Supergravity Backgrounds in IIB Matrix Model*)

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We construct wave functions and vertex operators in the type IIB matrix model by using a supersymmetric Wilson line operator. It is shown that wave functions can be intepreted as overlaps of the D-instanton boundary states with the closed string massless states. We then calculate the one-loop effective action for N D-instantons under the supergravity backgrounds by integrating out one D-instanton with an appropriate wave function.

§1. Introduction

Type IIB (IKKT) matrix model was proposed as a nonperturbative formulation of the type IIB superstring theory.[3] The action of the model is given by

$$S_{\mathrm{IKKT}} = -\frac{1}{4}\mathrm{tr}\ [A_\mu, A_\nu]^2 - \frac{1}{2}\mathrm{tr}\ \bar{\psi}\Gamma^\mu[A_\mu, \psi], \tag{1·1}$$

where A^μ $(\mu = 0, \cdots, 9)$ and ten-dimensional Majorana-Weyl fermion ψ are $N \times N$ bosonic and fermionic hermitian matrices, respectively. The action was originally derived from the Schild action for the type IIB superstring by regularizing the world sheet coordinates by matrices. In this model, the distribution of eigenvalues of bosonic matrices is interpreted to form space-time and fluctuations around it generate the dynamical fields on space-time. Therefore we can discuss dynamics of space-time directly in this model.[4)–7)] It is interesting that this action has the same form as the low energy effective action for N D-instantons.

It is important to understand how this model includes dynamics of the closed string modes. Here we investigate effective actions of the matrix model in the supergravity backgrounds generated by a D-instanton. To this end, we consider the system of N D-instantons which is embedded in larger size $(N+1) \times (N+1)$ matrices,[8]

$$\begin{pmatrix} ND(-1) & \\ & 1D(-1) \text{ as background for } ND(-1) \end{pmatrix}. \tag{1·2}$$

A single D-instanton on the lower right block (we call this a mean-field D-instanton) generates backgrounds for effective actions of N D-instantons through interactions mediated by off-diagonal components. By choosing an appropriate wave function for the mean-field D-instanton, the effective action under the supergravity background can be derived.

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§2. Wave functions and vertex operators

2.1. *Wave functions*

The degrees of freedom of a D-instanton are described by its coordinates, a ten-dimensional vector y_μ and a Majorana-Weyl fermion λ. Thus information of its state is encoded in wave functions of y_μ and λ. Here we consider wave functions corresponding to the supergravity modes in the form of $f_A(\lambda)e^{ik\cdot y}$ with a momentum k, where the index A specifies each mode. Wave functions are defined to form a multiplet of the following $d=10$ $\mathcal{N}=2$ supersymmetry transformations

$$\delta^{(1)}f(\lambda)=[\bar{\epsilon}_1 q_1, f(\lambda)]=\epsilon_1\frac{\partial}{\partial\lambda}f(\lambda), \tag{2·1}$$

$$\delta^{(2)}f(\lambda)=[\bar{\epsilon}_2 q_2, f(\lambda)]=(\bar{\epsilon}_2\not{k}\lambda)f(\lambda), \tag{2·2}$$

where the transformation parameters ϵ_i $(i=1,2)$ are the Majorana-Weyl spinors. Imposing the massless condition $k^2=0$ in order to derive wave functions for massless modes, $\not{k}\lambda$ in (2·2) has 8 independent degrees of freedom. Then there are 2^8 independent wave functions for λ. They form the type IIB supergravity multiplet containing a complex dilaton $\Phi(\lambda,k)$, a complex dilatino $\tilde{\Phi}(\lambda,k)$, a complex antisymmetric tensor field $B_{\mu\nu}(\lambda,k)$, a complex gravitino $\Psi_\mu(\lambda,k)$, a real graviton $h_{\mu\nu}(\lambda,k)$ and a real 4th-rank self-dual antisymmetric tensor field $A_{\mu\nu\rho\sigma}(\lambda,k)$.

