Size Scaling and Bursting Activity in Thermally Activated Breakdown of Fiber Bundles

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We study subcritical fracture driven by thermally activated damage accumulation in the framework of fiber bundle models. We show that in the presence of stress inhomogeneities, thermally activated cracking results in an anomalous size effect; i.e., the average lifetime $\langle t_f \rangle$ decreases as a power law of the system size $\langle t_f \rangle \sim L^{-z}$, where the exponent z depends on the external load σ and on the temperature T in the form $z \sim f(\sigma/T^{3/2})$. We propose a modified form of the Arrhenius law which provides a comprehensive description of thermally activated breakdown. Thermal fluctuations trigger bursts of breakings which have a power law size distribution.

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Subcritical rupture, occurring under a constant load below the fracture strength of materials, is of fundamental importance in a wide range of physical [1,2], biological [1], and geological systems [3], and has an enormous technological impact. Recent investigations led to the surprising discovery that thermally activated microcrack nucleation plays a crucial role in subcritical rupture being responsible for the finite lifetime of specimens [1,2,4–7]. Thermally activated slow crack advancements also affect the surface roughness of growing cracks [8], and even the emergence of earthquake sequences [3]. Macroscopic failure as a consequence of thermally activated damage accumulation often occurs as a sudden, unexpected event. In order to forecast the imminent failure, it is of high importance to derive relations of observables which characterize the approach to the critical point.

Quenched structural disorder in the form of defects, flaws, or microcracks gives rise to strong sample-to-sample fluctuations of the fracture strength and the yield stress of materials. These macroscopic characteristics show also a strong size effect; i.e., their average values decrease with increasing sample size, which is of high importance in applications and is extensively exploited by industrial design [9,10]. During the last decade major progress has been achieved in the understanding of the role of quenched disorder in the size scaling of materials strength [9–11]. However, under subcritical loads, the interplay of annealed disorder (thermal noise) and of the inhomogeneous stress field in the rupture process still remained an open fundamental problem.

In the present Letter we study the effect of the inhomogeneous stress distribution around failed regions of materials on the process of subcritical rupture driven by thermally activated microcrack nucleation. Based on a fiber bundle model (FBM) of disordered materials, we demonstrate that varying the temperature T and the external load σ, the relative importance of annealed disorder and stress concentration can be controlled in the failure process tuning the system from the mean field limit to the extreme of catastrophic failure. Model calculations show that in the realistic situation of highly inhomogeneous stress distributions, the simple Arrhenius law of rupture life does not hold. The lifetime is found to exhibit an anomalous size scaling; i.e., our calculations revealed a power law decrease of lifetime with the sample size. Thermally driven local failures trigger bursts of breakings due to the redistribution of the excess load. The size distribution of bursts has a power law behavior whose exponent changes from one to two as the stress concentration becomes dominating over the effect of thermal noise when the external load approaches the critical point.

Our theoretical approach is based on the fiber bundle model, where we consider N parallel, brittle fibers with identical Young modulus E. The bundle is subject to a constant external stress σ parallel to the fibers' direction. There are several possible ways to introduce time-dependent rupture in stochastic fracture models. After the pioneering works of Coleman [12] on time-dependent FBM, the models were further extended to a broad class of time-dependent damage accumulation laws and fiber strength by Phoenix and Curtin [13,14]. In our work we follow the approach of Guarino *et al.*; i.e., we assume that the local load σ_i of fibers has time-dependent fluctuations $\xi_i(t)$ due to the presence of thermal noise so that the actual load of fiber i at time t reads as $\sigma_i(t) = \sigma_i^0(t) + \xi_i(t)$, where $\sigma_i^0(t)$ denotes the local stress arising due to the external load and to load transfer following breaking events. The fibers have a finite strength characterized by a failure threshold σ_{th}^i, which is, in general, a random variable. A fiber fails when the total load on it $\sigma_i(t)$ exceeds the respective threshold value σ_{th}^i. We assume that the system consists of homogeneous fibers; i.e., all the breaking thresholds are the same $\sigma_{th}^i = \sigma_{th}$, $i = 1, \ldots, N$, where $\sigma_{th} = 1$ is set. Note that the quenched disorder of fibers' strength can have an important effect on the evolution of the system with an experimental relevance, which will be explored elsewhere. The thermal noise has a Gaussian distribution with zero mean and a variance controlled by the temperature T of the system $p(\xi) = p(\xi; T) = (1/\sqrt{2\pi T})\exp(-\xi^2/(2T))$, from which the complementary cumulative distribution follows as $P(\xi; T) = \int_{\xi}^{+\infty} p(x; T)dx$. After a breaking event, the load of the failed fiber has to be overtaken by the remaining intact ones. Thermally activated breakdown has extensively been studied by means of FBMs in the limit of equal load sharing (ELS), where all the fibers share the same load resulting in a homogeneous stress distribution in the system [4,15,16]. To reveal the effect of stress inhomogeneity on thermally activated breakdown, in our model the fibers are organized on a square lattice of size $L \times L$ and localized load sharing (LLS) is considered [17]: the load of broken fibers is redistributed over their nearest intact neighbors, giving rise to high stress concentrations.

