Relativistic formalism for computation of irrotational binary stars in quasiequilibrium states

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We present relativistic hydrostatic equations for obtaining irrotational binary neutron stars in quasiequilibrium states in the $3+1$ formalism. The equations derived here are different from those previously given by Bonazzola, Gourgoulhon, and Marck, and have a simpler and more tractable form for computation in numerical relativity. We also present hydrostatic equations for computation of equilibrium irrotational binary stars in the first post-Newtonian order. [S0556-2821(98)00914-X]

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I. INTRODUCTION

Preparation of reliable theoretical models on the late inspiraling stage of binary neutron stars is one of the most important issues for gravitational wave astronomy. This is because they are one of promising sources for gravitational wave detectors such as the Laser Interferometric Gravitational Wave Observatory (LIGO) [1], VIRGO [2], GEO600 [3], and TAMA [4]. From their signals, we will get a wide variety of physical information on neutron stars such as their mass, spin, and so on if we have a theoretical template of them [5]. In particular, a signal from the very late inspiraling stage just prior to merging may contain physically important information on neutron stars, such as their radius [5], which will be utilized for determining the equation of state of neutron stars [6].

Binary neutron stars evolve due to the radiation reaction of gravitational waves, so that they never settle down to equilibrium states. However, the emission time scale will be always longer than the orbital period outside their innermost stable circular orbit (ISCO), so that we may consider that they are in quasiequilibrium states in their inspiraling phase even near the ISCO. Motivated by this idea, there have been several works in which the sequence of equilibrium states of binary neutron stars is computed and the sequence is regarded as an evolutionary track; for example, we have obtained corotational equilibrium states in the first post-Newtonian approximation [7]; Baumgarte *et al.* have obtained corotational equilibrium states in a relativistic framework using the conformal flat approximation [8]. Up to now, however, all relativistic works have been done assuming a corotational velocity field [9]. As pointed out previously [10], corotation is not an adequate assumption for the velocity field of realistic binary neutron stars, because the effect of viscosity is negligible for the evolution of neutron stars in a binary and, as a result, their velocity fields are expected to be irrotational (or nearly irrotational).

For computation of realistic quasiequilibrium states of coalescing binary neutron stars just prior to merging, Bonazzola, Gourgoulhon, and Marck (BGM) [11] recently presented a relativistic formalism. In their formulation, they assume a helicoidal Killing vector ℓ^μ, and then project relativistic hydrodynamic equations onto a hypersurface orthogonal to ℓ^μ. After that, they impose their irrotational condition on the hypersurface and derive hydrostatic equations for the irrotational fluid. We think, however, that there were several inadequate treatments in their work. The first one is their definition of the irrotational condition, because their irrotational condition is nothing but a necessary condition for irrotation even in the case when we assume the existence of ℓ^μ [12]. In the general case, their condition is not identical to the irrotational condition. Second, in numerical relativity, we usually solve equations such as the Hamiltonian constraint, momentum constraint, and equations for gauge conditions, using spatial coordinates on the hypersurface Σ_t, which is perpendicular to the unit normal n^ν. For this reason, they had to reproject their equations onto Σ_t. As a result, their equations for determining the velocity field have a complicated form. Finally, in their formalism, it is necessary to solve a complicated vector Poisson equation for relativistic cases, which should be unnecessary for an irrotational fluid. Although we may get correct results using their formalism, we had better obtain a simpler and more tractable formalism. The purpose in this paper is to present such a one.

In Sec. II, we derive hydrostatic equations for an irrotational fluid from relativistic hydrodynamic equations. We use a $3+1$ formalism and project the hydrodynamic equations onto Σ_t. Then, we impose an irrotational condition on Σ_t, which agrees with the relativistic irrotational condition [12]. As a result of the projection onto Σ_t, we obtain hydrostatic equations on Σ_t, and hence, they have suitable forms to be solved in numerical relativity. Also, in our formalism, we need to solve only one Poisson-type equation for a scalar field for the determination of the vector field. In Sec. III, taking the Newtonian limit, we show that well-known Newtonian hydrostatic equations are derived from the present formalism. In Sec. IV, we give first post-Newtonian hydrostatic equations for an irrotational fluid as well as gravitational potentials to be solved. Section V is devoted to a summary. Throughout this paper, c denotes the speed of light, and we use units in which the gravitational constant is unity. We use units $c=1$ in Sec. II for convenience and recover c in Secs. III and IV. Latin and Greek indices denote three-dimensional (3D) spatial components (1–3) and four dimensional (4D) components (0–3), respectively. As spatial coordinates, we use the Cartesian coordinates $x^k=(x^1,x^2,x^3)$.

