YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Black holes in brane gravity

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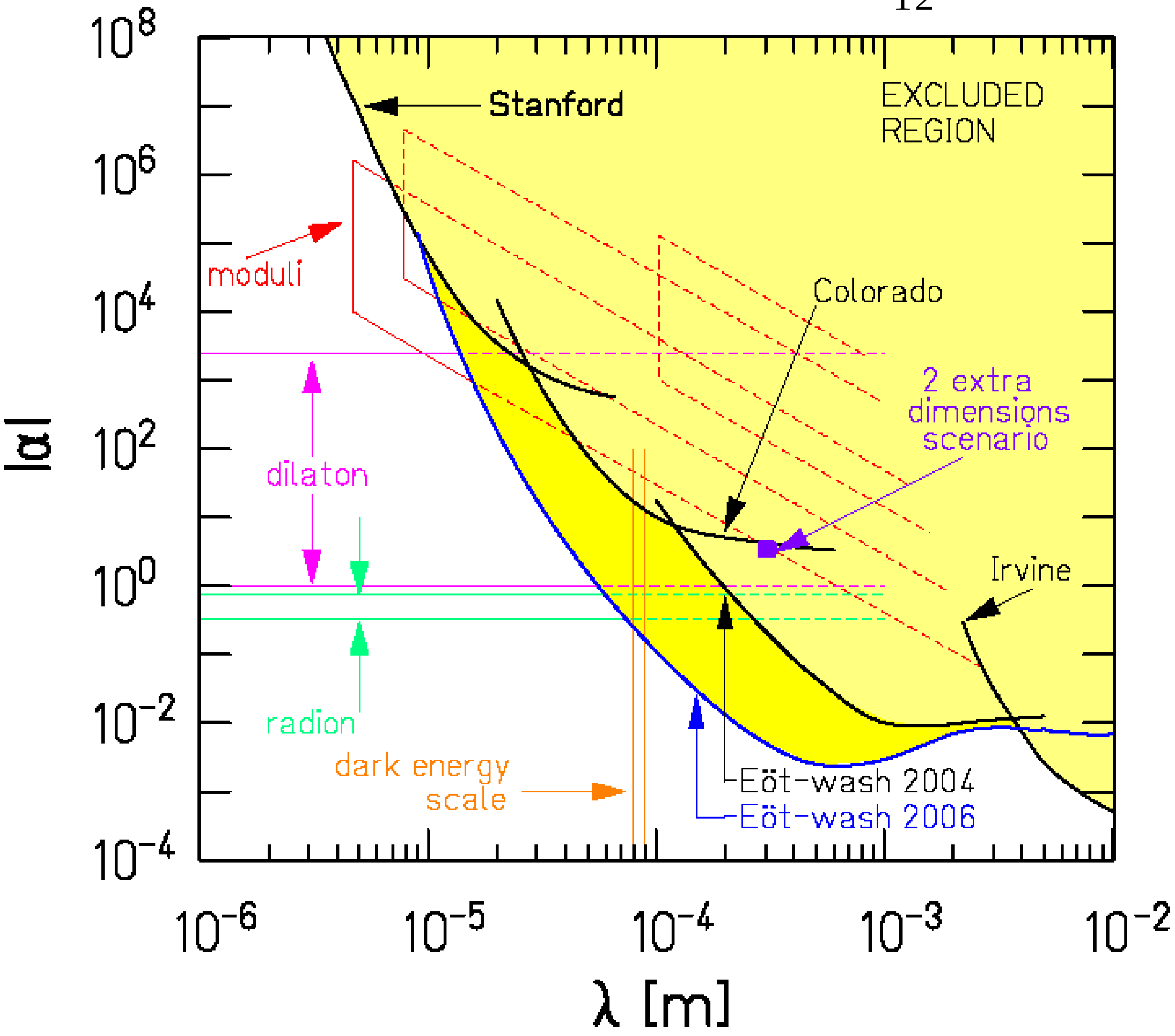
Modified gravity models

