NR/HEP: roadmap for the future

NR/HEP: roadmap for the future[*]

Vitor Cardoso[1,2]**, Leonardo Gualtieri**[3]**, Carlos Herdeiro**[4]**,
Ulrich Sperhake (editors)**[1,5,6]**, Paul M Chesler**[7]**, Luis Lehner**[8,9]**,
Seong Chan Park**[10]**, Harvey S Reall**[11,30]**,
Carlos F Sopuerta (section coordinators)**[5]**, Daniela Alic**[12]**,
Oscar J C Dias**[13]**, Roberto Emparan**[14,15]**, Valeria Ferrari**[3]**,
Steven B Giddings**[16]**, Mahdi Godazgar**[11]**, Ruth Gregory**[17]**,
Veronika E Hubeny**[17]**, Akihiro Ishibashi**[18,19]**, Greg Landsberg**[20]**,
Carlos O Lousto**[21]**, David Mateos**[14,15]**, Vicki Moeller**[22]**,
Hirotada Okawa**[23]**, Paolo Pani**[1]**, M Andy Parker**[22]**, Frans Pretorius**[24]**,
Masaru Shibata**[23]**, Hajime Sotani**[25]**, Toby Wiseman**[26]**, Helvi Witek**[1]**,
Nicolas Yunes**[27,28] **and Miguel Zilhão**[29]

[1] Departamento de Física, Centro Multidisciplinar de Astrofísica—CENTRA, Instituto Superior
Técnico—IST, Universidade Técnica de Lisboa—UTL, Av. Rovisco Pais 1, 1049-001 Lisboa,
Portugal
[2] Department of Physics and Astronomy, The University of Mississippi, MS 38677-1848, USA
[3] Dipartimento di Fisica, Università di Roma 'Sapienza' & Sezione, INFN Roma1, P A Moro 5,
00185, Roma, Italy
[4] Departamento de Física da Universidade de Aveiro and I3N, Campus de Santiago, 3810-183
Aveiro, Portugal
[5] Institut de Ciències de l'Espai (CSIC-IEEC), Facultat de Ciències, Campus UAB, Torre C5
parells, Bellaterra, 08193 Barcelona, Spain
[6] California Institute of Technology, Pasadena, CA 91125, USA
[7] Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
[8] Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2 L 2Y5, Canada
[9] Department of Physics, University of Guelph, Guelph, Ontario N1G 2W1, Canada
[10] Department of Physics, Sungkyunkwan University, Suwon 440-746, Korea
[11] DAMTP, Centre for Mathematical Sciences, Wilberforce Road, Cambridge CB3 0WA, UK
[12] Albert-Einstein-Institut, Max-Planck-Institut für Gravitationsphysik, D-14476 Golm, Germany
[13] Institut de Physique Théorique, CEA Saclay, CNRS URA 2306, F-91191 Gif-sur-Yvette,
France
[14] Departament de Física Fonamental and Institut de Ciències del Cosmos, Universitat de
Barcelona, Martí i Franquès 1, E-08028 Barcelona, Spain
[15] Institució Catalana de Recerca i Estudis Avançats (ICREA) Passeig Lluís Companys 23,
E-08010 Barcelona, Spain
[16] Department of Physics, University of California, Santa Barbara, CA 93106, USA
[17] Department of Mathematical Sciences, Centre for Particle Theory, Durham University,
South Road, Durham DH1 3LE, UK
[18] Theory Center, Institute of Particle and Nuclear Studies, High Energy Accelerator Research
Organization (KEK), Tsukuba, 305-0801, Japan
[19] Department of Physics, Kinki University, Higashi-Osaka 577-8502, Japan
[20] Department of Physics, Brown University, Providence, RI 02912, USA
[21] Center for Computational Relativity and Gravitation and School of Mathematical Sciences,
Rochester Institute of Technology, 85 Lomb Memorial Drive, Rochester, NY 14623, USA
[22] Cavendish Laboratory, JJ Thomson Ave, Cambridge CB3 0HE, UK
[23] Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

[24] Department of Physics, Princeton University, Princeton, NJ 08544, USA

[25] Division of Theoretical Astronomy, National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan

[26] Theoretical Physics Group, Blackett Laboratory, Imperial College, London SW7 2AZ, UK

[27] MIT Kavli Institute for Astrophysics and Space Research, Cambridge, MA 02139, USA

[28] Department of Physics, Montana State University, Bozeman, MT 59717, USA

[29] Departamento de Física e Astronomia, Centro de Física do Porto—CFP, Faculdade de Ciências da Universidade do Porto—FCUP, Rua do Campo Alegre, 4169-007 Porto, Portugal

E-mail: vitor.cardoso@ist.utl.pt

Abstract

Physic in curved spacetime describes a multitude of phenomena, ranging from astrophysics to high-energy physics (HEP). The last few years have witnessed further progress on several fronts, including the accurate numerical evolution of the gravitational field equations, which now allows highly nonlinear phenomena to be tamed. Numerical relativity simulations, originally developed to understand strong-field astrophysical processes, could prove extremely useful to understand HEP processes such as trans-Planckian scattering and gauge–gravity dualities. We present a concise and comprehensive overview of the state-of-the-art and important open problems in the field(s), along with a roadmap for the next years.

PACS numbers: 04.25.D−, 11.25.−w, 11.25.Tq, 04.50.−h, 04.25.−g, 04.70.−s

(Some figures may appear in colour only in the online journal)

Contents

List of acronyms

AdS   Anti-de Sitter
BH   Black hole
BSSN   Baungarte–Shapiro–Shibata–Nakamura
CFT   Conformal field theory
EFT   Effective field theory
GL   Gregory Laflamme
GR   General relativity
GW   Gravitational wave
PN   Post–Newtonian
SMT   String theory/M-theory
TeV   Tera-electron volt

1. Introduction

Numerical relativity (NR), the gauge/gravity duality and trans-Planckian scattering, as well as high-energy physics (HEP) in general, have been tremendously active and successful research areas in recent years. Strong motivation for the combined study of these fields has arisen from direct experimental connections: gravitational wave detection, probing strong interactions at the Large Hadron Collider (LHC) and Relativistic Heavy Ion Collider (RHIC) and possibly black hole (BH) production at LHC. Inspired in part by the fairly recent advent of techniques to evolve BH spacetimes numerically and the consequent unprecedented opportunities to expand and test our understanding of fundamental physics and the universe, a group of leading experts in different fields (HEP, astrophysics, general relativity (GR), NR and phenomenology of gravitational effects on curved backgrounds) got together from 29 August to 3 September 2011 in Madeira island (Portugal) to review some of the most exciting recent results and to discuss important future directions. The meeting was both an excuse and an opportunity to place at the same table colleagues from formerly disjoint fields and to discuss the vast number of possibilities that exist at the interface.

This overview, written in a 'white-paper' style, is a summary of the many interesting discussions at the meeting. Online presentations can be found at the meeting's website http://blackholes.ist.utl.pt/nrhep/.

We have divided the meeting's discussions into six (not-really disjoint) parts, each culminating in a round-table with all participants discussing the state of the field and visions for the future. Each round-table was coordinated by one participant who has also been in charge of putting together a write-up for that particular section. The (somewhat artificial) division has resulted in the following topics (round-table organizers' names in parentheses): numerical methods and results (Lehner), BH solutions in generic settings (Reall), trans-Planckian scattering (Park), gauge/gravity duality (Chesler), alternative theories of gravity (Gualtieri) and approximation methods (Sopuerta). We feel that the rich content of each of these sections alone is sufficient to impart to the reader the exciting times that lie ahead.

Finally, we end this short introduction by thanking the European Research Council, Fundação Calouste Gulbenkian and FCT—Portugal for their generous support of this workshop. Special thanks go to Ana Sousa and Rita Sousa for their invaluable help, and to Sérgio Almeida and Luís Ferreira for providing technical and visual assistance.

2. Strong gravity, high energy physics and numerical relativity

(Coordinator: Luis Lehner)

Extensive experimental and observational programs are underway to test our understanding of gravity in 'extreme' regimes. These extreme regimes are naturally encountered in cosmology, violent astrophysical events and possibly at high-energy particle accelerators if nature is described by certain higher dimensional scenarios. Furthermore, gauge/gravity dualities provide an intriguing opportunity to understand various physical phenomena, naturally described by field theories, in terms of gravity in anti-de-Sitter (AdS) spacetime and vice versa.

For all these scenarios it is vital to understand gravity in the 'strong', i.e. fully nonlinear regime. Currently, the only path towards obtaining such an understanding is the use of numerical techniques which facilitate the generation of solutions to the Einstein equations, or their analogues in alternative theories of gravity, with no approximation other than those arising from a discretization of the equations; in particular, there is no fundamental restriction such as a linearization of the equations in perturbative studies.

Doing so requires the ability to obtain accurate solutions from the underlying gravitational theory which, in turn, often requires suitable numerical simulations. Fortunately, in the case of GR in asymptotically flat, four-dimensional spacetimes, this task is largely under control [1–3] and will aid in future astrophysical and cosmological observational prospects. Thus we here concentrate on summarizing the status and open issues of efforts towards understanding gravity, at the classical level, in energetic phenomena related to higher dimensional scenarios. In particular, we focus our attention on efforts to understand gravity within holographic studies and TeV-gravity scenarios with the aim to report both on the state-of-the-art and ongoing investigations of unresolved issues. We mainly restrict our discussion to studies in the framework of GR; cf section 6 for a discussion of alternative theories of gravity. As efforts on this front are just beginning, our discussion cannot possibly be exhaustive and it is likely that new issues will arise as progress is made. Indeed, drawing from the experience gained in the four-dimensional setting, it is safe to expect that the challenges and outcomes will be as, if not more, exciting as can currently be anticipated.

2.1. Framework

We concentrate on gravitational studies required in the context of two main encompassing themes: *TeV gravity* and *holography*. TeV gravity arises by attempts to explain the hierarchy problem, i.e. the relative weakness of gravity by about 40 orders of magnitude compared to the other fundamental interactions. As first noted in [4–7], this problem can be addressed by considering large volumes and/or warping within higher dimensional scenarios [8, 9]. One way to achieve this goal arises in the context of so-called braneworld models, where all fundamental interactions except gravity are confined to a four-dimensional brane while gravity permeates through a bulk higher dimensional spacetime. Unification with gravity can take place at energy scales $\simeq$ TeV, a regime which can be probed at the most powerful currently available accelerator (LHC). Experiments might thus allow for testing this possibility and open up a window to new physics [9]. Furthermore, TeV-scale gravity scenarios can be realized within string theory and thus are tied to a prospective quantum theory of gravity. We could therefore be on the verge of experimentally detecting clues from quantum gravity behaviour in the near future.

Holography, and in particular AdS/CFT dualities [10], refer to a remarkable relation between D-dimensional field theories and gravity in AdS in $(D+1)$ dimensions [10–12]. It is conjectured that this relation can be exploited to gain an insight into physics on either side of the relation via studying properties of the other. It provides ways to understand, or re-interpret, the behaviour of condensed matter, plasmas, etc in terms of AdS BHs interacting with particular fields. Indeed, for (particular limits of) field theories in $D = 4$, their dual corresponds to AdS $D = 5$ gravity in the classical limit. Furthermore, it is commonly believed that the dualities so far uncovered may be the first examples of a much broader class of gauge/gravity dualities.

Clearly, both themes, TeV gravity and holography, involve understanding gravity in dimensions higher than 4, and possibly additional higher curvature corrections depending on the specific regimes under study. Unfortunately, our understanding of gravity in four dimensions does not necessarily help in providing a good intuition of the expected behaviour in higher dimensional settings. For example, the non-existence of stable circular orbits for point particles around BHs (the 'centrifugal' potential barrier becomes negligible beyond $D = 4$), non-uniqueness of BH solutions (e.g. see [13–16]) and generic violation of cosmic censorship [17] illustrate the richer phenomenology of gravity in $D > 4$ dimensions. Developing an intuitive understanding for $D > 5$ is likely to require coordinated efforts involving numerical as well as (semi-)analytic techniques such as perturbation theory or point-particle calculations; cf section 7. In the following, we will highlight the role of NR for achieving these goals, drawing, whenever possible, parallels from the corresponding effort in $D = 4$.

2.2. Challenges ahead

In the past decade, the field of NR has made great strides in modelling $D = 4$ asymptotically flat spacetimes and is now able to solve strongly gravitating/highly dynamical scenarios. A considerable portion of this was driven by the goal of guiding detection and analysis strategies for gravitational wave astronomy (see [18] and references cited therein), although progress has been achieved on many fronts: gravitational waves, cosmology, fundamental studies of gravity, astrophysics, etc For further details, we refer the reader to some reviews [19–26] and textbooks [27–29] on the subject.

This success has been achieved by the combination of several ingredients as follows.

 (i) A good understanding on how to frame the problem in a manner that is both physically complete and mathematically well posed.

(ii) The availability of well-understood approximations to describe the state of the solution for stages where the dynamics is relatively mild. This helps not only in constructing physical initial stages but also in finding an (approximate) description of systems where nonlinear effects have little relevance (through e.g. post–Newtonian or effective field theory approaches, see also section 7).

(iii) The knowledge, or expectation, of the expected late-time behaviour of the system. Understanding the fact that ultimately a stable Kerr BH would most likely describe the asymptotic solution, motivated coordinate and boundary conditions to aid the numerical implementation. Furthermore, it allows one to devise an effective perturbative description of the solution (through BH perturbation theory), which in turn alleviates the computational cost to obtain a solution and improves our ability to understand the expected generic behaviour.

(iv) A well-understood and robust set of numerical techniques to discretize Einstein equations.

(v) Adequate computational resources to deploy the implementation, and obtain results in a reasonable time for further tests as well as fine-tuning the implementation and obtain relevant solutions.

The points above are generic requirements for any implementation, and as we argue in the following discussions, the status of these points in $D > 4$ is less than what would be desired— even with respect to point (i) above. We next discuss several relevant questions related to the particular applications we have in mind. Some of these are common to both TeV gravity and holography, while others are tied to just one of these main themes.

TeV-scale gravity. This putative description of nature does not have a unique theoretical predictive formulation; rather it is a generic framework where different theories can be formulated such that they naturally address the hierarchy problem and possibly connect with TeV physics. While this facilitates the broad theoretical discussion towards achieving a fundamental description of nature, at a practical level it makes it difficult to choose which option to concentrate on. Furthermore, in many cases the full problem cannot be completely defined at a physical level, let alone a mathematically rigorous one. Broadly speaking, one has, in a sense, too many (!) options: GR, modifications of GR or alternative gravitational theories and possibly even the introduction of quantum corrections for the treatment of some basic phenomena. Among the set of possible theories, GR is unambiguously defined, can be shown to define well-posed problems and provides a unique framework to work on. As far as alternatives to GR are concerned, it is unclear which theory to pursue. Some are incomplete, e.g. have higher curvature corrections which have not been explicitly presented; others are either known to be ill posed—but are actively pursued nonetheless—or not yet fully understood regarding the extent to which they might be mathematically sound (e.g. [30–32]); see section 6 for a discussion on this subject. Naturally, well posedness is a necessary condition for a successful numerical implementation. While imposing this requirement rules out a number of alternative theories of gravity, several ones remain in principle valid. These theories involve a high degree of complexity, and their associated computational cost in practical applications is likely to be high. An alternative or complementary way could be envisioned taking a page out of the 'parametrized post-Newtonian' approach (PPN) [33, 34] developed within $D = 4$ gravitational theories, see section 7. An analogous framework for higher dimensional gravitational studies would be extremely beneficial. Our current focus is not to investigate this option, but rather to discuss the role that numerical simulations can play in this problem.

For concreteness, we shall in the following concentrate on GR and the main systems of interests which could be realized in accelerators (e.g. [35–39]). In particular, for large enough BHs, one can expect that GR does provide the generic high-energy behaviour as higher curvature corrections become sub-dominant. Of utmost interest is the understanding of $D > 4$ collisions, with a non-zero impact parameter, in order to improve the modelling of microscopic BH production in Monte Carlo event generators used in the analysis of data taken at the LHC. Note that these processes are essentially local, and can therefore be studied with $\Lambda = 0$ (asymptotically flat scenarios). There remain delicate questions, however, which need to be addressed in order to define concrete target problems; e.g. 'which is the dimensionality of the process?', 'which other non-vacuum interactions need to be considered?'; 'how to treat interactions restricted to the 4D brane as well as possibly incorporate higher curvature corrections?' From a physical point of view, one is interested in understanding high-speed collisions or scattering, as well as the behaviour (stability, decay, etc) of BHs possibly resulting from highly energetic collisions. We note that understanding the stability of possible BHs cannot only bring important clues about the physical behaviour of the system, but also help in achieving stability in the numerical implementation.

Holography. Here the situation is more concrete. The theoretical framework is most commonly defined with $AdS_5 \times S^5$ (though other scenarios including asymptotically flat spacetimes are under study [40]). The presence of the S^5 symmetry helps in reducing the practical dimensionality of the problem. It is crucial to understand the limiting cases of the correspondence where the gravitational side is captured by classical GR in AdS (possibly coupled to gauge fields) and identify what states on the CFT side they correspond to. Most questions of interest on the CFT side involve understanding BH solutions in AdS. Rigorous results about the stability of generic BHs are lacking with the exception of Schwarzschild AdS BHs interacting with a scalar field in spherical symmetry [41]. The presence of the AdS time-like boundary allows for fields to propagate away from the central region, bounce off the boundary and return to interact. As a result, *asymptotically flat intuition* does not translate to the $\Lambda < 0$ case. Indeed, as recently shown in [42] and further argued in [43], a complex 'turbulent-like' phenomenology arises which renders pure AdS nonlinearly unstable to BH formation regardless of how weak the initial perturbation is. This is in stark contrast to the known stability of Minkowski spacetime in asymptotically flat scenarios [44] (see also [45]). This difference between asymptotically flat and AdS spacetimes also indicates that richer phenomenology might be found in the latter type of spacetimes and carry with it interesting ties to a diverse CFT behaviour. Indeed, since the proposal of dualities, a plethora of work has been presented indicating connections across many areas of physics: fluids, plasmas, condensed matter theory, etc (see e.g. [12, 46–48, 11, 49–51] for some recent discussions).

With such a large number of exciting physical arenas involved, it would be useful to understand where numerical studies can make the highest impact. In order to address this question, it is important to comprehend which 'real world' applications (e.g. quark–gluon plasmas (QGPs) [52, 53], superconductors [54, 55]) or generic physical behaviour (e.g. turbulence [42]) can be accurately captured through a CFT model so as to study essential physical features of the system one is trying to model and identify those that can be studied within a reduced dimensionality (see section 2.3).

A particularly delicate issue for a numerical implementation concerns the AdS boundary. Indeed, as opposed to the asymptotically flat case where numerical implementations typically deal with boundaries using a combination of techniques to ensure they do not play a role in the dynamics, here the situation is markedly different. Not only is the boundary in causal contact with the interior domain, but also the field behaviour in its vicinity is of special interest. Indeed,

one goal required from a numerical implementation is to allow for extracting the asymptotic behaviour and connecting with the boundary theory. As we will discuss later, the mathematical understanding of how to pose correct boundary conditions is lacking. Further details can also be found in section 5.

2.3. Understanding strongly gravitating, dynamical systems in D > 4

The problems of interest concern energetic events involving BH formation, BH interactions and/or highly perturbed BHs. For specific limits, e.g. ultra-high-speed collisions, particular approximations have been proved quite useful in exploring the phenomenology of the system [56]. Here fields of particles can be described by the Aichelburg–Sexl [57] solution[31] and show BHs form for a range of impact parameters. For more general scenarios, perturbative studies might be unable to capture the sought after behaviour correctly, especially during highly nonlinear stages. Solutions are thus needed within the full theory which requires NR.

Obviously, the study of general scenarios in higher dimensions is computationally more demanding. At a rough level, the computational cost scales as the Dth power of the resolution for general situations in D dimensions. Putting the requirements into perspective, $D = 4$ scenarios without symmetries are currently routinely studied, and typical vacuum (binary BH) simulations demand a few weeks of continued run on several dozen processors[32]. Extrapolating from this observation, one can argue that general scenarios could be studied in $D = 5$ with current and near future resources. However, this will come at a significant cost, especially if progress on this front might require, as has been the case in $D = 4$, some experimentation to achieve a robust code. Thus, higher dimensional cases with no symmetries will in practice be out of the question for a long time. It is safe, therefore, to expect that NR will have its highest impact in studying scenarios where certain symmetries can be adopted, which reduces the dimensionality of the computational domain to more tractable two and three spatial dimensions. Particularly relevant cases are those in which $SO(D - 2)$ and/or $(D - 2)$ planar symmetries can be assumed such that the dynamics of interest takes place in a dimensionally reduced setting [13].

To date, a small number of NR efforts have been carried out beyond spherically symmetric scenarios to study specific questions in higher dimensional spacetimes. Within large extra dimension scenarios, these have concentrated on examining BH instabilities [58–60] and high-speed collisions [61–65]. These implementations have mostly followed the two successful approaches within the Cauchy formulation of Einstein equations used in $D = 4$: the generalized harmonic and BSSN formulations together with direct extensions of $D = 4$ gauge conditions. These efforts have further taken advantage of generic computational infrastructure developed to handle parallelization and adaptive regridding so as to ensure an efficient usage of computational resources (e.g. the publicly available Cactus/Carpet, PAMR, HAD [66–68]). Within holographic studies, published works of BH formation have relied on the characteristic formulation of Einstein equations [69, 70] and ongoing work is also exploiting the generalized-harmonic approach [71].

Interestingly, within the Cauchy approach, directly exploiting symmetries (say in p of the dimensions) assumed at the onset, and defining a reduced problem has given some difficulties, possibly related to the use of curvilinear coordinates for a subspace of the manifold and the

---

[31] This analytic solution describes a spherically symmetric solution with large boost. In the limit of a Schwarzschild black hole of zero rest mass boosted to the speed of light, it is given by a shockwave in an otherwise Minkowskian spacetime and thus amenable to straightforward superposition.

[32] Details depending on the accuracy requirements; non-vacuum scenarios may require significantly more resources depending on the complications introduced by additional physical ingredients involved.

ensuing singular nature of the radial coordinate at its origin; cf appendix B in [72]. There
is no fundamental impediment to deal with this issue, however, and successful runs have
been presented in the literature for five-dimensional spacetimes [61–64]. On the other hand,
implementing symmetries in an effective way following the 'cartoon method' (where the
problem is treated formally as D dimensional, but the equations are integrated in a $(D - p)$
sector and derivatives of this sector are accounted for via the symmetries) has provided robust
implementations in a rather direct, though perhaps not elegant way [58–60]. This observation
stresses that more work at the foundational level is still required to translate the success
obtained in $D = 4$ to higher dimensional settings. Beyond these issues, we next discuss some
particular points of relevance to our two encompassing themes.

In the context of TeV-scale gravity scenarios, the intrinsic computational problem in
vacuum is quite similar to that in $D = 4$ and existing computational infrastructure can be
readily exploited. In essence, one deals with an *initial value problem* and computational
boundaries need just to be dealt with in a stable manner and placed sufficiently far from the
region of interest so they do not causally influence it. After dealing with the dimensionally
reduced problem with either of the above options, one can fine tune the implementation
(adjusting gauge conditions, refining algorithms, etc) and begin studying problems of interest
provided the correct physical ingredients are incorporated in the model.