These wave functions were constructed in Ref. 1). In the construction, we assumed that the dilaton wave function is $\exp(-ik\cdot y)$ which is annihilated by the supersymmetry q_1 (2·1). Then the other wave functions are determined by the supersymmetry transformations. Some of concrete forms of the wave functions are as follows.

$$\Phi(\lambda,k)=1, \tag{2·3}$$

$$B_{\mu\nu}(\lambda,k)=-\frac{1}{2}b_{\mu\nu}(\lambda), \tag{2·4}$$

$$h_{\mu\nu}(\lambda,k)=\frac{1}{96}b_\mu{}^\rho b_{\rho\nu}(\lambda), \tag{2·5}$$

$$A_{\mu\nu\rho\sigma}(\lambda,k)=-\frac{i}{32(4!)^2}b_{[\mu\nu}b_{\rho\sigma]}(\lambda), \tag{2·6}$$

where we defined a fermion bilinear as $b_{\mu\nu}=k_\rho\bar{\lambda}\Gamma_{\mu\nu\rho}\lambda$. For complete expressions of the wave functions and details of their derivations, see Ref. 1).

2.2. *Stringy interpretation of wave functions*

The wave functions given in the previous subsection have a definite physical meaning in string theory side. We use the Green-Schwarz formalism in the light-cone gauge and a description of a D-instanton by boundary states. The D-instanton is a half BPS state and break a half of supersymmetries. Hence we can construct a supersymmetry multiplet by acting broken supersymmetry generators on the boundary state of a D-instanton. The ordinary D-instanton couples only to scalar fields, the dilaton and axion. But states in this supersymmetry multiplet interact with various supergravity modes such as gravitons or antisymmetric tensor fields through derivative couplings.[9)–16)] These derivative couplings also appear in block-block interactions at the one-loop in the IIB matrix model. We show that the wave functions coincide with overlaps between states which are obtained by acting broken supercharges on the D-instanton boundary state and closed string massless states.

We adopt the Green-Schwarz formalism in the light-cone gauge. In this gauge, the IIB superstring theory is described by eight bosons $X^i(\tau,\sigma)$ $(i=1,\cdots,8)$ and two $SO(8)$ spinors $S^{Aa}(\tau,\sigma)$ $(A=1,2;\ a=1,\cdots,8)$ on the world sheet. $\mathcal{N}=2$ supersymmetry is generated by the following supercharges,

$$Q^{Aa}=\int_0^{2\pi}\frac{d\sigma}{2\pi}\sqrt{2p^+}S^{Aa}, \tag{2·7}$$

$$Q^{1\dot{a}}=\int_0^{2\pi}\frac{d\sigma}{2\pi}\frac{1}{\sqrt{p^+}}\left(\partial_\tau-\partial_\sigma\right)X^i\gamma^i_{\dot{a}a}S^{1a}, \tag{2·8}$$

$$Q^{2\dot{a}}=\int_0^{2\pi}\frac{d\sigma}{2\pi}\frac{1}{\sqrt{p^+}}\left(\partial_\tau+\partial_\sigma\right)X^i\gamma^i_{\dot{a}a}S^{2a}. \tag{2·9}$$

The boundary state for the D-instanton is defined by the following boundary conditions,

$$\partial_\sigma X^i|D(-1)\rangle=0, \tag{2·10}$$

$$Q^{+a}|D(-1)\rangle\equiv\left(Q^{1a}+iQ^{2a}\right)|D(-1)\rangle=0, \tag{2·11}$$

$$Q^{+\dot{a}}|D(-1)\rangle\equiv\left(Q^{1\dot{a}}+iQ^{2\dot{a}}\right)|D(-1)\rangle=0. \tag{2·12}$$

The D-instanton boundary state preserves a half of supersymmetries Q^{+a} and $Q^{+\dot{a}}$, and breaks the other half $Q^{-a}\equiv Q^{1a}-iQ^{2a}$ and $Q^{-\dot{a}}\equiv Q^{1\dot{a}}-iQ^{2\dot{a}}$. The broken and unbroken supercharges satisfy the algebra

$$\begin{aligned}\{Q^{+a},Q^{-b}\}&=4p^+\delta_{ab}, & \{Q^{+a},Q^{-\dot{b}}\}&=2\sqrt{2}\gamma^i_{a\dot{b}}p^i,\\ \{Q^{+\dot{a}},Q^{-b}\}&=2\sqrt{2}\gamma^i_{\dot{a}b}p^i, & \{Q^{+\dot{a}},Q^{-\dot{b}}\}&=2P^-\delta_{\dot{a}\dot{b}},\end{aligned} \tag{2·13}$$

where p^+ and p^i are components of the momentum and P^- is the light-cone hamiltonian. The other anticommutators vanish.