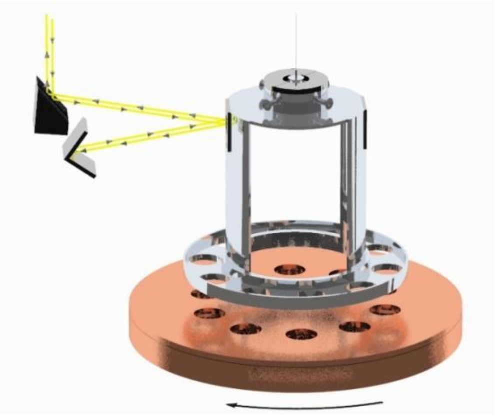
as an alternative to dark energy/matter
as a test bench of general relativity

Modified gravity models should be arranged to satisfy various tests.

Constraint on deviation from Newton’s law

- Short rage force $U = -\frac{Gm_1m_2}{r_{12}}\left[1+\alpha\exp\left(-r_{12}/\lambda\right)\right]$



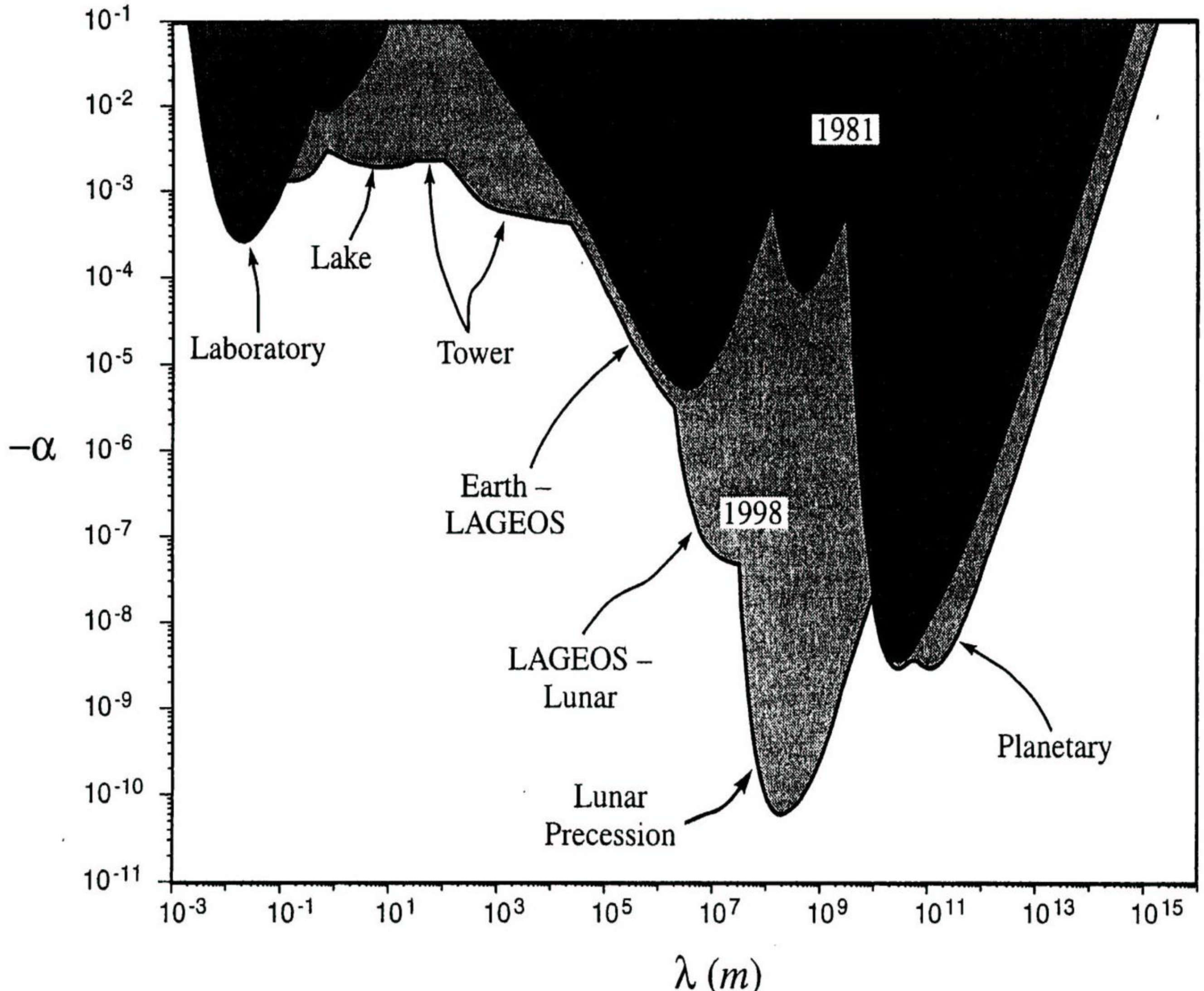


Capner et al
hep-ph/0611184

Constraint on Gravity

- intermediate rage

$$U = -\frac{Gm_1m_2}{r_{12}}\left[1+\alpha\exp\left(-r_{12}/\lambda\right)\right]$$



Fischbach & Talmadge

“The Search for Non-Newtonian Gravity”

(1998)

Constraint on Gravity

- Parameterized post-Newton

Ref) Will gr-qc/9811036

