Black hole dynamics in generic spacetimes

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Outline

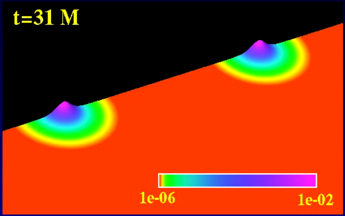
1. Collisions of black holes in higher dimensional spacetimes
2. Black holes in a box
3. Conclusions and Outlook

Black hole collisions in higher dimensional spacetimes

High Energy Collision of Particles

Consider particle collsions with $E = 2\gamma m_0 c^2 > M_{Pl}$

- Hoop - Conjecture (Thorne '72)
  $\Rightarrow$ BH formation, if circumference of particle $< 2\pi r_S$
- Collisions of shock waves
  (Penrose '74, Eardley & Giddings '02)
  $\Rightarrow$ BH formation if $b \leq r_S$
- numerical evidence in ultra relativistic collision of boson stars
  $\Rightarrow$ BH formation if boost $\gamma_c \geq 2.9$



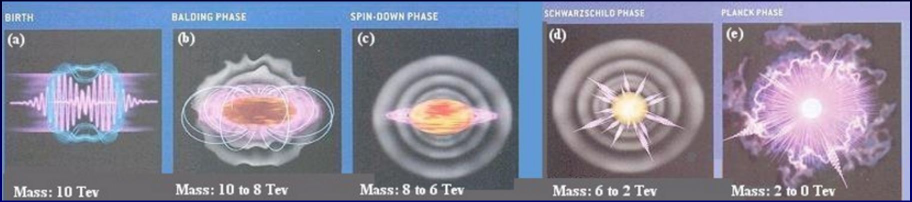
Low Lorentz boost, $\gamma = 1$
Large Lorentz boost, $\gamma = 4$

Choptuik & Pretorius '10

$\Rightarrow$ black hole formation in high energy collisions of particles

- higher dimensional theories of gravity $\Rightarrow$ TeV gravity scenarios
- signatures of black hole production in high energy collision of particles
  - at the Large Hadron Collider
  - in ultra-high relativistic Cosmic rays interactions with the atmosphere

Life cycle of Mini Black Holes



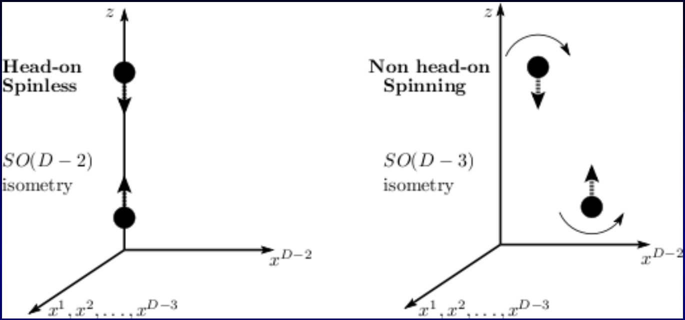
1. Formation
   - lower bound on BH mass from area theorem (Yoshino & Nambu '02)
2. Balding phase: end state is Myers-Perry black hole
3. Spindown phase: loss of angular momentum and mass
4. Schwarzschild phase: decay via Hawking radiation
5. Planck phase: $M \sim M_{Pl}$

Goal: more precise understanding of black hole formation
$\Rightarrow$ compute energy and angular momentum loss in gravitational radiation
Toy model: black hole collisions in higher dimensions

Numerical Relativity in $D > 4$ Dimensions

- Yoshino & Shibata, Phys. Rev. **D80**, 2009,
  Shibata & Yoshino, Phys. Rev. **D81**, 2010
- Okawa, Nakao & Shibata, Phys. Rev. **83**, 2011
- Lehner & Pretorius, Phys. Rev. Lett. **105**, 2010
- Sorkin & Choptuik, GRG **42**, 2010; Sorkin, Phys. Rev. **D81**, 2010
- Zilhão et al., Phys. Rev. **D81**, 2010, Witek et al, Phys. Rev. **D82**, 2010, Witek et al, Phys. Rev. **D83**, 2011, Zilhão et al., Phys. Rev. **D84**, 2011.

