

Inverse reconstruction of jet structure from off-axis gamma-ray burst afterglows

Kazuya Takahashi & Kunihiro Ioka
(Yukawa Institute for Theoretical Physics,
Kyoto University)

Neutron Star Merger Event GW170817



**Gravitational wave
+ EM counterparts**

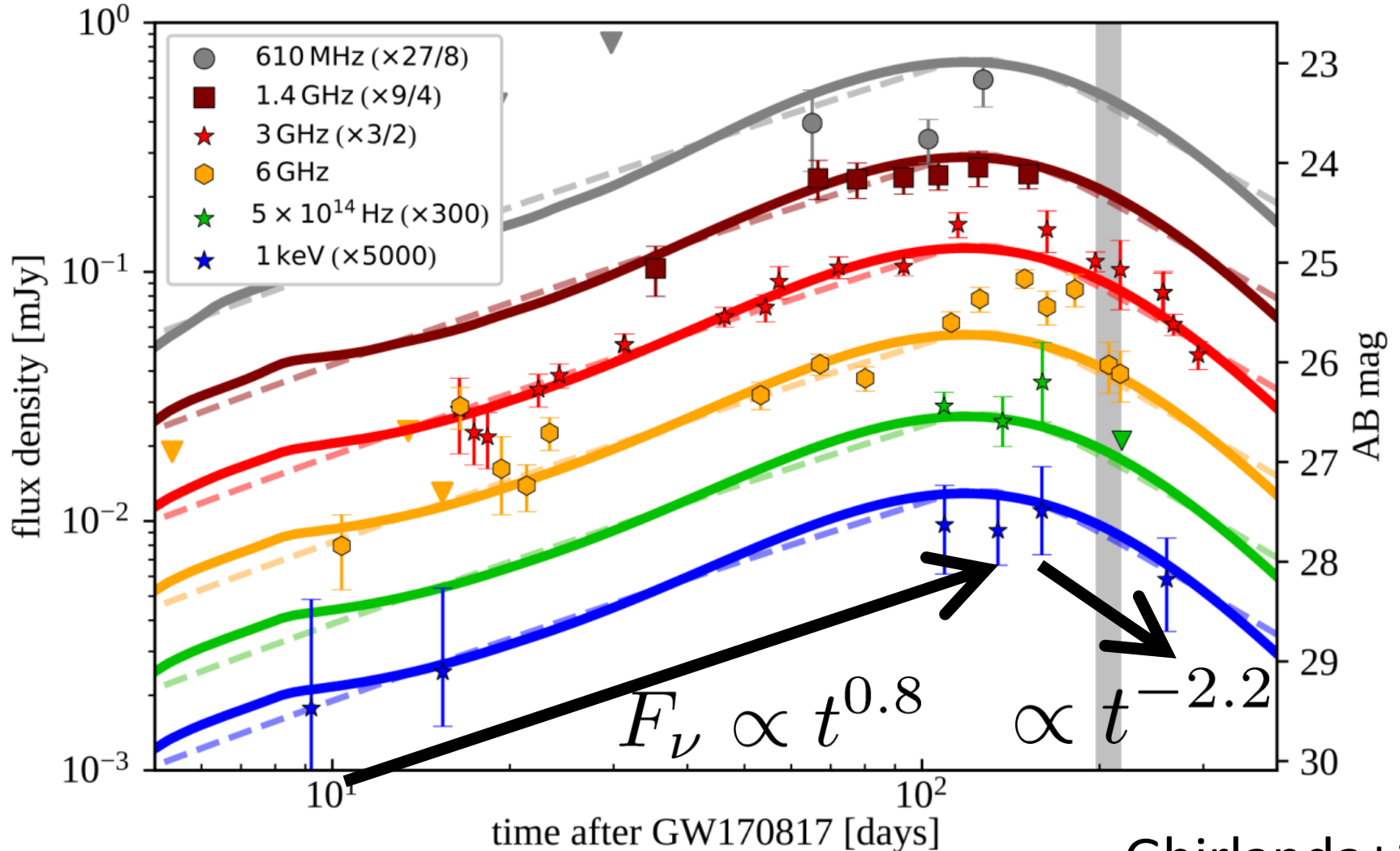
(dim) short GRB

macronova (kilonova)

Radio - X-ray afterglow

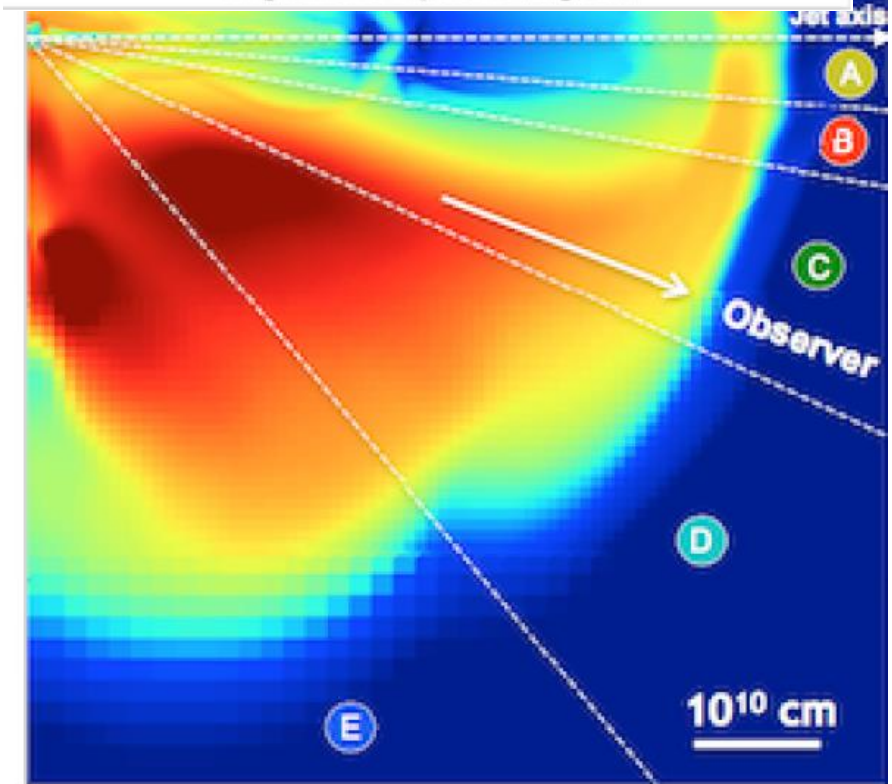
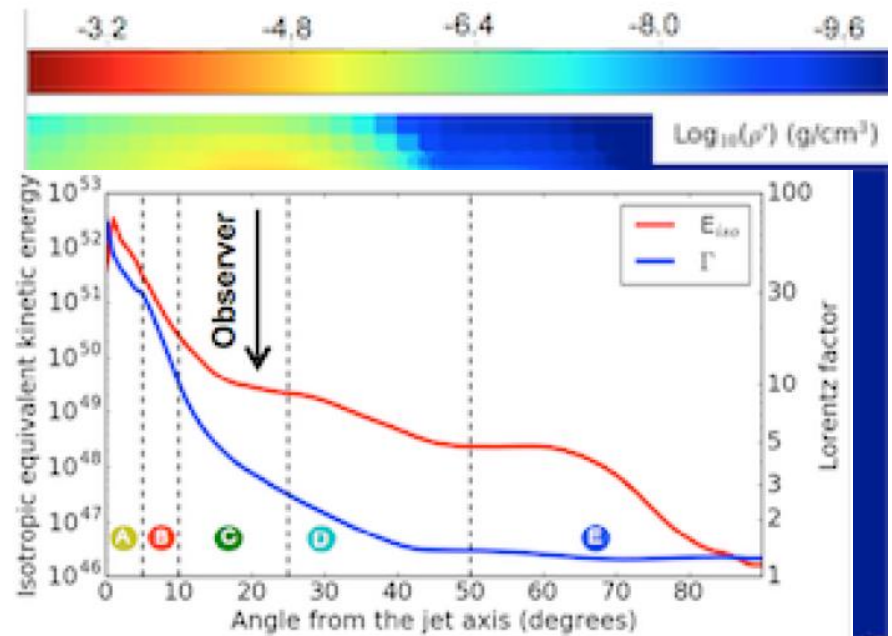
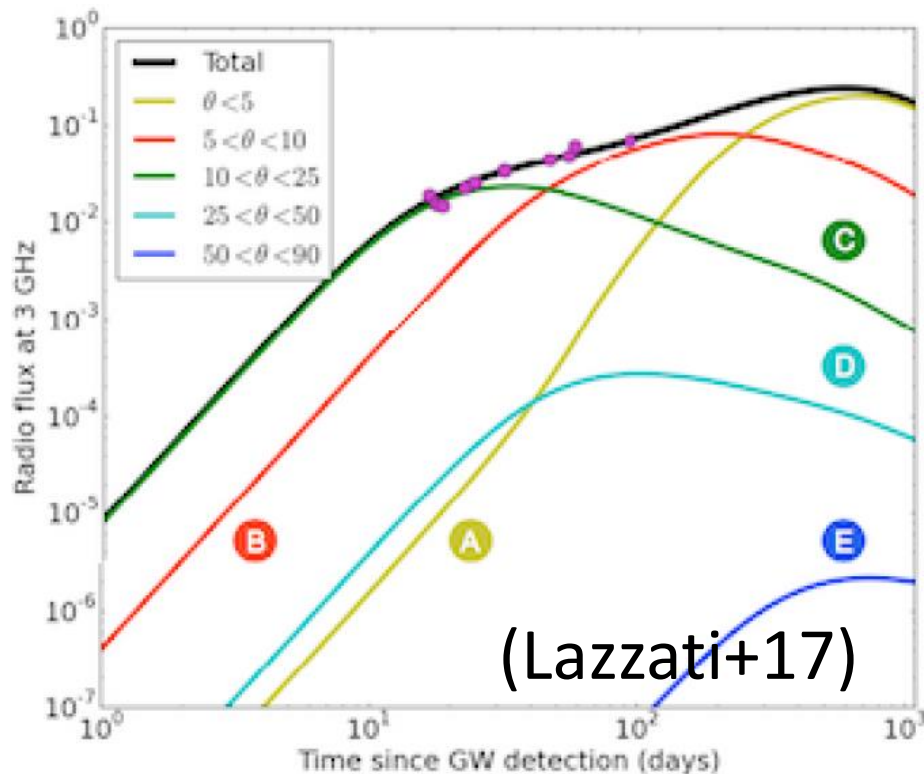
Artist conception of the moment two neutron stars collide.