In the context of AdS-CFT *dualities*, the above picture is more complex as the problem
is intrinsically an *initial boundary value problem*. The AdS time-like boundary is causally
connected with the bulk region and must be carefully treated as it plays a central role in
the solution sought. Understanding how to deal with time-like boundaries in $D = 4$ in the
absence of a cosmological constant required major theoretical efforts which culminated in
a series of mathematically sound options guaranteeing the well-posedness of the underlying
problem [73, 74]. An analogue of such work is absent in AdS (even in $D = 4$) which is
rendered more delicate due to the diverging behaviour of the spacetime metric at the boundary.
We note, however, the derivation of boundary conditions for the linearized field equations in
AdS [75] and studies of the fall-off behaviour of the metric in particular coordinate systems
[76, 77]. Extension of this work towards establishing well-posedness of the full equations will
be of vital importance for a robust numerical treatment of general problems. In the meantime,
however, interesting advances have been presented exploring the characteristic formulation
of GR—where the spacetime is foliated by incoming null hypersurfaces emanating from
the AdS boundary—[69, 70]. Numerical schemes based on the characteristic approach are
particularly robust, a property that has been observed in $D = 4$ as well, but more restricted in
applicability as caustics will render the coordinate system employed singular. At present there
is no general purpose infrastructure to ensure efficient parallelization and dynamic regridding
for characteristic approaches, though a proof-of-principle work has demonstrated there is
no fundamental obstacle for developing one [78]. Within a Cauchy approach—where the
spacetime is foliated by space-like hypersurfaces—strategies have been specifically tailored
for special cases, see section 5.

Regardless of the nature of the foliation adopted, a related question concerns the boundary
topology. It is well known that within the global 5D-AdS spacetime (with boundary topology
$R \times S^3$) a Poincaré patch can be defined through a suitable transformation. The boundary of
this patch is R^4. From the perspective of aiming for a numerical implementation, the options
of adopting a Poincaré patch or global coordinates bring up non-trivial issues related to both
the possibility of achieving a robust implementation and its application to generic problems.
Namely, if a global AdS picture is adopted, can all physics be extracted in standard fashion
through a Poincaré patch after a suitable coordinate transformation? As a matter of principle,
a calculation performed in global AdS can always capture the physics of a calculation in the

Poincaré patch, although in practice it may be a rather inefficient and involved manner of doing so. The converse, however, might not always be possible. In Euclidean space, the global and Poincaré patches are related to each other through a rather straightforward coordinate transformation in a unique way. However, in Lorentzian spacetimes the situation is more subtle. Excitations or objects evolving in global AdS may cross the Cauchy horizon of the Poincaré patch, entering or leaving it. (Their energies, measured in Poincaré-patch time, are redshifted to zero as they reach the Cauchy horizon.) As long as the relevant physics occurs entirely inside the Poincaré patch, this is not problematic, but for phenomena where properties on global AdS are required, Poincaré-patch evolutions appear to be ill suited. Although in principle analyticity should allow for recovering Lorentzian-time global-AdS correlation functions from Poincaré-patch ones, one must have a detailed understanding of the prescriptions involved and the approach is likely unsuitable for numerical results. Moreover, relating the Poincaré picture to that of the global picture may require infinite resolution, and thus be difficult in numerical approaches. It also remains to be seen whether particular conditions are required in the boundary treatment of global AdS in order to make such a mapping possible.

We also note that some applications require 'operator insertions' in the CFT which correspond to particularly 'deformed' AdS boundaries. For these spacetimes, all issues with respect to fall-off behaviour, boundary regularization, well posedness of the resulting problem, need to be investigated again. While some of these questions (as for example that of using a Poincaré patch or not) may have simple answers from the physics perspective, they might introduce delicate numerical issues.

Fields, what fields?.   An issue related to both efforts—TeV scale gravity or holography— concerns the choice of fields to be included for studying relevant scenarios. For instance, in the context of AdS/CFT, the inclusion of a gauge field is natural and well motivated, as it allows for studying the effect of a non-zero chemical potential, i.e. finite density, in the boundary theory. Scalar fields with suitable potentials are also of interest and have been studied to model in a phenomenological manner the effects of confinement ('soft-wall' models).

- For TeV-scale gravity, considering charged collisions, or those involving other fields, will allow for an exploration of other observational signatures to be searched for in accelerator experiments. In $D = 4$, simulations involving scalar and vector interactions, including charged BHs, are well under control (e.g. [79–82]). In higher dimensions, however, these fields would be confined to branes and the treatment becomes considerably more complicated.

- In the context of holography, studying the interaction of BHs with other fields is important for exploring dualities within a broad physics scope. This includes the 'condensation' of fields outside BHs, which has proven important for studies of superconductors and generic (holographic) condensed matter systems. Of particular importance, for instance, is to understand the global behaviour driven by super-radiance of scalar or even tensor fields [83–85] and its final outcome, likely a hairy BH or non-trivial BH solutions.

While exciting work is proceeding to address problems of interest, there is undoubtedly still plenty of room for resolving fundamental questions that will help in the construction of robust and general methods to simulate relevant systems and to extract useful physics from them.

2.4. Targets of opportunity

As mentioned, gravity in higher dimensions brings about several new delicate issues, some of which can be addressed via numerical simulations while others must be taken into account analytically in order to achieve a robust implementation. Indeed, essentially every single point discussed in section 2.2 would benefit from further work. In addition, there are important quantum issues not covered here, such as the behaviour of Hawking evaporation and non-perturbative quantum gravity effects. On the one hand, these issues make the understanding of these systems more difficult; on the other hand, they represent targets of opportunity. From a practical standpoint, we loosely group these targets as follows.

- *Mathematical.* Understand the correct setting for guaranteeing a well-posed initial boundary value problem in AdS. Understand BH topologies that might arise in higher dimensional contexts [86].
- *Mathematical/Physical.* Very basic issues about the stability of higher dimensional BHs remain unresolved, and at present we can only expect to address them through numerical studies. Results thus obtained can guide us towards establishing general results from a mathematical point of view. As discussed earlier, this is particularly important for the scenarios in which BHs can be formed at colliders. Two outstanding open (and related) questions in the context of vacuum BHs (with $\Lambda = 0$) are: (i) Black rings with large angular momenta are unstable, but is there a window of stability for (5D) black rings at moderate values of the spin? (ii) For given values of M and J, is there more than one stable BH? If the answer is yes, it would have consequences, for instance, for BH production at colliders. If not, this would allow us to recover a notion of uniqueness: the only neutral, asymptotically flat BHs with a connected horizon, would be Myers–Perry BHs with angular momentum below a certain bound.

   More generally, numerical analysis, in a less intensive manner (soft numerics) is necessary for obtaining information about the space of BH solutions in higher dimensions that cannot be obtained through other methods. A good example of this is the progress in the past decade in understanding BHs and black strings in Kaluza–Klein theory. Very similar problems arise in any other higher dimensional theory of gravity, either in asymptotically flat vacuum or with a cosmological constant or additional fields.
- *Physical.* List scenarios relevant for TeV gravity and holography in both vacuum and non-vacuum spacetimes. To this end, describe desired initial conditions required to study the future development of the solution. Furthermore, investigate which particularly interesting cases are amenable to a reduced dimensionality description. Develop methods that could shed further light on the linearized stability of many BH solutions. For instance, recognize mappings between different solutions and known cases (as in the case of mapping close horizon geometries to the GL instability e.g. [87–89]), extend and comprehend possible limitations of thermodynamical arguments (e.g. [90, 91, 88]), construct suitable approximation schemes able to capture the system's behaviour in appropriate limits (e.g. extensions of PN and EFT [92], blackfolds [93], etc).
- *Numerical.* Several open problems should be solved numerically, both to obtain static/stationary solutions or particular initial data as well as studying the dynamical behaviour of relevant cases. In the following sections, we discuss some particularly interesting issues on this front.

Initial data: stationary/static solutions and perturbations thereof. In many applications of NR to TeV-gravity or the AdS/CFT correspondence, BHs play a vital role and need to be

accounted for in the construction of initial data. Whereas the popular puncture initial data [94] can be generalized to asymptotically de Sitter spacetimes with positive cosmological constant [95–97] using McVittie's solution for Schwarzschild–de Sitter [98], a similar procedure is not known for AdS spacetimes. A number of other single-BH solutions in AdS potentially suitable for initial data construction are known, but there remains the question of their stability properties which are not yet fully understood; cf section 3. In some cases, important headway has been made by entropic arguments and recognized mappings to known instabilities, most notably the Gregory–Laflamme (GL) instability [99, 100]. With the exception of the GL instability, however, present options (short of performing a full nonlinear study) are linearized perturbation analysis or considering scenarios describing BH perturbations violating Penrose's inequality [91]. To date, linearized stability of many of the known BH solutions remains an open question. This issue is further complicated because many relevant solutions are only known numerically. Indeed, many situations of interest relate to static or stationary BH solutions [101] whose derivation requires significant work at analytical and numerical levels.

Dynamical behaviour.   Understanding of the full nonlinear behaviour of BH perturbations at both classical and quantum levels is paramount for establishing a connection with possible observations in the framework of TeV-scale gravity models as well as elucidating holographic connections with equilibration in the dual picture. Furthermore, studies of dynamical scenarios in higher dimensional gravity will help in building up intuition and guide further work. For instance, numerical simulations have already shown (see also [102])

- That negative energy 'bubbles of nothing' do not give rise to naked singularities, and initially expanding ones continue doing so, and furthermore that gauge fields can significantly affect the dynamics and give rise to a static bubble solution dual to black strings [103, 104].
- That large static BHs exist in AdS_5 with the Schwarzschild AdS as the boundary metric [105, 106], thus disproving a conjecture that no such solutions are admitted [107, 108].
- That a higher dimensional class of unstable AdSs—black strings—[99, 100] display rich dynamics leading to a self-similar behaviour that ultimately gives rise to naked singularities [59, 17] under rather generic conditions. Thus cosmic censorship does not hold beyond $D = 4$.
- The unstable behaviour of rapidly spinning BHs [87], which can lose enough angular momentum through gravitational waves so as to cross to the stable branch [58, 60].
- That the amount of energy radiated in $D = 5$ head-on BH collisions agrees well with the value obtained from (extrapolations of) linearized, point-particle calculations [64, 109].
- That within the holographic picture, the collision of gravitational shock waves in AdS gives rise to a dynamical behaviour consistent with that expected from hydrodynamics in the boundary theory. Furthermore, the behaviour in the gravity sector was exploited to compute the time required for thermalization in the system [70].

These are just a few examples of interesting physics that can be extracted from the dynamical behaviour of relevant systems. *A priori* there is a vast number of interesting problems, and this number will likely grow as new, and probably unexpected, behaviour is uncovered. Obviously, prospects for exciting physics lie ahead. Indeed, within large extra dimensions, a thorough understanding of how a newly formed—but unstable—BH decays to its eventual fate; what that fate is and how it depends on dimensionality can have profound observational implications and/or signal difficulties in prospective models. Furthermore, the influence of the strongly gravitating/highly dynamical regime explored by such a process

on additional fields could induce tractable signals that can provide important additional observational channels.

Within holographic studies for static situations there is a clear prescription on the relationship between the CFT thermal state and properties of the BH horizon. However, for dynamical scenarios—related to both formation and stabilization of BHs—is there a sensible, unique way to define such a map? Furthermore, what is the preferred way to connect CFT and horizon properties as a function of time? The answer to this question is inevitably tied to the choice of a preferred slicing, which need not be the most convenient one for the numerical implementation. Additionally, complex dynamics might be driven by the super-radiance 'instability'. To date, however, a study of such scenarios even in $D = 4$ is absent. Related work has presented possible end-state solutions describing hairy BHs [110–114], though their stability has not yet been established nor is known whether they are indeed attractor solutions. The super-radiance instability need not be the only or main effect driving the dynamics. For instance, recent work indicating 'turbulent-like' behaviour in AdS gravity [42, 43] highlights surprises that might still be lurking in dynamical scenarios beyond what might be understood at low perturbation orders. For instance, the turbulent-like behaviour takes place on the gravity side in all dimensions, whereas the Navier–Stokes equations (appearing to lowest order on the field theory side) display a behaviour markedly different in $D = 3$ from that in higher dimensions. This indicates that different phenomena are awaiting our understanding in the context of strongly coupled quantum field theory (QFT).

Drawing a path from analogies to 4D.   As mentioned above, many cases of interest involve understanding the interaction of BHs in a fully nonlinear framework. It is important to recall once more that simulations provide information about one single, isolated BH system per run. Having a sufficient number of such simulations available, certain phenomenological models could be constructed to capture more generic properties of systems across the parameter space. Undoubtedly, generating an exhaustive set of numerical simulations will be a time-consuming task. An alternative (or complementary) approach has been pursued successfully in $D = 4$, namely to employ different approximations to understand the system in separate (early and late) regimes and exploit numerical simulations to bridge the gap in between. Techniques used for this purpose are post-Newtonian (and related) approximations when the compact objects move at slow velocities and perturbation theory around a suitable BH solution for the post-merger (or BH formation) stage (e.g. [115–121] for $D = 4$ studies and [122] for the first step in this direction for $D > 4$.). A judicious phenomenological match between the two phases, motivated and further tuned via gradual generation of numerical solutions, can thus provide for an efficient way to encode the system's behaviour. This approach should be given consideration in higher dimensional scenarios. However, to this end the following issues must be kept in mind.

- PN and EFT approximations must be developed to the appropriate order. In particular to incorporate radiation-reaction effects. Currently, this is only available to first order (and not in the AdS case) [92]. It is important to note that the order at which this can be done without internal effects playing a role depends on dimensionality. Nevertheless, where possible, the knowledge of the trajectories can be directly exploited in obtaining reasonable approximations to the spacetime metric by suitable superpositions (see e.g. [123–126]).
- Perturbations of BHs only make sense if the stability of the BH is understood. As mentioned, in many cases this is still a question to be addressed.

13

For a detailed discussion of the use of approximation methods for ($D \geqslant 4$)-dimensional BH spacetimes see section 7.

3. Higher dimensional black holes

(Coordinator: Harvey Reall)

3.1. Motivation

This section will discuss classical properties of stationary BH solutions of the vacuum Einstein equation in higher dimensions.

The review article [13] listed several motivations for the study of BH solutions in more than four spacetime dimensions.

 (i) Statistical calculations of BH entropy using string theory. This was first achieved for certain five-dimensional BHs and later extended to 4D black holes. Each entropy calculation is a check on the theory, irrespective of the dimension. Hence the study of higher dimensional black holes is a worthwhile contribution to developing a theory of quantum gravity.

 (ii) The gauge/gravity correspondence relates the properties of black holes in D dimensions to strongly coupled, finite temperature, QFT in $D - 1$ dimensions. This provides a way of calculating certain field theory quantities which cannot be determined by any other method.

(iii) Certain 'large extra dimension' scenarios predict that microscopic higher dimensional black holes might be formed at the LHC. However, LHC results discussed at the workshop give no evidence in favour of 'large extra dimension' scenarios. It was also emphasized that if black holes were formed at the LHC when run at higher energy, they would not be in a semi-classical regime, so quantum gravity would be required to study them. Therefore, it seems that this motivation for the study of classical higher dimensional gravity is lessened.

(iv) Higher dimensional BH spacetimes might have interesting mathematical properties. For example, analytically continued versions of BH solutions have been used to obtain explicit metrics on compact Sasaki–Einstein spaces [127].

 (v) Just as it is valuable to consider QFT with field content different from that of the Standard Model (or any conceivable extension), it might be worthwhile considering higher dimensions when studying black holes. For example, there might exist explicit higher dimensional solutions that provide a clean example of some effect in GR. A nice example of this is the frame-dragging effect exhibited by the 'black Saturn' solution [128]. Perhaps there are examples in which an exact calculation in higher dimensions can be used to check a calculation that has to be done perturbatively in 4D.

(vi) Some things are simpler in higher dimensions. For example, in 4D the asymptotic symmetry group of null infinity in an asymptotically flat spacetime is the infinite-dimensional Bondi–Metzner–Sachs group. However, in higher dimensions it is simply the Poincaré group [129, 130]. This makes it possible to define angular momentum at null infinity in higher dimensions [131]. In 4D this appears to be difficult.

The focus of this paper will be on stationary BH solutions and their properties. Time-dependent processes are of great interest for the applications, but will only be touched on here (see sections 2 and 5 for further details on this topic).

3.2. State of the art

Explicit solutions. We restrict this discussion to the vacuum Einstein equations with vanishing cosmological constant. There are two families of explicit asymptotically flat BH solutions with a connected horizon. There are the Myers–Perry solutions [132], known for any spacetime dimension D, and the three-parameter black ring solution [133, 134], for $D = 5$. The MP solutions have horizons with topologically spherical cross section. The black ring solution has topology $S^1 \times S^2$.

There are also explicit solutions with disconnected horizons. The 'black Saturn' solution [128] describes a black ring with an MP BH sitting at the centre of the ring. There are solutions involving a pair of black rings, e.g., the 'black di-ring' [135].

The MP solutions have been generalized to include a cosmological constant for $D = 5$ [136] and $D \geqslant 6$ [137]. These solutions describe rotating, topologically spherical BHs in an asymptotically (anti-)de Sitter spacetime.

Classification. It has been shown that a static, asymptotically flat, BH must be described by the Schwarzschild solution in any number of dimensions [138].

Hawking's topology theorem has been generalized to higher dimensions [139], with the conclusion that a cross section of the event horizon of a stationary BH must be a positive Yamabe space, i.e. it must admit a metric with positive Ricci scalar. In 4D, this implies spherical topology. In 5D, it restricts the topology to a connected sum in which each component is either $S^1 \times S^2$ or a quotient of S^3. For $D > 5$ there are many more possibilities.

Hawking's rigidity theorem asserts that a stationary black hole must admit a rotational symmetry. This has been extended to higher dimensions [140]. But it guarantees only one rotational symmetry, whereas all known explicit solutions have multiple rotational symmetries. Combining the rigidity and topology theorems further restricts the possible topologies in 5D [141].

For $D = 5$, one can classify stationary black holes with two commuting rotational symmetries according to their 'rod structure': there exists at most one black hole with given mass, angular momenta and rod structure [142]. If one also assumes S^3 topology, then the black hole must be a Myers–Perry solution [143]. Solutions with lens space topology are consistent with this classification. It is not known whether such solutions exist. (They do not appear in the blackfold approach discussed below.)

Related to the problem of classification is the characterization of the phase space of black hole solutions: what are the different families of black hole solutions that exist, and how they branch-off or merge at different points in the phase space. Even if we do not have a complete classification of all possible BHs, one would like to know how the known phases (explicit or approximate) relate to each other, at least qualitatively.

In this direction, one expects that the main features of the phase space of neutral, asymptotically flat, higher dimensional BHs are controlled by solutions in three different regions as follows.

  (i) Large angular momenta.
 (ii) Bifurcations in phase space.
(iii) Topology-changing transition regions.

The regime (i) is captured by the blackfold effective theory [144, 93] discussed below. Regions (ii) are controlled by zero-mode perturbations of BHs that give rise to bifurcations into new families of solutions. The initial conjectures about these points [87, 145] have been

confirmed and extended in [89]. For the regions (iii) in $D \geqslant 6$, [146] has provided local models for the critical geometries that effect the topology change. The critical behaviour in the five-dimensional case is qualitatively different.

Stability.   Singly spinning MP BHs in $D \geqslant 6$ have no upper bound on the angular momentum J for a given mass. Reference [87] conjectured that such BHs should be classically unstable for large enough J. Strong evidence for this was found in [89, 147], where it was shown that a regular stationary perturbation appears at a critical value of J. This is believed to be a 'threshold' mode indicating the presence of exponentially growing perturbations for larger J.

Reference [88] considered the most symmetrical case of MP BHs with odd D and all angular momenta equal. Such solutions have an upper bound on J at which the BH becomes extreme. It was shown that, close to extremality, there exist linearized perturbations that grow exponentially with time. Reference [148] showed how the threshold mode of this instability connects to the threshold mode in the singly spinning case by considering unequal angular momenta.

The studies just described considered only instabilities that preserve the rotational symmetry of the BH that arises from the rigidity theorem. However, in the singly spinning case, it is known that a non-rotationally symmetric instability appears at a lower value of J than the rotationally symmetric instability. Reference [58] performed a full nonlinear evolution of the Einstein equation starting from initial data describing a singly spinning MP BH with a non-rotationally symmetric perturbation. For large enough J, it was found that the BH became very asymmetrical, resulting in significant gravitational wave emission, and then settled down to a (presumably stable) MP BH with J always smaller than some critical value. It was found that this kind of instability is present for $D = 5$ (despite the upper bound on J) as well as for $D \geqslant 6$.

With a negative cosmological constant, MP-AdS BHs suffer a super-radiant instability when $\Omega \ell > 1$, where Ω is the angular velocity of the horizon and ℓ the AdS radius [84]. This instability breaks rotational symmetry. A rotationally symmetric instability can also appear at even larger angular velocity [149].

References [150, 151] presented heuristic arguments indicating a classical instability of 'fat' black rings. This was confirmed by [91] using an argument based on Penrose inequalities. This instability preserves rotational symmetry. Heuristic arguments also suggest that sufficiently thin black rings will suffer a GL instability, which would break rotational symmetry [133].

Approximate techniques.   Heuristic arguments suggest that black ring solutions should also exist for $D > 5$. These are made more precise by the 'blackfold' approach [145, 144, 93] which constructs solutions perturbatively in a derivative expansion, valid when the geometry of the solution has a large hierarchy of scales, e.g., a black ring with topology $S^1 \times S^{D-3}$ where the radius of the S^1 is large compared to that of the S^{D-3}. This approach indicates the existence of BH solutions with a variety of different topologies, for example a product $S^{p_1} \times \cdots \times S^{p_k} \times s^{D-p-2}$, where p_i are odd with $\sum p_i = p$ and S^{p_i} denotes a sphere of 'large' radius and s^{D-p-2} a sphere of 'small' radius. It also indicates the existence of 'helical' black ring solutions with just the single rotational symmetry predicted by the rigidity theorem, i.e. less symmetry than any known higher dimensional black hole [14].

Another approximate technique which also points to the existence of new solutions is to consider perturbations of a known solution. If one finds a perturbation which is stationary, regular, and does not correspond simply to a variation of parameters of the known solution,

then it may correspond to the bifurcation of a new family of stationary BH solutions. The studies of MP perturbations just discussed do indeed provide evidence for such bifurcations. For singly spinning MP BHs with $D \geqslant 6$, a stationary perturbation appears at the critical value of the angular momentum beyond which the BH is unstable [89]. This is taken to indicate the existence of a new family of 'pinched' MP BHs. Further bifurcations appear at larger angular momentum. All of these perturbations preserve the symmetries of the original MP solution.

The same technique applied to cohomogeneity-1 MP BHs also suggests the bifurcation of a new family of solutions at the critical value of angular momentum beyond which the solution becomes unstable [88]. However, in this case, the perturbation generically breaks all of the rotational symmetries of the MP solution except for the symmetry guaranteed by the rigidity theorem. In contrast with helical black rings, the new family of solutions would be topologically spherical. Furthermore, counting the number of free parameters in the perturbation suggests that the new solutions should have many parameters (e.g. 70 in $D = 9$ whereas the 9D MP solution has only five parameters).

In summary, approximate techniques indicate the existence of many stationary vacuum BH solutions in $D > 5$ dimensions. These include solutions with only a single rotational symmetry, solutions with topology different from any known solution and solutions with many more parameters than the known solutions.

In asymptotically AdS spacetimes, it appears that things can be even more complicated. Reference [84] suggested the possible existence of stationary, non-static, vacuum BHs without any rotational symmetry. Evidence in favour of this was found in [43].

Numerical solutions. The study of asymptotically flat BHs gives one a stepping stone to understand more complicated theories of gravity relevant for phenomenology or holography. In the asymptotically flat setting, the static solutions are simply Schwarzschild and are known to be unique [152]. However in more exotic settings, such as compact extra dimensions or braneworlds, even the static BHs often have a complicated structure. For example in the simplest toy model, Kaluza–Klein theory, where we are interested in vacuum geometries that asymptote to $\mathbb{R}^{1,3} \times S^1$, there are three distinct classes of the static solution. As reviewed in [16], these are the homogeneous black string, the inhomogeneous black string and the localized black hole. Only the first is known analytically. The second and third class may be constructed perturbatively in various limits, but generally are only known from numerical computations (the most recent being [153]). The inhomogeneous black strings were originally predicted by Gregory and Laflamme [154] and are generated from the homogeneous strings by the marginally stable static Gregory–Laflamme perturbation [155]. Moving along this solution branch the horizon appears to degenerate to pinch off and change the topology to that of a spherical horizon [156]. Kol has argued [157] that on general grounds a singular cone geometry provides a local model for the part of the horizon where the pinch off occurs, and that one may resolve the cone to pass through to the spherical topology horizons of the localized static solution branch. This localized branch is thought to interpolate between this topology changing point and very low mass spherical BHs that are near their horizon approximate five-dimensional Schwarzschild and were first studied by Myers [158] and may be constructed in perturbation theory [159, 160]. This picture is indeed supported by the numerical evidence [161, 153]. This topology change of the horizon through a conical transition is also thought to be relevant in understanding the space of asymptotically flat stationary solutions [145, 146]. Further work to improve these numerical solutions and confirm the cone topology change picture is an interesting future direction.

