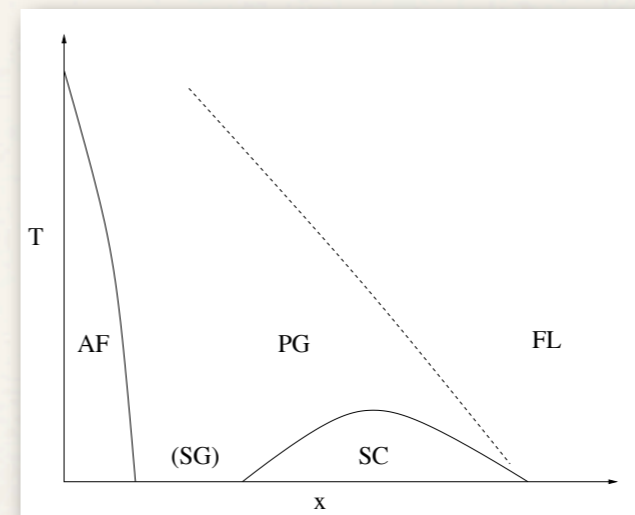
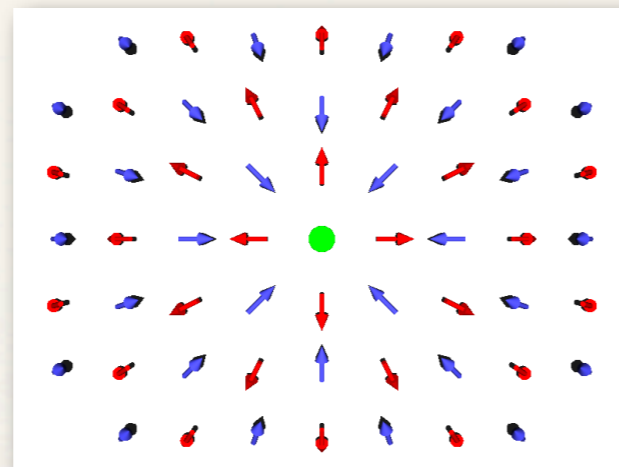


# Half-skyrmion theory of high- $T_c$ cuprates

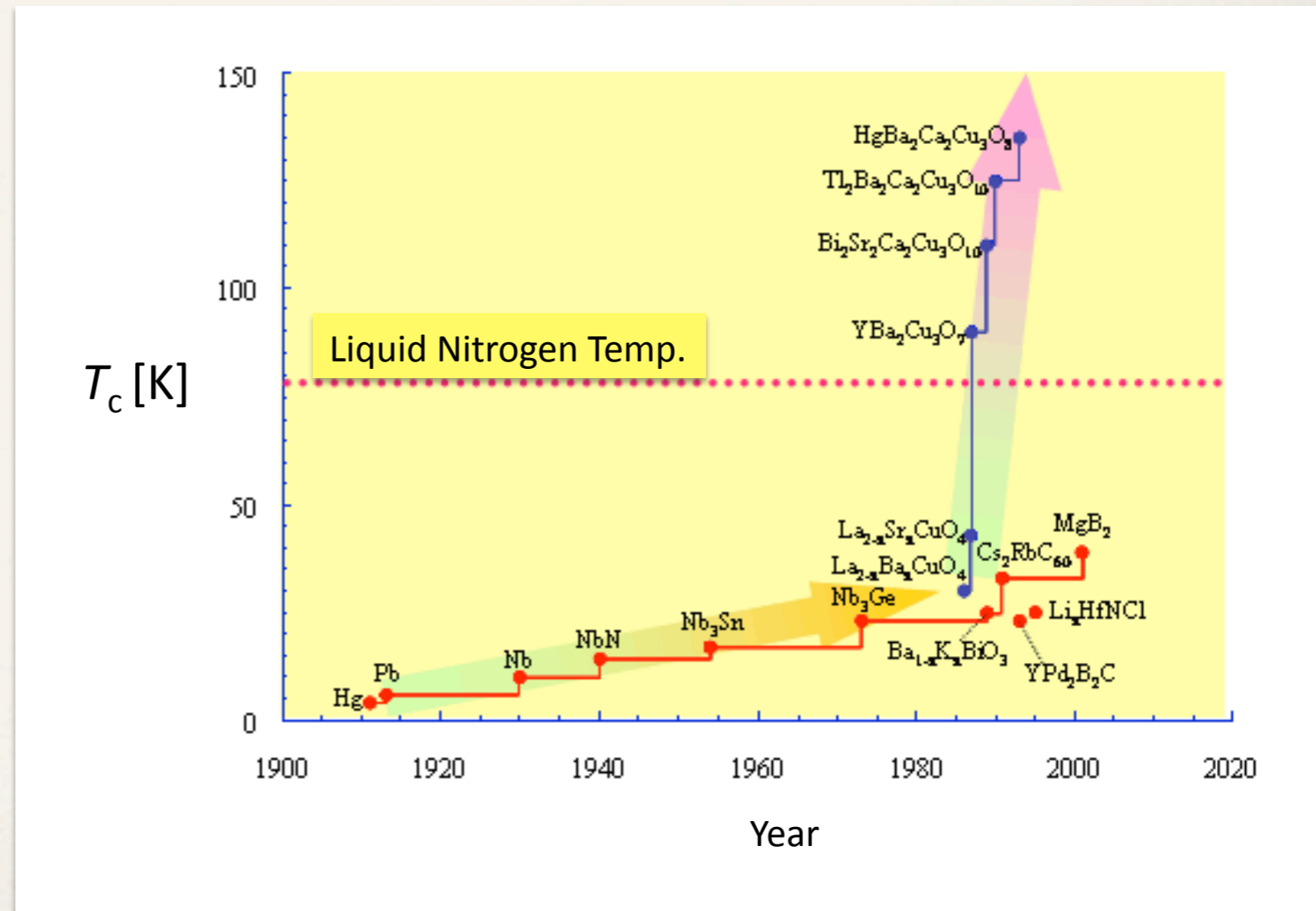
Takao Morinari

Yukawa Institute for Theoretical Physics  
Kyoto Univ., JAPAN



# The high- $T_c$ cuprates

- ❖ Bednorz and Müller, 1986  
 $\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$   
 $T_c=30\text{K}$
- ❖ Wu et al., 1987  
 $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$   
 $T_c=93\text{K}$
- ❖ Guo et al., 1994  
 $\text{HgBa}_2\text{Ca}_2\text{Cu}_3\text{O}_{8+x}$   
 $T_c=164\text{K}$  (under pressure)



Source: Akimitsu group homepage

# Other superconductors?

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## Iron-based superconductors

- \* Kamihara et al., 2008 (February)

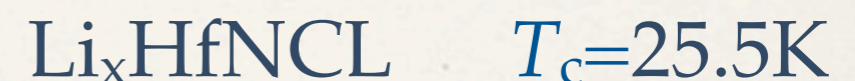


$$T_c=26\text{K}$$

- \* Ren et al., 2008 (April)



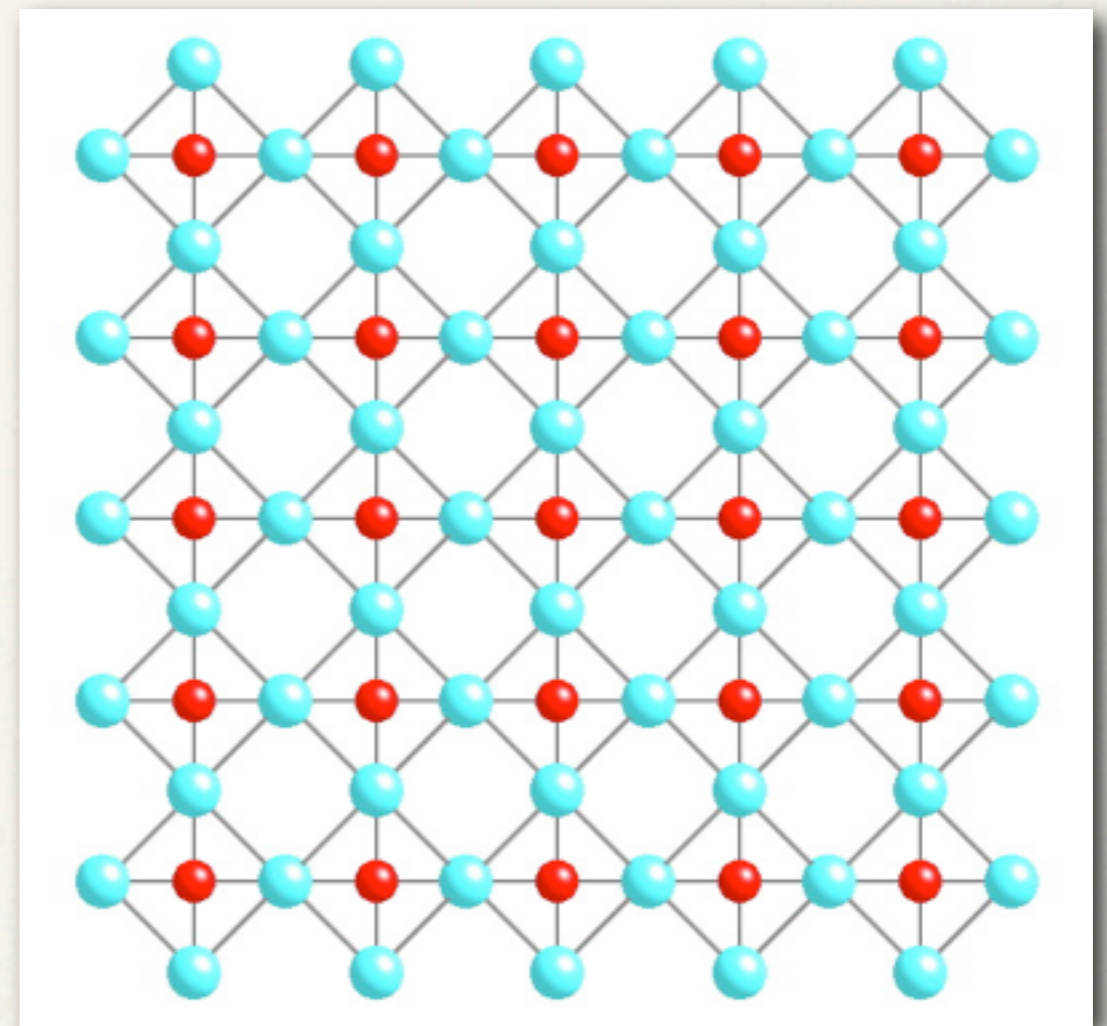
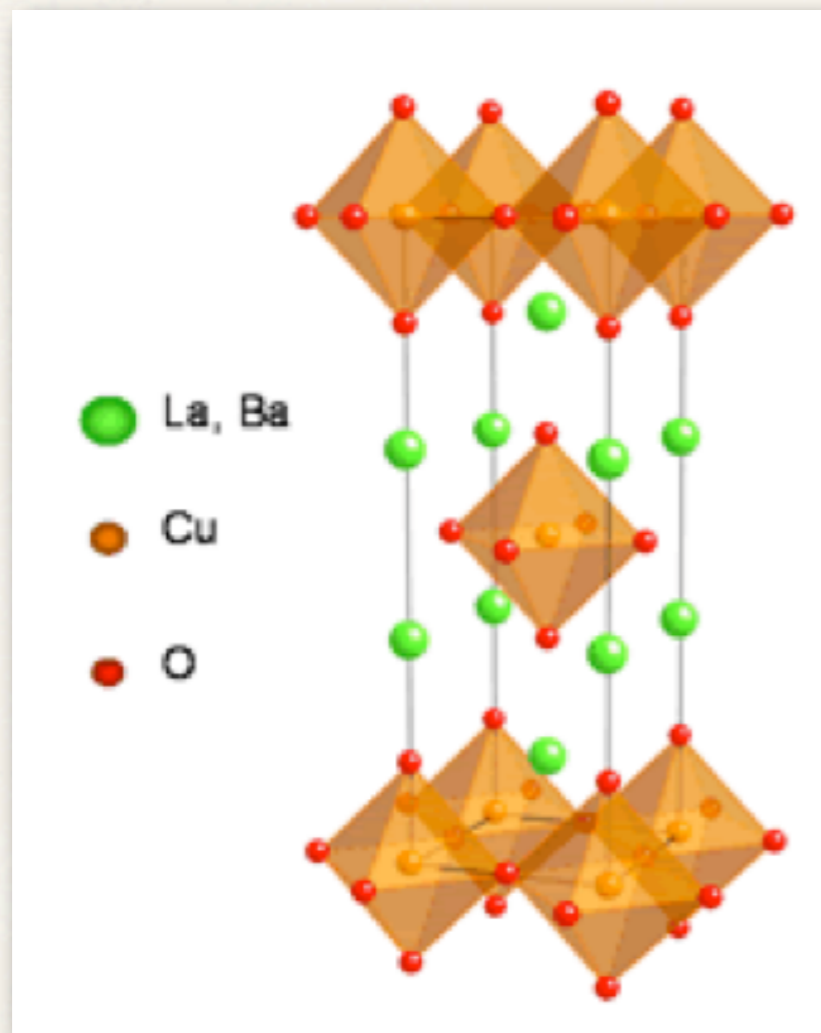
$$T_c=55\text{K}$$



