Gravitational waves from protoneutron stars

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collaborated with

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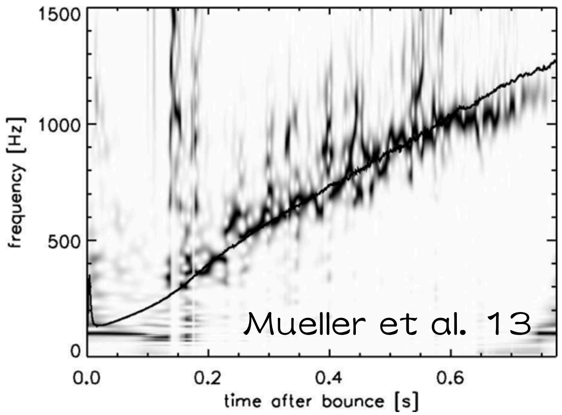
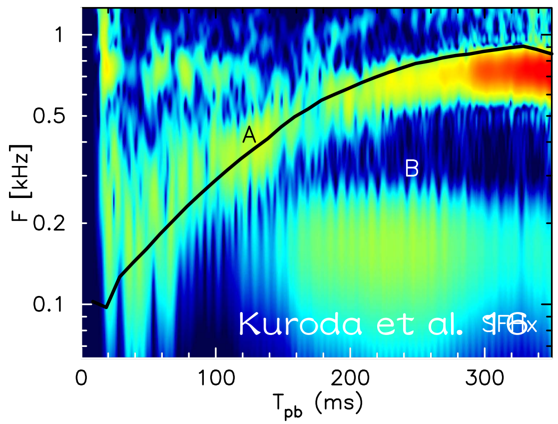
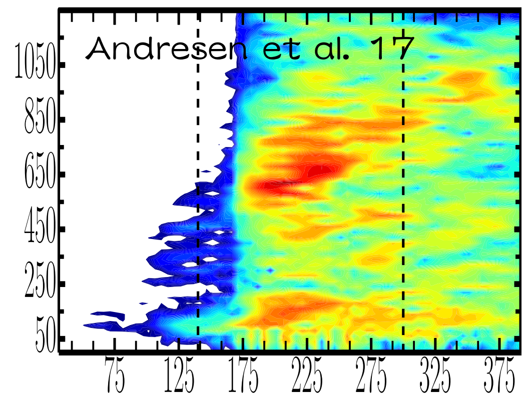
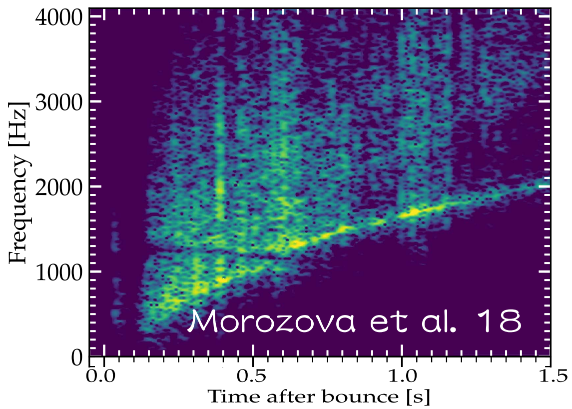
2nd candidate as GW sources

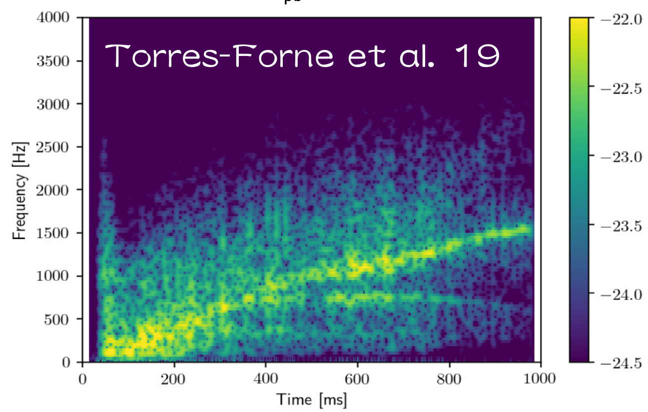
- supernovae
  - event rate : ~1/100 yr in our galaxy
  - compered to binary merger, system is more spherically symmetric
    - less energy of gravitational waves
  - many numerical simulations show the existence of GW signals





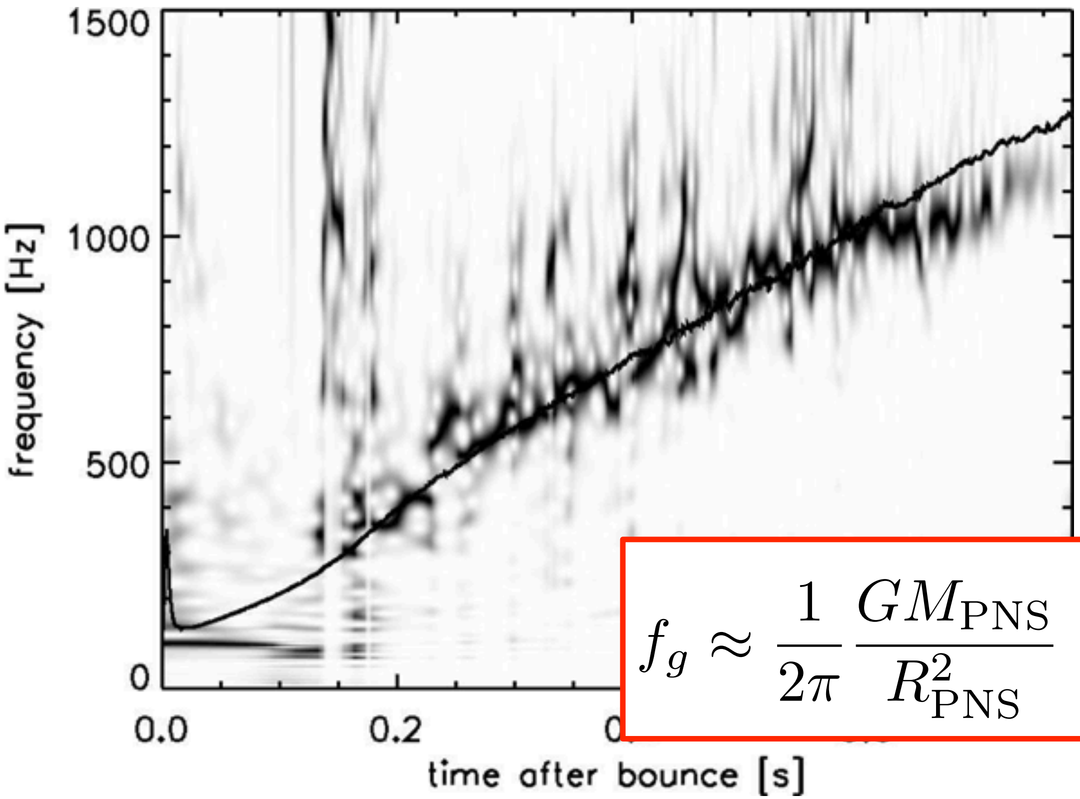


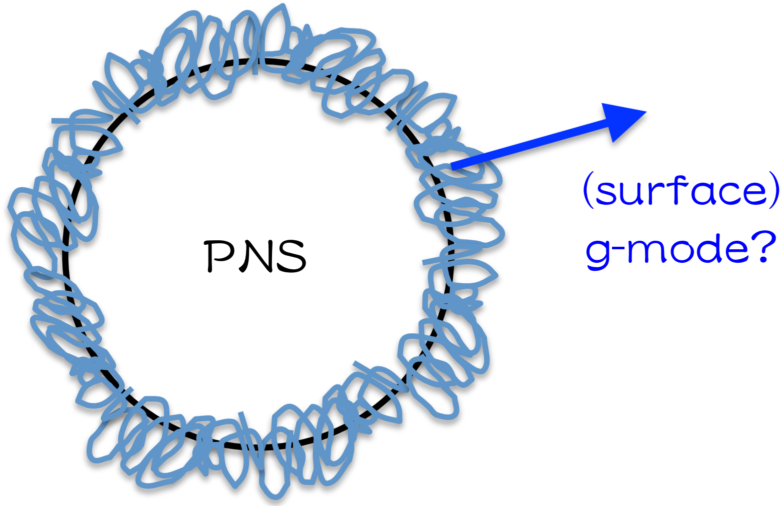




(surface) g-mode oscillations?

- 2D non-rotation with convection by Mueller et al. (2013)
  → excitations of specific frequency





the convection & the standing accretion-shock instability

$$f_g \approx \frac{1}{2\pi}\frac{GM_{\rm PNS}}{R_{\rm PNS}^2}\left(\frac{1.1 m_n}{\langle E_{\bar{\nu}_e}\rangle}\right)^{1/2}\left(1-\frac{GM_{\rm PNS}}{c^2 R_{\rm PNS}}\right)^2$$

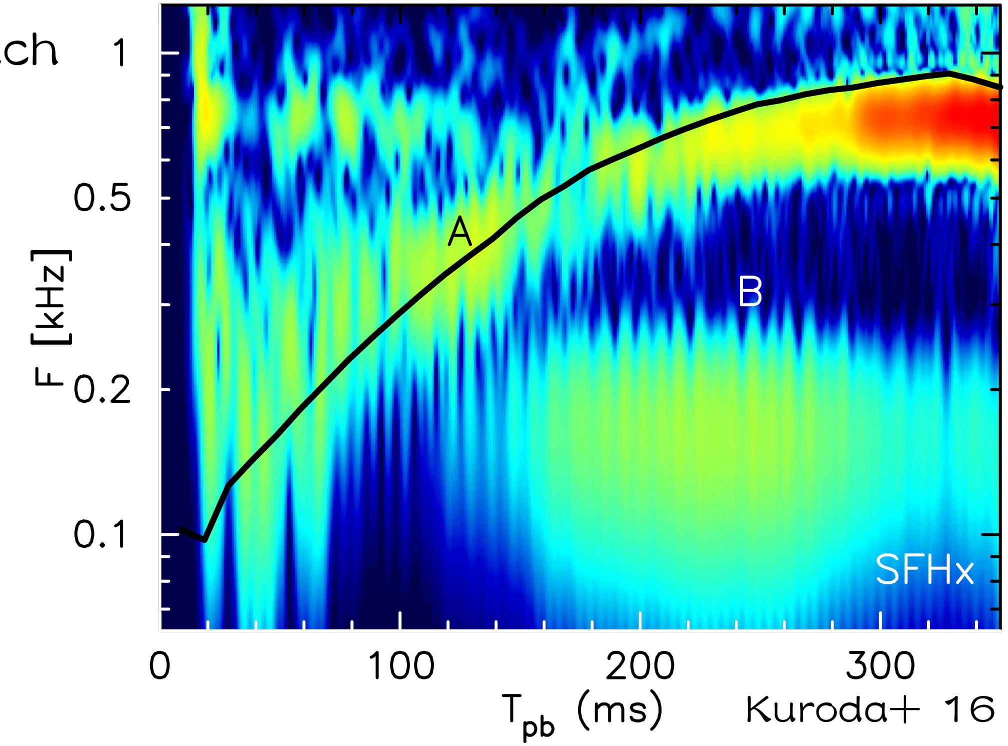
BV frequency @$r = R_{\rm PNS}$

(similar expression is also given in Cerda-Duran+13)