In order to see the correspondence between boundary states and the wave functions systematically, we introduce a fermionic coherent state by acting $\exp(-\lambda Q^{-a})$ on $|D(-1)\rangle$;

$$|\lambda\rangle=\exp(-\lambda^a Q^{-a})|D(-1)\rangle, \tag{2·14}$$

where λ is a $SO(8)$ spinor and can be related with the fermionic coordinates of a D-instanton as seen below. This state satisfies modified boundary conditions

$$Q^{+a}|\lambda\rangle=4p^+\lambda^a|\lambda\rangle, \tag{2·15}$$

$$Q^{+\dot{a}}|\lambda\rangle=2\sqrt{2}p^i\gamma^i_{\dot{a}a}\lambda^a|\lambda\rangle. \tag{2·16}$$

The wave functions given in the previous subsection can be written as

$$f_A(\lambda)=\langle A|\lambda\rangle, \tag{2·17}$$

for each supergravity mode A. This can be understood as follows. When the momentum k is taken as $k^\mu = \frac{1}{\sqrt{2}}(p^+, 0, \cdots, 0, p^+)$, the supercharges q_1 and q_2 for a D-instanton, (2·1) and (2·2), have the following forms,

$$q_1^a = -i\frac{\partial}{\partial\lambda^a}, \qquad q_2^a = \sqrt{2}ip^+\lambda^a, \tag{2·18}$$

and satisfy the algebra

$$\left\{q_1^a,\ q_2^b\right\} = \sqrt{2}p^+\delta_{ab}, \qquad \text{others} = 0. \tag{2·19}$$

On the other hand, as far as massless states are concerned, this algebra is equivalent to the ones among the supercharges $Q^{\pm a}$ and $Q^{\pm\dot{a}}$ with $p^i = P^- = 0$, eq.(2·13). Actually actions of q_i^a on the wave functions (2·17) can be regarded as insertions of Q^{-a} and Q^{+a} which act on the massless state of the supergravity modes $|A\rangle$ as follows,

$$q_1^a f_A(\lambda) = -i\frac{\partial}{\partial\lambda^a} f_A(\lambda) = i\left(\langle A|Q^{-a}\right)|\lambda\rangle,$$
$$q_2^a f_A(\lambda) = \sqrt{2}ip^+\lambda^a f_A(\lambda) = \frac{i}{2\sqrt{2}}\left(\langle A|Q^{+a}\right)|\lambda\rangle,$$

where we have used eqs.(2·15) and (2·16). Hence a construction of the supergravity multiplet by acting $Q^{\pm a}$ on the closed string massless state $\langle A|$ corresponds to the one by acting q_i^a on wave functions $f_A(\lambda)$ and the wave functions we constructed describe the (derivative) couplings between a D-instanton and various supergravity modes.

2.3. *Vertex operators*

In the IIB matrix model, interactions corresponding to the closed string modes are induced as quantum effects and their couplings are described through the vertex operators. Here we consider vertex operators only for the supergravity modes.[17)]

Vertex operators $V_A(A^\mu, \psi; k)$ covariantly transform under the following $\mathcal{N} = 2$ supersymmetry of the IIB matrix model,

$$\left\{\begin{array}{l} \delta^{(1)}A_\mu = i\bar{\epsilon}_1\Gamma_\mu\psi, \\ \delta^{(1)}\psi = -\frac{i}{2}[A_\mu, A_\nu]\Gamma^{\mu\nu}\epsilon_1, \end{array}\right. \qquad \left\{\begin{array}{l} \delta^{(2)}A_\mu = 0, \\ \delta^{(2)}\psi = \epsilon_2 1_N. \end{array}\right. \tag{2·20}$$