- $\gamma : g_{ij}$ components
  - light bending

$$\delta\theta = \frac{1}{2}(1+\gamma)\times 1.75''$$

VLBI unpublished?

$$\frac{1}{2}(\gamma - 1) \approx 1.6\times 10^{-4}$$

  - Cassini radar ranging

$$\frac{1}{2}(\gamma - 1) \approx 10^{-5} \pm 10^{-5}$$

- β : 1PN g_{00} components

$$g_{00} = -1 + 2U - 2\beta U^2$$

  - perihelion shift

43 arcsec/100yr

$$\beta - 1 \approx 3\times 10^{-3}$$

  - Nordtvedt effect

$$\beta - 1 \approx 6\times 10^{-4}$$

Modified gravity effect might be significant in strongly gravitating systems.

Black holes, (compact stars)

Braneworld models:

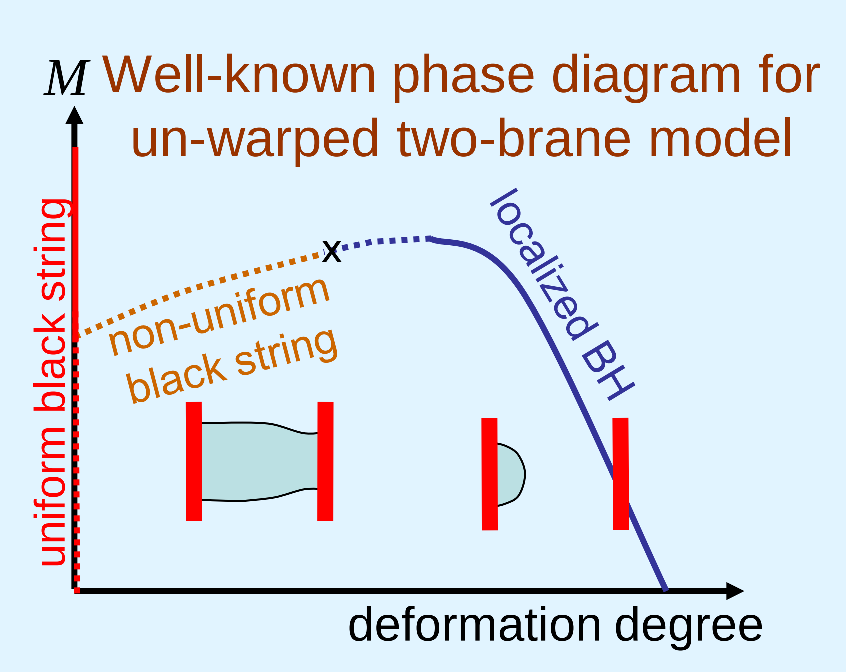
Large extra-dimensions

Black string instability

(Gregory & Laflamme ('93))

