GRAVITATIONAL WAVES FROM TRIPLE SYSTEMS

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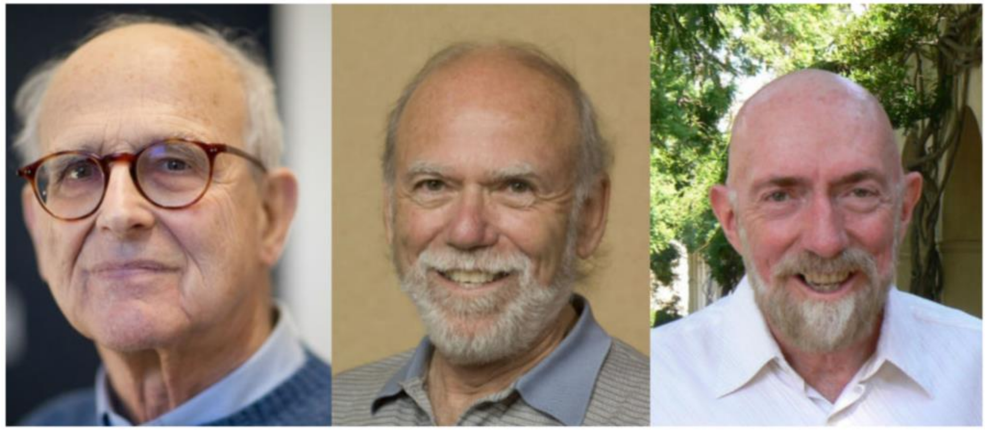
INTRODUCTION

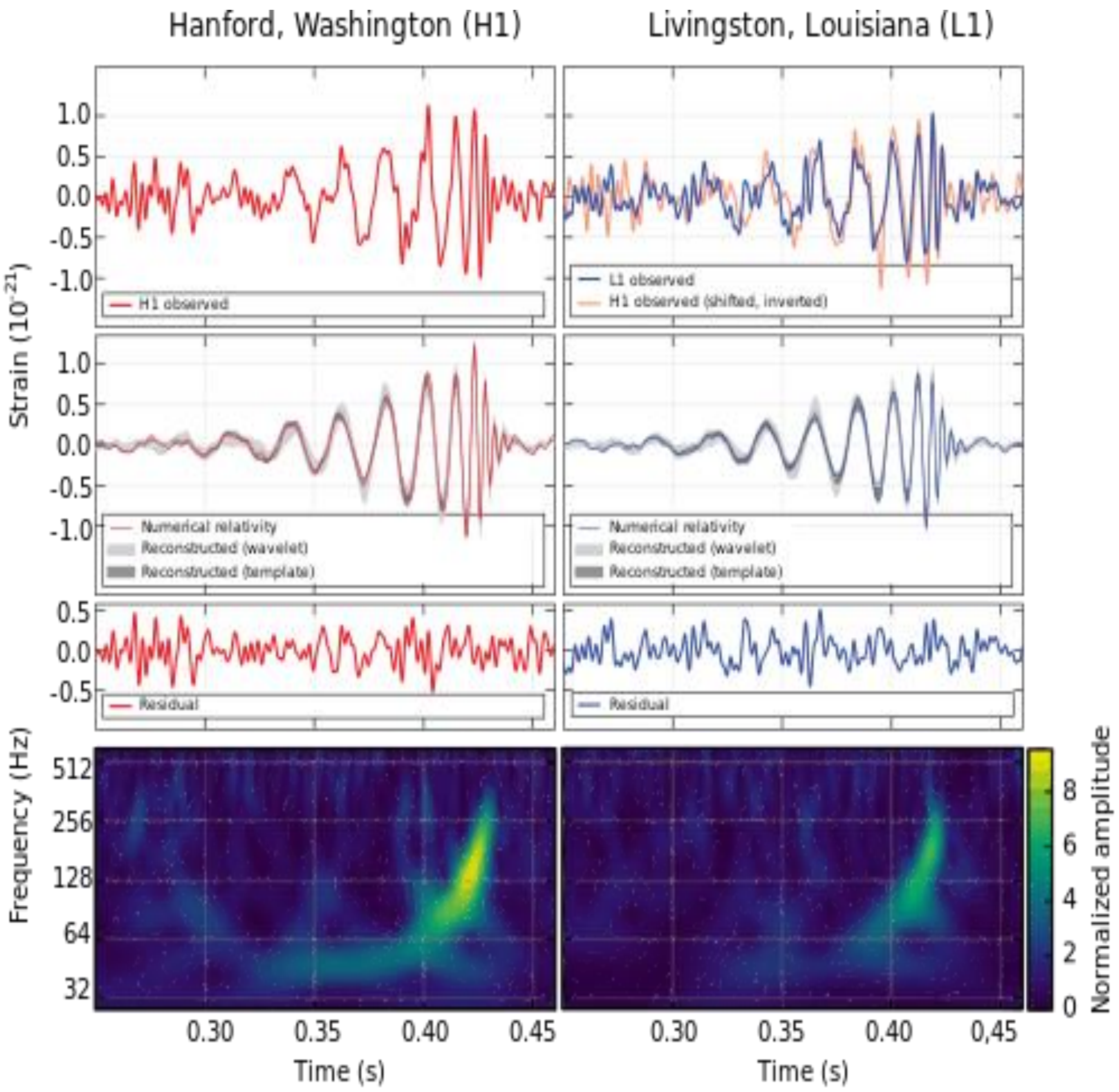
First Direct Observation
GW150914

The waveform, detected by both LIGO observatories, matched the predictions of general relativity for a gravitational wave emanating from the inward spiral, merger of a pair of black holes and the subsequent "ringdown" of the single resulting black hole.