**The high- $T_c$  cuprates are clearly distinguished from other superconductors!**

# The key structure

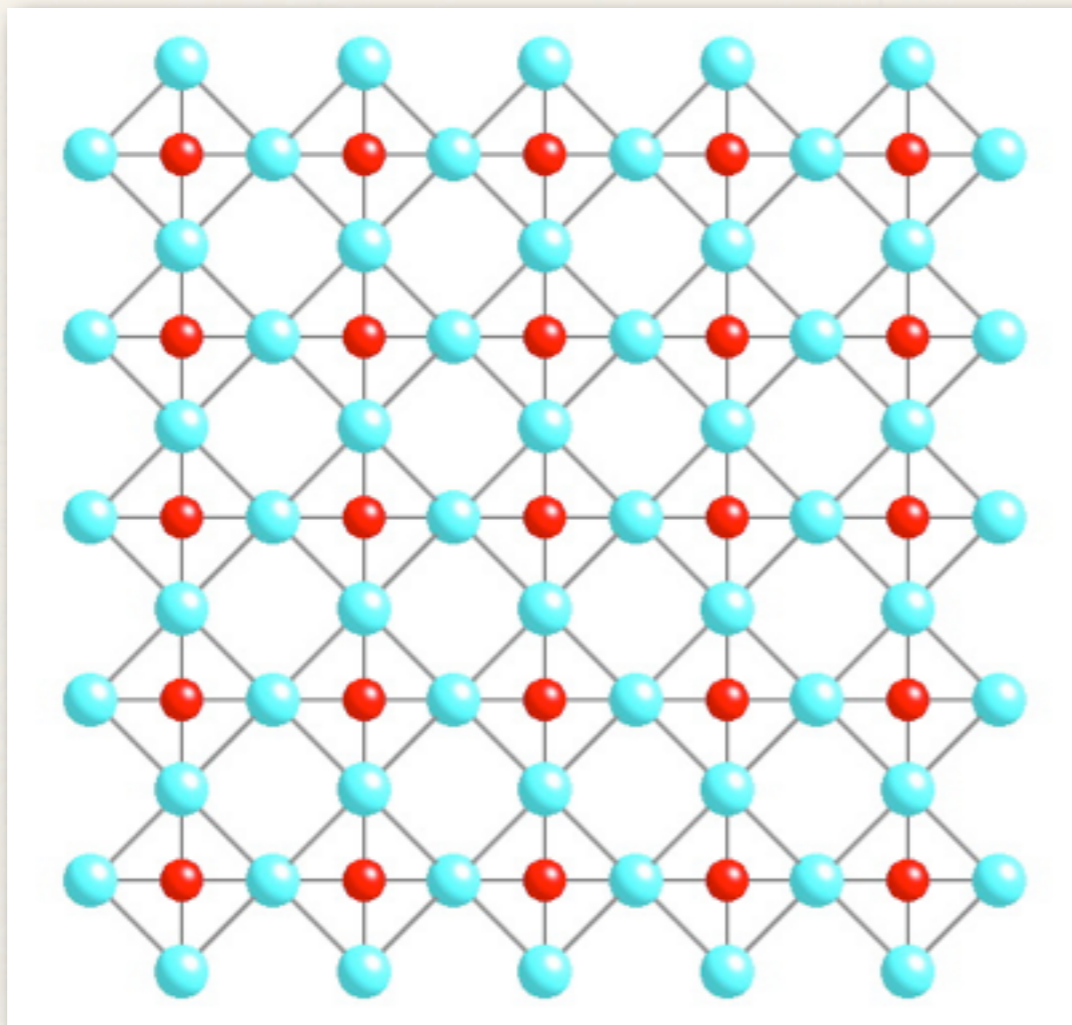
---



$\text{CuO}_2$  plane

# Pure $\text{CuO}_2$ plane

---

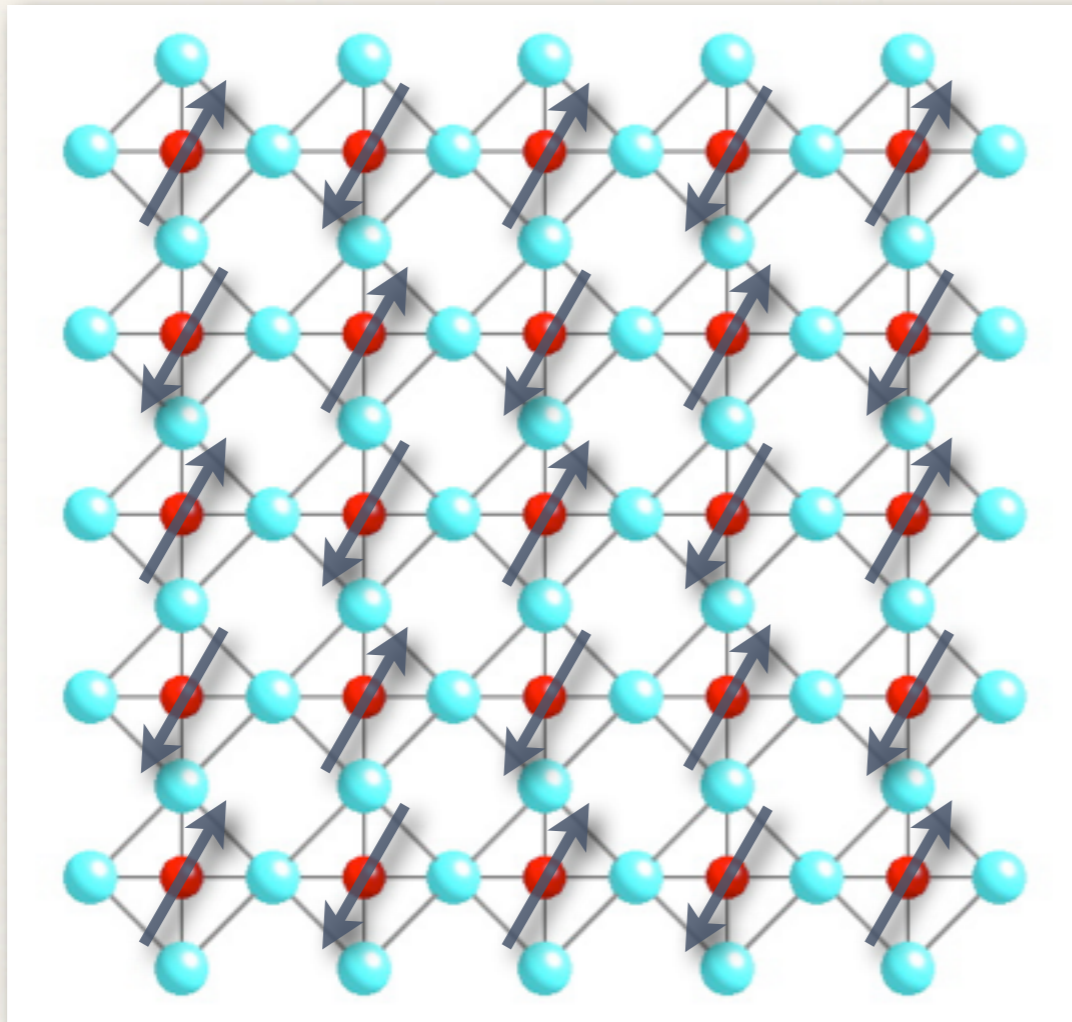


$\text{CuO}_2$  plane

$\text{Cu}^{2+}: 3d^9$

$3d_{x^2-y^2}$

# Pure $\text{CuO}_2$ plane



$\text{CuO}_2$  plane

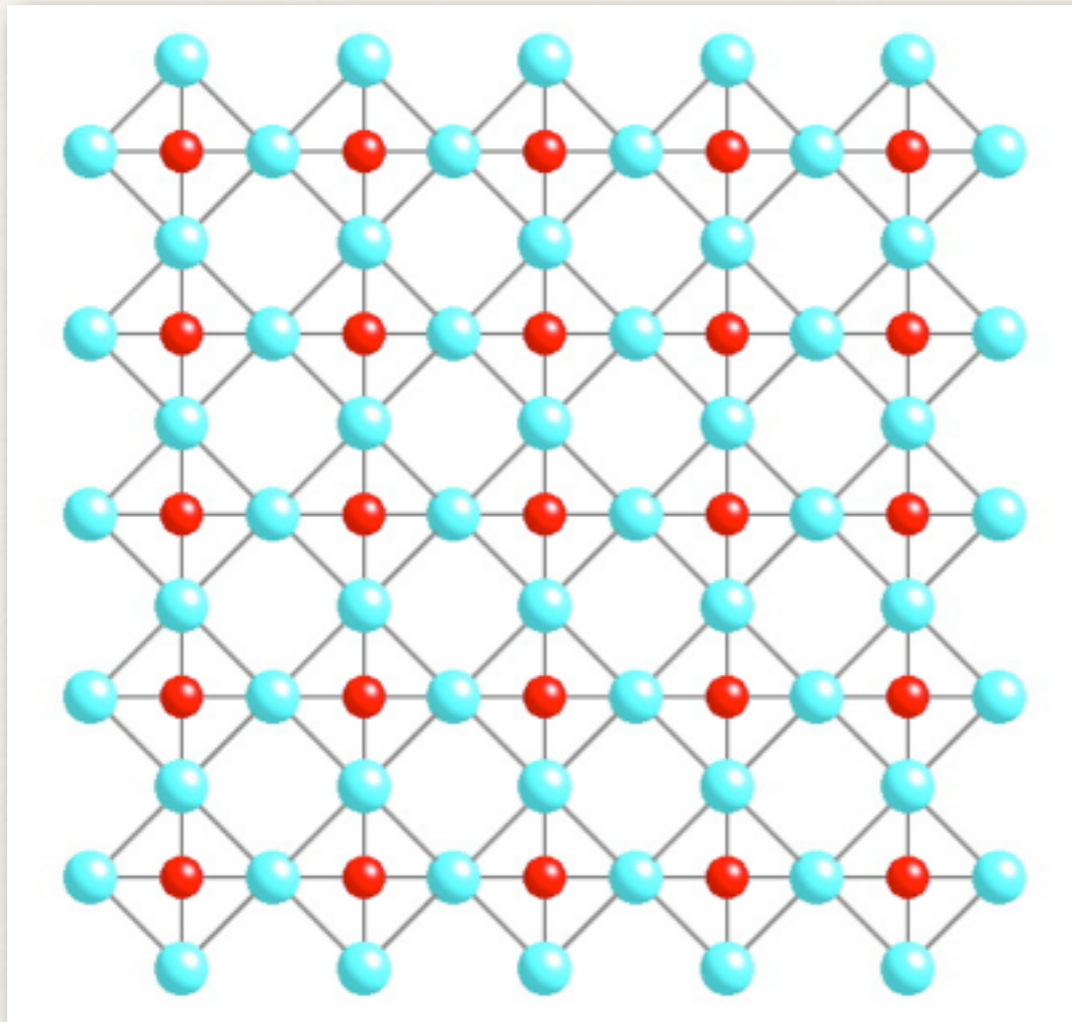
$\text{Cu}^{2+}: 3d^9$

$3d_{x^2-y^2}$

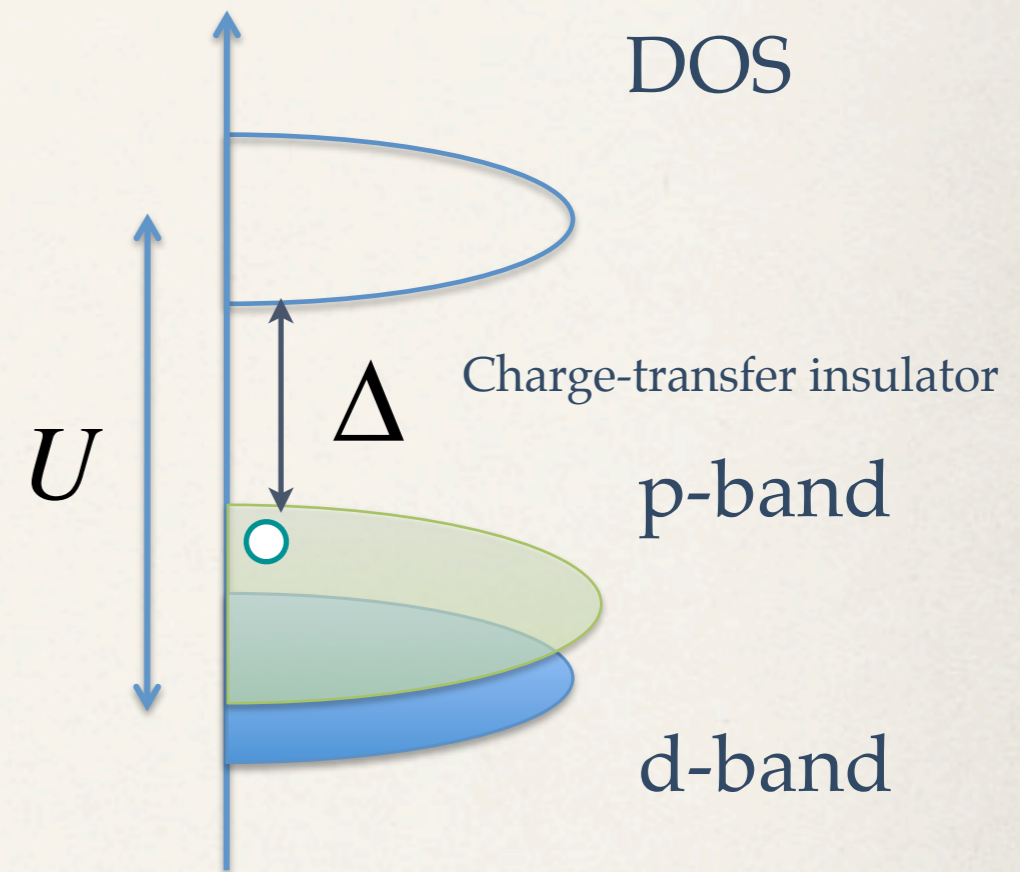
$S=1/2$   $J = 1300\text{K}$

$\text{O}^{2-}: 2p^6$

# Strong correlations

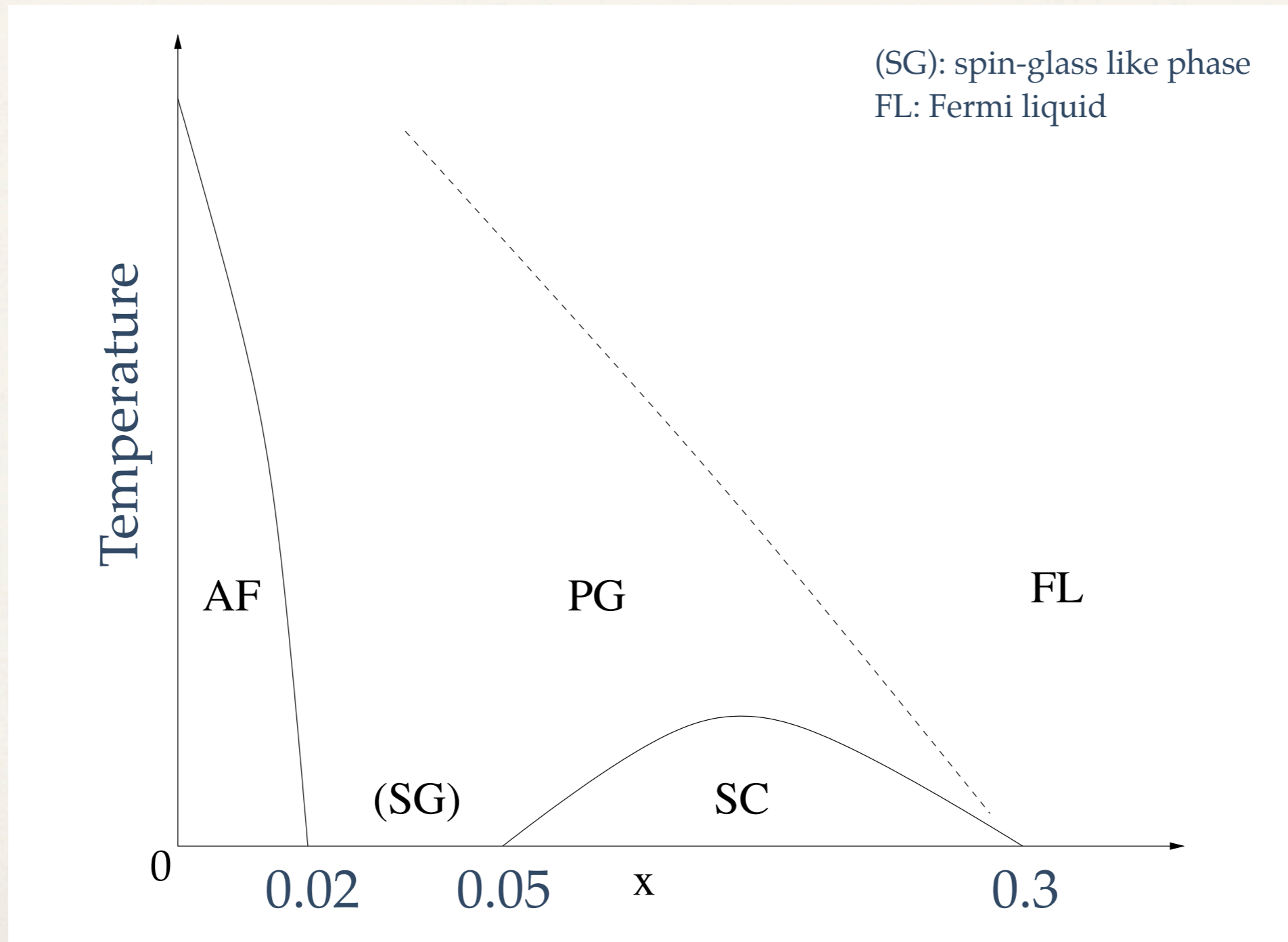


CuO<sub>2</sub> plane



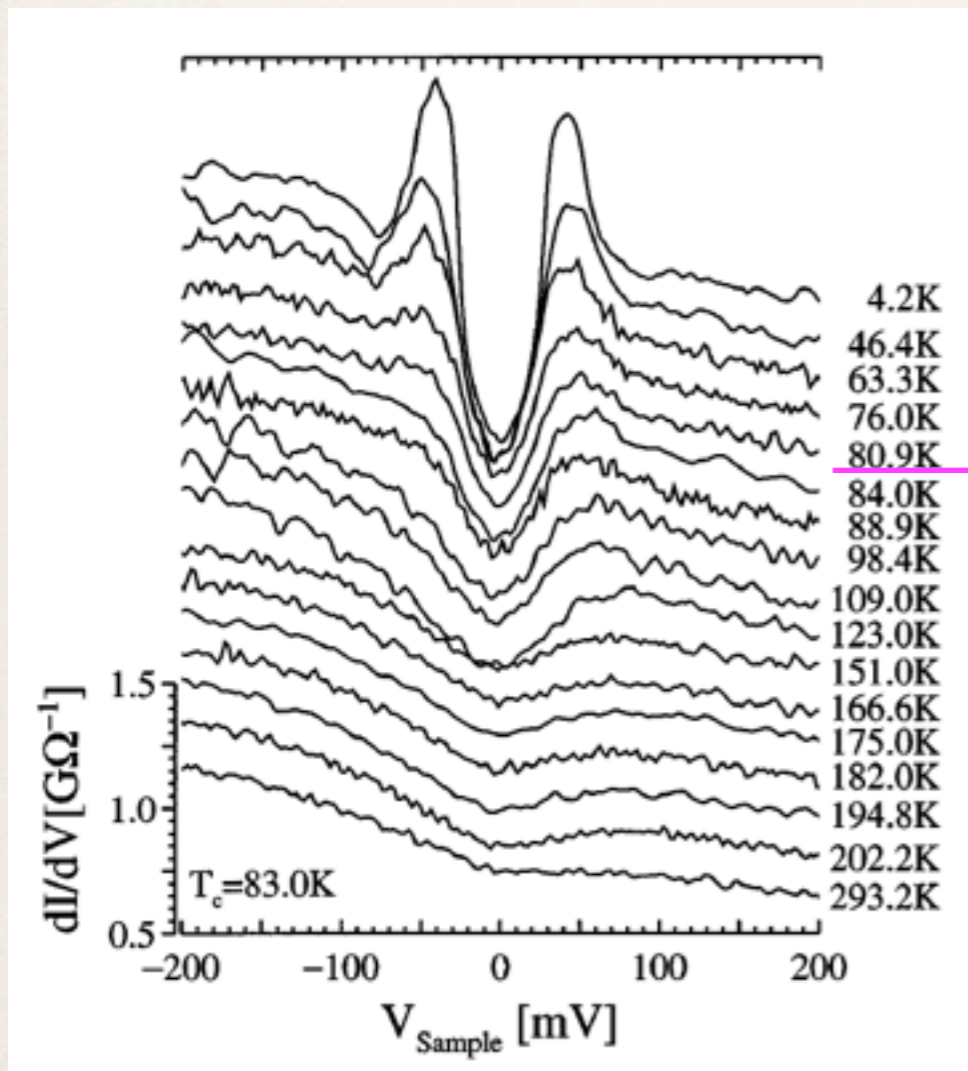
C.T. Chen et al., PRL 66, 104 (1991)

# Phase diagram



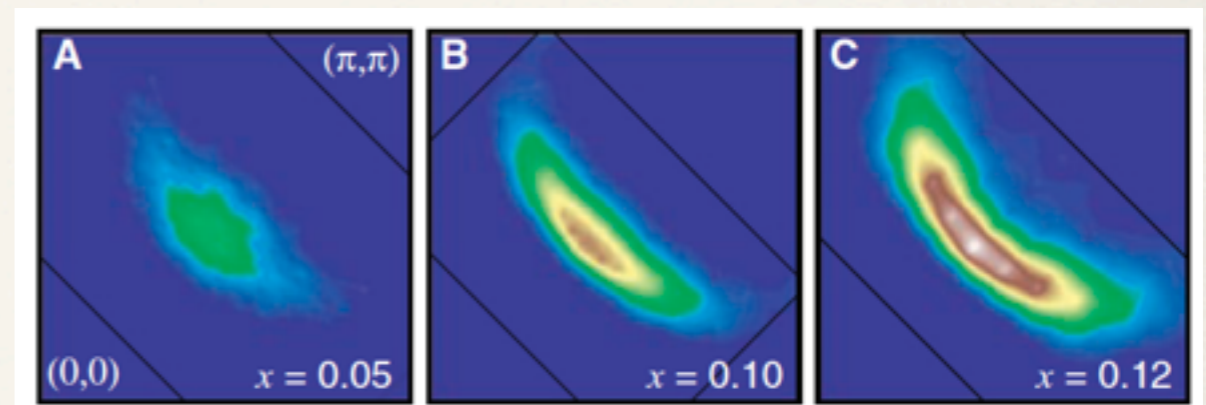
# Pseudogap?