II. RELATIVISTIC FLUID EQUATIONS IN THE 3+1 FORMALISM

Since we use the $3+1$ formalism in general relativity, we write the line element as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$
$$= (-\alpha^2 + \beta_k \beta^k) dt^2 + 2\beta_i dx^i dt + \gamma_{ij} dx^i dx^j, \quad (2.1)$$

where $g_{\mu\nu}$, α, $\beta_i = \gamma_{ij}\beta^j$, and γ_{ij} are the 4D metric, lapse function, shift vector, and 3D spatial metric, respectively. Using the unit normal to the 3D spatial hypersurface Σ_t,

$$n^\mu = \left(\frac{1}{\alpha}, -\frac{\beta^i}{\alpha}\right) \quad \text{and} \quad n_\mu = (-\alpha, 0,0,0), \quad (2.2)$$

γ_{ij} is written as

$$\gamma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu. \quad (2.3)$$

Hereafter, we use ∇_μ and D_i as the covariant derivatives with respect to $g_{\mu\nu}$ and γ_{ij}, respectively.

We assume the energy momentum tensor of the perfect fluid as

$$T^{\mu\nu} = \rho\left[1 + \varepsilon + \frac{P}{\rho}\right] u^\mu u^\nu + P g^{\mu\nu}, \quad (2.4)$$

where ρ, ε, P, and u^μ denote the rest mass density, specific internal energy, pressure, and four-velocity, respectively. We assume the polytropic equation of state $P = (\Gamma - 1)\rho\varepsilon$, where $\Gamma = 1 + 1/n$ and n is the polytropic index. From the adiabatic condition, we also get $P = K\rho^\Gamma$, where K is a constant. For the following, we define h as

$$h = 1 + \varepsilon + \frac{P}{\rho} = 1 + \frac{K\Gamma}{\Gamma - 1}\rho^{\Gamma - 1} = 1 + \int \frac{dP}{\rho}. \quad (2.5)$$

From the conservation equation for the energy momentum tensor,

$$\nabla_\mu T^\mu_\nu = 0, \quad (2.6)$$

we get the hydrodynamic equation as

$$u^\mu \nabla_\mu \tilde{u}_\nu + \nabla_\nu h = 0, \quad (2.7)$$

where $\tilde{u}_\nu = h u_\nu$, and we use the conservation equation for rest mass density as

$$\nabla_\mu(\rho u^\mu) = 0. \quad (2.8)$$

To rewrite the hydrodynamic equation, we decompose u^μ as

$$u^\mu = u^0(\ell^\mu + V^\mu), \quad (2.9)$$

and assume that (1) ℓ^μ is a timelike vector of its component $(1, \ell^i)$, and (2) V^μ is a spatial vector, $V^\mu n_\mu = 0$, i.e., $V^\mu = (0, V^i)$. By using ℓ^μ and V^μ, we get the following relations:

$$\gamma_i^\nu \ell^\mu \nabla_\mu \tilde{u}_\nu$$
$$= \gamma_i^\nu [\mathcal{L}_\ell \tilde{u}_\nu - \tilde{u}_\mu \nabla_\nu \ell^\mu] = \gamma_i^\nu \left[\mathcal{L}_\ell \tilde{u}_\nu - \tilde{u}_\mu \nabla_\nu \left(\frac{u^\mu}{u^0} - V^\mu\right)\right]$$
$$= \gamma_i^\nu \left[\mathcal{L}_\ell \tilde{u}_\nu + h \nabla_\nu \left(\frac{1}{u^0}\right) + \tilde{u}_\mu \nabla_\nu V^\mu\right]$$
$$= \gamma_i^\nu \mathcal{L}_\ell \tilde{u}_\nu + h D_i \left(\frac{1}{u^0}\right) + {}^{(3)}\tilde{u}_k D_i V^k + \gamma_i^\nu \eta_\sigma \tilde{u}^\sigma V^\mu \nabla_\nu \eta_\mu$$
$$(2.10)$$

and

$$\gamma_i^\nu V^\mu \nabla_\mu \tilde{u}_\nu = V^k D_k {}^{(3)}\tilde{u}_i - \gamma_i^\nu \eta_\sigma \tilde{u}^\sigma V^\mu \nabla_\mu \eta_\nu, \quad (2.11)$$