3.3. Future directions

Explicit solutions. The approximate methods discussed above point to the existence of many stationary vacuum BH solutions in higher dimensions in addition to the known Myers–Perry and 5D black ring solutions. It is of obvious interest to determine such solutions explicitly. This is a long-term goal. To do this, new techniques for solving the Einstein equation in higher dimensions must be developed.

So far, the technique which has led to the most interesting results is the Belinskii–Zakharov inverse scattering method for solving the Einstein equation when there are $D - 3$ commuting rotational symmetries, as well as time-translation symmetry. This method led to the discovery of doubly spinning black rings and black Saturn and its generalizations. But these symmetry assumptions are consistent with asymptotic flatness only for $D = 4, 5$. Furthermore, this method does not work with a cosmological constant. However, if 5D 'black lenses' exist, then this method could be used to find them.

An alternative approach is based on the algebraic classification of the Weyl tensor. The Kerr solution was discovered in search for solutions with a Weyl tensor that, in the Petrov classification, is algebraically special. There is a large literature [162] on exploiting the algebraically special property as a tool for solving the Einstein equation in 4D. The most spectacular example of this is the determination of all vacuum type D solutions [163]. This technique is not restricted to the case of vanishing cosmological constant.

In higher dimensions, there are various notions of 'algebraically special' that have been explored. These have been reviewed in [164]. The most widely used definition is that of Coley, Milsson, Pravda and Pravdova (CMPP) [165]. So far, the only attempt to use the CMPP 'algebraically special' property as a tool for solving the Einstein equation is [166] which determined all *axisymmetric* algebraically special solutions of the vacuum Einstein equation (with cosmological constant) where 'axisymmetric' is defined as the existence of an $SO(D-4)$ symmetry with S^{D-3} orbits. There is a significant scope for extending this approach of combining the algebraically special property with some symmetry assumptions. An obvious case of interest is $U(1)^{D-2}$ symmetry.

A more systematic exploitation of the algebraically special property probably will require further development of the general theory of algebraically special solutions. The key result lacking is a higher dimensional generalization of the Goldberg–Sachs theorem. Partial progress has been made (reviewed in [164]), but this is an area in which plenty remains to be done.

An alternative definition of 'algebraically special' involves spinors. This works only for $D = 5$ (and $D = 4$). This definition is independent of the CMPP classification (although the MP solutions are algebraically special according to both definitions) and therefore could lead to rather different results if exploited as a tool to solve the Einstein equation. The first attempt at exploiting the spinorial definition of algebraically special to solve the 5D Einstein equation was made in [167]. Only the most special algebraic classes were considered. It would be very interesting to consider more general classes.

Another property that might be exploited to solve the Einstein equation is the existence of a Killing or Killing–Yano tensor. It is known that the wave equation is separable in an MP spacetime and this is because of the existence of a conformal Killing–Yano tensor. Furthermore, it has been shown that a certain generalization of the MP solution is the most general spacetime admitting a 'principal' conformal Killing–Yano tensor [168, 169]. This result shows that one can exploit the existence of such a tensor to solve the Einstein equation. In this case, it leads to a known solution, but maybe the assumptions here could be weakened. Is it possible to make progress if the 'principal' condition is dropped? Does this lead to new solutions?

Classification. A long-term goal is to solve the classification problem: classify all stationary, asymptotically flat, vacuum BH solutions in $D > 4$ dimensions. This seems far beyond the current techniques.

So far, classification results for higher dimensional BHs are restricted to $D = 5$ with two rotational symmetries. Since the general case of one rotational symmetry appears intractable at present, one could consider other special cases with enhanced symmetry. For example, can one classify cohomogeneity-1 vacuum BH solutions? Are there any such solutions other than Schwarzschild and equal angular momentum, odd D Myers–Perry? What about cohomogeneity-2 solutions in general D? These include singly spinning MP BHs. Can such solutions be classified?

Another simplifying assumption is extremality. Any extreme BH admits a near-horizon geometry with more symmetry than the full BH solution [170]. Can we classify possible near-horizon geometries? (See [171, 172] for results in this direction.) Can this be used to classify extreme BHs? Performing this would involve understanding the global question of what restrictions asymptotic flatness imposes on a near-horizon geometry.

Although BHs with gauge fields are not within the scope of this paper, it should be noted that *supersymmetry* is more restrictive than extremality. So perhaps one could attempt to classify supersymmetric BHs in a higher dimensional supergravity theory. So far, this has been investigated only for the simplest 5D theory [173].

Stability. A long-term goal is to determine the classical stability of the known BH solutions, i.e. Myers–Perry and black rings. Where instabilities exist, the endpoint should be determined.

As discussed above, there are now several results concerning instabilities of MP BHs. So far, studies of linearized perturbations have considered only perturbations preserving the rotational symmetry that arises from the rigidity theorem. However, for a singly spinning BH, numerical evolution finds that the instability which appears first (i.e. at smallest J) breaks this symmetry. Since numerical evolution is hard, it would be interesting to study this non-rotationally symmetric instability using linearized theory. It would also be nice to know which type of instability is dominant at very large J.

A target for future research would be to map out the stable regions of the MP parameter space. This could be done via a combination of linearized analysis and full numerical evolution. Although unsuccessful so far [174], perhaps one could derive a master equation describing MP perturbations, a higher dimensional analogue of the Teukolsky equation.

The stability of black rings against non-rotationally symmetric perturbations is an important open question that future work should address. Are there any stable vacuum black rings?

At present, the endpoint of the super-radiant instability of MP-AdS BHs is unknown, although it is conceivable that no regular endpoint exists [43]. It would be interesting to determine the time evolution of this instability numerically.

Numerical methods. Given that we expect higher dimensions to admit many new solutions, many without much symmetry, it is unlikely that we will succeed in constructing them all analytically. Numerically solving the Einstein equation is likely to become an important tool in finding new stationary BH solutions. New methods have been developed recently for solving the Einstein equation to obtain stationary solutions, see [101] for a review.

Of course, numerical work has its limitations: one usually must have a good idea of the properties of the solution one wishes to find: it will be hard to go 'fishing' for new solutions; as discussed above, there may exist BH solutions with a large number of parameters;

it would be hard to explore such a parameter space numerically. Nevertheless, numerical methods can be used to confirm the existence of solutions for which the evidence is otherwise indirect or perturbative in nature. For example, the arguments for the existence of BHs with a single rotational symmetry are based on perturbation theory. If we cannot find such solutions explicitly, then numerical analysis of the Einstein equation provides the only way of confirming the existence of such solutions at the full nonlinear level. (Actually, this is not quite true: one might be able to prove existence of solutions without finding them explicitly, but such an approach may not provide much information about properties of the solutions.) Finding numerical solutions describing stationary $D > 4$ BHs with a single rotational symmetry should be a goal for future work.

An obvious approach to finding new BH solutions is to try to form them in a time-dependent simulation of gravitational collapse. However, this approach probably will only find classically stable solutions and many higher dimensional solutions are expected to be unstable. (This does not make them uninteresting: the timescale for the instability might be long e.g. compared to the time-scale of Hawking evaporation.) On the other hand, there is no reason that new stable solutions should not exist, particularly in the less well-understood case of asymptotically AdS spacetimes.

The recent discovery of the absence of a threshold for BH formation [42] emphasizes how little is understood about time-dependent processes in AdS. Numerical simulations will be invaluable in developing our intuition for what is possible. It will be interesting to see how the results of [42] are modified if one relaxes the assumption of spherical symmetry.

The case of asymptotically AdS spacetimes is particularly interesting because of the range of possible boundary conditions that one can consider. For example, there has been recent progress in constructing solutions in which the conformal boundary is the Schwarzschild spacetime [105], which can be used to study the behaviour of strongly coupled field theory in a BH background. This idea has been discussed more generally in [175], which argues for the existence of different types of 'funnel' and 'droplet' solutions when the boundary metric describes a BH. Future numerical work should investigate such solutions. It will also be interesting to construct numerical solutions which describe an AdS BH localized on an internal space e.g. on the S^5 of $AdS_5 \times S^5$ (see [176] for a discussion of the instability of Schwarzschild–$AdS_5 \times S^5$ and its implications for the dual gauge theory).

Approximate methods.   The equations of the blackfold approach described above, which allow us to construct approximate BH solutions, have been solved only in the simplest cases with a high degree of symmetry that allows us to solve the equations algebraically. This, however, is far from being a systematic and exhaustive study and has uncovered only a very small part of the solutions that presumably exist in $D \geqslant 6$. For instance, it would be important to study solutions with a lower degree of symmetry, since they will play a role in connecting different BH phases, as well as providing new classes of BHs with less symmetry than the Cartan subgroup of $SO(D - 1)$. Novel horizon topologies may also be discovered.

Instead of a case-by-case study of specific ansatze for the solutions, a more systematic approach to the blackfold equations would be highly desirable. This approach should give very valuable results towards the classification of BHs, or at least towards a useful characterization of them.

The blackfold methods can also be usefully applied to the study of the stability properties of BHs whenever their horizons possess two separate length scales [177]. The connection it makes to hydrodynamics has made it possible to simplify the study of the GL instability of black branes and possibly will yield a better understanding of the nonlinear evolution of this instability towards its endpoint. Having an analytical tool to study this problem seems
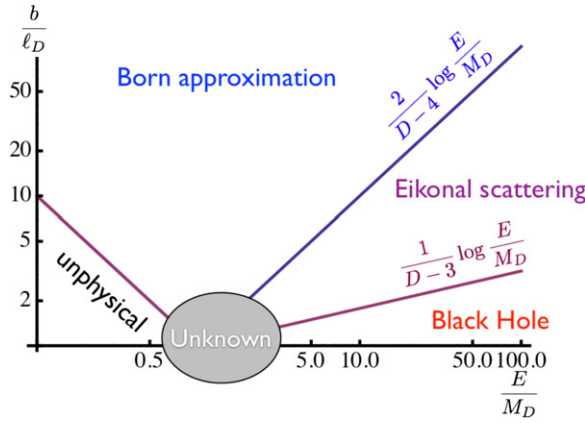
Figure 1. A proposed 'phase diagram' of different regimes for gravitational scattering (from [181]).

very valuable, since so far it has only been tackled via massive numerical calculations with supercomputers.

4. Trans-Planckian scattering

(Coordinator: Seongchan Park)

The ultimate aim of physics is to discover the fundamental laws of nature. According to the uncertainty principle, $\Delta x \geqslant \hbar/\Delta p$, higher energies are needed to probe smaller distances. Ultimately, however, when the energies involved in physical processes exceed the Planck energy, we enter the profoundly mysterious trans-Planckian regime. In trans-Planckian processes, gravity dominates the scattering at increasing distances [178, 179], and it has been argued that it prevents probing distances shorter than the Planck length [180, 35],

$$\ell_D \equiv \left(\frac{G_D \hbar}{c^3} \right)^{\frac{1}{D-2}} . \tag{1}$$

Specifically, in the regime with centre of mass energy $\sqrt{s} \gg M_D$, where the Planck mass in D-dimensions is $M_D = \left(\frac{\hbar^{D-3} c^{5-D}}{G_D} \right)^{\frac{1}{D-2}}$, the Schwarzschild–Tangherlini BH radius (R_s) exceeds the de Broglie wavelength,

$$R_s = \left(\frac{16\pi}{(D-2)\Omega_{D-2}} \right)^{\frac{1}{D-3}} \left(\frac{G_D \sqrt{s}}{c^4} \right)^{\frac{1}{D-3}} \gg \lambda = \frac{4\pi\hbar}{\sqrt{s}} \tag{2}$$

and the scattering range grows as a power of energy. Here, general features of the dynamics are expected to be well approximated by semiclassical scattering and/or horizon formation.

One can characterize the scattering in terms of the centre-of-mass energy $E = \sqrt{s}$ and impact parameter $b = J/\sqrt{s}$, where J is the angular momentum. A proposed 'phase diagram' describing the relevant physics in different regions of the (E, b) plane is summarized in figure 1 (see [181]). The left lower part is unphysical as $\Delta x < \hbar/\Delta p$. The grey part near the Planck scale $\sqrt{s} \sim M_D$ is largely unknown and nonlinear quantum gravity effects dominate, so that we need the full theory of quantum gravity to describe physics in this domain. On the other hand, the large impact parameter regime appears well approximated by the Born or eikonal approximation, as long as the scattering angle is 'small', $\theta \sim (R_s/b)^{D-3} \ll 1$. Lowering the impact parameter further to the value of Schwarzschild–Tangherlini radius, gravity becomes

strong and highly nonlinear, and BH formation is expected, even though a full quantum description of this process remains unknown. Specifically, in the classical theory, high-energy particles are well approximated by colliding Aichelburg–Sexl solutions [57], and in such a collision, at impact parameters $b \lesssim R_s(\sqrt{s})$, one can show that a trapped surface forms [182, 56, 183]. (One can supply additional arguments that quantum effects do not modify these statements [184].) Simulations in NR have confirmed this prediction of BH formation in high-energy collisions [79, 185–187].

Once a BH is formed, it decays through Hawking radiation. This process would enable the LHC to probe BH signatures in TeV-gravity models with extra dimensions [35, 36]. (For reviews, see for instance [9, 38, 37, 39, 188].)

In this section, we review the current status of trans-Planckian physics focusing on the LHC search for BHs and other trans-Planckian signatures.

The trans-Planckian physics could be relevant to the LHC experiment if M_D is as low as a TeV scale. Indeed, several models of TeV scale gravity have been proposed based on extra dimension(s) with a large volume [4, 5] or a large warp factor [6, 7] to address 'the hierarchy problem'. The hierarchy problem is often casted in terms of the huge ratio between the fundamental scale of gravity and the electroweak scale, which essentially determines the relative weakness of the gravitational interaction compared to the other gauge interactions among elementary particles. It is problematic in particle physics since the mass of a scalar boson, such as the Higgs boson, is quadratically sensitive to the fundamental scale, so that a fine tuning of the order of the ratio $M_W^2/M_G^2 \sim \mathcal{O}(10^{-34})$ is required to realize a weak scale scalar mass. Recently some hints of the Higgs mass around 125 GeV have been found by the ATLAS [189] as well as CMS detectors [190] of the LHC.

The large volume of flat extra dimensions helps to understand the hierarchy because the Planck scale in four dimension, M_4, and the scale in $D > 4$ dimension, M_D, are related as $M_4^2/M_D^2 = M_D^n V_n$. If the volume of extra dimensions is big enough, say, as large as $M_D^n V_n \sim M_W^2/M_G^2$, one can find that the fundamental gravity scale in higher dimensions may not be very different from the electroweak scale so that the hierarchy problem could be nullified. The warped extra dimension also helps, since an energy scale runs with respect to the position along the warped extra dimension as $\Lambda(y) \propto e^{-ky}$, so that the big hierarchy between the scale of UV ($\Lambda_{UV} \sim M_G$) and the scale of IR ($\Lambda_{IR} \sim M_w$) is generated by the warp factor $e^{-k\ell} \sim M_w/M_G$ with a moderate distance between the UV boundary and the IR boundary of the warped space ($\ell \sim 34/k$). For more discussions on Trans–Planckian physics and its implications to the LHC, one may refer a recent review [188].

4.1. Is trans-Planckian physics testable in the near future?: the experimental status of TeV-gravity models

The critical question for the experimental observation of trans-Planckian physics is the size of the fundamental Planck scale. In extra-dimensional models with large and/or highly warped extra dimensions, the scale of gravity can be as low as $M_D = \mathcal{O}(1)$ TeV, eliminating a large hierarchy between this scale and the electroweak scale. The LHC has begun to exclude the low end of the parameter space for TeV-gravity models by searching for signatures in various channels. What is the current status of the exclusion? What are the future expectations?

ATLAS searches. At the time of writing, the most recent results from the ATLAS experiment at the LHC come from an integrated luminosity of around 1fb^{-1}, acquired during the first part of 2011. These results will be updated with around 5 pb^{-1} available in early 2012. ATLAS

has focused its searches on channels with leptons, because of the expected democratic nature of the gravitational coupling. This means that one would expect particle production to be in proportion to the number of states available (with appropriate grey-body factors), and not related to the gauge quantum numbers of the states. However, it is possible to construct models, such as split branes [191], where this is not the case.

In [192], a search for microscopic BHs and string ball states was performed, using final states with three or more high transverse momentum objects, at least one of which was required to be an electron or muon. No deviations were observed from the standard model expectations, which were estimated using a combination of data-driven and Monte-Carlo-based techniques. The dominant background sources come from vector boson production, either directly or from top decays. A smaller background arises from QCD events which contain a fake lepton. The events were studied as a function of the scalar sum of the transverse momenta of the final state particles (Σp_T). In the highest bin, with $\Sigma p_T > 1500$ GeV, there were eight (2) electron (muon) events observed, with standard model expectations of $10.2 \pm 1.4 \pm 2.6$ ($2.8 \pm 0.5 \pm 1.1$), respectively.

In [193], events are selected containing two muons of the same charge. This channel is expected to have low Standard Model backgrounds while retaining good signal acceptance. Isolated muons (i.e. muons with very little activity around them in the detector) can be produced directly from the BH or from the decay of heavy particles such as W or Z bosons. Muons from the semi-leptonic decays of heavy-flavour hadrons produced from the BH can have several other particles nearby and can therefore be non-isolated. In order to maintain optimal acceptance for a possible signal, only one of the muons is required to be isolated in this analysis, thereby typically increasing the acceptance in the signal region by 50%. The decay of the BH to multiple high-p_T objects is used to divide the observed events into background-rich and potentially signal-rich regions. This is done by using the number of high-p_T charged particle tracks as the criterion to assign events to each region. BH events typically have a high number of tracks per event (N_{trk}), while Standard Model processes have sharply falling track multiplicity distributions. In the background-rich region, where only small signal contributions are expected, data and Monte Carlo simulations are used to estimate the number of events after selections. This background estimate is validated by comparing to data. The expected number of events from Standard Model processes in the signal-rich region is then compared with the measured number, and a constraint on the contribution from BH decays is inferred. Good agreement is observed between the measured distributions and the background expectations. No excess over the Standard Model predictions is observed in the data.

Although it is clear that there is no anomalous signal in the data, setting limits on BH production is problematic. The LHC experiments are searching at the limit of its current energy range, and hence, by definition, cannot explore well beyond the current limit on the bulk Planck mass. This means that predictions for the rate of BH production using semi-classical approximations are not valid: a full theory of quantum gravity is required.

The experiment tackles this issue in two ways. Firstly, using the number of events observed in data, and the background expectations, upper limits are set on $\sigma \times \mathrm{BR} \times A$, where σ is the cross section, BR is the branching ratio to the signal channel and A is the acceptance of non-Standard Model contributions in this final state in the signal region. These limits are reasonably model independent. However, to make use of these limits, theorists need to know the relevant value of A for their model scenario. Representative values of A are therefore published by the experiments. The acceptance tends to be high for the very high mass states from the BH decay, and so this approach is useful; however, such limits must be used with caution when applied to extreme scenarios, such as two-body decays.

The second approach is to set limits on benchmark scenarios. This is done in a plane of M_D versus M_{TH}, where M_D is the bulk Planck mass. For the purpose of limit setting, it is assumed that BH production only occurs above a certain scale, denoted M_{TH}, which should be well above M_D for the semiclassical calculations used in the event generators to be valid. This approach produces 90% confidence limits up to $M_D \approx 1.5$ TeV depending on the model used. However, there is a strong dependence on the modelling of the final remnant decay, which is a pure quantum gravity effect, and can dominate the final state in this mass range.

CMS Searches.   The CMS experiment has pioneered accelerator searches for BHs in 2011 and published a paper [194] based on data collected in the first 7 TeV running period of the LHC (March–November 2010), corresponding to an integrated luminosity of 36 pb^{-1}. Despite the relatively small statistics, the sensitivity of the search was high enough to largely disfavour the possibility of BH production at a 7 TeV centre-of-mass energy. The analysis conducted by the CMS collaboration was done in the inclusive final state, thus maximizing the sensitivity to BH production and decay. Semi-classical BHs are expected to evaporate in a large ($\sim$10) number of energetic particles, emitted nearly isotropically, with the major fraction of the emitted particles being quarks and gluons, which are detected as jets in the CMS detector. Quantum effects and grey-body factors may change the relative fraction of emitted quarks and gluons, but generally it is expected that these particles appear most often even in decays of quantum BHs, due to a large number of (colour) degrees of freedom that quarks and gluons possess, compared to the other Standard Model particles (e.g., photons, leptons, neutrinos and gauge bosons).

The discriminating variable between the signal and the dominating QCD multijet background used in the search was the scalar sum of transverse energies[33] of all particles in the event, for which transverse energy exceeds 50 GeV. This variable, S_T, was further corrected for any missing transverse energy in the event by adding the missing transverse energy to the S_T variable, if the former exceeds 50 GeV. The choice of S_T as the discriminating variable is extremely robust and rather insensitive to the particle content in the process of BH evaporation, as well to the details of the final, sub-Planckian evaporation phase. The addition of the missing transverse energy to the definition of S_T further ensures high sensitivity of the search for the case of stable non-interacting remnant with the mass of order of the fundamental Planck scale, which may be produced in the terminal stage of the evaporation process.

The main challenge of the search is to describe the inclusive multijet background in a robust way, as the BH signal corresponds to a broad enhancement in S_T distribution at high end, rather than a narrow peak. Since the BH signal is expected to correspond to high multiplicity of final-state particles, one has to reliably describe the background for large jet multiplicities, which is quite challenging theoretically, as higher order calculations to fully describe multijet production simply do not exist. Thus, one cannot rely on the Monte Carlo simulations to reproduce the S_T spectrum correctly.

The CMS collaboration developed and used a novel method of predicting the QCD background directly from collision data to overcome this problem. It has been noted empirically, first via Monte-Carlo-based studies, and then from the analysis of data at low jet multiplicities that the shape of the S_T distribution for the dominant QCD multijet background does not depend on the multiplicity of the final state, above the turn-on threshold. This observation, motivated by the way parton shower is developed via nearly collinear emission, which conserves S_T, allows one to predict S_T spectrum of a multijet final state using

---

[33] Transverse energy of a particle, E_T, is defined as the energy of the particle E times the sine of the polar angle of the particle direction with respect to the counterclockwise proton beam.
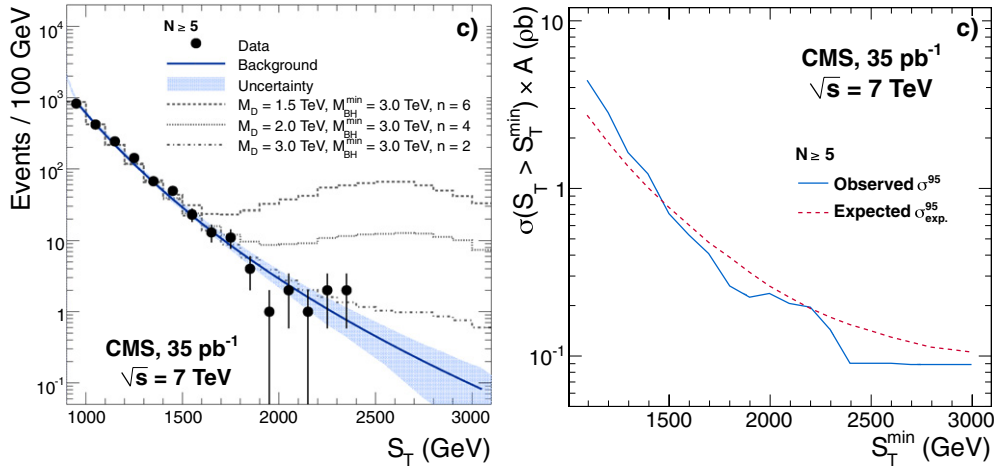
Figure 2. Left: predicted QCD multijet background with its uncertainties (the shaded band), data and several reference BH signal benchmarks, as a function of S_T in the final state with the multiplicity of 5 or more particles. Right: model-independent upper limits at 95% confidence level on a cross section of a new physics signal decaying in the final state with 5 or more particles, as a function of the minimum S_T requirement. From [194].

low-multiplicity QCD events, e.g. dijets or three-jet events. This provides a powerful method of predicting the main background for BH production by taking the S_T shape from the diet events, for which signal contamination is expected to be negligible, and normalizing it to the observed spectrum at high multiplicities at the low end of the S_T distribution, where signal contamination is also negligible even for large multiplicities of the final-state objects. The results are shown in figure 2 (left).