Numerical Relativity in D Dimensions



- consider highly symmetric problems
- dimensional reduction by isometry on a (D-4)-sphere

general metric element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \lambda(x^\mu) d\Omega_{D-4}$$

Numerical Relativity in D Dimensions

D dimensional vacuum Einstein equations $G_{AB} = R_{AB} - \frac{1}{2} g_{AB} R = 0$ **imply**

$$
\begin{aligned}
{}^{(4)}T_{\mu\nu} &= \frac{D-4}{16\pi\lambda}\left[\nabla_\mu\nabla_\nu\lambda - \frac{1}{2\lambda}\partial_\mu\lambda\partial_\nu\lambda - (D-5)g_{\mu\nu} + \frac{D-5}{4\lambda}g_{\mu\nu}\nabla_\alpha\lambda\nabla^\alpha\lambda\right] \\
\nabla^\mu\nabla_\mu\lambda &= 2(D-5) - \frac{D-6}{2\lambda}\nabla^\mu\lambda\nabla_\mu\lambda
\end{aligned}
$$

- $4D$ Einstein equations coupled to scalar field

Formulation of EEs as Cauchy Problem in $D > 4$

- 3+1 split of spacetime ${}^{(4)}\mathcal{M} = \mathbb{R} + {}^{(3)}\Sigma$ (Arnowitt, Deser, Misner '62)
  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \left(-\alpha^2 + \beta_k \beta^k\right) dt^2 + 2\beta_i dx^i dt + \gamma_{ij} dx^i dx^j$
- 3+1 split of 4D Einstein equations with source terms
  $\Longrightarrow$ Formulation as initial value problem with constraints (York 1979)

Evolution Equations

$$
\begin{aligned}
\partial_t \gamma_{ij} &= -2\alpha K_{ij} + \mathcal{L}_\beta \gamma_{ij} \\
\partial_t K_{ij} &= -D_i D_j \alpha + \alpha \left({}^{(3)}R_{ij} - 2K_{il}K^l_j + K K_{ij} \right) + \mathcal{L}_\beta K_{ij} \\
& \quad -\alpha \frac{D-4}{2\lambda} \left(D_i D_j \lambda - 2K_{ij}K_\lambda - \frac{1}{2\lambda} \partial_i \lambda \partial_j \lambda \right) \\
\partial_t \lambda &= -2\alpha K_\lambda + \mathcal{L}_\beta \lambda \\
\partial_t K_\lambda &= -\frac{1}{2} \partial^l \alpha \partial_l \lambda + \alpha \left((D-5) + K K_\lambda + \frac{D-6}{\lambda} K_\lambda^2 \right. \\
& \quad \left. - \frac{D-6}{4\lambda} \partial^l \lambda \partial_l \lambda - \frac{1}{2} D^l D_l \lambda \right) + \mathcal{L}_\beta K_\lambda
\end{aligned}
$$

Wave Extraction in $D > 4$

Generalization of Regge-Wheeler-Zerilli formalism by Kodama & Ishibashi '03

Master function

$$\Phi_{,t} = (D-2) r^{(D-4)/2} \frac{2rF_{,t} - F^r_t}{k^2 - D + 2 + \frac{(D-2)(D-1)}{2} \frac{r_S^{D-3}}{r^{D-3}}} \,, \qquad k = l(l+D-3)$$

Energy flux & radiated energy

$$\frac{dE_l}{dt} = \frac{(D-3)k^2(k^2-D+2)}{32\pi(D-2)} (\Phi^l_{,t})^2 \,, \qquad E = \sum_{l=2}^{\infty} \int_{-\infty}^{\infty} dt \frac{dE_l}{dt}$$

Momentum flux & recoil velocity

$$\frac{dP^i}{dt} = \int_{S_\infty} d\Omega \frac{d^2E}{dt d\Omega} n^i \,, \qquad v_{recoil} = \left| \int_{-\infty}^{\infty} dt \frac{dP}{dt} \right|$$

Numerical Setup

- use Sperhake's extended LEAN code (Sperhake '07, Zilhão et al '10)
  - 3+1 Einstein equations with scalar field
  - Baumgarte-Shapiro-Shibata-Nakamura formulation with moving punctures dynamical variables: χ, $\tilde{\gamma}_{ij}$, K, $\tilde{A}_{ij}$, $\tilde{\Gamma}^i$, ζ, K_ζ
  - modified puncture gauge

$$\begin{aligned} \partial_t \alpha &= \beta^k \partial_k \alpha - 2\alpha(K + (D-4)K_\zeta) \\ \partial_t \beta^i &= \beta^k \partial_k \beta^i - \eta_\beta \beta^i + \eta_\Gamma \tilde{\Gamma}^i + \eta_\lambda \frac{D-4}{2\zeta} \tilde{\gamma}^{ij} \partial_j \zeta \end{aligned}$$