Subjecting the bundle to a constant external load σ, two competing physical mechanisms contribute to the failure of fibers: When the load is small enough $\sigma \ll \sigma_{\rm th}/N$, even a single fiber can sustain the entire load, and the load increments arising in the vicinity of failed fibers are not sufficient to trigger further breakings. Hence, in this load regime, the failure process is dominated by the thermal fluctuations and there is practically no difference between ELS and LLS calculations since the range of interaction is irrelevant. However, at high load values $\sigma \to \sigma_{\rm th}$, the load redistributions give rise to considerable increments of the local load on intact fibers leading to additional breakings. In the initial state of the system all the fibers have the same load $\sigma_i^0 = \sigma$, $i = 1, \ldots, N$. When a fiber breaks due to thermal noise $\sigma_i^0 + \xi_i > \sigma_{\rm th}$, the load σ_i^0 is transferred to its four intact neighbors resulting in the increment $\Delta\sigma^0 = \sigma/4$. If the updated load exceeds the breaking threshold $5\sigma/4 > \sigma_{\rm th}$, the fibers break again transferring the load further to their intact neighbors. Once this breaking sequence starts, removing all four neighbors of the initial one, it does not stop until all fibers break leading to macroscopic fracture. It follows that due to the localized stress transfer, the system has a critical load $\sigma_c = 4\sigma_{\rm th}/5$ above which even a single fiber breaking triggers the immediate collapse.

The most important macroscopic characteristic quantity of the system is the average lifetime $\langle t_f \rangle$ which has a finite value even at zero external stress $\sigma = 0$ if the temperature is finite $T > 0$. Under the assumption of equal load sharing it has been shown analytically in FBMs with a fixed breaking threshold $\sigma_{\rm th}$ that $\langle t_f \rangle$ follows the Arrhenius law $\langle t_f \rangle \simeq (\sqrt{2\pi T}/\sigma)\exp((\sigma_{\rm th} - \sigma)^2/2T)$, without any dependence on the system size N [15,18]. Using a different modelling approach, a similar functional form was obtained in Refs. [13,14]. Further analytical studies and computer simulations with randomly distributed breaking thresholds have revealed that the above functional form prevails also in the presence of quenched disorder, only the effective temperature shifts to a higher value [16]. Figure 1(a) presents the scaling plot of lifetime obtained by our computer simulations with the LLS FBM at the system size $L = 1024$ varying the load σ and the temperature T. No data collapse is obtained in the figure, which implies that the simple Arrhenius law does not hold when stress concentrations are present.

Our analytical and numerical calculations revealed that the interplay of stress concentrations and annealed disorder results in an anomalous size effect of the lifetime of the system, which is responsible for the discrepancy observed in Fig. 1(a). At zero stress $\sigma = 0$, the average number of intact fibers $\langle N_i(t) \rangle$ after t time steps reads as $\langle N_i(t) \rangle = N[1 - P(\sigma_{\rm th}; T)]^t$. The average lifetime $\langle t_f \rangle$ of the system can be obtained as a sum $\langle t_f \rangle = \langle t_1 \rangle + \langle t_0 \rangle$, where $\langle t_1 \rangle$ is the average time of breaking the first $N - 1$ fibers (i.e. $\langle N_i(t_1) \rangle = 1$ holds), while $\langle t_0 \rangle$ denotes the average time to break the last fiber. Finally, we get,

