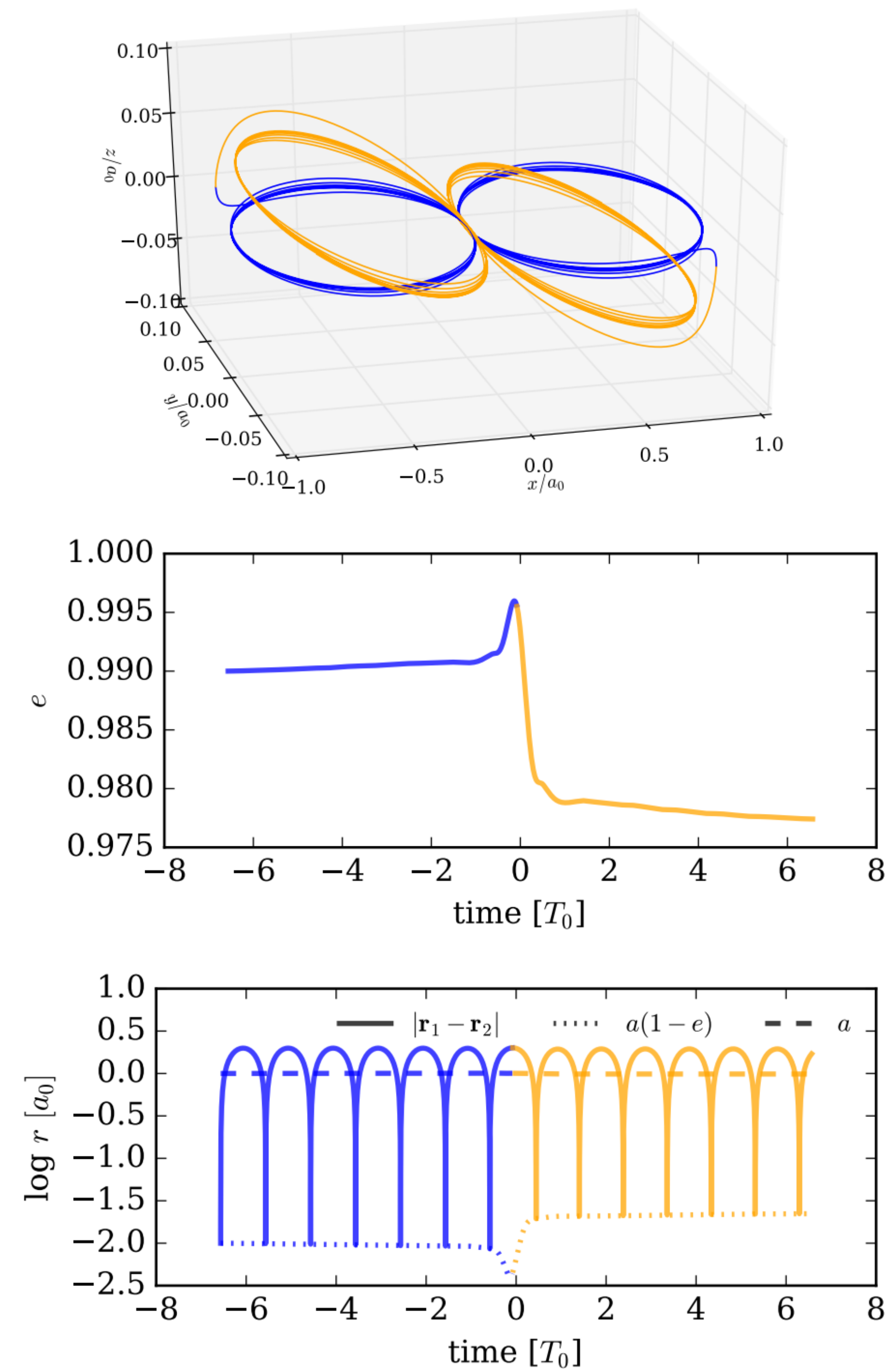
Terms of order	Number of terms in Δe	
	$E = 1$	$E > 1$
ϵ_{SA} (all)	16	60
ϵ_{SA}	2	8
$\epsilon_{SA}\,\epsilon_{oct}$	14	52
ϵ_{SA}^2 (all)	193	55,895
ϵ_{SA}^2	17	1,871
$\epsilon_{SA}^2\,\epsilon_{oct}$	60	16,035
$\epsilon_{SA}^2\,\epsilon_{oct}^2$	116	37,989
ϵ_{SA}^3 (all)	1,146	2,931,541
ϵ_{SA}^3	54	38,366
$\epsilon_{SA}^3\,\epsilon_{oct}$	175	289,496
$\epsilon_{SA}^3\,\epsilon_{oct}^2$	311	856,072
$\epsilon_{SA}^3\,\epsilon_{oct}^3$	606	1,747,607

Merger Type: Secular-processes

PN effects?