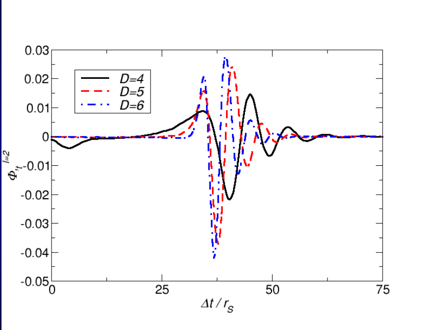
- measure lengths in terms of r_S with

$$r_S^{D-3} = \frac{16\pi}{(D-2)A^{S^{D-2}}} M$$

Equal mass head-on in $D = 4, 5, 6$

Brill-Lindquist type initial data

$$\psi = 1 + r_{S,1}^{D-3}/4r_1^{D-3} + r_{S,2}^{D-3}/4r_2^{D-3}$$



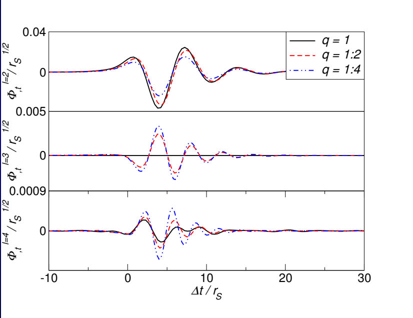
- Key (technical) issues:
  - modification of gauge conditions
  - modification of formulation
- increase in E/M with D
  $\Rightarrow$ qualitative agreement with PP calculations
  (Berti et al, 2010)

D	$r_S\,\omega(l=2)$	$E/M(\%)$
4	$0.7473 - \mathrm{i}\,0.1779$	0.055
5	$0.9477 - \mathrm{i}\,0.2561$	0.089
6	$1.140 - \mathrm{i}\,0.304$	0.104

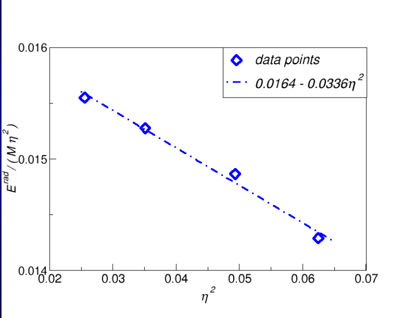
Unequal mass head-on in $D = 5$

- consider mass ratios $q = r_{S,1}^{D-3}/r_{S,2}^{D-3} = 1, 1/2, 1/3, 1/4$

Modes of $\Phi_{,t}$



$E/M \sim \eta^2$ (M.Lemos '10, MSc thesis)



- fitting function $\frac{E}{M\eta^2} = 0.0164 - 0.0336\eta^2$
- within $< 1\%$ agreement with point particle calculation (Berti et al, 2010)

Initial data for boosted BHs in $D > 4$

- construct initial data by solving the constraints
- assumption: $\bar{\gamma}_{ab} = \psi^{\frac{4}{D-3}} \delta_{ab}\,,\ \bar{K} = 0\,,\ \bar{K}_{ab} = \psi^{-2} \hat{A}_{ab}$
- constraint equations

$$\partial_a \hat{A}^{ab} = 0\,, \quad \hat{\triangle}\psi + \frac{D-3}{4(D-2)} \psi^{-\frac{3D-5}{D-3}} \hat{A}^{ab} \hat{A}_{ab} = 0\,, \quad \text{with } \hat{\triangle} \equiv \partial_a \partial^a$$

- analytic ansatz for $\hat{A}_{ab}$ $\rightarrow$ generalization of Bowen-York type initial data
- elliptic equation for ψ $\rightarrow$ puncture method (Brandt & Brügmann '97)

$$\psi = 1 + \sum_i r_{S(i)}^{D-3} / 4 r_{(i)}^{D-3} + u$$

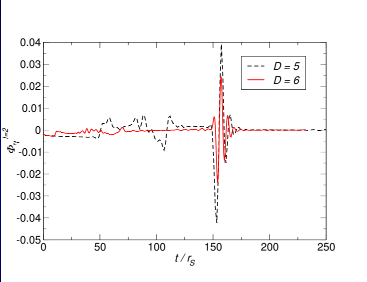
$\Rightarrow$ Hamiltonian constraint becomes

$$\hat{\triangle} u + \frac{D-3}{4(D-2)} \hat{A}^{ab} \hat{A}_{ab} \psi^{-\frac{3D-5}{D-3}} = 0$$

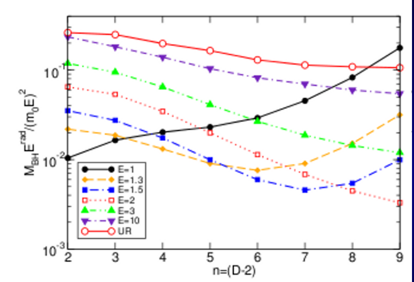
Head-on of boosted BHs (preliminary results)