The CMS data at high final-state multiplicities are well fit by the background shape obtained from the diet events, no excess characteristic of a BH production is seen in the data. This lack of an apparent signal can be interpreted in a model-independent way by providing limit on a cross section for any new physics signal for S_T values above a certain cutoff, for any given inclusive final state multiplicity. An example of such a limit is shown in figure 2 (right), for the final-state multiplicity $N \geqslant 5$. For signals corresponding to large values of S_T (above 2 TeV or so), the cross-section limit reaches ~ 100 fb. These limits can be compared with production cross section for the BHs in a variety of models and used to set limits on the minimum BH masses (M_{BH}) that can be produced in these models. In the original publication [194], the CMS collaboration used just a few simplest semi-classical benchmark models that are expected to break down at the values of BH masses near the exclusion, given that they are of the same order as the probed values of the fundamental Planck scale. Despite these obvious limitations, the benchmark models probed in this search show that the results are rather insensitive to the details of the BH decay and that BH masses as high as 4.5 TeV are probed in this analysis. The negative results of this search largely exclude the possibility of observing BH production at the 7 TeV LHC. An updated analysis using full 2011 statistics is expected to be published soon.

In figure 3 the indicative lower limit on the minimum black hole mass is set from the CMS data using the 35 pb^{-1} in 7 TeV collisions [194]. The black holes are assumed to be semi-classical and Schwarzschild-like. It should be noticed that the mass scale in the limit is too close to the fundamental scale, so that the result should be considered as illustration only.
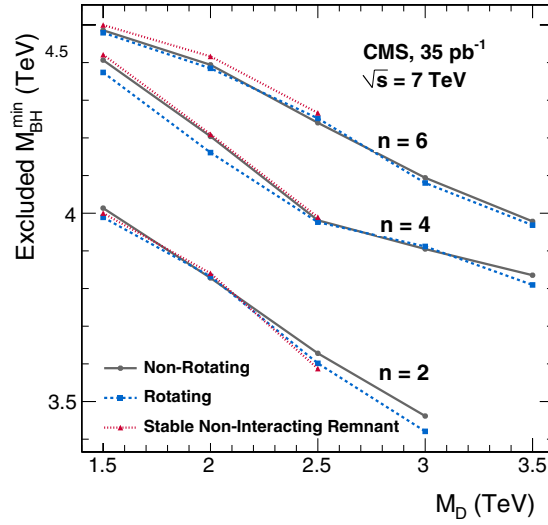
Figure 3. Limits on the minimum BH mass as a function of the fundamental Planck scale for a few semi-classical benchmark models with and without BH rotation and non-evaporating remnant. Note that the semi-classical approximation used in setting these limits is not expected to hold for BH masses so close to the Planck scale, so this plot should be considered as illustration only. From [194].

Theoretical issues. In order to observe strong-gravitational scattering, approximated as classical BH formation, scattering energies must significantly exceed the Planck energy. There are two reasons for this: (1) an object must have a mass significantly exceeding M_D in order to be well approximated as a semiclassical BH; the expansion parameter justifying the semiclassical approximation is M_D/E and (2) not all of the energy of colliding partons goes into BH formation; some escapes in radiation [195, 35, 56].

It is thus important to determine the threshold for onset of BH formation, for a given Planck mass. There are different aspects of this problem. Classically, an important question is to determine the inelasticity, the amount of energy lost to radiation as compared to the mass of the resulting BH. The trapped surface constructions [182, 56, 183] places lower bounds on the mass of the BH; these have been modestly improved in [196]. One can also estimate the radiated energy perturbatively [195, 197–201, 109, 202–204]. Here, NR calculations could ultimately provide more definitive answers, by computing the amount of radiated energy, in higher dimensional gravity, and as a function of impact parameter.

Another aspect of the problem is to determine where quantum effects lead to large deviations from semiclassical BH behaviour. This requires greater knowledge of quantum gravity. One possible criterion for onset of BH behaviour is the onset of thermodynamic behaviour, characterized by the entropy [35].

4.2. How can we improve our understanding of trans-Planckian scattering?

There are a number of important questions on the subject of trans-Planckian scattering. As noted above, we would like to determine classical characteristics of high-energy gravitational collisions. In particular, we would like to find the radiated energy and BH mass as a function of collision energy and impact parameter. In addition to give the inelasticity factors, this would

also provide information about the cross section for BH formation. It would also be interesting to determine further characteristics of the radiation, and the evolution of the BH as it goes through the 'balding' phase [35], asymptoting to a Myers–Perry [132] BH. Also of interest is determining the classical evolution (e.g. radiation details) in the larger impact parameter regime, where a BH does not form.

A second class of questions regards corrections to the classical analysis. Specifically, given a classical description of scattering in the eikonal and BH regimes, one would like to understand the size and nature of quantum corrections. In the eikonal regime $\theta \ll 1$, one would for example like to check what effect quantum corrections have on the classical scattering plus radiation, and also check the expectation that the basic features of the scattering are largely determined by classical physics.

In the BH regime, we know there are important quantum corrections to the classical analysis. These include Hawking radiation. However, so far we lack any quantum description of the full evolution of the BH, and in fact encounter a conflict of basic physical principles in attempting to describe such evolution. This conflict (for some more discussion see [181] and references therein) has been called the 'information paradox' or 'unitarity crisis,' and seems to indicate the need for fundamental revision of the foundations of physics. Beyond theoretical study of this regime, one can anticipate that experimental data on trans-Planckian scattering, should it be found, could provide further clues.

Interplay between perturbative and numerical methods. Despite being a well-defined problem, the computation of the exact solution for the collision of two ultra-relativistic particles in GR is out of reach and approximation techniques have to be used. Then, obtaining estimates from different techniques becomes fundamental for cross-checking. Let us discuss three such techniques and compare their results.

The oldest method consists of modelling the gravitational field of the colliding particles by Aichelburg–Sexl (AS) shock waves [57] and computing the apparent horizon that forms, as first done in four dimensions by Penrose [182]. From the apparent horizon construction, a lower bound for the energy loss into gravitational radiation and estimate of the threshold impact parameter for BH formation in D-dimensional collisions can be obtained [56]. This method does not require the knowledge of the geometry in the future light cone of the collision and can only provide bounds. Its main advantage is its technical and conceptual simplicity, although the critical impact parameter estimates in $D > 4$ require numerical solutions [183]. This method suggests that the amount of radiation loss increases (as a ratio of $\sqrt{s}$) with D. An improvement of the method, for determining the cross section, that considers an apparent horizon *on* the future light-cone (but not inside) was given in [196].

The second method, which may be regarded as a refinement of the first one, also uses the superposition of two AS shocks, but attempts to compute the geometry *in* the future light cone of the collision by a perturbative expansion. In four dimensions, the method was carried out to second order in perturbation theory [195, 197, 198], in the case of a head-on collision, yielding $0.164\sqrt{s}$ for the energy loss. The case with impact parameter is technically harder, due to the loss of symmetry, and has not been fully discussed. The generalization of this method to higher D head-on collisions was done recently, in first-order perturbation theory [199, 205]. Interestingly, the trend exhibited for the energy loss is the same as that obtained from the first method and suggests that the two methods converge as $D \to \infty$.

The third method is to model the colliding particles as BHs and to perform high-energy collisions using NR techniques. The use of BHs is, in principle, only for simplicity (i.e. one uses the vacuum Einstein equations). Using any other lumps of energy (such as boson

stars) should yield similar results in the trans-Planckian regime, since the process should be dominated by the energy of the objects; in other words, the individual phase space of the colliding objects is irrelevant as compared to the phase space of the entire collision process, and the detailed structure of the objects is hidden behind their mutual event horizon. It would be extremely interesting to confirm this expectation; in four dimensions, since high-energy head-on collisions of both BHs [185] and boson stars [79] have been performed, such comparison is within reach. The high-energy collisions performed so far have reached $\gamma \sim 3$ [185]; these authors extrapolated to $\gamma \to \infty$ using the numerical points and a fit based on the zero frequency limit, yielding, in the ultra-relativistic regime $(0.14 \pm 0.03)\sqrt{s}$ for the energy loss (the error bars are dominated by the numerical errors, rather than the specific fit). Remarkably, this overlaps the second-order perturbation theory result described above, which may be interpreted as lending credibility to both methods. Thus, two important directions are to obtain both the perturbative computation to second order in higher D and the corresponding numerical simulations. Some important steps concerning the latter have been already given [206, 72, 63, 64, 207], but it seems harder to reach the same γs in higher dimensions, due to stability, but also perhaps fundamental [65] problems.

4.3. Black hole search at the LHC: future improvements

There are various theoretical developments needed to refine BH searches in high-energy collisions. As noted above, it is important to better determine BH mass as a function of energy and impact parameter, which also supplies cross-sectional information, as well as to characterize the classical radiation.

To improve study of BH signatures, one needs further information about the BH decay. In particular, the full evolution of a BH through the 'spindown' and 'Schwarzschild' phases [35], ending with the 'Planck phase' $M \sim M_D$, where the semiclassical approximation breaks down, has not been determined. One would like to determine from this the spectra of decay products, as well as other features such as angular distributions, indicative of BH spin. First estimates of these basic features for a spinning BH were given in [35], and there has been a lot of work on the important problem of refining calculations of grey-body factors [208–224]. However, we still lack a calculation of the grey-body factors for graviton emission from spinning BHs, and so a complete picture of BH evolution is lacking (see [211] for the evolution in the spin-down and Schwarzschild phases). Such a calculation in particular is important to determine the amount of radiation in visible particles.

Finally, in addition to the questions associated with the unitarity crisis (described above) and Planck phase, there are questions of the dependence of the BH decay on the detailed microphysical model of TeV-scale gravity. Such details could contribute additional signatures. For this reason, further study of viable models for TeV-scale gravity would be helpful.

The data samples accumulated during 2011 are likely to provide the strongest limits obtainable on BH production for the LHC at a centre-of-mass energy of 7 TeV. The results are already constrained by the lack of theoretical understanding of the final stages of the BH decay. Furthermore, the fundamental Planck scale M_D has been constrained over the years, up to the present LHC data. The main sources of bounds come from (i) deviations from Newtonian gravity in torsion balance experiments [225, 226]; (ii) collider searches for Kaluza–Klein graviton production (monojet or photon with missing energy) [227–229] and KK graviton mediated dilepton or diphoton production [230–234]; (iii) astrophysical or cosmological KK graviton production in supernovae and in the early universe [235, 236]. The most stringent laboratory bounds from current LHC searches indicate $M_D \gtrsim 2.5$–3.5TeV for $D > 6$, whereas observational bounds from astrophysics and cosmology only allow for such small M_D for

$D > 7$. (However there are many model-dependent uncertainties on the latter.) Because of the theoretical uncertainties and the limits on M_D indicated by these searches, it is important to perform a wide-ranging search in the data, using all available channels to look for TeV-gravity effects. It would be dangerous, for example, to use limits from dijet production to infer the absence of the signal in a leptonic channel, since a TeV-gravity model would be needed to link the observations together. The experiments have already looked at dijet, multijet, single lepton and dilepton data, and this approach should be continued. Once the full 2011 dataset has been analysed, further progress will probably have to await an increase in the LHC beam energy.

On the theoretical front, it would be useful to try to constrain the range of allowed scenarios near the bulk Planck mass. The cross section is predicted to rise by many orders of magnitude, at a very rapid rate, in this region. From past experience with the onset of new physics, such as that observed in low-energy hadron scattering near the QCD scale, one would expect resonant behaviour near threshold, settling to the semi-classical prediction as the energy increases. Such behaviour could greatly enhance the sensitivity of the experiments, since the peak cross sections could be very large. It would be useful, if it were possible, to make generic predictions for the maximum allowed cross section for BH production, perhaps based on unitarity considerations, or even on the maximum allowed rate of change of the cross section near threshold. Such predictions could then be used instead of sampling parameter space in regions where the approximations required are known to be violated.

Finally, studies of the possible BH solutions of higher dimensional gravity over the last decade have revealed a vast number of possible solutions besides the Myers–Perry family of BHs (see section 3). These include five-dimensional black rings and black Saturns (which are known in exact explicit form) as well as a large number of other BHs in six or more dimensions with more complicated topologies (which have been constructed using approximate techniques). The potential formation and detection of one of these BHs would provide a new window into the study of higher dimensional gravity. However, many of these solutions are expected to be dynamically unstable when their angular momentum is sufficiently large and therefore would not appear as states after the balding phase (although they might play a role in the approach to this phase). For moderate angular momenta, currently it is not yet known whether a black ring (arguably the simplest and most important among the new kinds of BHs) may be dynamically stable and thus have observational signatures in a collision. It is expected that progress in this stability problem will be achieved in the near future via numerical studies. Before that, it seems premature to make any predictions about the possibility of observing them in collider experiments.

5. Strong gravity and high-energy physics

(Coordinator: Paul M Chesler)

5.1. Dynamics in holographic quantum field theories from numerical relativity

The study of real-time dynamics in QFT is a theoretically diverse topic, with available tools ranging from perturbative QFT and kinetic theory to string theory and gauge/gravity duality. It is also a challenging topic to study. Prior to the discovery of gauge/gravity duality, the only theoretically controlled regime of real-time dynamics in QFT was that of asymptotically weak-coupling, where QFTs often admit an effective kinetic theory description in terms of weakly interacting quasiparticles. In this regime, one can systematically study real-time dynamics via perturbative expansions. However, the domain of utility of real-time perturbative expansions

is more often than not quite limited, with extrapolations to $O(1)$ couplings converging poorly.

One interesting phenomenon where strongly coupled dynamics appears to be relevant is that of heavy-ion collisions at the RHIC and the LHC. There, collisions are believed to produce a strongly coupled QGP, which behaves as a nearly ideal liquid [237]. During the initial stages of a collision, when the QGP is produced, the system is surely far from equilibrium. However, the success of near-ideal hydrodynamic models of heavy-ion collisions suggests that the time required for the far-from-equilibrium initial state to thermalize may be as short as 1 fm c^{-1}, the time it takes for light to traverse the diameter of a proton [238].

Quantum chromodynamics (QCD) is the accepted theory of the strong interactions and therefore should describe the dynamics of heavy-ion collisions. Understanding the dynamics of the QGP produced in heavy-ion collision—from small viscosities to short thermalization times—from QCD has been a challenge. This is not to say that QCD is an incorrect description of nature, but rather that theorists simply do not know how to perform controlled calculations in QCD when the coupling is large.

Holography, or gauge/gravity duality, has emerged as a powerful tool to study real-time dynamics in strongly coupled QFTs from first principle calculations [10]. The utility of gauge/gravity duality lies in the fact that it maps the dynamics of some strongly coupled QFTs (i.e. holographic QFTs) in D spacetime dimensions onto the dynamics of semiclassical gravity in $D + 1$ spacetime dimensions. From a utilitarian perspective this means strongly coupled QFT dynamics can be mapped onto classical PDEs in $D + 1$ spacetime dimensions, which can be solved numerically if needed.

Holography is a unique tool. There is no other tool available which provides controlled access to dynamics in strongly coupled QFTs. However, it is also a tool of limited applicability [239]. Currently, there are no known theories of nature with dual gravitational descriptions. However, this has not deterred the construction of holographic toy models of QCD or other QFTs. While holography does not provide systematic access to strongly coupled QCD, it does provide systematic access to a regime of QFT never accessible before and therefore should be explored to the fullest possible extent. Moreover, holographic toy models of QCD can provide valuable qualitative insight into strongly coupled dynamics in QCD (for a useful review see [52]). Perhaps the most celebrated example is that of the shear viscosity in strongly coupled holographic QGP. All QGPs with holographic descriptions have the same shear viscosity to entropy density ratio $\frac{\eta}{s} = \frac{1}{4\pi}$ [240]. Such a small viscosity is a hallmark of a strongly coupled QFT. Indeed, recent models of heavy-ion collisions suggest that the viscosity of the QGP produced is within a factor of 2 of $\frac{\eta}{s} = \frac{1}{4\pi}$ [241]. No other systematic calculation of the viscosity has yielded values close to this.

Much can be learned about holographic QFTs via analytical techniques. Indeed, many other transport coefficients than the shear viscosity can be computed analytically [242, 243]. However, there are many dynamical processes where one must resort to numerical techniques. For example, the creation of a QGP via collisions of sheets of matter in $3 + 1$ spacetime dimensions is dual to the creation of a BH via the collision of gravitational waves in $4 + 1$ dimensions. To study such a complicated process one must have a NR toolkit suitable for spacetimes relevant to holography. The development of such a toolkit would be a valuable addition to the quantum field theorist's toolbox. However, as we discuss below in section 5.2, stable algorithms for NR in holographic spacetimes have been difficult to construct. As a consequence of this, very little progress has been made in the application of NR to holography and strongly coupled dynamics in QFT. Notable examples of successful applications of NR in the context of holography include [244, 70, 42, 71].

5.2. The challenge of numerical relativity in asymptotically AdS *geometries*

The simplest and most widely studied example of holography is the AdS/CFT correspondence, which maps the dynamics of non-Abelian CFTs in D spacetime dimensions onto semiclassical gravity in asymptotically AdS$_{D+1}$ spacetime. In particular, the AdS metric encodes the expectation value of the stress $\langle T_{\mu\nu} \rangle$ in the dual CFT [245] and metric correlation functions (which satisfy linearized Einstein equations) encode stress correlation functions in the dual CFT [246]. For concreteness, we shall consider the case of $D = 3 + 1$. In Fefferman–Graham coordinates, the metric of asymptotically AdS$_5$ takes the form

$$ds^2 = r^2 g_{\mu\nu}(x^\mu, r) \, dx^\mu \, dx^\nu + \frac{dr^2}{r^2}, \tag{3}$$

where x^μ are the $3 + 1$ spacetime directions of the dual CFT and r is the radial direction of the AdS geometry. The boundary of the AdS geometry, which is where the dual CFT can be thought of as living, is at $r = \infty$.

As is evident from the metric (3), a light-like signal from some finite r can reach $r = \infty$ in a finite amount of time. As a consequence of this, boundary conditions must be imposed on the metric at the boundary. To shed light on what the requisite boundary conditions should be, it is useful to solve Einstein's equations with a series expansion in r near the boundary. Doing so, one finds that near the boundary the metric has the asymptotic expansion [245]

$$g_{\mu\nu}(x^\mu, r) = g^{(0)}_{\mu\nu}(x^\mu) + \cdots + g^{(4)}_{\mu\nu}(x^\mu)/r^4 + \cdots, \tag{4}$$

where $g^{(0)}_{\mu\nu}(x^\mu)$ and $g^{(4)}_{\mu\nu}(x^\mu)$ are two independent constants of integration. Via holography, the expansion coefficient $g^{(0)}_{\mu\nu}(x^\mu)$ has the physical interpretation of the $(3+1)$-dimensional metric of the geometry that the dual CFT lives in and the expansion coefficient $g^{(4)}_{\mu\nu}(x^\mu)$ is related to the expectation value of the dual CFT stress tensor via [245]

$$\langle T_{\mu\nu}(x^\mu) \rangle = \text{const.} \times g^{(4)}_{\mu\nu}(x^\mu). \tag{5}$$

From the perspective of the dual CFT, the required boundary conditions on the metric $g_{\mu\nu}(x^\mu, r)$ are obvious. The CFTs described by AdS/CFT are not gravitating: they do not backreact on the geometry they live in. For a *given* geometry that the CFT lives in, the evolution of the expectation value of the stress tensor is governed by the Hamiltonian of the CFT. Therefore, natural boundary conditions consist of fixing $g^{(0)}_{\mu\nu}(x^\mu)$ and letting $g^{(4)}_{\mu\nu}(x^\mu)$ be dynamically determined. The simplest possible choice is $g^{(0)}_{\mu\nu}(x^\mu) = \eta_{\mu\nu}$ where $\eta_{\mu\nu}$ is the metric of Minkowski space. It should be emphasized that fixing $g^{(4)}_{\mu\nu}(x^\mu)$ can lead to unstable evolution (and even the appearance of naked singularities). Such unphysical behaviour is easy to interpret from the perspective of the dual CFT, as one must judiciously choose the evolution of $g^{(4)}_{\mu\nu}(x^\mu)$, so that it is consistent with the CFT Hamiltonian.

In addition to imposing the boundary condition $\lim_{r \to \infty} g_{\mu\nu}(x^\mu, r) = $ fixed, there is another type of boundary condition that must be imposed at $r = \infty$. Four components of Einstein's equations are radial constraint equations [70]. If they are satisfied at one value of r, then the other Einstein equations imply that they are satisfied at all values of r. [34] In addition to imposing $\lim_{r \to \infty} g_{\mu\nu}(x^\mu, r) = $ fixed, one must also demand that the four radial constraint equations are satisfied at $r = \infty$. As we will discuss below in section 5.3, this is tantamount to demanding that the CFT stress tensor is conserved.

Even with the correct boundary conditions imposed, finding stable NR algorithms in asymptotically AdS spacetime is a challenge. The challenge lies in the fact that Einstein's

---

[34] In particular, if the radial constraint equations are satisfied at one value of r, then the other components of Einstein's equations imply that the radial derivative of the radial constraint equations also vanishes. This can easily be seen from the Bianchi identities.

equations for the rescaled metric $g_{\mu\nu}(x^\mu, r)$ contain a regular singular point at $r = \infty$. Because of this, at $r = \infty$ one must not only impose boundary conditions, but one must also solve Einstein's equations *very* well near $r = \infty$ if one is to peel off the expansion coefficient $g^{(4)}_{\mu\nu}(x^\mu)$ and obtain the expectation value of the (conserved) stress in the dual CFT. The simplest possible approach is to put a cutoff on the geometry at some $r = r_{max}$ and impose the boundary condition $g_{\mu\nu}(x^\mu, r_{max}) = \eta_{\mu\nu}$. However, such an approach generically does not lead to stable evolution. Einstein's equations imply that gravitational radiation propagating towards $r = \infty$ cannot change the boundary geometry at $r = \infty$ [35]. However, gravitational radiation propagating up from the bulk can change the geometry at r_{max}. An arbitrary boundary condition at r_{max} is generally inconsistent with Einstein's equations and thus the evolution of the stress tensor in the dual CFT. For suitably large r_{max} one would expect this effect to be small and controllable by taking r_{max} larger. However, because Einstein's equations are singular at $r = \infty$, any discontinuity in the metric at r_{max} can be amplified by the singular point in Einstein's equation and lead to numerical evolution which quickly breaks down. Therefore, one of the challenges of doing NR in asymptotically AdS spacetime lies in imposing boundary conditions at $r = \infty$. One must find an algorithm which is consistent with Einstein's equations and which is suitably accurate near $r = \infty$, so that the asymptotics of the metric can be peeled off and the dual CFT stress tensor can be determined.

5.3. The characteristic formulation of Einstein's equations

One successful approach to NR in asymptotically AdS has been the characteristic formulation [244, 69, 70]. While, as we discuss more below in section 5.4, the characteristic formulation does have limitations, it has provided the first stable numerical solutions to Einstein's equations in asymptotically AdS [244] and lends itself to many interesting problems with relative computational ease.

