Local Equivalence of Ensembles

M. Cramer
Ulm University

on work with

F.G.S.L. Brandão
Microsoft Research and University College London

M. Guta
University of Nottingham

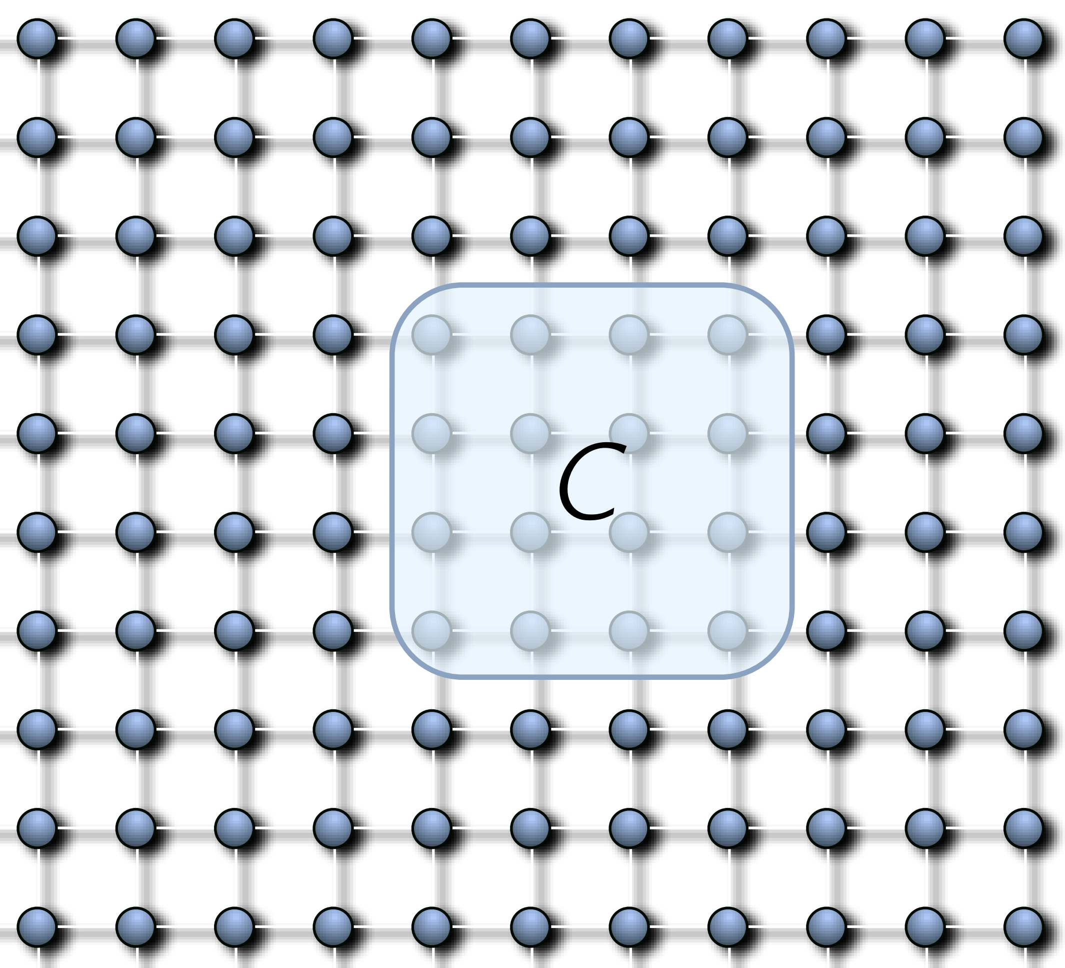
$$\hat{\varrho}_T = \mathrm{e}^{-\hat{H}/T}/Z$$

lack of knowledge, ignorance

Jaynes' principle

$$\hat{\varrho}_T = \mathrm{e}^{-\hat{H}/T}/Z$$

- part of a large (closed) system $\hat{\varrho}_C = \mathrm{tr}_{\setminus C}[\hat{\varrho}]$



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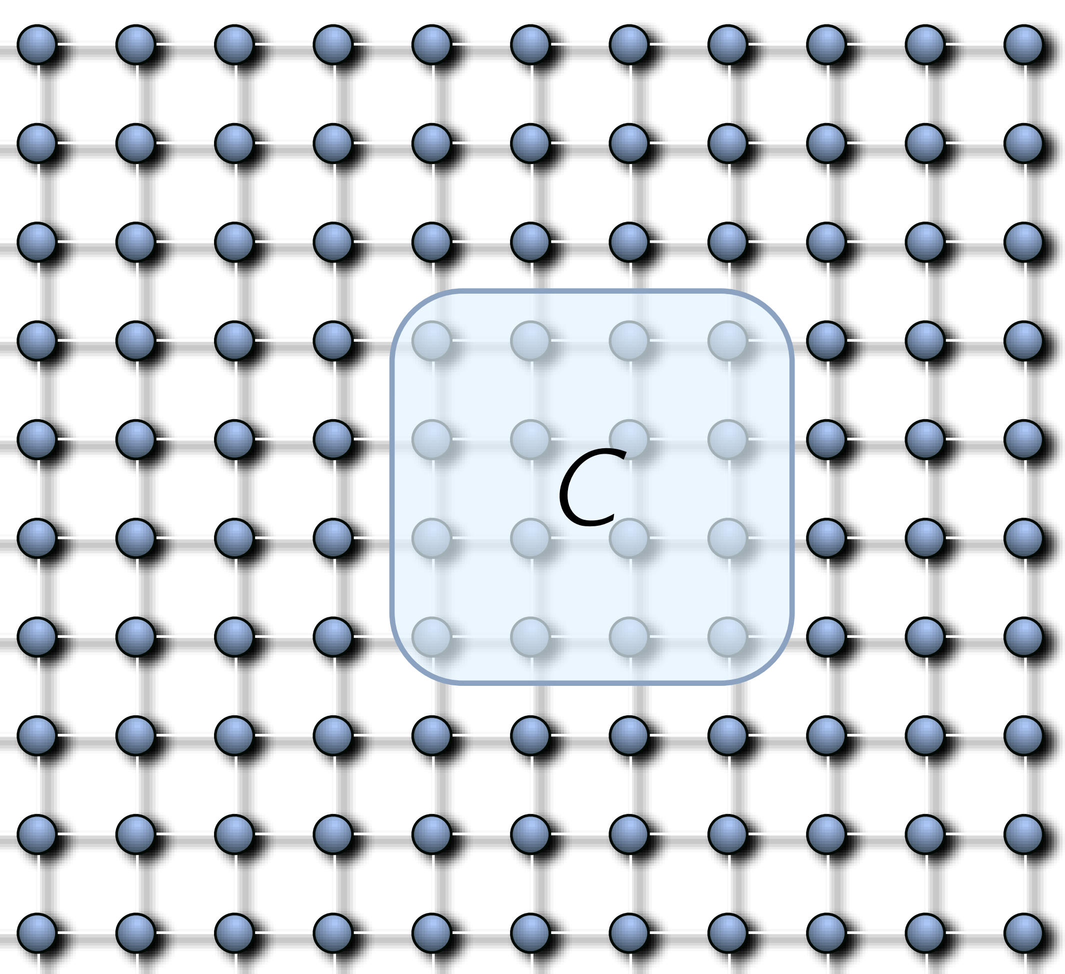
$$\approx \mathrm{e}^{-\hat{H}_C/T}/Z$$



