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Gravitational waves from supermassive stars collapsing to a supermassive black hole

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We derive the gravitational waveform from the collapse of a rapidly rotating supermassive star (SMS) core leading directly to a seed of a supermassive black hole (SMBH) in axisymmetric numerical-relativity simulations. We find that the peak strain amplitude of gravitational waves emitted during the black hole formation is $\approx 5 \times 10^{-21}$ at the frequency $f \approx 5$ mHz for an event at the cosmological redshift $z = 3$, if the collapsing SMS core is in the hydrogen-burning phase. Such gravitational waves will be detectable by space laser interferometric detectors like eLISA with signal-to-noise ratio ≈ 10, if the sensitivity is as high as LISA for $f = 1$–10 mHz. The detection of the gravitational wave signal will provide a potential opportunity for testing the direct-collapse scenario for the formation of a seed of SMBHs.

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I. INTRODUCTION

Clarifying the formation process of supermassive black holes (SMBHs), which are often observed in the center of galaxies [1], is one of the longstanding problems in astrophysics [2]. One possible scenario for this is the so-called direct-collapse scenario, in which one supposes that a supermassive star (SMS) of mass $\gtrsim 10^5 M_\odot$ would be the progenitor for the formation of a seed of SMBHs. Recent star-formation calculations [3,4] have suggested that if a high mass-accretion rate with $\gtrsim 0.1\ M_\odot$/yrs is preserved in the period of nuclear burning $\sim 2 \times 10^6$ yrs, a SMS (in nuclear burning) with mass $\gtrsim 2 \times 10^5 M_\odot$ could be indeed formed. Subsequently, the SMS core (i.e., a central high-density isentropic region) would collapse directly to a black hole by general-relativistic quasiradial instability [5–7]. The high mass-accretion rate, necessary for the formation of SMSs, requires primordial gas clouds with virial temperature $\gtrsim 10^4$ K. There are several scenarios proposed to achieve this condition such as Lyman-Werner radiation from a nearby local star formation region [8,9] or shock heating in cold accretion flows in the forming first galaxies [10,11]. However, these possibilities have not been tested by the observation yet. To test the direct-collapse scenario, a certain observation is necessary.

The formation process of a SMBH after the collapse of a SMS is determined by the initial condition at which the general-relativistic quasiradial instability sets in for the SMS. In reality, it is natural to consider that SMSs are rotating because they are likely to be formed in a non-symmetric environment of a galactic center as indicated by recent numerical simulations for the collapse of an atomic cooling halo in the early Universe (e.g., Refs. [12–14]). These simulations have suggested that protostellar disks initially formed in the central gas could be gravitationally unstable and fragment into several clumps, preventing straightforward growth of mass of the central protostar. However, the fragments are likely to subsequently migrate inward by gas drags and fall onto the central protostar [15,16], leading eventually to a rotating SMS.

Motivated by this possibility, we determined the realistic conditions for the onset of the general-relativistic quasiradial instability of rigidly rotating SMS cores in nuclear-burning phases [17]. Here, the reason that the SMS cores are supposed to be rigidly rotating is that they are in nuclear-burning phases and hence in a strongly convective phase, resulting in a uniform rotation as in massive stars (see, e.g., Ref. [18]). We find the following: (i) The equation of state (EOS) for the SMS cores in hydrogen- and helium-burning phases that are close to a marginally stable state against the general-relativistic quasiradial instability can be approximated by a polytropic EOS with the adiabatic index $\Gamma \approx 1.335$. (ii) The SMS cores in maximally rigid rotation are unstable against the gravitational collapse if their mass exceeds $\approx 6.3 \times 10^5 M_\odot$ in the hydrogen-burning phase and $\approx 2.3 \times 10^5 M_\odot$ in the helium-burning phase. (iii) The dimensionless spin parameter for such SMS cores is ≈ 0.8. These marginally stable states, on which we focus in this paper, are plausible initial conditions for the collapse to a seed of SMBHs.