where $\mathcal{L}_\ell$ denotes the Lie derivative with respect to ℓ^μ and ${}^{(3)}\tilde{u}_i$ is a spatial vector defined as $\gamma_i^k \tilde{u}_k$. Using these relations, the projection of Eq. (2.7) onto the 3D hypersurface Σ_t becomes

$$u^0\left[\gamma_i^\nu \mathcal{L}_\ell \tilde{u}_\nu + V^k D_k {}^{(3)}\tilde{u}_i + {}^{(3)}\tilde{u}_k D_i V^k + h D_i\left(\frac{1}{u^0}\right)\right] + D_i h = 0. \quad (2.12)$$

We can rewrite this equation as

$$\gamma_i^\nu \mathcal{L}_\ell \tilde{u}_\nu + D_i\left(\frac{h}{u^0} + {}^{(3)}\tilde{u}_k V^k\right) + V^k(D_k {}^{(3)}\tilde{u}_i - D_i {}^{(3)}\tilde{u}_k) = 0. \quad (2.13)$$

Besides the conservation equation of the energy momentum tensor, we have the conservation equation for rest mass density (2.8). We note that for the case of a barotropic equation of state such as $P = K\rho^\Gamma$, Eq. (2.8) is also derived from the conservation equation of the energy momentum tensor. This implies that if we solve Eq. (2.8), we do not have to take into account the n^μ component of Eq. (2.7). Using Eq. (2.9), Eq. (2.8) is written as

$$\alpha[\mathcal{L}_\ell(\rho u^0) + \rho u^0 \nabla_\mu \ell^\mu] + D_i(\rho \alpha u^0 V^i) = 0. \quad (2.14)$$

Now, we assume that ℓ^μ is a Killing vector such that $\nabla_\mu \ell_\nu + \nabla_\nu \ell_\mu = 0$, $\mathcal{L}_\ell \tilde{u}_\nu = 0$ and $\mathcal{L}_\ell(\rho u^0) = 0$, and we write its component as $(1, -\Omega x^2, \Omega x^1, 0)$, where Ω is identified with the orbital angular velocity with respect to a distant inertial observer. We note that the fluid exists inside the light cylinder $|x^k| \ll c\Omega^{-1}$, and the existence of the Killing vector is assumed within it. We also note that ℓ^μ defined here is identical to the helicoidal Killing vector defined by BGM [11]. If the Killing vector exists, we can derive hydrostatic equations for the two interesting cases. One is the corotational case where we simply set $V^i = 0$. Then, we get a well-known result as [13]

$$\frac{h}{u^0} = \text{const}, \quad (2.15)$$

and the continuity equation is trivially satisfied in this case.

The other is the case where ${}^{(3)}\tilde{u}_i$ satisfies an ''irrotational condition'' defined as

$$W_{ij} \equiv D_i {}^{(3)}\tilde{u}_j - D_j {}^{(3)}\tilde{u}_i = 0, \tag{2.16}$$

and hence

$${}^{(3)}\tilde{u}_i = D_i \phi, \tag{2.17}$$

where ϕ is a scalar field. Then, the hydrodynamic equation (2.13) is integrated to give

$$\frac{h}{u^0} + {}^{(3)}\tilde{u}_k V^k = \text{const.} \tag{2.18}$$

Note that V^k and u^0 are written as

$$V^k = -\ell^k - \beta^k + \frac{1}{hu^0}\gamma^{kl} D_l \phi, \tag{2.19}$$

$$u^0 = \frac{1}{\alpha}[1 + h^{-2}\gamma^{kl} D_k \phi D_l \phi]^{1/2}, \tag{2.20}$$

so that we can rewrite the left-hand side of Eq. (2.18) as

$$\frac{h}{u^0} + {}^{(3)}\tilde{u}_k V^k = h\alpha^2 u^0 - (\ell^k + \beta^k){}^{(3)}\tilde{u}_k. \tag{2.21}$$

By substituting Eq. (2.19) into Eq. (2.14), we get a Poisson-type equation for determining ϕ as

$$D_i(\rho\alpha h^{-1} D^i \phi) - D_i\{\rho\alpha u^0(\ell^i + \beta^i)\} = 0. \tag{2.22}$$

Hence, the hydrodynamic equations which should be solved for the determination of equilibrium states reduce to only two hydrostatic equations (2.18) and (2.22). We do not have to solve any equations for vector potentials which were introduced in the formalism of BGM [11].