- evolution of puncture with $z/r_S = \pm 30.185$ with $P/r_S^{D-3} = 0.4$

$l = 2$ mode of $\Phi_{,t}$



PP calculations (Berti et al, 2010)



Issues:

- dependence of radiated energy on D and boost
- long-term stable evolutions for larger boosts
- adjustment of (numerical) gauge
- requirement of very high resolution in wavezone for reasonable accuracy

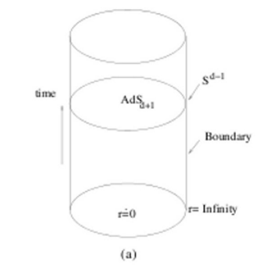
Black hole binaries in a box

AdS / CFT correspondence (Maldacena '97)

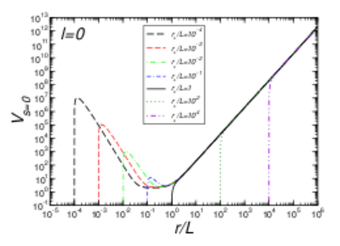
- Anti-de Sitter spacetime
  $\Rightarrow$ spacetime with negative cosmological constant
- duality between
  theory with gravity on $AdS_d \times X$

  $\Updownarrow$

  conformal field theory on conformal boundary



- consider scalar field propagation in SAdS background
  - $\frac{d^2}{dr_*^2}\psi + (\omega^2 - V)\psi = 0$
  - potential $V \longrightarrow \infty$ as $r \longrightarrow \infty$
    $\Rightarrow$ view AdS as boxed spacetime



Berti, Cardoso & Starinets '09

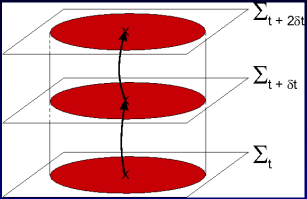
$\Rightarrow$ toy model: BH evolution in a box

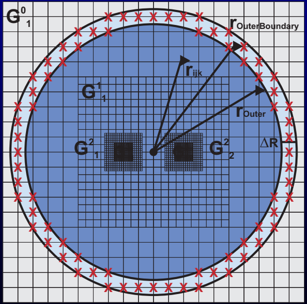
Black Hole stability

- superradiant scattering of rotating BH (Penrose '69, Christodoulou '70, Misner '72)
  - impinging wave amplified as it scatters off a BH if $\omega < m\Omega_H$
  - extraction of energy and angular momentum of the BH by superradiant modes
- "black hole bomb" (Press & Teukolsky '72)
  - consider Kerr BH surrounded by a mirror
  - subsequent amplification of superradiant modes
- stability of Kerr-AdS BHs (Hawking & Reall '99, Cardoso et al. '04)
  - AdS infinity behaves as box $\Rightarrow$ amplification of superradiant instabilities?
  - large Kerr-AdS BH: stable
  - small Kerr-AdS BH: unstable

BBHs in a Box - Setup

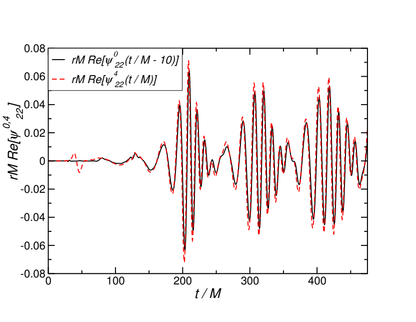
- mimic AdS-BH spacetime by BHs in a box
- impose reflecting b.c. via $\partial_t K_{ij} = 0$ and $\partial_t \gamma_{ij} = 0$ at boundary
- wave extraction: Newman Penrose scalars Ψ_4 (outgoing) and Ψ_0 (ingoing)
- initial data: non-spinning, equal mass BHs
- head-on collision $\Rightarrow$ non-spinning final BH
  quasi-circular inspiral $\Rightarrow$ spinning final BH with $a/M = 0.69$
- Movie