GW signals in numerical simulations

- Numerical simulations are very powerful, but...
  - still difficult to extract GW signal, especially from protoneutron stars (PNSs).
  - additionally, not easy to understand the physics in PNSs directly from numerical results
- linear analysis is another approach
  - GW asteroseismoogy in PNS
  - *what is the physics behind the GW signals ?*

similar motivation in the talks
by Cerda-Duran in the 1st week &
by Torres-Forne in the 2nd week



Kuroda+ 16

linear analysis (asteroseismology)

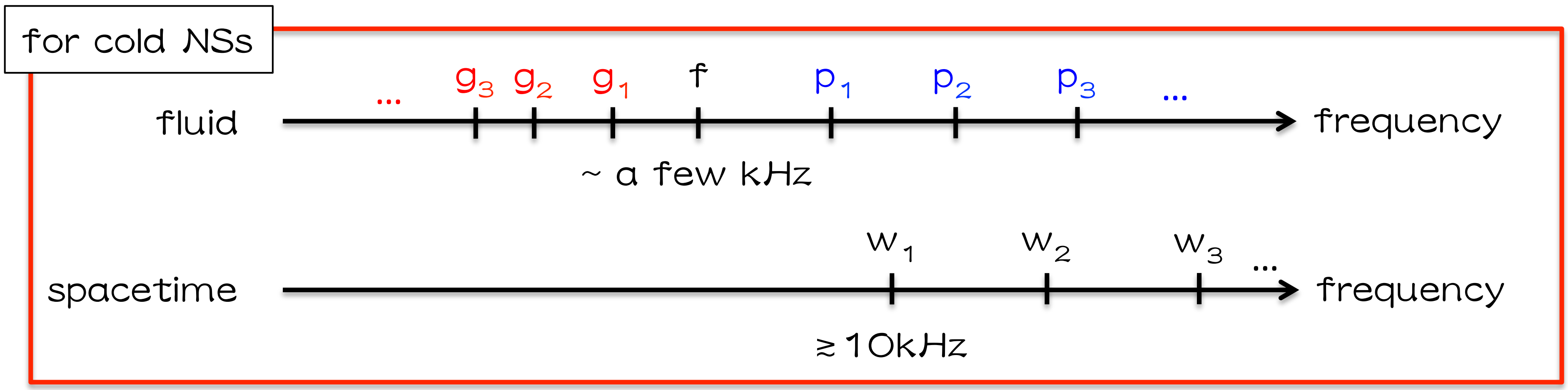
- variables = background + perturbations

$$A = A_0 + \delta A$$

- expand the perturbed variables

$$\delta A(t,r,\theta,\phi) = \delta A(r) e^{i\omega t} Y_{lm}(\theta,\phi)$$

  - if background is spherically symmetric, the perturbations are independent from *m*
  - ω is an eigenfrequencies of star for each *l*, where f = $\omega/2\pi$
  - subscript denotes the number of radial nodes in eigenfunction



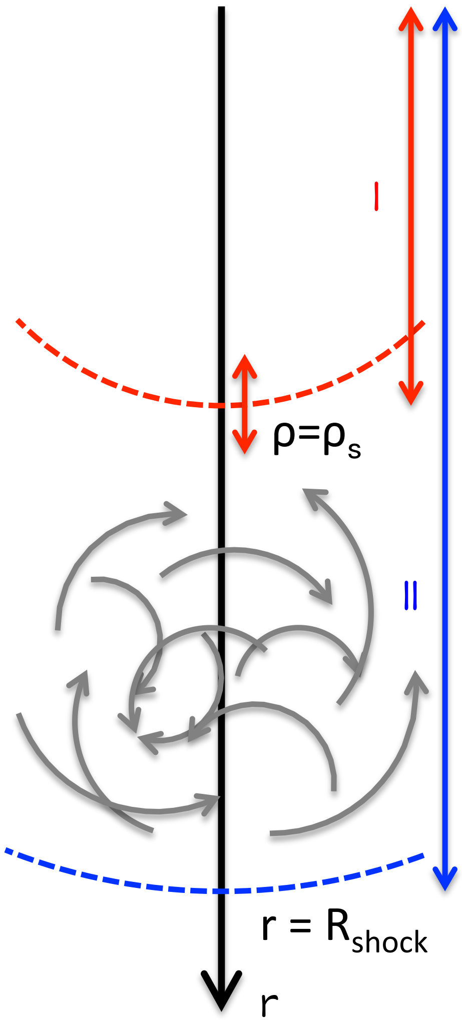
different two approaches

- PNS models, whose surface defined with a specific surface density, ρ_s (Model I)
  - Sotani+16; 1D-Newton, without rotation
  - Sotani+17; 3D-GR, without rotation
  - Morozova+18; 2D-effective GR, without rotation
  - Radice+19; 3D-effective GR, without rotation
  - Sotani+19; 3D-GR, without rotation

- Numerical region up to the shock radius, R_{shock} (Model II)
  - Torres-Forne+18; 2D-GR, with rotation
  - Torres-Forne+19a; 2D-GR, with rotation/2D-effective GR, without rotation
  - Torres-Forne+19b; 1D-Newton/effective GR/GR, without rotation

- With either I or II, to prepare the background PNS model for linear analysis, the numerical data is averaged in the angular direction, assuming the static solution at each time step.
  - linear analysis on the static, spherically background model.

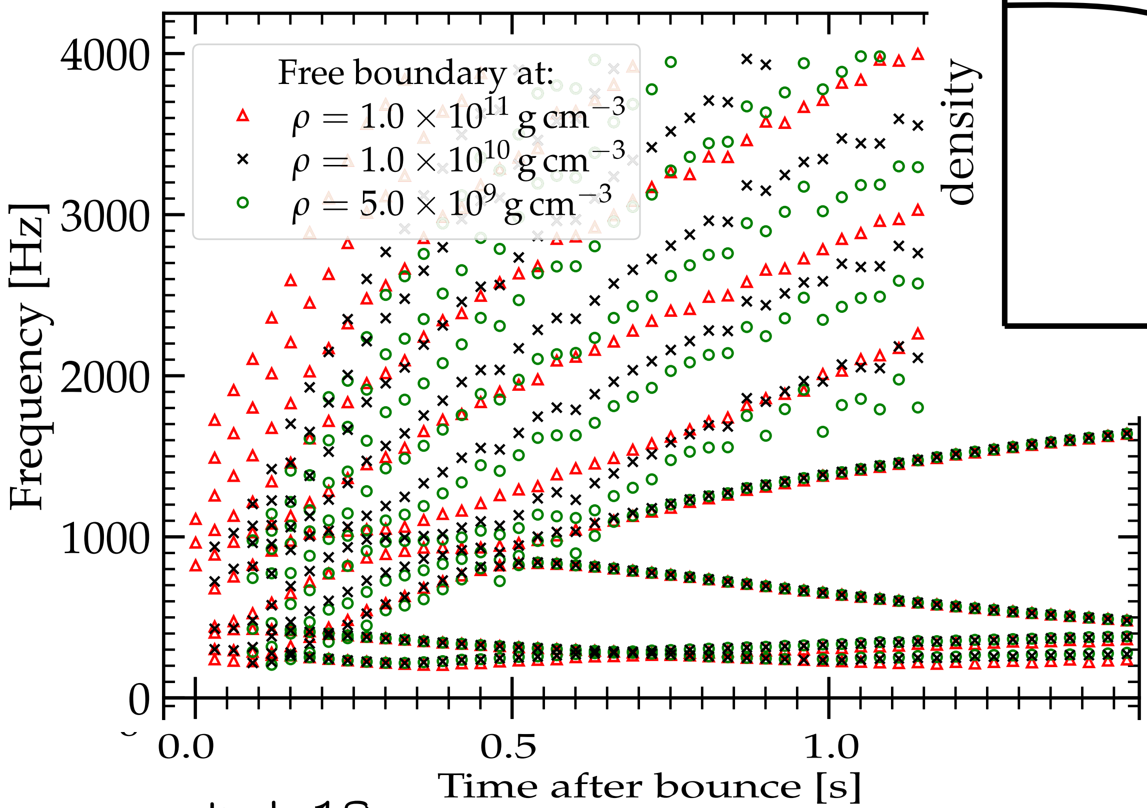
Difference in two approaches

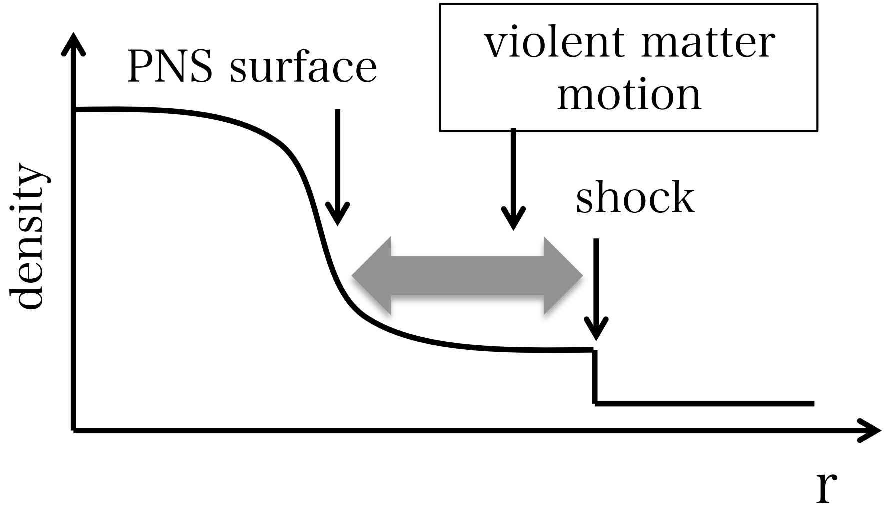


- computational domain
  - Model I : only inside R_{PNS} defined by ρ_s
  - Model II : $\delta\xi^r = 0$ @r = R_{shock}
- Boundary condition for solving the eigenvalue problem
  - Model I : $\Delta p = 0$ @r = R_{PNS}
  - Model II : $\delta\xi^r = 0$ @r = R_{shock}
  - mathematically, problem to solve is complete different
- advantage
  - Model I : matter motion is relatively small
    mode classification is as usual
  - Model II : boundary is uniquely determined
- disadvantage
  - Model I : uncertainty in choice of ρ_s
  - Model II : matter motion may not be negligible outside R_{PNS}
    mode classifications is different from the standard one.