Using infalling Eddington–Finkelstein (EF) coordinates, the metric of asymptotically AdS$_5$ may be written as

$$\mathrm{d}s^2 = -A\,\mathrm{d}v^2 + \Sigma^2 g_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j + 2F_i\,\mathrm{d}x^i\,\mathrm{d}v + 2\,\mathrm{d}r\,\mathrm{d}v, \tag{6}$$

where v is EF time, x^i are the CFT spatial directions and $\det g_{ij} = 1$. Lines of constant v represent infalling null geodesics from $r = \infty$. Moreover, the radial coordinate r is an affine parameter for these geodesics. The metric (6) is invariant under the residual diffeomorphism $r \rightarrow r + \xi(v, \mathbf{x})$, where $\xi(v, \mathbf{x})$ is an arbitrary function.

Einstein's equations read

$$R^{MN} - \tfrac{1}{2}G^{MN}(R - 2\Lambda) = 0, \tag{7}$$

where G_{MN} is the 5D metric and $\Lambda = -6/L^2$ is the cosmological constant, and L is the AdS curvature radius (which can be set to 1). Upon substituting the metric (6) into Einstein's equations one reaches the following conclusions.

(1) Time derivatives only appear in the combination $d_+ \equiv \partial_v + \tfrac{1}{2}A\partial_r$. This is the directional derivative along an outgoing null radial geodesic.

(2) Given g_{ij} on a slice of constant v, the $(0, 0)$, $(0, i)$, $(0, 5)$ and (i, j) components of Einstein's equations are a system of linear ODEs in r for the fields Σ, F_i, d_+g_{ij} and A at time v. These ODEs can be integrated in from the boundary and thereby have all boundary conditions imposed at $r = \infty$. Once these ODEs are solved, one can compute $\partial_v g_{ij} = d_+g_{ij} - \tfrac{1}{2}A\partial_r g_{ij}$ and g_{ij} on the next time slice.

---

[35] In other words, the expectation value of the CFT stress tensor does not change the geometry that the CFT lives in.

(3) The $(i, 5)$ and $(5, 5)$ components of Einstein's equations are the radial constraint equations. If they are satisfied at one value of r, then the other components of Einstein's equations imply that they are satisfied at all values of r. These equations must be implemented as the radial boundary conditions at $r = \infty$ in the aforementioned nested linear system of ODEs.

To determine the requisite boundary conditions for the nested system of ODEs and how the radial constraint equations must be implemented, it is again useful to solve Einstein's equations with a series expansion near $r = \infty$. Doing so, one finds that the required boundary conditions on the nested system of linear ODEs consist of the boundary geometry *and* the expectation value of the conserved densities $\langle T^{0\mu} \rangle$ at time v. Moreover, one then finds that the radial constraint equations are equivalent to the condition that the CFT stress is conserved $\partial_\mu \langle T^{\mu\nu} \rangle = 0$. Therefore, the algorithm for solving Einstein's equation in the characteristic formulation consists of

(1) Specify $g_{ij}(x, r)$ and $\langle T^{0\mu}(x) \rangle$ at time v.
(2) Solve the $(0, 0)$, $(0, i)$, $(0, 5)$ and (i, j) components of Einstein's equations for Σ, F_i, $d_+ g_{ij}$ and A at time v with the boundary condition that the CFT momentum density is $\langle T^{0\mu}(x) \rangle$ and with a given boundary geometry.
(3) Compute the field velocity $\partial_v g_{ij}$.
(4) Compute the field velocity $\partial_v \langle T^{0\mu}(x) \rangle$. This is given by the energy–momentum conservation equation $\partial_v \langle T^{0\mu}(x) \rangle = -\nabla_i \langle T^{i\mu}(x) \rangle$, which is equivalent to the radial constraint equations. At time v the spatial components $\langle T^{ij}(x) \rangle$ are determined by the near-boundary asymptotic of g_{ij}.
(5) Compute g_{ij} and $\langle T^{0\mu}(x) \rangle$ on the next time step and repeat the above process.

The utility of the characteristic formulation of Einstein's equations lies in the fact that Einstein's equations are integrated in from the boundary along infalling null radial geodesics. Therefore, any numerical error made at the boundary *instantaneously* falls to finite r away from the singular point in Einstein's equations. While this tames the singular point in Einstein's equations at $r = \infty$, it does not completely ameliorate it. One must still solve Einstein's equations very well near $r = \infty$. Two successful approaches thus far are to (i) solve Einstein's equations semi-analytically for $r > r_{\max}$ and match the semi-analytic solution onto the numerical solution at $r = r_{\max}$ [244] or to (ii) discretize Einstein's equations using pseudospectral methods [70]. In the latter approach, one can directly impose boundary conditions at $r = \infty$, as the exponential convergence of pseudospectral methods outpaces the power-law singularities in Einstein's equations.

As a proof by example that the characteristic method can be stable in asymptotically AdS, figure 4, taken from [70], shows the energy density and transverse and longitudinal pressures for a collision of two translationally invariant sheets of matter in a holographic CFT. Translational invariance in the two directions transverse to the collision axis implies that the corresponding NR problem can be dimensionally reduced to $2 + 1$ dimensions. In the dual gravitational description, this problem is equivalent to the formation of a black brane via the collision of two gravitational waves[36].

The fluid gravity correspondence [242] implies that at suitably late times the evolution of the CFT stress tensor will be governed by hydrodynamics. The charm of the gravitational

---

[36] We note that AdS$_5$ contains a horizon at $r = 0$, the Poincaré horizon. Because of this, a horizon exists for all times, even in the infinite past before the collision event. As a consequence of this and the fact that the horizon has the topology of a plane, the residual diffeomorphism invariance of the metric (6) can be removed by setting the position of the apparent horizon to be at $r = 1$.
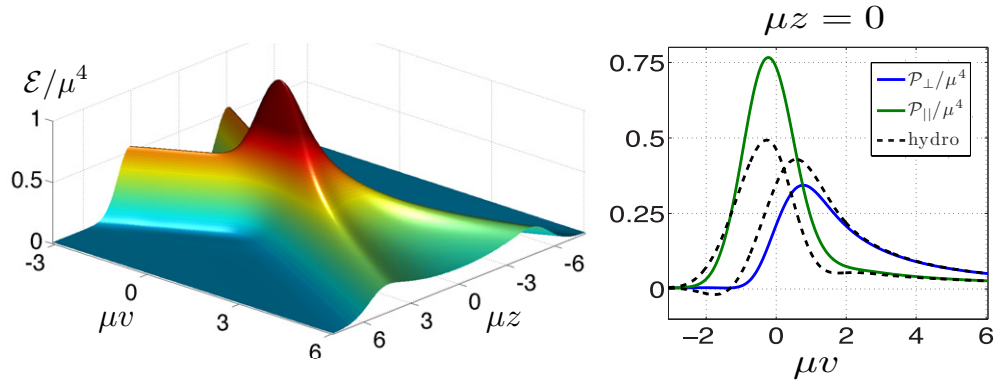
Figure 4. Left: the energy density $\mathcal{E}$ of two colliding sheets of matter in a holographic CFT. Right: the transverse and longitudinal pressures $\mathcal{P}_\perp$ and $\mathcal{P}_{||}$ at $z = 0$. The colliding sheets are translationally invariant in the two directions orthogonal to the collision axis z. The scale μ sets the energy density per unit area of the sheets. The sheets propagate at the speed of light and collide at time $v = 0$. Near $v = 0$, the system is very far from equilibrium and the transverse and longitudinal pressures are very different. However, after a few units of $1/\mu$, the dynamics of the debris left over from the collision is governed by viscous hydrodynamics with increasing accuracy as time progresses.

calculation is that Einstein's equations encode *all* physics: from far-from-equilibrium dynamics during the initial stages of the collision to the onset of hydrodynamics at late times. As is evident from the right panel in figure 4, the total time required for the collision to take place and for the system to relax to a hydrodynamic description is $\Delta_{\text{hydro}} v \sim 4/\mu$, where the scale μ is related to the energy density per unit area of the shocks $\mu^3 (N_c^2/2\pi^2)$, with N_c being the number of colours of the CFT gauge group. Crudely modelling heavy-ion collisions at RHIC with translationally invariant sheets of matter, one can estimate $\mu = 2.3$ GeV, which yields a relaxation time $\Delta v_{\text{hydro}} \sim 0.35$ fm c^{-1}.

While demonstrating that NR in asymptotically AdS geometries can be stable, the above example also demonstrates that thermalization times < 1 fm c^{-1} are not unnatural in strongly coupled QFTs.

5.4. Conclusions

The marriage of NR and holography is still in its infancy. There are many unexplored QFT and gravity problems to study, such as turbulence, QGP formation and thermalization in non-conformal QFTs, and Choptuik critical phenomena and its holographic interpretation.

Much work remains to be done in order to develop a robust NR toolkit applicable to holography. While the characteristic method discussed in section 5.3 has proven to be stable, it is not without its limitations. One limitation is that the characteristic method breaks down when caustics form. This limits the applicability of the characteristic method to black brane geometries, where the horizon of the black brane has the topology of a plane. Clearly, it would be desirable to develop alternative NR algorithms which lend themselves to more general geometries.

It cannot be over emphasized how valuable a robust NR toolkit would be to the study of dynamics in QFT. The first QFTs studied were written down a generation ago. And yet in that generation the vast majority of the progress made in understanding dynamics in

QFT has come from weak-coupling expansions. With the discovery of gauge/gravity duality, dynamics in strongly coupled QFTs is now accessible in a systematic and controlled setting. The strongly coupled regime of QFT should be explored to the fullest extent possible. While NR in holographic spacetimes may be challenging, it is vastly easier than first principle calculations in strongly coupled QFTs.

6. Alternative theories of gravity

(Coordinator: Leonardo Gualtieri)

The theory of GR is one of the greatest achievements in Physics. Almost one century after its formulation, it still stands as an impressive construction which captures in a compelling and successful way most of what is known about the universe on a large scale. And yet, both theory and observations hint at the incompleteness of GR.

On the theoretical side, GR is conceptually disjoint from another remarkable and successful explanation of nature: *Quantum Field Theory*, which explains with great accuracy small-scale physics. It is believed that GR and QFT should be limits of a more fundamental theory, which despite a decades-long effort devoted to its understanding, remains as yet elusive. Adding to this, GR is plagued by singularities and other 'weird' phenomena such as causality violations which appear to reveal a breakdown of the theory at very small length scales, even though such occurrences are probably always hidden by horizons.

On the experimental/observational side, important open problems may be associated with deviations from GR. Among them, the most compelling regard cosmology: observations have shown that our Universe is filled with dark matter and dark energy, which seem to be difficult to incorporate elegantly in GR[37].

One possible way out of this conundrum is to accept that Einstein's theory is valid only in the weak-field regime, but has to be modified for a description of strong fields. In fact, all experiments and observations so far have only probed the weak-field regime of gravity [250–253][38] and while Einstein's theory passes all these tests with flying colours, extrapolating GR to the elusive strong-field regime may be dangerous.

Testing the strong-field limit of GR is one of the main objectives of many future astrophysical missions [254, 255]. In the context of testing alternative theories of gravity, arguably the most promising of these missions would be the space-based GW detector LISA/eLISA, since it would be sensitive enough to give birth to the so-called gravitational-wave astronomy, i.e. observing the universe through a new window, the most appropriate to look at violent, strong-field phenomena occurring in our Universe. Unfortunately, after recent decisions from NASA and ESA it is not clear if this detector is going to be realized in the next decade. Several other projects to probe the strong-field regime of GR are currently in construction or under design study. Detectors in construction include the second generation ground-based GW detectors Advanced LIGO/Virgo [256] and, on a longer timescale, the underground cryogenic detector KAGRA [257]; we should also mention the Pulsar Timing Array project to detect GWs indirectly by observing an array of pulsars using radio telescopes [258]. Detectors in design study include the third generation detector ET [259], more advanced

---

[37] This is a debated issue. See e.g. [247], where it is argued that the cosmological constant should be considered as a natural part of Einstein's equations. Some attempts to incorporate dark energy in GR have also been done, see e.g. [248, 249].

[38] Even tests from binary pulsars [251, 253] verify the weak-field limit of GR, because the strong gravitational field near the stellar surface does not significantly affect the orbital motion, and the gravitational compactness is smaller than 1 part in 10^5 for the most relativistic (double) binary pulsar.

space-based detectors (DECIGO, BBO) [260, 261], next generation x-ray and microwave detectors to study BHs and the cosmic microwave background (IXO, NICER) [262].

Further experiments target the observation of specific violations of GR, like violations of Lorentz symmetry, of the equivalence principle, of the expected polarization and speed of GWs, etc. We expect then to have soon a large amount of data, which will allow us to test, for the first time, the strong-field limit of gravity. A signature of new physics in these experiments and observations could be a 'message in a bottle' coming from a more fundamental theory, standing at energies far beyond our reach.

In this context, it is not surprising that the community of theorists has been paying, in recent years, more and more attention to possible deviations of GR, and more generally to alternative theories of gravity. Many theories have been proposed, such as scalar–tensor theories [263], scalar–tensor–vector theories [264], massive graviton theories [265, 266], braneworld models [6, 7, 267], $f(R)$ theories [268], quadratic curvature corrections (e.g. Gauss–Bonnet gravity [269], Chern–Simons gravity [270]) or Lifshitz-type theories [271]. Given this plethora of possible corrections to GR, it is crucial to devise some guideline which allows us to make contact between theoretically conceivable models and upcoming observations.

In the following we discuss some important issues about alternative theories of gravity in four dimensions, with the aim to report both on the state-of-the-art and on the ongoing discussion on these topics. We will not discuss here alternative theories of gravity in $D > 4$; we only mention that some effort has been done to study deviations from GR in higher dimensions, mainly in the context of theories with quadratic curvature corrections [272–274].

6.1. Which kind of GR deviations should we expect?

An answer, even partial, to this question would be extremely valuable. Indeed, on the one hand, the realm of possible alternative theories of gravity is too large to be the starting point for systematic tests of GR. On the other hand, choosing one specific alternative theory of gravity could become a merely formal exercise, unless the choice is made on physical grounds. Actually, many alternative theories of gravity claim to be 'string inspired', since they have features which also appear, in different contexts, in string theory/M-theory (SMT). This is the case, for instance, for scalar–tensor theory [263], in which, as in SMT, the spacetime metric is non-minimally coupled to a scalar field, of theories with quadratic curvature corrections [269, 270], which arise in several low-energy truncations of SMT, of braneworld theories where, as in SMT, gauge fields can be confined on submanifolds (branes) of the spacetime, and so on.

These are merely inspirations and analogies, however. It would be extremely useful, in the study of possible GR modifications, to have a guidance from candidate fundamental theories (SMT, loop quantum gravity, etc), but actual SMT constraints on the low-energy theory are still poor, because we do not know the symmetry breaking mechanism which would yield our world. Many deviations from GR can be accommodated in some model of SMT. Actually, it would be more appropriate to speak of 'predictions of an SMT model' rather than 'SMT predictions', since today SMT should be viewed as a framework on which different models can be defined, rather than a well-defined theory.

Given that SMT is one of the few candidates for a unified theory, it is perhaps wise to extract from it general indications, which may be useful (*if* it is the correct route to the fundamental theory unifying GR and QFT) as a possible, preliminary guideline. In this context, the most likely and potentially interesting GR corrections would be associated with couplings with other fields (scalars, vectors, etc). Another interesting possibility is that GR corrections would involve curvature invariants. The dimensionful coupling constant of such

curvature invariants may, however, be suppressed by the Planck scale, making these corrections irrelevant for astrophysical phenomena. On the other hand, both the hierarchy problem and the cosmological constant problem suggest that possible couplings to gravity may be very different from those expected from standard field theory. Thus, if for some reason these corrections are not Planck-suppressed, alternative theories like Chern–Simons gravity or Gauss–Bonnet gravity may have important implications for gravitational-wave astronomy.

Alternatively, one could adopt an 'agnostic' attitude, not trying to infer general indications from candidate fundamental theories, and instead considering whether gravitational wave observations or other gravitational experiments can provide any constraints on gravity theories beyond that of Einstein. One route towards this goal is to think about what possible symmetries or fundamental GR principles one could compromise to study whether nature violates them. This is the case, for instance, of generic breaking of parity invariance (captured in Chern–Simons gravity [270]), or of kinematical violations of Lorentz–Symmetry in the propagation of gravitational modes [275]. Whether one expects such deviations from fundamental principles or not, it might still be worthwhile to consider what current or future experiments have to say about these topics.

6.2. Strategies to find a GR deviation in experiments and observations

A possible strategy to find GR deviations may be building a parametrization as general as possible of metric theories of gravity, modelling in this framework the relevant (strong-field) astrophysical processes and determining observable signatures in terms of the parameters of the theory. This could be made in two ways (which have been dubbed, respectively, top–down and bottom–up [276]): parametrizing the action and looking at the phenomenological consequences, or parametrizing a phenomenological description of the observations, and inferring the consequences on the underlying theory. An attempt to pursue the former approach has been made in [277], where a parametrization of the action in terms of polynomials in the curvature tensor has been proposed. An example of the latter, instead, is the parametrization of the BH spacetime (in a general theory of gravity) in terms of multipole momenta, which has been proposed in the context of extreme mass-ratio inspirals [278, 279]. Another example is given by the ppE formalism [280, 281], in which a parametrization of the gravitational waveform emitted in BH binary coalescences is proposed. All these approaches are attempts to build a general parametrization of the gravitational theory, to be compared with observations.

On the other hand, a general parametrization of gravitational theories could be difficult to implement in the analysis of data (for instance from gravitational interferometers). Indeed, it could depend on too many parameters to preserve practicality of the matched filtering process: too many free parameters in the template space can introduce degeneracies in the parameter estimation process and also lead to larger uncertainties in those estimations, i.e. they could raise the false-alarm rate. It should further be remarked that having a precise template will be crucial for the determination of the physical parameters, but should be less important for detection, [282]. Therefore, it will probably be the outcome of the experiments which dictate how we should proceed or which strategies will work best in testing the behaviour of the gravitational interaction.

Nevertheless, there exists strong motivation for embedding the alternative theories of gravity proposed so far in a large class of theories. A clever strategy could be to first find the general signature of a theory (or of a class of theories), then identify the experimental/observational setup in which such a signature is enhanced. Thus, the search for 'smoking guns' of alternative theories (for instance spontaneous scalarization [283] and floating orbits [284] in scalar–tensor theories, birefringence of GWs in parity-violating theories

[285]) would be a solid route to establish or rule out many candidate theories. Knowing 'where to look' would also be extremely useful to conceive new experiments and to fine-tune the experiments now in the commissioning and building stages. We would be able, once a deviation from the GR prediction is detected or observed, to understand its theoretical implications, and translate the new observation into a deeper understanding of the fundamental laws of nature.

It is also worth saying that, even in the absence of any observational evidence to date for deviations from GR, experimental/observational data are already providing useful information on alternative theories of gravity. Indeed, negative results (even in weak-field processes) enable us to constrain the set of allowed theories [286, 250, 287–290].

6.3. GR deviations in astrophysical processes

To test deviations from GR, we should look at the most violent astrophysical processes dominated by gravitational fields, which are the coalescences of compact binary systems formed by BHs and/or neutron stars, either of comparable masses [291–293] or of extreme mass ratio [294–296]. BH oscillations are a promising process, too, since they encode in a clean way the features of the underlying theory of gravity [297]. Such processes provide an optimal testbed for GR, as they probe the pure gravitational interaction. Indeed, most of the literature on phenomenological bounds of alternative theories of gravity refer to such processes.

Possible insights from BH and neutron star physics would be complementary to the large amount of literature on alternative theories in cosmological scenarios. Indeed, cosmological observations could also provide a valuable tool to study alternative theories of gravity. For instance, some of these theories (braneworld models, $f(R)$ gravity) could account for the acceleration of the universe without the need of including dark energy [298–300]; these models can then be tested against the large amount of observational data on supernovae, cosmic microwave background, gravitational lensing, galaxy clustering, etc [301] (see also [302]). Other theories (such as scalar–vector–tensor theories) could explain the galaxy rotation curves without the need of including dark matter [264] (see also [303]). This topic is beyond the scope of this paper; for a comprehensive review we refer the reader to [304].

Some of these processes, however, especially those involving BHs, do not probe a key aspect of GR, namely the coupling of gravitation to matter. Possible constraints in this sector may come from different kinds of strong-gravity processes, like those involving the internal dynamics of neutron stars. Unfortunately, the uncertainty about the behaviour of matter at extreme density in the core of a neutron star makes it difficult to disentangle the effects of an alternative theory of gravity from those due to a different equation of state. Within the next few years, astrophysical and gravitational wave observations may be able to shed more light onto the neutron star equation of state. This will open up the possibility of making precision tests of the coupling of gravity with matter [305–307]. Accretion of matter into BHs has also been studied in alternative theories of gravity [308, 309], but the uncertainties in our knowledge of this process make it difficult to use it as a testbed of GR against other theories.

More generally, rather than constraining small corrections to standard gravity, it could be useful to look for effects which only appear in alternative theories, but identically vanish in GR [283–285]. Such effects may provide clear observational signatures and thus prove effective in constraining or ruling out alternative theories.

6.4. Numerical relativity and alternative theories of gravity

NR may be a powerful tool to study alternative theories of gravity and to understand their phenomenology in strong-field astrophysical processes. Numerical simulations in this field,

however, have so far almost exclusively been restricted to highly symmetric configurations such as the spherically symmetric gravitational collapse of dust [310–312] or processes involving scalar fields in spherical symmetry [313]. We note, however, the recent study of BHs in scalar–tensor theory by Healy *et al* [314]. With that exception, though, the use of NR following the 2005 breakthroughs [1, 3, 2] in the study of alternative theories of gravity remains essentially uncharted territory. Aside from the rather well-studied case of scalar–tensor theories (see also [315, 316]), it is not even clear to what extent the stability properties of the present state-of-the-art codes in $(3+1)-$dimensional and higher dimensional spacetimes carry over to modifications of GR. Furthermore, for those cases where the action contains higher order polynomials of the Riemann tensor, the existence of a well-posed initial value formulation of the field equations is not known. For this class of theories, which includes for example Chern–Simons gravity, the field equations contain derivatives higher than second order which may drastically change the mathematical structure of the theory (see e.g. [30]).

There may be reason, however, for some optimism in this regard. We emphasize that the following items cannot be regarded as mathematical arguments, but merely as an intuitive motivation for carrying out such investigations with some level of confidence. First, if a given theory correctly describes real physical processes, then we should expect there to exist a well-posed form of this theory. Second, we typically consider scenarios which only mildly deviate from GR. Given that GR itself has well-posed initial-value formulations, we might hope that this remains the case, at least in a 'neighbourhood around GR in the space of theories'. Third, modified theories of gravity often arise as the low-energy limits of more fundamental theories. It could then be acceptable if a modified theory is well posed in certain regimes only.

Deriving fully nonlinear numerical evolutions in the framework of alternative theories of gravity will require substantial effort, but the wealth of physical systems thus opened up for systematic study almost certainly justifies the effort. For instance, we would be able to model processes with low degree of symmetry or those involving complex matter distributions and/or other fields, whose study is not possible using semi-analytic approaches such as perturbation theory or parametrized post-Newtonian expansions.

The answer to the above questions and issues crucially depends on the specific alternative theory to be studied. Two recent works on different directions allow for optimism. In the case of GR coupled with a scalar and/or vector field, the mathematical structure of the evolution equations is expected to be preserved [315, 316]. In line with this expectation, NR simulations of binary BH coalescences in scalar/tensor theory have recently been performed [314]. A second exciting development concerns a well-known conjecture [107, 108, 317] that large static BHs do not exist in type II Randall–Sundrum scenarios of the modified gravity model. These works used indirect arguments to claim static solutions do not exist, and gravitational collapse would yield a dynamical BH that would 'evaporate' classically due to (classical) gravitational radiation. A counter-example to this conjecture was recently presented where static BHs were numerically generated [106, 105]. These works, albeit preliminary, show that NR can be successfully applied to model alternative theories of gravity, in order to understand their phenomenology in strong-field astrophysical processes.