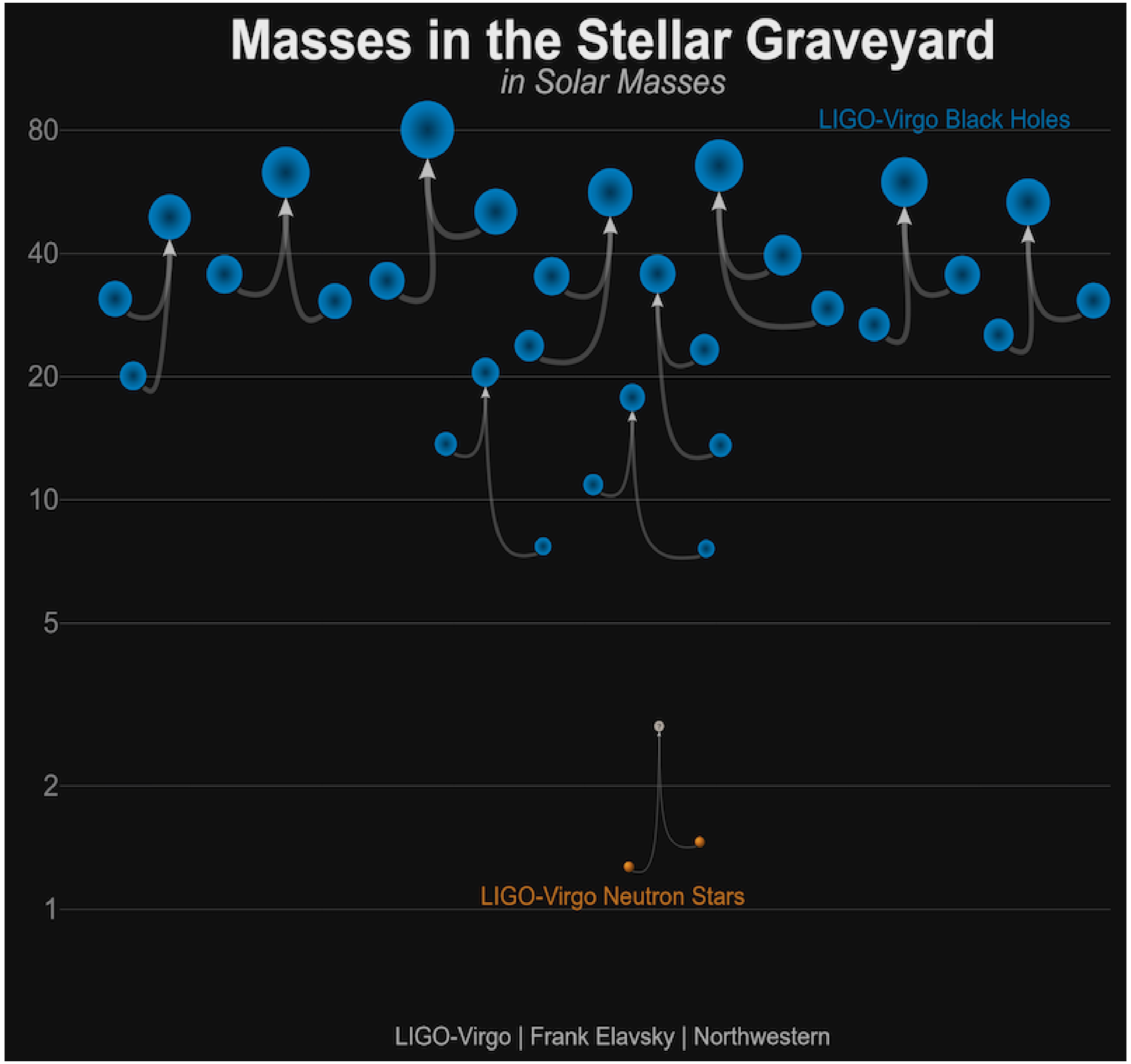


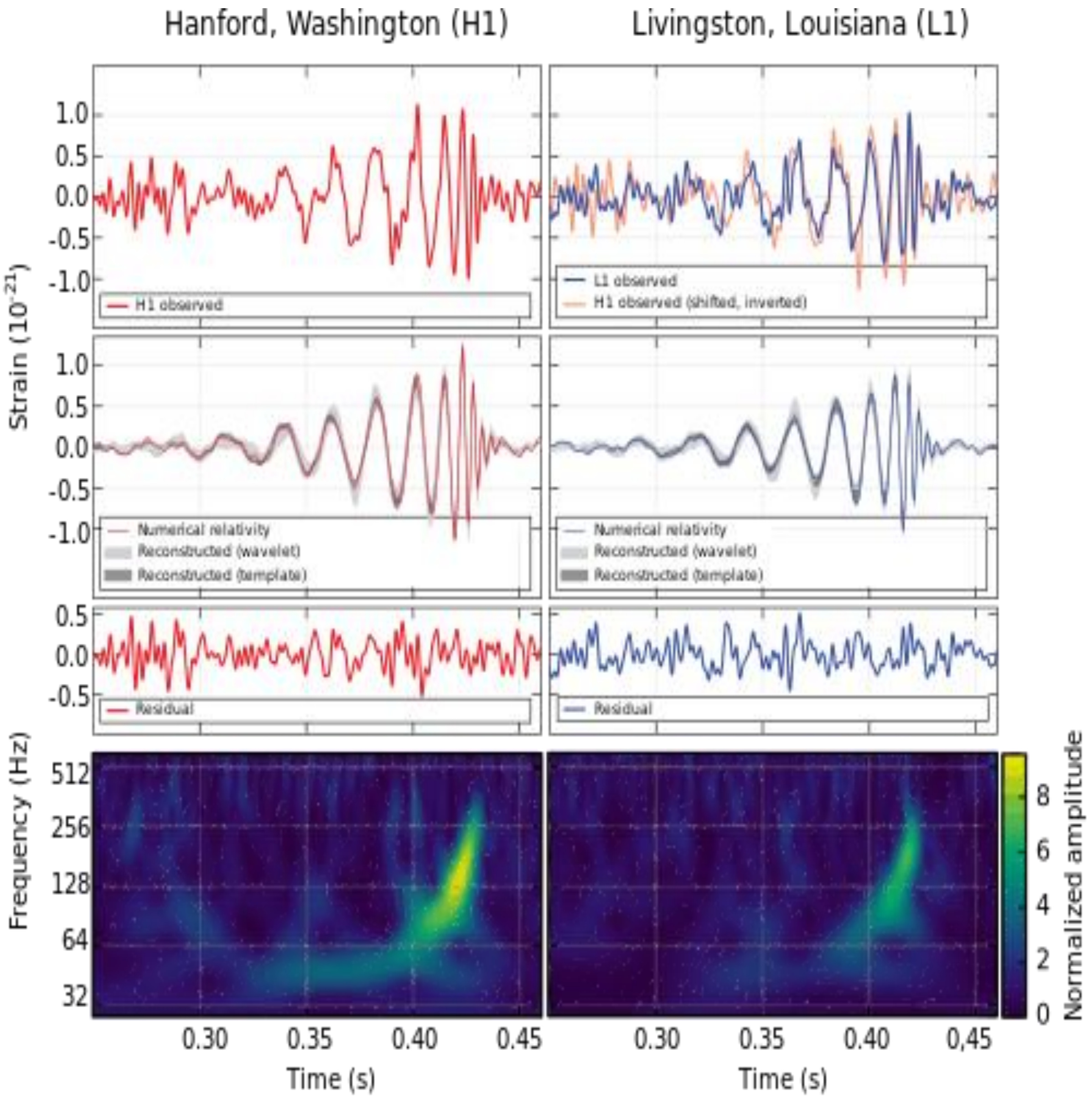
Rainer Weiss   Barry Barish   Kip Thorne