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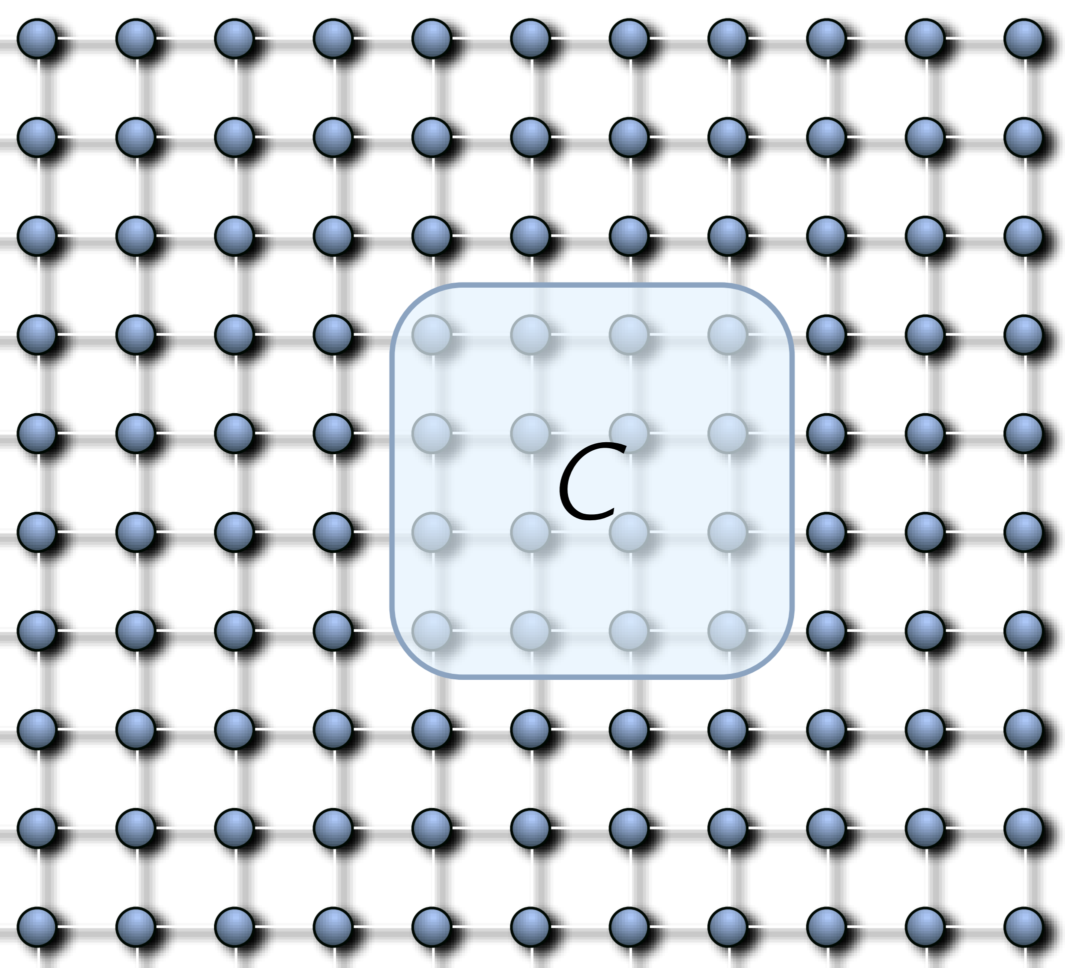
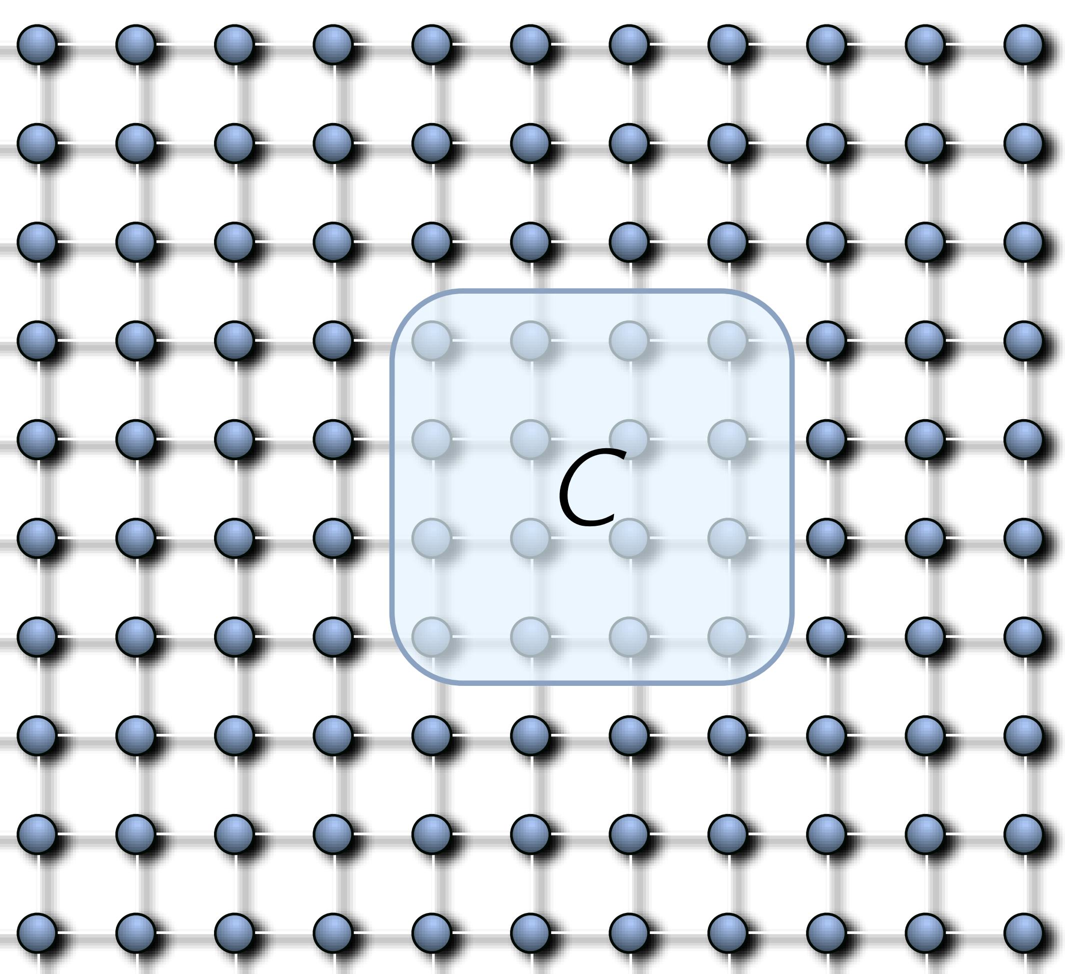