STS

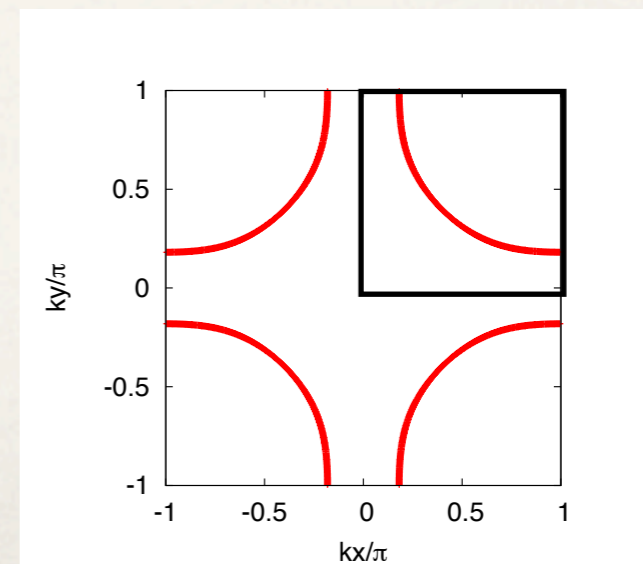


Renner et al., Phys. Rev. Lett. 80, 149 (1998)

ARPES

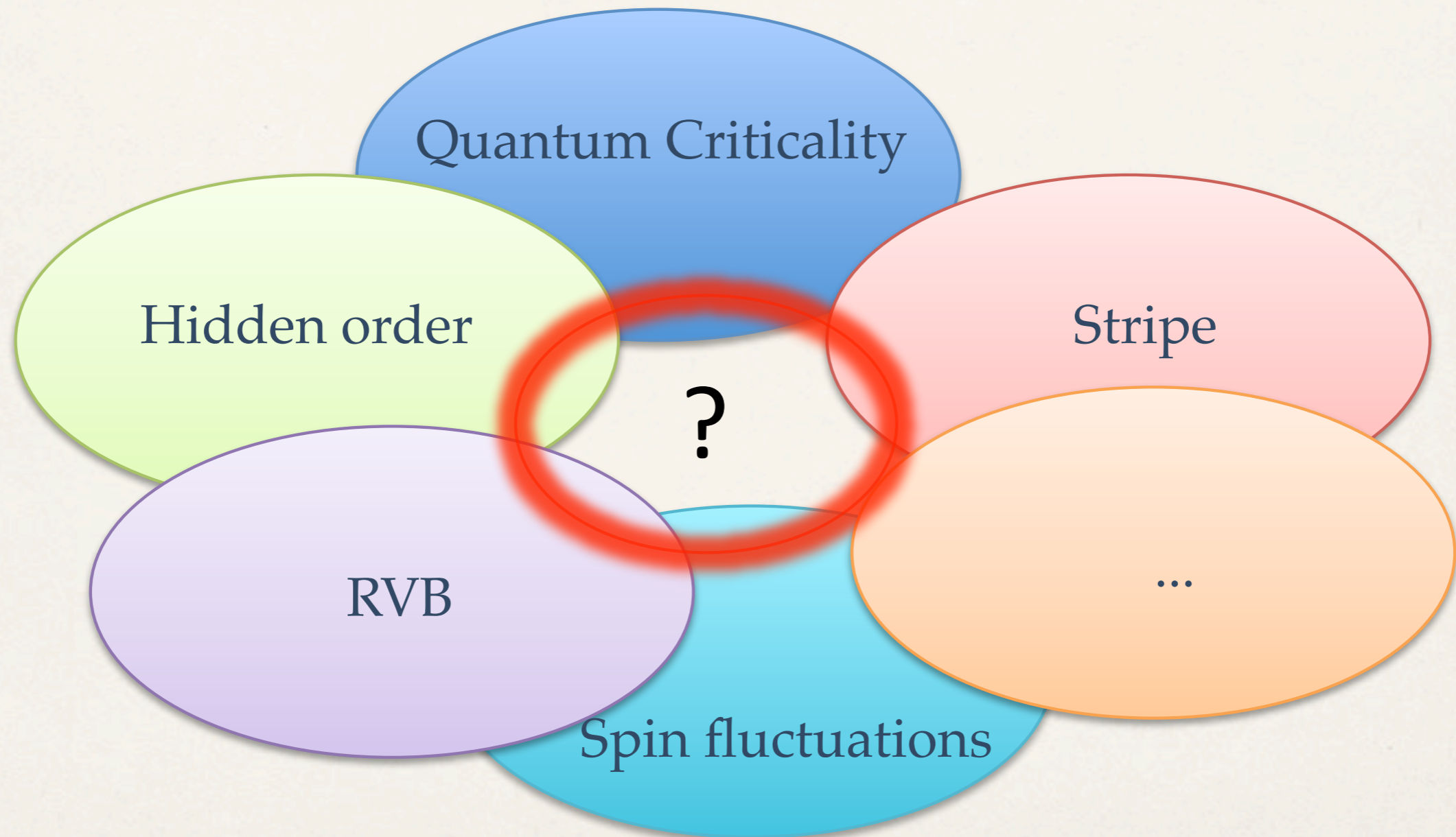


Shen et al., Science 307, 901 (2005)



# Theory of high- $T_c$ ?

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P. W. Anderson, G. Baskaran, S. Chakravarty, V. J. Emery, S. Kivelson, P. A. Lee, A. Millis, N. Nagaosa, M. Norman, S. Sachdev, D. Scalapino, C. Varma, J. Zaanen, ...

# Theory of high- $T_c$ ?

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No established theory yet!

What is the key?

# Comparison with FQHS

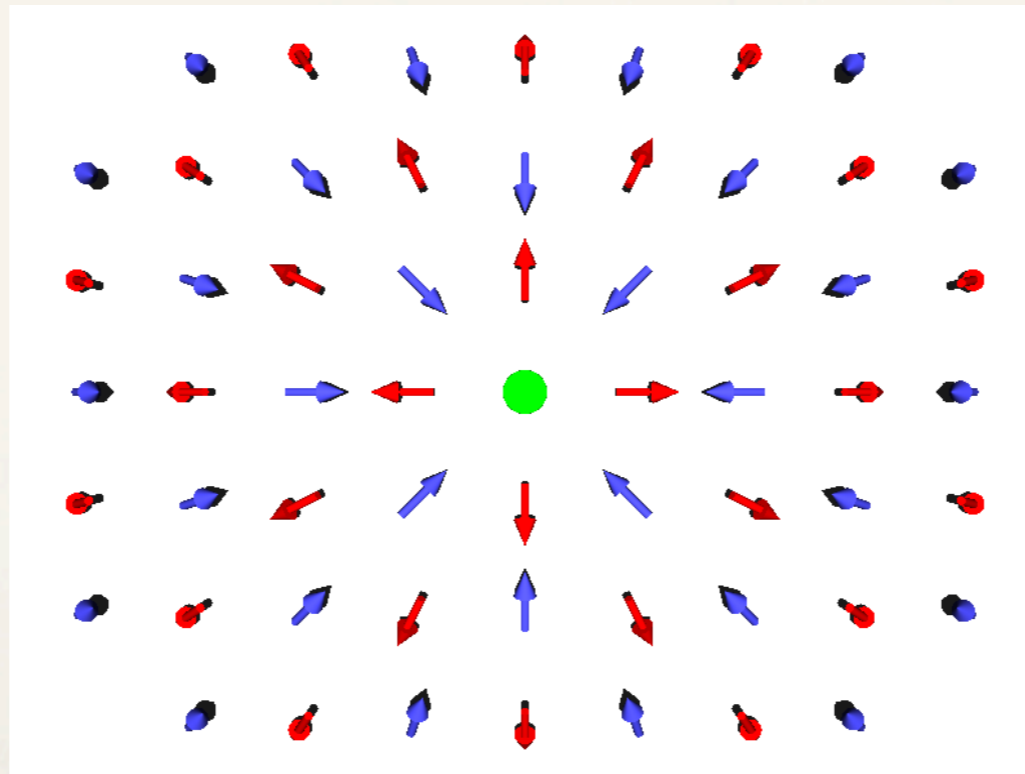
Strongly correlated electron system	FQHS	High-Tc
Model	2D electron gas under magnetic field	d-p model? t-J model? Hubbard model?
Strong correlations	Haldane's pseudopotential	on-site U
G.S. wave function	Laughlin w.f. $\Psi = \prod (z_i - z_j)^m \exp\left(-\sum \frac{ z_j ^2}{4}\right)$	x=0: Néel x≠0: ???
The key idea	composite particle	???

# Comparison with FQHS

Strongly correlated electron system	FQHS	High-Tc
Model	2D electron gas under magnetic field	d-p model? t-J model?
Strong correlations	Haldane's pseudopotential	on-site U
G.S. wave function	Laughlin w.f. $\Psi = \prod (z_i - z_j)^m \exp\left(-\sum \frac{ z_j ^2}{4}\right)$	x=0: Néel x≠0: ???
The key idea	composite particle	Half-Skyrmion

# Half-skyrmion theory

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# Half-skyrmion theory

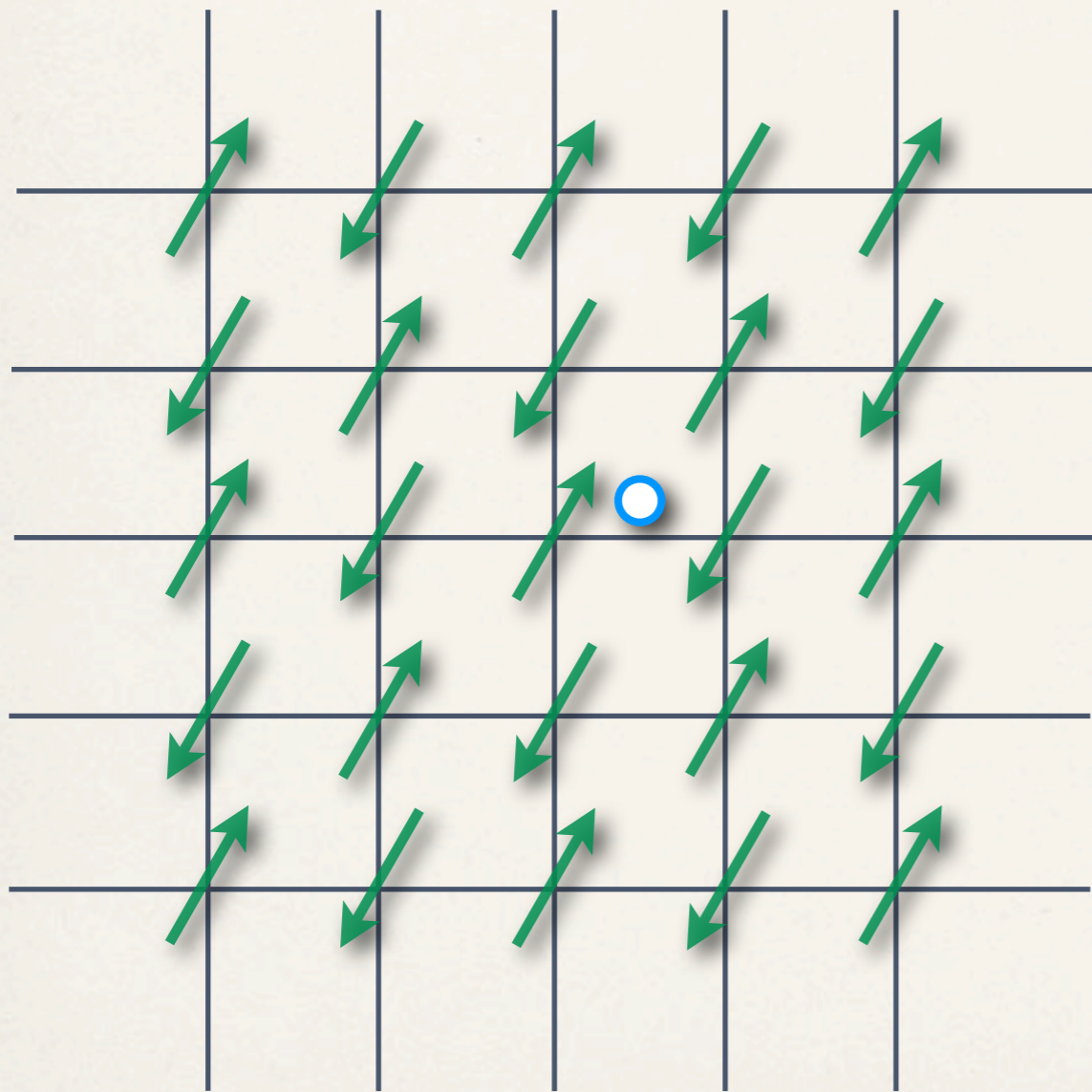
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- ❖ Why half-skyrmion?
- ❖ Superconductivity?
- ❖ Pseudogap?

# Why half-skyrmion?

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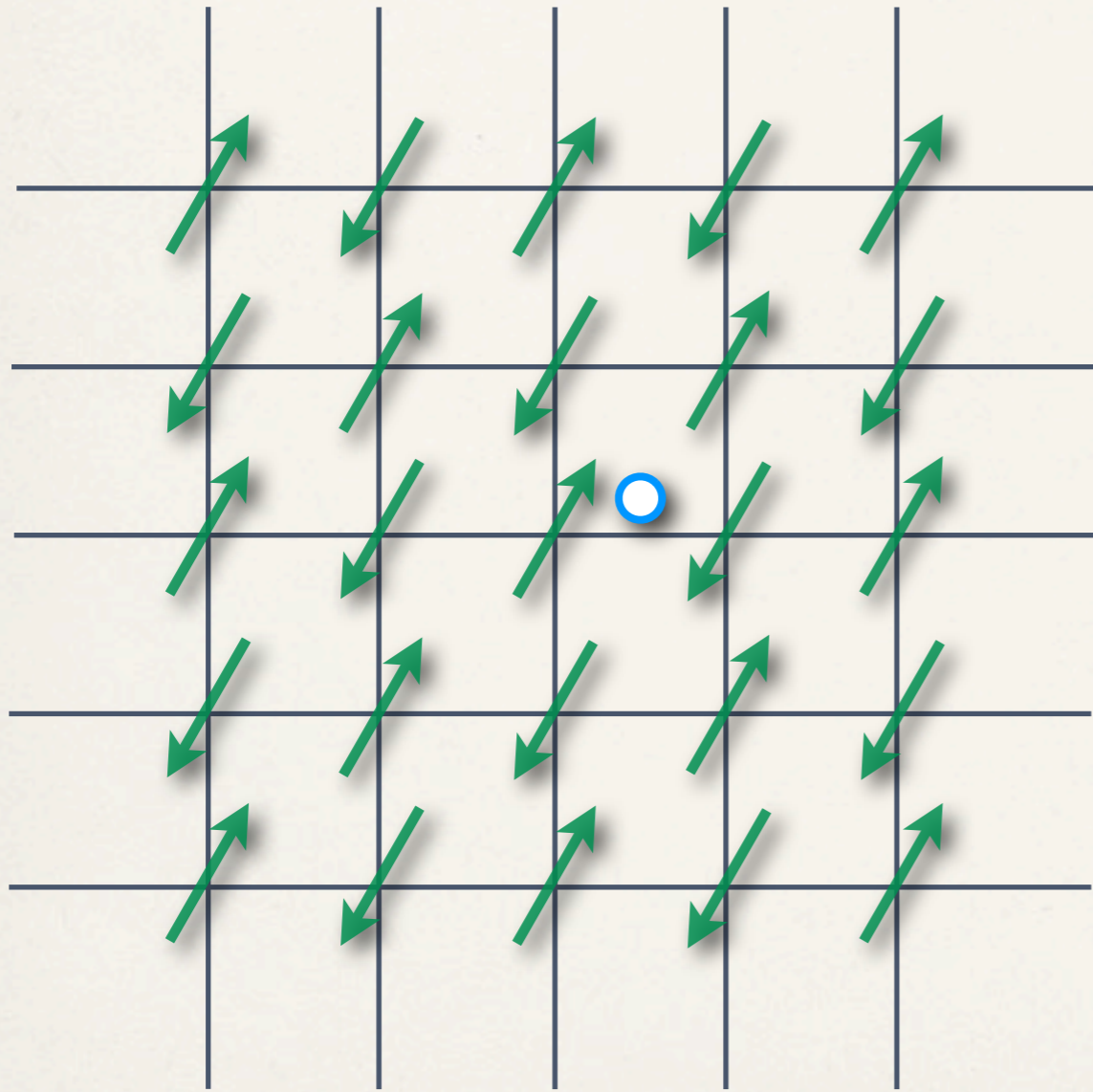


Introduce a single hole

How does the hole affect the surrounding spins?

# Why half-skyrmion?

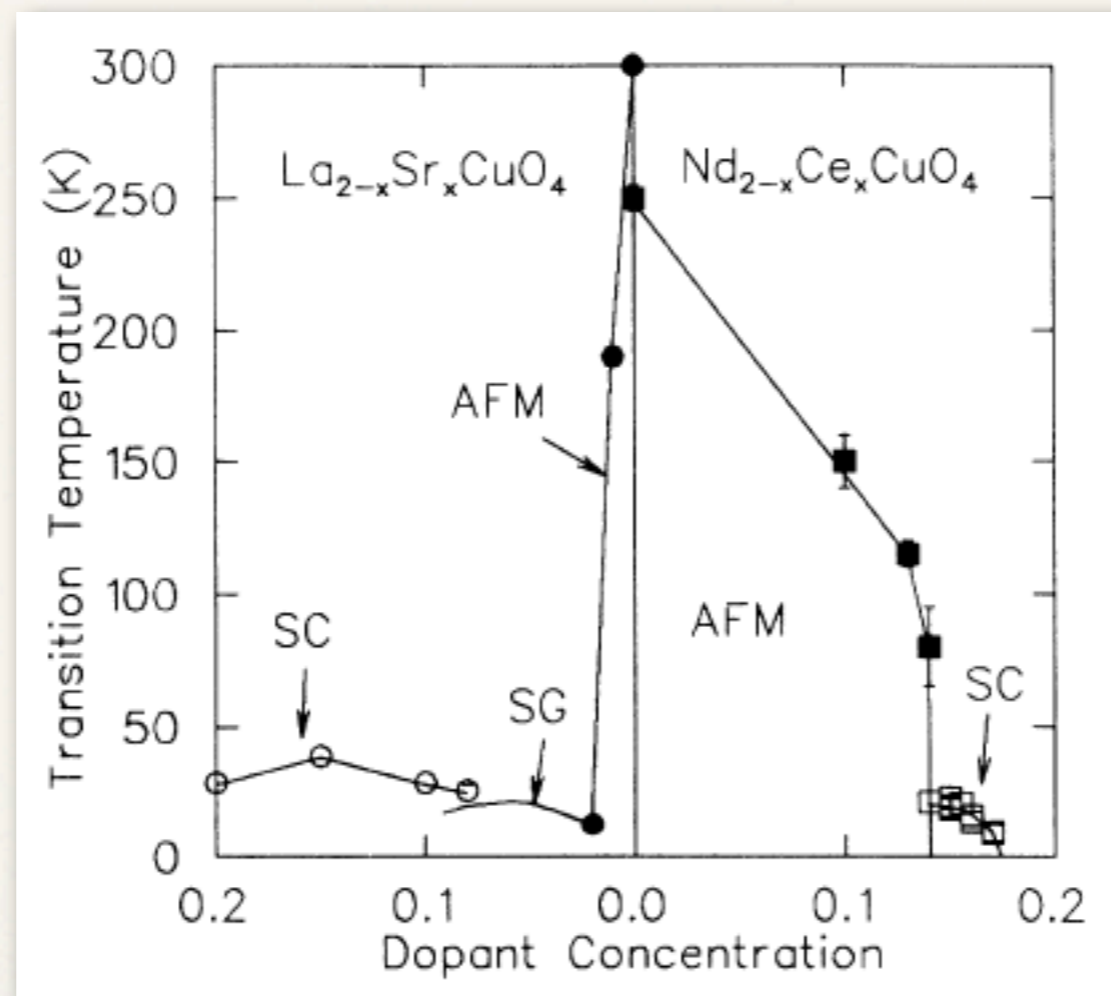
---



Nothing happened?