Nobel Prize 2017

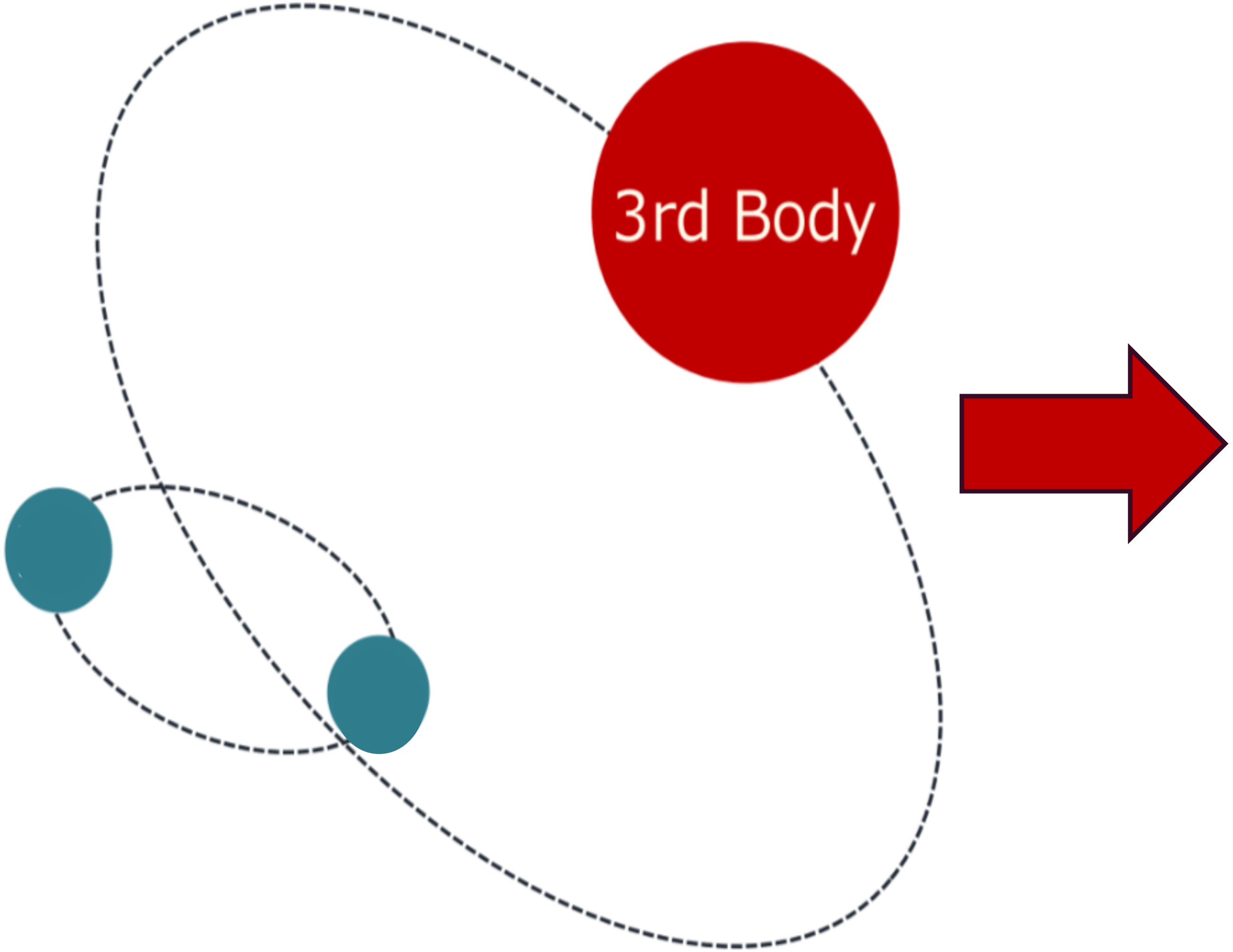


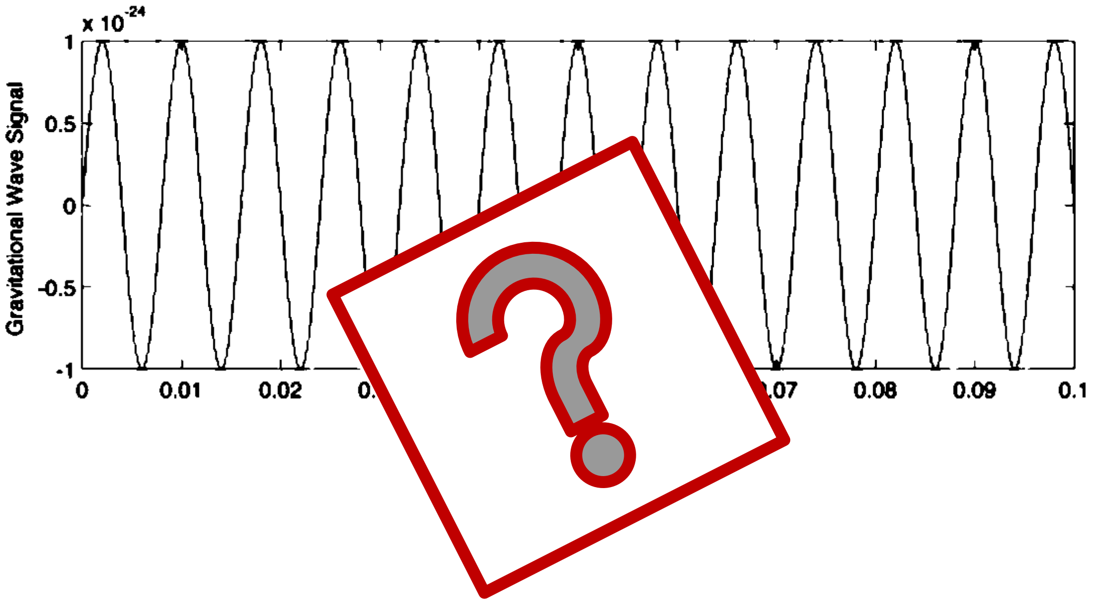
Observations during O1 and O2





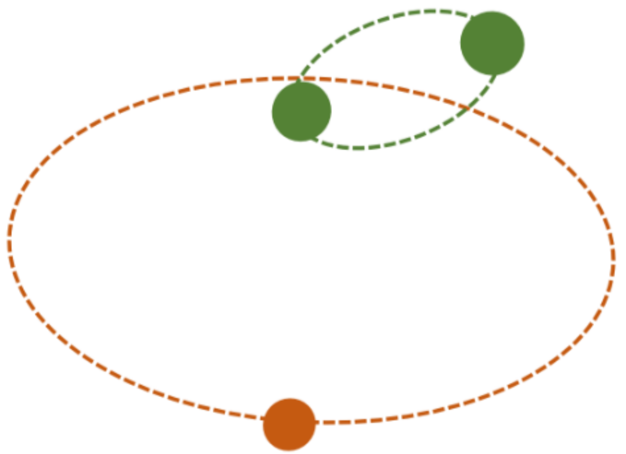
MOTIVATION


3rd Body

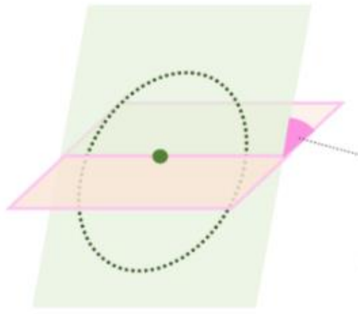

x 10^-24
1
0.5
0
-0.5
-1
Gravitational Wave Signal
0
0.01
0.02
0.08
0.09
0.1

KOZAI - LIDOV MECHANISM

Dynamical phenomenon affecting the orbit of a binary system perturbed by a distant third body under certain conditions.