2.5





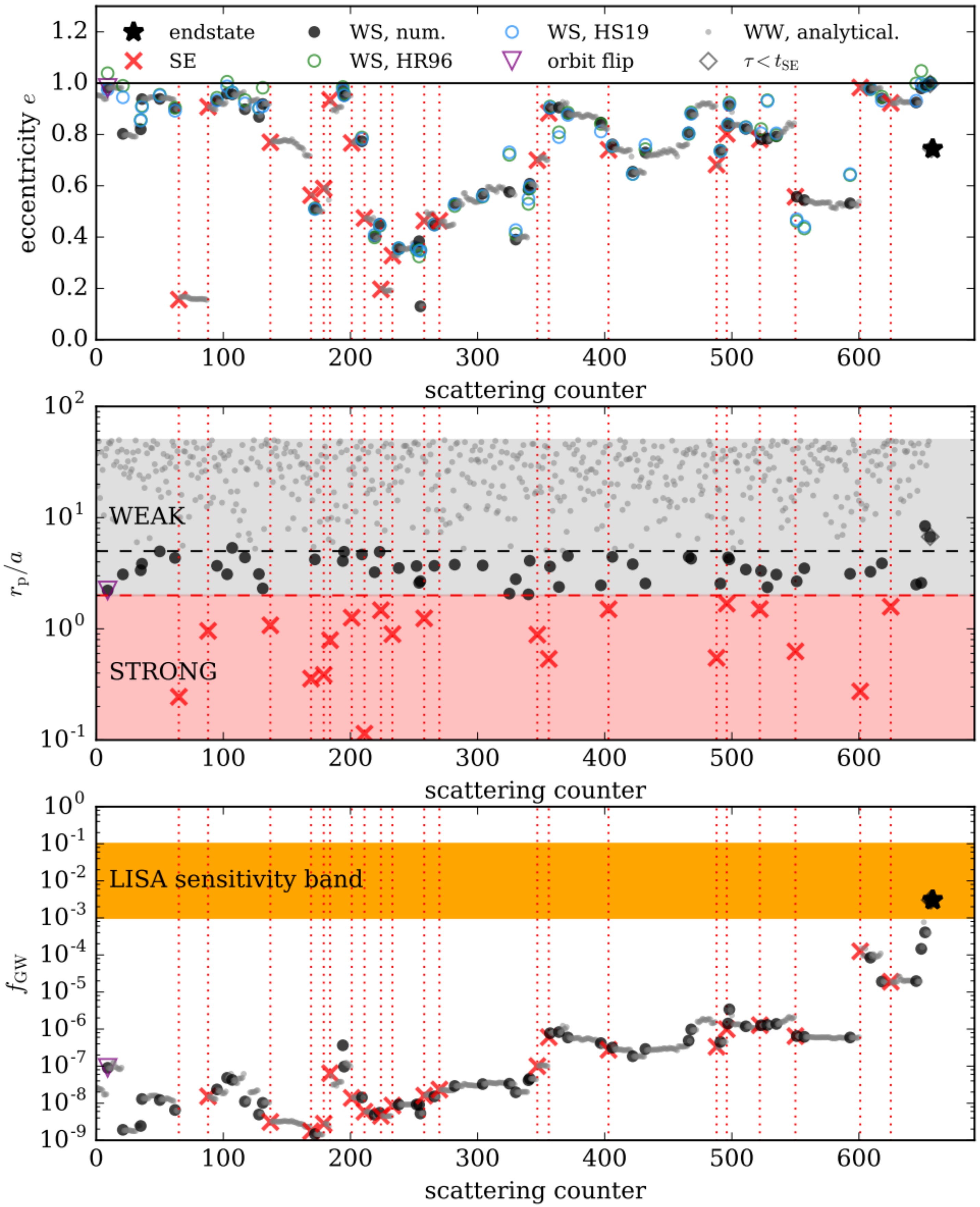
1,2

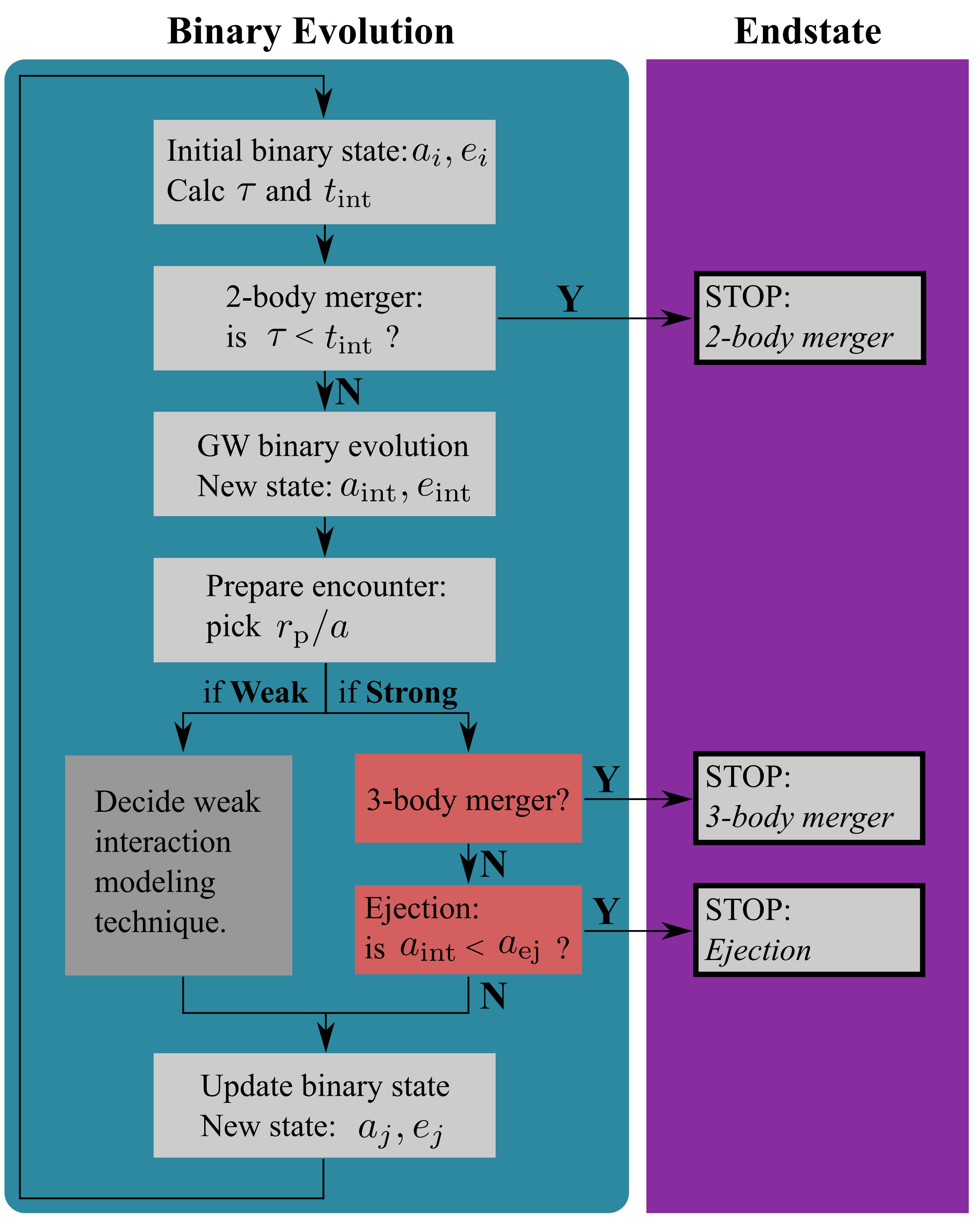