We denote the generator of $\delta^{(i)}$ $(i = 1, 2)$ as Q_i $(i = 1, 2)$, respectively. Since the $\mathcal{N} = 2$ supersymmetry algebra closes only on shell, in this section we assume that the $N \times N$ matrices A_μ and ψ satisfy the equations of motion for the IKKT action (1·1),

$$[A^\nu, [A_\mu, A_\nu]] - \frac{1}{2}(\Gamma_0\Gamma_\mu)_{\alpha\beta}\{\psi_\alpha, \psi_\beta\} = 0, \tag{2·21}$$
$$\Gamma^\mu[A_\mu, \psi] = 0. \tag{2·22}$$

In order to construct vertex operators systematically, we use a supersymmetric Wilson line operator.[18] Since we are interested in the massless multiplet, we here consider the simplest straight Wilson line operator with a global momentum k;

$$\omega(\lambda, k) = e^{\bar{\lambda}Q_1}\ tre^{ik\cdot A}\ e^{-\bar{\lambda}Q_1}. \tag{2·23}$$

The vertex operators are obtained by expanding $\omega(\lambda, k)$ in terms of the wave functions $f_A(\lambda)$ as

$$\omega(\lambda, k) = \sum_A f_A(\lambda)\ V_A(A_\mu, \psi;\ k). \tag{2·24}$$

Some of concrete forms of vertex operators are as follows.

- dilaton

$$V^\Phi = \mathrm{tr}\ e^{ik\cdot A}, \tag{2·25}$$

- antisymmetric tensor field

$$V^B_{\mu\nu} = \mathrm{Str}\ e^{ik\cdot A}\left(\frac{1}{16}k^\rho(\bar{\psi}\cdot\Gamma_{\mu\nu\rho}\psi) - \frac{i}{2}[A_\mu, A_\nu]\right), \tag{2·26}$$

- graviton

$$\begin{aligned} V^h_{\mu\nu} = 2\ \mathrm{Str}\ e^{ik\cdot A}\Big\{&[A_\mu, A^\rho]\cdot[A_\nu, A_\rho] + \frac{1}{4}\bar{\psi}\cdot\Gamma_{(\mu}[A_{\nu)}, \psi] - \frac{i}{8}k^\rho\bar{\psi}\cdot\Gamma_{\rho\sigma(\mu}\psi\cdot[A_{\nu)}, A^\sigma] \\ &-\frac{1}{8\cdot 4!}k^\lambda k^\tau(\bar{\psi}\cdot\Gamma_{\mu\lambda}{}^\sigma\psi)\cdot(\bar{\psi}\cdot\Gamma_{\nu\tau\sigma}\psi)\Big\}, \end{aligned} \tag{2·27}$$

- 4-th rank self-dual antisymmetric tensor field

$$\begin{aligned} V^A_{\mu\nu\rho\sigma} = -i\ \mathrm{Str}\ e^{ik\cdot A}\Big\{&F_{[\mu\nu}\cdot F_{\rho\sigma]} - \frac{1}{3}\bar{\psi}\cdot\Gamma_{[\mu\nu\rho}[A_{\sigma]}, \psi] + \frac{i}{4}k^\lambda\bar{\psi}\cdot\Gamma_{\lambda[\mu\nu}\psi\cdot F_{\rho\sigma]} \\ &-\frac{1}{8\cdot 4!}k^\lambda k^\tau(\bar{\psi}\cdot\Gamma_{\lambda[\mu\nu}\psi)\cdot(\bar{\psi}\cdot\Gamma_{\rho\sigma]\tau}\psi)\Big\}. \end{aligned} \tag{2·28}$$

§3. Condensation of the supergravity modes

3.1. *One-loop effective action*

In this section, we investigate condensation of the supergravity fields in the type IIB matrix model. We consider a matrix model with $(N+1)\times(N+1)$ bosonic matrices A'_μ $(\mu = 0, \cdots, 9)$ and fermionic ones ψ'. We decompose them into backgrounds (X_μ, Φ) and fluctuations (a_μ, φ) as,

$$A'_\mu = X_\mu + a_\mu, \qquad \psi' = \Phi + \varphi. \tag{3·1}$$

Then we take backgrounds which consist of $N\times N$ and 1×1 blocks, and the fluctuations which are represented by N-vectors (α_μ, ϕ),

$$X_\mu = \begin{pmatrix} x_\mu 1_N + A_\mu & 0 \\ 0 & y_\mu \end{pmatrix}, \qquad \Phi = \begin{pmatrix} \psi & 0 \\ 0 & \xi \end{pmatrix}, \tag{3·2}$$

$$a_\mu = \begin{pmatrix} 0 & \alpha_\mu \\ \alpha_\mu^\dagger & 0 \end{pmatrix}, \qquad \varphi = \begin{pmatrix} 0 & \phi \\ \phi^\dagger & 0 \end{pmatrix}. \tag{3·3}$$