Uncertainty from ρ_s in Model I

- in the late phase after core bounce, e.g., ~ 500ms, f-mode freq. becomes almost independent of the choice of ρ_s



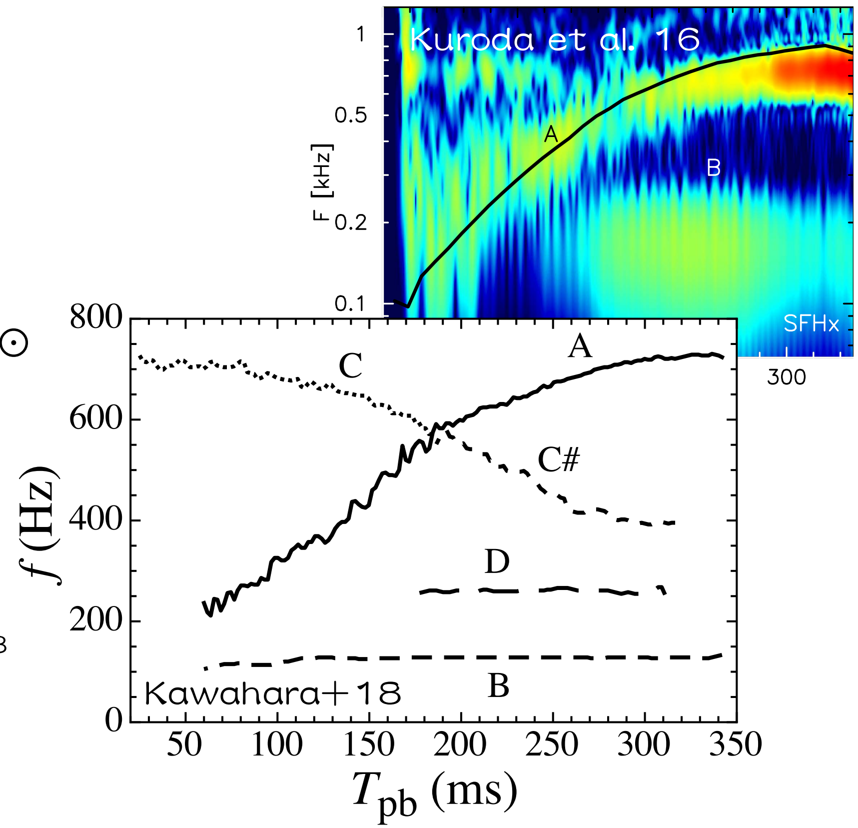


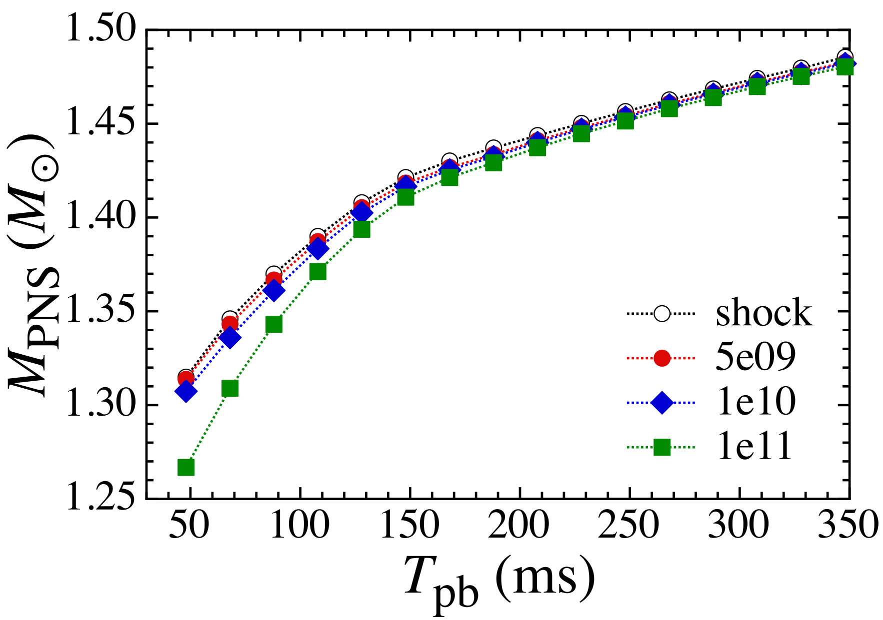
Morozova et al. 18

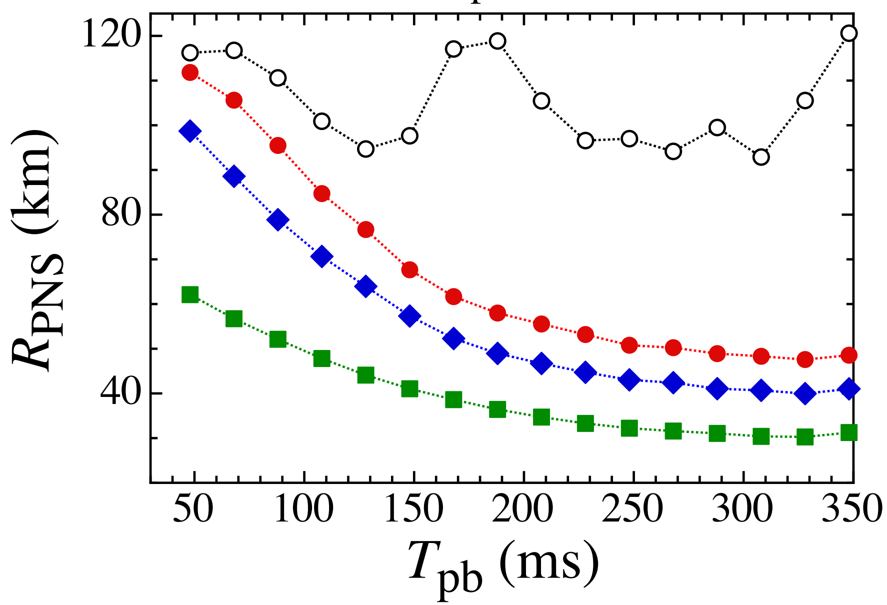
Comparison between I & II

PNS models (HS+19)

- 3D-GR core-collapse simulations with a 15M⊙ progenitor model and with SFHx EOS
  - sequence A corresponds to the so-called surface g-mode
- calculate the both Models I and II
  - for Model I, ρ_s = 5×10⁹, 10¹⁰, and 10¹¹ g/cm³

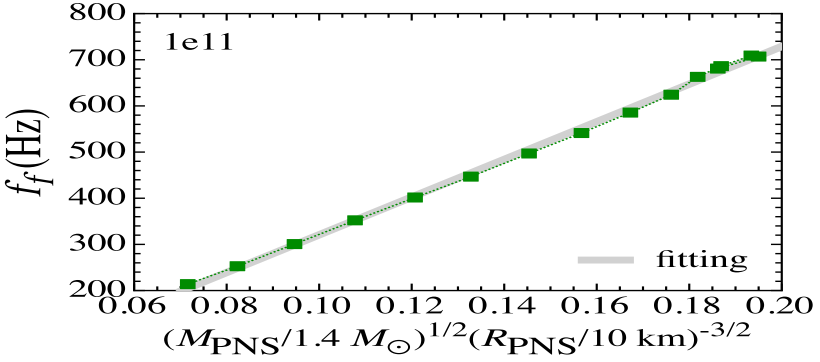




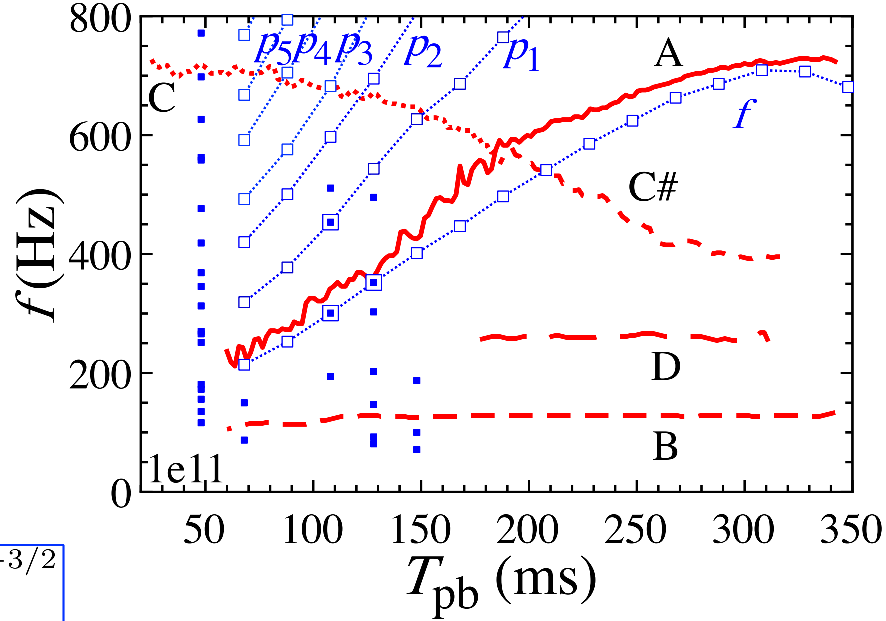


Model I (ρ_s=10^{11} g/cm^3)

- sequence A agrees well with f-mode oscillations
  - via GW observations, one could extract the PNS properties, i.e., average density

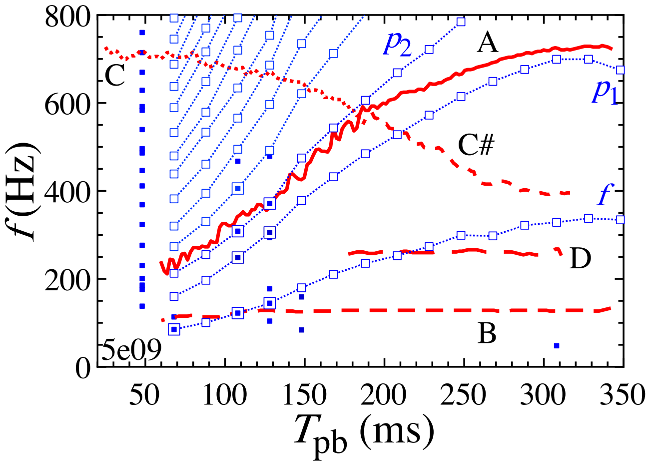


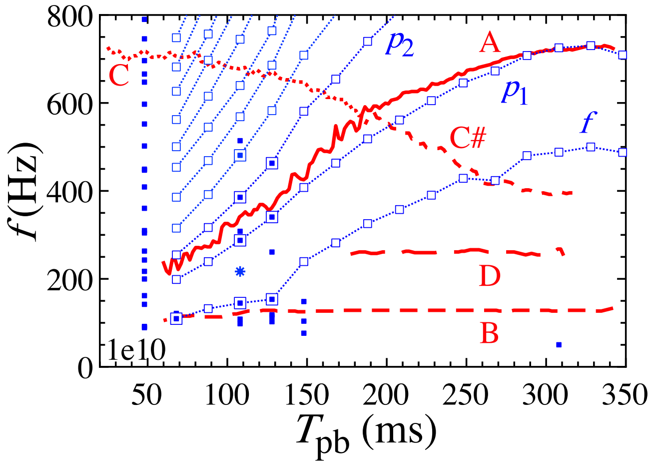
$$f_f\ (\text{Hz}) = -87.34 + 4080.78 \left(\frac{M_{PNS}}{1.4M_\odot}\right)^{1/2} \left(\frac{R_{PNS}}{10\text{ km}}\right)^{-3/2}$$



- we can not find a specific correspondence between modes and sequences of B, D, and C (C#)
- Also, we can not find the g-mode oscillations in this study
  - this is NOT due to the numerical code for eigenvalue problem
  - maybe the frequency is too small, or background data is not general ?