$$\langle t_f \rangle = -(\ln N + 1)/\ln[1 - P(\sigma_{\rm th}; T)]. \quad (1)$$

Figure 2(a) presents the numerical verification of the above analytic result, i.e., independently of the range of load sharing, in the zero stress limit the average lifetime increases logarithmically with the system size $\langle t_f \rangle \sim \ln N$. Simulations performed at $\sigma = 10^{-5}$ both for ELS and LLS demonstrate in Fig. 2(a) that at finite load values the size effect drastically changes: the logarithmic correction gradually disappears and the lifetime $\langle t_f \rangle$ tends to a constant value as the system size N increases. At the other extreme, in the limiting case of high loads $\sigma \to \sigma_c$, the breaking of a single fiber can trigger the failure of the entire system. Hence, the average lifetime of the bundle is equal to the average time of the first breaking [18]

$$\langle t_f \rangle = -L^{-2}/\ln[1 - P(\sigma_{\rm th} - \sigma; T)], \quad (2)$$

which shows a power law decrease $\langle t_f \rangle \sim L^{-2}$ with exponent 2. In order to explore the regime of intermediate load and temperature values, we carried out computer simula-

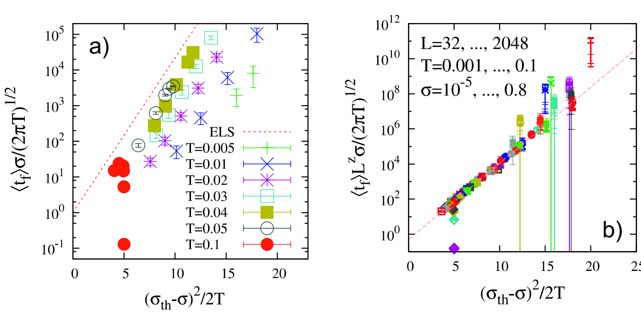

FIG. 1 (color online). (a) Scaling plot of lifetime obtained at different temperatures T and external loads σ for a fixed system size $L = 1024$. Based on the simple Arrhenius law [15,18], no data collapse is obtained. (b) Taking into account the anomalous size scaling of lifetime, a high quality data collapse is obtained which verifies the modified Arrhenius law Eq. (4).

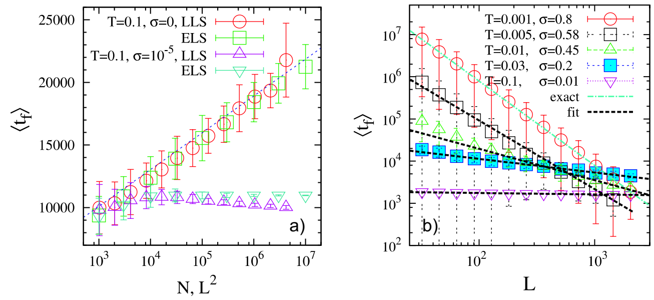

FIG. 2 (color online). Size scaling of lifetime. (a) At zero load, $\langle t_f \rangle$ increases logarithmically with the system size. The dashed line represents the analytic solution Eq. (1). (b) When stress concentrations dominate, power law dependence is obtained with a high precision. Note that $\sigma_c = 0.8$ in the model.

tions varying the system size in a broad range $L = 32$–2048. It can be observed in Fig. 2(b) that at any finite load value σ the average lifetime of the system exhibits a strong size effect, namely, it decreases as a power law of the lattice size

$$\langle t_f \rangle \sim L^{-z(T,\sigma)}. \tag{3}$$

The value of the exponent falls between the limits determined above analytically $0 \leq z(T, \sigma) \leq 2$ depending both on the temperature T and on the external load σ. It can be observed in Fig. 2(b) that at high temperature when thermal fluctuations dominate in the failure process, the size effect disappears $z \to 0$ as in the absence of stress concentrations (ELS) [16,18]. In the other extreme of high load values and reduced temperatures, fiber breakings due to stress enhancements in the vicinity of failed regions drive the system to rapid failure resulting in a large value of the exponent $z \to 2$. We note that in Refs. [13,14], using a different type of damage law in a one dimensional fiber bundle model, a power law dependence of $\langle t_f \rangle$ was found on the logarithm of the system size $\langle t_f \rangle \sim (\ln L)^{-z'}$. Our 2D model cannot be solved analytically; hence, a careful numerical analysis revealed that the power law functional form Eq. (3) provides a better quality fit of the simulation results than the log-power law over the entire $\sigma - T$ parameter regime considered. We propose a modified form of the Arrhenius law which takes into account the size scaling of the lifetime

$$\langle t_f \rangle \simeq \frac{L^{-z(T,\sigma)}}{2} \frac{\sqrt{2\pi T}}{\sigma} \exp\left(\frac{(\sigma_{\text{th}} - \sigma)^2}{2T}\right). \tag{4}$$