7. Approximation methods in GR

(Coordinator: Carlos F Sopuerta)

7.1. Relativistic perturbation theory

In the theory of GR, gravity does no longer appear as a force but as a manifestation of the geometry of spacetime, which is a dynamical entity. As such, the gravitational dynamics is

encoded by the spacetime metric tensor, a spin-2 field that satisfies the Einstein field equations. The main difficulties that arise in GR are related to the nonlinearity and the diffeomorphism invariance of the theory, which complicate solving the field equations and understanding the solutions. Most of what we have learned about GR comes from the detailed studies of a few exact solutions (see, e.g. [318, 162]), in particular Minkowski, Schwarzschild, Kerr, Friedmann–Lemaître–Robertson–Walker, etc). These exact solutions can also be taken as the basis for the study of more complex physical situations, e.g. oscillations of compact stars and BHs, cosmological structure formation, etc. The reason for this is that many of these physical situations admit a perturbative analysis where the zeroth-order solution (usually called the background geometry) is one of the exact solutions mentioned above. The starting point of relativistic perturbation theory is to consider two different spacetimes, the *physical* one, which describes the actual physical system, and the *background* one, which corresponds to a simpler idealized situation. We can relate these two spacetimes using different maps (which identify points of the background and physical spacetimes and can be used to transport the tensorial structure between them), each of them corresponding to a *gauge* choice and the transformation between maps is known as a gauge transformation. Using one such mapping we can write the metric tensor of the physical spacetime, $g_{\mu\nu}$, as

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} + \delta g_{\mu\nu} \qquad (\mu, \nu = 0, \dots, D-1),\tag{8}$$

where D is the spacetime dimension, $\tilde{g}_{\mu\nu}$ denotes the background spacetime metric and $\delta g_{\mu\nu}$ are the metric perturbations, which can be split into a first-order piece (that satisfies the Einstein equations linearized with respect to the background metric), a second-order piece, etc. Experience tells us that the gauge freedom in the identification between the background and physical spacetimes can cause problems; in fact, many authors talk about the *gauge problem*, e.g. [319, 320]. These problems arise in the physical interpretation of results, and in particular when one works with a family of gauges instead of a unique gauge. For instance, considering only first-order perturbations (something similar applies to higher perturbative orders), the Lorenz gauge condition $\tilde{\nabla}^{\mu}(\delta g_{\mu\nu} - (1/2)\tilde{g}_{\mu\nu}\tilde{g}^{\rho\sigma}\delta g_{\rho\sigma}) = 0$ does not identify a unique gauge, since we can perform gauge transformations whose generating vector field ξ^{μ} satisfies $\Box\xi^{\mu} \equiv \tilde{g}^{\rho\sigma}\tilde{\nabla}_{\rho}\tilde{\nabla}_{\sigma}\xi^{\mu} = 0$. These gauge transformations respect the Lorenz gauge condition but can change the metric perturbations, and hence one must be careful when dealing with perturbations in this family of gauges. To avoid these potential problems, a usual procedure is to look for gauge-invariant quantities, i.e. quantities that have the same values independent of the gauge one is working in. For details on the mathematical formulation of relativistic perturbation theory see [321–327]. In the case of 4D spacetimes, it has been applied and developed for different physical systems, mainly for the following.

(i) *Perturbations of flat spacetime* (see, e.g. [328]). These are the most simple type of perturbations that one can think of since the background spacetime geometry is the Minkowski flat spacetime. They are useful for several types of studies, in particular to describe the propagation of gravitational waves far away from the sources. Perturbations of flat spacetime are also at the core of the so-called post-Minkowskian approximation, where one is interested in gravitational weak-field phenomena.

(ii) *Cosmological perturbation theory* [329–336]. As the name indicates, here one perturbs a spacetime of cosmological character. Most of the work done in the literature has focused on the development of perturbations of the Friedmann–Lemaître–Robertson–Walker cosmological models. They are one of the cornerstones of the present standard model of cosmology as they are fundamental for the description of many physical phenomena in cosmology. The formation and growth of structures in the early universe within the inflationary paradigm, the description of anisotropies in the cosmic microwave background

(e.g. the Sachs–Wolfe effect [337]), etc. Recently, second-order perturbation theory has been used to argue that the acceleration of the Universe (the so-called dark energy problem) could be explained as a backreaction effect (for a review see, e.g. [338]), although some other studies indicate that this is unlikely [339, 340].

(iii) *BH perturbation theory.* Here we have to distinguish between non-rotating (Schwarzschild) and rotating (Kerr) BHs (for textbook reviews on BHs and BH perturbations see [341, 342]). In the case of non-rotating BHs, the formalism has been developed based on metric perturbation variables [343–345]. Spherical symmetry is a key ingredient, since it allows decomposition in tensor spherical harmonics and the equations for each harmonic can be decoupled in terms of gauge-invariant master functions that satisfy wave-type equations in 1+1 dimensions. Once these equations are solved, all metric perturbations can be reconstructed from the master functions. The situation in the rotating case is significantly different. First of all, partly due to the lack of the spherical symmetry, we do not have any metric-perturbation-based formalism to decouple the first-order perturbative equations. Nevertheless, we do have a formalism based on curvature variables due to Teukolsky [346, 347] that provides master equations (the Teukolsky equation) for the Weyl tensor components that can be associated with ingoing and outgoing gravitational radiation. In both cases, rotating and non-rotating, we can compute gravitational waveforms, and energy and momentum fluxes radiated at infinity from the master functions. What remains to be done in the rotating case is to establish, in general, the reconstruction of the metric perturbations from the curvature-based variables, although significant progress has already been made (see [348–351]).

BH perturbation theory has been applied to study the stability of BHs [352–354], the computation of quasi-normal modes of BHs [355–357], etc. In an analogous way to the BH case, perturbations and quasi-normal oscillations of relativistic stars have been extensively studied (see [358] for a review). It also has been applied to the description of the dynamics of binary systems with an extreme mass ratio and their gravitational wave emission, the so-called *extreme mass-ratio inspirals*. This is a very demanding subject in terms of the perturbation theory technology needed for the computations. Usually, one describes the small compact object as a point mass (which is at odds with the full theory but allows us some simplifications) that induces perturbations on the geometry of the large one, considered to be a (supermassive) BH. Then, the inspiral can be described in terms of the action of a local force, called the *self-force*, that can be constructed from the gradients of the first-order perturbations. However, the point-like description of the small compact object leads to singularities in the perturbative solution that must be regularized. Procedures to regularize the solutions have been devised in the Lorenz gauge (see, e.g. [359]), but working in this gauge complicates the computations as we no longer have some of the advantages associated with the well-known Regge–Wheeler gauge in the case of non-rotating BHs, as for instance the decoupling of the metric perturbations; something similar happens in the case of spinning BHs. At present, the gravitational self-force has been computed for the case of a non-rotating BH first using time-domain techniques [360, 361] and later with frequency-domain techniques [362]. These calculations have allowed the study of some physical consequences of the self-force [363–365]. Progress is being made towards calculations for the case of a spinning BH [366]. In any case, given the amount of cycles required for extreme mass-ratio inspirals GWs (it scales with the inverse of the mass ratio, which can be in the range 10^{-7}–10^{-3}), we cannot expect to generate complete gravitational waveform template banks by means of full self-force calculations. Instead, the goal of these studies should be to understand all the details of the structure of the self-force, so that we can formulate efficient and precise algorithms to create the waveforms

needed for gravitational-wave observatories like LISA [367] (see [368–370] for reviews on the progress in the self-force programme). Observations of extreme mass-ratio inspirals have a great potential for improving our understanding of BHs and even the theory of gravity (see, e.g. [295]).

Usually, approximation methods require the introduction of a smallness parameter to establish in which sense the perturbations are small. In relativistic perturbation theory, this is done in an implicit way, in the sense that we could replace $\delta g_{\mu\nu}$ by $\lambda \delta g_{\mu\nu}$, where λ is a formal perturbation parameter without having a direct physical meaning (as in cosmology, in backreaction problems, or in the study of quasi-normal modes of stars and BHs), although there are situations in which we can assign to it a specific physical meaning, as in the study of BH mergers via the close limit approximation, in the analysis of quasi-normal mode excitation by a physical source, or in the modelling of perturbations generated by the collapse of a rotating star.

From these basic ideas, one can develop other approximation schemes in GR. In general, this involves adding extra assumptions in the approximations, combining different schemes, etc. One example of this is post-Newtonian theory (see, e.g. [371–373]). In general, a post-Newtonian approximation can describe the GR dynamics in the regimes where the speeds involved are smaller than the speed of light ($v \ll 1$) and the gravitational interaction is *weak* ($M/R \ll 1$, where R denotes the size of each body or the typical orbital separations). Moreover, it is well known that the post-Newtonian approximation is valid only in the vicinity of the massive objects (a region around the bodies that is small as compared with the wavelength of the gravitational waves emitted by the system). One can use a post-Minkowskian approximation (where only the weak gravitational field condition $M/R \ll 1$ is imposed) to describe the gravitational field outside the near zone. Then, one can match the two expansions, the post-Newtonian and post-Minkowskian ones, by means of the method of matched asymptotic expansions, which is a key ingredient to connect the orbital motion and the gravitational-wave emission. This process is quite involved in practice and has been developed during many years (see [371] for a review). The result is a framework in which we can in particular model the general relativistic dynamics of compact binary systems, including their gravitational-wave emission. By comparing with recent NR simulations, it has been found that post-Newtonian computations provide a good description of the inspiral of a binary system close to the merger phase. In order to improve these results, new schemes that use post-Newtonian theory have been proposed. In particular, the effective-one-body scheme [374, 375] has been developed to a point where one can construct gravitational waveforms for the whole binary BH coalescence [376], including merger and ringdown in the case of non-rotating BHs; for first applications of the effective-one-body method to spinning binaries see also [119, 377].

Another relativistic computation scheme that has been useful in GR is the so-called *close-limit* approximation [378]. This is based on the realization that binary BH coalescence can be divided into three stages: a long and relatively slow inspiral, a short nonlinear merger phase and finally the ringdown of the final BH towards a stationary BH state. Then, it turns out that in the last two stages of this process (or at least including a part of the merger phase where we are close to the formation of a common apparent horizon) the binary BH system can be seen as a single deformed BH. The idea is then to map the two-BHs system to a single BH (we must read its mass and eventually its spin from this mapping) plus perturbations (which are also read from the mapping and this is the key part of the computation). Then, by using the standard techniques of BH perturbation theory that we have described above, we can evolve the system until the final BH is settled, and from this evolution we can compute physical quantities such as energy and momentum emission in gravitational waves. It has been shown [379, 380] that in the case of head-on collisions of BHs, the close limit approximation

provides accurate results in comparison with NR results. The close limit approximation scheme has been developed by a number of authors [381–388, 125, 389], and it has been applied to astrophysical problems like the computation of recoil velocities (see e.g. [390, 391]) of BHs formed in binary BH collisions, in the past for head-on collisions [392] and recently for non-rotating BHs in circular and in small eccentricity orbits [393, 394] (see also [117, 395]). Since the close limit approximation scheme applies to the final part of the binary BH coalescence, it offers considerable potential for problems in HEP that involve BH collisions or similar systems (see the following subsection). In essence, the close limit approximation is a linear approximation around the final, equilibrium BH state. In the context of gauge–gravity duality, it has been recently applied in [396].

Up to now, we have described approximation methods that use the most common background spacetimes and the main perturbative schemes that have been developed from them to study a variety of physical phenomena. However, this is not all what has been done. There are other 4D spacetimes that have received attention and perturbations of them have been studied. Among those, we note the study of oscillations (quasi-normal modes) of Schwarzschild–dS BHs [397], Schwarzschild–AdS BHs [398–401], Reissner–Nordstrom–AdS BHs [402] and Kerr–AdS BHs [403]. Oscillations of BHs in alternative theories of gravity, such as higher derivative gravity, have also been studied [404–408] (for a review, see [409]).

7.2. Perturbations of higher dimensional spacetimes

All that we have described until now is standard relativistic perturbation theory in the case of 4D spacetimes. Nevertheless, it is clear that the basic ideas and foundations of relativistic perturbation theory can be transferred without problems to higher dimensional spacetimes. Here, we will only consider higher dimensional spacetimes governed by Einstein's equations, $G_{\mu\nu}[g_{\rho\sigma}] = 8\pi T_{\mu\nu}$, where $G_{\mu\nu}$ is the Einstein curvature tensor and $T_{\mu\nu}$ is the stress–energy tensor.

In the last decades, there has been growing interest in physical phenomena in higher dimensions motivated by the emergence of new theoretical models in HEP, in particular, on theoretical models either based on string theory or motivated by it. An interesting feature of many of these scenarios is that they involve higher dimensional spacetime geometries where the extra dimensions need not be compactified or have a large curvature radius. Many of these theories deal with physical situations that involve energies beyond those associated with the standard model, and given that some of them suggest that the *fundamental* Planck mass may be small (as low as the TeV range, which can be a solution to the so-called *hierarchy problem* [4]), BHs and other *dark* objects (containing horizons) in spacetimes with different number of dimensions can play an important role and as such are an important subject of investigation (see sections 2, 3 and 4 for further details). There is also a strong motivation coming from the correspondence between $\mathcal{N} = 4$ super Yang–Mills theory in the large N limit and type-IIB string theory in $\mathrm{AdS}_5 \times S^5$, the AdS/CFT correspondence [10]. The idea is that in the low-energy limit, string theory reduces to classical supergravity and the AdS/CFT correspondence becomes a tool to calculate the gauge field-theory correlation functions in the strong-coupling limit leading to non-trivial predictions on the behaviour of gauge theory fluids, which has a lot of applications that have made this correspondence one of the main subjects of current research; see also section 5.

Cosmological perturbations have also been analysed in certain theoretical scenarios, as for instance in braneworld models. These models involve at least one extra dimension. Two scenarios where these cosmological perturbations have been studied extensively are the Randall–Sundrum type II scenario [7] (see, e.g. [410]) and a closely related one, the Dvali–

Gabadadze–Porrati model [267]. Again, the interest here is in the early Universe dynamics where the energies can make these models deviate from the usual GR cosmological dynamics (see [411] for a review).

In higher dimensions, many geometrical properties that are valid in 4D no longer hold (and some hold in an appropriate form [140, 412]). In the context of HEP, it is particularly relevant that the topology of connected components of space-like sections of event horizons does not need to be that of a sphere, as happens in 4D [413, 318] (see also [141]). This opens the door to many topologically different objects with horizons [13]. Not all these objects need to be stable, and actually many of them are not, and this strongly motivates the study of deviations from all those geometries; cf the discussion of stability in section 3. Therefore, despite the fact that the basics of relativistic perturbation theory are the same independently of the dimensionality, the development of a complete perturbative scheme in the sense of constructing gauge-invariant quantities and their equations for describing the physics associated with the perturbations is a task that needs to be performed for each background spacetime geometry or at least for families of background spacetime geometries.

As we have discussed above, in four dimension the perturbative schemes for BHs are quite developed. For higher dimensional BHs there is still a long way to reach a complete formulation for perturbations. Nevertheless, significant progress has already been made. For static BHs in arbitrary higher dimensions, a set of decoupled master equations, which correspond to the Regge–Wheeler–Zerilli equations in 4D, have been found [414–416]. In this case, for a $D = 2 + d$ BH background geometry, where d is the number of dimensions of the internal space (which is an Einstein space; in $D = 4 \Leftrightarrow d = 2$, it is the 2-sphere). Then, the perturbative variables are classified according to their tensorial behaviour on the internal space and gauge-invariant variables are introduced. Furthermore, for each type of perturbations, decoupled master equations are found for scalar master functions on the two-dimensional sector of the BH background spacetime. Using these techniques, it has been established [417] that the Schwarzschild solution is mode-stable against linearized gravitational perturbations for all dimensions $D > 4$. More specifically, it was shown that the master equation for each tensorial type of perturbations does not admit normalizable negative modes which would describe unstable solutions. It was also shown that there exists no static perturbation which is regular everywhere outside the event horizon and well behaved at spatial infinity, which is a check, within the perturbation framework, of the uniqueness of the higher dimensional spherically symmetric, static, vacuum BH.

The situation is different for the rotating case, where we have different families of solutions. So far, studies of linearized perturbations of higher dimensional rotating BHs have exploited isometries of BH spacetimes, e.g. enhancement of symmetry of the Myers–Perry family of solutions [132] by choosing some of the intrinsic angular momenta to coincide. There has been some effort to extend the 4D Teukolsky formalism to the case of the Myers–Perry family of solutions. To that end, in order to identify similar perturbative variables as in the Teukolsky case, the use of the Petrov classification for higher dimensional spacetimes [165] has been made. And in order to look for decoupled equations, generalizations of the Geroch–Held–Penrose formalism [418] (a generalization of the Newman–Penrose formalism [419] for cases with algebraically special spacetimes) have been proposed [420]. Unfortunately, decoupling does not occur in higher dimensional BH spacetimes (see [84, 174, 421]), except for near-horizon geometries (see also [422]) and some special cases. For instance, in $D = 5$ dimensions a master equation for a part of the metric perturbations that are relevant for the study of stability has been derived [423]. For a detailed discussion of the stability properties of BHs in $D > 4$, we refer to section 3.

Apart from the study of BH-like dark objects, other types of dark objects have been investigated. In particular, this has led to the discovery of the well-known Gregory–Laflamme instability of black strings and p-branes [99, 100]; see [424] for a review.

We also mention that there are a number of studies of quasi–normal modes of black objects in 4D and in higher dimensions (for reviews see [425, 409, 426]), and that gravitational radiation in *D*-dimensional spacetimes has been studied in [427].

7.3. Conclusions and prospects for the future

While relativistic perturbation theory has been developed for many years and even though it has achieved great success in many areas of astrophysics, cosmology and fundamental physics, there remains a lot of work to be done, even in the realm of 4D GR. Among the main challenges are the development of tools for higher order perturbation theory, and especially for the treatment of backreaction, which is important for extreme mass-ratio inspirals and the particular case of cosmology where it has been invoked to try to explain the acceleration of the Universe. It would also be important to extend approximation methods currently applied to GR, to the study of alternative theories of gravity; see section 6.

In the case of higher dimensional spacetimes, there has been significant progress in the case of static non-rotating BHs; unfortunately, such progress has not yet been extended to rotating BHs or other black objects, where there is a pressing need for new tools. The fact that there are no decoupled master equations that describe all the gravitational degrees of freedom is a challenge for future work. Given that significant progress has also been made in nonlinear numerical studies, it is important for the near future to develop tools in perturbation theory to complement the numerical computations and aid in their physical interpretation. In this sense, it would be interesting to develop more sophisticated perturbative schemes adapted to the different problems. For instance, the close limit approximation could be developed for higher dimensional BHs in spacetimes with different asymptotic properties, which may be of interest for physical applications like ultrarelativistic collisions of BHs and the AdS/CFT conjecture in relation with QGPs. Also, in this line of research, point particle calculations, which model BH collisions when one of them is much smaller than the other, give remarkably accurate results when extrapolated to the equal-mass case [200, 201, 109]. Because this kind of computation relies on linear perturbation theory, the extension of classical results to different asymptotics is clearly desirable.

References

[1] Pretorius F 2005 *Phys. Rev. Lett.* **95** 121101 (arXiv:gr-qc/0507014)
[2] Campanelli M, Lousto C O, Marronetti P and Zlochower Y 2006 *Phys. Rev. Lett.* **96** 111101 (arXiv:gr-qc/0511048)
[3] Baker J G, Centrella J, Choi D-I, Koppitz M and van Meter J 2006 *Phys. Rev. Lett.* **96** 111102 (arXiv:gr-qc/0511103)
[4] Arkani-Hamed N, Dimopoulos S and Dvali G R 1998 *Phys. Lett.* B **429** 263 (arXiv:hep-ph/9803315)
[5] Antoniadis I, Arkani-Hamed N, Dimopoulos S and Dvali G R 1998 *Phys. Lett.* B **436** 257 (arXiv:hep-ph/9804398)
[6] Randall L and Sundrum R 1999 *Phys. Rev. Lett.* **83** 3370 (arXiv:hep-ph/9905221)
[7] Randall L and Sundrum R 1999 *Phys. Rev. Lett.* **83** 4690 (arXiv:hep-th/9906064)
[8] Giddings S B, Kachru S and Polchinski J 2002 *Phys. Rev.* D **66** 106006 (arXiv:hep-th/0105097)
[9] Giddings S B 2007 *AIP Conf. Proc.* **957** 69 (arXiv:0709.1107)
[10] Maldacena J M 1998 *Adv. Theor. Math. Phys.* **2** 231 (arXiv:hep-th/9711200)
[11] Polchinski J 2010 arXiv:1010.6134
[12] McGreevy J 2010 *Adv. High Energy Phys.* **2010** 723105 (arXiv:0909.0518)
[13] Emparan R and Reall H S 2008 *Living Rev. Rel.* **11** 6 (arXiv:0801.3471)
[14] Emparan R, Harmark T, Niarchos V and Obers N A 2010 *J. High Energy Phys.* JHEP04(2010)046 (arXiv:0912.2352)
[15] Emparan R and Figueras P 2010 *J. High Energy Phys.* JHEP11(2010)022 (arXiv:1008.3243)
[16] Horowitz G T and Wiseman T 2011 arXiv:1107.5563
[17] Lehner L and Pretorius F 2011 arXiv:1106.5184
[18] Aylott B *et al* 2009 *Class. Quantum Grav.* **26** 114008 (arXiv:0905.4227)
[19] Reula O A 1998 *Living Rev. Rel.* **1** 3
[20] Lehner L 2001 *Class. Quantum Grav.* **18** R25 (arXiv:gr-qc/0106072)
[21] Berger B K 2002 *Living Rev. Rel.* **5** 1 (arXiv:gr-qc/0201056)
[22] Pretorius F 2009 *Binary Black Hole Coalescence Physics of Relativistic Objects in Compact Binaries: From Birth to Coalescence* ed M Colpi *et al* (New York: Springer) (arXiv:0710.1338 [gr-qc])
[23] Shibata M and Taniguchi K 2011 *Living Rev. Rel.* **14** 6
[24] Sperhake U, Berti E and Cardoso V 2011 arXiv:1107.2819 [gr-qc]
[25] Yoshino H and Shibata M 2011 *Prog. Theor. Phys. Suppl.* **190** 282

[26] Yoshino H and Shibata M 2011 *Prog. Theor. Phys. Suppl.* **189** 269

[27] Alcubierre M 2008 *Introduction to 3+1 Numerical Relativity* (Oxford: Oxford University Press)

[28] Bona C, Palenzuela-Luque C and Bona-Casas C 2009 *Elements of Numerical Relativity and Relativistic Hydrodynamics: From Einstein's Equations to Astrophysical Simulations (Lecture Notes in Physics)* (New York: Springer)

[29] Baumgarte T W and Shapiro S L 2010 *Numerical Relativity: Solving Einstein's Equations on the Computer* (Cambridge: Cambridge University Press)

[30] Motohashi H and Suyama T 2011 arXiv:1110.6241

[31] Garfinkle D, Pretorius F and Yunes N 2010 *Phys. Rev. D* **82** 041501 (arXiv:1007.2429)

[32] Cardoso V and Gualtieri L 2009 *Phys. Rev. D* **80** 064008 (arXiv:0907.5008)

[33] Will C M 2005 *Ann. Phys.* **15** 19 (arXiv:gr-qc/0504086)

[34] Will C M 1989 *9th Moriond Workshop: Tests of Fundamental Laws (Particle Physics, Astrophysics, Atomic Physics) (Les Arcs, France, 21–28 Jan 1989)*

[35] Giddings S B and Thomas S D 2002 *Phys. Rev. D* **65** 056010 (arXiv:hep-ph/0106219)

[36] Dimopoulos S and Landsberg G L 2001 *Phys. Rev. Lett.* **87** 161602 (arXiv:hep-ph/0106295)

[37] Cavaglia M 2003 *Int. J. Mod. Phys.* A **18** 1843 (arXiv:hep-ph/0210296)

[38] Kanti P 2004 *Int. J. Mod. Phys.* A **19** 4899 (arXiv:hep-ph/0402168)

[39] Landsberg G L 2006 *J. Phys. G: Nucl. Part. Phys.* G **32** R337 (arXiv:hep-ph/0607297)