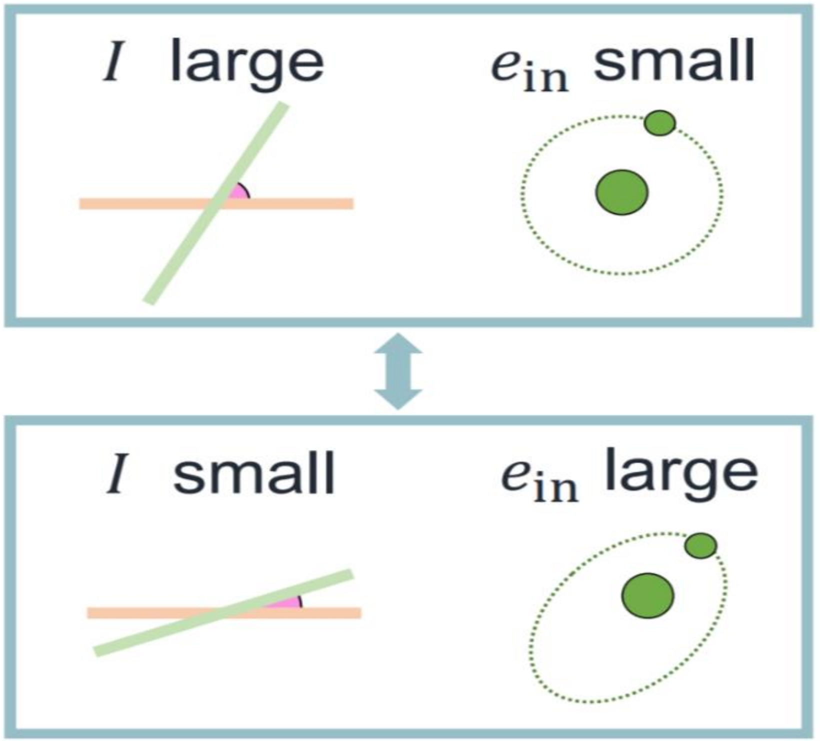


Hierarchical triplet : Binary + tertiary companion



Newtonian TPQ

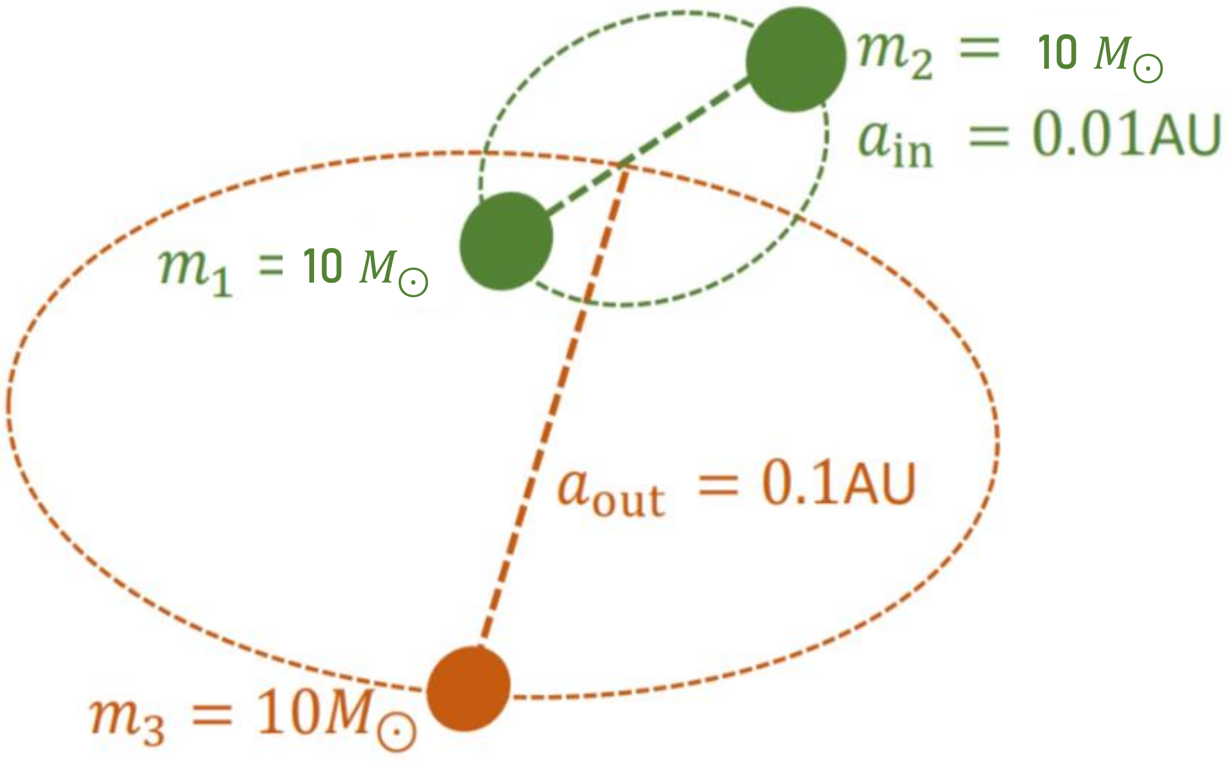
$$\Theta = \sqrt{1 - e_{\rm in}^2}\cos I = const.$$



Leads to a periodic exchange between eccentricity and inclination.

Model + Timescales

Hierarchical triplet : Binary + tertiary companion



$$e_{in,0} = e_{out,0} = 0$$

$$I_0 = 90\ ^\circ$$

$$t_{KL} \simeq \frac{16}{15} \frac{a_{out}^3}{a_{in}^{3/2}} \sqrt{\frac{m_1}{G m_3^2}} (1 - e_{out}^2)^{\frac{3}{2}}$$

Kozai-Lidov Timescale

Time Scales for our Model

$$P_{in} = 0.258\ days$$
$$P_{out} = 3.334\ days$$
$$\tau_{KL} \sim 200\ days$$
$$\tau_{merger} \sim 10^9 years$$

$$P_{in} \ll P_{out} \ll \tau_{KL} \ll \tau_{merger}$$

Orbit Evolution Method

1st order post-Newtonian equation of motion

Einstein-Infeld-Hoffmann equation

Lorentz & Droste,, 1917

$$
\begin{aligned}
\frac{d\boldsymbol{v}_k}{dt} = & -G\sum_{n\neq k} m_n \frac{\boldsymbol{x}_k-\boldsymbol{x}_n}{|\boldsymbol{x}_k-\boldsymbol{x}_n|^3} \\
& \times\left[1-4G\sum_{n'\neq k}\frac{m_{n'}}{|\boldsymbol{x}_k-\boldsymbol{x}_{n'}|}-\sum_{n'\neq n}\frac{m_{n'}}{|\boldsymbol{x}_n-\boldsymbol{x}_{n'}|}\left\{1-\frac{(\boldsymbol{x}_k-\boldsymbol{x}_n)\cdot(\boldsymbol{x}_n-\boldsymbol{x}_{n'})}{2|\boldsymbol{x}_n-\boldsymbol{x}_{n'}|^2}\right\}+v_k^2\right. \\
& \left.+2v_n^2-4\boldsymbol{v}_k\cdot\boldsymbol{v}_n-\frac{3}{2}\left\{\frac{(\boldsymbol{x}_k-\boldsymbol{x}_n)\cdot\boldsymbol{v}_n}{|\boldsymbol{x}_k-\boldsymbol{x}_n|}\right\}^2\right] \\
& -G\sum_{n\neq k} m_n\frac{\boldsymbol{v}_k-\boldsymbol{v}_n}{|\boldsymbol{x}_k-\boldsymbol{x}_n|^3}(\boldsymbol{x}_k-\boldsymbol{x}_n)\cdot(3\boldsymbol{v}_n-4\boldsymbol{v}_k) \\
& -\frac{7}{2}G^2\sum_{n\neq k}\frac{m_n}{|\boldsymbol{x}_k-\boldsymbol{x}_n|}\sum_{n'\neq n}\frac{m_{n'}(\boldsymbol{x}_n-\boldsymbol{x}_{n'})}{|\boldsymbol{x}_n-\boldsymbol{x}_{n'}|}
\end{aligned}
$$

m_k, v_k, z_k are the mass, velocity and coordinates of k-th component of the system.

integrate with 6th order Implicit Runge-Kutta method

※no GW back reaction

Gravitational Waveform

We use Quadrupole formula for the evolution of Gravitational Waveform.

$$[h_{ij}]_{quad} = \frac{1}{r}\frac{2G}{c^4}\ddot{Q}_{ij}\left(t - \frac{r}{c}\right)$$

$$Q^{ij} = M^{ij} - \frac{1}{3}\delta^{ij}M_{kk} = \int d^3x\, \rho(t,x)(x^i x^j - \frac{1}{3}r^2\delta^{ij})$$