In this article, we first report a result of our new numerical-relativity simulations for the collapse of a rapidly rotating SMS core, focusing on gravitational waves emitted during the black hole formation. The direct collapse of rigidly rotating SMSs to a SMBH has been studied by several groups [19–21] (see also Refs. [22–24] for a different scenario), but no group has derived the accurate gravitational waveform associated with the black hole formation in the scenario that we suppose. We show that the gravitational wave signal is characterized by a ringdown oscillation of the formed black hole with the frequency $\approx 20(M/6.3\times 10^5 M_\odot)^{-1}(1+z)^{-1}$ mHz and the strain amplitude $\approx 5\times 10^{-21}(M/6.3\times 10^5 M_\odot)(D/25\ \mathrm{Gpc})^{-1}$, where M is the mass of the SMS core, z is the cosmological redshift of the source, and D is its luminosity distance (which is ≈26 Gpc for $z=3$ in the ΛCDM model). Since the best sensitivity frequency band of eLISA and LISA is ≈1–10 mHz [25–27], such a gravitational wave signal is one of their possible targets. We then emphasize that the detection of this characteristic gravitational wave signal will be used for testing the direct-collapse scenario for the seed-SMBH formation and that improving the sensitivity of eLISA around 1–10 mHz is crucial for this purpose. Throughout this paper, we employ the units of $c=1=G$, where c and G are the speed of light and gravitational constant, respectively.

II. NUMERICAL RESULTS

Our method for a solution of Einstein's equation is the same as that in Ref. [28]. We employ the original version of Baumgarte-Shapiro-Shibata-Nakamura formulation with a puncture gauge [29]. The gravitational field equations are solved in the standard fourth-order finite differencing scheme. The axial symmetry is imposed using a fourth-order cartoon method [28,30,31] because nonaxisymmetric deformation is unlikely to be excited during the collapse in the rigidly rotating initial condition (see, e.g., Ref. [32]). Gravitational waves are extracted from the outgoing component of the complex Weyl scalar Ψ_4, which is expanded by a spin-weighted spherical harmonics of weight -2, ${}_{-2}Y_{lm}(\theta,\varphi)$, with $m=0$ in axisymmetric spacetime (see, e.g., Ref. [33]). In this work, we focus only on the quadrupole mode with $l=2$ (denoted by Ψ_{20} in the following) because it is the dominant mode.

A rigidly rotating SMS core near the mass shedding limit is employed as the initial condition with the polytropic EOS, $P=\kappa\rho^\Gamma$, where κ, ρ, and P are the polytropic constant, the rest-mass density, and the pressure, respectively, and we choose $\Gamma=1.335$ because we found it a realistic value [17]. Here, Γ is approximately written as $\Gamma = 4/3 + 3.7\times 10^{-3}(M/10^5 M_\odot)^{-1/2}(Y_T/1.69)$, where M is the mass of the SMS core and Y_T is the total particle number per baryon, which is 1.69 for pure hydrogen plasma and 0.75 for pure helium plasma. For rigidly rotating SMS cores that are at the mass shedding limit and marginally stable against general-relativistic gravitational collapse, M is $\approx 6.3\times 10^5 M_\odot$ and $2.3\times 10^5 M_\odot$ in the hydrogen-burning and helium-burning phases, respectively. By appropriately setting κ, we choose $M = 6.3\times 10^5 M_\odot$ in this paper (see Table I for physical quantities of the SMS core). In this model, the central temperature is $\approx 10^{8.2}$ K, which agrees with that in the stellar evolution calculation [4]. The initial ratio of the polar to equatorial axis lengths is ≈2/3. During numerical evolution, we employ the Γ-law EOS, $P=(\Gamma-1)\rho\varepsilon$, where ε is the specific internal energy. This is a good approximation for the realistic EOS of the SMS core as long as we focus on the phase up to black hole formation because the effects of nuclear burning and neutrino emission are minor in the standard scenario for metal-poor SMSs [21]. To slightly accelerate the collapse, we initially reduce the pressure by 1% or 2% uniformly (we refer to each model as D1 and D2, respectively). We checked that for these two depletion cases the resulting gravitational waveforms agree well (see Fig. 2).

TABLE I. Quantities for SMS core employed in this article and for remnant black hole. Γ: adiabatic index. M: gravitational mass of the system. β: ratio of rotational kinetic energy to gravitational potential energy. J: angular momentum. R_e: equatorial circumferential radius. $M_{\rm BH}$ and $a_{\rm BH}$: mass and dimensionless spin of the black hole eventually formed, respectively.

