Quantum Information and Space-time

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A pedestrian description of possible connection between quantum information theory and relativity theory is given. I picked up topics of synchronization by entanglement and causality of completely positive maps in special relativity, while the problems of information loss in the black hole evaporation and the possible quantum state at the space-like singularities are briefly touched upon in general relativity but in short of quantum gravity.

§1. Introduction

When Einstien published his celebrated paper on special relativity one hundred years ago, he considered the adjustment of two clocks located at different places by sending light signals back and forth.(1) This operational and even epistemological way of exposition of physical theory is rare and strangely has not been used in the textbooks of relativity after relativity theory is established.

Actually as far as classical theory is concerned, epistemology is more or less straightforward and can be a matter of taste, because physical quantity has one to one correspondence to what can be observed by experiments. However, in quantum world this is not the case. We have to take into account how we perform experiments. As Einstein Podolsky and Rosen(2) got an insight in their paper of EPR "paradox", physical reality and communication are not separable in quantum world and epistemology has come into play of essential role. I honestly do not know whether this direction of research opens up breakthrough to the long standing problem of quantum gravity. But at least it is amusing to collect materials to look at quantum gravity from quantum epistemology, i.e. quantum information theory. First I will quickly summarize basic things in quantum information theories and its important consequences. Then I will pick up topics on quantum information theoretical aspects of special relativity and general relativity.

§2. Quantum Information

In the standard undergraduate course, students are normally taught that quantum mechanics is based on the principles, (1) superposition principle (2) Schrödinger evolution (3) probabilistic interpretation and perhaps in more advanced course (4) many particle states are tensor product of single particle state. Quantum information theory is more concerned with the last two principles.

One of the power of quantum information theory is its ability to state such and such for all possible experiments, normally given in a form of inequality, example of which will be shown shortly later. Most general physical operation including measurement is described by completely positive (CP) map Λ of a quantum state to another state: $\rho \to \rho' = \Lambda(\rho)$[3).4)] Positive map sends any positive operators to positive operators defined in the Hilbert space H for the system, while completely positivity demands further that the mapping $\Lambda \otimes 1$ is positive even for all trivially extended Hilbert space $H \otimes H'$. This mathematical definition of CP maps has a simple physical meaning that the extended Hilbert spaces correspond to detectors and possible environments. Operationally, the CP map can be described by a combination of a unitary transformation of compound system of interest and the detector/environment and then tracing out over the additional states.

One of the important result of this precise treatment of measurement is Ozawa's uncertainty relation.[5)] The famous Heisenberg's uncertain principle has been presented in an unsatisfactory way; either an intuitive but imprecise way on the basis of gedanken experiment by a gamma ray microscope or mere Schwarz inequality. Ozawa gave a rigorous inequality among the error of measured position of a particle δq, the disturbance of momentum δp, by the position measurement and the standard deviation of them, $\Delta q, \Delta p$,

$$\delta p \delta q + \Delta p \delta q + \delta p \Delta q \geq \hbar/2. \tag{2·1}$$

In particular, the momentum δp does not necessarily diverge even if we have made a precise measurement $\delta q = 0$ in contrast with the conventional uncertainty relation.

Fundamental resource of quantum information science is the entanglement. Long ago, Einstein Podolsky and Rosen (EPR)[2)]wrongly argued that the apparent nonlocality of an entangled sate,

$$|\psi>= (|0>_A |1>_B - |1>_A |0>_B)/\sqrt{2} \tag{2·2}$$

leads to an incomplete description of "physical reality" if we insist on Copenhagenian interpretation of quantum mechanics. However, EPR opened up the research field which has been now flourishing as quantum information science via important theoretical and experimental works[6),7).8)]

In what follows we review possible implication of quantum information theory to special relativity and general relativity.[9)]

§3. Special relativity

3.1. *Synchronization*

Let us first consider the synchronization of two clocks located at different places. Apart from its fundamental importance, it is intriguing also practically e.g., for GPS system. Some years ago, Jozsa proposed a use of entanglement to adjust two clocks.[10)] Basic idea is more similar to the way of Eddington[11)]rather than Einstein's way.[1)] Eddington's way is simply carrying the clocks adjusted at the same place to different places.

Consider a two spin-halves system with the Hamiltonian

$$H = \Omega(S_z^A + S_z^B) \tag{3·1}$$

and prepare the singlet state, which is the EPR state,[2)]

$$|\psi_0>=|->_A|+>_B-|+>_A|->_B \tag{3·2}$$

at $t=0$ with $|\pm>=\frac{|0>\pm|1>}{\sqrt{2}}$ is the eigen state of the x component of the spin σ_x with the eigen values ±1.