Sequence of solutions was found numerically

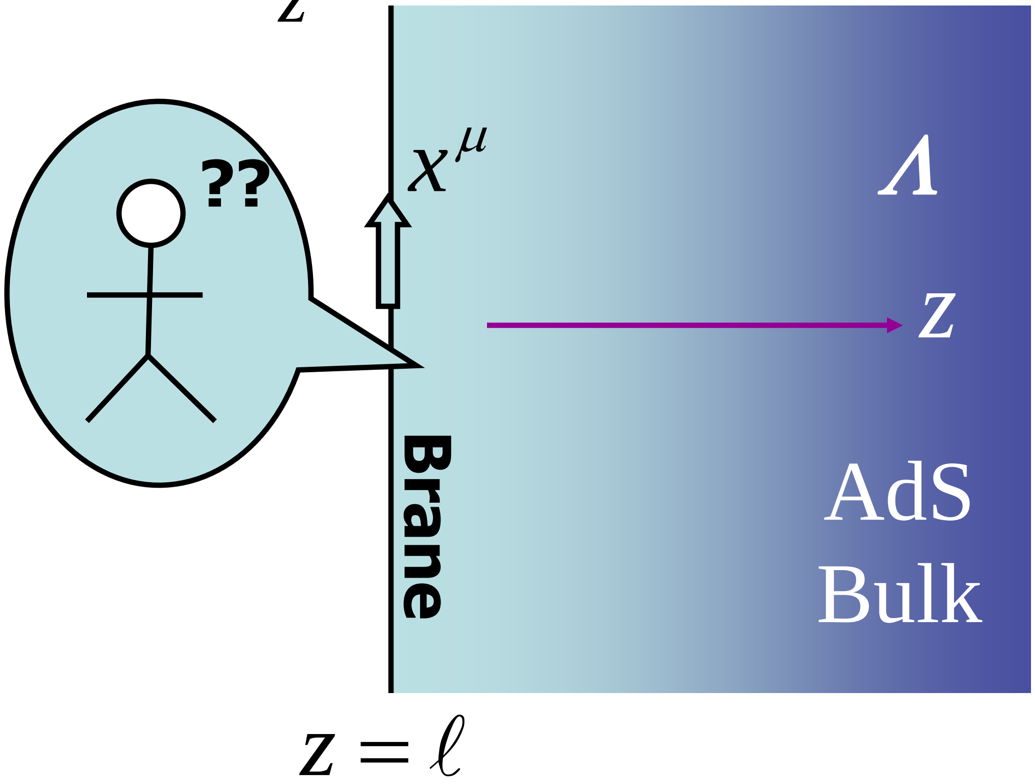
(e.g. Kudoh & Wiseman ('05))



Infinite extra-dimension: Randall-Sundrum II model

Volume of the bulk is finite due to warped geometry although its extension is infinite.

$$ds^2 = \frac{\ell^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$



ℓ : AdS curvature radius

$\Lambda = -\frac{6}{\ell^2}$ Negative cosmologica
constant

$\sigma = \frac{3}{4\pi G_5 \ell}$ Brane tension

- Extension is infinite, but 4-D GR seems to be recovered!

Gravity on the brane looks like 4D GR approximately, BUT for many years Schwarzschild-like BH solution had been unknown.