Since interactions corresponding to the closed string modes appear as quantum effects in the matrix model, we first integrate over the fluctuations a_μ and φ. The free energy at the one-loop is derived by expanding the action up to the second order of the fluctuations and integrating over the fluctuations,[19),20)]

$$F(X,\Phi) = -\ln \int da_\mu d\varphi dbdc\ e^{-(S_{IKKT}+S_{g.f.+ghost})}$$
$$= S_{IKKT} + F_b + F_f, \tag{3·4}$$
$$F_b = \frac{1}{2}\mathcal{T}r \ln\left(\delta_{\mu\nu}\tilde{X}^2 + 2\tilde{F}_{\mu\nu}\right) - \frac{1}{4}\mathcal{T}r\ln\left(\tilde{X}^2 + \frac{1}{2}\Gamma_{\mu\nu}\tilde{F}_{\mu\nu}\frac{1+\Gamma_{11}}{2}\right)$$
$$-\mathcal{T}r\ln\tilde{X}^2, \tag{3·5}$$
$$F_f = \frac{1}{2}\mathcal{T}r\ln\left[\delta_{\mu\nu} + \left(\frac{1}{\tilde{X}^2+2\tilde{F}}\right)_{\mu\rho}\tilde{\bar{\Phi}}\Gamma_\rho\frac{1}{\Gamma\cdot\tilde{X}}\Gamma_\nu\tilde{\Phi}\right], \tag{3·6}$$

where we denoted the adjoint action of a general operator O as $\tilde{O}S \equiv [O,S]$ and $\mathcal{T}r$ is the trace of the adjoint operators. $S_{g.f.+ghost}$ is added to the IKKT action to fix a gauge. The free energy obtained in this way is a function of the diagonal components $A_\mu, x_\mu, y_\mu, \psi$ and ξ, $F(A,x,\psi;y,\xi)$, and includes various interactions between these two diagonal blocks through the closed string modes, in particular the supergravity modes. Hence we can see in the free energy that a D-instanton couples to the supergravity modes through derivative couplings.

In the following calculations, we expand the free energy with respect to $1/|d|$ ($d_\mu \equiv x_\mu - y_\mu$). Since $|d|$ is a distance between N D-instantons and a single D-instanton, this expansion is valid when the contributions for the free energy mainly come from configurations in which the single D-instanton is far separated from the other N D-instantons.

We next choose wave functions $f_A(y,\xi)$ for a D-instanton described by (y_μ,ξ) and integrate over y_μ and ξ. Then we obtain a modified effective action $S_{eff}(A,x,\psi;f_A)$ under condensation of the supergravity modes;

$$e^{-S_{eff}(A,x,\psi;f_A)} = \int dyd\xi\ e^{-F(A,x,\psi;y,\xi)}f_A(y,\xi). \tag{3·7}$$

In what follows, we mainly look at terms without fermionic matrices ψ and replace all fermionic variables by the D-instanton fermionic coordinate ξ.

3.2. *Condensation of the antisymmetric tensor $B_{\mu\nu}$*

We now calculate the effective action with an insertion of a wave function describing the antisymmetric tensor field $B_{\mu\nu}$. We use the following wave function for the antisymmetric tensor field;

$$f_B(y,\xi) = \int d^{10}k e^{-ik\cdot y}\zeta_{\mu\nu}(k)k_\rho\left(\Gamma_{\mu\nu\rho}\Gamma_0\right)_{\alpha\beta}\frac{\partial}{\partial\xi_\alpha}\frac{\partial}{\partial\xi_\beta}\left(\prod_{\gamma=1}^{16}\xi_\gamma\right), \tag{3·8}$$

where $\zeta_{\mu\nu}(k)$ is a polarization tensor, $\zeta_{\mu\nu}(k) = -\zeta_{\nu\mu}(k)$.