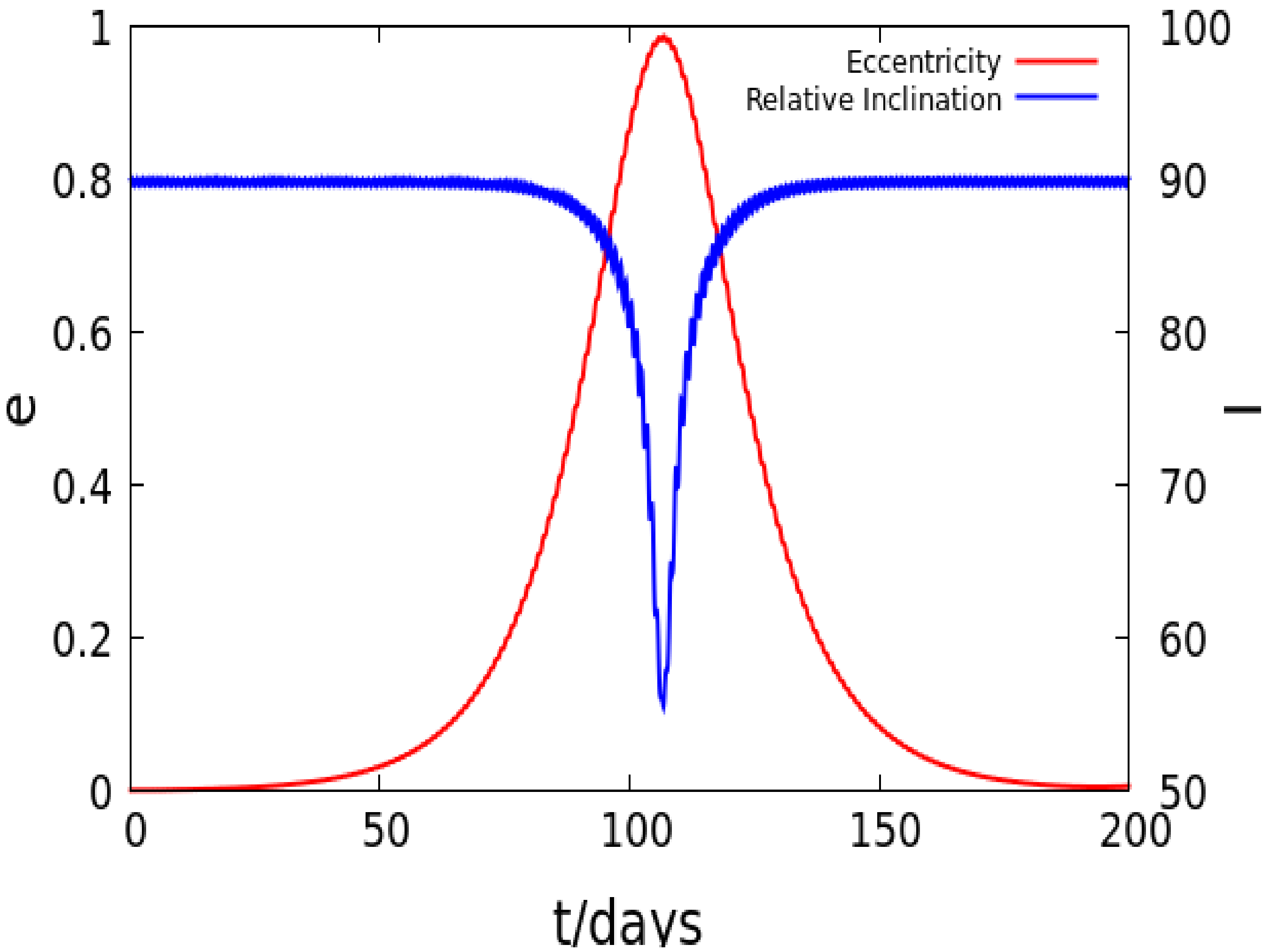
Q_{ij} *is the Quadrupole tensor.*

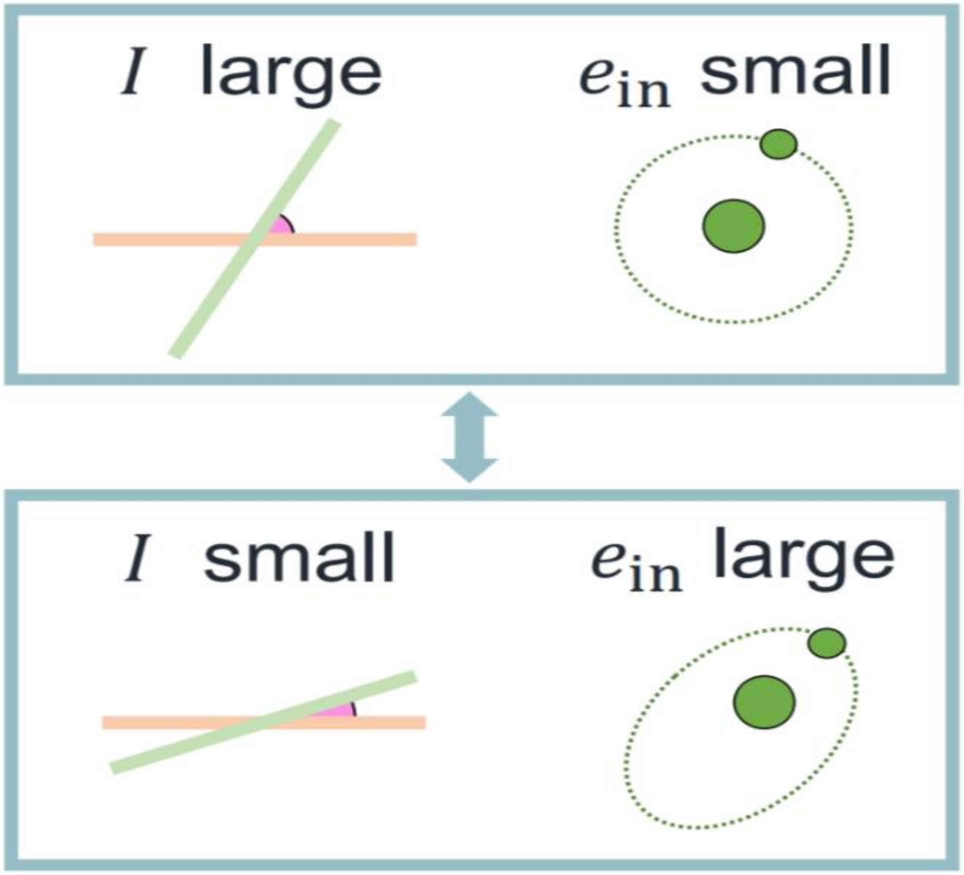
M_{ij} *is the second mass moment.*

Energy spectra is evaluated as

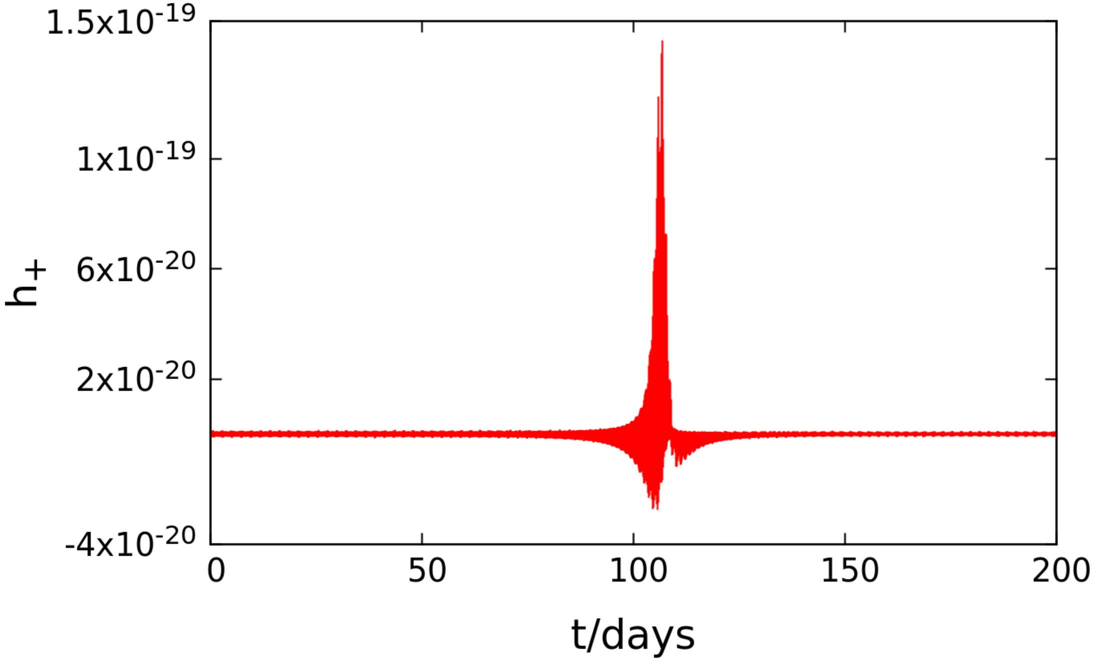
$$\frac{dE}{d\omega} = \frac{G\,\omega^6}{5\,\pi\,c^5}\,\tilde{Q}_{ij}\,(\omega)\tilde{Q}^*{}_{ij}(\omega)$$