[40] Guica M, Hartman T, Song W and Strominger A 2009 *Phys. Rev. D* **80** 124008 (arXiv:0809.4266)

[41] Holzegel G and Smulevici J 2011 arXiv:1103.3672

[42] Bizon P and Rostworowski A 2011 *Phys. Rev. Lett.* **107** 031102 (arXiv:1104.3702)

[43] Dias O J, Horowitz G T and Santos J E 2011 arXiv:1109.1825

[44] Christodoulou D and Klainerman S 1993 *The Global Nonlinear Stability of the Minkowski Space* (Princeton, NJ: Princeton University Press)

[45] Lindblad H and Rodnianski I 2004 arXiv:math/0411109

[46] Gubser S S 2009 *Nucl. Phys.* A **830** 657C (arXiv:0907.4808)

[47] Schafer T and Teaney D 2009 *Rep. Prog. Phys.* **72** 126001 (arXiv:0904.3107)

[48] Janik R A 2011 *Lect. Notes Phys.* **828** 147 (arXiv:1003.3291)

[49] Horowitz G T 2010 arXiv:1002.1722

[50] Bernamonti A and Peschanski R 2011 *Nucl. Phys.* B **216** 94 (arXiv:1102.0725)

[51] Hartnoll S A 2011 arXiv:1106.4324

[52] Casalderrey-Solana J, Liu H, Mateos D, Rajagopal K and Wiedemann U A 2011 arXiv:1101.0618

[53] Mateos D 2007 *Class. Quantum Grav.* **24** S713 (arXiv:0709.1523)

[54] Hartnoll S A 2009 arXiv:0903.3246

[55] Hartnoll S A, Herzog C P and Horowitz G T 2008 *Phys. Rev. Lett.* **101** 031601 (arXiv:0803.3295)

[56] Eardley D M and Giddings S B 2002 *Phys. Rev. D* **66** 044011 (arXiv:gr-qc/0201034)

[57] Aichelburg P and Sexl R 1971 *Gen. Rel. Grav.* **2** 303

[58] Shibata M and Yoshino H 2010 *Phys. Rev. D* **81** 021501 (arXiv:0912.3606)

[59] Lehner L and Pretorius F 2010 *Phys. Rev. Lett.* **105** 101102 (arXiv:1006.5960)

[60] Shibata M and Yoshino H 2010 *Phys. Rev. D* **81** 104035 (arXiv:1004.4970)

[61] Zilhao M *et al* 2011 *J. Phys.: Conf. Ser.* **314** 012103

[62] Witek H *et al* 2011 *J. Phys.: Conf. Ser.* **314** 012104

[63] Witek H *et al* 2010 *Phys. Rev. D* **82** 104014 (arXiv:1006.3081)

[64] Witek H *et al* 2011 *Phys. Rev. D* **83** 044017 (arXiv:1011.0742)

[65] Okawa H, Nakao K-i and Shibata M 2011 *Phys. Rev. D* **83** 121501 (arXiv:1105.3331)

[66] Cactus www.cactuscode.org/ 2001

[67] Pamr http://laplace.phas.ubc.ca/Doc/pamr/ 2003

[68] HAD www.had.liu.edu/ 2003

[69] Chesler P M and Yaffe L G 2010 *Phys. Rev. D* **82** 026006 (arXiv:0906.4426)

[70] Chesler P M and Yaffe L G 2011 *Phys. Rev. Lett.* **106** 021601 (arXiv:1011.3562)

[71] Bantilan H, Pretorius F and Gubser S S 2012 *Phys. Rev. D* **85** 084038 (arXiv:1201.2132)

[72] Zilhao M *et al* 2010 *Phys. Rev. D* **81** 084052 (arXiv:1001.2302)

[73] Kreiss H O, Reula O, Sarbach O and Winicour J 2007 *Class. Quantum Grav.* **24** 5973

[74] Reula O and Sarbach O 2011 *Int. J. Mod. Phys. D* **20** 767 (arXiv:1009.0589)

[75] Ishibashi A and Wald R M 2004 *Class. Quantum Grav.* **21** 2981 (arXiv:hep-th/0402184)

[76] Fefferman C and Graham C R 1985 *Conformal invariants Sémin. Lyon 1984: Élie Cartan et les Mathématiques d'aujourd'hui, Lyon 1984: the Mathematical Heritage of Élie Cartan (Astérisque, Numéro Hors Sér.)* pp 95–116

[77] Henneaux M and Teitelboim C 1985 *Commun. Math. Phys.* **98** 391
[78] Pretorius F and Lehner L 2004 *J. Comput. Phys.* **198** 10 (arXiv:gr-qc/0302003)
[79] Choptuik M W and Pretorius F 2010 *Phys. Rev. Lett.* **104** 111101 (arXiv:0908.1780)
[80] Palenzuela C, Olabarrieta I, Lehner L and Liebling S L 2007 *Phys. Rev.* D **75** 064005 (arXiv:gr-qc/0612067)
[81] Neilsen D *et al* 2011 *Proc. Natl Acad. Sci.* **108** 12641 (arXiv:1012.5661)
[82] Zilhao M, Cardoso V, Herdeiro C, Lehner L and Sperhake U 2012 arXiv:1205.1063 [gr-qc]
[83] Cardoso V and Dias O J C 2004 *Phys. Rev.* D **70** 084011 (arXiv:hep-th/0405006)
[84] Kunduri H K, Lucietti J and Reall H S 2006 *Phys. Rev.* D **74** 084021 (arXiv:hep-th/0606076)
[85] Witek H *et al* 2010 *Phys. Rev.* D **82** 104037 (arXiv:1004.4633)
[86] Galloway G J 2011 arXiv:1111.5356
[87] Emparan R and Myers R C 2003 *J. High Energy Phys.* JHEP09(2003)025 (arXiv:hep-th/0308056)
[88] Dias O J, Figueras P, Monteiro R, Reall H S and Santos J E 2010 *J. High Energy Phys.* JHEP05(2010)076 (arXiv:1001.4527)
[89] Dias O J C, Figueras P, Monteiro R, Santos J E and Emparan R 2009 *Phys. Rev.* D **80** 111701 (arXiv:0907.2248)
[90] Gubser S S and Mitra I 2000 arXiv:hep-th/0009126
[91] Figueras P, Murata K and Reall H S 2011 *Class. Quantum Grav.* **28** 225030 (arXiv:1107.5785)
[92] Kol B and Smolkin M 2008 *Class. Quantum Grav.* **25** 145011 (arXiv:0712.4116)
[93] Emparan R, Harmark T, Niarchos V and Obers N A 2010 *J. High Energy Phys.* JHEP03(2010)063 (arXiv:0910.1601)
[94] Brandt S and Brügmann B 1997 *Phys. Rev. Lett.* **78** 3606 (arXiv:gr-qc/9703066)
[95] Nakao K-i, Nakamura T, Oohara K-i and Maeda K-i 1991 *Phys. Rev.* D **43** 1788
[96] Nakao K-i, Yamamoto K and Maeda K-i 1993 *Phys. Rev.* D **47** 3203
[97] Zilhao M *et al* 2012 (arXiv:1204.2019 [gr-qc])
[98] McVittie G 1933 *Mon. Not. R. Astron. Soc.* **93** 325
[99] Gregory R and Laflamme R 1993 *Phys. Rev. Lett.* **70** 2837 (arXiv:hep-th/9301052)
[100] Gregory R and Laflamme R 1994 *Nucl. Phys.* B **428** 399 (arXiv:hep-th/9404071)
[101] Wiseman T 2011 arXiv:1107.5513
[102] Yoshino H M S and Shibata M 2011 *Prog. Theor. Phys. Suppl.* **189** 269
[103] Sarbach O and Lehner L 2004 *Phys. Rev.* D **69** 021901 (arXiv:hep-th/0308116)
[104] Sarbach O and Lehner L 2005 *Phys. Rev.* D **71** 026002 (arXiv:hep-th/0407265)
[105] Figueras P, Lucietti J and Wiseman T 2011 *Class. Quantum Grav.* **28** 215018 (arXiv:1104.4489)
[106] Figueras P and Wiseman T 2011 arXiv:1105.2558
[107] Tanaka T 2003 *Prog. Theor. Phys. Suppl.* **148** 307 (arXiv:gr-qc/0203082)
[108] Emparan R, Fabbri A and Kaloper N 2002 *J. High Energy Phys.* JHEP08(2002)043 (arXiv:hep-th/0206155)
[109] Berti E, Cardoso V and Kipapa B 2011 *Phys. Rev.* D **83** 084018 (arXiv:1010.3874)
[110] Dias O J, Horowitz G T and Santos J E 2011 *J. High Energy Phys.* JHEP07(2011)115 (arXiv:1105.4167)
[111] Stotyn S, Park M, McGrath P and Mann R B 2011 arXiv:1110.2223
[112] Basu P *et al* 2010 *J. High Energy Phys.* JHEP10(2010)045 (arXiv:1003.3232)
[113] Gentle S A, Rangamani M and Withers B 2011 arXiv:1112.3979
[114] Dias O J *et al* 2011 arXiv:1112.4447
[115] Pullin J 1999 *Prog. Theor. Phys. Suppl.* **136** 107 (arXiv:gr-qc/9909021)
[116] Campanelli M, Kelly B J and Lousto C O 2006 *Phys. Rev.* D **73** 064005 (arXiv:gr-qc/0510122) (Updated version of original Lazarus paper (arXiv:gr-qc/0104063)
[117] Le Tiec A and Blanchet L 2010 *Class. Quantum Grav.* **27** 045008 (arXiv:0910.4593)
[118] Jaranowski P *et al* 2008 *Class. Quantum Grav.* **25** 114020 (arXiv:0710.5658)
[119] Pan Y *et al* 2010 *Phys. Rev.* D **81** 084041 (arXiv:0912.3466)
[120] Santamaria L *et al* 2010 *Phys. Rev.* D **82** 064016 (arXiv:1005.3306)
[121] Sperhake U, Brügmann B, Müller D and Sopuerta C F 2011 *Class. Quantum Grav.* **28** 134004 (arXiv:1012.3173 [gr-qc])
[122] Yoshino H, Shiromizu T and Shibata M 2005 *Phys. Rev.* D **72** 084020 (arXiv:gr-qc/0508063)
[123] Bishop N T, Isaacson R, Maharaj M and Winicour J 1998 *Phys. Rev.* D **57** 6113 (arXiv:gr-qc/9711076)
[124] Marronetti P *et al* 2000 *Phys. Rev.* D **62** 024017 (arXiv:gr-qc/0001077)
[125] Sarbach O, Tiglio M and Pullin J 2002 *Phys. Rev.* D **65** 064026 (arXiv:gr-qc/0110085)
[126] Johnson-McDaniel N K, Yunes N, Tichy W and Owen B J 2009 *Phys. Rev.* D **80** 124039 (arXiv:0907.0891)
[127] Martelli D and Sparks J 2005 *Phys. Lett.* B **621** 208 (arXiv:hep-th/0505027)
[128] Elvang H and Figueras P 2007 *J. High Energy Phys.* JHEP05(2007)050 (arXiv:hep-th/0701035)
[129] Hollands S and Ishibashi A 2005 *J. Math. Phys.* **46** 022503 (arXiv:gr-qc/0304054)
[130] Tanabe K, Kinoshita S and Shiromizu T 2011 *Phys. Rev.* D **84** 044055 (arXiv:1104.0303)

[131] Tanabe K, Tanahashi N and Shiromizu T 2011 *J. Math. Phys.* **52** 032501 (arXiv:1010.1664)
[132] Myers R C and Perry M 1986 *Ann. Phys.* **172** 304
[133] Emparan R and Reall H S 2002 *Phys. Rev. Lett.* **88** 101101 (arXiv:hep-th/0110260)
[134] Pomeransky A and Sen'kov R 2006 arXiv:hep-th/0612005
[135] Iguchi H and Mishima T 2007 *Phys. Rev.* D **75** 064018 (arXiv:hep-th/0701043)
[136] Hawking S, Hunter C and Taylor M 1999 *Phys. Rev.* D **59** 064005 (arXiv:hep-th/9811056)
[137] Gibbons G, Lu H, Page D N and Pope C 2005 *J. Geom. Phys.* **53** 49 (arXiv:hep-th/0404008)
[138] Gibbons G W, Ida D and Shiromizu T 2002 *Phys. Rev. Lett.* **89** 041101 (arXiv:hep-th/0206049)
[139] Galloway G J and Schoen R 2006 *Commun. Math. Phys.* **266** 571 (arXiv:gr-qc/0509107)
[140] Hollands S, Ishibashi A and Wald R M 2007 *Commun. Math. Phys.* **271** 699 (arXiv:gr-qc/0605106)
[141] Hollands S, Holland J and Ishibashi A 2011 *Ann. Henri Poincare* **12** 279 (arXiv:1002.0490)
[142] Hollands S and Yazadjiev S 2008 *Commun. Math. Phys.* **283** 749 (arXiv:0707.2775)
[143] Morisawa Y and Ida D 2004 *Phys. Rev.* D **69** 124005 (arXiv:gr-qc/0401100)
[144] Emparan R, Harmark T, Niarchos V and Obers N A 2009 *Phys. Rev. Lett.* **102** 191301 (arXiv:0902.0427)
[145] Emparan R, Harmark T, Niarchos V, Obers N A and Rodriguez M J 2007 *J. High Energy Phys.* JHEP10(2007)110 (arXiv:0708.2181)
[146] Emparan R and Haddad N 2011 arXiv:1109.1983
[147] Dias O J C, Figueras P, Monteiro R and Santos J E 2010 *Phys. Rev.* D **82** 104025 (arXiv:1006.1904)
[148] Dias O J, Monteiro R and Santos J E 2011 *J. High Energy Phys.* JHEP08(2011)139 (arXiv:1106.4554)
[149] Dias O J, Figueras P, Monteiro R and Santos J E 2010 *J. High Energy Phys.* JHEP12(2010)067 (arXiv:1011.0996)
[150] Arcioni G and Lozano-Tellechea E 2005 *Phys. Rev.* D **72** 104021 (arXiv:hep-th/0412118)
[151] Elvang H, Emparan R and Virmani A 2006 *J. High Energy Phys.* JHEP12(2006)074 (arXiv:hep-th/0608076)
[152] Gibbons G W, Ida D and Shiromizu T 2003 *Prog. Theor. Phys. Suppl.* **148** 284 (arXiv:gr-qc/0203004)
[153] Headrick M, Kitchen S and Wiseman T 2010 *Class. Quantum Grav.* **27** 035002 (arXiv:0905.1822)
[154] Gregory R and Laflamme R 1988 *Phys. Rev.* D **37** 305
[155] Gubser S S 2002 *Class. Quantum Grav.* **19** 4825 (arXiv:hep-th/0110193)
[156] Wiseman T 2003 *Class. Quantum Grav.* **20** 1137 (arXiv:hep-th/0209051)
[157] Kol B 2005 *J. High Energy Phys.* JHEP10(2005)049 (arXiv:hep-th/0206220)
[158] Myers R C 1987 *Phys. Rev.* D **35** 455
[159] Harmark T 2004 *Phys. Rev.* D **69** 104015 (arXiv:hep-th/0310259)
[160] Gorbonos D and Kol B 2004 *J. High Energy Phys.* JHEP06(2004)053 (arXiv:hep-th/0406002)
[161] Sorkin E 2006 *Phys. Rev.* D **74** 104027 (arXiv:gr-qc/0608115)
[162] Stephani H, Kramer D, MacCallum M, Hoenselaers C and Herlt E 2003 *Exact Solutions of Einstein's Field Equations* (Cambridge: Cambridge University Press)
[163] Kinnersley W 1969 *J. Math. Phys.* **10** 1195
[164] Reall H S 2011 arXiv:1105.4057
[165] Coley A, Milson R, Pravda V and Pravdova A 2004 *Class. Quantum Grav.* **21** L35 (arXiv:gr-qc/0401008)
[166] Godazgar M and Reall H S 2009 *Class. Quantum Grav.* **26** 165009 (arXiv:0904.4368)
[167] Godazgar M 2010 *Class. Quantum Grav.* **27** 245013 (arXiv:1008.2955)
[168] Houri T, Oota T and Yasui Y 2007 *Phys. Lett.* B **656** 214 (arXiv:0708.1368)
[169] Krtous P, Frolov V P and Kubiznak D 2008 *Phys. Rev.* D **78** 064022 (arXiv:0804.4705)
[170] Kunduri H K, Lucietti J and Reall H S 2007 *Class. Quantum Grav.* **24** 4169 (arXiv:0705.4214)
[171] Kunduri H K and Lucietti J 2009 *J. Math. Phys.* **50** 082502 (arXiv:0806.2051)
[172] Hollands S and Ishibashi A 2010 *Ann. Henri Poincaré* **10** 1537 (arXiv:0909.3462)
[173] Reall H S 2003 *Phys. Rev.* D **68** 024024 (arXiv:hep-th/0211290)
[174] Durkee M and Reall H S 2011 *Class. Quantum Grav.* **28** 035011 (arXiv:1009.0015)
[175] Hubeny V E, Marolf D and Rangamani M 2010 *Class. Quantum Grav.* **27** 095015 (arXiv:0908.2270)
[176] Hubeny V E and Rangamani M 2002 *J. High Energy Phys.* JHEP05(2002)027 (arXiv:hep-th/0202189)
[177] Camps J, Emparan R and Haddad N 2010 *J. High Energy Phys.* JHEP05(2010)042 (arXiv:1003.3636)
[178] 't Hooft G 1987 *Phys. Lett.* B **198** 61
[179] Giddings S B, Schmidt-Sommerfeld M and Andersen J R 2010 *Phys. Rev.* D **82** 104022 (arXiv:1005.5408)
[180] Banks T and Fischler W 1999 arXiv:hep-th/9906038
[181] Giddings S B 2011 arXiv:1105.2036
[182] Penrose R 1969 unpublished
[183] Yoshino H and Nambu Y 2003 *Phys. Rev.* D **67** 024009 (arXiv:gr-qc/0209003)
[184] Giddings S B and Rychkov V S 2004 *Phys. Rev.* D **70** 104026 (arXiv:hep-th/0409131)

[185] Sperhake U, Cardoso V, Pretorius F, Berti E and Gonzalez J A 2008 *Phys. Rev. Lett.* **101** 161101 (arXiv:0806.1738)
[186] Shibata M, Okawa H and Yamamoto T 2008 *Phys. Rev.* D **78** 101501 (arXiv:0810.4735)
[187] Sperhake U *et al* 2009 *Phys. Rev. Lett.* **103** 131102 (arXiv:0907.1252)
[188] Park S C 2012 *Prog. Part. Nucl. Phys.* at press (arXiv:1203.4683) (invited review article)
[189] Aad G *et al* (ATLAS Collaboration) 2012 *Phys. Lett.* B **710** 49 (arXiv:1202.1408)
[190] Chatrchyan S *et al* (CMS Collaboration) 2012 arXiv:1202.1488
[191] Dai D-C, Starkman G D and Stojkovic D 2006 *Phys. Rev.* D **73** 104037 (arXiv:hep-ph/0605085)
[192] ATLAS Collaboration 2011 *ATLAS Conf. Notes* ATLAS-CONF-2011-147
[193] ATLAS Collaboration 2011 arXiv:1111.0080
[194] CMS Collaboration 2011 *Phys. Lett.* B **697** 434 (arXiv:1012.3375)
[195] D'Eath P and Payne P 1992 *Phys. Rev.* D **46** 658
[196] Yoshino H and Rychkov V S 2005 *Phys. Rev.* D **71** 104028 (arXiv:hep-th/0503171)
[197] D'Eath P and Payne P 1992 *Phys. Rev.* D **46** 675
[198] D'Eath P and Payne P 1992 *Phys. Rev.* D **46** 694
[199] Herdeiro C, Sampaio M O and Rebelo C 2011 *J. High Energy Phys.* JHEP07(2011)121 (arXiv:1105.2298)
[200] Cardoso V and Lemos J P 2002 *Phys. Lett.* B **538** 1 (arXiv:gr-qc/0202019)
[201] Berti E *et al* 2010 *Phys. Rev.* D **81** 104048 (arXiv:1003.0812)
[202] Constantinou Y, Gal'tsov D, Spirin P and Tomaras T N 2011 *J. High Energy Phys.* JHEP11(2011)118 (arXiv:1106.3509)
[203] Galtsov D V, Kofinas G, Spirin P and Tomaras T N 2010 *J. High Energy Phys.* JHEP05(2010)055 (arXiv:1003.2982)
[204] Gal'tsov D V, Kofinas G, Spirin P and Tomaras T N 2010 *Phys. Lett.* B **683** 331 (arXiv:0908.0675)
[205] Coelho F S, Herdeiro C and Sampaio M O 2012 *Phys. Rev. Lett.* **108** 181102 (arXiv:1203.5355)
[206] Yoshino H and Shibata M 2009 *Phys. Rev.* D **80** 084025 (arXiv:0907.2760)
[207] Zilhao M *et al* 2011 *Phys. Rev.* D **84** 084039 (arXiv:1109.2149)
[208] Ida D, Oda K-y and Park S C 2003 *Phys. Rev.* D **67** 064025 (arXiv:hep-th/0212108)
[209] Ida D, Oda K-y and Park S C 2005 arXiv:hep-ph/0501210
[210] Ida D, Oda K-y and Park S C 2005 *Phys. Rev.* D **71** 124039 (arXiv:hep-th/0503052)
[211] Ida D, Oda K-y and Park S C 2006 *Phys. Rev.* D **73** 124022 (arXiv:hep-th/0602188)
[212] Harris C and Kanti P 2006 *Phys. Lett.* B **633** 106 (arXiv:hep-th/0503010)
[213] Duffy G, Harris C, Kanti P and Winstanley E 2005 *J. High Energy Phys.* JHEP09(2005)049 (arXiv:hep-th/0507274)
[214] Casals M, Kanti P and Winstanley E 2006 *J. High Energy Phys.* JHEP02(2006)051 (arXiv:hep-th/0511163)
[215] Casals M, Dolan S, Kanti P and Winstanley E 2007 *J. High Energy Phys.* JHEP03(2007)019 (arXiv:hep-th/0608193)
[216] Casals M, Dolan S, Kanti P and Winstanley E 2008 *J. High Energy Phys.* JHEP06(2008)071 (arXiv:0801.4910)
[217] Sampaio M O 2010 *J. High Energy Phys.* JHEP02(2010)042 (arXiv:0911.0688)
[218] Sampaio M O 2009 *J. High Energy Phys.* JHEP10(2009)008 (arXiv:0907.5107)
[219] Rogatko M and Szyplowska A 2009 *Phys. Rev.* D **79** 104005 (arXiv:0904.4544)
[220] Kanti P and Pappas N 2010 *Phys. Rev.* D **82** 024039 (arXiv:1003.5125)
[221] Herdeiro C, Sampaio M O and Wang M 2012 *Phys. Rev.* D **85** 024005 (arXiv:1110.2485)
[222] Cardoso V, Cavaglia M and Gualtieri L 2006 *Phys. Rev. Lett.* **96** 071301 (arXiv:hep-th/0512002)
[223] Cornell A S, Naylor W and Sasaki M 2006 *J. High Energy Phys.* JHEP02(2006)012 (arXiv:hep-th/0510009)
[224] Kanti P, Kodama H, Konoplya R, Pappas N and Zhidenko A 2009 *Phys. Rev.* D **80** 084016 (arXiv:0906.3845)
[225] Kapner D J *et al* 2007 *Phys. Rev. Lett.* **98** 021101 (arXiv:hep-ph/0611184)
[226] Tu L-C, Guan S-G, Luo J, Shao C-G and Liu L-X 2007 *Phys. Rev. Lett.* **98** 201101
[227] Aaltonen T *et al* (CDF collaboration) 2008 *Phys. Rev. Lett.* **101** 181602 (arXiv:0807.3132)
[228] Aad G *et al* (ATLAS Collaboration) 2011 *Phys. Lett.* B **705** 294 (arXiv:1106.5327)
[229] Chatrchyan S *et al* (CMS Collaboration) 2011 *Phys. Rev. Lett.* **107** 201804 (arXiv:1106.4775)
[230] Adam W *et al* 2004 (LEP Exotica Working Group) http://lepexotica.web.cern.ch/LEPEXOTICA/notes/2004-03/ed_note_final.ps.gz
[231] Abazov V M *et al* (D0 Collaboration) 2009 *Phys. Rev. Lett.* **102** 051601 (arXiv:0809.2813)
[232] Abazov V M *et al* (D0 Collaboration) 2005 *Phys. Rev. Lett.* **95** 161602 (arXiv:hep-ex/0506063)
[233] Aad G *et al* (ATLAS Collaboration) 2011 arXiv:1112.2194
[234] Chatrchyan S *et al* (CMS Collaboration) 2011 arXiv:1112.0688
[235] Hannestad S and Raffelt G G 2002 *Phys. Rev. Lett.* **88** 071301 (arXiv:hep-ph/0110067)
[236] Hannestad S 2001 *Phys. Rev.* D **64** 023515 (arXiv:hep-ph/0102290)