Introduce some modification?

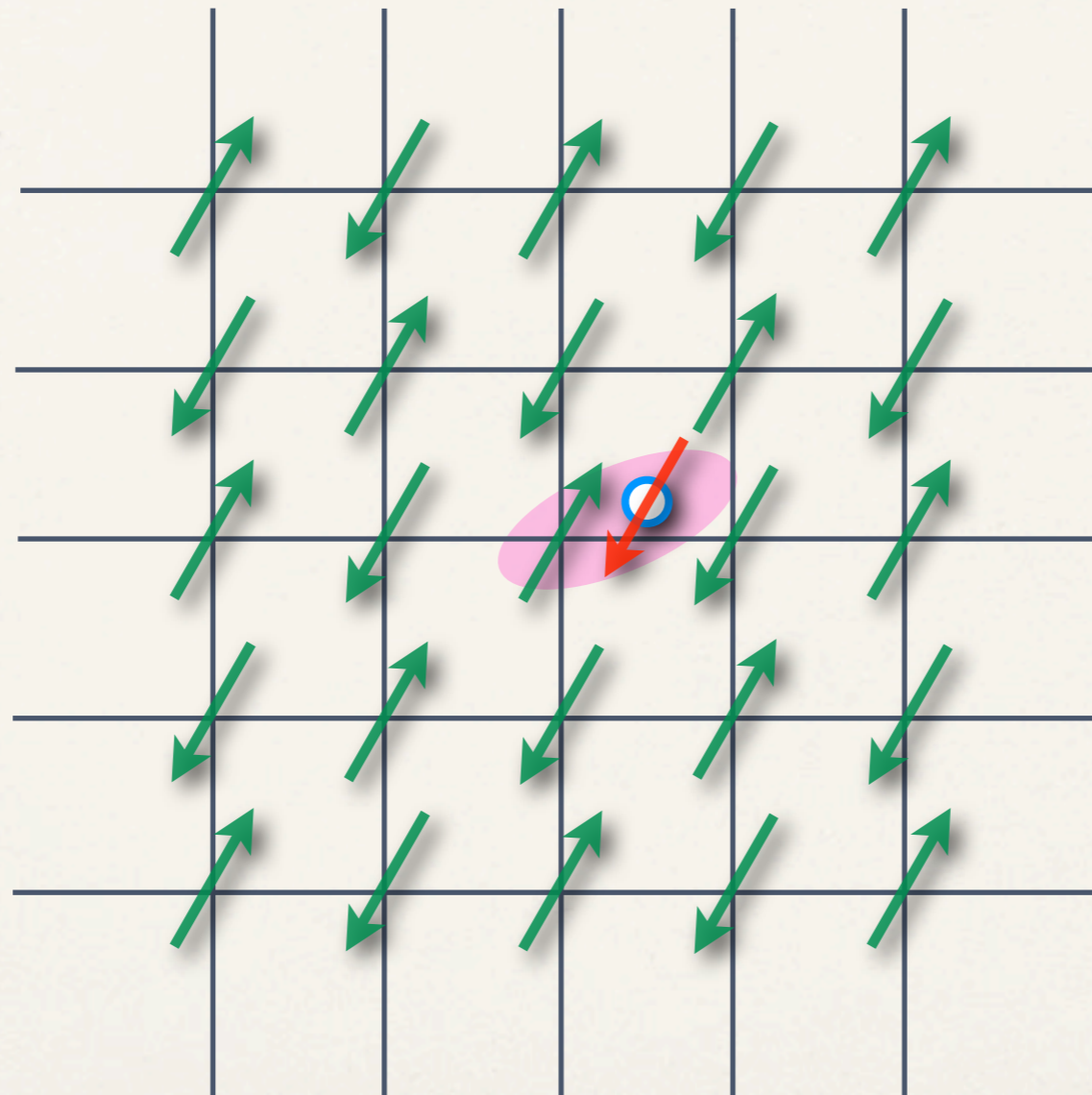
# Rapid suppression of Neel ordering



G. M. Luke et al., Phys. Rev. B 42, 7981 (1990)

# Interaction between hole spin and copper spin

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Kondo coupling

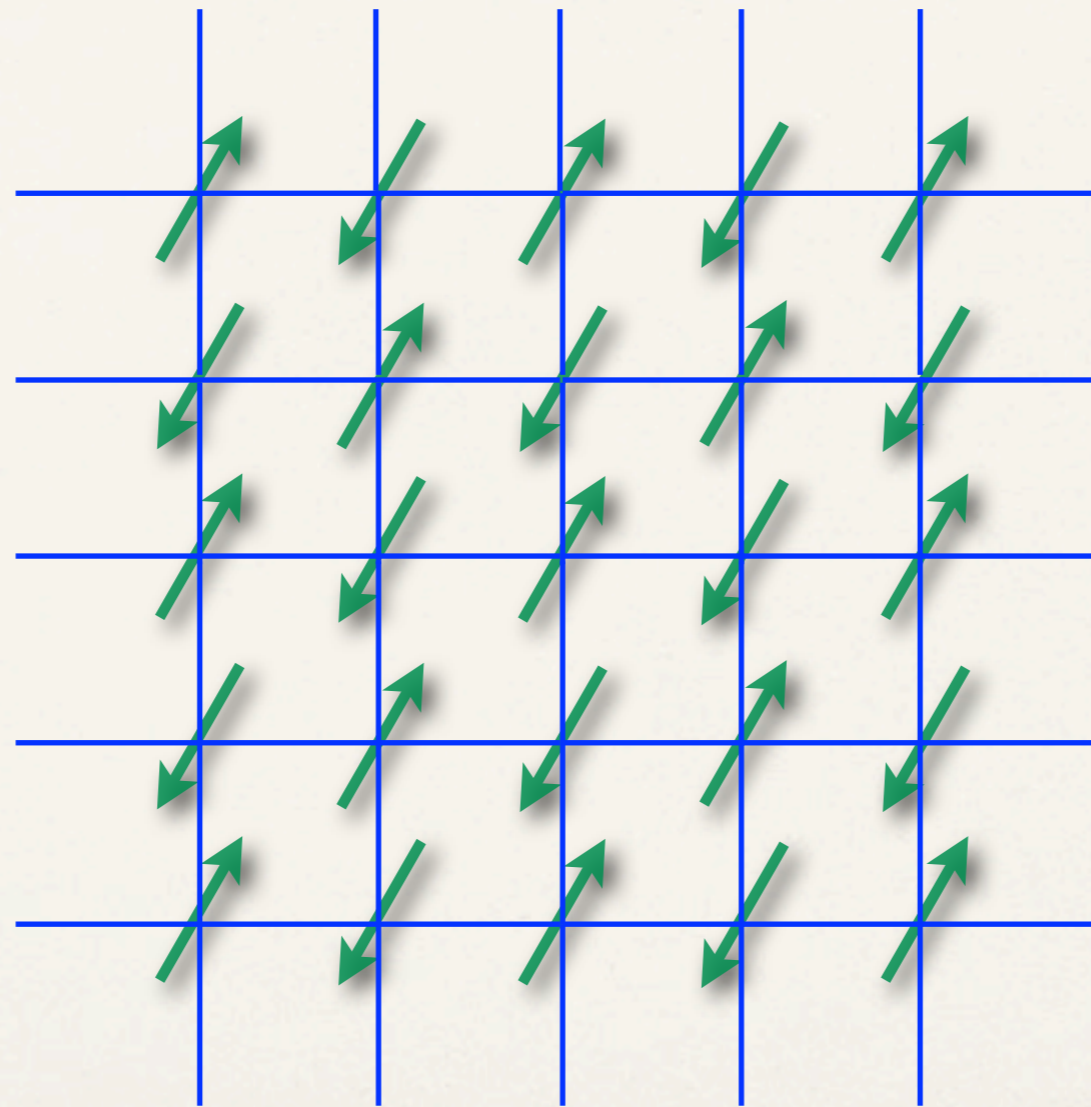
Zhang and Rice, PRB 37, 3759 (1988)

$$J_K \approx 1\text{eV}$$

Zhang-Rice singlet?