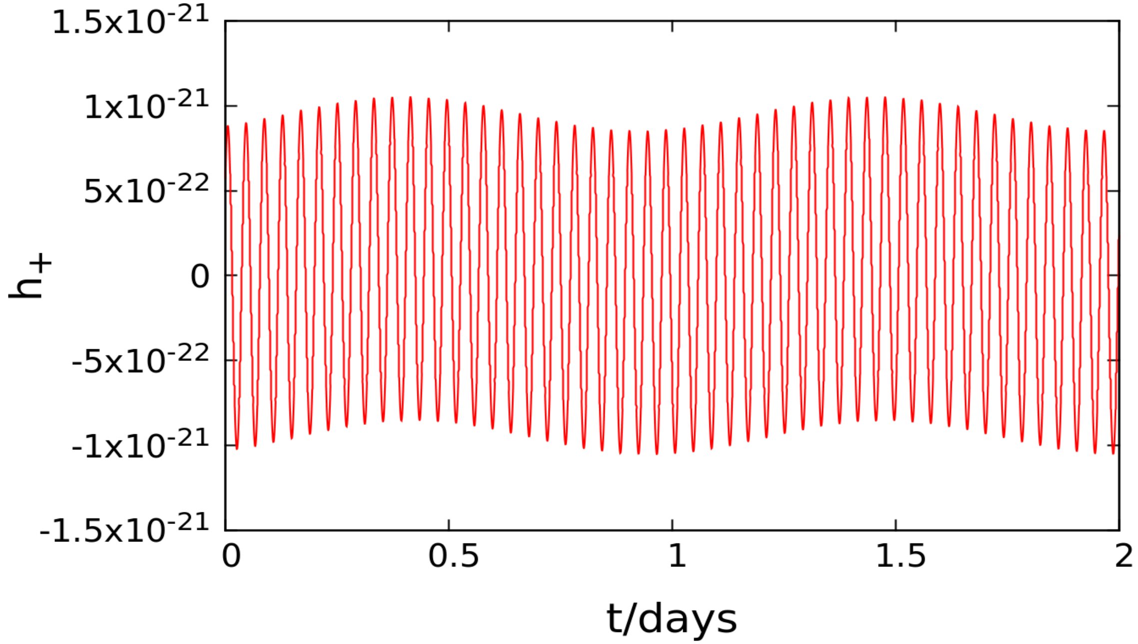
RESULTS

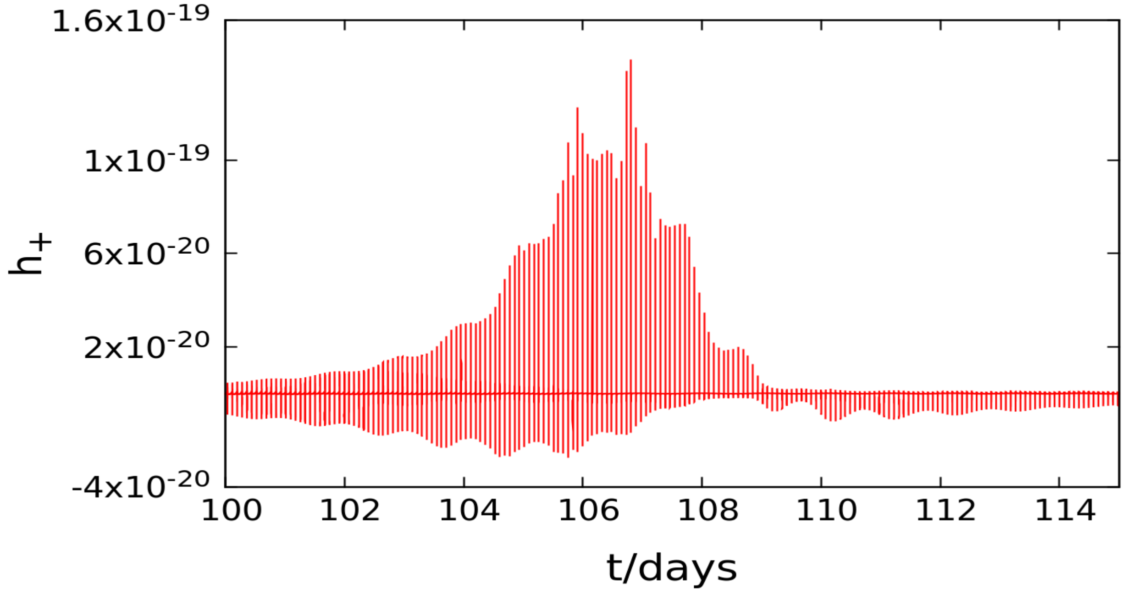

Eccentricity
Relative Inclination
e
I
t/days
0
0.2
0.4
0.6
0.8
1
50
60
70
80
90
100
0
50
100
150
200