Γ	$M(M_\odot)$	β	J/M^2	R_e/M	$M_{\rm BH}(M_\odot)$	$a_{\rm BH}$
1.335	6.3×10^5	0.0090	0.80	423	6.0×10^5	0.68

Numerical simulations are performed in cylindrical coordinates (X, Z), and a nonuniform grid is used for X and Z. Specifically, we employ the following grid spacing (the same profile is chosen for Z): For $X\le X_{\rm in}$, $\Delta X = \Delta X_0 = \mathrm{const}$ and for $X > X_{\rm in}$, $\Delta X_i = \eta\Delta X_{i-1}$. Here, ΔX_0 is the grid spacing in an inner region and $X_{\rm in}\approx 2M$. $\Delta X_i{:=}X_{i+1}-X_i$ with X_i the location of ith grid. At $i=\mathrm{in}$, $\Delta X_i = \Delta X_0$. Here, η determines the nonuniform degree of the grid spacing. We employ $\Delta X_0 = 0.046M$ and $\eta = 1.018$ for a low-resolution run and $\Delta X_0 = 0.037M$ and $\eta=1.015$ for a high-resolution run. We confirm that the numerical results depend only weakly on the grid resolution. In the following, we show the results by the high-resolution run.

Figure 1 displays snapshots of density profiles for the SMS core collapse to a black hole surrounded by a torus for the D2 model. The SMS core collapses directly to a black hole (see first through fourth panels of Fig. 1), and 95.5% of the total rest mass falls into the black hole eventually irrespective of the initial pressure depletion factor. The properties of the black hole are determined by analyzing the area and circumferential radii of apparent horizons after the black hole relaxes to a stationary state. We find that the final black hole mass is $M_{\rm BH}\approx 6.0\times 10^5 M_\odot$ and the dimensionless spin is $a_{\rm BH}\approx 0.68$. All these values do not depend on the initial pressure depletion factor and also agree approximately with those predicted from the initial condition in the assumption that the specific angular momentum of each fluid element is conserved [17]. Since the SMS is rapidly rotating in this model, ≈4.5% of the rest mass eventually constitutes a torus surrounding the central

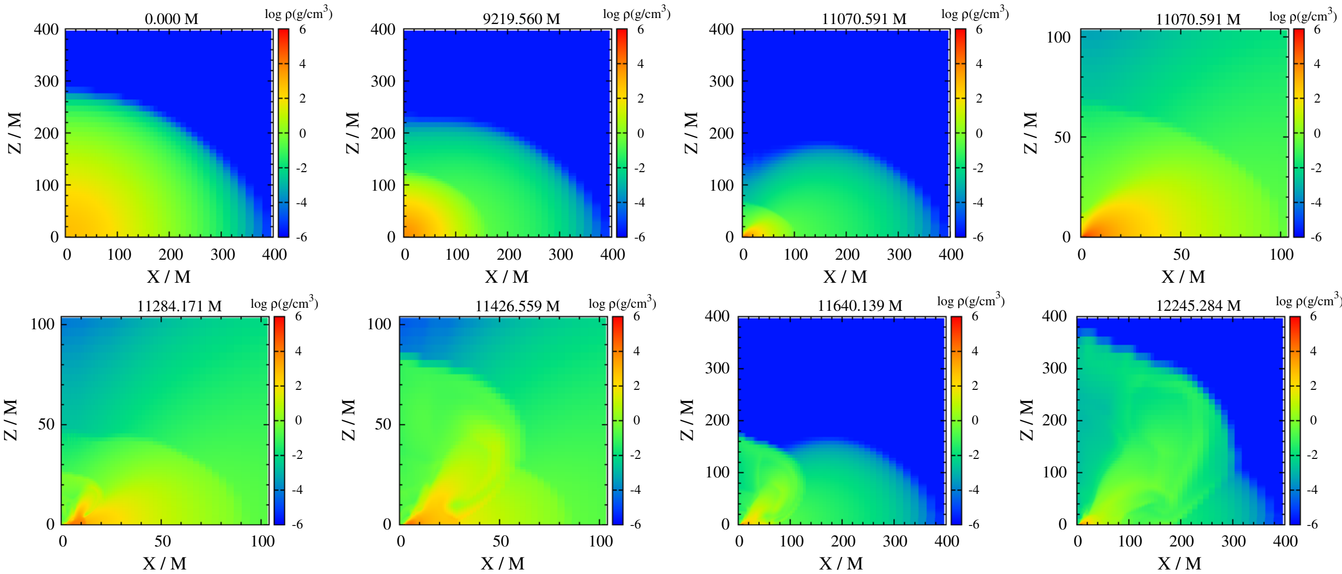

FIG. 1. Snapshots of density profiles for the SMS core collapse to a black hole surrounded by a torus for the D2 model. The black hole is formed at $t \approx 11050M$ (near the time for the third and fourth panels) in this model. Outer boundaries of the computational domain are located at $533M$ along each axis. The fourth through sixth panels show zoom-in views of the central region, and the fourth panel is a zoom-in of the third one.

black hole. The geometrical thickness of the torus is high because shocks are formed and heat up the matter in an inner region of the torus during its formation (see the fifth panel of Fig. 1). The inner part of the torus relaxes to a stationary state in $1000M$ after the formation of the black hole, while the envelope expands due to the heating by the early shock (see the sixth through eighth panels of Fig. 1). Exploring the subsequent evolution of the torus and resulting signals is one of the interesting future issues.