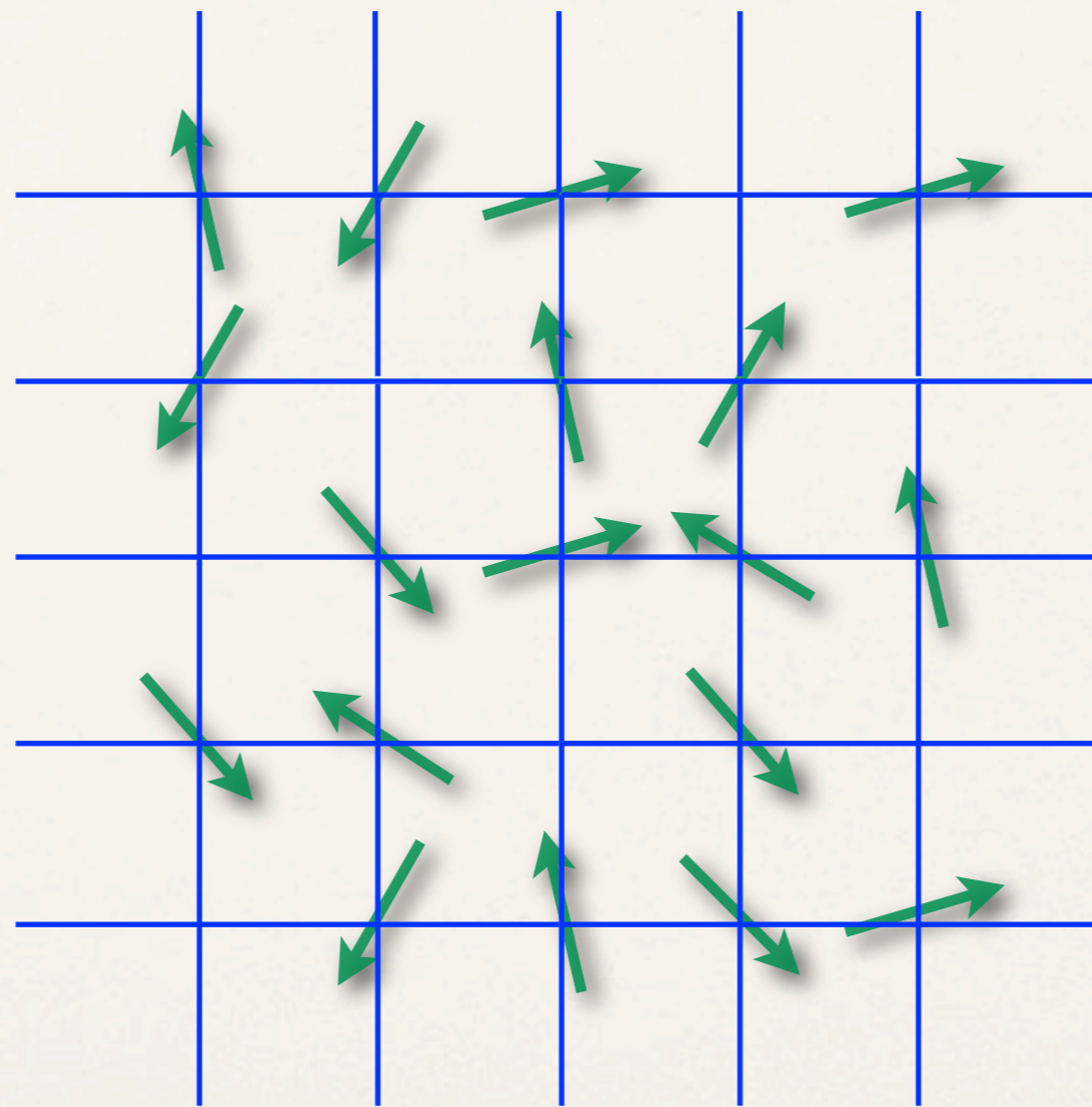
# Remove sites

---



# Remove sites

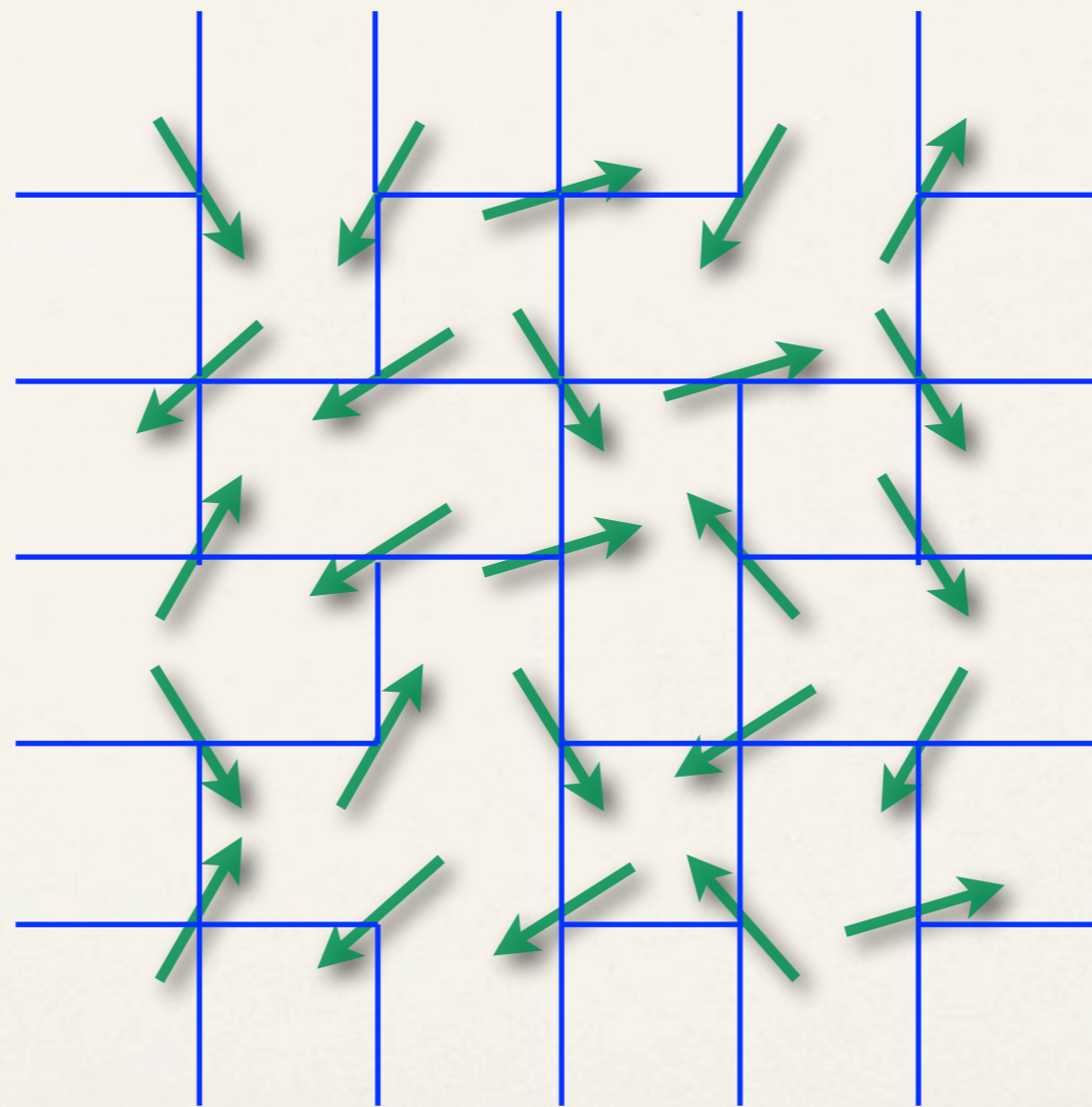
---



$$x_c \sim 0.3$$

# Remove bonds

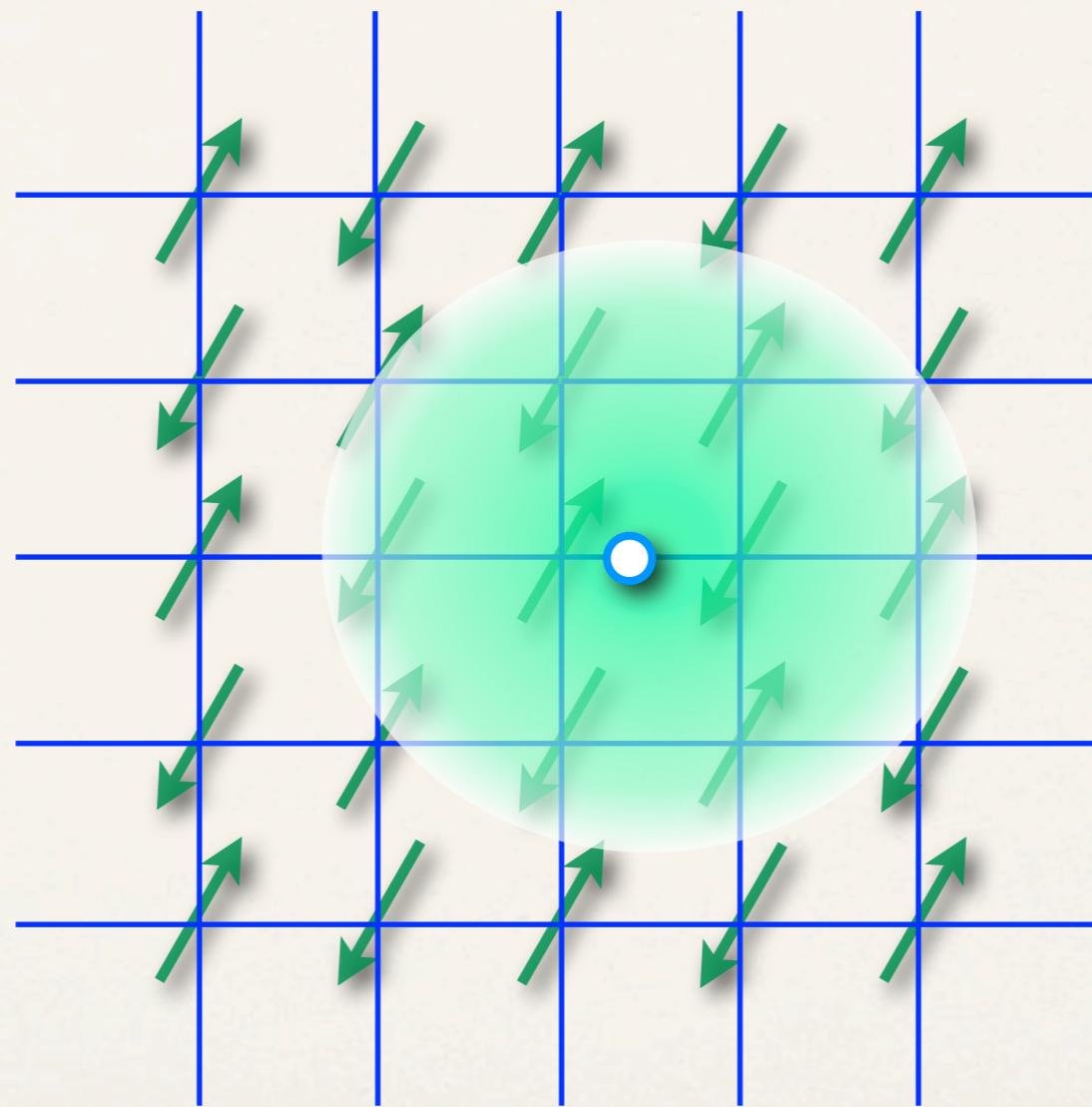
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$$x_c \sim 0.3$$

# Texture creation

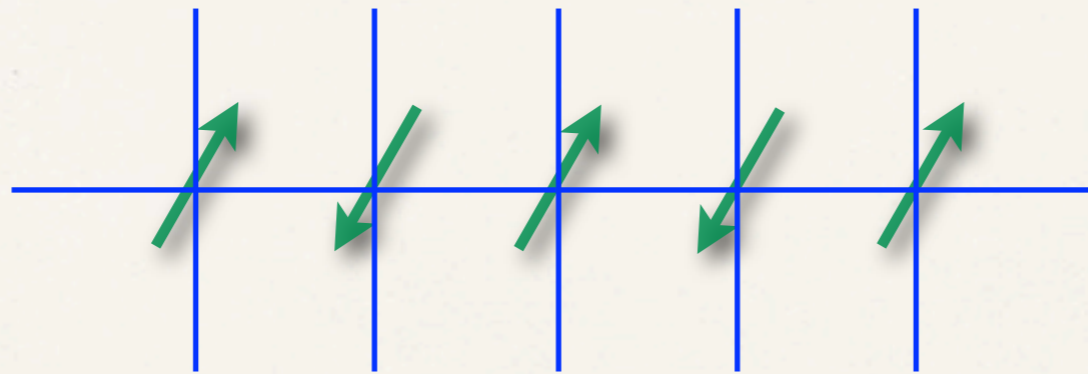
---



What kind of texture  
is formed?

# Boson representation

---



$$S_{jz} = \frac{1}{2} (b_{j\uparrow}^\dagger b_{j\uparrow} - b_{j\downarrow}^\dagger b_{j\downarrow})$$

Schwinger bosons

D. P. Arovas and A. Auerbach,  
PRB 38, 316 (1988)



$$\langle b_{\uparrow}^\dagger b_{\uparrow} \rangle = 1$$



$$\langle b_{\downarrow}^\dagger b_{\downarrow} \rangle = 1$$

Constraint

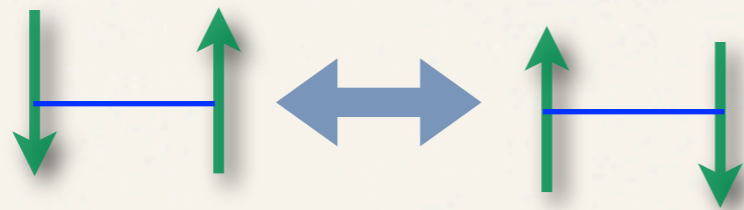
$$b_{j\uparrow}^\dagger b_{j\uparrow} + b_{j\downarrow}^\dagger b_{j\downarrow} = 1$$

# Bose-Einstein condensation analogy

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Néel ordering

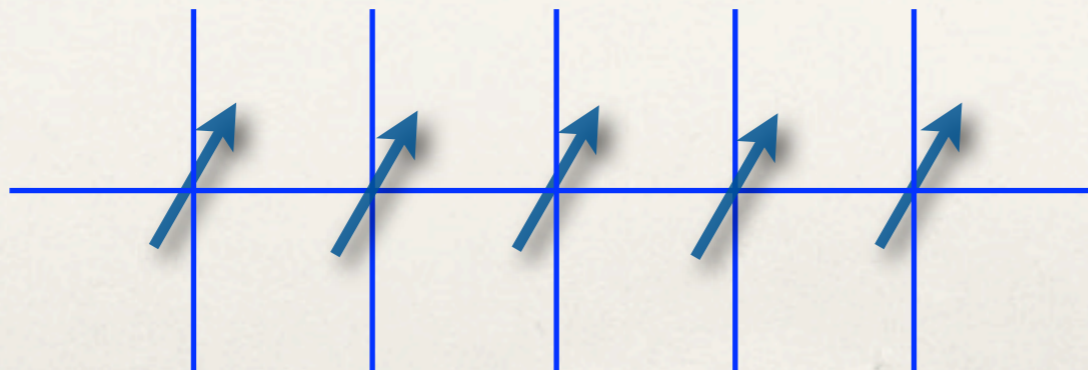
$$J\mathbf{S}_i \cdot \mathbf{S}_j = \frac{J}{2}(S_i^+ S_j^- + S_i^- S_j^+) + JS_i^z S_j^z$$



phase coherence

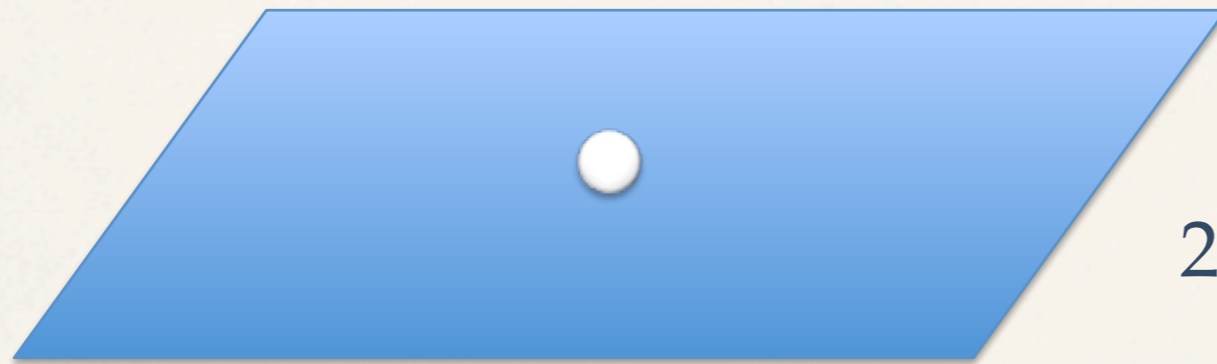
Quantum fluctuations

Ferromagnetic LRO



boson number is fixed

# Hole in a BEC



2D Gross-Pitaevskii equation

Fetter and Walecka, *Quantum Theory of Many-Particle Systems*, pp. 498-499



Vortex !

$$\langle \mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_k) \rangle \neq 0$$

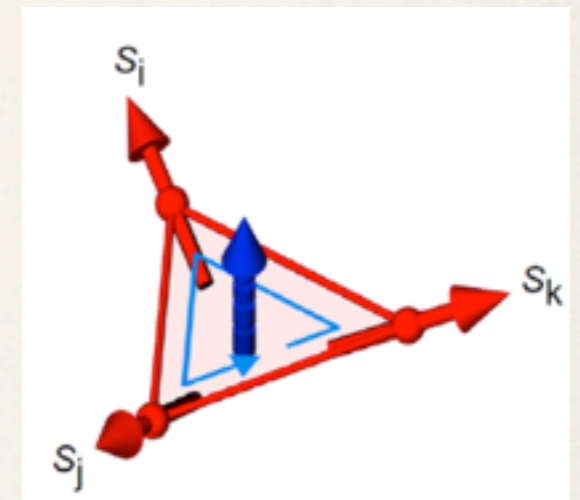
Flux quantization in “charge” 2 bosons

$$\langle b_{i\uparrow} b_{j\downarrow} - b_{i\downarrow} b_{j\uparrow} \rangle$$

T. K. Ng, *Phys. Rev. Lett.* 82, 3504 (1999)

Half-vortex

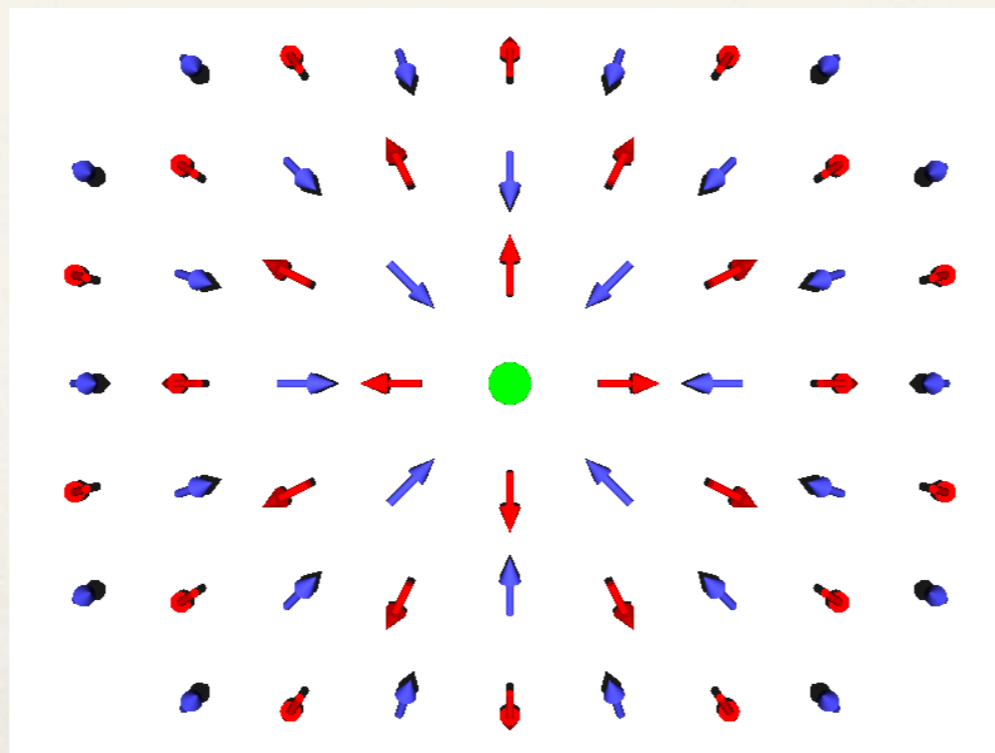
Half-Skyrmion spin texture



# Short summary of half-skyrmion

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- \* Rapid suppression of Néel ordering  $x \sim 0.02$   
Texture creation
- \* Boson representation of spins  
Néel ordering  $\longrightarrow$  Bose-Einstein condensation
- \* A hole creates a vortex



Half-skyrmion

# Half-skyrmion dynamics?

T.M., Phys. Rev. B 72, 104502 (2005)

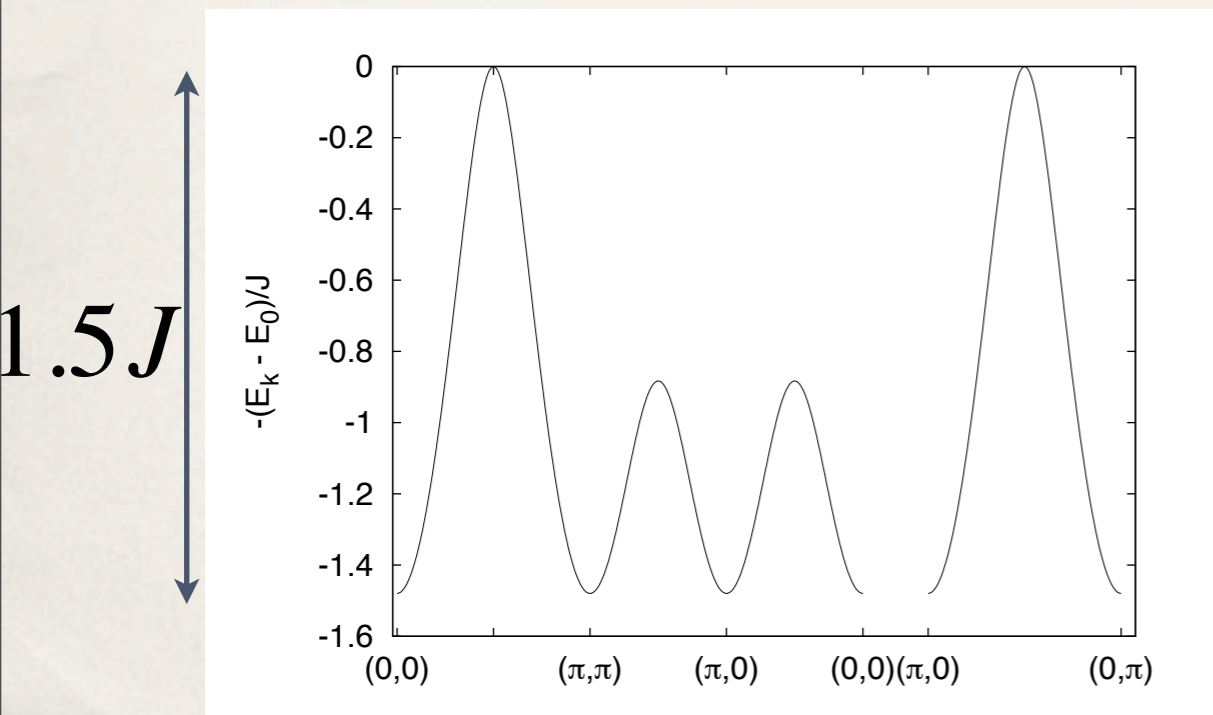
❖ Non-linear sigma model

S. Chakravarty, B. I. Halperin, and D. R. Nelson  
Phys. Rev. Lett. 60, 1057 (1988)

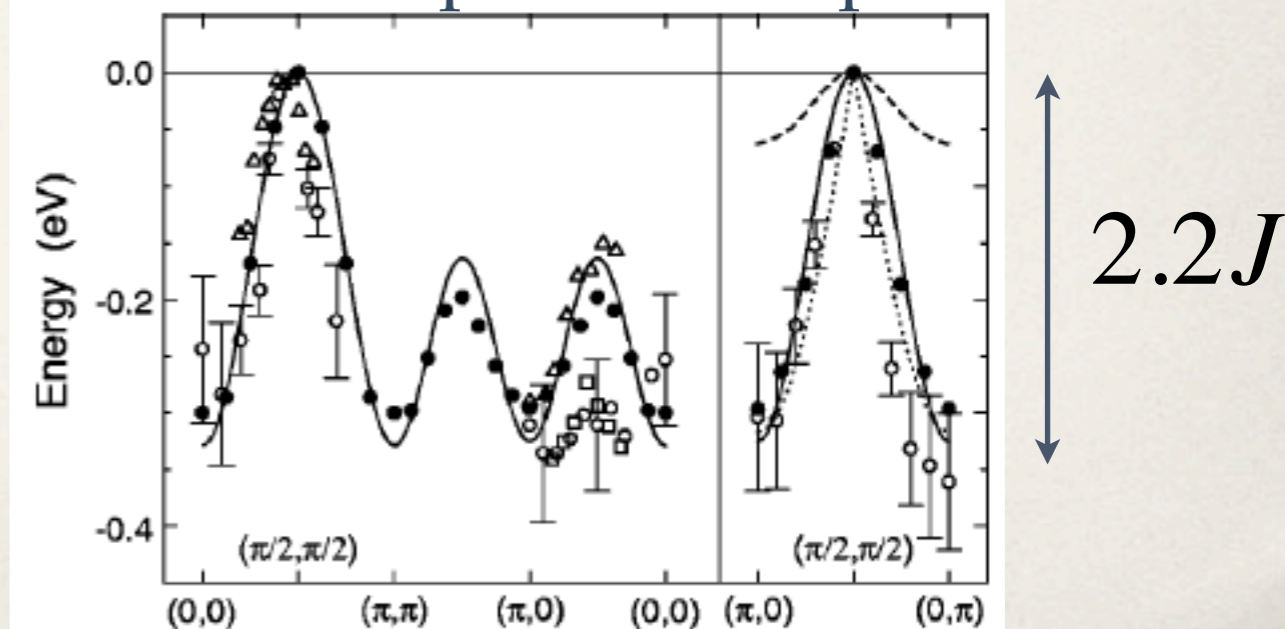
❖ Belavin-Polyakov solution

❖ Lorentz boost

$$E_{\mathbf{k}} = \sqrt{c_{sw}^2 (\cos^2 k_x + \cos^2 k_y) + E_0^2} \quad E_0 = 2\pi\rho_s \approx J$$