I large
e_{in} small
I small
e_{in} large

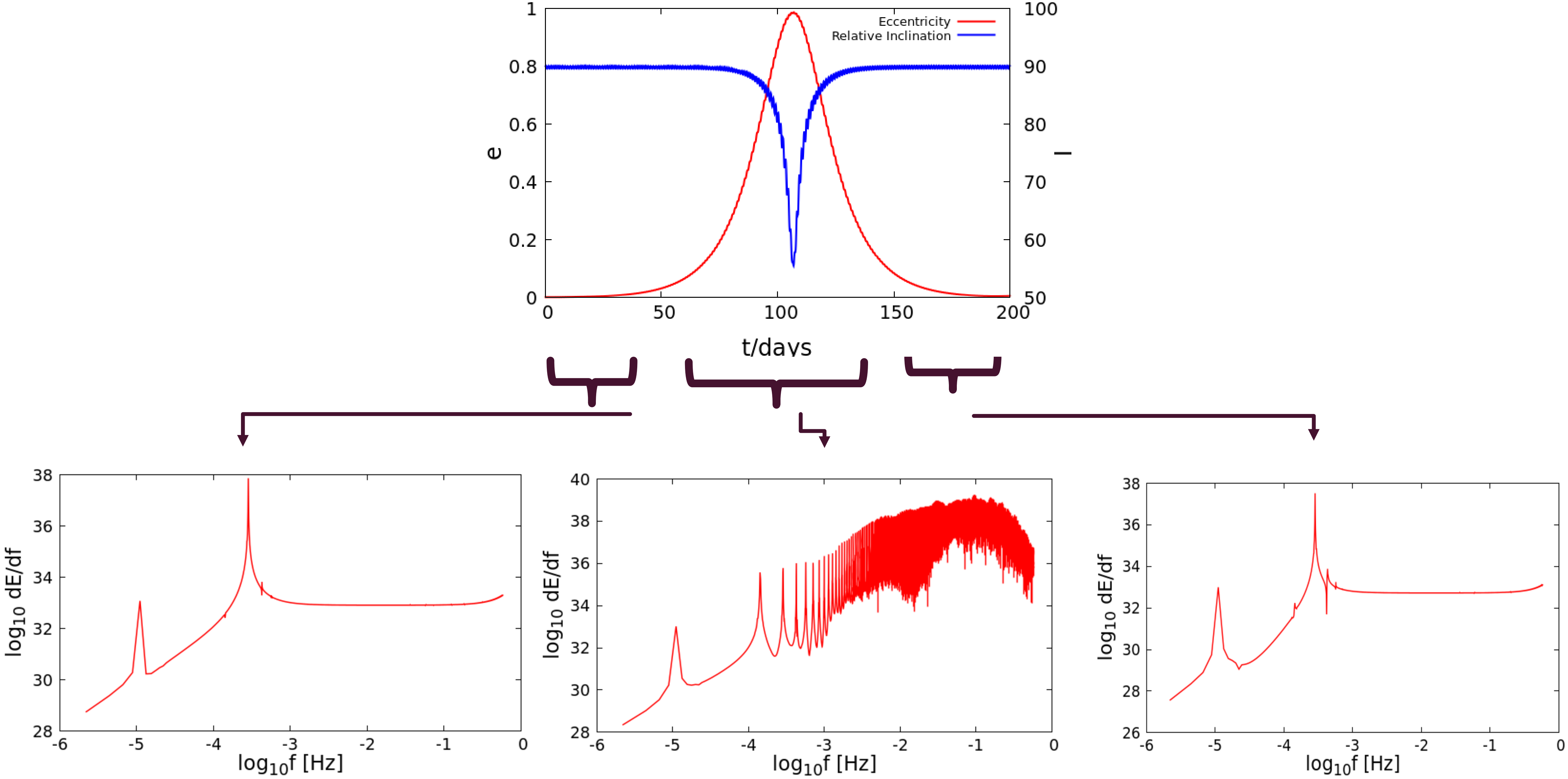
Result – Gravitational Waveform







Result – Energy Spectra



Result – Observability

Frequency Evolution

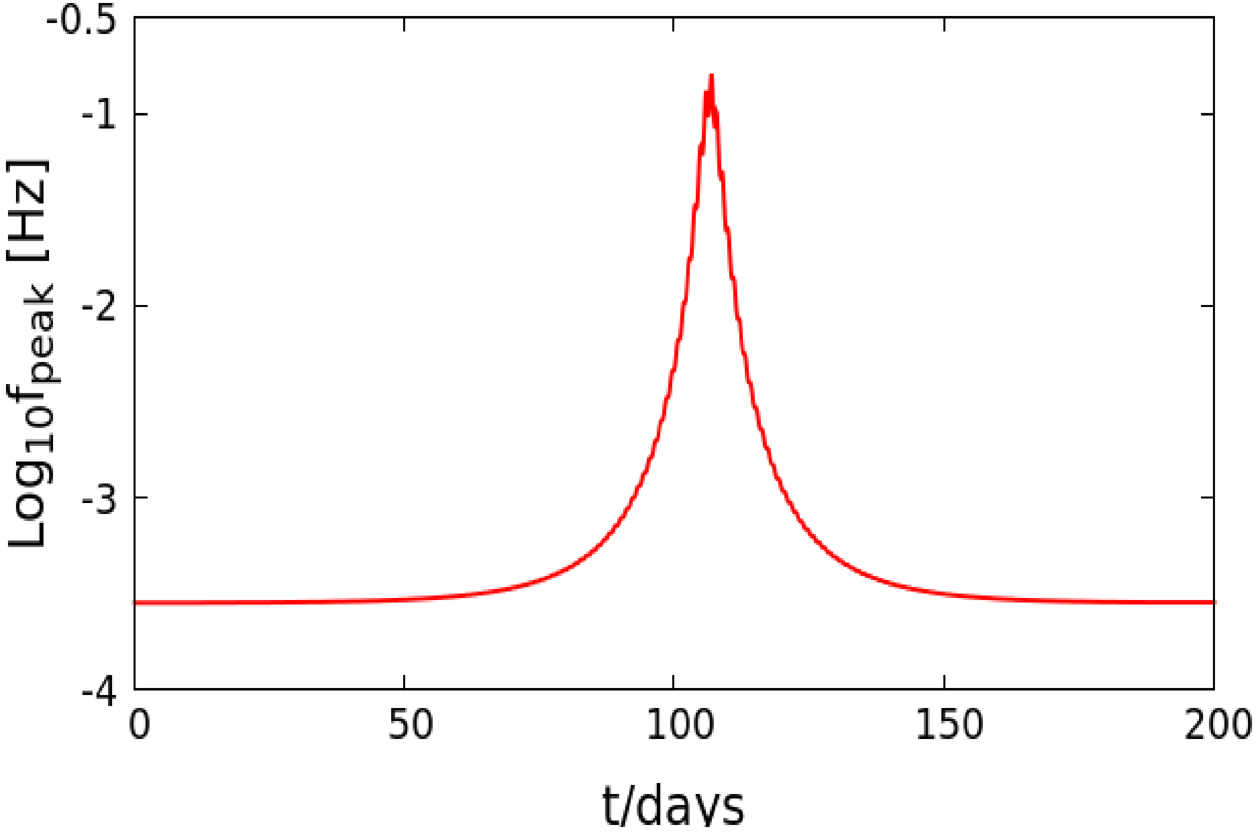
$$f_{\text{peak}} \sim 3 \times 10^{-8} n_m \left(\frac{m_1 + m_2}{M_\odot}\right)^{\frac{1}{2}} \left(\frac{\text{AU}}{a_{\text{in}}}\right)^{\frac{3}{2}} \text{ Hz}$$

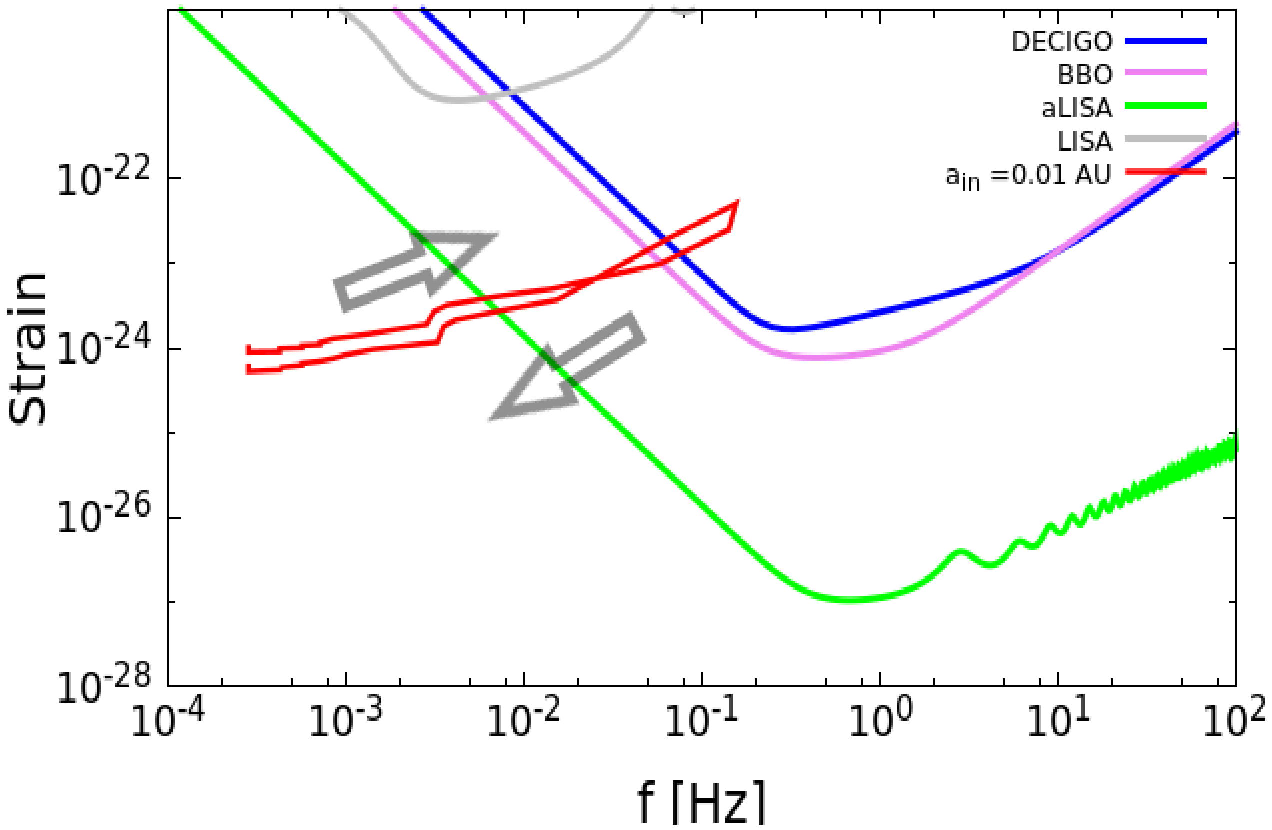
$$n_m = \frac{2(1+e)^{1.1954}}{(1-e^2)^{1.5}}$$