Figure 2 displays the gravitational waveform (Ψ_{20}) as a function of the retarded time, $t_{\rm ret} := t - [r_A + 2M \ln(r_A/2M - 1)]$, where r_A is the circumferential radius defined

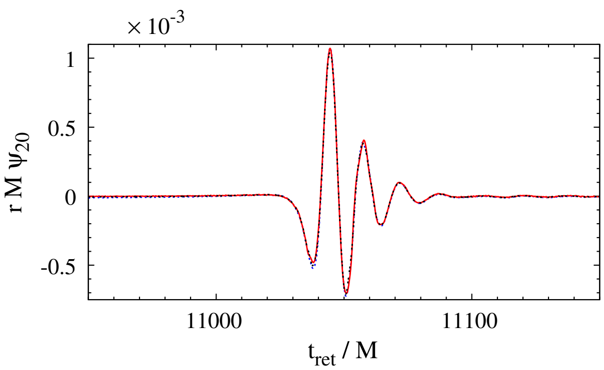

FIG. 2. Gravitational waveforms ($l = 2$ axisymmetric mode of Ψ_4) as a function of retarded time. Gravitational waves are extracted at $50M$ (dotted blue curve) and $75M$ (solid red curve) for the D2 model and at $75M$ (dot-dot black curve) for the D1 model. It is found that these three curves agree well with each other. Note that a black hole is first formed at $t_{\rm ret} \approx 11050M$ for the D2 model and at $t_{\rm ret} \approx 15538M$ for the D1 model, and hence, the waveform for the D1 model is shifted by $4488M$.

from the coordinate radius, r, by $r_A := r(1 + M/2r)^2$. Note that this mode has the maximum amplitude for the observer located on the equatorial plane and the amplitude vanishes if the observer is located along the rotation axis. It is found that gravitational waves are composed of a precursor associated with long-term collapse of the SMS and of a ringdown oscillation associated with the formed black hole. The period of the ringdown oscillation is $\approx 16M_{\rm BH}$ and it agrees well with the result of a linear-perturbation analysis for the black hole quasinormal mode [34] with $a_{\rm BH} \approx 0.68$. The corresponding rest-frame frequency is $\approx 1/16M_{\rm BH} \approx 21(M_{\rm BH}/6\times10^5 M_\odot)^{-1}$ mHz. The total energy of gravitational waves emitted is $\Delta E \approx 1.1 \times 10^{-6} M$. Thus, the emissivity is much smaller than those in binary black hole mergers in which $\sim 0.1M$ can be radiated (e.g., Ref. [35]).

Figure 3 shows the Fourier spectrum, $|h(f)f|$, of gravitational waves. Here, $h(f)$ is derived from

$$h(f) = -{}_{-2}Y_{20} \int dt \frac{2\Psi_{20}(t)}{(2\pi f)^2} \exp(-2\pi f t) dt. \tag{2.1}$$

Figure 3 is generated for the average value of ${}_{-2}Y_{20} \propto \sin^2\theta$ (i.e., setting the average $\langle \sin^2\theta \rangle = 2/3$). We choose cosmological redshifts as $z = 1$, 2, and 3 for which the luminosity distance is $D \approx 6.7$, 15.8, and 25.9 Gpc, respectively, in the standard ΛCDM model. We also plot the noise curve of eLISA [25,26] and a proposed optimal one (N2A5MxL4 of Ref. [27]). This shows that the gravitational wave frequency associated with the ringing oscillation of the formed black hole, $\approx 21(1+z)^{-1}$ $(M_{\rm BH}/6\times10^5 M_\odot)^{-1}$ mHz, is in the most sensitive