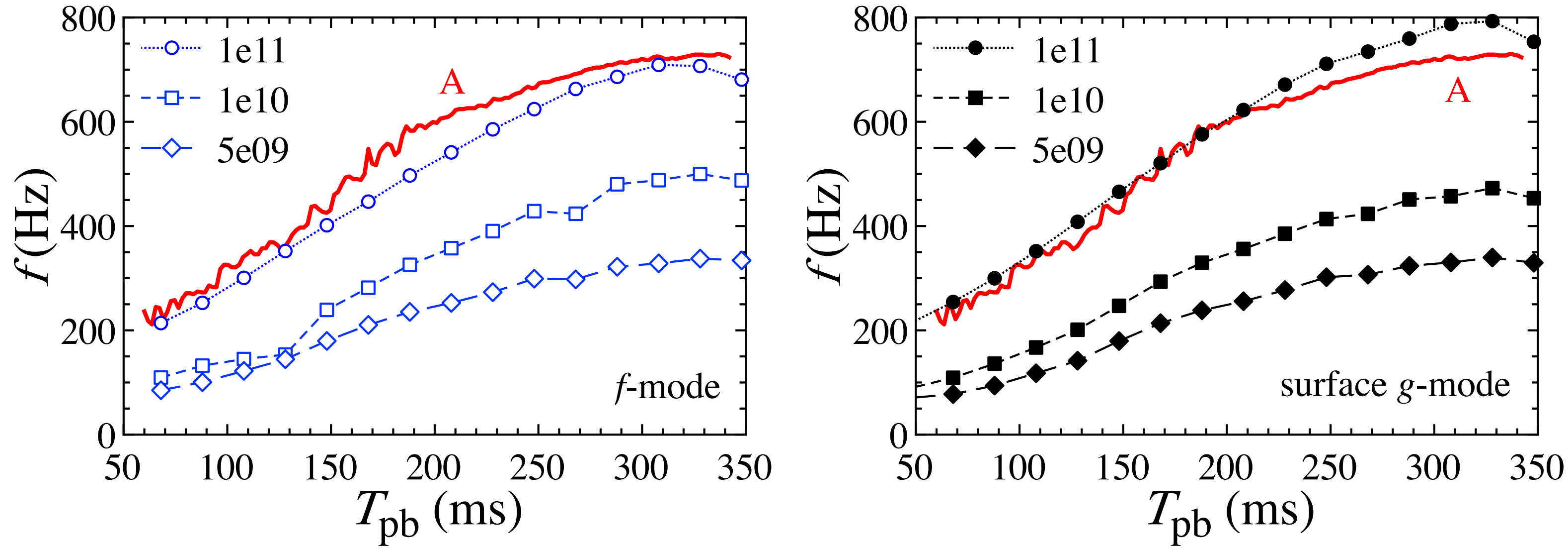
Model I (ρ_s=5x10^9 & 10^{10} g/cm^3)





- eigenfrequncies depend on ρ_s
  - as ρ_s decreases, PNS average density also decreases, which leads to the lower f- and p_i-mode GWs
- this dependence could appear only in the early postbounce phase
  - in the phase later than ~500ms after bounce, Morozova et al. (2018) showed that the eigenfrequencies are almost independent from the selection of ρ_s
  - this could be because the density gradient in the vicinity of PNS surface becomes steeper in the later phase, making the average density less sensitive to the selection of ρ_s

f-mode or surface g-mode?



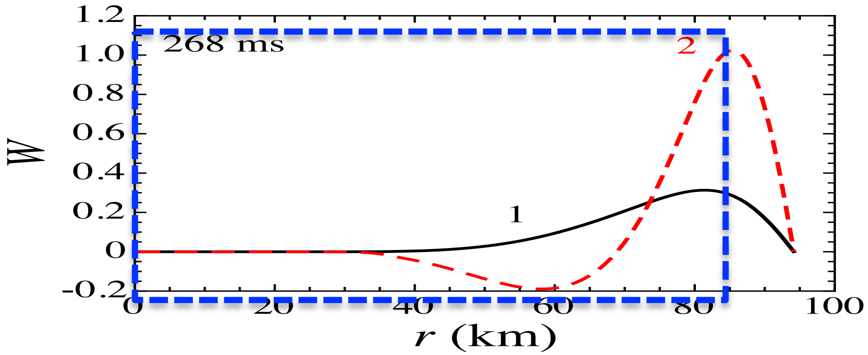
- The both frequencies strongly depend on ρ_s
  - agree well with the GW signal of A for PNS with $\rho_s = 10^{11}$ g/cm^3

$$f_g \approx \frac{1}{2\pi}\frac{GM_{\rm PNS}}{R_{\rm PNS}^2}\left(\frac{1.1 m_n}{\langle E_{\bar{\nu}_e}\rangle}\right)^{1/2}\left(1-\frac{GM_{\rm PNS}}{c^2 R_{\rm PNS}}\right)^2$$

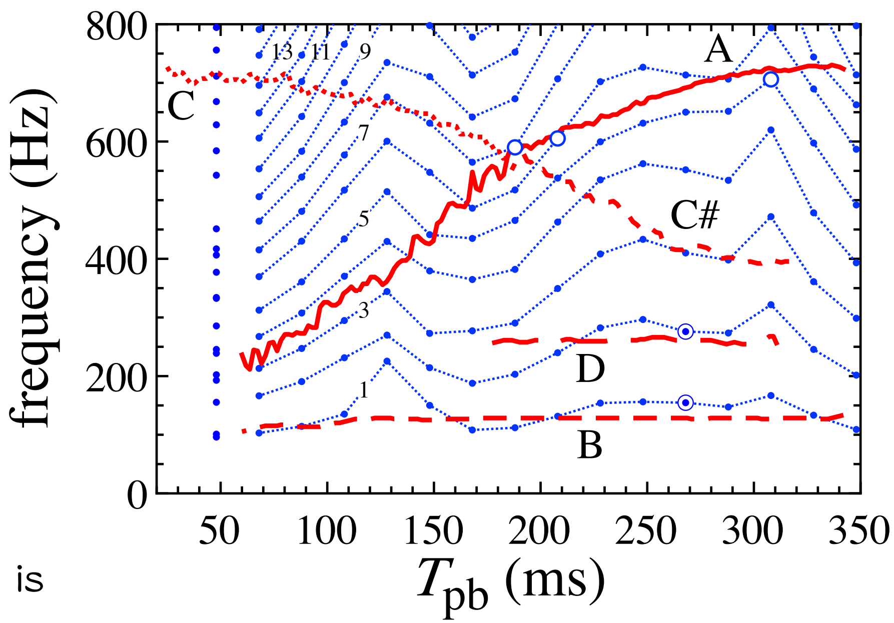
- Even so, since the surface g-mode (BV frequency @r=R_{PNS}) is local value, while f-mode is the global oscillations of PNS, it may be more natural that the GW signal A is considered as a result of the f-mode oscillations
  - by comparing to the GW signal in the later phase, one may conclude which modes (surface g- or f-) are suitable for the GW signals from PNS

Model II

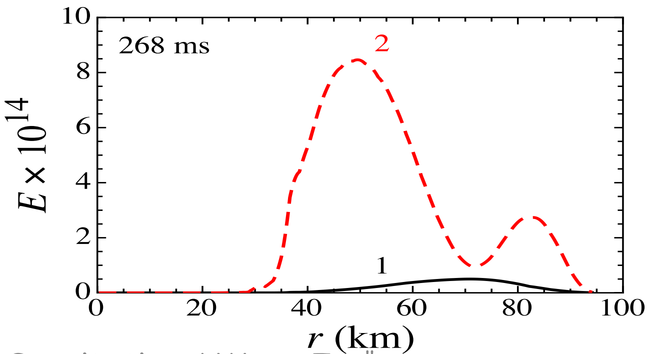
- nodal number monotonically increases from bottom to top



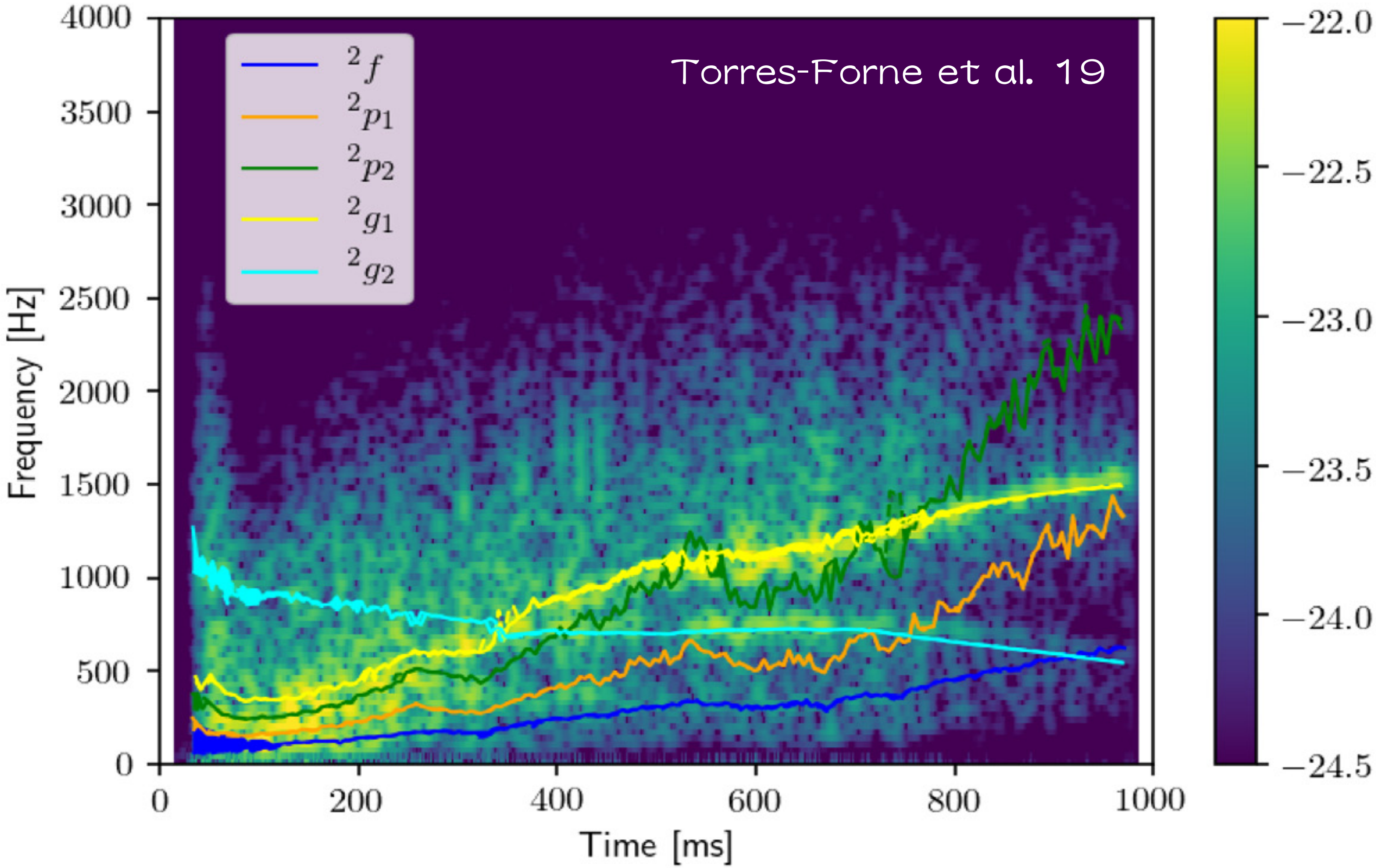
  - focus only on the dotted box, behavior is very similar to f- and p_1-modes