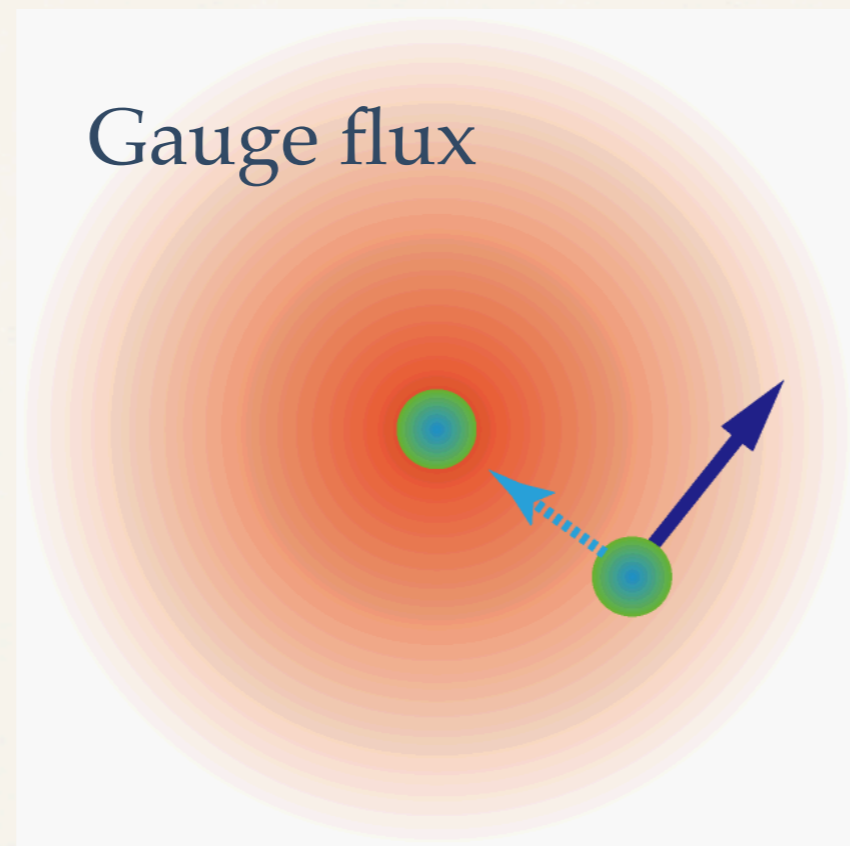
ARPES in the parent compound



T.Tohayama and S.Maekawa, Supercond. Sci. Technol. 13, R17 (2000)

# Mechanism of d-wave superconductivity

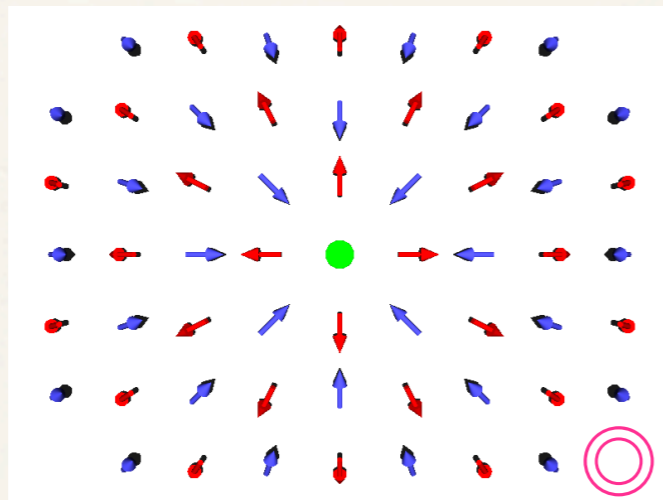
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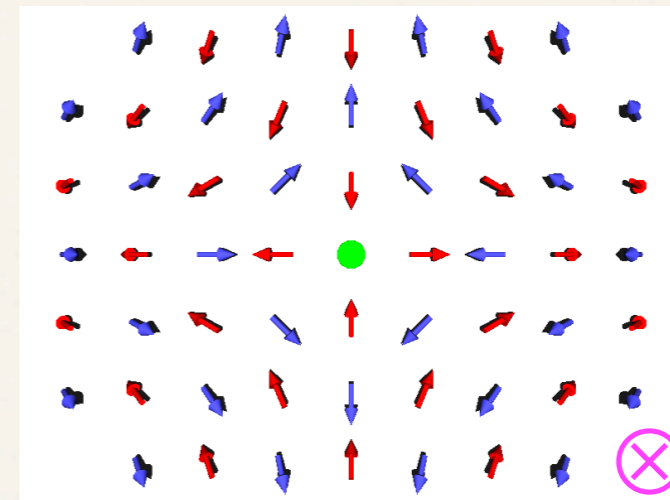
Lorentz force

# Mechanism of d-wave superconductivity

T.M., Phys. Rev. B 73, 064504 (2006)

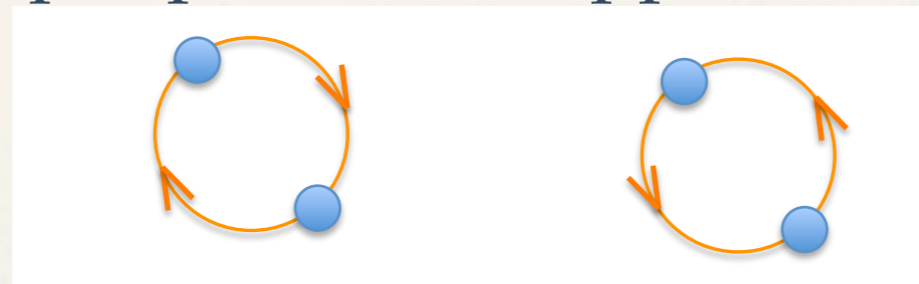


half-skyrmion



anti-half-skyrmion

Cooper pairs with opposite chiralities



$d_{x^2-y^2}$  - wave SC

# BCS mean field theory

T.M., Phys. Rev. B 73, 064504 (2006)

---

Integrate out half-skyrmion degrees of freedom

Effective interaction between holes

BCS gap equation

$$\Delta_{k,s,\sigma}^{(+)} = +\frac{1}{\Omega} \sum_{\mathbf{k} \neq \mathbf{k}'} V_{\mathbf{k},\mathbf{k}'}^{(s,\sigma)} \langle c_{-\mathbf{k}',s,\sigma} c_{\mathbf{k}',s,\sigma} \rangle$$

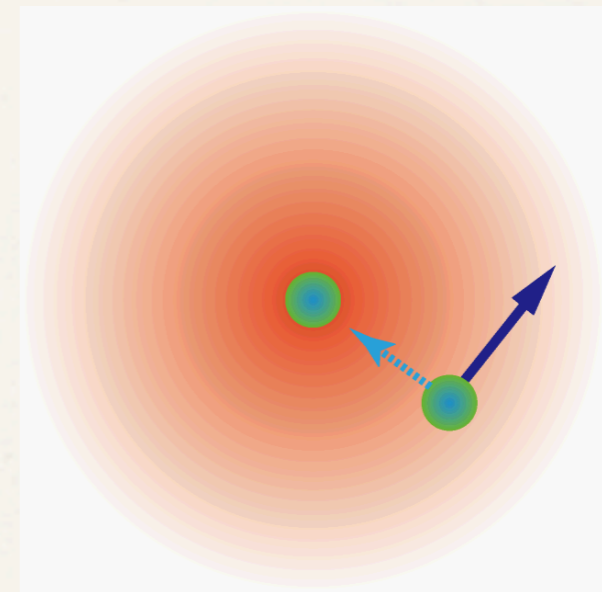
$$\Delta_{k,s,\sigma}^{(-)} = -\frac{1}{\Omega} \sum_{\mathbf{k} \neq \mathbf{k}'} V_{\mathbf{k},\mathbf{k}'}^{(s,\sigma)} \langle c_{-\mathbf{k}',-s,-\sigma} c_{\mathbf{k}',s,\sigma} \rangle$$

$$V_{\mathbf{k},\mathbf{k}'}^{(s,\sigma)} = \frac{2\pi i}{m\Omega} s\sigma \frac{k_x k'_y - k_y k'_x}{(\mathbf{k} - \mathbf{k}')^2} + V_{\mathbf{k},\mathbf{k}'}^C$$

# Mechanism of d-wave superconductivity

---

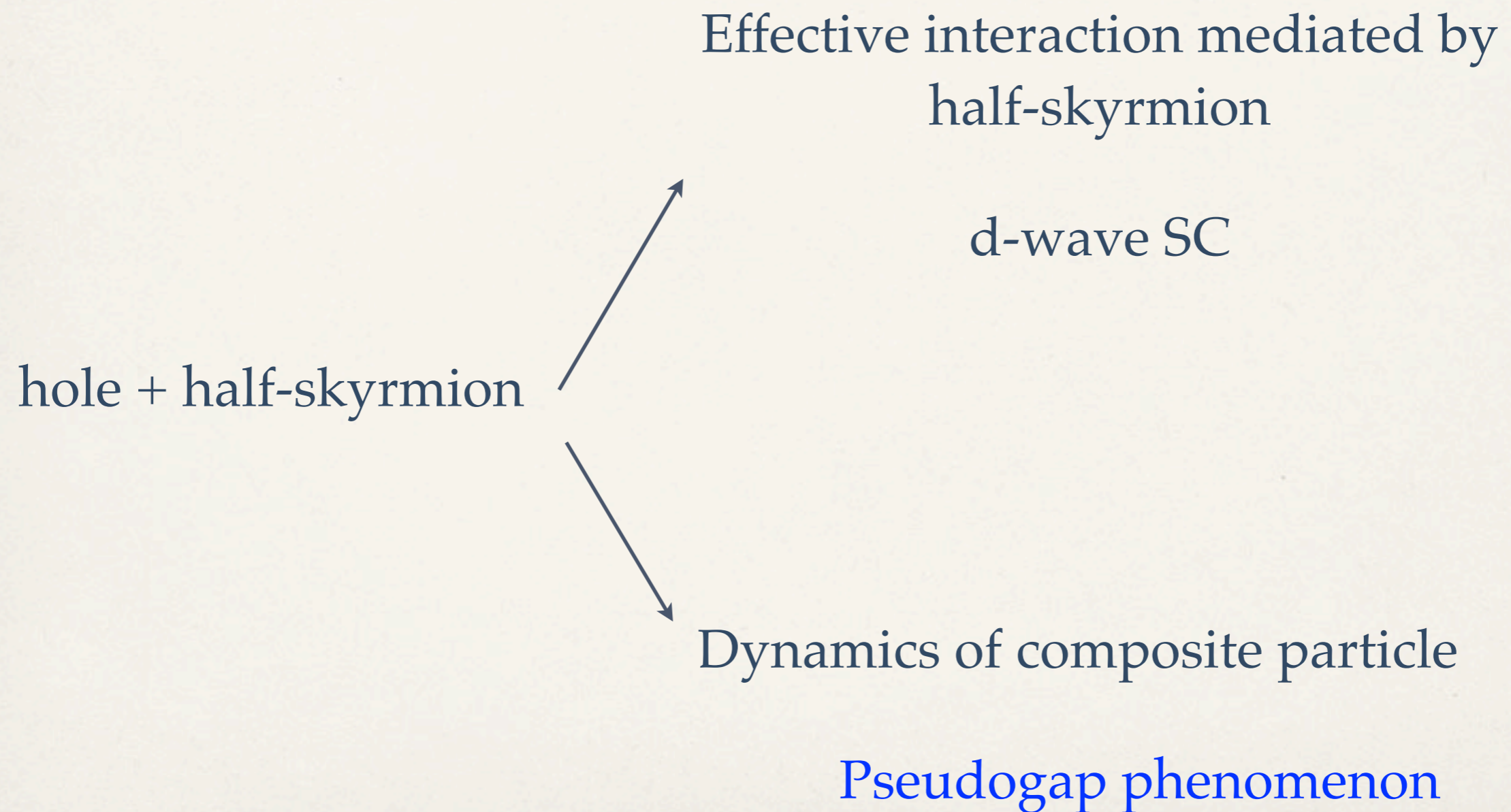
- ❖ Holes carry a gauge flux.
- ❖ Another hole feels the Lorentz force.
- ❖ Two chiral Cooper pairing



d-wave pairing

# Pseudogap?

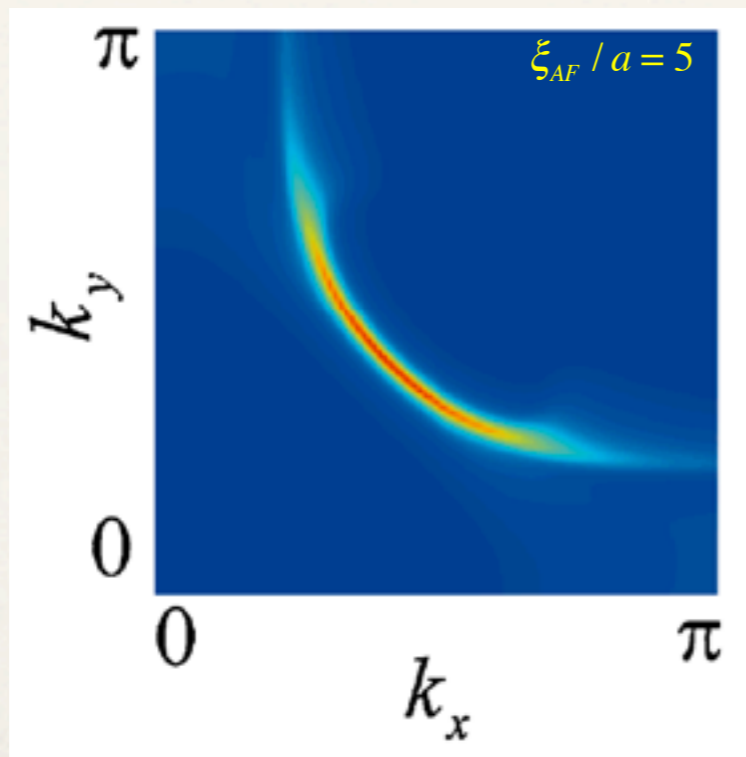
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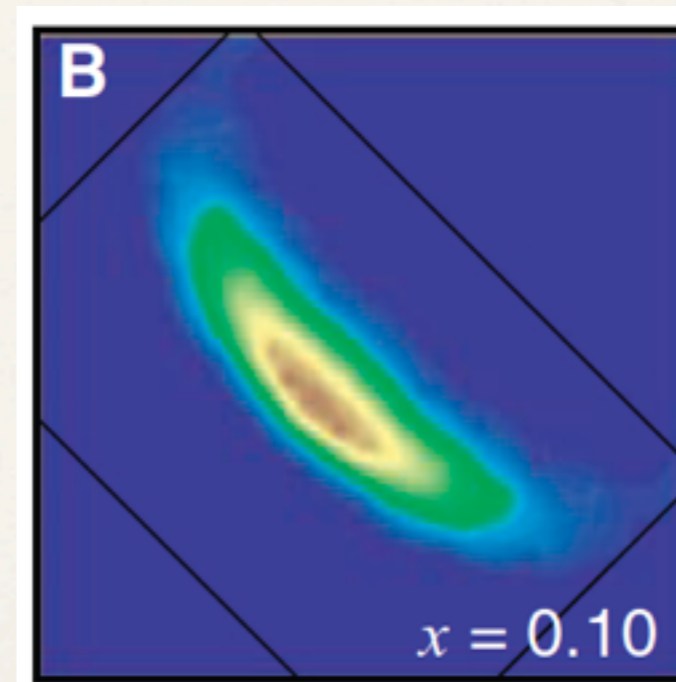
# Dynamics of composite particle

$$\mathcal{H}_{\text{composite particle}} = \mathcal{H}_{\text{hole}} + \mathcal{H}_{\text{half-skyrmion}}$$

Creates a partial gap



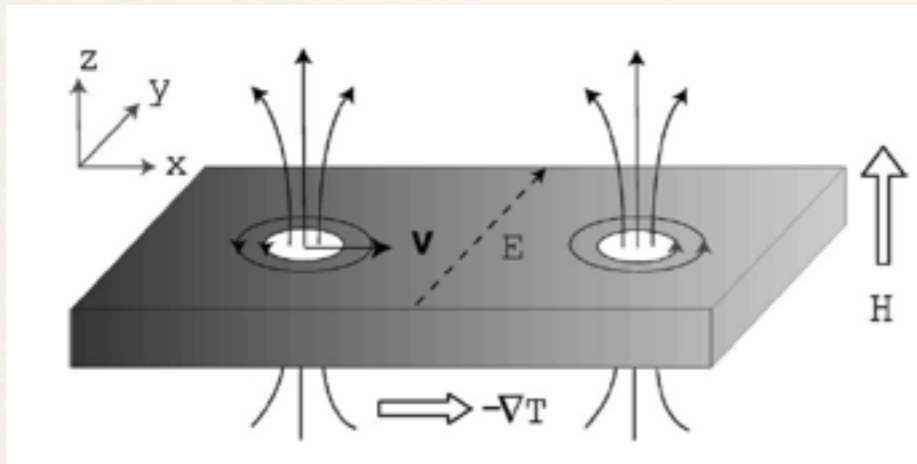
T.M. J. Phys. Soc. Jpn. 78, 054708 (2009)