Unexpectedly slow-rising light curve of GRB170817A



Ghirlanda+19
Mooley+18

We see a structured jet from off-axis.



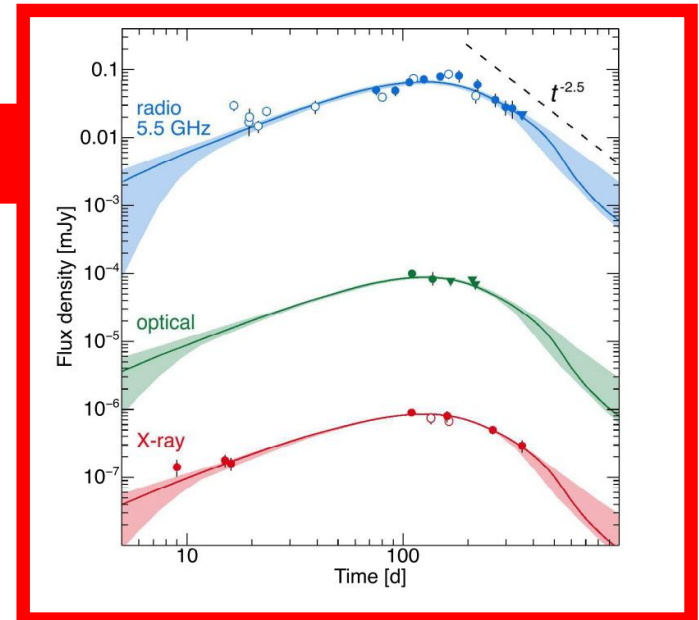
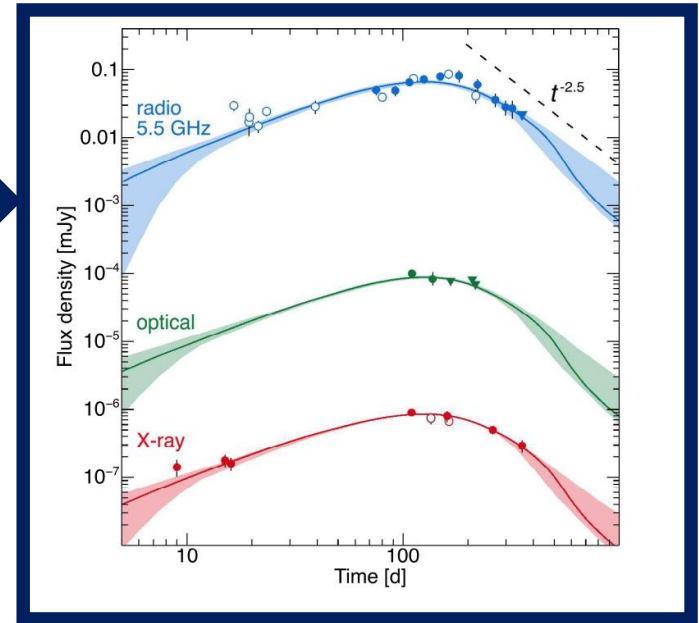
Previous studies (e.g., Troja+19, Ghirlanda+19)