- part of a large (closed) system $\hat{\varrho}_C \approx \mathrm{tr}_{\setminus C}\left[\mathrm{e}^{-\hat{H}/T}/Z\right]$

- in contact with heat bath

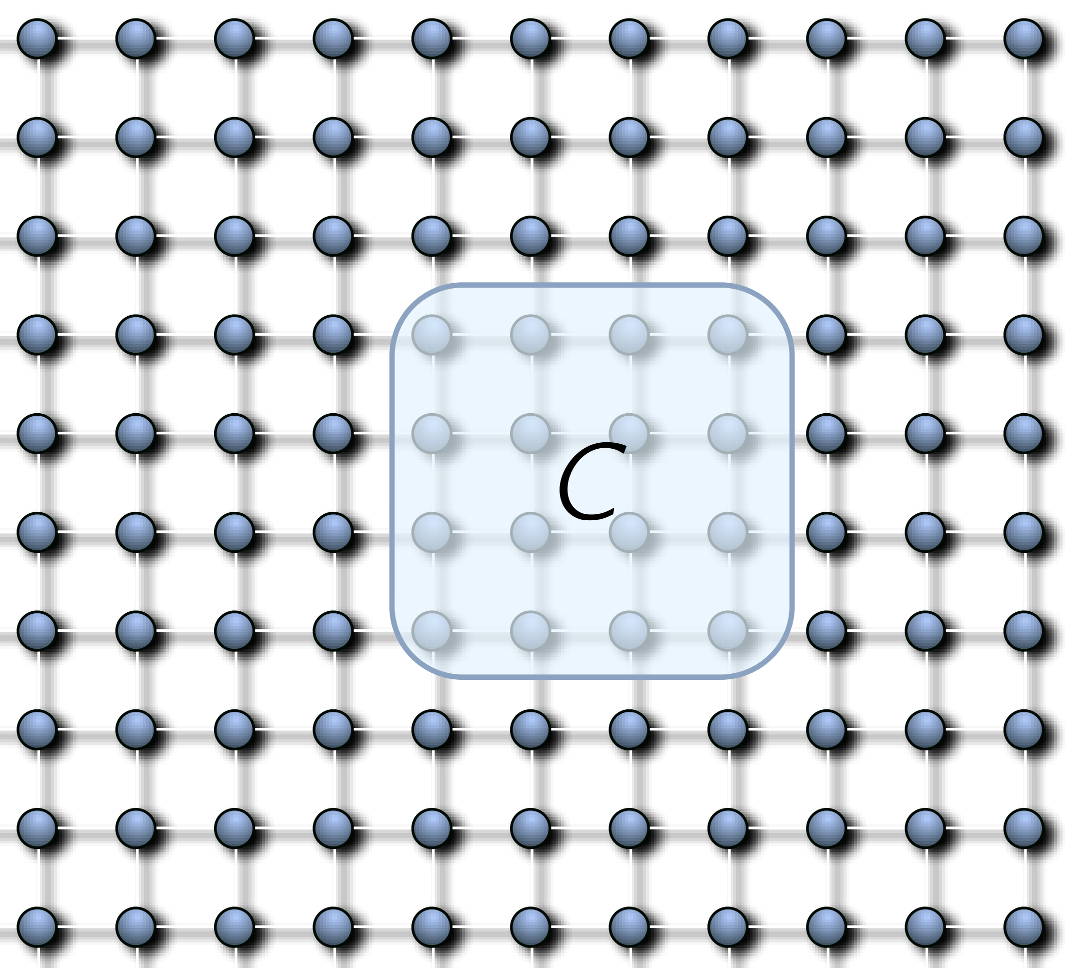
$$\hat{\varrho}_C(0) \otimes \hat{\varrho}_B$$



- part of a large (closed) system $\hat{\varrho}_C \approx \mathrm{tr}_{\setminus C}\left[\mathrm{e}^{-\hat{H}/T}/Z\right]$