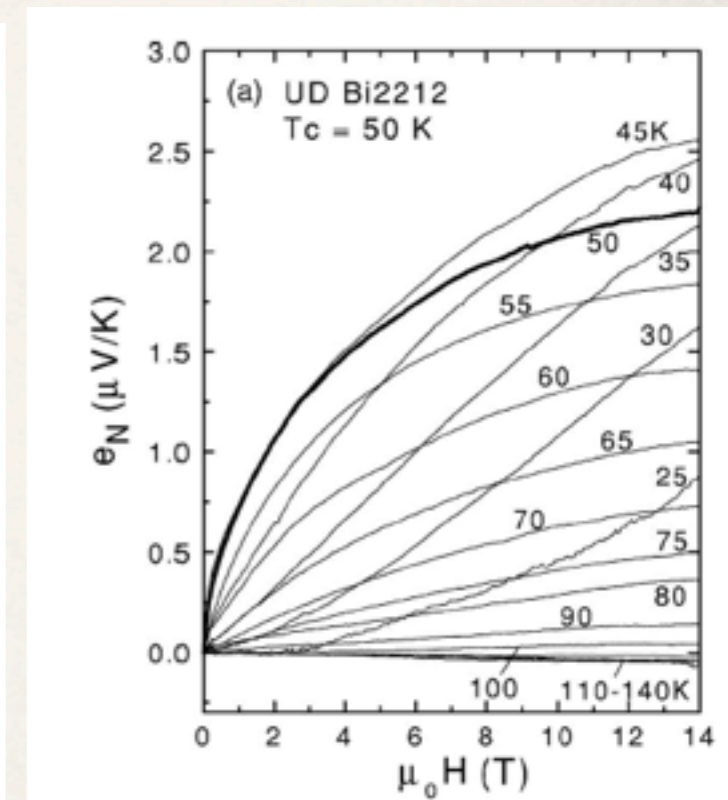
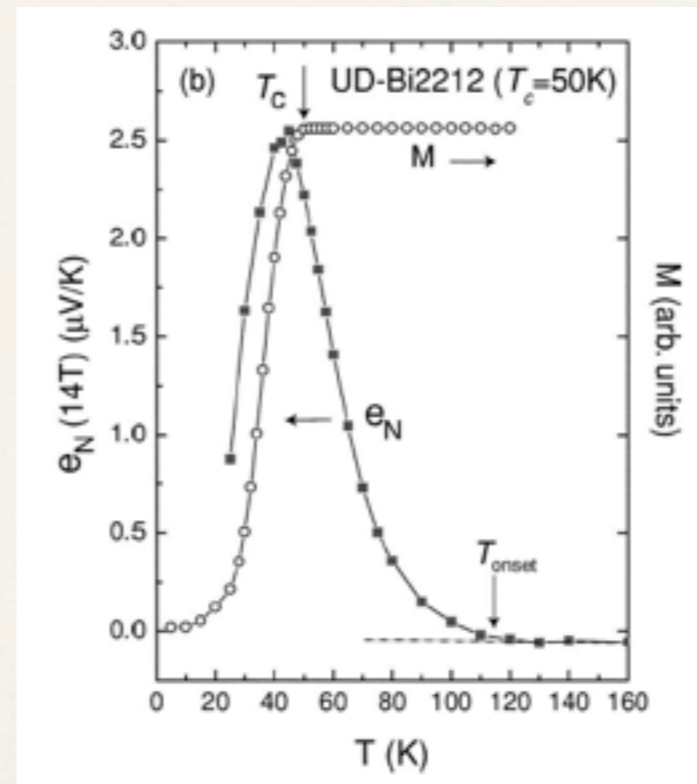
Shen et al., Science 307, 901 (2005)

# Nernst effect

Y. Wang, L. Li, and L. P. Ong, PRB 73, 024510 (2006)

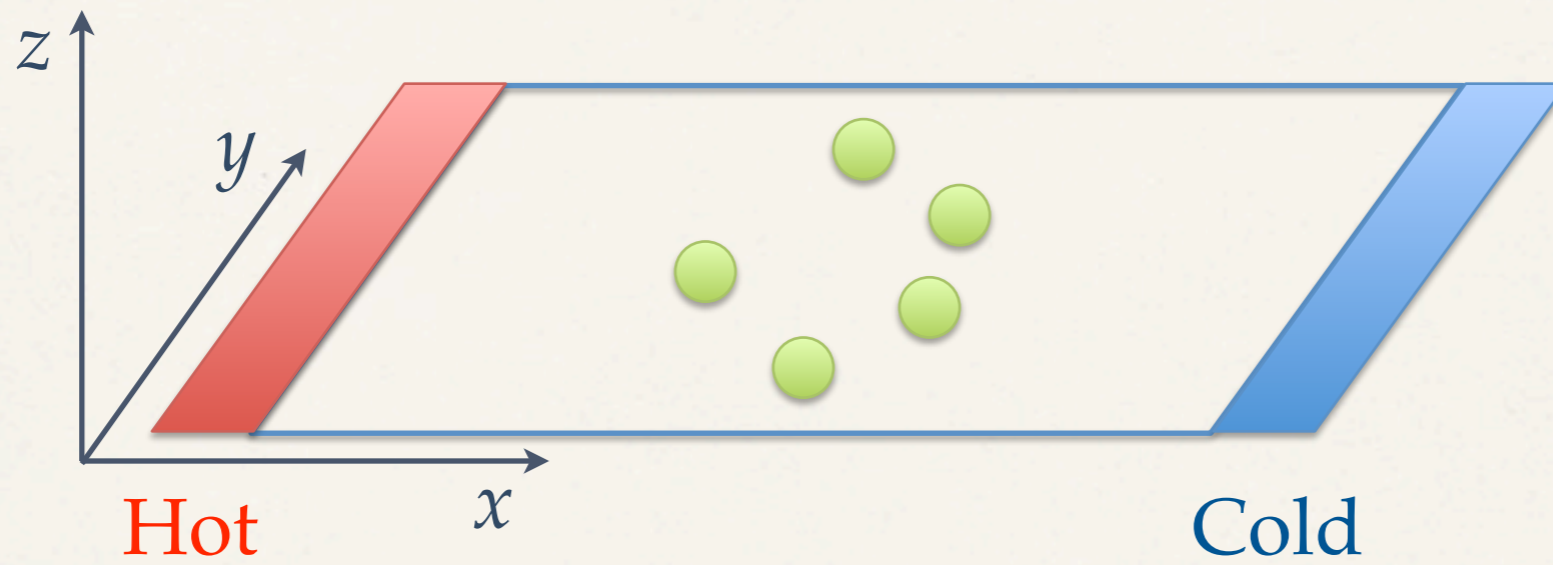


$$e_N = \frac{E_y}{-\nabla_x T}$$

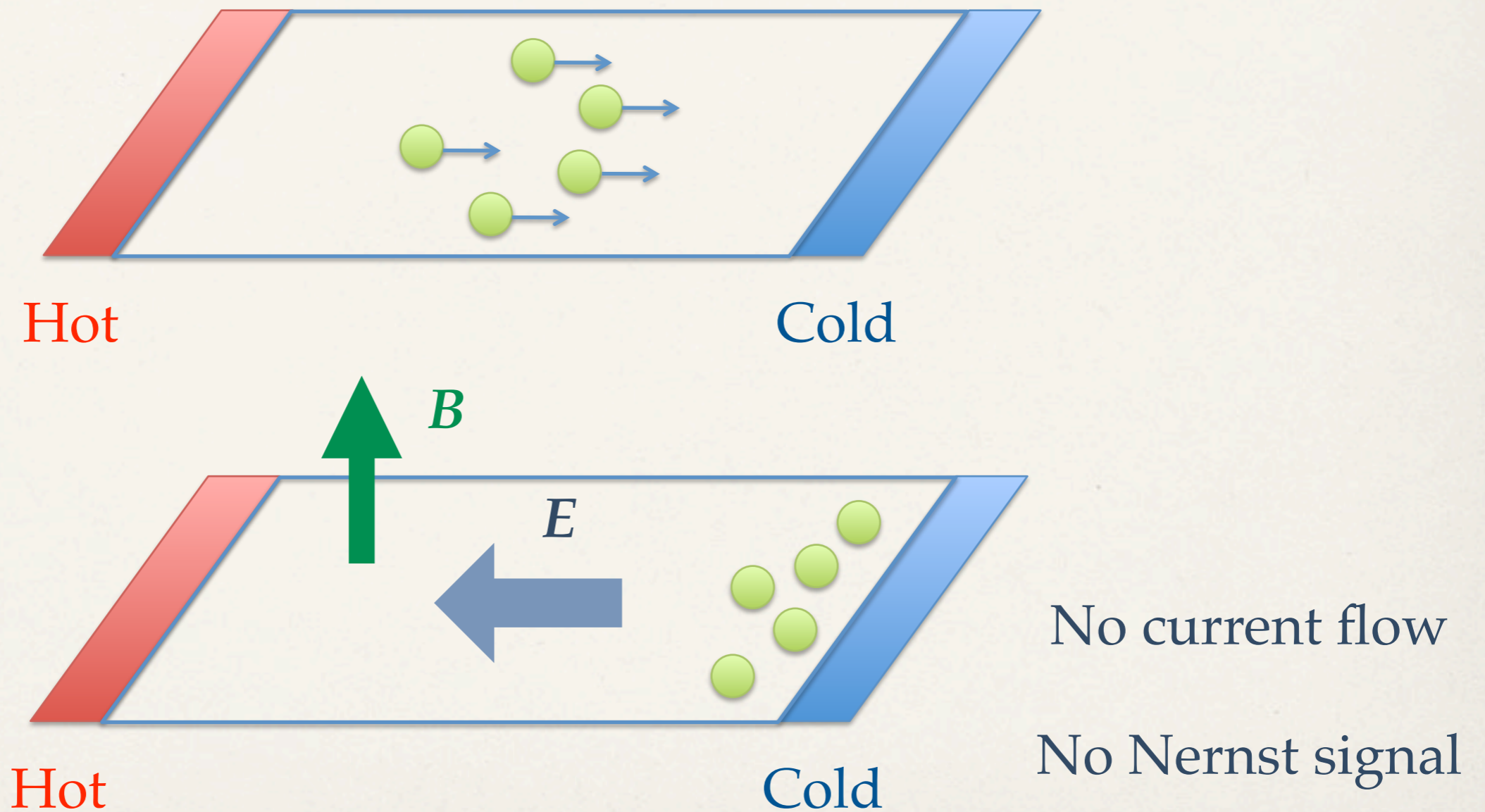


# What is Nernst effect?

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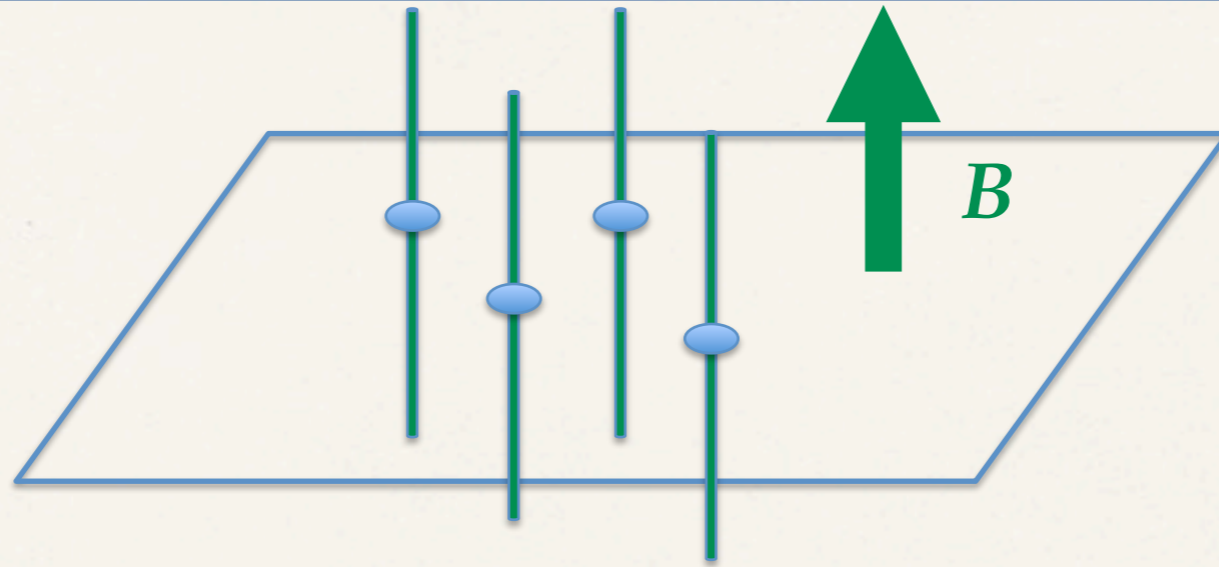
# What is Nernst effect?



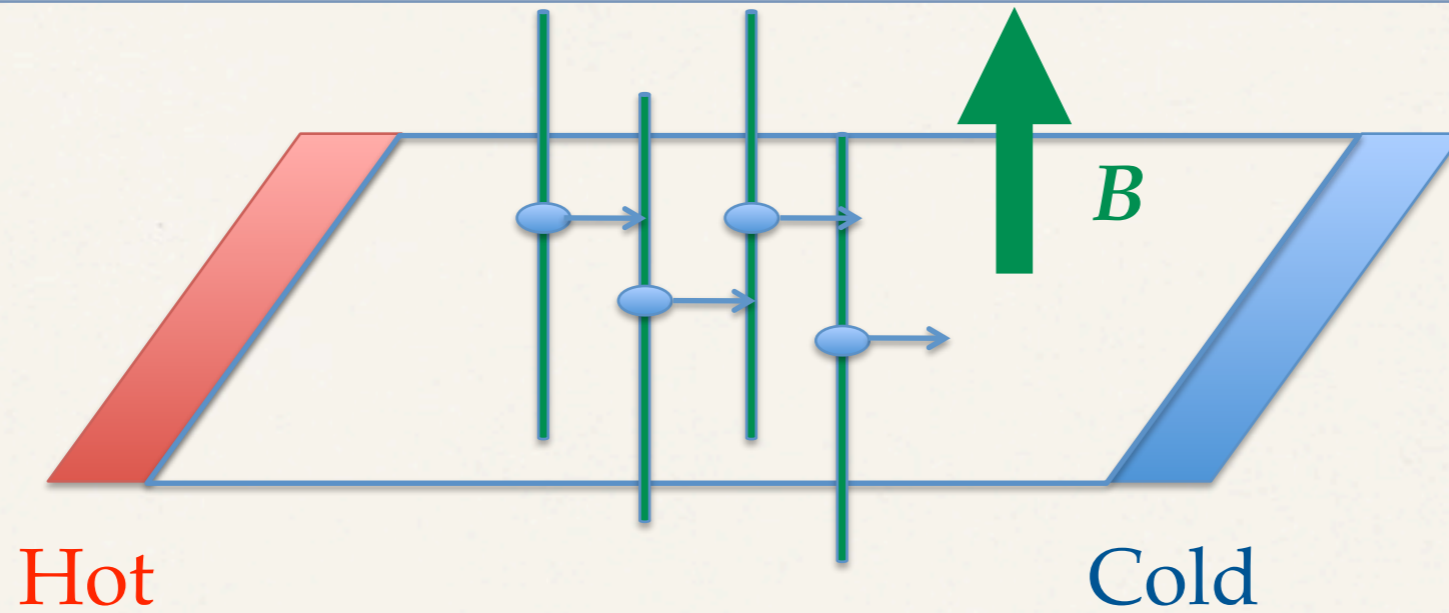
$$qE - k_B \partial_x T = 0$$

# Nernst effect by vortex-flow

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# Nernst effect by vortex-flow



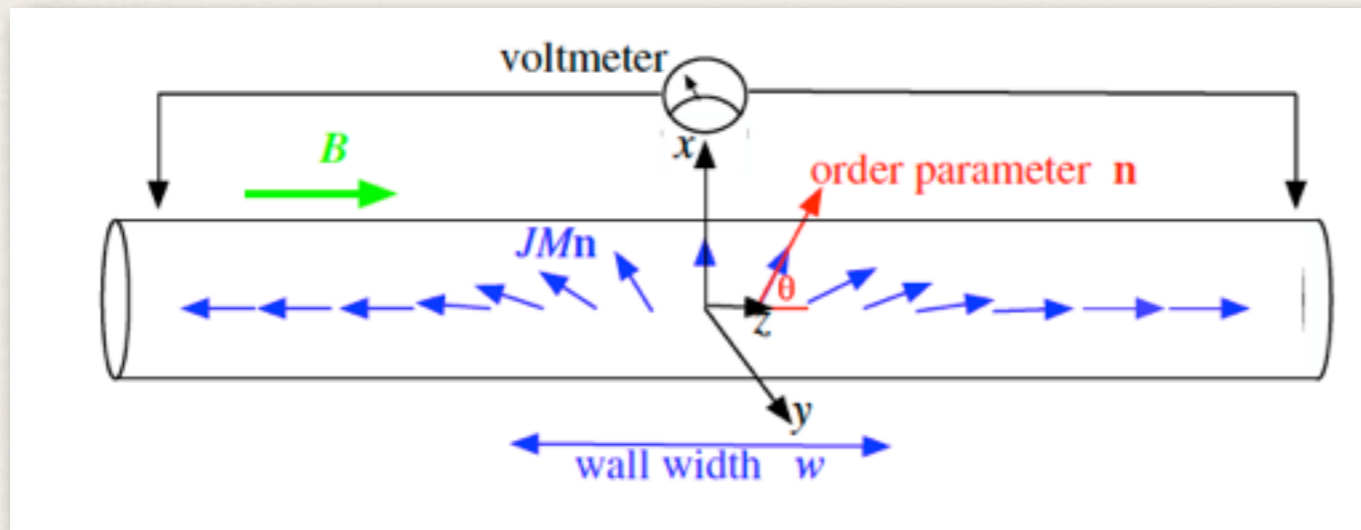
Steady vortex current

$$e_N = \frac{E_y}{-\nabla_x T} \neq 0$$

Vortex formation above  $T_c$ ?