$$E(\theta) = E_c \exp\left(-\frac{\theta^2}{2\theta_c^2}\right)$$

This study

$$E(\theta) = ?$$

Inverse problem

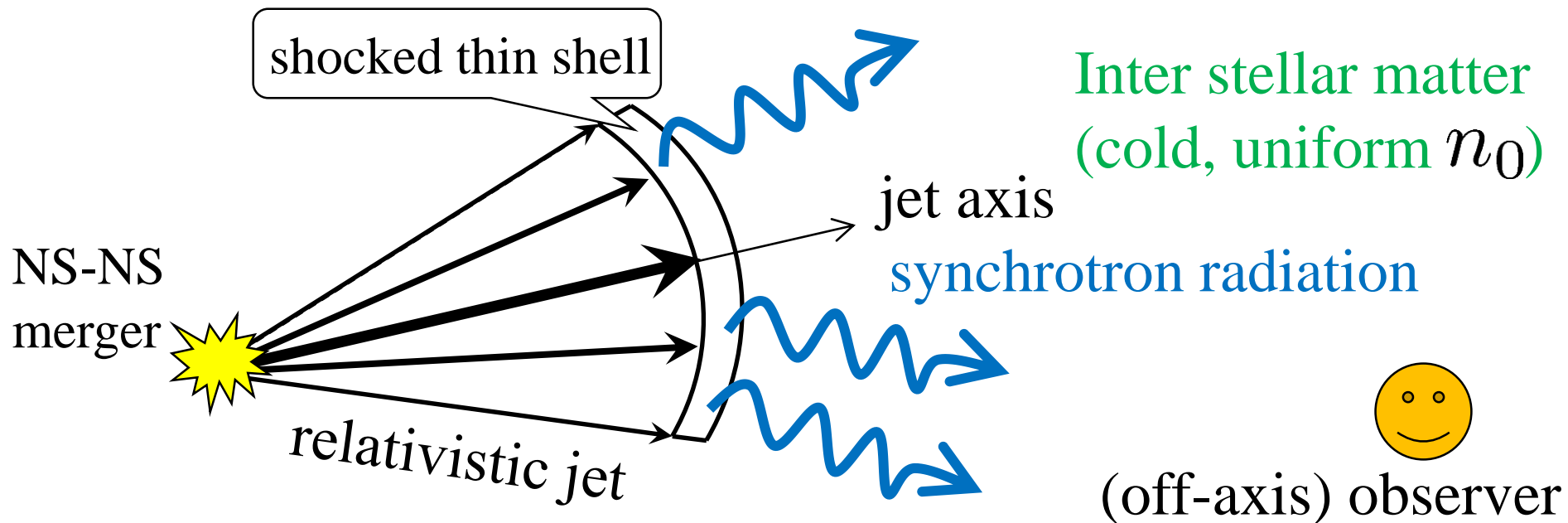


Method

A novel method
to inversely
reconstruct $E(\theta)$

A model of structured jets

(Troja+19, van Eerten+10)



The jet energy has an angle dependence: $E = E(\theta)$

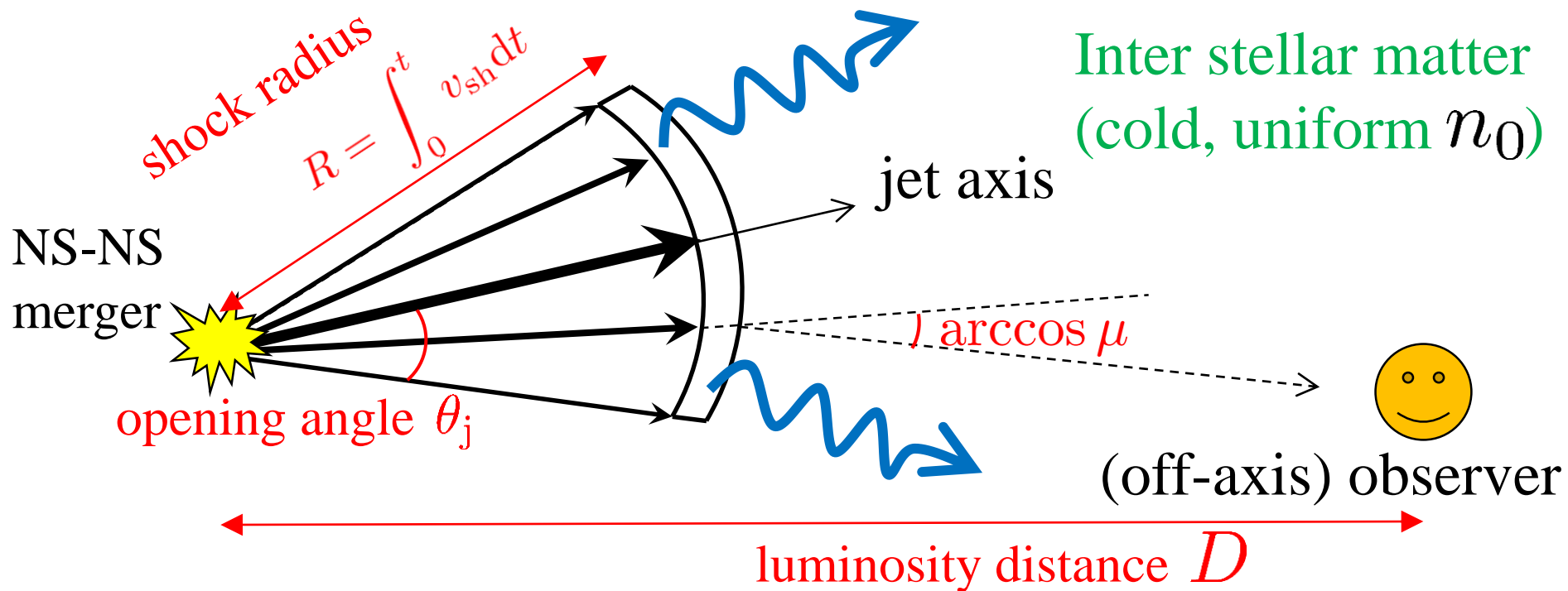
Each segment expands as if it were a portion of a spherically expanding shell (e.g., Kumar & Granot 2003):

$$\Gamma_{\text{sh}}\beta_{\text{sh}} \sim \sqrt{\frac{17E(\theta)}{8\pi n_0 m_p c^5}} t^{-3/2} \quad (\text{cf. Blandford-McKee 1976})$$

The synchrotron emissivity is given by the standard model of Sari+98: $\epsilon'_{\nu'} = \epsilon'_{\nu'}(n_0, \epsilon_B, \epsilon_e, p)$

A model of structured jets

(Troja+18, van Eerten+10)



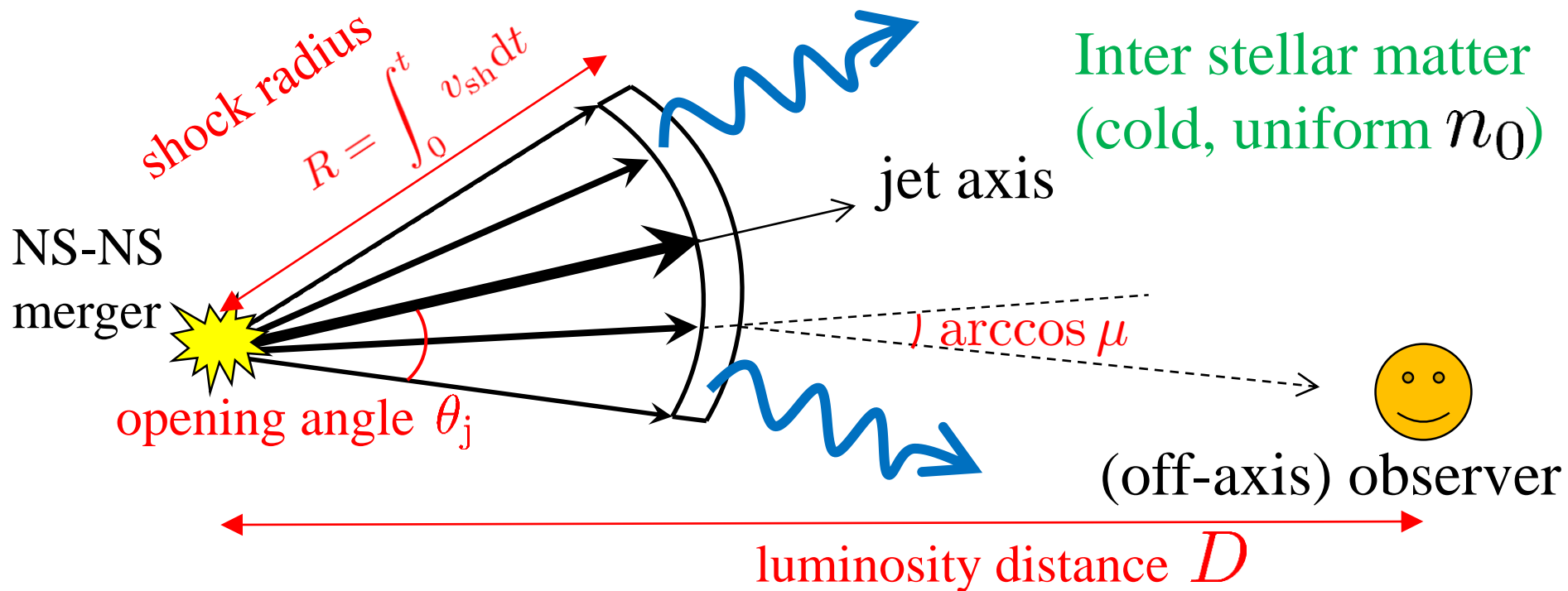
The observed flux density at T:

$$F_\nu(T) = \int_0^{\theta_j} d\theta \int_0^{2\pi} d\phi \frac{\sin \theta R^3 \epsilon'_{\nu'}}{4\pi D^2 \cdot 12\Gamma^4 (1 - \beta\mu)^2 (1 - \beta_{sh}\mu)} \Big|_{t=T + \frac{R\mu}{c}}$$

