Please acknowledge YITP
and register your papers:

- The workshop number is:

  YITP-T-11-08

  I will send it to you via email

- Please go to our workshop home page and follow the instruction

Close connection to Axiverse project:

- Project outline and Targets: Kodama’s talk
  Latest results: Yoshino’s talk

- black hole superradiance instabilities
  Helvi’s, Vitor’s and Oscar’s talks

- Very welcome to join the Axiverse sphere!

Towards singularity theorems in asymptotically anti-de Sitter spaces

Akihiro Ishibashi

5 April 2012 at YITP

Work in progress w/ Kengo Maeda

Instability in AdS spaces

- Numerical (in part, analytical) results

  Bizon – Rostworowski 2011
  (massless scalar field, spherical system)
  Dias – Horowitz – Santos 2011
  (vacuum, gravitational d.o.f. excited)

- Non-linear effects

- Oscar's talk : "AdS is linearly stable but non-linearly unstable.
  Generic small perturbations of AdS become large and eventually form black holes"

So far only a few pieces of supporting evidence and still speculative

Need further study by both numerical and analytical methods to proceed

- One of their motivations: Hawking-Penrose's singularity theorem in closed universe

"This theorem does not apply to AdS directly (because spacelike surfaces are not compact) but morally speaking, the negative cosmological constant acts like a confining box for fields inside. So one expects that generic solutions will be singular."

Dias-Horowitz-Santos '11

- Outline:

1. AdS instability
2. Singularity theorems
3. What we wish to show
4. A singularity theorem in AdS
5. Towards more general theorems

Singularity theorems

- "Singularity" in singularity theorem is defined as

  "incomplete causal" geodesic curve

- Show the existence of such an incomplete curve under

  1. Generic, and energy (convergence) conditions
  2. Global conditions (Causality)
  3. Strong-gravity (trapped region) condition

- Theorem 1 - (null geodesic incompleteness)

(1) null convergence

(2) non-compact Cauchy surface

(3) closed trapped surface

- Theorem 2 – (timelike or null incompleteness)

  (1) convergence for causal curves

  (2) generic condition

  (3) chronology condition

  (4) trapped set

    (i) compact, achronal set without edge

    (ii) closed trapped surface

    (iii) point p s.t. the null geodesics from p are focussed and start to reconverge

- Thoerem 3 - (past incomplete causal geodesic)

(1) convergence for every causal curves

(2) strong causality

(3) some past-dir unit timelike vector W at p s.t.

- Theorem 4 – (timelike incomplete geodesic)

(1) convergence for every causal curve

(2) compact spacelike hypersurface S (without edge)

(3) unit normals to S are everywhere converging on S

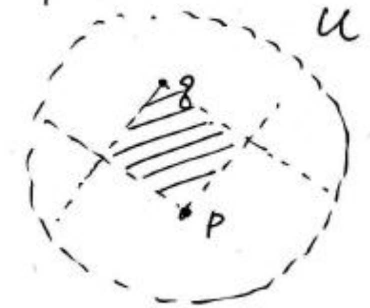
(2) Can be replaced w/ (2’) S is a Cauchy surface

Key notion I

① Global Hyperbolicity

- **Def.** $\mathcal{U}$ (open set) : globally hyperbolic

$:\Longleftrightarrow$
① $\forall$ pair of $p, q \in \mathcal{U}$.
$I^+(p) \cap I^-(q)$ has compact clousure.



② Strong causality.

- importance 😺 in physics

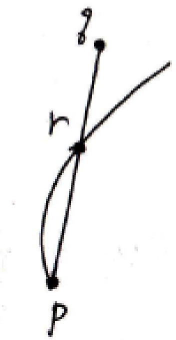
$\rightarrow$ $\exists$ Cauchy surface in $\mathcal{U}$.

Key notion II.

(ii) Conjugate points

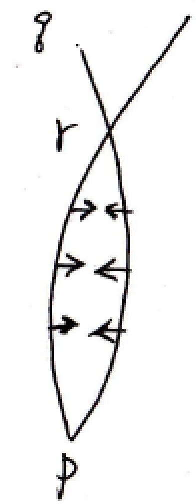
► **Def.** Point "r" is conjugate to "p"

$:\Leftrightarrow$ $\exists$ infinitesimally neighboring geodesic from "p" which intersects "r" in $(p, q]$



► Gravity is attractive.

$$\frac{d\theta}{ds} = -R_{ab}k^a k^b - \sigma_{ab}\sigma^{ab} - \frac{1}{n}\theta^2$$

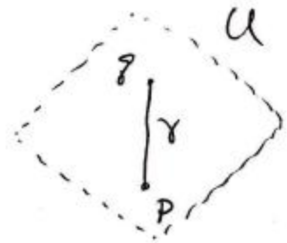


Significance of (I) Global Hyperbolicity & (II) Conjugate points.