We note that the definition of irrotation in the 4D covariant form should be [12]

$$\begin{aligned}\omega_{\mu\nu} &= P^\mu_\sigma P^\nu_\lambda(\nabla_\mu u_\nu - \nabla_\nu u_\mu) \\ &= h^{-1}(\nabla_\mu \tilde{u}_\nu - \nabla_\nu \tilde{u}_\mu) = 0,\end{aligned} \tag{2.23}$$

where $P^\mu_\sigma = g^\mu_\sigma + u_\sigma u^\mu$, and we use Eq. (2.7) to rewrite the first line into the second line. When $\omega_{\mu\nu}$ is vanishing initially for a fluid element, it remains zero along the trajectory of the fluid element for the perfect fluid [12]. Hence, $\omega_{\mu\nu}=0$ is just the irrotational condition. In our present irrotational condition (2.16), Eq. (2.23) is satisfied on the 3D hypersurface Σ_t trivially. However, it is not trivial whether or not the projection of Eq. (2.23) to the $n^\mu \gamma^\nu_k$ component is satisfied. (Projection to the $n^\mu n^\nu$ component is trivially satisfied.) We show here that it is really guaranteed due to Eq. (2.16). By operating $n^\mu \gamma^\nu_k$ to $\nabla_\mu \tilde{u}_\nu - \nabla_\nu \tilde{u}_\mu$, we get

$$\begin{aligned}n^\mu \gamma^\nu_k(\nabla_\mu \tilde{u}_\nu - \nabla_\nu \tilde{u}_\mu) &= \gamma^\nu_k \mathcal{L}_n \tilde{u}_\nu - \gamma^\nu_k(\tilde{u}_\mu \nabla_\nu n^\mu + n^\mu \nabla_\nu \tilde{u}_\mu) \\ &= \gamma^\nu_k \mathcal{L}_n \tilde{u}_\nu - D_k(n^\mu \tilde{u}_\mu) \\ &= \gamma^\nu_k \mathcal{L}_n \tilde{u}_\nu + D_k(h\alpha u^0) \equiv W_k,\end{aligned} \tag{2.24}$$

where we use $n^\mu \tilde{u}_\mu = -h\alpha u^0$. From a straightforward calculation, we can rewrite $\gamma^\nu_k \mathcal{L}_n \tilde{u}_\nu$ as

$$\begin{aligned}\gamma^\nu_k \mathcal{L}_n \tilde{u}_\nu = \frac{1}{\alpha}[&\gamma^\nu_k \mathcal{L}_\ell \tilde{u}_\nu + h\alpha u^0 D_k \alpha - (\beta^j + \ell^j)\partial_j {}^{(3)}\tilde{u}_k \\ &- {}^{(3)}\tilde{u}_j \partial_k(\beta^j + \ell^j)],\end{aligned} \tag{2.25}$$

where ∂_k denotes a partial derivative on Σ_t. Hence,

$$\begin{aligned}W_k = \frac{1}{\alpha}[&\gamma^\nu_k \mathcal{L}_\ell \tilde{u}_\nu - (\beta^j + \ell^j)\partial_j {}^{(3)}\tilde{u}_k - {}^{(3)}\tilde{u}_j \partial_k(\beta^j + \ell^j) \\ &+ \partial_k(h\alpha^2 u^0)].\end{aligned} \tag{2.26}$$

Using the hydrodynamic equation (2.13) and an identity (2.21), we obtain

$$\begin{aligned}W_k &= \frac{1}{\alpha}(V^j + \beta^j + \ell^j)(-\partial_j {}^{(3)}\tilde{u}_k + \partial_k {}^{(3)}\tilde{u}_j) \\ &= \frac{1}{\alpha}(V^j + \beta^j + \ell^j) W_{kj}.\end{aligned} \tag{2.27}$$

Equation (2.27) implies that $W_k = 0$ if Eq. (2.16) is satisfied. Note that to derive Eq. (2.27) we have not assumed the fact that ℓ^μ is a Killing vector. Therefore, Eq. (2.16) is the necessary and sufficient condition for the irrotational condition in the general case. Note that Eq. (2.18) itself does not mean irrotation in general. Even for the case when a Killing vector ℓ^μ exists, it is nothing but a necessary condition for irrotation.