Complicated function of $E(\theta)$

A model of structured jets

(Troja+18, van Eerten+10)



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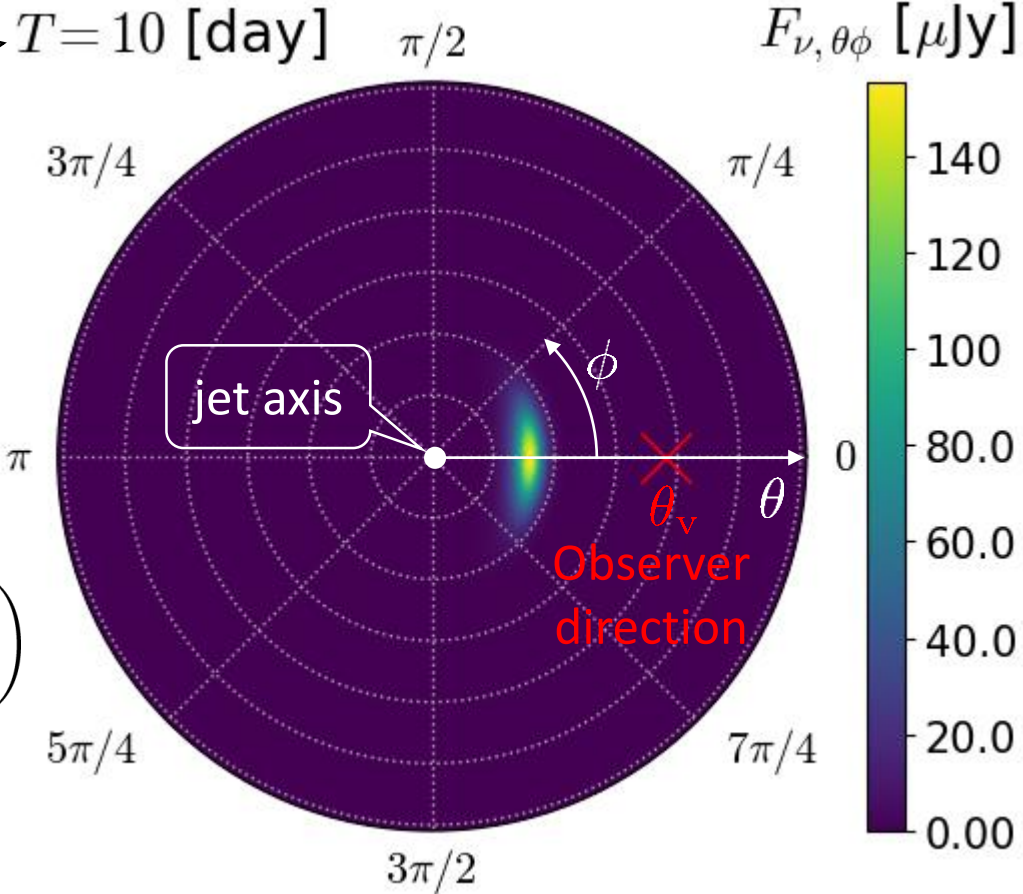
How to reconstruct?

Complicated function of $E(\theta)$

$$F_\nu(T) = \int_0^{\theta_j} d\theta \int_0^{2\pi} d\phi \frac{\sin \theta R^3 \epsilon'_\nu}{4\pi D^2 \cdot 12\Gamma^4 (1 - \beta\mu)^2 (1 - \beta_{\text{sh}}\mu)} \Big|_{t=T + \frac{R\mu}{c}}$$

$$\equiv F_{\nu, \theta\phi}$$

Observer time
after the merger



For a Gaussian jet:

$$E(\theta) = E_c \exp\left(-\frac{\theta^2}{2\theta_c^2}\right)$$

$$E_c = 10^{52.5} \text{ erg}$$

$$\theta_c = 0.0565$$

Only a limited region contributes to the observed flux
(Why?)