- we can not find a correspondence between A and a specific mode
  - difficult to explain A with a specific mode with Model II at lease our PNS model
- lower modes appear close to B and D
  - pulsation energy density concentrates r ~ 40-100km, while the energy of B and D effectively comes from r ~ 20 km
  - lower modes here do not physically correspond



discrepancy

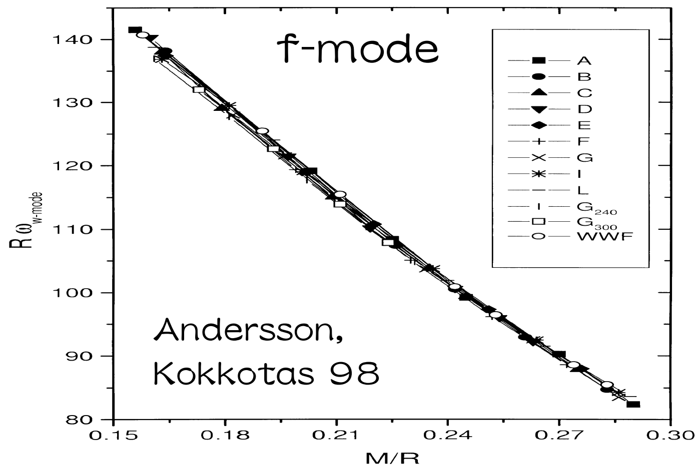


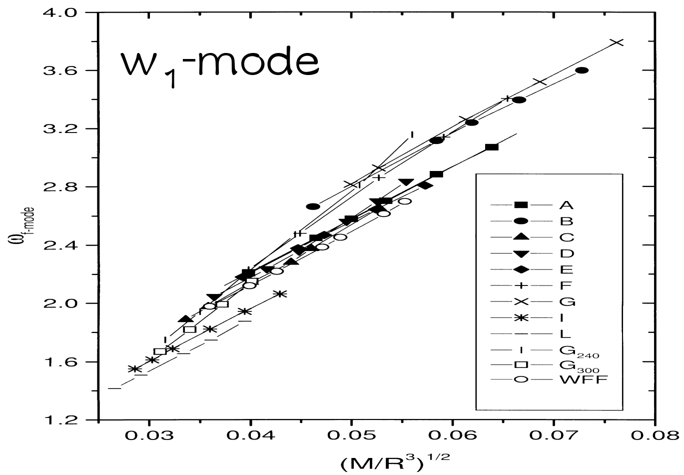
- Still, we can not understand this discrepancy.
- Our background data is not general?

PNS asteroseismology

what we learn from GW obs.

- From asteroseismological point of view, via direct observations of GWs, one may extract the PNS properties.
- In fact, it is known for cold neutron stars that
  - f-mode, which is a acoustic oscillation, is characterized by the stellar average density
  - w-mode, which is a spacetime oscillation, is characterized by the stellar compactness

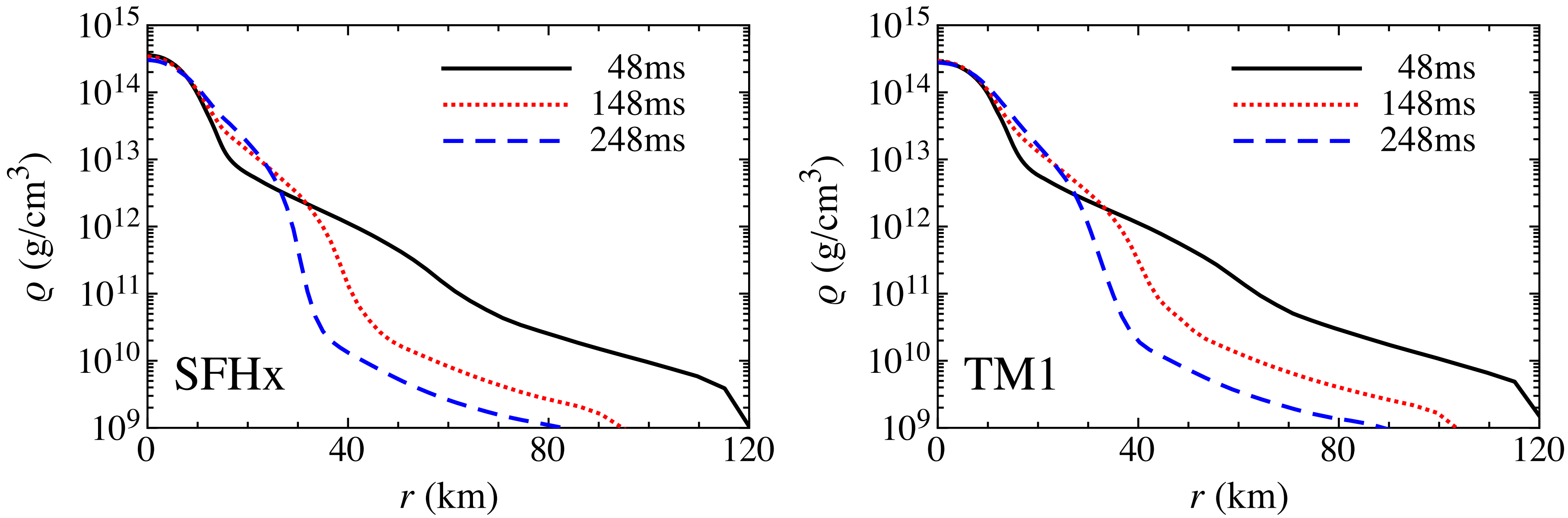




- If similar characterization is possible, one could extract the PNS average density and compactness, via the simultaneous observations of f- and w-modes GWs.

PNS models (HS+17)

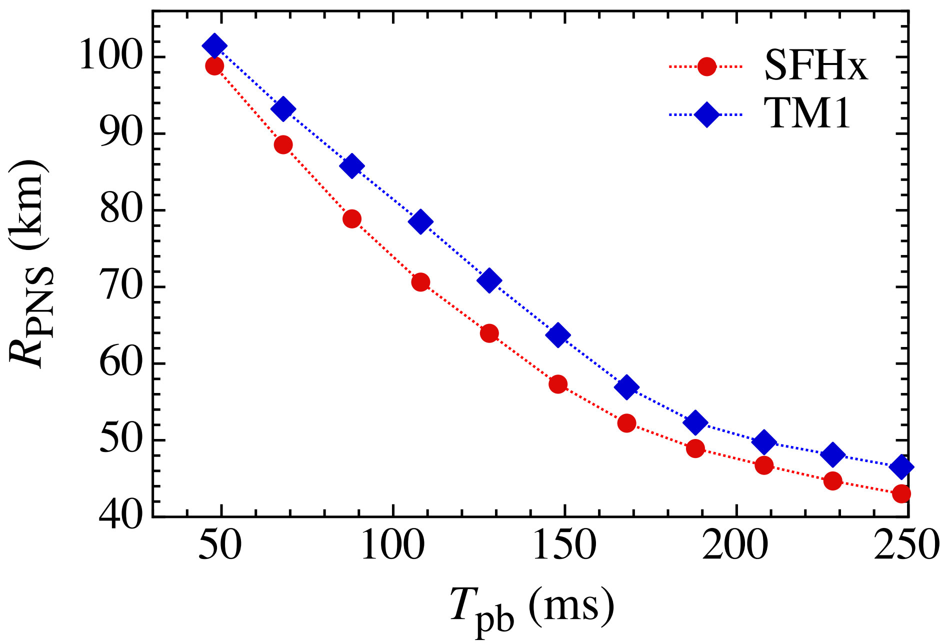
- we adopt the results of 3D-GR simulations of core-collapse supernovae (Kuroda et al. 2016)
  - progenitor mass = $15M_{\odot}$
  - EOS : SFHx ($2.13M_{\odot}$) & TM1 ($2.21M_{\odot}$)

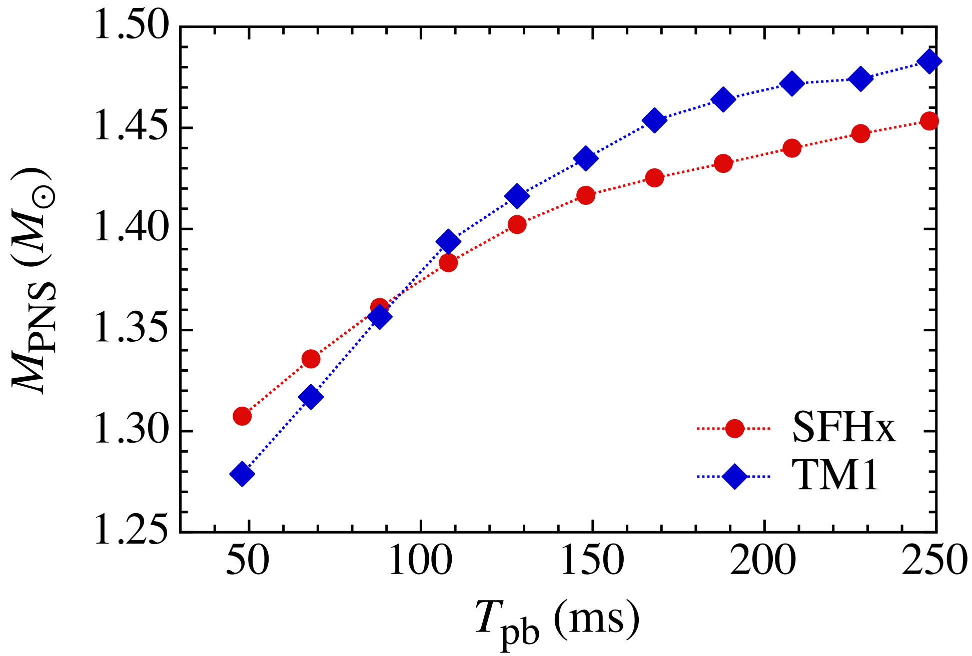


  - R_{PNS} is defined with $\rho_s = 10^{10}$ g/cm^3
  - using the radial profiles as a background PNS model, the eigen-frequencies are determined.