With the choice of the wave function (3·8), we need to look at second order terms of ξ in the free energy. The contributions start from the order $1/d^8$ and terms at the order $1/d^8$ and $1/d^9$ have the following forms,

$$
\begin{aligned}
&-\frac{1}{2d^8}\left(\bar{\xi}\Gamma_{\mu\rho\sigma}\xi\right) tr\left[A_\nu, F_{\mu\nu}\right] F_{\rho\sigma} \\
&-12\frac{d_\lambda}{d^{10}}\left(\bar{\xi}\Gamma_{\mu\nu\lambda}\xi\right) \mathrm{tr}\left(F_{\mu\rho}F_{\rho\sigma}F_{\sigma\nu} - \frac{1}{4}F_{\mu\nu}F_{\rho\sigma}F_{\sigma\rho}\right) \\
&+\frac{2}{d^{10}}\left(\bar{\xi}\Gamma_{\mu\rho\sigma}\xi\right) \mathrm{tr}\left(\left[A_\nu, F_{\mu\nu}\right](d\cdot A)F_{\rho\sigma} + \left[A_\nu, F_{\mu\nu}\right] F_{\rho\sigma}(d\cdot A)\right).
\end{aligned}
\tag{3·9}
$$

The second line represents an interaction through the vertex operator for (the charge conjugation of) the antisymmetric tensor field. Since the interaction between one D-instanton and the antisymmetric tensor field contains one derivative, $d_\mu/d^{10} \sim \partial_\mu\left(1/d^8\right)$, this term appears at $\mathcal{O}(1/d^9)$. A meaning of the other terms is given below.

After the integration of y_μ and ξ with the wave function (3·8), we obtain the effective action under condensation of the antisymmetric tensor field,

$$
\begin{aligned}
S_{eff}(A,x,\psi;f_B) = S_{IKKT} - i\int d^{10}k f_{\mu\nu\rho}(k)e^{ik\cdot x}\Big\{ &tr\left[A_\sigma, F_{\mu\sigma}\right] F_{\nu\rho}, \\
&-3ik_\rho tr\left(F_{\mu\sigma}F_{\sigma\lambda}F_{\lambda\nu} - \frac{1}{4}F_{\mu\nu}F_{\sigma\lambda}F_{\lambda\sigma}\right) \\
&+\frac{1}{2}tr\left(\left[A_\sigma, F_{\mu\sigma}\right](ik\cdot A)F_{\nu\rho} + \left[A_\sigma, F_{\mu\sigma}\right]F_{\nu\rho}(ik\cdot A)\right)\Big\},
\end{aligned}
\tag{3·10}
$$

where $f_{\mu\nu\rho}(k) = i(k_\mu\zeta_{\nu\rho} + k_\nu\zeta_{\rho\mu} + k_\rho\zeta_{\mu\nu})/k^2$. Here we assumed an appropriate regularization in the infrared region of y_μ integration and renormalized the polarization tensor $\zeta_{\mu\nu}(k)$. The first and third terms can be combined into a form

$$
-i\int d^{10}k f_{\mu\nu\rho}(k)e^{ik\cdot x}\mathrm{S}tr\ e^{ik\cdot A}\left[A_\sigma, F_{\mu\sigma}\right]\cdot F_{\nu\rho},
\tag{3·11}
$$

up to the sixth order of A_μ.

This effective action shows that the Chern-Simons-like term (3·11) is induced by an effect of condensation of the antisymmetric tensor field. This phenomenon is similar to the Myers effect,[21)] but there is a difference. In the case of the Myers effect for D0-branes, a cubic term of bosonic matrices is induced in the RR three-form background. This term can be interpreted as a vertex operator for the RR potential. In our case, however, this term (3·11) is different from the expected vertex operator for the charge conjugation of the antisymmetric tensor field[2)] (the vertex operator is on the second line of eq.(3·10)). The reason can be understood as follows. If we also calculate the fermionic term containing ψ, we would expect to obtain a term like tr $\left(\bar{\psi}\Gamma_\mu\psi\right)F_{\nu\rho}$ and the leading order term in (3·10) with this fermionic term would be cancelled by using the equation of motion (2·21) of the original IKKT action. This kind of terms can not be seen in the vertex operators since we have assumed the equation of motion (2·21) and (2·22) in their construction. Here, since we are interested in investigating the effective actions under condensation of the antisymmetric tensor fields, we do not use the equations of motion of the original IKKT action and the Chern-Simons like term in (3·10) should not be omitted.