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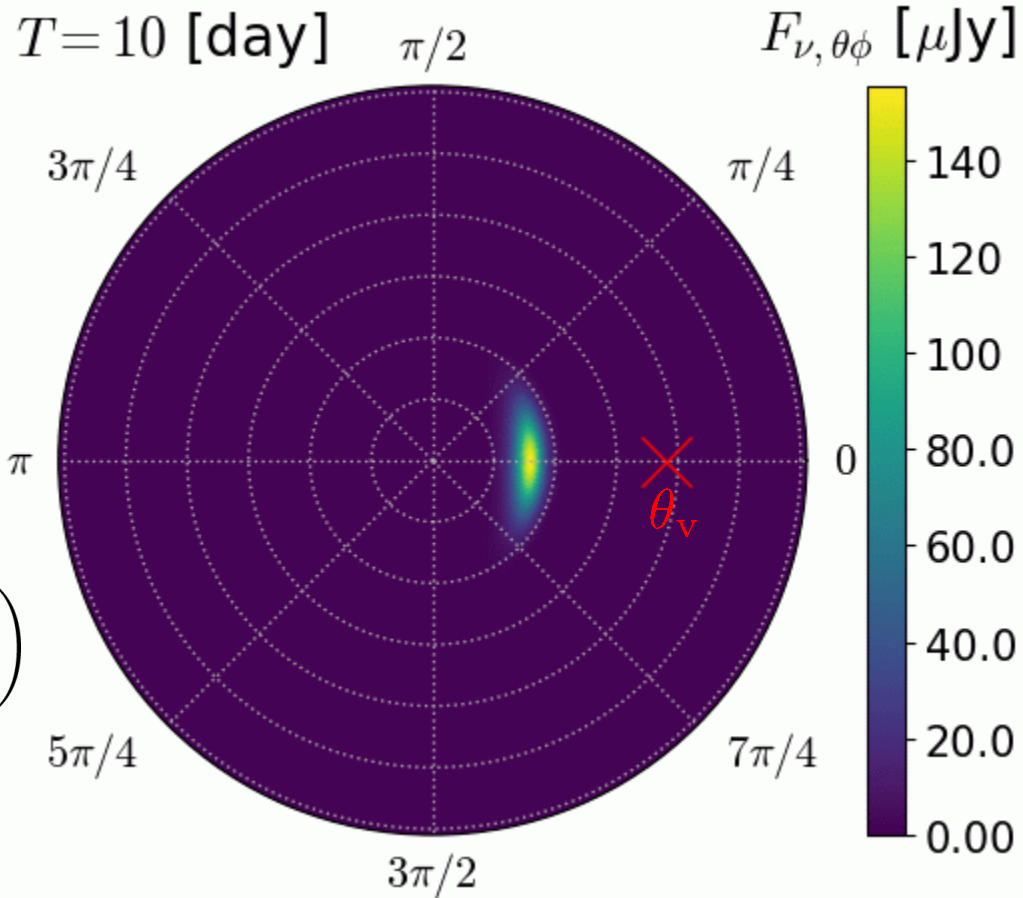
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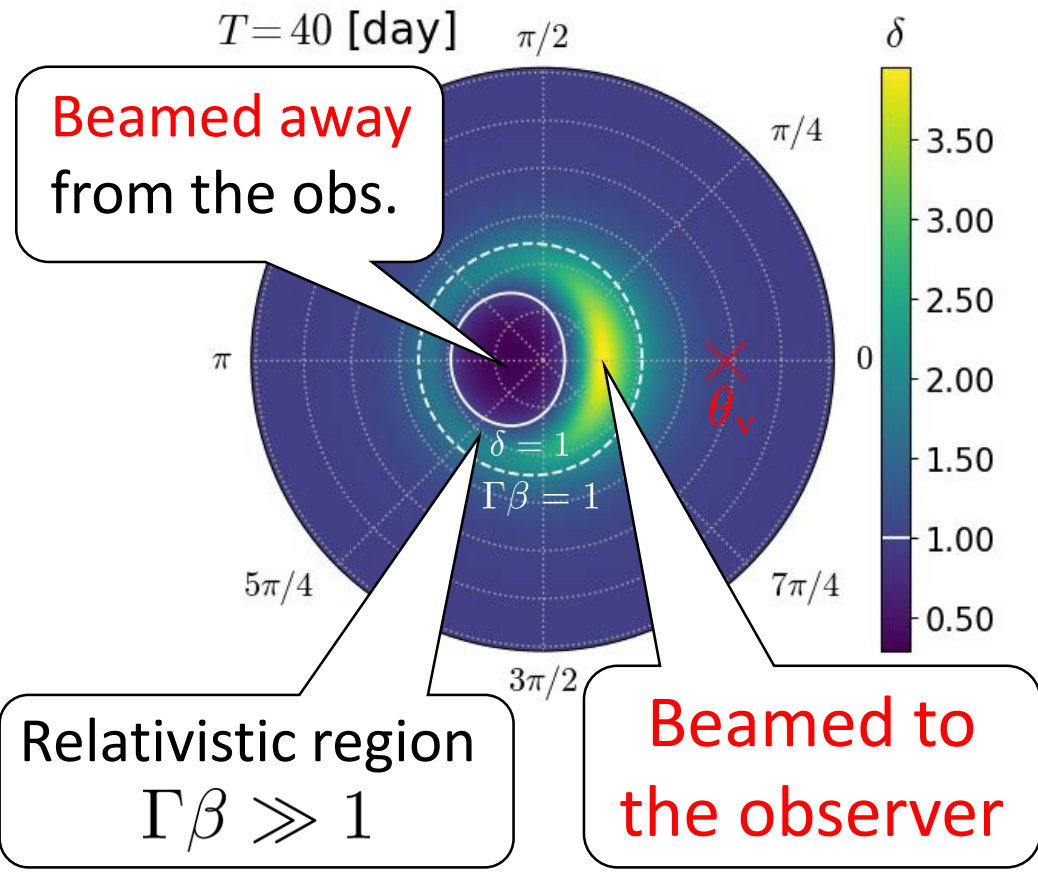
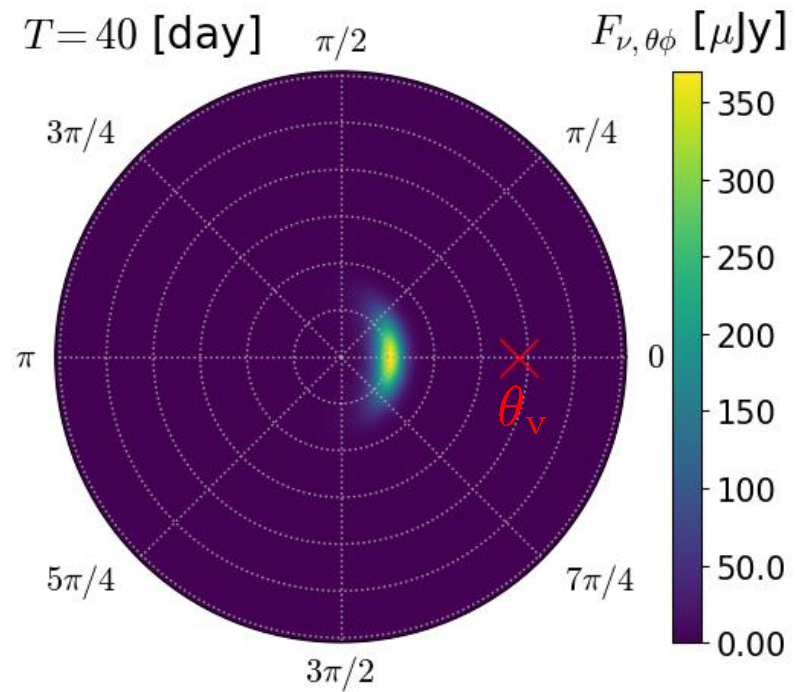
$$\theta_c = 0.0565$$



The contributing region gradually approach the jet axis, as the time passes. (Why?)

Contribution to the flux **strongly related** Beaming factor

$$F_{\nu, \theta\phi} \propto E(\theta) \frac{3p+5}{10} \delta^{\frac{2(10-p)}{5}} \sin \theta \quad \longleftrightarrow \quad \delta = \frac{1}{\Gamma(1 - \beta\mu)}$$

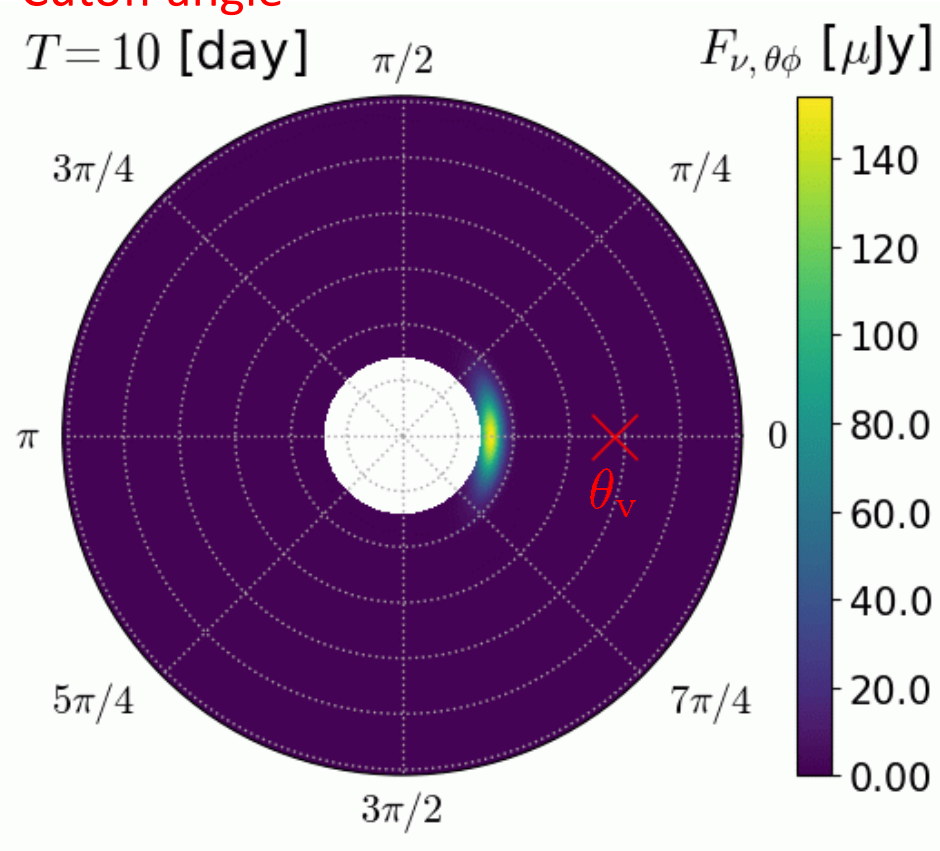


The contributing region gradually approach the jet axis as the jet is decelerated, due to beaming effects.