AdS/CFT correspondence

(Maldacena ('98))
(Gubser ('01))
(Hawking, Hertog, Reall ('00))

- $$Z\underbrace{[q]}_{\text{Boundary metric}}=\int d[\phi]\,\exp(-S_{CFT}[\phi,q])$$
  $$=\int d[g]\,\exp(-S_{HE}-S_{GH}\underbrace{+S_1+S_2+S_3}_{\text{Counter terms}})\equiv\exp(-W_{CFT}[q])$$

$$S_{EH}=-\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left({}^{(5)}R+\frac{12}{\ell^2}\right)$$

$$S_{GH}=-\frac{1}{\kappa_5^2}\int d^4x\sqrt{-q}\;K$$

$$S_1=-\frac{3}{\kappa_5^2\ell}\int d^4x\sqrt{-q}$$

$$S_2=-\frac{\ell}{4\kappa_5^2}\int d^4x\sqrt{-q}\;{}^{(4)}R$$

$$S_3=\cdots$$

- $\underbrace{z_0}_{\text{Brane position}}\longrightarrow 0$ limit is well defined with the counter terms

brane tension

- $$\int d[g]\,\exp(-S_{RS})=\int d[g]\,\exp(-2(S_{EH}+S_{GH})+2S_1-S_{matter})$$
  $$=\exp(-2S_2-S_{matter}-2(W_{CFT}+S_3))$$

4D Einstein-Hilbert action

z_0 ⇔ cutoff scale parameter

Classical black hole evaporation conjecture

(T.T. ('02), Emparan et al ('02))

4D Einstein+CFT with the lowest order quantum correction

$$\left(\text{number of field of CFT}\right) \approx \frac{\ell^2}{\kappa_4^2}$$

AdS/CFT correspondence

equivalent

Classical 5D dynamics in RS II model

4D BH with CFT

equivalent

5D BH on brane

Hawking radiation in 4D Einstein+CFT picture

equivalent

Classical evaporation of 5D BH

However, static brane-localized black hole was finally obtained numerically

Pau Figueras, James Lucietti, Toby Wiseman (2011)

Shape of the horizon in the bulk:
The boundary metric is conformal to Schwarzschild BH