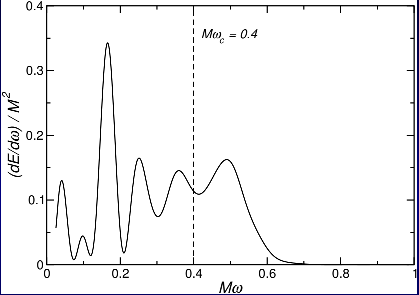


BBHs in a Box - Waveforms

$\Re(\Psi^4_{22})$ and $\Re(\Psi^0_{22})$



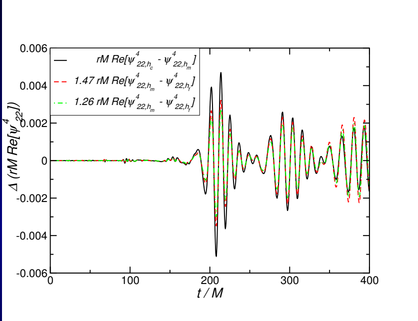
spectrum of first outgoing pulse



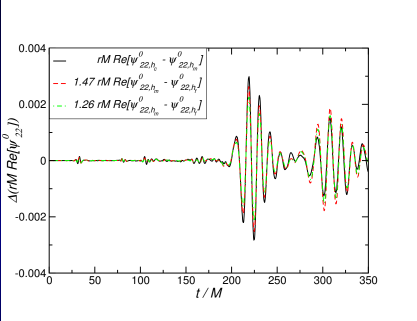
- spectrum of Ψ^4_{22} after merger and before interaction with boundary
- assume quasi Kerr BH
  $\Rightarrow$ critical frequency for superradiance $M\omega_C = m\Omega = 0.4$
- signal contains frequencies within and above superradiance regime

BBHs in a Box - Convergence Test

$\Re(\Psi^4_{22})$



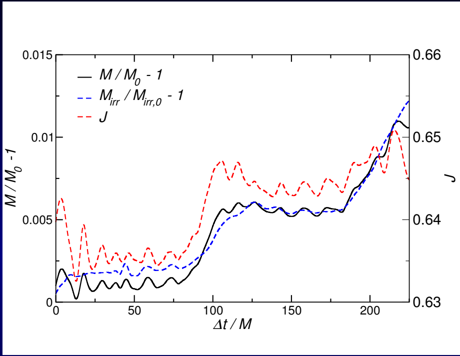
$\Re(\Psi^0_{22})$



- simulations at 3 different resolutions:
  $h_c = 1/48M$, $h_m = 1/52M$, $h_f = 1/56M$
- 4^{th} order accurate in merger signal
- 2^{nd} order convergence in 1^{st} & 2^{nd} reflection
- loosing convergence afterwards $\Rightarrow$ consider first 3 cycles

BBHs in a Box - Area and Spin

Relative horizon mass and spin



- successive increase in horizon area, mass and angular momentum during interaction between BH and gravitational radiation
- absorption of $\sim 15\%$ of GW energy per cycle
- increase of $\sim 5\%$ in angular momentum in *first* cycle
- *no* indication of superradiant amplification

BBHs in a Box

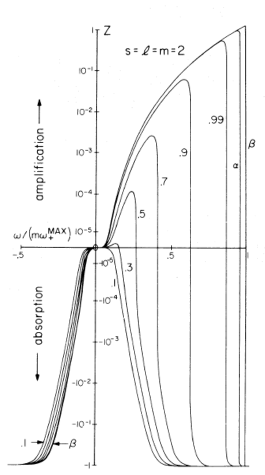
- IDEA:
  complementary phenomena
  - high frequency modes:
    absorbtion of mass and angular momentum from radiation

    $\Updownarrow$

  - low frequency modes:
    amplification of superradiant modes
    with $\omega < m\Omega_H$
- frequencies in superradiant and absorption regime
  $\Rightarrow$ at transition point between stable and superradiant regime?

ToDo:

- improvement of bcs $\Rightarrow$ long-term evolution
- model highly spinning BHs



Press & Teukolsky '74

Conclusions I

Evolutions of BH head-on collisions in $D = 5, 6$ dimensions

- good agreement between PP and numerical results for unequal mass binaries
- collisions from rest: increase in radiated energy with increasing dimension
- setup of initial data for boosted BH solving the constraints
- evolutions of collisions with small boost parameter

Issues & ToDo list:

- dependence of radiated energy on D and boost
- go beyond $D = 6$
- long-term stable evolutions for large boosts
- adjustment of gauge conditions
- modification of formulation

Conclusions II

mimic AdS-BH spacetimes by BHs in a box

- “BH bomb” like setup
- monitor interaction between Kerr BH and gravitational radiation
- evidence for absorption of radiation by the BH

Issues & ToDo list:

- no clear evidence for superradiant amplification
- systems becomes numerically unstable after few reflections
- improvement of boundary conditions
  $\Rightarrow$ long-term modelling of system
- increasing amplification rate with increasing spin
  $\Rightarrow$ evolve highly spinning BHs

Arigato!

http://blackholes.ist.utl.pt