III. NEWTONIAN LIMIT

In the Newtonian limit, metric variables can be expanded as

$$\alpha = 1 - \frac{U}{c^2} + O(c^{-4}), \tag{3.1}$$

$$\beta^k = O(c^{-3}), \tag{3.2}$$

$$\gamma_{ij} = \delta_{ij} + O(c^{-2}), \tag{3.3}$$

where U denotes the Newtonian potential which satisfies

$$\Delta U = -4\pi\rho, \tag{3.4}$$

and Δ is the flat Laplacian. By using $v^i \equiv u^i/u^0$, the components of u^μ which we need here are also expanded as

$$u^0=1+\frac{1}{c^2}\left\{\frac{1}{2}v^2+U\right\}+O(c^{-4}), \tag{3.5}$$

$$u^i=u_i=\frac{v^i}{c}+O(c^{-3}), \tag{3.6}$$

where $v^2=\Sigma_i v^i v^i$. Note also that $\ell^\mu=(1,\ell^i/c)$ and $V^\mu=(0,V^i/c)$. For the corotational case ($V^i=0$), we get the Newtonian limit of the left-hand side of Eq. (2.15) as

$$\left[\frac{h}{u^0}\right]_{\text{full relativity}}\to 1+\frac{1}{c^2}\left[-\frac{v^2}{2}-U+\int\frac{dP}{\rho}\right]. \tag{3.7}$$

Since $V^k=0$, v^k is equal to ℓ^k, and $v^2=R^2\Omega^2$ where $R^2=(x^1)^2+(x^2)^2$. Substituting this relation of v^2 into Eq. (3.7), we get a well-known result

$$-\frac{R^2\Omega^2}{2}-U+\int\frac{dP}{\rho}=\text{const.} \tag{3.8}$$

For the irrotational case, the Newtonian limit of the left-hand side of Eq. (2.18) becomes

$$\left[\frac{h}{u^0}+{}^{(3)}\tilde{u}_k V^k\right]_{\text{full relativity}}\to 1+\frac{1}{c^2}\left[-\frac{v^2}{2}-U+\int\frac{dP}{\rho}+\sum_k v^k(-\ell^k+v^k)\right]. \tag{3.9}$$

In Newtonian order, $v^k=\partial_k\phi_{\rm N}$, where ϕ is expanded as $\phi_{\rm N}/c+O(c^{-3})$, so that we get

$$\frac{1}{2}\sum_k(\partial_k\phi_{\rm N})^2-U+\int\frac{dP}{\rho}-\sum_k\ell^k\partial_k\phi_{\rm N}=\text{const.} \tag{3.10}$$

Equation (3.10) agrees with that of BGM [11].

From continuity equations in Newtonian order, we obtain equations for $\phi_{\rm N}$ as

$$\rho\Delta\phi_{\rm N}+\sum_k(\partial_k\phi_{\rm N}-\ell^k)\partial_k\rho=0. \tag{3.11}$$

Equation (3.11) is solved under the boundary condition,

$$\sum_k(\partial_k\phi_{\rm N}-\ell^k)\partial_k\rho=0, \tag{3.12}$$

at the stellar surface.

IV. FIRST POST-NEWTONIAN EQUATIONS

In this section, we derive hydrostatic equations in first post-Newtonian order. The equations for the corotational case agree with those shown in previous papers [14,7], so that we derive here equations only for the irrotational case. In the first post-Newtonian approximation, the metric in the standard post-Newtonian gauge can be expanded as [15]

$$\alpha=1-\frac{U}{c^2}+\frac{1}{c^4}\left[\frac{U^2}{2}+X\right]+O(c^{-6}), \tag{4.1}$$

$$\beta^k=\frac{1}{c^3}\hat{\beta}_k+O(c^{-5}), \tag{4.2}$$

$$\gamma_{ij}=\delta_{ij}\left[1+\frac{2}{c^2}U\right]+O(c^{-4}), \tag{4.3}$$

where X and $\hat{\beta}_k$ are obtained from

$$\Delta X=4\pi\rho\left(2U+2\sum_k(\partial_k\phi_{\rm N})^2+\varepsilon+\frac{3P}{\rho}\right), \tag{4.4}$$

$$\hat{\beta}_k=-\frac{7}{2}P_k+\frac{1}{2}\left(\partial_k\chi+\sum_j x^j\partial_k P_j\right), \tag{4.5}$$

and

$$\Delta P_k=-4\pi\rho\partial_k\phi_{\rm N}, \tag{4.6}$$

$$\Delta\chi=4\pi\rho\sum_k(\partial_k\phi_{\rm N})x^k. \tag{4.7}$$

Note that to derive these Poisson equations, we use a relation in Newtonian order, $v^k=\partial_k\phi_{\rm N}$.