Figure 1(b) demonstrates that the modified Arrhenius law provides an excellent scaling of the numerical data obtained by computer simulations of LLS FBM varying the system size L, the temperature T, and the external load σ in the ranges $L = 32$–2048, $T = 0.001$–0.1, and $\sigma = 10^{-5} - 0.8$, respectively. Our numerical analysis showed that no data collapse can be obtained when log-power size scaling is assumed in Eq. (4) (not presented).

In order to determine the value of the exponent z as a function of the external load σ and temperature T, we explored the $\sigma - T$ plane by means of computer simulations. Since at low loads and low temperatures the lifetime takes huge values, while in the opposite extreme it becomes very short, simulations performed on a super computer were restricted to a stripe of the parameter plane with feasible computation times and good statistics. Figure 3 presents that plotting z, determined at different σ and T, as a function of $\sigma/T^{3/2}$ an excellent data collapse occurs which implies the functional structure $z(\sigma, T) \sim f(\sigma/T^{3/2})$ of the exponent. This functional form is the consequence of the diffusive growth of the largest cracks which drive the macroscopic failure.

Fibers initially break due to thermal fluctuations, then the load increments on intact fibers trigger additional breakings at fixed values of the fluctuating loads $\xi_i(t)$. This way thermally activated breakdown proceeds in bursts even when the stress distribution is homogeneous (ELS). For equal load sharing, the average burst size $\langle \Delta(\sigma^0) \rangle$ can be obtained analytically as a function of the load of single fibers σ^0 in the form $\langle \Delta(\sigma^0) \rangle = \frac{P(\sigma_{\text{th}} - \sigma^0; T)}{1 - P(\sigma_{\text{th}} - \sigma^0; T)} \frac{\sigma}{\sigma^0} N$. It is important to emphasize that $\langle \Delta(\sigma^0) \rangle$ has a minimum at the location σ^*, which is an increasing function of the temperature T, but is independent of the external load σ. The size distribution of bursts $D(\Delta)$ can be cast into the form $D(\Delta(t)) \simeq [1 - P(\sigma_{\text{th}} - \sigma^0(t); T)] |\frac{d\Delta(t)}{dt}|^{-1} \delta t^{-1}$. In two limiting cases the asymptotic behavior of the distribution $D(\Delta)$ can be determined analytically: at low load values $\sigma \ll \sigma^*$, $D(\Delta)$ can be rewritten as $D(\Delta) \simeq \Delta^{-1}[1 - P(\sigma_{\text{th}}; T)]/\ln[1 - P(\sigma_{\text{th}}; T)]$, which yields the power law behavior $D(\Delta) \sim \Delta^{-1}$. At high loads $\sigma \gg \sigma^*$ and low temperatures $T \ll \sigma_{\text{th}}^2$; however, the general ex-

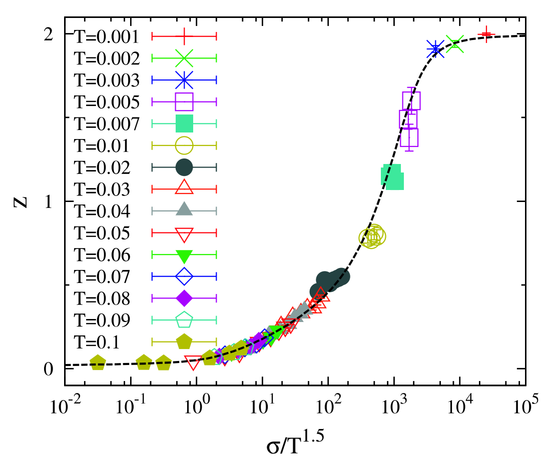

FIG. 3 (color online). The size-scaling exponent z is a unique function of $\sigma/T^{3/2}$. At each temperature, simulations were performed for three σ values varying the lattice size in the range $L = 32$–2048. The line is to guide the eye.