$$F_\nu(T) = \int_{\Theta(T)}^{\theta_j} d\theta \int_0^{2\pi} d\phi F_{\nu,\theta\phi} \Big|_{t=T+\frac{R\mu}{c}}$$

Cutoff angle

$T = 10$ [day]

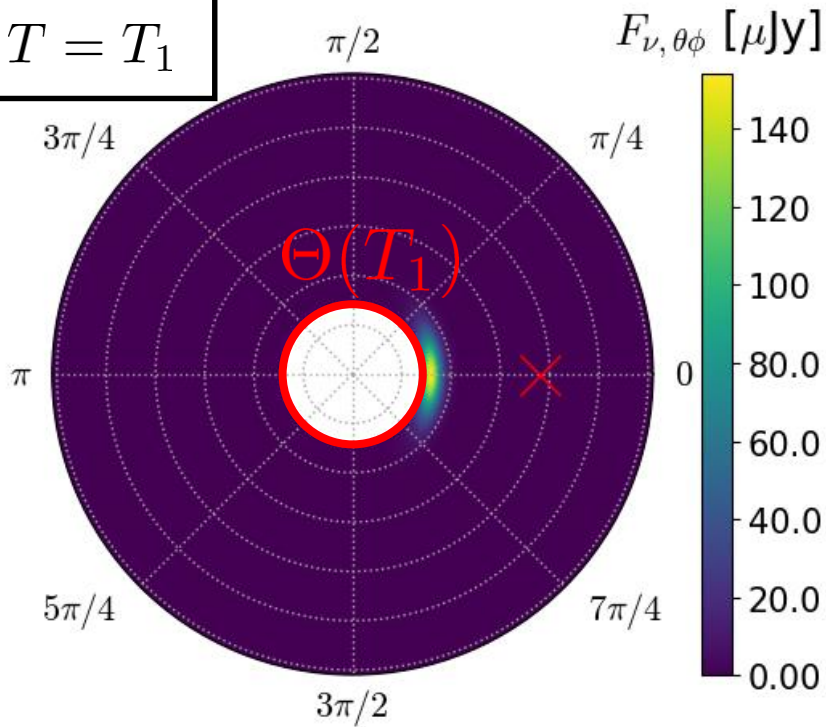


General expression for the cutoff angle:

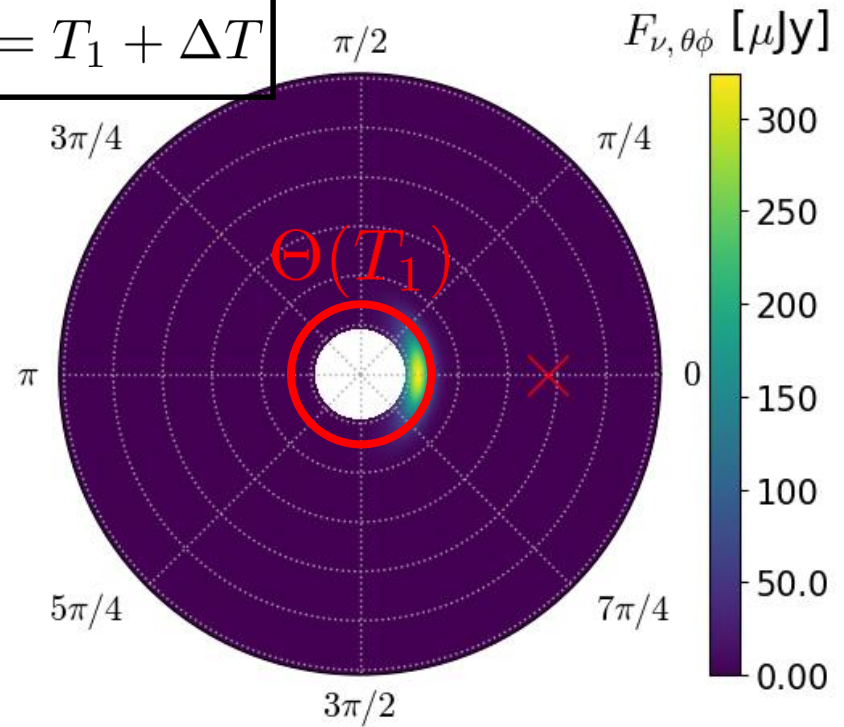
$$\Theta = \theta_\nu - 28 \left(\frac{\pi n_0 m_p c^5}{17 \cdot 57^3} \right)^{1/8} E^{-1/8}(\Theta) T^{3/8}$$

n_0 : ambient matter number density

$$T = T_1$$



$$T = T_1 + \Delta T$$



All the contributions to the observed flux $F_\nu(T_1 + \Delta T)$

— Contribution from the region that

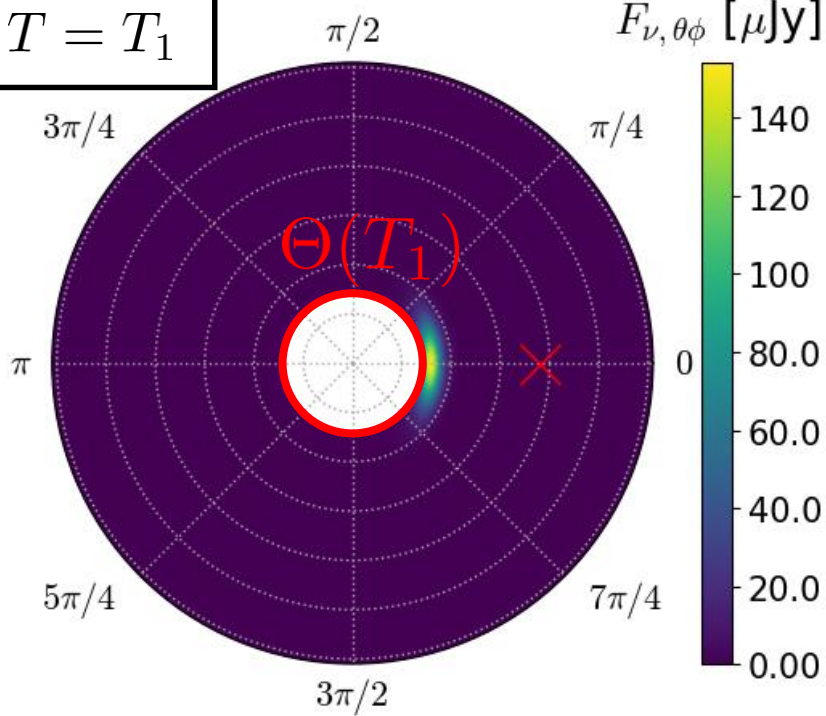
has been seen before: $\theta \geq \Theta(T_1)$