- in contact with heat bath, unitary evolution

$$\mathrm{e}^{-\mathrm{i}t\hat{H}}\left(\hat{\varrho}_C(0) \otimes \hat{\varrho}_B\right)\mathrm{e}^{\mathrm{i}t\hat{H}}$$



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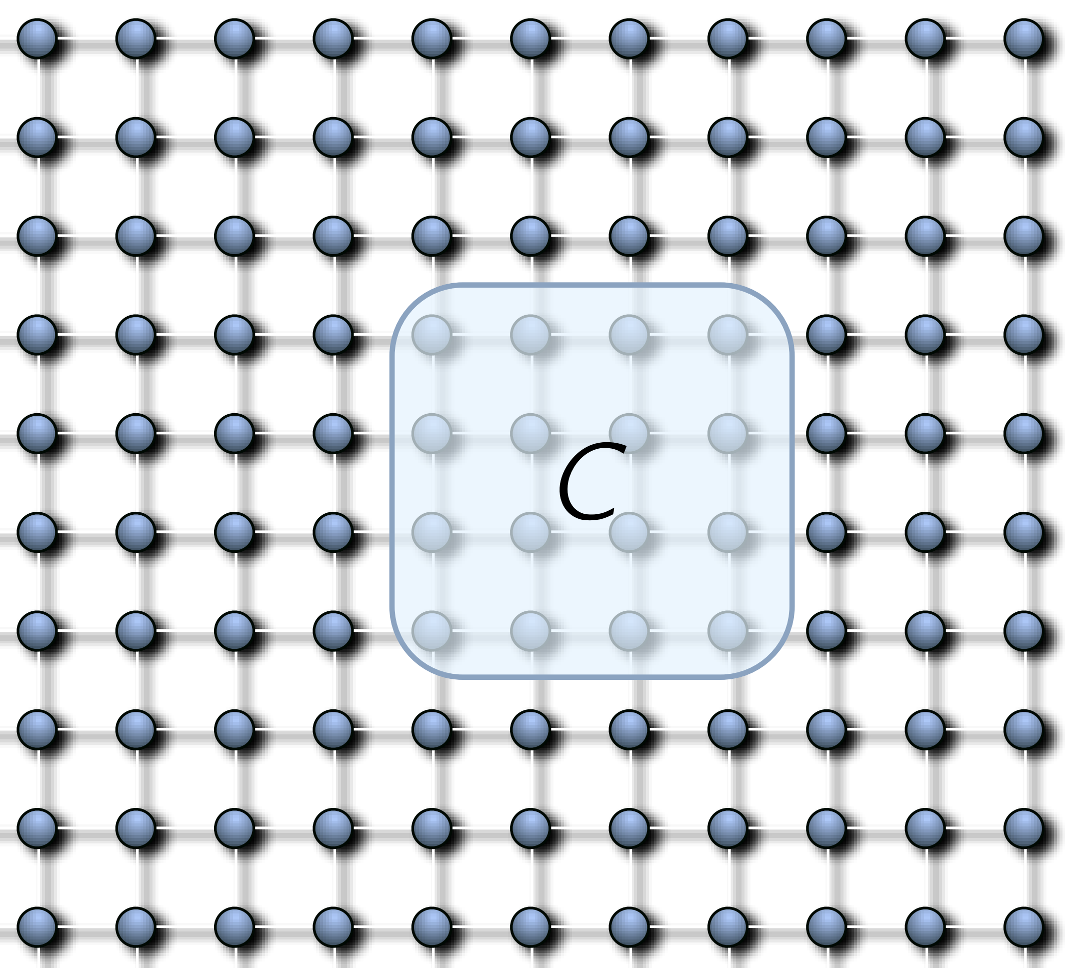

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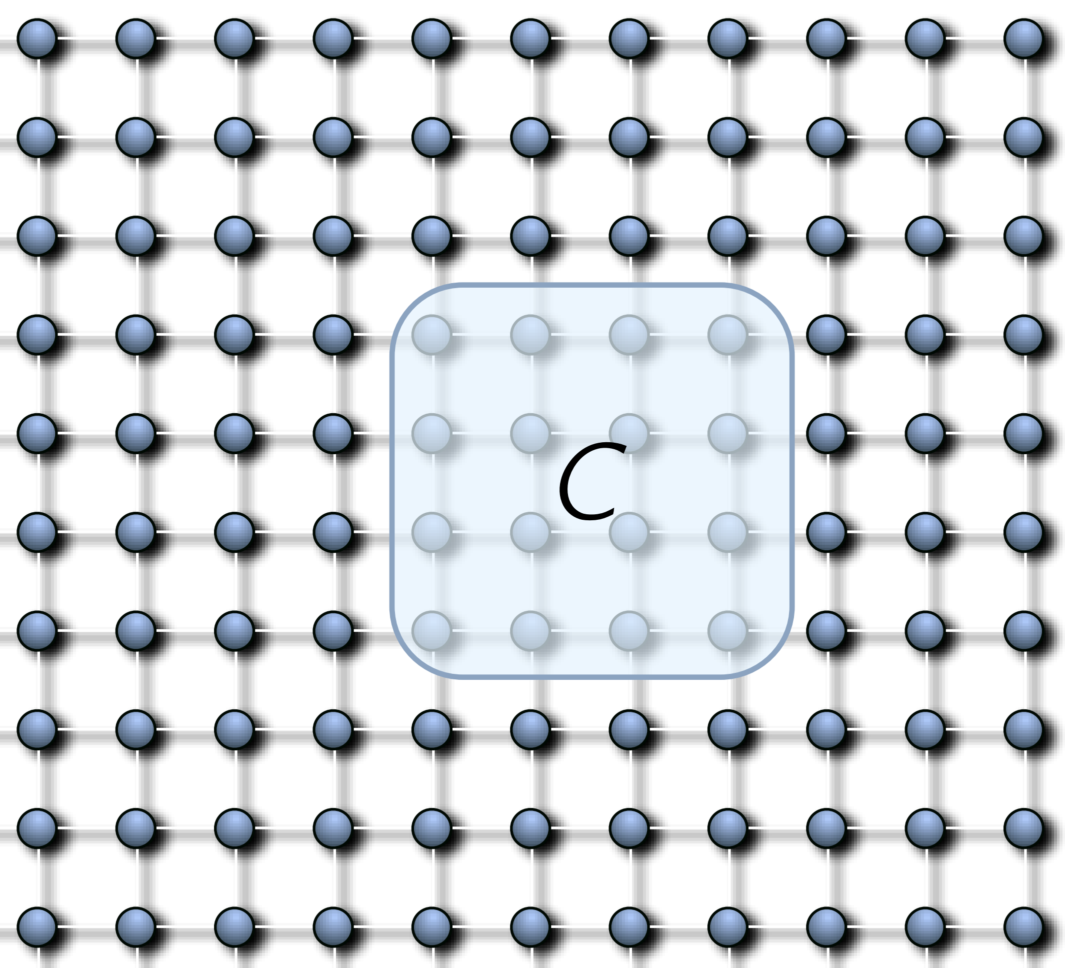