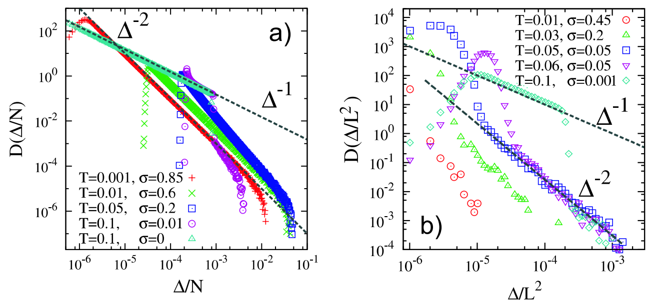

FIG. 4 (color online). Burst size distributions $D(\Delta)$ for homogeneous (ELS) (a) and inhomogeneous (LLS) (b) stress distributions varying the load σ and the temperature T at the system size $L = 1024$ with $\sigma_{\rm th} = 1$. In (a) the characteristic load σ^* has values 0.0128, 0.0412, 0.0932, 0.130, for the temperatures 0.001, 0.01, 0.05, 0.1, respectively. In (b) the load values should be compared to the critical load $\sigma_c = 0.8$.

pression of $D(\Delta)$ reduces to the form $D(\Delta) \sim [1 - P(\sigma_{\rm th} - \sigma^0(t); T)]\Delta^{-2}$, which has also a power law asymptotics but with a different exponent $D(\Delta) \sim \Delta^{-2}$. It can be observed in Fig. 4(a) that the burst size distributions $D(\Delta)$ obtained by computer simulations with homogeneous stress distribution (ELS), are in an excellent agreement with the above analytic predictions: $D(\Delta)$ has a power law behavior $D(\Delta) \sim \Delta^{-\alpha(T,\sigma)}$, where the value of the exponent $\alpha(T, \sigma)$ recovers 1 and 2 in the limiting cases discussed above. In the regime $\sigma \approx \sigma^*$, no analytical results could be obtained; computer simulations provided power law distributions over 3–4 orders of magnitude with exponents slightly above 2 [see Fig. 4(a)]. The distribution for the parameters $T = 0.1$, $\sigma = 0.01$ seems to switch from the power law of exponent 1 to a steeper regime for large bursts. Such switching occurs for external loads $\sigma \lesssim \sigma^*$ due to bursts generated when the local load of single fibers becomes significantly larger than σ^*. In the presence of stress inhomogeneities (LLS) no analytical results could be derived; however, computer simulations revealed that $D(\Delta)$ has a Gaussian form for small bursts with a power law decay for the large ones [see Fig. 4(b)]. It is important to emphasize that the power law exponent α of the LLS case does not vary continuously, instead, it suddenly switches from $\alpha = 1$ to $\alpha = 2$ when the external load approaches the critical value $\sigma \rightarrow \sigma_c$, which is accompanied by the shrinking of the Gaussian regime. In the vicinity of σ_c, the system becomes very sensitive to the thermal fluctuations and cannot tolerate large bursts, which is expressed by the higher value of the exponent α [Fig. 4(b)]. This is an important unique feature of thermally driven creep rupture; when quenched disorder dominates the rupture process the opposite effect occurs; i.e., the burst exponent decreases when approaching catastrophic failure [19].

In summary, based on a fiber bundle model we showed that stress inhomogeneities play a crucial role in the process of thermally activated subcritical rupture giving rise to a broad spectrum of novel behaviors. Stress concentrations, arising in the vicinity of failed regions of the material, make the system more sensitive to thermal fluctuations. Consequently, an astonishing size effect emerges where the average time-to-failure of the model system decreases as a power law of the system size. On the microlevel, thermally driven breakdown proceeds in bursts of breakings, whose size distribution has a power law behavior. The value of the exponent strongly depends on the temperature T and on the external load σ and goes to two when the system approaches the regime of sudden failure. Monitoring the evolution of the system by means of acoustic emission technique, the change of the burst size exponent with the parameters of the system can be exploited to forecast the imminent failure event.

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