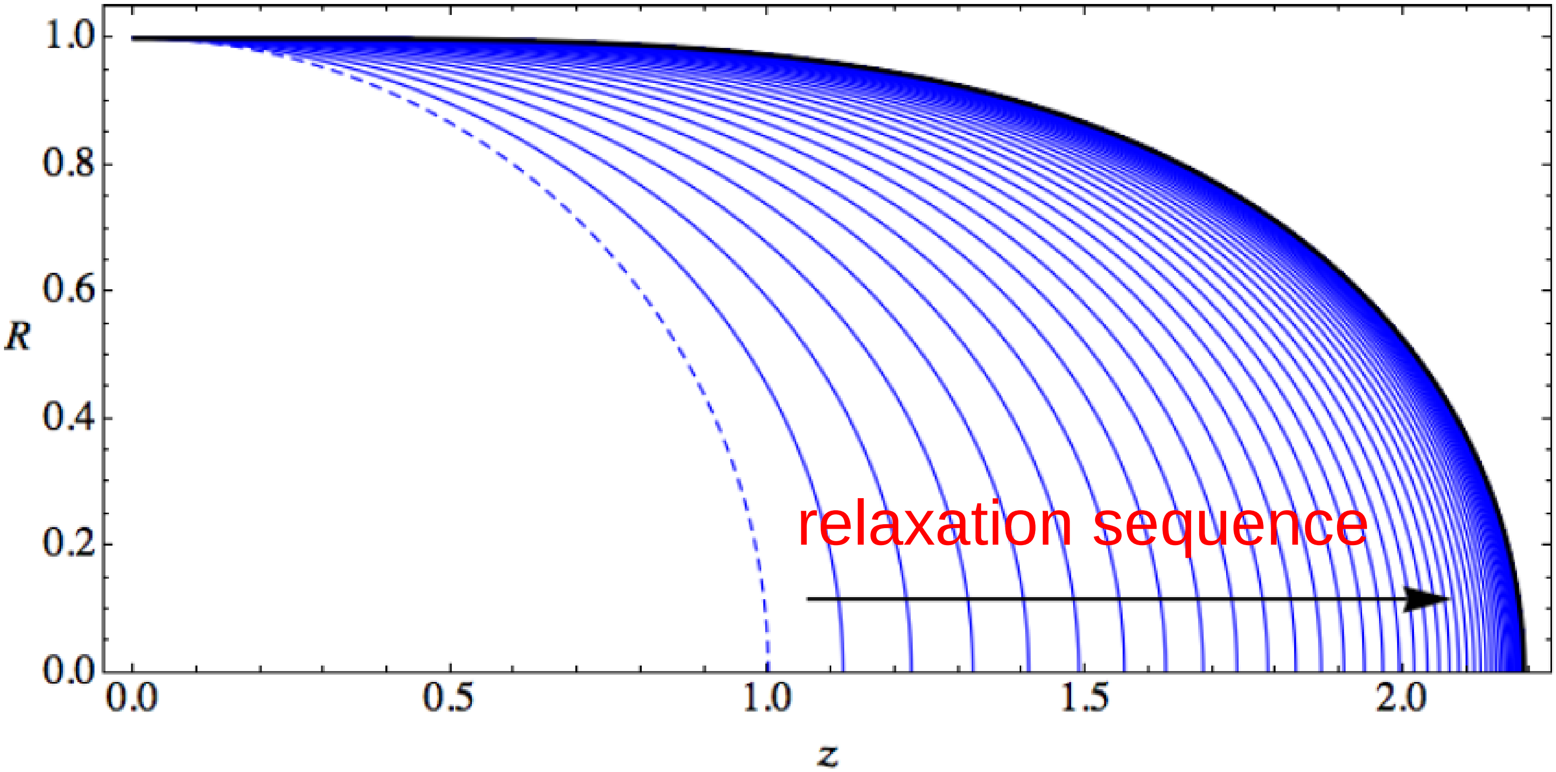
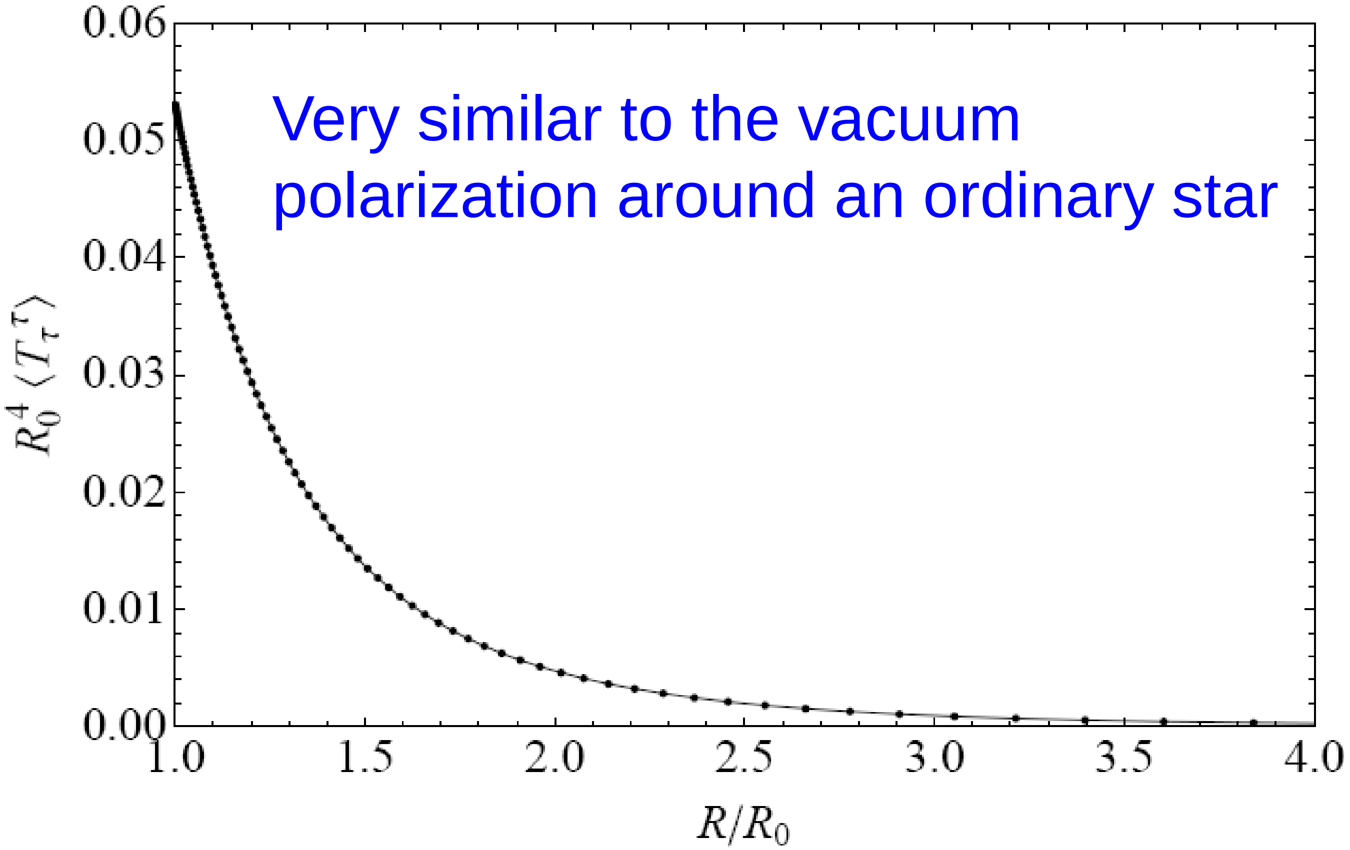