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- quantum quench

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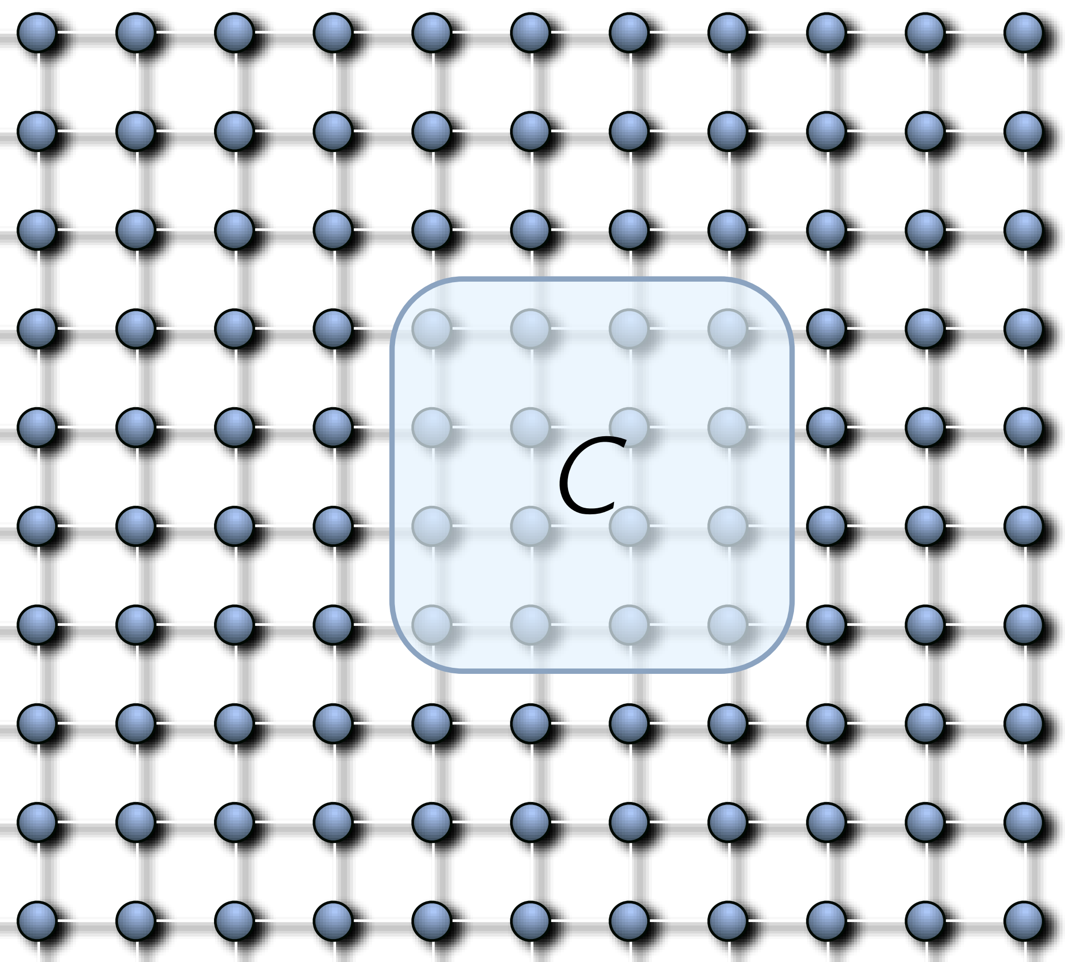

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gen. can. principle

for random $|\psi\rangle \in \mathcal{H}_R \subset \mathcal{H}_C \otimes \mathcal{H}_B$

with high probability $\hat{\varrho}_C \approx \mathrm{tr}_{\setminus C}[\mathbb{1}_R/d_R]$

…thermal?