$$\frac{r_{\rm p}}{a_0} \gtrsim \frac{a_0^{2/3}}{\mathscr{R}_{\rm m}^{2/3}}(1-e_0^2)^{2/3}$$

$$\gtrsim 10^3 \times \left(\frac{a_0}{0.5\mathrm{AU}}\right)^{2/3} \left(\frac{m}{20M_\odot}\right)^{-2/3} \left(1-(e_0/0.99)^2\right)^{2/3}$$

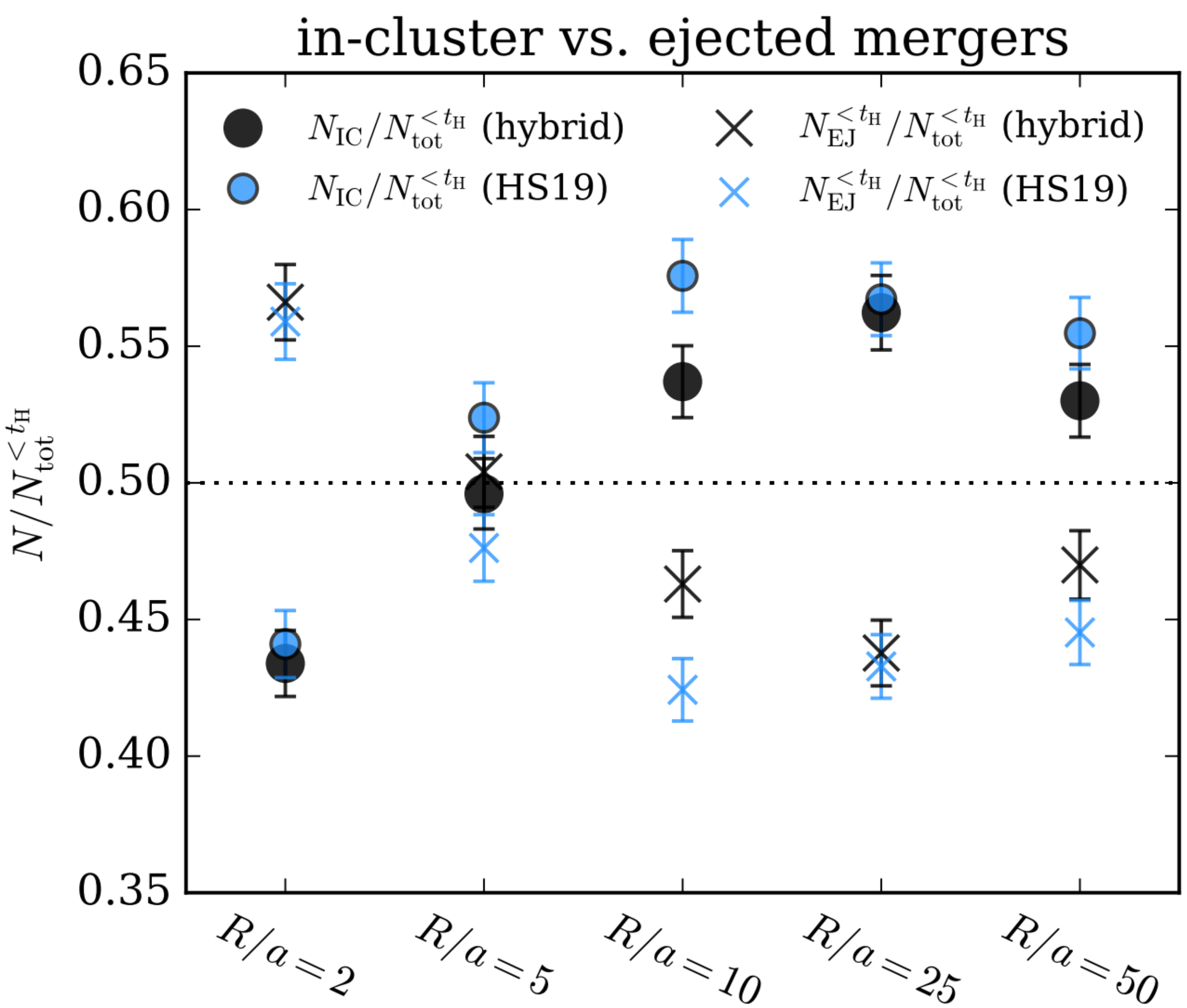
Merger Type: Secular-processes