Using a post-Newtonian relation

$$\alpha u^0=1+\frac{1}{2c^2}\sum_k(\partial_k\phi_{\rm N})^2+\frac{1}{c^4}\left[-\frac{1}{8}\left(\sum_k(\partial_k\phi_{\rm N})^2\right)^2+\sum_k\partial_k\phi_{\rm N}\partial_k\phi_{\rm PN}-(\eta+U)\sum_k(\partial_k\phi_{\rm N})^2\right]+O(c^{-6}), \tag{4.8}$$

where we expand ϕ as $\phi_{\rm N}/c+\phi_{\rm PN}/c^3+O(c^{-5})$ and $\eta=\varepsilon+P/\rho$, the first post-Newtonian expansion of Eq. (2.18) becomes

$$\begin{aligned}\left[\frac{h}{u^0}+{}^{(3)}\tilde{u}_k V^k\right]_{\text{full relativity}}&\to 1+\frac{1}{c^2}\left[\eta-U+\frac{1}{2}\sum_k(\partial_k\phi_{\rm N})^2-\sum_k\ell^k\partial_k\phi_{\rm N}\right]\\&+\frac{1}{c^4}\Bigg[-\eta U+\frac{1}{2}U^2+X-\frac{1}{2}(\eta+3U)\sum_k(\partial_k\phi_{\rm N})^2\\&-\frac{1}{8}\left(\sum_k(\partial_k\phi_{\rm N})^2\right)^2+\sum_k\partial_k\phi_{\rm N}\partial_k\phi_{\rm PN}-\sum_k\ell^k\partial_k\phi_{\rm PN}\\&-\sum_k\hat{\beta}_k\partial_k\phi_{\rm N}\Bigg]=\text{const.}\end{aligned} \tag{4.9}$$

The first post-Newtonian expansion of continuity equation is also derived as

$$\sum_i \partial_i(\rho A_i)=0, \tag{4.10}$$

where

$$A_i=-\ell^i+\partial_i\phi_{\rm N}+\frac{1}{c^2}\times\left\{-\ell^i\left(\frac{1}{2}\sum_k(\partial_k\phi_{\rm N})^2+3U\right)-\eta\partial_i\phi_{\rm N}-\hat{\beta}_i+\partial_i\phi_{\rm PN}\right\}. \tag{4.11}$$

If $\phi_{\rm N}$ is obtained from Eq. (3.11), Eq. (4.10) is regarded as an equation for $\phi_{\rm PN}$ and solved under the boundary condition

$$\sum_i A_i\partial_i\rho=0 \tag{4.12}$$

at the stellar surface.

Equations (4.9) and (4.10) with Poisson equations (4.4), (4.6), (4.7), and (3.4) are the basic equations for the computation of irrotational equilibrium states in first post-Newtonian order.

V. SUMMARY

In this paper, we have derived relativistic hydrostatic equations for obtaining irrotational (quasi)equilibrium configurations of binary neutron stars using a $3+1$ formalism. In order to derive the hydrostatic equations, we first projected hydrodynamic equations onto Σ_t and then imposed the irrotational condition to obtain the hydrostatic equations. As a result, the hydrostatic equations obtained are simple and suitable for numerical relativity, compared with a previous formalism [11]. Also, as a natural consequence, in our formalism there is no vector Poisson equation to be solved and only a scalar Poisson equation is needed to be solved for the determination of the velocity field not only for Newtonian, but also for relativistic cases.

We also give hydrostatic equations as well as Poisson equations for the gravitational potentials needed for the computation of irrotational equilirium states in a first post-Newtonian approximation. We think that as a first step toward a fully relativistic study, we had better construct post-Newtonian configurations for a firm investigation of the relativistic effect on binary neutron stars. In reality, we have been able to obtain much information on the relativistic effect in binary neutron stars from first post-Newtonian studies [7,16,17]. Up to now, however, our attention has been paid only to corotational binary neutron stars. The present formalism makes it possible to extend previous studies to the irrotational case. As a first work, we plan to obtain incompressible, irrotational equilibrium states of binary stars as we carried out for the corotational case previously [16].

Note added in proof. After this paper was posted, the author noticed that Teukolsky [18] presented a similar formalism, which is shown to be essentially the same as the formalism given here in Ref. [19].

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