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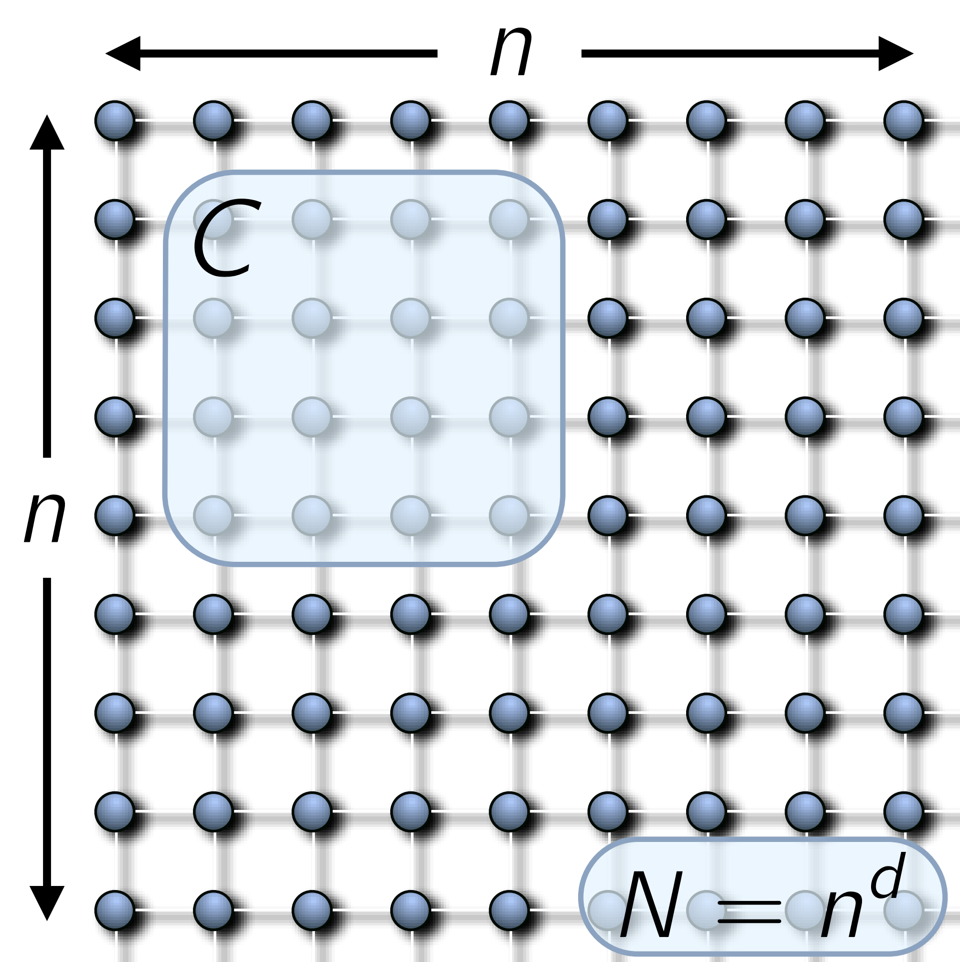
integrable

no thermalization
instead: generalized Gibbs ensemble

non-integrable

“equilibrium state”, close to it for most times
…thermal? time scale?

- $\hat{H} = \sum_{ij} (\hat{b}_i^\dagger A_{ij} \hat{b}_j + \hat{b}_i B_{ij} \hat{b}_j + \text{h.c.})$ local, t.i.
- $\hat{\varrho}(0) \in \mathcal{H}_C \otimes \mathcal{H}_B$ sufficiently clustering (not necessarily Gaussian)



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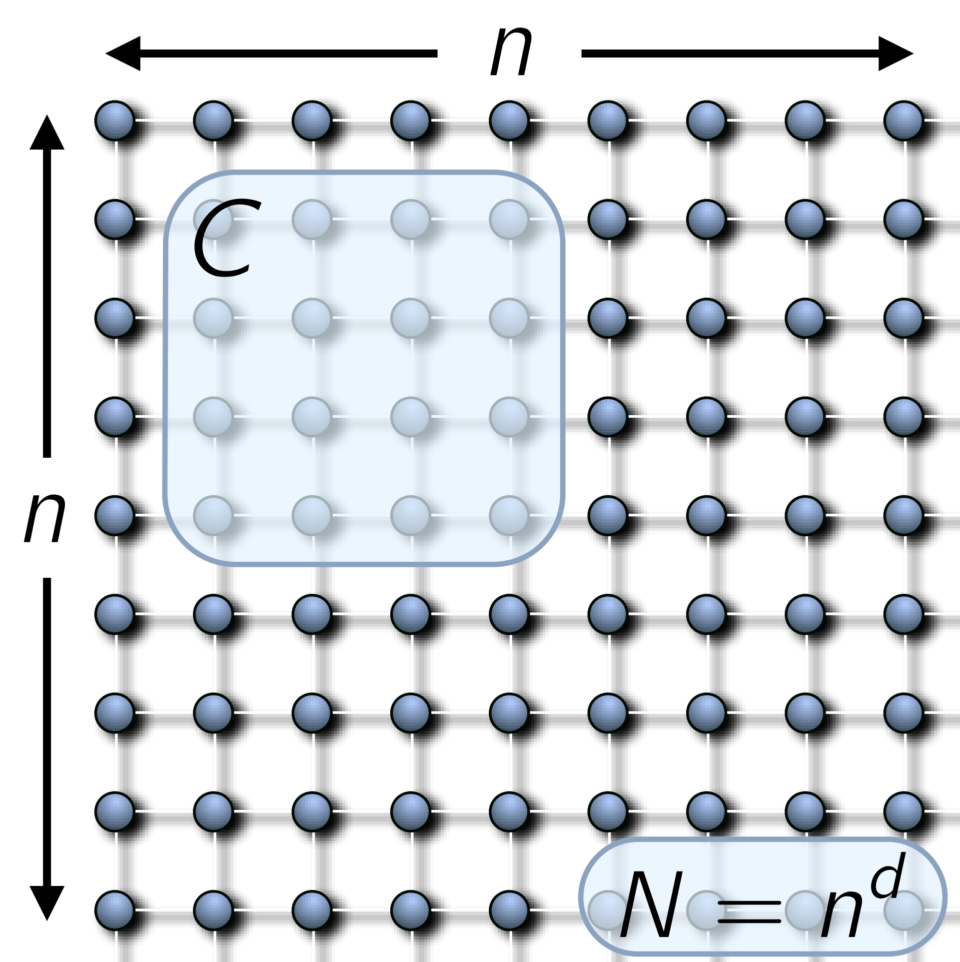
$$\|\hat{\varrho}_C(t) - \hat{G}(t)\|_{\text{tr}} \leq \epsilon \quad \text{for all} \quad t \in [t_1(\epsilon, N), t_2(\epsilon, N)]$$