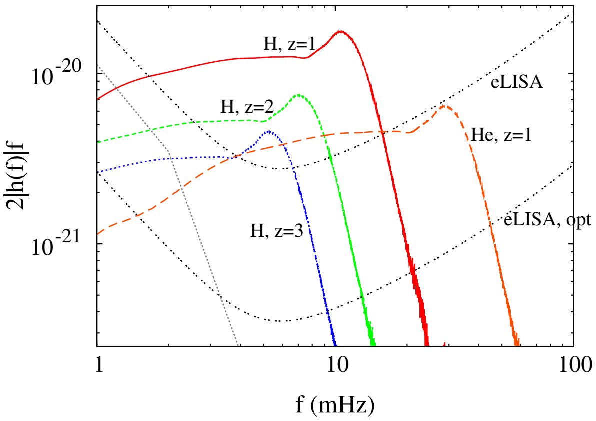

FIG. 3. Fourier spectrum of gravitational waves: The dotted, solid, dashed, and long dashed curves denote the results for the hydrogen-burning model at $z = 1$ (H, $z = 1$), at $z = 2$ (H, $z = 2$), at $z = 3$ (H, $z = 3$), and helium-burning model at $z = 1$ (He, $z = 1$). We plot $2|h(f)|f$ because the signal-to-noise ratio (SNR) is written by $\int_0^\infty d\ln f(2|h(f)|f)^2/(S_n(f)f)$, where $S_n(f)$ is the one-sided noise spectrum density. The dot-dot curves denote the planned noise curve of eLISA (upper) [25] and proposed optimal one (lower; N2A5MxL4 of Ref. [27] for which the arm length is assumed to be 5 million km). Here, plotted is $\sqrt{S_n(f)f}$. The short-dotted curve denotes a model for the expected unresolved part of the gravitational wave signals emitted by galactic binaries [27]. Note that the SNR for the hydrogen-burning cases is 5.4 and 2.2 at $z = 1$ and 2 for eLISA, while it is ≈43, 17, and 10 at $z = 1$, 2, and 3 for the optimally designed one.

frequency band of these proposed space interferometers. Because the gravitational wave amplitude (observed along the most optimistic direction) is approximately written as $4(M\Delta E)^{1/2}/D$, it is of the order 10^{-20} for the cosmological scale with $D \sim 7$ Gpc $(z \sim 1)$. Since the SNR $[= \int_0^\infty df(2|h(f)|)^2/S_n(f)$ with $S_n(f)$ the one-sided noise spectrum density] for it is ≈5 at $z = 1$, the sensitivity of originally planned eLISA may not be high enough for a confident detection of these gravitational waves, if they are emitted for $z \gtrsim 1$. However, if the sensitivity is improved by a factor of ∼10 as discussed in Ref. [27] (i.e., if the sensitivity is as high as the LISA project), the SNR would be $\gtrsim 10$ for $z \lesssim 3$, and a confident detection for them will be possible. Note that SMS could be formed only in an ultra-metal-poor environment, which would be present only for a high-redshift universe with $z \gtrsim 2$ (e.g., Ref. [36]). Thus, a detector as sensitive as LISA (not eLISA) will be necessary for testing the direct-collapse scenario by detecting gravitational waves emitted for $z \gtrsim 2$.

For the collapse of helium-burning SMS cores, the expected core mass is $\approx 2 \times 10^5 M_\odot$ [17]. In this case, the collapse process would be qualitatively the same as that for the hydrogen-burning SMS core. However, the mass of the black hole formed is about one-third of that in the collapse of the hydrogen-burning SMS. As a result, the peak frequency would be $\approx 60(1+z)^{-1}(M_{\rm BH}/2\times 10^5 M_\odot)^{-1}$ mHz with the peak strain amplitude approximately one-third as high as that in the collapse of the hydrogen-burning SMS core. Thus, the predicted gravitational wave signal with $z \gtrsim 1$ would be detectable only by an optimally designed eLISA. For the collapse of oxygen-burning SMS cores, the expected highest core mass is $\approx 2 \times 10^4 M_\odot$ [17], and hence, it will be difficult to detect the gravitational wave signal even by optimally designed eLISA. Since the planned sensitivity of DECIGO is better than that of eLISA for $f \gtrsim 20$ mHz [37], such gravitational waves may be a possible source for DECIGO.

To summarize, by a new numerical-relativity simulation, we derive the gravitational waveform from a rapidly rotating SMS core collapsing to a seed of a SMBH. The predicted frequency at the Fourier-spectrum peak is $\approx 20(1 + z)^{-1}$ mHz, and the peak amplitude is $\approx 5 \times 10^{-21}$ for an event at $z = 3$. This gravitational wave signal will be detectable by space interferometric gravitational wave detectors if its sensitivity is as high as LISA. The detection of this signal will provide a potential opportunity for testing the direct-collapse scenario for the formation of a seed of SMBHs.

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