Suppose Alice measures her own $\sigma_x^{(A)}$ to obtain ∓1. Note that it is not necessary to specify when she performs the measurement because the singlet state $|\psi_0>$ does not evolve in time. Upon the measurement the state collapses to

$$|\psi_1=\pm|\mp>_A|\pm>_B \tag{3·3}$$

After that the state evolves by the Hamiltonian above

$$|\psi_1(t)>=\pm\frac{1}{2}(|0>_A\mp e^{-i\Omega t}|1>_A)(|0>_B\pm e^{-i\Omega t}|1>_B) \tag{3·4}$$

We see that the time evolutions of the states of Alice and Bob are identical, which suggests the synchronization of the two clocks. Let us examine the consequence of the observation, Suppose Bob observes his own $\sigma_x^{(B)}$. The probability to get $+1$ is

$$P(|+>_B,t,|\mp>_A)=\frac{1}{2}(1\pm cos\Omega t). \tag{3·5}$$

On the other hand Alice sends the outcome $\pm1$of the measurement $\sigma_x^{(A)}$ to Bob by classical communication. On the basis of the information from Alice, Bob knows that which probability distribution he has to adopt. Repeat this process of measurements and classical communications of Alice and then the measurement by Bob. Collecting the outcomes of Bob's observation with the classical signal from Alice -1, the probability distribution to obtain $|+>$ at t_B has to be $\frac{1}{2}(1+cos\Omega t_B)$. so that Bob can deduce the time t_B. For N singlets the precision of time is $\delta t=\frac{1}{\sqrt{N}\Omega}$. Note that if Alice did not provide any classical information, Bob would get flat distribution of t_B.

It is interesting to point out that both Alice and Bob have to measure $\sigma_x^{(A)}$ and $\sigma_x^{(B)}$ adjusting the orientation of the x axes at different places before operation, which is also non-trivial.[14)]The synchronization a lá Einstein by exchange of light signals, which is normally used in GPS, has error through the change of light velocity due to the fluctuation of plasma density, while the method by EPR pair does not. However, it has a defect in the alignment of the coordinate axes. See[15)] for details. This synchronization by entanglement obviously motivated by the seminal work by Einstein uses local operation and classical communication (LOCC), a standard paradigm of quantum information.[4)] However, it seems unsatisfactory from the view point of fundamental physics. Everything including communication has to be fully quantum.

3.2. *Causality and Semi-localizable Operation*

As we stated before the most general quantum operation is the CP map. However, not all of the CP maps are physical. For example, a CP map operation at remote places cannot be done simultaneously, because of the relativistic causality. Therefore the natural question is what is the necessary and sufficient condition for the CP maps that satisfy the relativistic causality[16)17)]?

Let Alice' and Bob's states be $|n>_A \in \mathcal{H}_A$ and$|n>_B \in \mathcal{H}_B$, respectively. We also introduce the state $|a)$ of mediator Φ with a photon in mind as an example. Prepare the initial state

$$|\Psi_0 >= \sum_n \sqrt{p_n}|0) \cdot |n>_A \cdot |n>_B, \tag{3·6}$$

Here we slightly abuse the notation by writing as $|0) \cdot |n>_A$ if the particle Φ in the state $|0)$ is located far from the particle A in the sate $|n>_A$,while they become close we simply write it as $|0)|n>_A$. Suppose when Φ approaches A they interact and the state in $\mathcal{H}_\Phi \otimes \mathcal{H}_A$ undergoes a unitary transformation U,

$$\to |\Psi_1 >= \sum_n \sqrt{p_n} U(|0)|n>_A) \cdot |n>_B, \tag{3·7}$$

with

$$|\Psi_1 >= \sum_n \sum_{\alpha,m} \sqrt{c_n} U^\alpha_{nm} |\alpha)|m>_A \cdot |n>_B, \tag{3·8}$$

where $U^\alpha_{nm} = (\alpha|U|0)_{nm}$.

Then send the particle Φ to Bob;

$$\sum_n \sum_{\alpha,m} \sqrt{c_n} U^\alpha_{nm} |m>_A \cdot |\alpha)|n>_B, \tag{3·9}$$

Then Bob and Φ can interact to cause another unitary transformation V in $\mathcal{H}_\Phi \otimes \mathcal{H}_B$ as

$$\begin{aligned} \to |\Psi_2> &= \sum_n \sum_{\alpha,m} \sqrt{p_n} U^\alpha_{nm} |m>_A \cdot V(|\alpha)|n>_B), \\ &= \sum_n \sum_{\alpha,m} \sum_{\beta,l} \sqrt{p_n} U^\alpha_{nm} |m>_A \cdot V^{\beta\alpha}_{ln} |\beta)|l>_B . \end{aligned}$$

The final state of Bob is described by a density operator,

$$\rho'_B = Tr_{A,\Phi} V U \rho_B U^\dagger V^\dagger. \tag{3·10}$$

We call the above successive unitary operations as "semi-localizable operation". We assume that the unitary transformation U by Alice and V by Bob are commutable if the space-time points of the two operations are space-likely separate. This reminds us of the causality in quantum field theory is described by the commutativity of field variable at space-likely separated space-time points x, y,

$$[\phi(x), \phi(y)] = 0$$

Interchanging U and V in the expression of ρ'_B,we see that Bob's state

$$\rho'_B = Tr_{A,\Phi} U V \rho_B V^\dagger U^\dagger = Tr_{A,\Phi} V \rho_B V^\dagger. \tag{3·11}$$

does not depend on Alice' unitary operation U. Namely Bob cannot get any information from Alice. If the two operations U and V are performed in the domain of dependence, they are not commutable in general so that ρ'_B depends on U in general so that Bob can detect what Alice did.