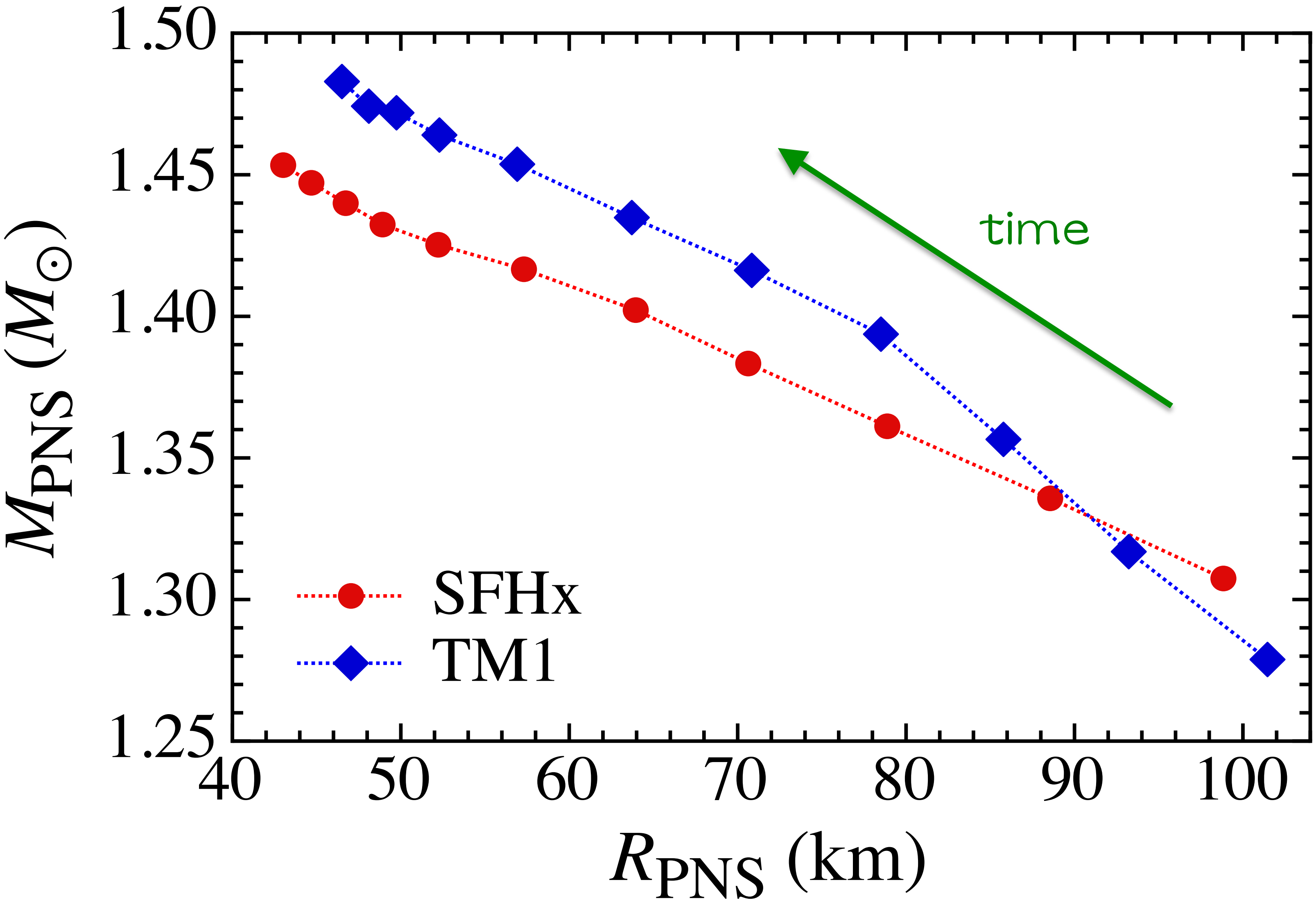
Mass & Radius

- M_{PNS} is increasing by mass accretion
- R_{PNS} is decreasing due to the relativistic effect





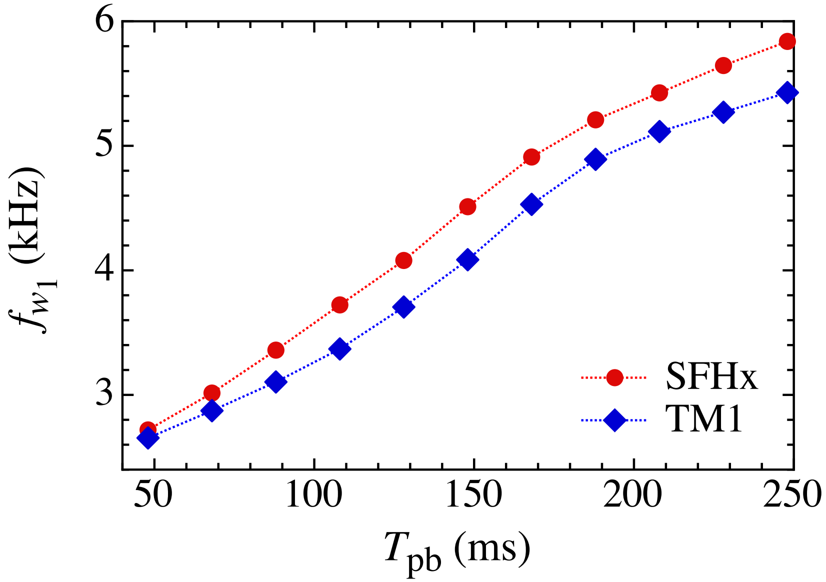
M-R evolution after core-bounce

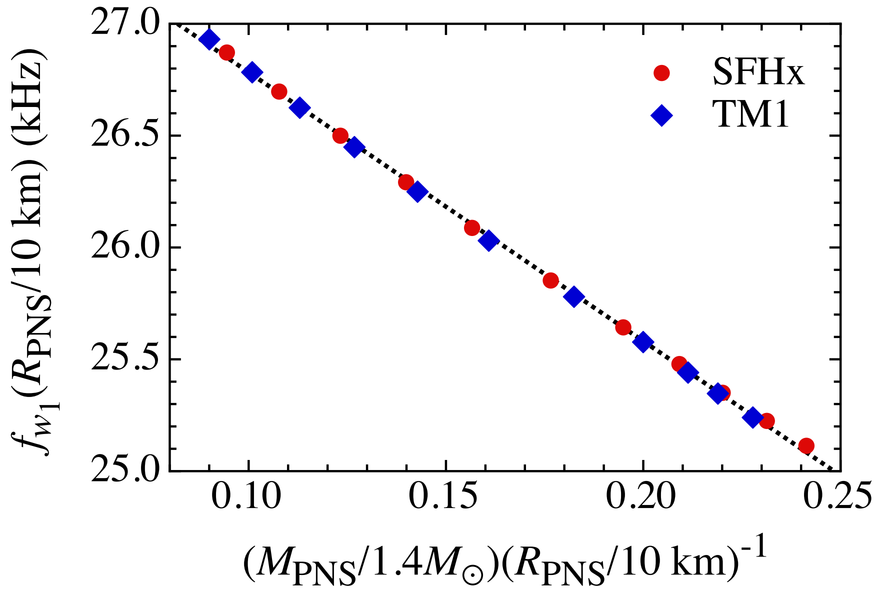


evolution of w_1-modes

- frequencies depend on the EOS.
  - increasing with time
  - can be characterized well by M_{PNS}/R_{PNS}
- as for cold NS, we can get the fitting formula, almost independent from EOS

$$f_{w_1}^{(PNS)}(\mathrm{kHz}) \approx \left[27.99 - 12.02\left(\frac{M_{PNS}}{1.4\ M_\odot}\right)\left(\frac{R_{PNS}}{10\ \mathrm{km}}\right)^{-1}\right] \times \left(\frac{R_{PNS}}{10\ \mathrm{km}}\right)^{-1}$$

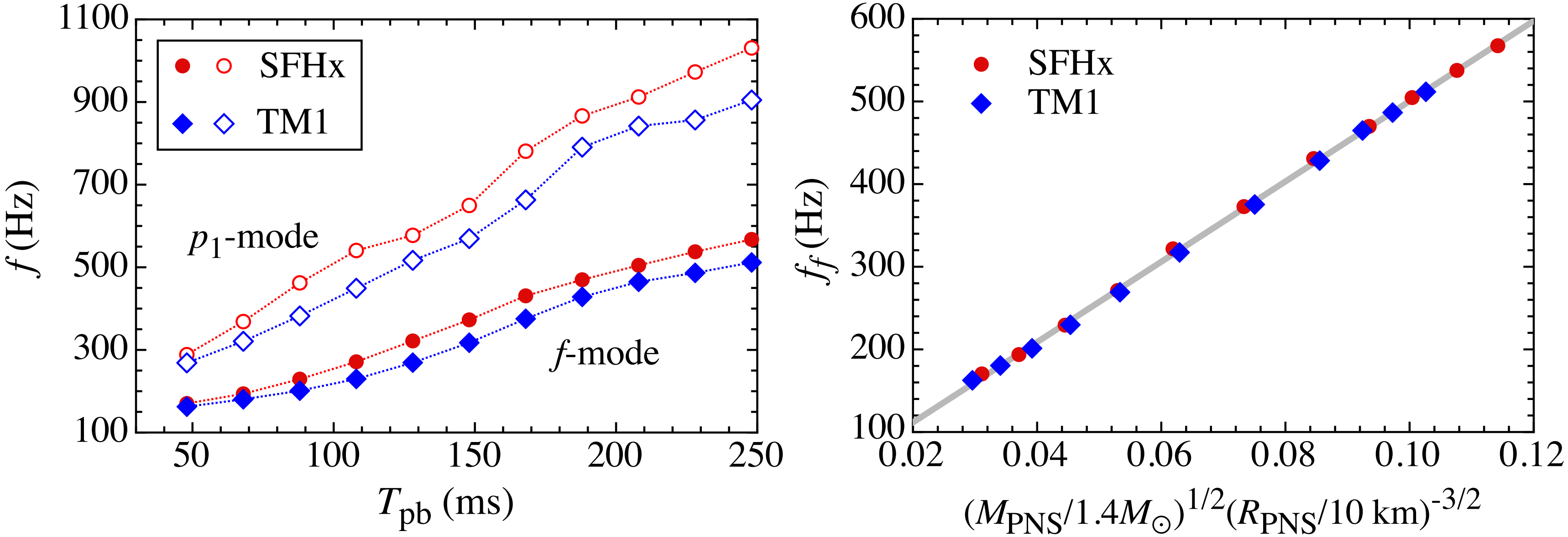




evolution of f-mode

- frequencies can be expressed well by the average density independent of the EOS (and progenitor mass)
- we derive the fitting formula as a function of M_{PNS}/R_{PNS}^3

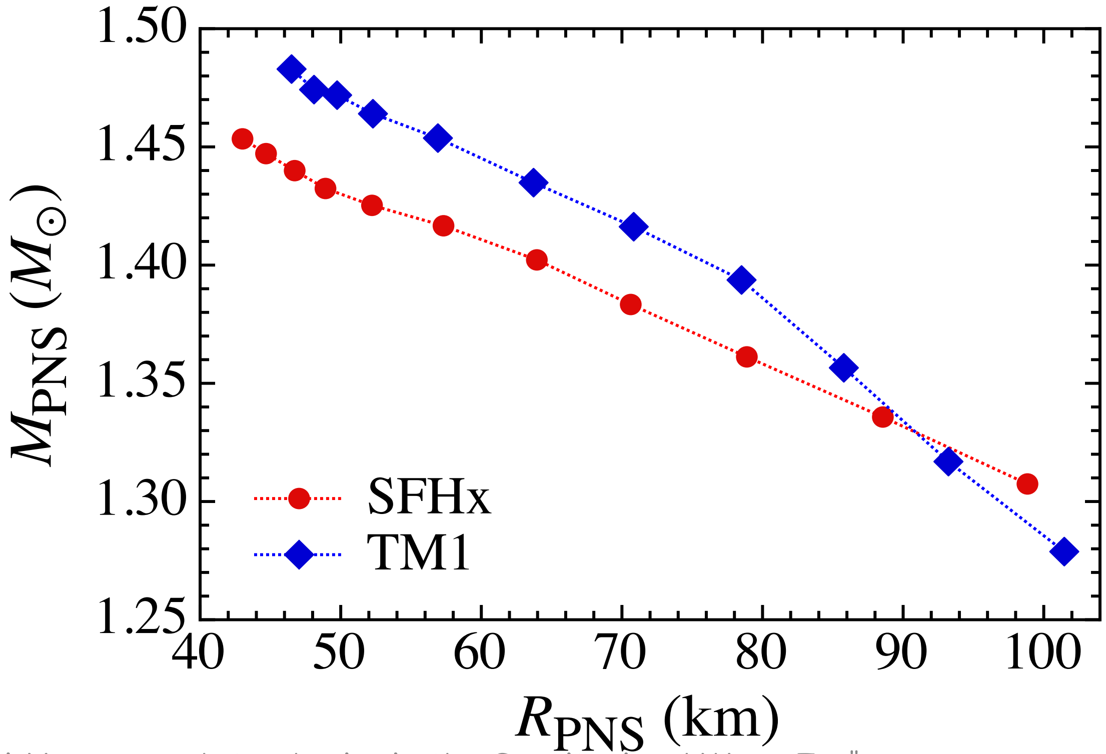
$$f_f^{(\mathrm{PNS})}(\mathrm{Hz}) \approx 14.48 + 4859 \left(\frac{M_{\mathrm{PNS}}}{1.4\ M_\odot}\right)^{1/2} \left(\frac{R_{\mathrm{PNS}}}{10\ \mathrm{km}}\right)^{-3/2}$$