= Contribution from $\Theta(T_1 + \Delta T) \leq \theta < \Theta(T_1)$

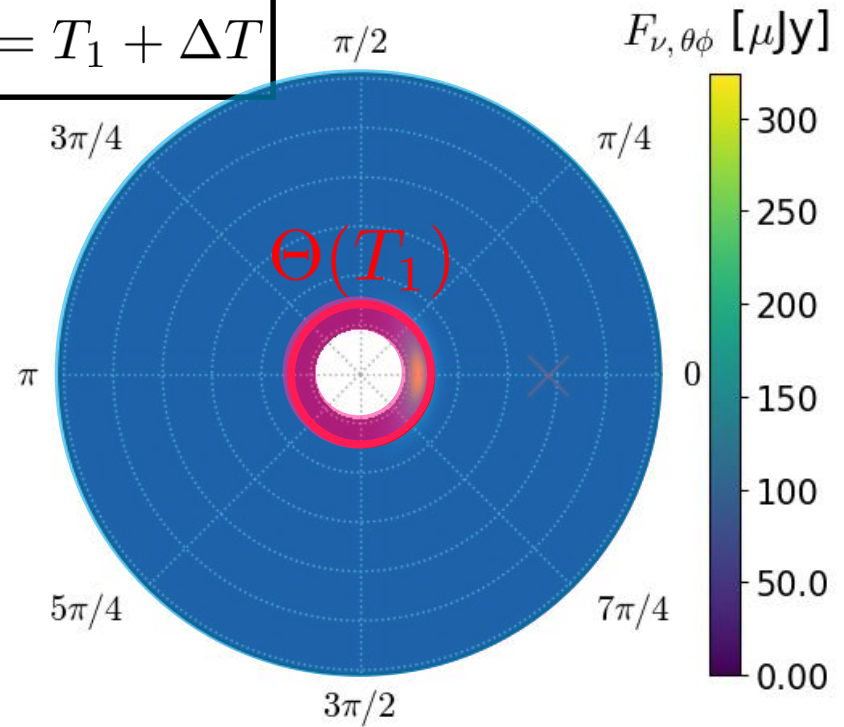
containing the information on

the energy distribution for $\Theta(T_1 + \Delta T) \leq \theta < \Theta(T_1)$

$$T = T_1$$



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Inversion Formula:

$$\frac{d \ln E}{d\Theta} = \frac{8}{\theta_v - \Theta} - \frac{3 \frac{K(T, \Theta; E(\Theta))}{F_\nu(T)}}{\frac{d \log F_\nu}{d \log T}(T) - \int_{\Theta}^{\theta_j} d\theta \frac{K(T, \theta; E(\theta))}{F_\nu(T)} \frac{d \log K}{d \log T}(T, \theta; E(\theta))}$$

Change rate
of the observed flux

Subtracted contribution
from the ever-seen region

where \mathbf{K} is defined by

$$F_\nu(T) = \int_{\Theta(T)}^{\theta_j} d\theta \int_0^{2\pi} d\phi F_{\nu, \theta\phi} \Big|_{t=T+R\mu/c} \equiv \int_{\Theta(T)}^{\theta_j} d\theta K(T, \theta; E(\theta))$$

Boundary condition (*assumption*):

$$E(\theta) \text{ for } \Theta(T_0) \leq \theta \leq \theta_j$$

jet truncation angle

The cutoff angle at a given initial time

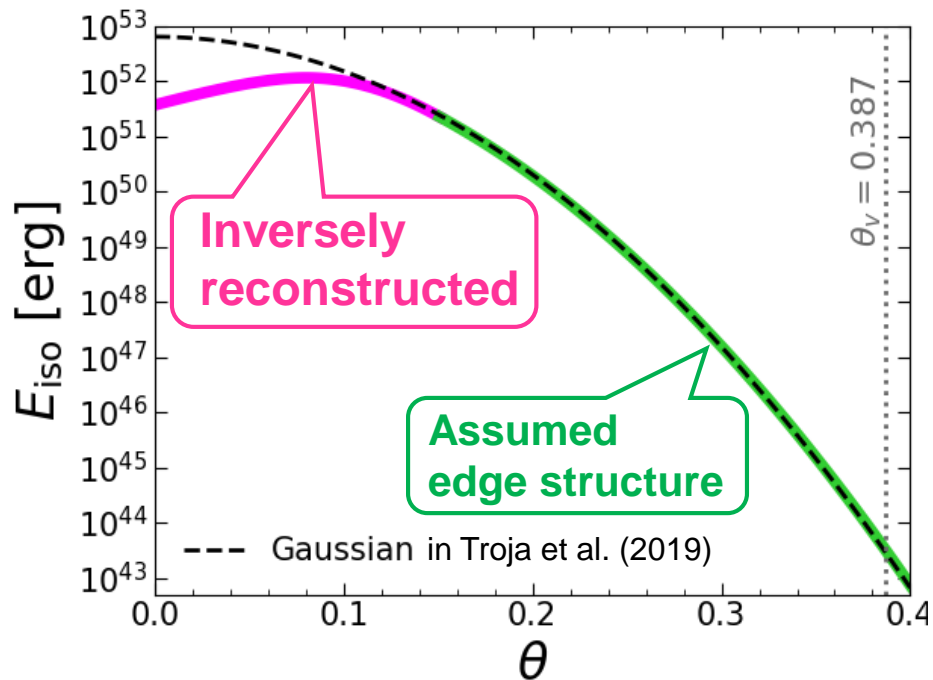
The jet structure is uniquely reconstructed by integrating the inversion formula from $\Theta(T_0)$ to $\Theta = 0$ (the jet axis).

Results

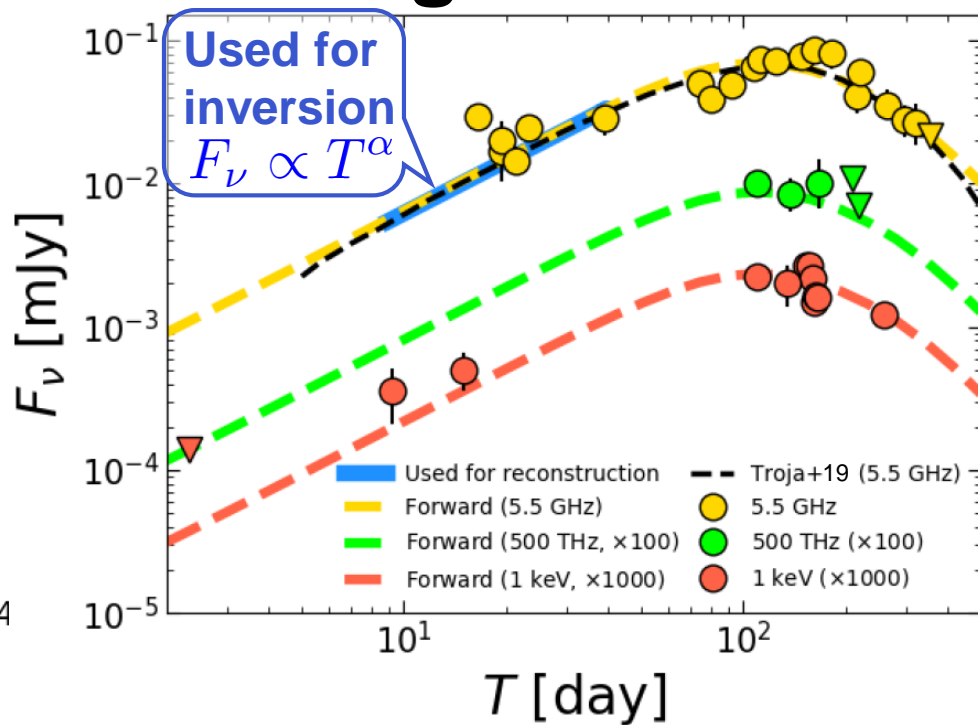
**A non-trivial jet
structure
obtained by our
method**

Non-trivial Result : Hollow-cone structure

Jet Structure



Light Curve



Parameters:

$$n_0 = 10^{-3.01} \text{ cm}^{-3}, \quad \varepsilon_B = 10^{-3.56} \quad \varepsilon_e = 10^{-1.39}, \quad p = 2.17,$$

$$\theta_v = 0.387, \quad D = 41 \text{ Mpc}, \quad \theta_j = 0.61$$

Possible origin of hollow-cone structures (only naïve consideration)

Jet acceleration mechanism?