In this way we see that the semi-localizability implies the causality, which conforms with our intuition, because the mediator M of the information respects the relativistic causality. Non-trivial is the converse; the causality implies the semi-localizability, which has been rigorously proven in Eggeling et al.18) It was argued that the Wilson loop operator is not observable in the sense of semi-localizability19)
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3.3. *Other topics in special relativity*

It was once discussed that the entanglement degree is not relativistically invariant concept by looking at the reduced density operator for spin part for the case of relativistic quantum mechanics, e.g. the Dirac theory of electron. However, it turns out the entanglement degree of full density matrix is invariant. That is, entanglement of spin part and and the spatial part of wave function is frame dependent as obvious from the Lorentz transformation to the Dirac field; much ado about nothing.20)

§4. General relativity

4.1. *Black hole evaporation*

Bekenstein,21) on the basis of information theoretical arguments in a gedanken experiment, proposed the generalized second law in the black hole spacetime prior to the discovery of the Hawking radiation22) and thus opened up black hole thermodynamics.23) It has been shown that there is an almost complete parallelism between black hole physics and thermodynamics from the zero-th to the third law. However, there remains a long standing problem: the apparent loss of information about the initial state by evaporation of the black hole.24) From our point of view, it is crucial to clarify the meaning of "information" to resolve this paradox.Recently, the information theoretical aspects in black hole physics have been reemphasized25) in the light of the entropy bound conjecture.

In the black hole thermodynamics the total entropy is the sum of the black hole entropy $S_{BH} = A/4$ (where A is the area of the black hole horizon, and $S_{BH} = 4\pi M^2$ for a spherical black hole of mass M) and of the ordinary matter entropy S_M, i.e. $S_T = S_{BH} + S_M$. The generalized second law is motivated by the paradox of Wheeler's demon: although the entropy S_M of the matter outside the black hole decreases by disposing it to the black hole, the total entropy ΔS_T increases. There is plenty of evidence to support it. For example, a gedanken experiment suggested by Unruh and Wald27) takes into account the Unruh effect, while Frolov and Page28) gave a general argument based on the EPR-like entanglement of the particle statesinside and outside the event horizon. In a previous work29)30) the present authors showed that, in a *quantum* version of the Geroch-Bekenstein gedanken experiment, for the outside region of a black hole the total entropy increases, while the matter entropy decreases when a detector is dropped into the black hole. The decrease of the matter entropy is more than compensated by the increase of the black hole entropy via the increase of the black hole mass which is ultimately attributed to the work done by the measurement. The present author showed that the increase of the generalized entropy is greater than or equal to the Holevo bound,3),31) which in turn is the upper bound to the classical information which can be obtained by quantum measurements. Entanglement plays an essential role in our argument and is a key concept of quantum information theory.

4.2. *Singularity*

Recently Horowitz and Maldacena33) proposed a quantum mechanical scenario that possibly resolves the information loss paradox of black hole evaporation assuming that the quantum state at classical singularity,32) quantum state of falling matter and ingoing Hawking radiation are maximally entangled. Recall that the ingoing and outgoing Hawking radiation states are also maximally entangled at the late high temperature stage of evaporation. The idea is closely related with the quantum teleportation. The two entanglements encode the information of initial state of falling matter in the outgoing Hawking radiation. The information can be in principle retrievable in the Hawking radiation. Of course, their proposal remains a conjecture and many points have to be clarified, e.g. the effect of interactions between infalling matter and the Hawking radiation. However, this novel approach to quantum state at the space-like singularity is suggestive and its application to initial singularity might be interesting. The scenario goes like this. At the big bang quantum state is maximally entangled and later on the matter part of quantum state has been decohered by their mutual interactions to become almost classical state as a whole, while the gravitation part remains entangled because of its weak interactions, which might be observed as a primordial gravitational wave ..... Probably the story goes too far at the moment but it stimulates our imagination.

4.3. *Summary*

I described possible connection between relativity and quantum information theories in a pedestrian way. I personally believe that the physical operation in quantum information has to incorporate relativistic causality and that quantum gravity should contain observational part as an built-in structure. In both ways, the two research fields seem to have stronger interactions in future. After all, in quantum world, the ontology (=space, time and matter) and the epistemology (=information) are not separate things but an integrated physical reality.

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