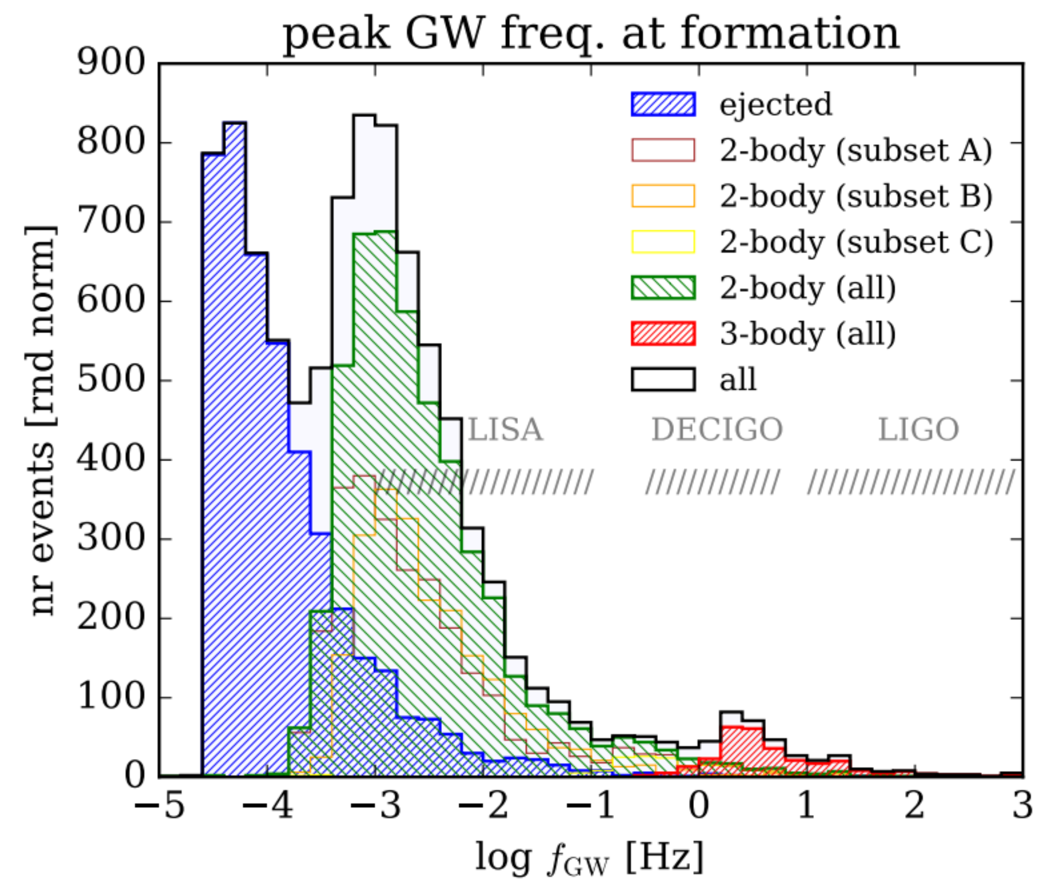
MC approach:

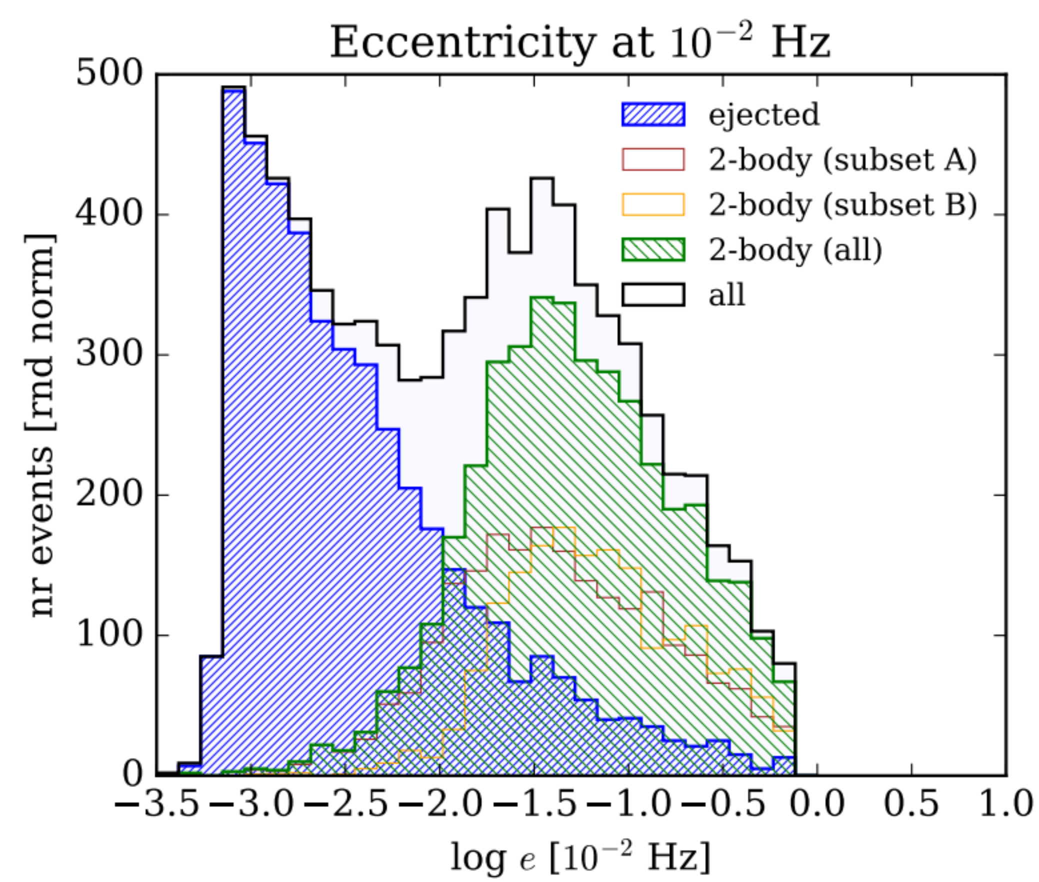