$\hat{G}(t)$: Gaussian with same second moments as $\hat{\varrho}_C(t)$



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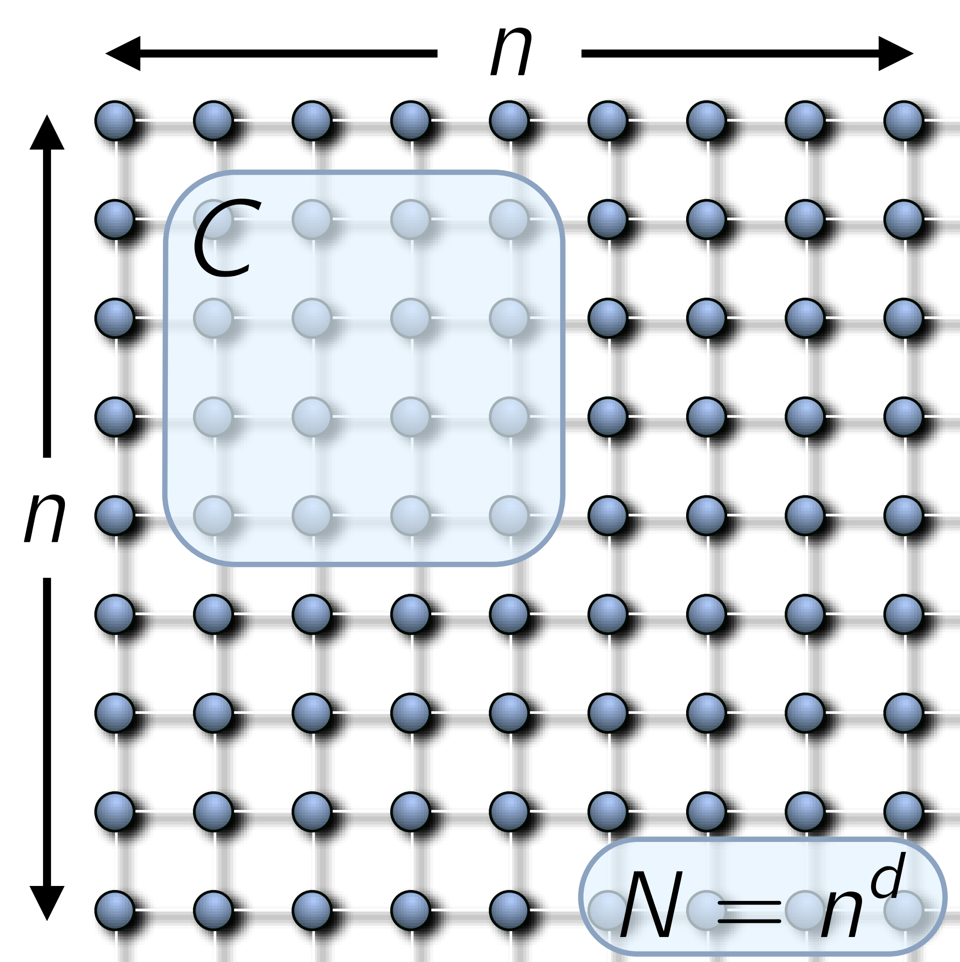
→ maximum entropy state



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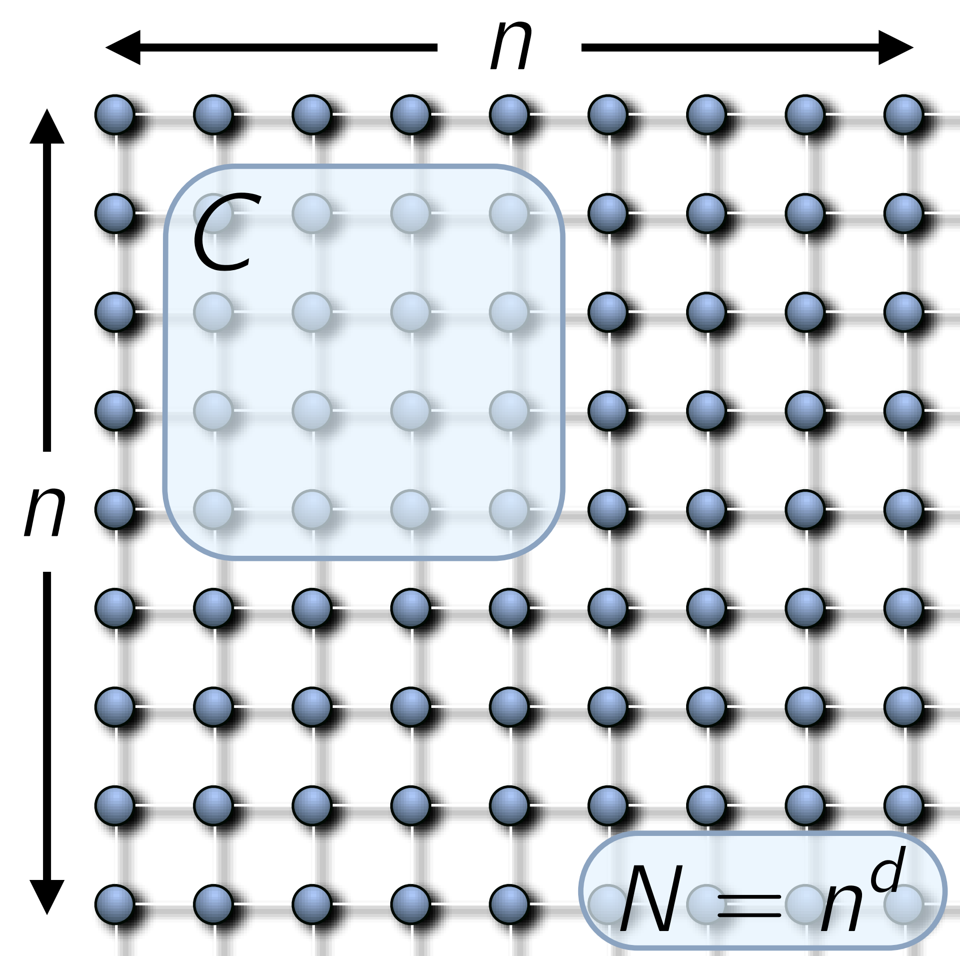
equilibration, non-thermal: Tegmark, Yeh (1994)