If the jet is launched by the Blandford-Znajek mechanism, the energy flux around the jet axis is low ($F_E \propto \sin^2 \theta$).
(e.g., McKinney 2006, Tchekhovskoy+ 2008,)

Structure created through the interaction with the ejecta during the propagation or at the shock breakout?

Some numerical simulations show the jet structure can be a hollow-cone type.

(e.g., Zhang et al. 2006, Mizuta & Ioka 2013)

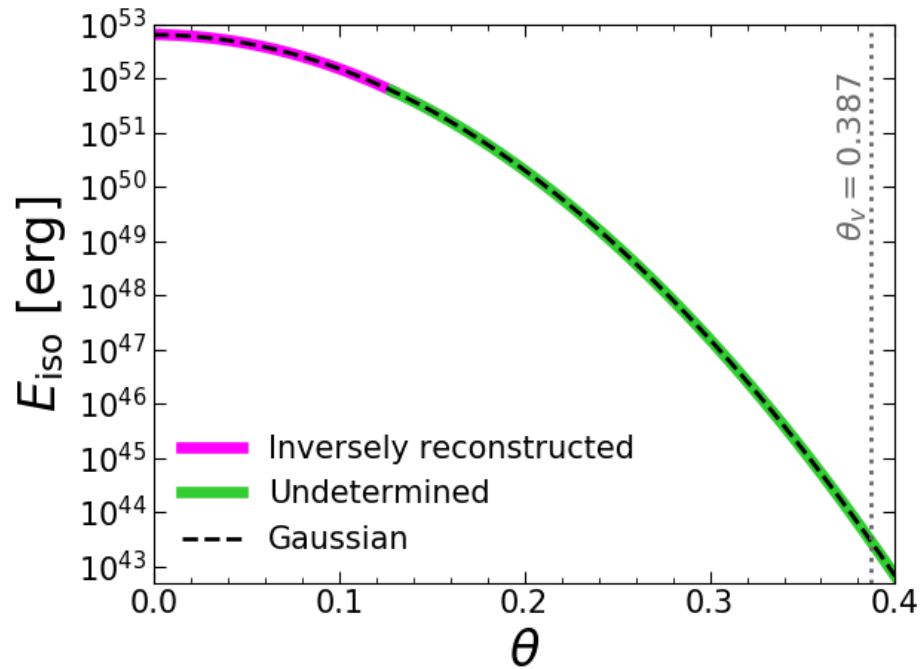
Summary

- We developed a **novel method to inversely reconstruct jet structures from off-axis GRB afterglows.**
 - Our method **does not assume the functional form of $E(\theta)$.**
 - Our method **uniquely obtains $E(\theta)$ for a given afterglow light curve and given parameters.**
- We found that a **hollow-cone jet structure** can be consistent with GRB170817A afterglow.
- The method can be **applicable to future off-axis GRB afterglows.**

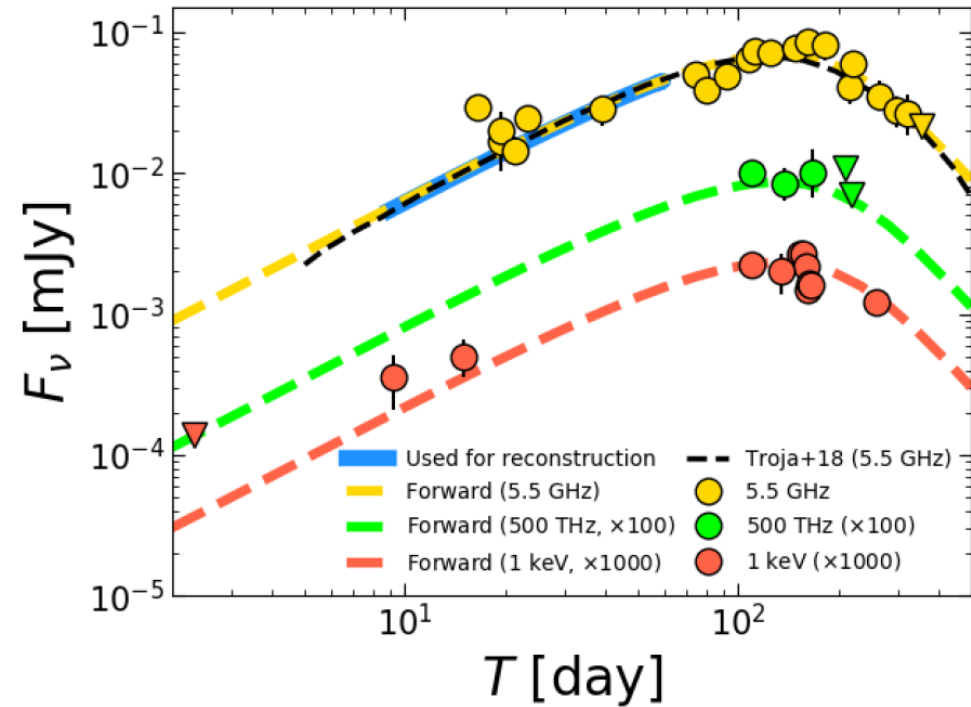
Appendix

Test problem : Gaussian structure

Jet Structure

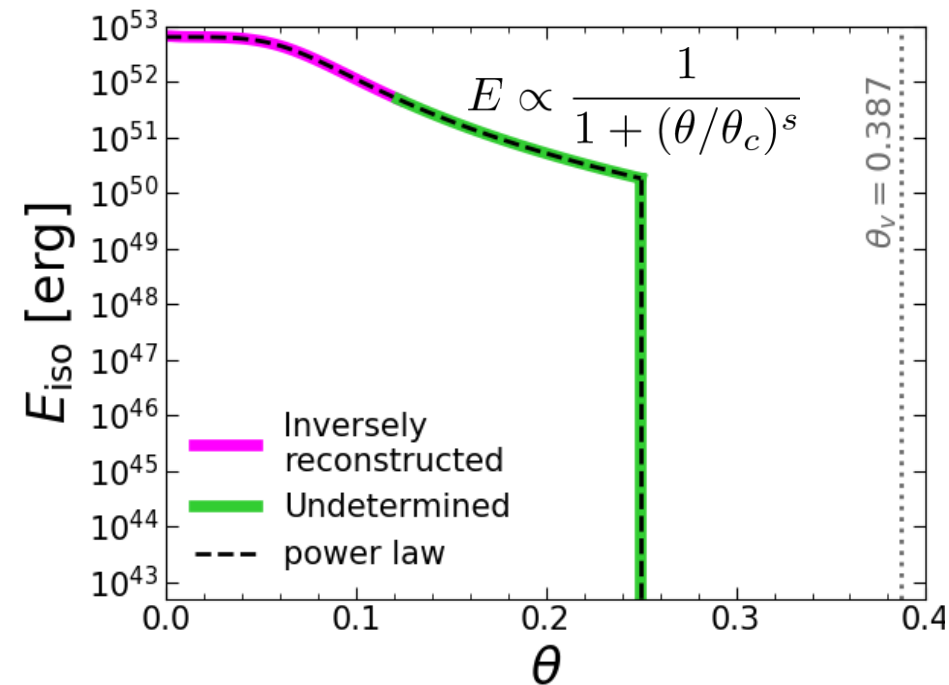


Light Curve



Test problem : Power-law structure

Jet Structure



Light Curve