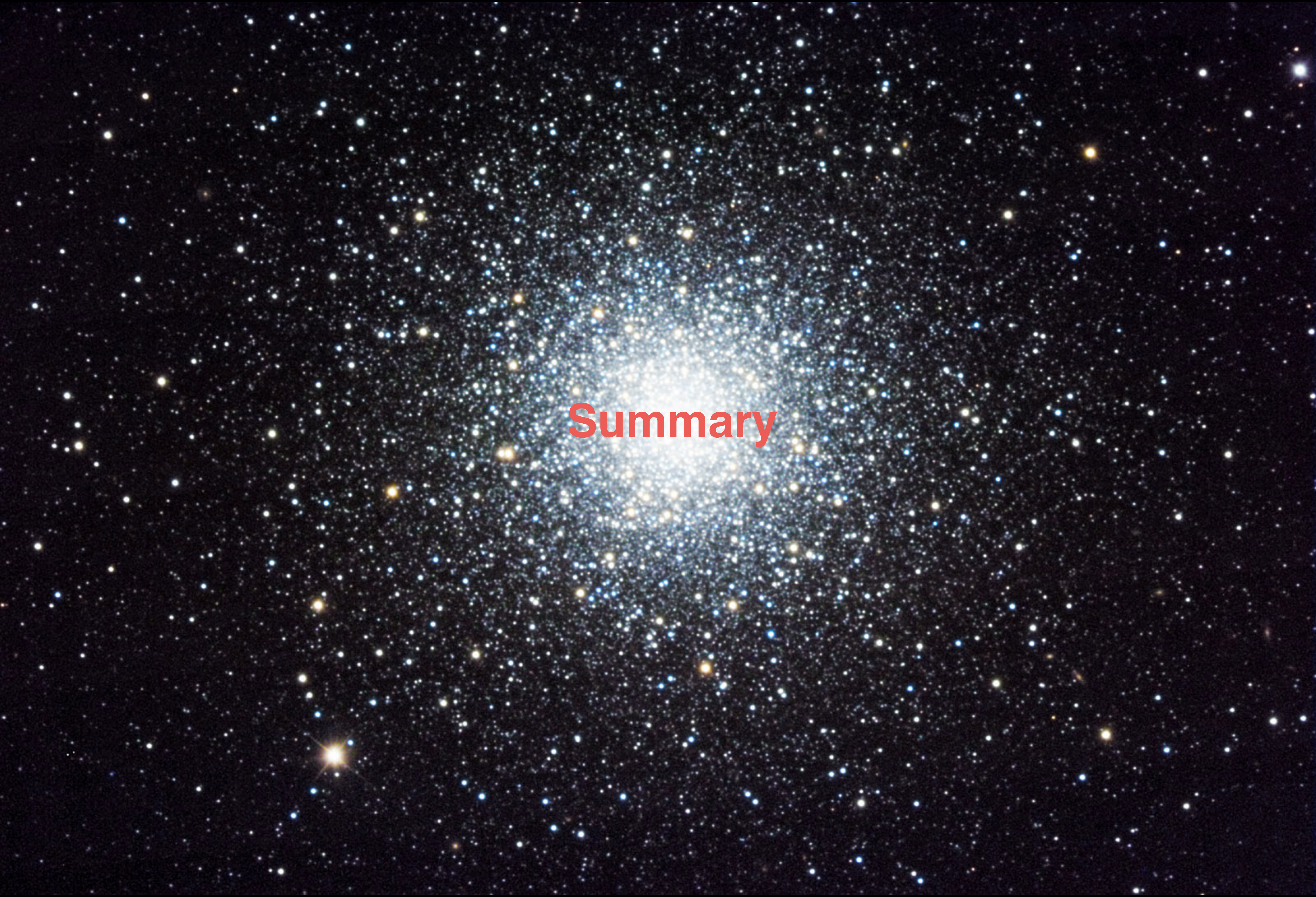
Results from our MC code







more BBHs are driven to merger!


Summary