(I) $\forall$ causally related $p, q \in \mathcal{U}$.

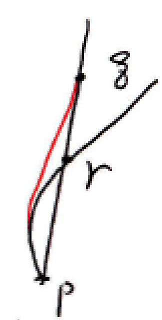
$\exists$ timelike (null) geodesic btween p and q

which maximizes the length of causal curves $p \to q$.



(II) If γ contains a pair of conj. p.t. (p, r)

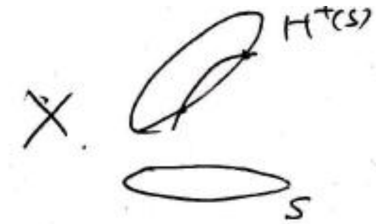
$\Rightarrow$ γ can be varied to a longer causal curve



- Hawking-Penrose Theorem shows:

There must be a globally hyperbolic region in which there should be a pair of conjugate points on every causal geodesic curve.

Proof. S: closed 3 achronal surface

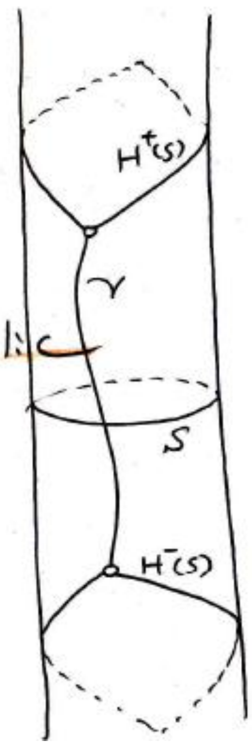


show: $H^+(S)$ is NON-compact or empty

$\Downarrow$

$\exists$ inextendible causal curve γ in $D(S)$

$\Uparrow$ globally hyperbolic



Let $\{x_n\}, \{y_n\}$ be sequences of points on γ. as in Fig.

$\exists$ timelike geodesic μ_n : $x_n \rightarrow y_n$: maximal length.

$\Downarrow$

$\exists$ timelike geodesic μ as a limit curve of $\{\mu_n\}$.

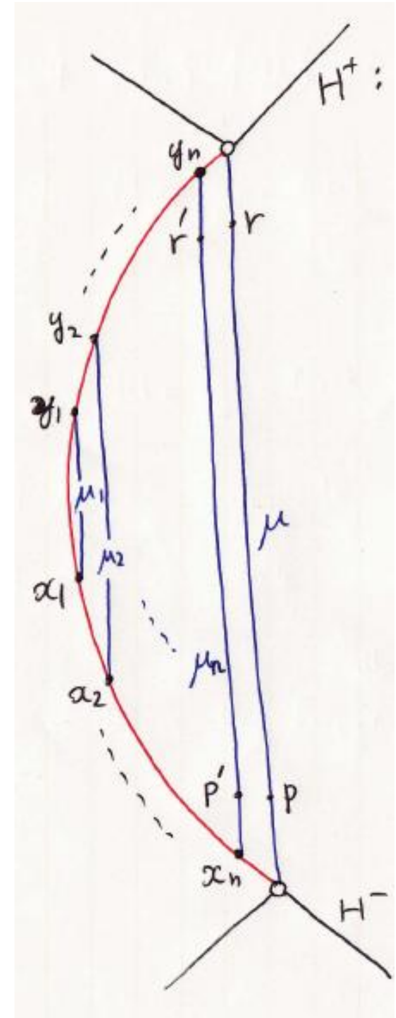
① If μ : in-complete $\rightarrow$ we are done!

② If μ : complete $\rightarrow$ μ contains pair conj. pts. (p, r)

$\Downarrow$

For sufficiently large n, μ_n would also contain pair conj. pts (p', r')

$\hookrightarrow$ contradiction!



What we wish to show

- Starting from No assumption of the trapped (strong-gravity) condition,

  Show the occurrence of a trapped set

(i) System.

{ • spherical
• perfect fluid.

• $ds^2 = -f\,dt^2 + h\,dr^2 + R^2 d\Omega^2$. $f,\ h,\ R : (t, r)$,

• $T_{ab} = \mu V_a V_b + P\,(g_{ab} + V_a V_b)$

t-coord. along fluid lines.

⇓

Raychaudhuri equation

$$\frac{d\theta}{ds} = -4\pi(\mu + 3P) - |\Lambda| - \sigma^2 - \frac{1}{3}\theta^2 + \nabla_a \dot{V}^a$$

Negative Λ enhances the contraction

$$\dot{V}^a \equiv V^c \nabla_c V^a.$$

Blackboard talk