* Note that we neglect the g-mode oscillations in this study

determination of EOS

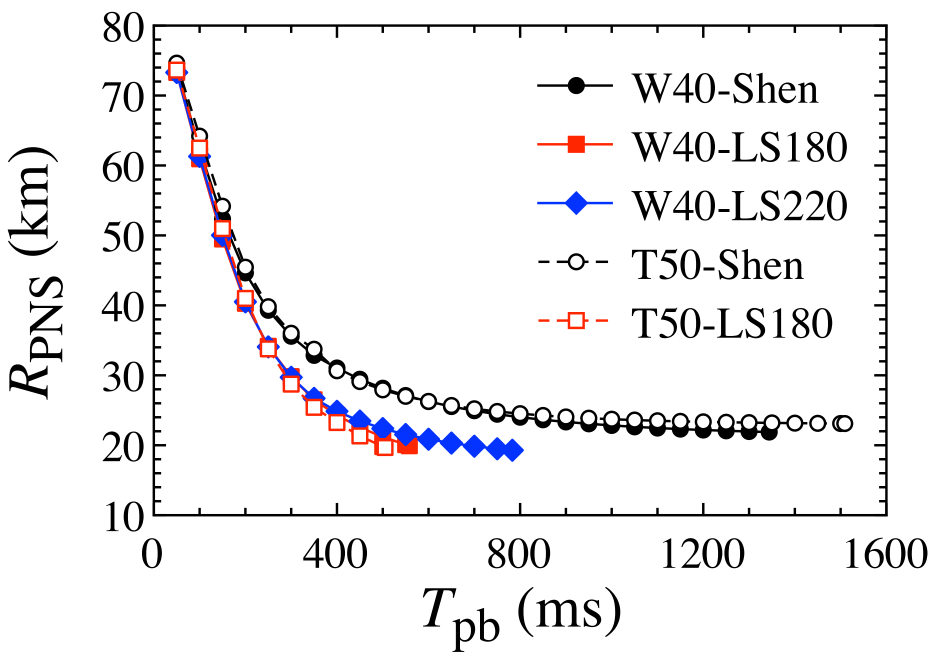
- GW spectra evolutions $f_f(t)$ & $f_{w1}(t)$
  → evolutions of M_{PNS}/R_{PNS}^3 & M_{PNS}/R_{PNS}
- one can determine (M_{PNS}, R_{PNS}) at each time after core bounce
  → determination of the EOS
- unlike cold NS cases, in principle one can determine the EOS even with ONE GW event !

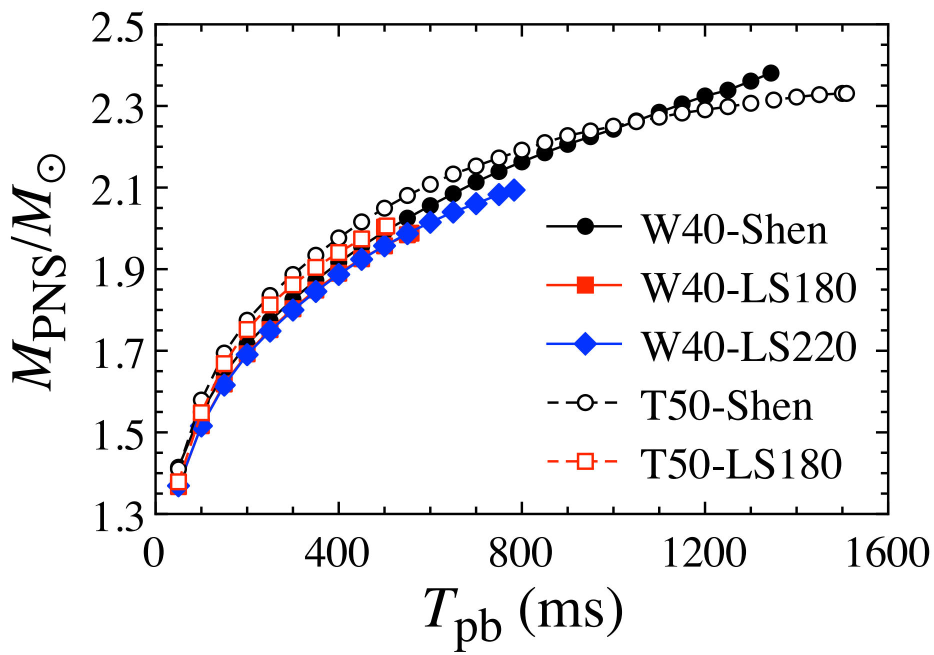


Case for BH formation

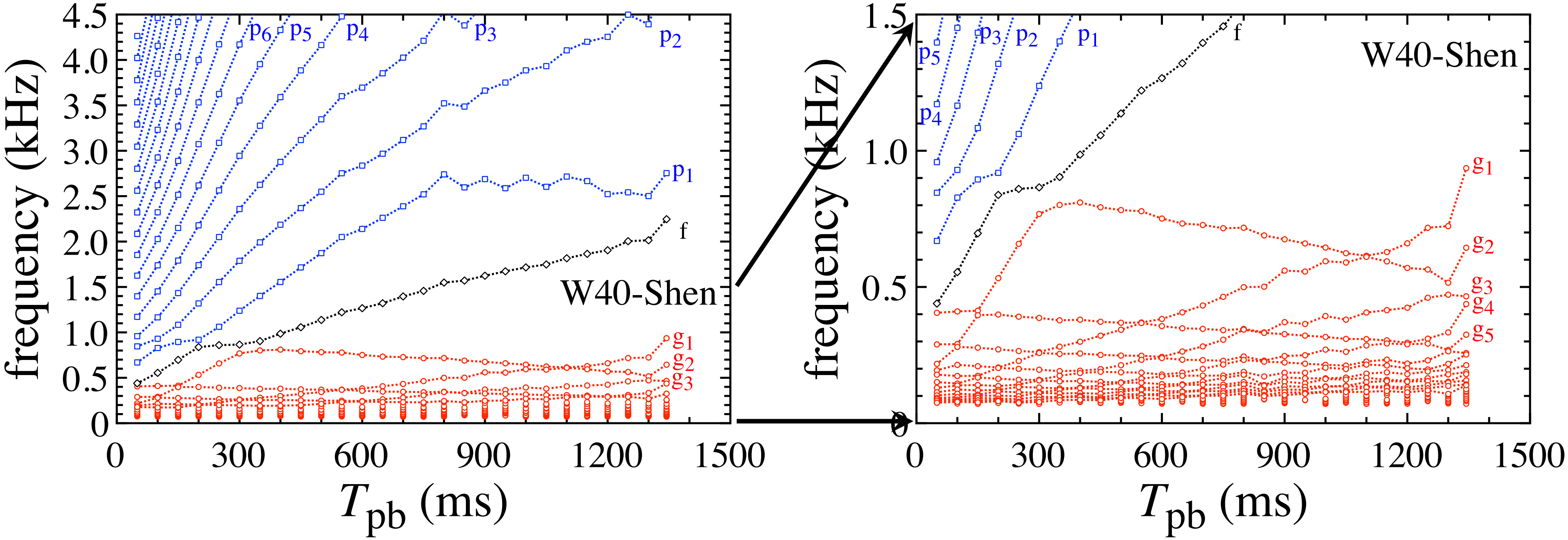
PNS models (HS & Sumiyoshi 19)

- 1D-GR core-collapse simulations (by Sumiyoshi)
  - 40M⊙ progenitor model (W40) based on Woosley & Weaver 95
  - 50M⊙ progenitor model (T50) based on Tominaga, Umeda & Nomoto 07
  - EOS: Shen (2.2M⊙), LS180 (1.8M⊙), LS220 (2.0M⊙)
  - surface density = 10^{11} g/cm^3





GW frequency for W40-Shen



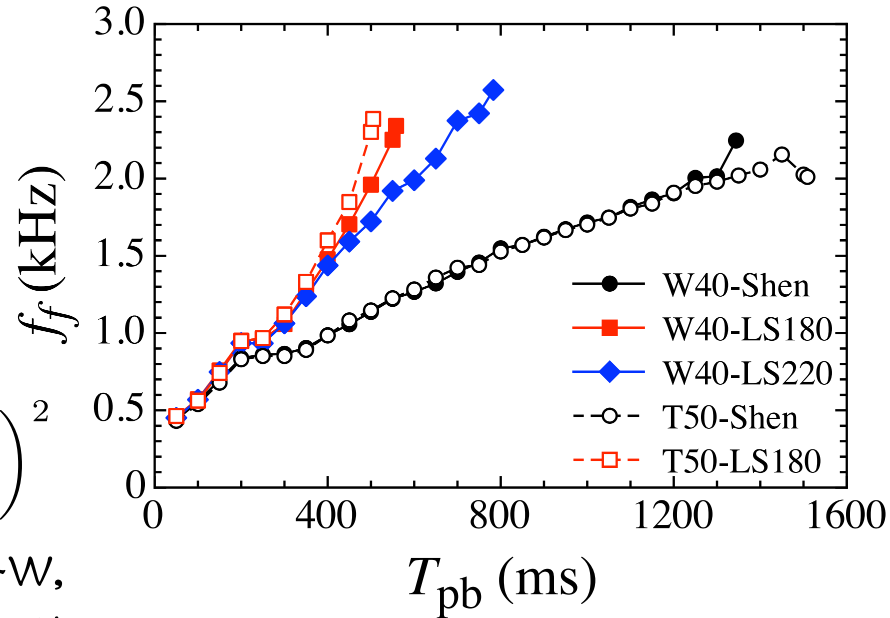
- One can clearly observe the phenomena of the avoided crossing in the evolution of GW frequency
  - focusing the f-mode GW,
    - T_{pb}~ 200 ms with p_1-mode,
    - T_{pb}~ (300-350) ms with g_1-mode