Wen 2003

Strain Evolution

$$h_c \sim n_m 10^{-33} \left(\frac{m_1 + m_2}{M_\odot}\right)^{\frac{5}{2}} \left(\frac{\text{AU}}{a_{\text{in}}}\right)^{\frac{5}{2}} \left(\frac{10\text{kpc}}{D}\right)$$





STABILITY & CONSTRAINTS

1) STABILITY OF HIERACHICAL TRIPLE SYSTEM

Mardeling et al. 2002

$$\frac{a_{\rm out}}{\rm AU} > 2.8\left[\left(1+\frac{m_3}{M_\odot}\frac{M_\odot}{m_1+m_2}\right)\right]^{\frac{2}{5}}\left(\frac{a_{\rm in}}{\rm AU}\right)$$

2) CONSTRAINT DUE TO GR PRECESSION

Blaes et al. 2002

$$\left(\frac{a_{\rm out}}{\rm AU}\right)^3 < 10^8\left(\frac{a_{\rm in}}{\rm AU}\right)^4\left(\frac{M_\odot}{m}\right)^2\left(\frac{m_3}{M_\odot}\right)\left(1-e_{\rm in}^2\right)^{3/2}$$

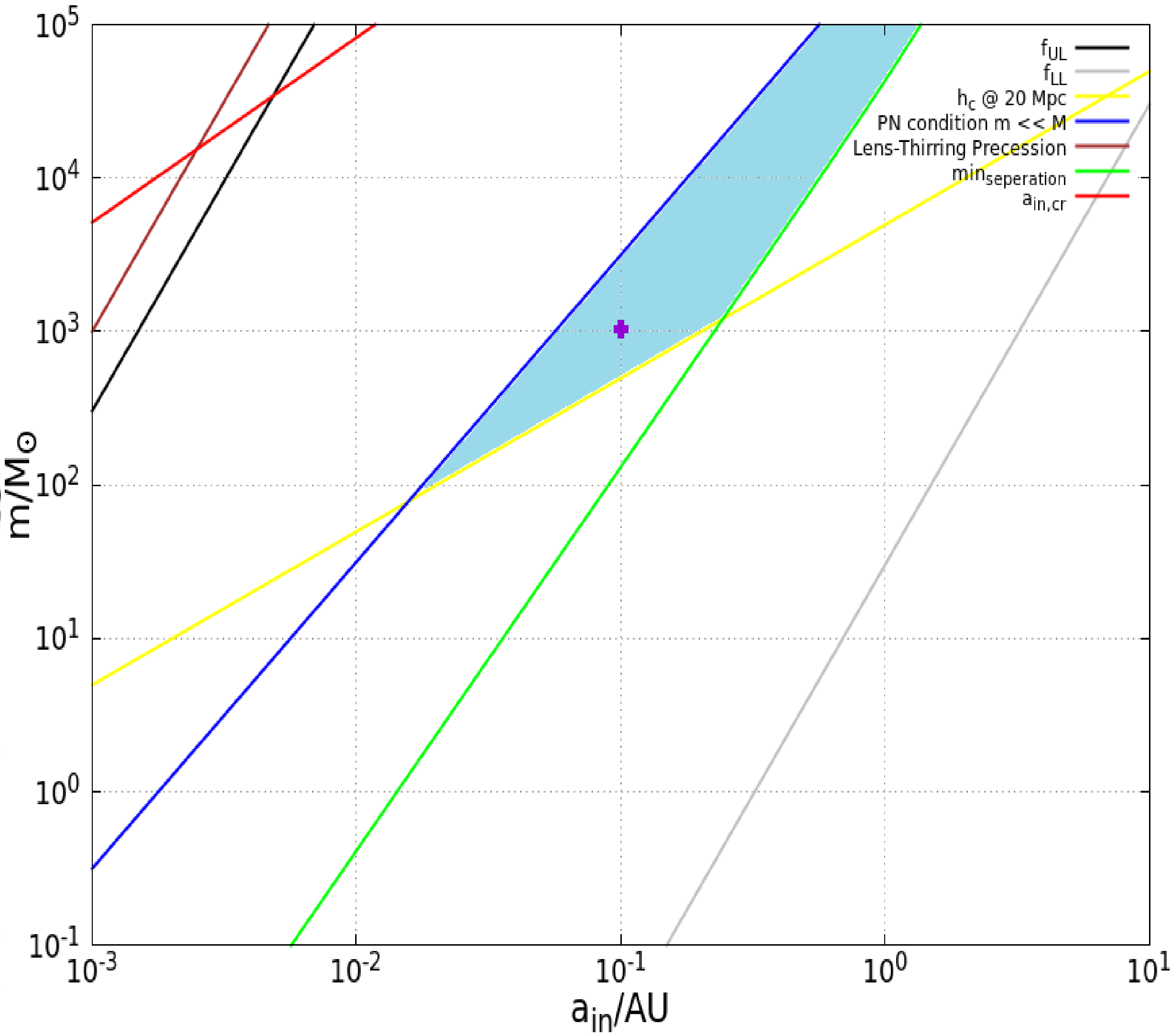
3) CONSTRAINT DUE TO LENSE-THIRRING EFFECT

Liu et al. 2019

$$\frac{\sigma_{L_{\rm out}S_3}}{\sigma_{KL}} = 10^{-12}\left(\frac{\rm AU}{a_{\rm in}}\right)^{3/2}\left(\frac{m_3}{M_\odot}\right)\left(\frac{m}{M_\odot}\right)^{1/2} < 1.$$

$$\sigma_{L_{\rm out}S_3} = \frac{GS_3(4+3m/m_3)}{2c^2a_{\rm out}^3(1-e_{\rm out}^2)^{3/2}}$$

$m_3 = 10^6 M_\odot$; $a_{out} = 10\ AU$



SUMMARY

- ✓ Kozai-Lidov effect can be seen in the time evolution of the waveform of the inner binary in a hierarchical triplet.
- ✓ Due to high eccentricity, we can see many harmonics in the energy spectra of the waveform.
- ✓ The frequency & strain range may lie in future space-based detectors (DECIGO, BBO).
- ✓ Commenting on event rate is hard due to many uncertainties.

  (Antonini et al 2015 , A.A Trani 2019) $[2-5\ Gpc^{-3}yr^{-1}]$
- ✓ Effect of K-L oscillations on indirect observation.

  (H. Suzuki, P. Gupta, H. Okawa, K. Maeda 2019)