Figure 4: Embedding into hyperbolic space, $ds^2 = \frac{\ell^2}{z^2}\left(dz^2 + dR^2 + R^2 d\Omega^2_{(2)}\right)$, of the spatial cross sections of the horizon along the flow as curves $R(z)$. The dashed line corresponds to the initial data, for which the horizon is round, and the thick black line is the embedding of the horizon of the fixed point. The snapshots are drawn at intervals of λ of 0.05.

CFT energy density profile on the AdS boundary



Very similar to the vacuum polarization around an ordinary star

Why CFT does not respect the periodicity of the Euclidean solution?

~ similar to thermal AdS state?

For flat boundary metric with a given temperature, there are two known solutions

$$ds^2 = -\left(-\frac{\mu}{r^2}+\frac{r^2}{l^2}\right)dt^2 + \left(-\frac{\mu}{r^2}+\frac{r^2}{l^2}\right)^{-1} dr^2 + r^2 ds_{(3)}^2$$

$$ds^2 = -\frac{r^2}{l^2}dt^2 + \left(\frac{r^2}{l^2}\right)^{-1} dr^2 + r^2 ds_{(3)}^2$$: thermal AdS state

CFT energy density vanishes.

dynamically stable but thermodynamically meta-stable.

Randall-Sundrum infinite braneworld

Classical BH evaporation conjecture

(T.T. ('02), Emparan et al ('02))

Static solution for small black holes

(Kudoh+ ('03), Yoshino ('09))

Static solution for large black holes

(P. Figueras+ ('11))

DGP braneworld

A concrete example of Vainstein mechanism (e.g. T.T. ('03))

But no black hole solution is known.

Mass screening is suggested. (G. Gabadadze and A. Iglesias, ('05))

Higher curvature gravity, scalar-tensor…

Too many possibilities

Mostly pathological? Ghost or other instability

Do we need to be afraid of ghost at the level of effective field theory?

Maybe logically there is no need if the ghost is very massive.

Ghost free Massive gravity

Again, is it necessary to require the absence of ghost?

→ Ghost free bi-gravity model

→ Graviton oscillation between two types of gravitons

→ Disappearance of gravitational waves

My interest

If we reduce the coupling of CFT, does the BH start to evaporate again?

Recall that Hawking radiation is derived only for *weakly interacting fields*.

Are we really sure that *weakly interacting field* emits Hawking radiation?

Already known since long time ago or completely wrong

⇒Nobody tells so.

Interesting observation

⇒ Please discuss with me!