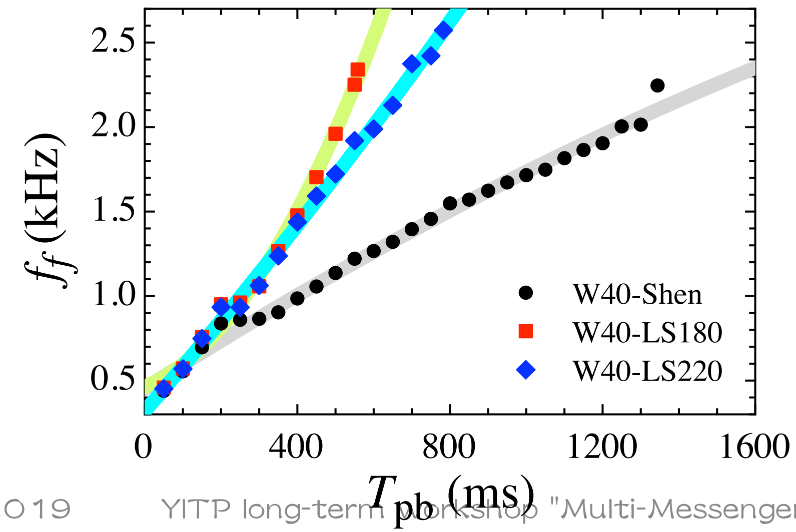
Dependence on PNS models

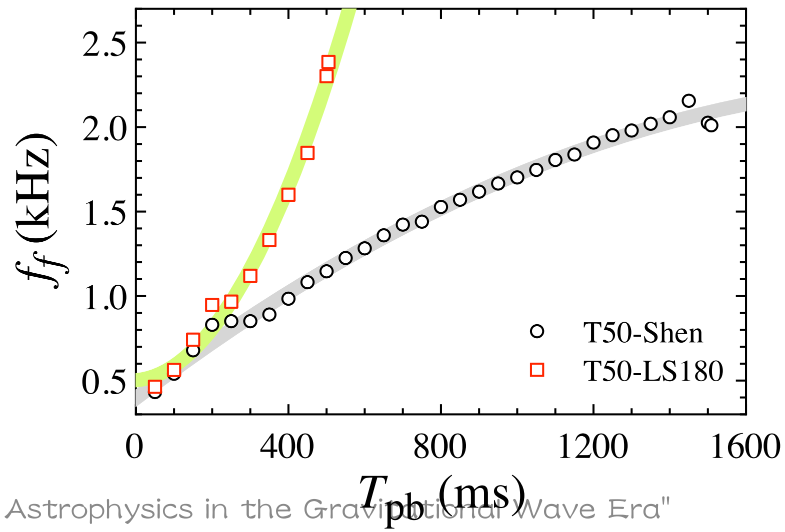
- Time evolution of f-mode GW strongly depends on the progenitor models.
- In any case, it can be well fitted as a function of T_{pb}, such as

$$f_f(\text{kHz}) = c_0 + c_1 \left(\frac{T_{\rm pb}}{1000\,\text{ms}}\right) + c_2 \left(\frac{T_{\rm pb}}{1000\,\text{ms}}\right)^2$$

  - one may expect high fre. f-mode GW, even though it is not detected directly.





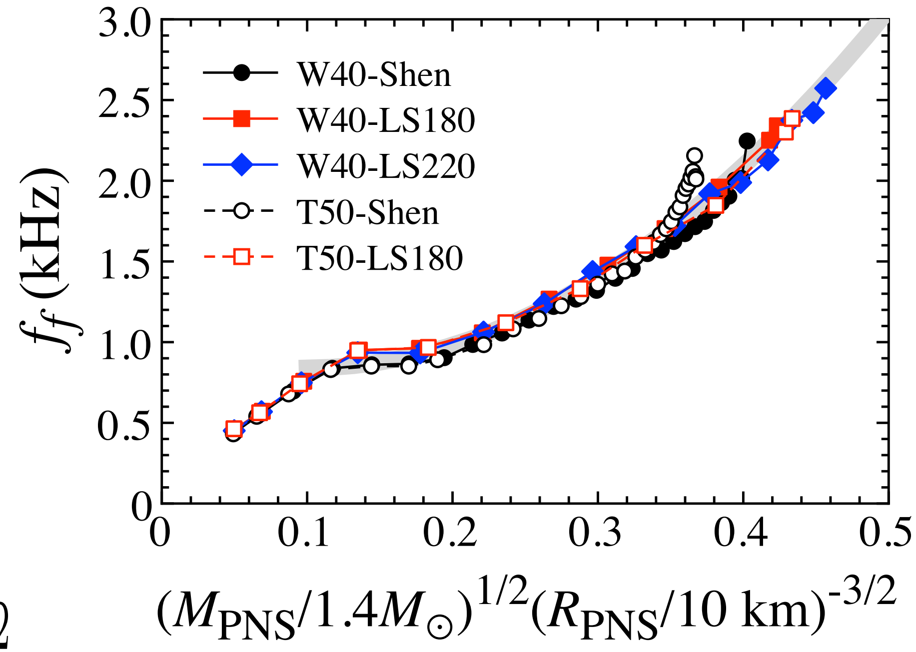


Universality in f-mode GWs

- The f-mode frequencies are well-expressed as a function of stellar average density, independently of progenitor models and EOSs.

$$f_f(\mathrm{kH}) = 0.9733 - 2.7171X + 13.7809X^2$$

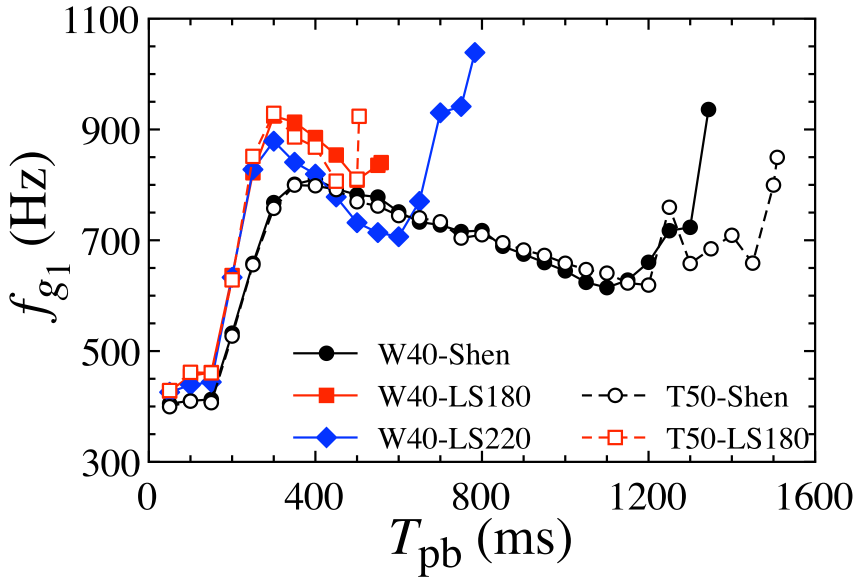
$$X \equiv (M_{\mathrm{PNS}}/1.4M_\odot)^{1/2}(R_{\mathrm{PNS}}/10\,\mathrm{km})^{-3/2}$$

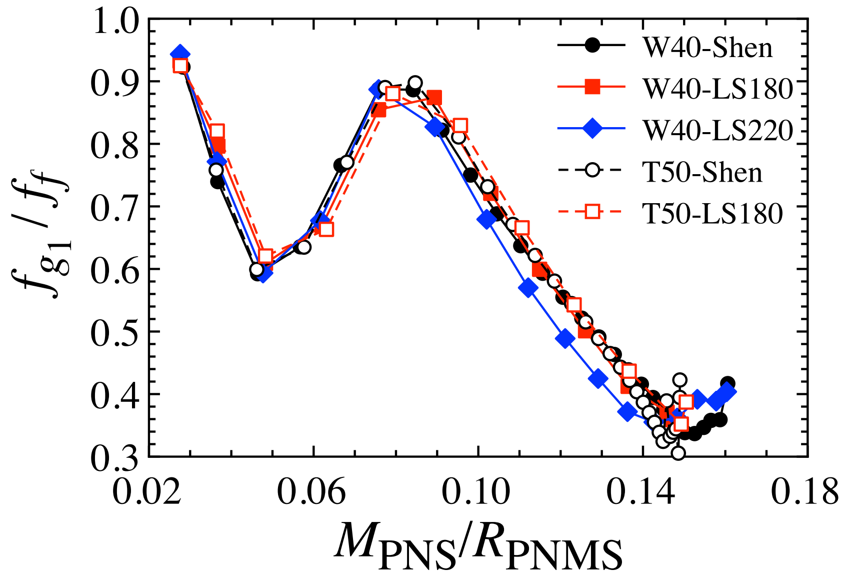


- Through the f-mode GW, one can extract the PNS average density, which leads to the time evolution of PNS average density.

g_1-mode GWs

- g_1-mode GW also strongly depends on the progenitor models.





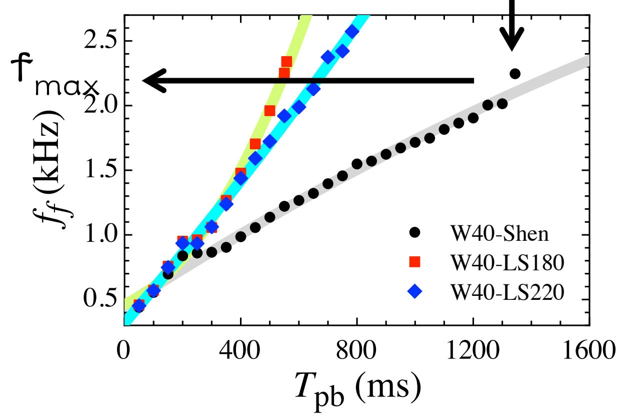
- Even so, we find that the ratio of g_1-mode to f-mode can be well-expressed as a function of PNS compactness, independently of the progenitor models.
- one can extract the PNS compactness via the simultaneous observations of g_1- and f-mode GWs.

PNS maximum mass

- PNS at the moment when it collapses to BH, corresponds to the PNS model with maximum mass.

  ↑ one can know via neutrino observation

- BUT, the f-mode frequency is too high to detector?



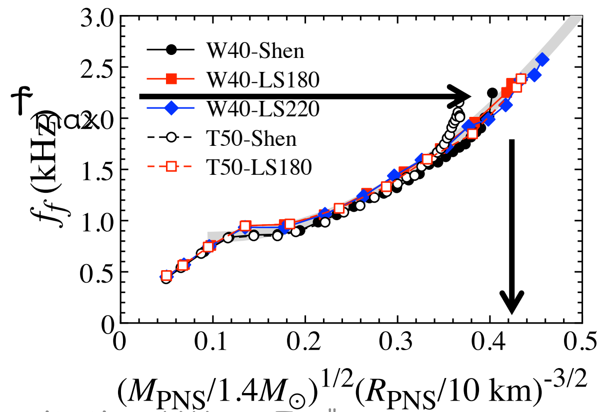
- How to determine the PNS property

① With the data of the f-mode GW, one can fit the time evolution of the f-mode GW

② Owning to the neutrino observation, one can know the moment when PNS collapses to BH

③ The f-mode frequency is expected via ① and ②

④ Via the universal relation of the f-mode, one can extract the average density of PNS with maximum mass



conclusion

- Asteroseismology could be a powerful technique for extracting the interior information.
- In the context of PNS asteroseismology, two different approaches are considered
  - The eigenvalue problem to solve is mathematically different each other
  - f-mode GW from PNS model with $\rho_s=10^{11}$ g/cm^3 agrees well with the GW signals obtained by the numerical simulation
- As for cold NSs, the f- and w_1-mode GWs from PNS can be characterized by the stellar average density and compactness, respectively.
  - via simultaneous observation of f- and w_1-mode GW, one can see the evolution of (M_{PNS}, R_{PNS}) after core bounce
  - in principle, even with ONE GW event from supernova, one might determine **the EOS** for high density region.
- we also consider the asteroseismology on the PNSs toward BH formation.
  - we find that, independently of the progenitor models,
    - the f-mode GW can be expressed as PNS average density, and
    - the ratio of g_1- to f-mode GWs can be expressed as PNS compactness.
  - owning to the neutrino obs., one would determine the average density of PNS with maximum mass by detecting the f-mode GW.