[237] Shuryak E V 2005 *Nucl. Phys.* A **750** 64 (arXiv:hep-ph/0405066)

[238] Heinz U W 2005 *AIP Conf. Proc.* **739** 163 (arXiv:nucl-th/0407067)

[239] Mateos D 2011 *J. Phys. G: Nucl. Part. Phys.* G **38** 124030 (arXiv:1106.3295)

[240] Kovtun P, Son D T and Starinets A O 2005 *Phys. Rev. Lett.* **94** 111601 (arXiv:hep-th/0405231)

[241] Luzum M and Romatschke P 2008 *Phys. Rev.* C **78** 034915 (arXiv:0804.4015)

[242] Bhattacharyya S, Hubeny V E, Minwalla S and Rangamani M 2008 *J. High Energy Phys.* JHEP02(2008)045
       (arXiv:0712.2456)

[243] Baier R, Romatschke P, Son D T, Starinets A O and Stephanov M A 2008 *J. High Energy
       Phys.* JHEP04(2008)100 (arXiv:0712.2451)

[244] Chesler P M and Yaffe L G 2009 *Phys. Rev. Lett.* **102** 211601 (arXiv:0812.2053)

[245] Haro S de, Solodukhin S N and Skenderis K 2001 *Commun. Math. Phys.* **217** 595 (arXiv:hep-th/0002230)

[246] Caron-Huot S, Chesler P M and Teaney D 2011 *Phys. Rev.* D **84** 026012 (arXiv:1102.1073)

[247] Bianchi E and Rovelli C 2010 arXiv:1002.3966

[248] Bernal A, Barranco J, Alic D and Palenzuela C 2010 *Phys. Rev.* D **81** 044031 (arXiv:0908.2435)

[249] Bernal A, Barranco J, Alic D and Palenzuela C 2010 *AIP Conf. Proc.* **1241** 335

[250] Bertotti B, Iess L and Tortora P 2003 *Nature* **425** 374

[251] Taylor J and Weisberg J 1982 *Astrophys. J.* **253** 908

[252] Weisberg J, Nice D and Taylor J 2010 *Astrophys. J.* **722** 1030 (arXiv:1011.0718)

[253] Kramer M *et al* 2006 *Science* **314** 97 (arXiv:astro-ph/0609417)

[254] NASA Gravitational Wave Mission http://beyondeinstein.gsfc.nasa.gov/studies/gravitational-wave-mission.php

[255] ESA eLISA/NGO www.elisa-ngo.org/articles/elisa-esa

[256] LIGO and Virgo www.ligo.org; http://www.ego-gw.it

[257] KAGRA http://gwcenter.icrr.u-tokyo.ac.jp/en/

[258] IPTA Consortium www.ipta4gw.org/

[259] Einstein Telescope www.et-gw.eu/

[260] Kawamura S *et al* 2011 *Class. Quantum Grav.* **28** 094011

[261] Crowder J and Cornish N J 2005 *Phys. Rev.* D **72** 083005 (arXiv:gr-qc/0506015)

[262] IXO and NICER ixo.gsfc.nasa.gov www.nasa.gov/topics/universe/features/nicer-science.html

[263] Fujii Y and Maeda K-I 2003 *The Scalar-Tensor Theory of Gravitation* (Cambridge: Cambridge University
       Press)

[264] Bekenstein J D 2004 *Phys. Rev.* D **70** 083509 (arXiv:astro-ph/0403694)

[265] Goldhaber A and Nieto M 1974 *Phys. Rev.* D **9** 1119

[266] Dubovsky S 2004 *J. High Energy Phys.* JHEP10(2004)076 (arXiv:hep-th/0409124)

[267] Dvali G R, Gabadadze G and Porrati M 2000 *Phys. Lett.* B **485** 208 (arXiv:hep-th/0005016)

[268] Sotiriou T P and Faraoni V 2010 *Rev. Mod. Phys.* **82** 451 (arXiv:0805.1726)

[269] Zwiebach B 1985 *Phys. Lett.* B **156** 315

[270] Alexander S and Yunes N 2009 *Phys. Rep.* **480** 1 (arXiv:0907.2562)

[271] Horava P 2009 *Phys. Rev.* D **79** 084008 (arXiv:0901.3775)

[272] Lovelock D 1971 *J. Math. Phys.* **12** 498

[273] Zegers R 2005 *J. Math. Phys.* **46** 072502 (arXiv:gr-qc/0505016)

[274] Charmousis C 2009 *Lect. Notes Phys.* **769** 299 (arXiv:0805.0568)

[275] Mirshekari S, Yunes N and Will C M 2011 arXiv:1110.2720

[276] Psaltis D 2009 *J. Phys.: Conf. Ser.* **189** 012033 (arXiv:0907.2746)

[277] Yunes N and Stein L C 2011 *Phys. Rev.* D **83** 104002 (arXiv:1101.2921)

[278] Ryan F D 1995 *Phys. Rev.* D **52** 5707

[279] Vigeland S, Yunes N and Stein L 2011 *Phys. Rev.* D **83** 104027 (arXiv:1102.3706)

[280] Yunes N and Pretorius F 2009 *Phys. Rev.* D **80** 122003 (arXiv:0909.3328)

[281] Cornish N, Sampson L, Yunes N and Pretorius F 2011 *Phys. Rev.* D **84** 062003 (arXiv:1105.2088)

[282] Cutler C *et al* 1993 *Phys. Rev. Lett.* **70** 2984 (arXiv:astro-ph/9208005)

[283] Damour T and Esposito-Farese G 1992 *Class. Quantum Grav.* **9** 2093

[284] Cardoso V, Chakrabarti S, Pani P, Berti E and Gualtieri L 2011 *Phys. Rev. Lett.* **107** 241101 (arXiv:1109.6021)

[285] Lue A, Wang L-M and Kamionkowski M 1999 *Phys. Rev. Lett.* **83** 1506 (arXiv:astro-ph/9812088)

[286] Talmadge C, Berthias J, Hellings R and Standish E 1988 *Phys. Rev. Lett.* **61** 1159

[287] Will C M 2006 *Living Rev. Rel.* **7** 3 www.livingreviews.org/lrr-2006-3

[288] Yunes N and Pretorius F 2009 *Phys. Rev.* D **79** 084043 (arXiv:0902.4669)

[289] Everitt C *et al* 2011 *Phys. Rev. Lett.* **106** 221101 (arXiv:1105.3456)

[290] Arvanitaki A and Dubovsky S 2011 *Phys. Rev.* D **83** 044026 (arXiv:1004.3558)

[291] Will C M 1998 *Phys. Rev.* D **57** 2061 (arXiv:gr-qc/9709011)

[292] Stavridis A and Will C M 2009 *Phys. Rev.* D **80** 044002 (arXiv:0906.3602)
[293] Berti E, Buonanno A and Will C M 2005 *Phys. Rev.* D **71** 084025 (arXiv:gr-qc/0411129)
[294] Sopuerta C F and Yunes N 2009 *Phys. Rev.* D **80** 064006 (arXiv:0904.4501)
[295] Sopuerta C F 2010 *Grav. Wave Notes* **4** 3 (arXiv:1009.1402)
[296] Yunes N, Pani P and Cardoso V 2011 arXiv:1112.3351
[297] Berti E, Cardoso V and Will C M 2006 *Phys. Rev.* D **73** 064030 (arXiv:gr-qc/0512160)
[298] Koyama K and Maartens R 2006 *J. Cosmol. Astropart. Phys.* JCAP01(2006)016 (arXiv:astro-ph/0511634)
[299] Kunz M and Sapone D 2007 *Phys. Rev. Lett.* **98** 121301 (arXiv:astro-ph/0612452)
[300] Carroll S M, Sawicki I, Silvestri A and Trodden M 2006 *New J. Phys.* **8** 323 (arXiv:astro-ph/0607458)
[301] Wang S, Hui L, May M and Haiman Z 2007 *Phys. Rev.* D **76** 063503 (arXiv:0705.0165)
[302] Koyama K, Taruya A and Hiramatsu T 2009 *Phys. Rev.* D **79** 123512 (arXiv:0902.0618)
[303] Skordis C 2008 *Phys. Rev.* D **77** 123502 (arXiv:0801.1985)
[304] Clifton T, Ferreira P G, Padilla A and Skordis C 2011 arXiv:1106.2476
[305] Sotani H and Kokkotas K D 2004 *Phys. Rev.* D **70** 084026 (arXiv:gr-qc/0409066)
[306] Sotani H and Kokkotas K D 2005 *Phys. Rev.* D **71** 124038 (arXiv:gr-qc/0506060)
[307] Pani P, Cardoso V and Delsate T 2011 *Phys. Rev. Lett.* **107** 031101 (arXiv:1106.3569)
[308] Campanelli M and Lousto C 1993 *Int. J. Mod. Phys.* D **2** 451 (arXiv:gr-qc/9301013)
[309] Bambi C and Barausse E 2011 *Phys. Rev.* D **84** 084034 (arXiv:1108.4740)
[310] Novak J 1998 *Phys. Rev.* D **57** 4789
[311] Novak J 1998 *Phys. Rev.* D **58** 064019
[312] Novak J and Ibáñez J M 2000 *Astrophys. J.* **533** 392
[313] Contaldi C R, Wiseman T and Withers B 2008 *Phys. Rev.* D **78** 044034 (arXiv:0802.1215)
[314] Healy J *et al* 2011 arXiv:1112.3928
[315] Salgado M 2006 *Class. Quantum Grav.* **23** 4719
[316] Salgado M, Martínez-del Río D, Alcubierre M and Nuñez D 2008 *Phys. Rev.* D **77** 104010
[317] Emparan R, Garcia-Bellido J and Kaloper N 2003 *J. High Energy Phys.* JHEP01(2003)079 (arXiv:hep-th/0212132)
[318] Hawking S W and Ellis G F R 1973 *The Large scale Structure of Space-Time* (Cambridge: Cambridge University Press)
[319] Sago N, Nakano H and Sasaki M 2003 *Phys. Rev.* D **67** 104017 (arXiv:gr-qc/0208060)
[320] Barack L and Lousto C O 2005 *Phys. Rev.* D **72** 104026 (arXiv:gr-qc/0510019)
[321] Stewart J M and Walker M 1974 *Proc. R. Soc. Lond.* A **341** 49
[322] Bruni M, Matarrese S, Mollerach S and Sonego S 1997 *Class. Quantum Grav.* **14** 2585 (arXiv:gr-qc/9609040)
[323] Bruni M and Sonego S 1999 *Class. Quantum Grav.* **16** L29 (arXiv:gr-qc/9906017)
[324] Bruni M, Gualtieri L and Sopuerta C F 2003 *Class. Quantum Grav.* **20** 535 (arXiv:gr-qc/0207105)
[325] Sopuerta C F, Bruni M and Gualtieri L 2004 *Phys. Rev.* D **70** 064002 (arXiv:gr-qc/0306027)
[326] Brizuela D, Martin-Garcia J M and Marugan G A Mena 2006 *Phys. Rev.* D **74** 044039 (arXiv:gr-qc/0607025)
[327] Brizuela D, Martin-Garcia J M, Sperhake U and Kokkotas K D 2010 *Phys. Rev.* D **82** 104039 (arXiv:1009.5605)
[328] Misner C W, Thorne K and Wheeler J A 1973 *Gravitation* (San Francisco, CA: Freeman)
[329] Lifshitz E 1946 *J. Phys., Moscow* **10** 116
[330] Bardeen J M 1980 *Phys. Rev.* D **22** 1882
[331] Brandenberger R, Kahn R and Press W H 1983 *Phys. Rev.* D **28** 1809
[332] Kodama H and Sasaki M 1984 *Prog. Theor. Phys. Suppl.* **78** 1
[333] Ellis G F R and Bruni M 1989 *Phys. Rev.* D **40** 1804
[334] Mukhanov V, Feldman H and Brandenberger R 1992 *Phys. Rep.* **215** 203
[335] Bruni M, Dunsby P K S and Ellis G F R 1992 *Astrophys. J.* **395** 34–53
[336] Malik K A and Wands D 2009 *Phys. Rep.* **475** 1–51
[337] Sachs R K and Wolfe A M 1967 *Astrophys. J.* **147** 73
[338] Buchert T 2008 *Gen. Rel. Grav.* **40** 467 (arXiv:0707.2153)
[339] Ishibashi A and Wald R M 2006 *Class. Quantum Grav.* **23** 235 (arXiv:gr-qc/0509108)
[340] Kumar N and Flanagan E E 2008 *Phys. Rev.* D **78** 063537 (arXiv:0808.1043)
[341] Chandrasekhar S 1992 *The Mathematical Theory of Black Holes* (New York: Oxford University Press)
[342] Frolov V P and Novikov I D 1997 *Black Hole Physics: Basic Concepts and New Developments* (Dordrecht: Kluwer)
[343] Regge T and Wheeler J A 1957 *Phys. Rev.* **108** 1063
[344] Zerilli F J 1970 *Phys. Rev. Lett.* **24** 737
[345] Moncrief V 1974 *Ann. Phys., NY* **88** 323
[346] Teukolsky S A 1972 *Phys. Rev. Lett.* **29** 1114

[347] Teukolsky S A 1973 *Astrophys. J.* **185** 635
[348] Chrzanowski P L 1975 *Phys. Rev.* D **11** 2042
[349] Wald R M 1978 *Phys. Rev. Lett.* **41** 203
[350] Kegeles L S and Cohen J M 1979 *Phys. Rev.* D **19** 1641
[351] Lousto C O and Whiting B F 2002 *Phys. Rev.* D **66** 024026 (arXiv:gr-qc/0203061)
[352] Vishveshwara C V 1970 *Phys. Rev.* D **1** 2870
[353] Kay B S and Wald R M 1987 *Class. Quantum Grav.* **4** 893
[354] Whiting B F 1989 *J. Math. Phys.* **30** 1301
[355] Vishveshwara C 1970 *Nature* **227** 936
[356] Press W 1971 *Astrophys. J. Lett.* **170** L105
[357] Chandrasekhar S and Detweiler S 1975 *Proc. R. Soc. Lond.* A **344** 441
[358] Kokkotas K D and Schmidt B 1999 *Living Rev. Rel.* **2**
[359] Barack L 2001 *Phys. Rev.* D **64** 084021 (arXiv:gr-qc/0105040)
[360] Barack L and Sago N 2009 *Phys. Rev. Lett.* **102** 191101 (arXiv:0902.0573)
[361] Barack L and Sago N 2010 *Phys. Rev.* D **81** 084021 (arXiv:1002.2386)
[362] Akcay S 2011 *Phys. Rev.* D **83** 124026 (arXiv:1012.5860)
[363] Barack L, Damour T and Sago N 2010 *Phys. Rev.* D **82** 084036 (arXiv:1008.0935)
[364] Barack L and Sago N 2011 *Phys. Rev.* D **83** 084023 (arXiv:1101.3331)
[365] Le Tiec A *et al* 2011 *Phys. Rev. Lett.* **107** 141101 (arXiv:1106.3278)
[366] Shah A G, Keidl T S, Friedman J L, Kim D-H and Price L R 2011 *Phys. Rev.* D **83** 064018 (arXiv:1009.4876)
[367] Danzmann K and Rudiger A 2003 *Class. Quantum Grav.* **20** S1
[368] Poisson E 2004 *Living Rev. Rel.* **7** 6 (arXiv:gr-qc/0306052)
[369] Barack L 2009 *Class. Quantum Grav.* **26** 213001 (arXiv:0908.1664)
[370] Thornburg J 2011 *Grav. Wave Notes* **5** (arXiv:1102.3647)
[371] Damour T 1987 The problem of motion in Newtonian and Einsteinian gravity *Three Hundred Years of Gravitation* ed S W Hawking and W Israel (Cambridge: Cambridge University Press) pp 128–98
[372] Blanchet L 2002 *Living Rev. Rel.* **5** 3 (arXiv:gr-qc/0202016)
[373] Poisson E 2007 *Post-Newtonian Theory for the Common Reader (Lecture Notes)* www.physics.uoguelph.ca/poisson/research/postN.pdf
[374] Buonanno A and Damour T 1999 *Phys. Rev.* D **59** 084006 (arXiv:gr-qc/9811091)
[375] Buonanno A and Damour T 2000 *Phys. Rev.* D **62** 064015 (arXiv:gr-qc/0001013)
[376] Damour T and Nagar A 2009 arXiv:0906.1769
[377] Barausse E and Buonanno A 2010 *Phys. Rev.* D **81** 084024 (arXiv:0912.3517)
[378] Price R H and Pullin J 1994 *Phys. Rev. Lett.* **72** 3297 (arXiv:gr-qc/9402039)
[379] Anninos P, Hobill D, Seidel E, Smarr L and Suen W-M 1993 *Phys. Rev. Lett.* **71** 2851 (arXiv:gr-qc/9309016)
[380] Anninos P, Price R H, Pullin J, Seidel E and Suen W-M 1995 *Phys. Rev.* D **52** 4462 (arXiv:gr-qc/9505042)
[381] Gleiser R J, Nicasio C O, Price R H and Pullin J 1996 *Phys. Rev. Lett.* **77** 4483 (arXiv:gr-qc/9609022)
[382] Gleiser R J, Nicasio C O, Price R H and Pullin J 2000 *Phys. Rep.* **325** 41 (arXiv:gr-qc/9807077)
[383] Krivan W and Price R H 1999 *Phys. Rev. Lett.* **82** 1358 (arXiv:gr-qc/9810080)
[384] Nicasio C O, Gleiser R J, Price R H and Pullin J 1999 *Phys. Rev.* D **59** 044024 (arXiv:gr-qc/9802063)
[385] Khanna G *et al* 1999 *Phys. Rev. Lett.* **83** 3581 (arXiv:gr-qc/9905081)
[386] Khanna G, Gleiser R, Price R and Pullin J 2000 *New J. Phys.* **2** 3 (arXiv:gr-qc/0003003)
[387] Gleiser R J and Dominguez E A 2002 *Phys. Rev.* D **65** 064018 (arXiv:gr-qc/0109018)
[388] Khanna G 2001 *Phys. Rev.* D **63** 124007 (arXiv:gr-qc/0101015)
[389] Khanna G 2002 *Phys. Rev.* D **66** 064004 (arXiv:gr-qc/0206010)
[390] Campanelli M, Lousto C O, Zlochower Y and Merritt D 2007 *Astrophys. J.* **659** L5 (arXiv:gr-qc/0701164)
[391] Gonzalez J, Hannam M, Sperhake U, Bruegmann B and Husa S 2007 *Phys. Rev. Lett.* **98** 231101 (arXiv:gr-qc/0702052)
[392] Andrade Z and Price R H 1997 *Phys. Rev.* D **56** 6336 (arXiv:gr-qc/9611022)
[393] Sopuerta C F, Yunes N and Laguna P 2006 *Phys. Rev.* D **74** 124010
[394] Sopuerta C F, Yunes N and Laguna P 2007 *Astrophys. J.* **656** L9
[395] Le Tiec A, Blanchet L and Will C M 2010 *Class. Quantum Grav.* **27** 012001 (arXiv:0910.4594)
[396] Heller M P, Mateos D, van der Schee W and Trancanelli D 2012 *Phys. Rev. Lett.* **108** 191601 (arXiv:1202.0981)
[397] Cardoso V and Lemos J P S 2003 *Phys. Rev.* D **67** 084020 (arXiv:gr-qc/0301078)
[398] Horowitz G T and Hubeny V E 2000 *Phys. Rev.* D **62** 024027 (arXiv:hep-th/9909056)
[399] Cardoso V and Lemos J P S 2001 *Phys. Rev.* D **64** 084017 (arXiv:gr-qc/0105103)
[400] Cardoso V, Konoplya R and Lemos J P S 2003 *Phys. Rev.* D **68** 044024 (arXiv:gr-qc/0305037)
[401] Bakas I 2009 *J. High Energy Phys.* JHEP01(2009)003 (arXiv:0809.4852)

[402] Berti E and Kokkotas K D 2003 *Phys. Rev.* D **67** 064020 (arXiv:gr-qc/0301052)
[403] Giammatteo M and Moss I G 2005 *Class. Quantum Grav.* **22** 1803 (arXiv:gr-qc/0502046)
[404] Moura F and Schiappa R 2007 *Class. Quantum Grav.* **24** 361 (arXiv:hep-th/0605001)
[405] Kanti P, Mavromatos N E, Rizos J, Tamvakis K and Winstanley E 1998 *Phys. Rev.* D **57** 6255
      (arXiv:hep-th/9703192)
[406] Torii T, Yajima H and Maeda K-i 1997 *Phys. Rev.* D **55** 739 (arXiv:gr-qc/9606034)
[407] Pani P and Cardoso V 2009 *Phys. Rev.* D **79** 084031 (arXiv:0902.1569)
[408] Molina C, Pani P, Cardoso V and Gualtieri L 2010 *Phys. Rev.* D **81** 124021 (arXiv:1004.4007)
[409] Berti E, Cardoso V and Starinets A O 2009 *Class. Quantum Grav.* **26** 163001 (arXiv:0905.2975)
[410] Kodama H, Ishibashi A and Seto O 2000 *Phys. Rev.* D **62** 064022 (arXiv:hep-th/0004160)
[411] Maartens R and Koyama K 2010 *Living Rev. Rel.* **13** 5 (arXiv:1004.3962)
[412] Hollands S and Ishibashi A 2009 *Commun. Math. Phys.* **291** 403 (arXiv:0809.2659)
[413] Hawking S 1972 *Commun. Math. Phys.* **25** 152
[414] Kodama H and Ishibashi A 2003 *Prog. Theor. Phys.* **110** 701 (arXiv:hep-th/0305147)
[415] Kodama H and Ishibashi A 2004 *Prog. Theor. Phys.* **111** 29 (arXiv:hep-th/0308128)
[416] Ishibashi A and Kodama H 2011 *Prog. Theor. Phys. Suppl.* **189** 165
[417] Ishibashi A and Kodama H 2003 *Prog. Theor. Phys.* **110** 901 (arXiv:hep-th/0305185)
[418] Geroch R P, Held A and Penrose R 1973 *J. Math. Phys.* **14** 874
[419] Newman E and Penrose R 1962 *J. Math. Phys.* **3** 566
[420] Durkee M, Pravda V, Pravdova A and Reall H S 2010 *Class. Quantum Grav.* **27** 215010 (arXiv:1002.4826)
[421] Murata K 2011 *Prog. Theor. Phys. Suppl.* **189** 210
[422] Durkee M and Reall H S 2011 *Phys. Rev.* D **83** 104044 (arXiv:1012.4805)
[423] Murata K and Soda J 2008 *Class. Quantum Grav.* **25** 035006 (arXiv:0710.0221)
[424] Harmark T, Niarchos V and Obers N A 2007 *Class. Quantum Grav.* **24** R1 (arXiv:hep-th/0701022)
[425] Cardoso V, Lemos J P S and Yoshida S 2004 *Phys. Rev.* D **69** 044004 (arXiv:gr-qc/0309112)
[426] Konoplya R and Zhidenko A 2011 *Rev. Mod. Phys.* **83** 793 (arXiv:1102.4014)
[427] Cardoso V, Dias O J C and Lemos J P S 2003 *Phys. Rev.* D **67** 064026 (arXiv:hep-th/0212168)