Let us see an effect of the induced term in (3·10) for a particular form of the polarization tensor. Assuming that the region $k \sim 0$ is dominant in the k-integration and that the coefficient $\int d^{10}k f_{\mu\nu\rho}(k) e^{ik\cdot x}$ is proportional to ϵ_{ijk} with a specific direction $(i,j,k) = (1,2,3)$, the modified matrix model action becomes

$$S_{eff}(A, x, \psi; f_B) = S_{IKKT} - i\alpha\epsilon_{ijk} tr\, [A_\nu, F_{i\nu}]\, F_{jk}, \tag{3·12}$$

with a constant coefficient α. This action has a fuzzy sphere classical solution; $A_i = \frac{1}{10\alpha} L_i$ $(i = 1,2,3)$ and the others vanish. The radius of the fuzzy sphere is in inverse proportion to the coefficient α.

In addition to the fuzzy sphere solution, flat D-branes

$$[A_\mu,\ A_\nu] = i\theta_{\mu\nu} 1_N \qquad (\theta_{\mu\nu} = -\theta_{\nu\mu}), \tag{3·13}$$

with a constant $\theta_{\mu\nu}$ are also classical solutions of the effective action (for an infinite N). It will be interesting to compare stabilities of these solutions to the fuzzy sphere solution by calculating loop corrections around them.

3.3. *Condensation of the graviton*

Effects of the condensation of gravitons can be seen from the fourth order terms of ξ in the effective action. The leading order terms start from the $\mathcal{O}(1/d^{10})$,

$$-\frac{1}{d^{12}} c_{\mu\rho} c_{\rho\nu} \mathrm{tr}\ F_{\mu\sigma} F_{\sigma\nu}, \tag{3·14}$$

where $c_{\mu\nu}(\xi) \equiv d_\rho \left(\bar{\xi}\Gamma_{\mu\nu\rho}\xi\right)$. It represents a derivative coupling of a single D-instanton to the graviton vertex operator constructed from the N D-instantons; $d_\mu d_\nu / d^{12} \sim \partial_\mu \partial_\nu \left(1/d^8\right)$. If we insert the graviton wave function and integrate over the single D-instanton coordinates, we can obtain the graviton vertex operator as an induced term in the effective action.

Similarly interactions mediated by the 4th-rank self-dual antisymmetric tensor field would appear, but such terms vanish in the leading order because of the cyclic property of the trace and the Jacobi identity.

§4. Conclusion

We have considered effective actions under the supergravity backgrounds in the IIB matrix model. We first constructed wave functions for the mean-field D-instanton and vertex operators in the matrix model which couple to the supergravity modes. These are obtained as massless representations of $\mathcal{N} = 2$ supersymmetry by using the supersymmetric Wilson line operator. It was shown that the wave functions can be interpreted as overlaps of the D-instanton boundary states with the supergravity states.

We then considered the one-loop effective actions for the N D-instanton system under backgrounds generated by the mean-field D-instanton by applying the wave functions we constructed. We showed that the vertex operators are induced in the effective action as leading contributions, if the equations of motion of the IKKT matrix model are assumed. If we do not assume them, a Chern-Simons like term is induced in the leading order of perturbations under backgrounds of the antisymmetric tensor field. Though this term is quintic with respect to A_μ, a fuzzy sphere becomes a solution to the equations of motion. In this sense this is a similar phenomenon to the Myers effect.

It is important to investigate dynamics of the closed string modes in the IIB matrix model further. In particular, it is interesting to derive the equations of motion for the supergravity fields from the matrix model. Loop equations or the large N renormalization group in the matrix model are expected to play a crucial role in such studies.

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