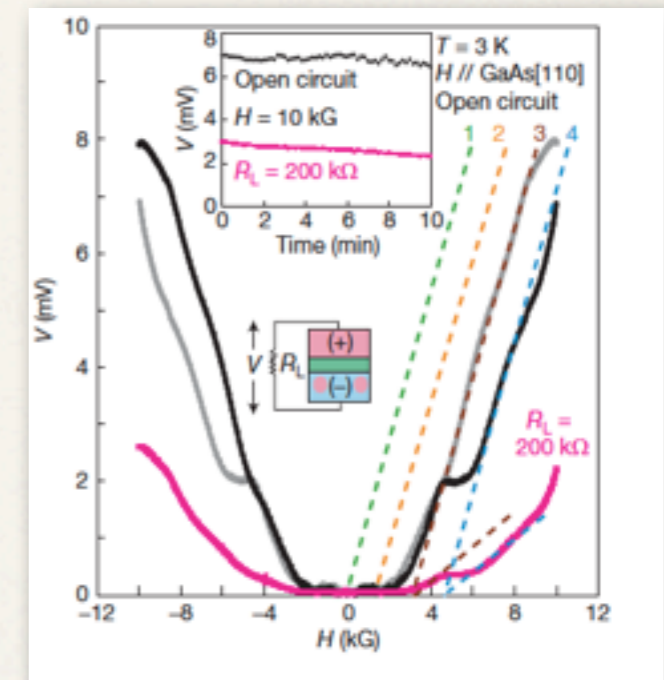
# Berry phase effect under magnetic field

## Domain wall



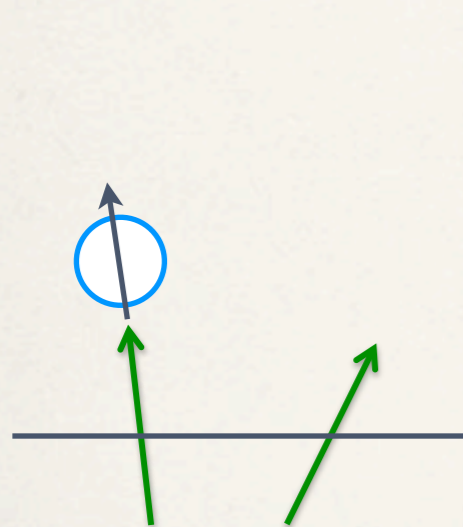
S. E. Barnes and S. Maekawa, PRL 98, 246601 (2007)

## Experiment



P. N. Hai et al., Nature 458, 489 (2009)

# Nernst effect by half-skyrmion



$$\mathbf{n} = (n_x, n_y, n_z)$$

$$\mathbf{m} = \frac{1}{\sqrt{2(1+n_z)}} (n_x, n_y, n_z)$$

$$\mathbf{e} = -2\hbar\boldsymbol{\sigma} \cdot (\partial_t \mathbf{m} \times \nabla \mathbf{m})$$

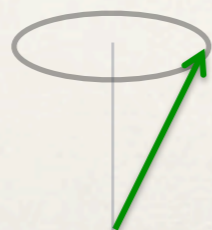
$$\mathbf{b} = \hbar \varepsilon_{ijk} \sigma_i (\nabla m_j) \times (\nabla m_k)$$

V. Korenman, J. L. Murray, and R. E. Prange,  
Phys. Rev. B, 16, 4032 (1977)

G. Volovik, J. Phys. C, 20, L83 (1987)

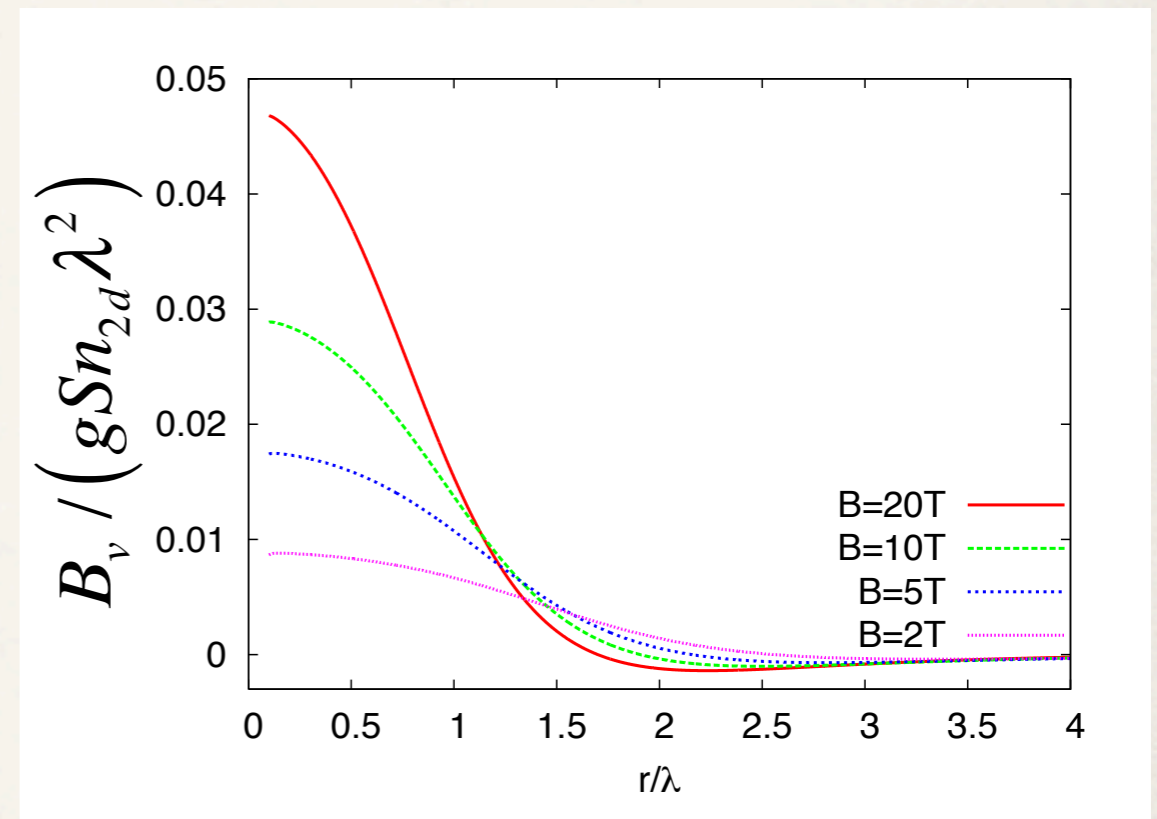
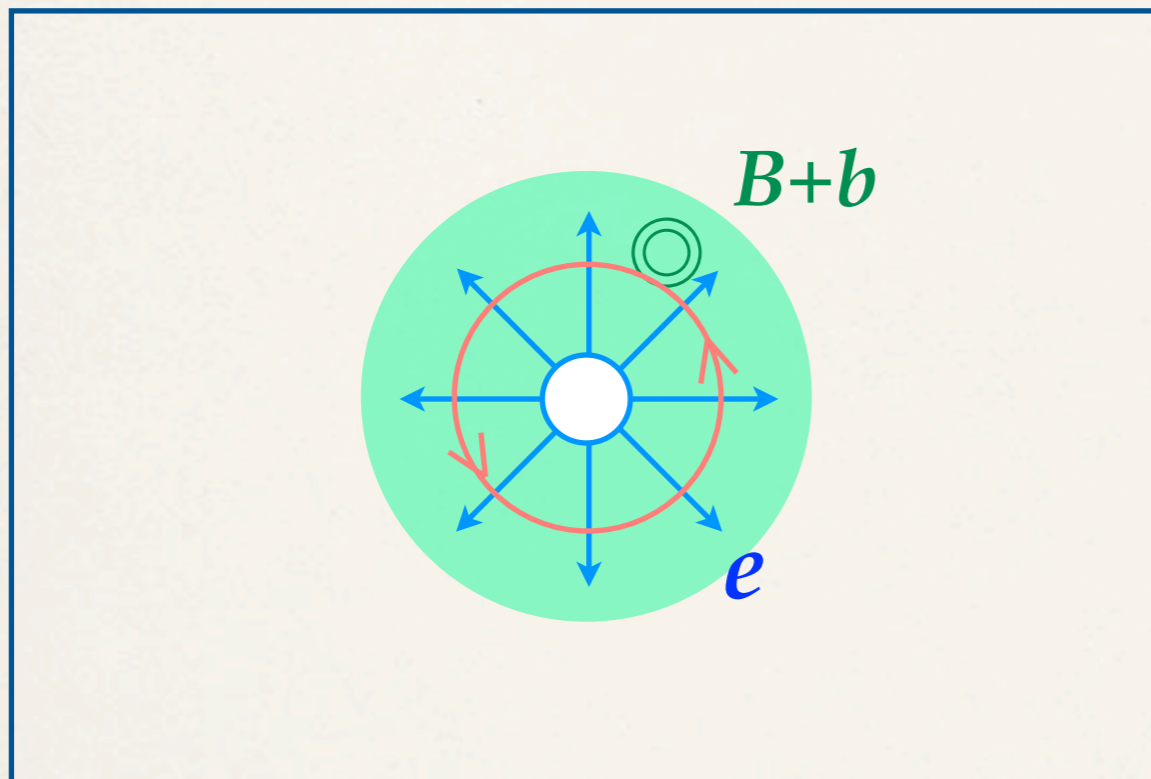
Y. Tserkovnyak and M. Mecklenburg,  
Phys. Rev. B, 77, 134407 (2008).

$$B \neq 0$$



$$\partial_t \mathbf{m} = \frac{g\tilde{S}\mu_B}{\hbar} \mathbf{B} \times \mathbf{m}$$

# Nernst effect by half-skyrmion

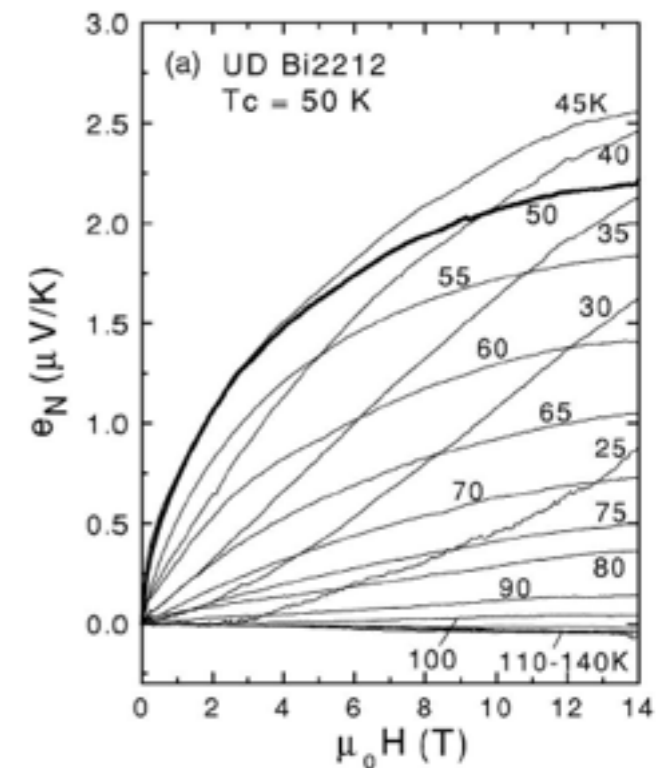
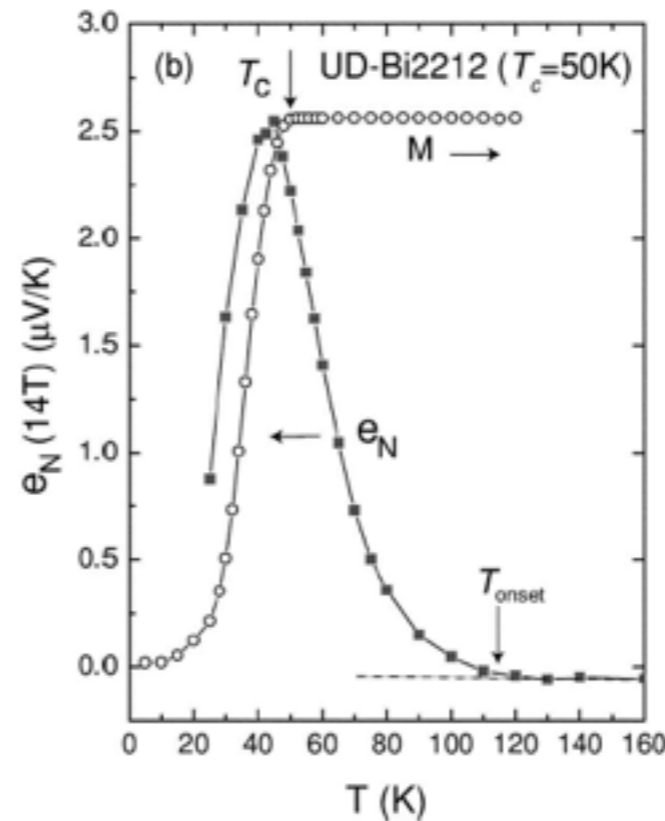
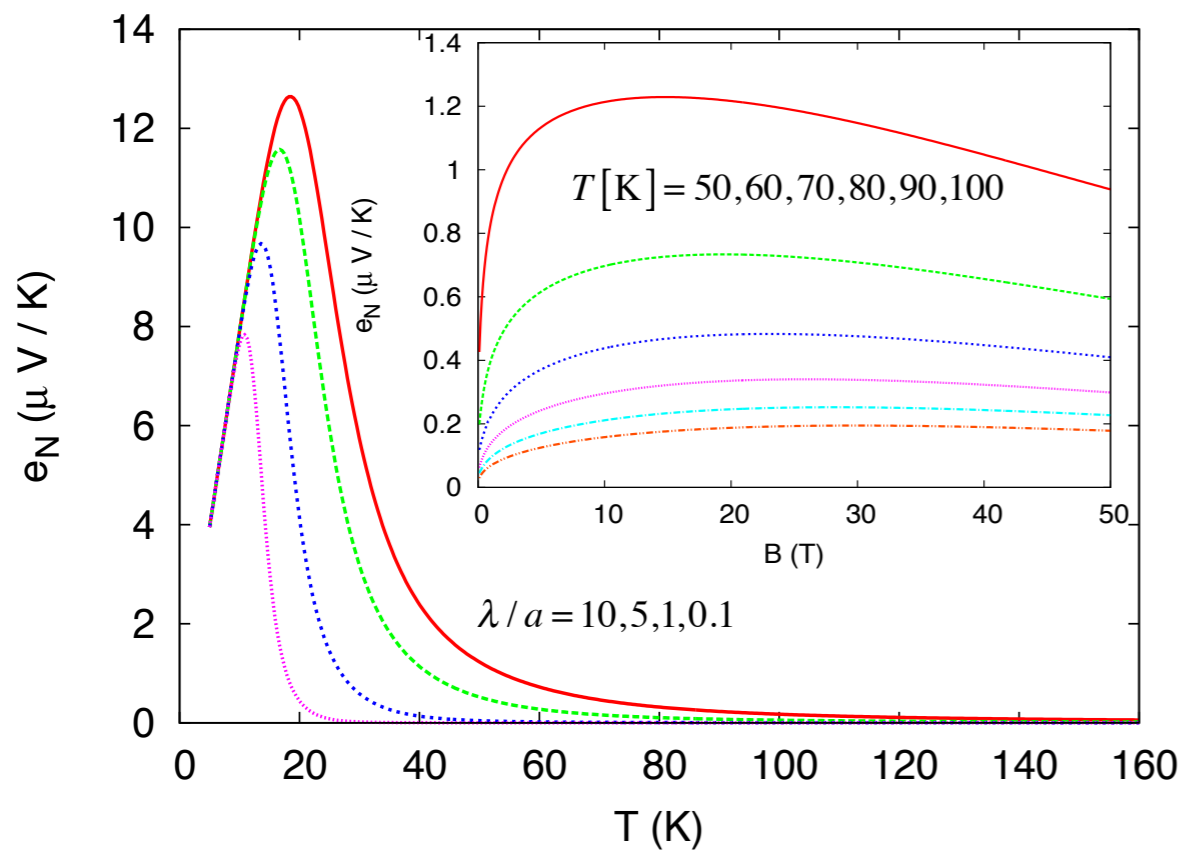


T.M. arXiv:1007.4047

# Nernst effect by half-skyrmion

$$\mathbf{j} = \frac{e}{\Omega} \sum_{\mathbf{k}} \left( -\frac{\partial f_{\mathbf{k}}^{(0)}}{\partial \varepsilon_{\mathbf{k}}} \right) \frac{1}{1 + (\omega_c \tau)^2} (\varepsilon_{\mathbf{k}} - \mu) \tau \mathbf{v}_{\mathbf{k}} \mathbf{v}_{\mathbf{k}} \cdot \left( -\frac{\nabla T}{T} \right)$$

$$m^* / m = 2, \tau = 2 \times 10^{-13} \text{ s}$$

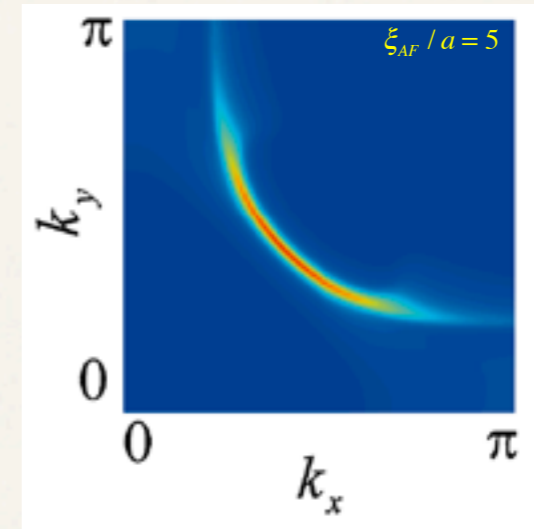


T.M. unpublished

$$T_{onset} = J_{eff}$$

# Pseudogap by half-skyrmion

- ❖ Dynamics of composite particle



T.M. J. Phys. Soc. Jpn. 78,

- ❖ Berry phase effect under magnetic field

T.M. arXiv:1007.4047

- ❖ Vortex creation without superconductivity correlations

—————> Nernst effect

- ❖ Onset temperature

$$T_{onset} = J_{eff}$$

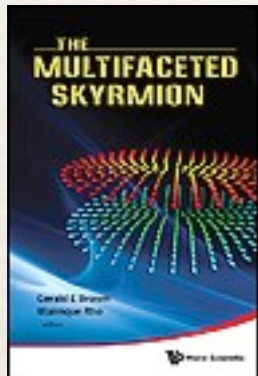
# Summary

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- ❖ Doped hole creates a half-skyrmion spin texture.

Excitation spectrum is consistent with ARPES in the parent comp.

- ❖ Lorentz force leads to d-wave superconductivity
- ❖ Dynamics of composite particle leads to pseudogap.



T.M., “Half-Skyrmion theory for high-temperature superconductivity,”  
in "The Multifaceted Skyrmion" (World Scientific, 2010) eds. G.E. Brown and M. Rho.

arXiv:0908.3385