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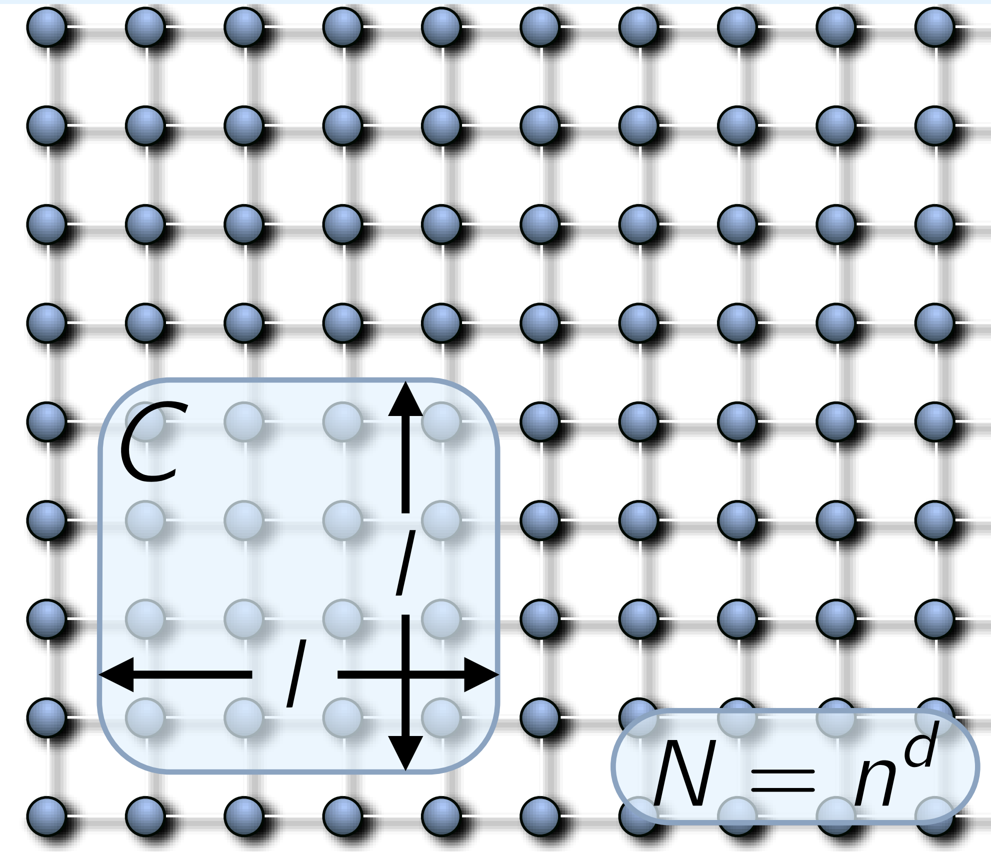

$$\|\hat{\varrho}_C - \hat{\tau}_C\|_{\mathrm{tr}} \leq \epsilon \quad ?$$

$$\hat{\tau}: \frac{|\langle \hat{A}\hat{B}\rangle - \langle \hat{A}\rangle\langle \hat{B}\rangle|}{\|\hat{A}\|\|\hat{B}\|} \leq N^z e^{-L/\xi}$$

for which states $\hat{\varrho}$
(and which l) is

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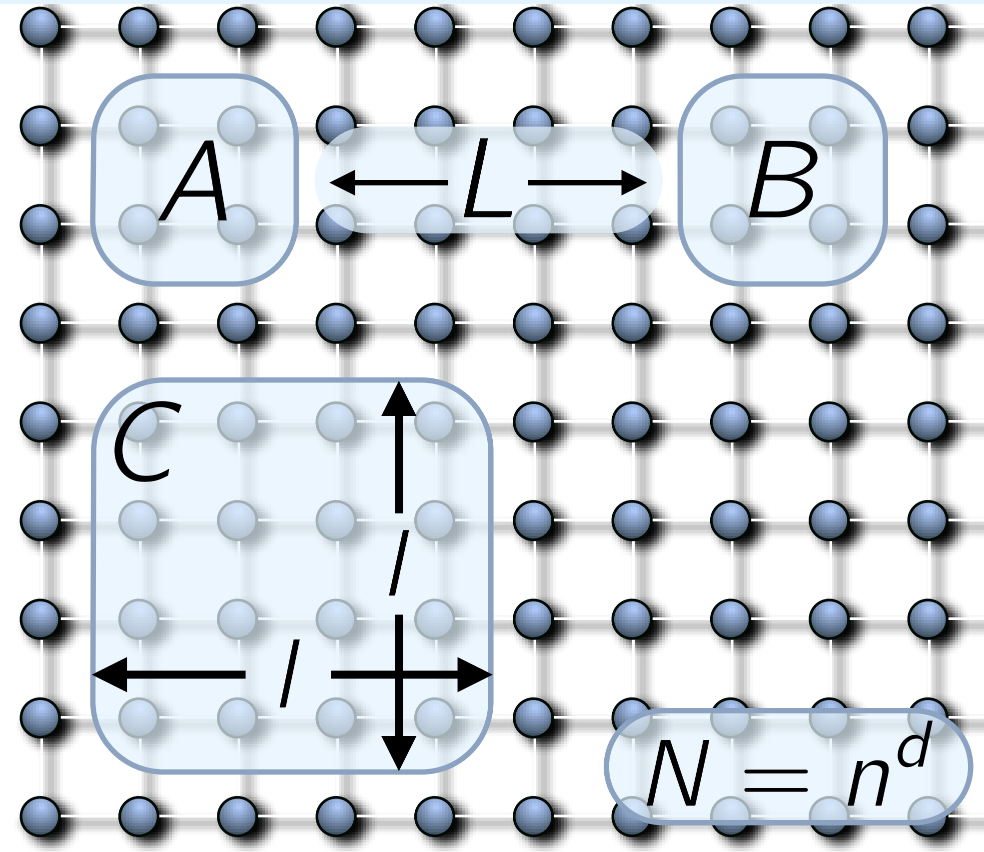

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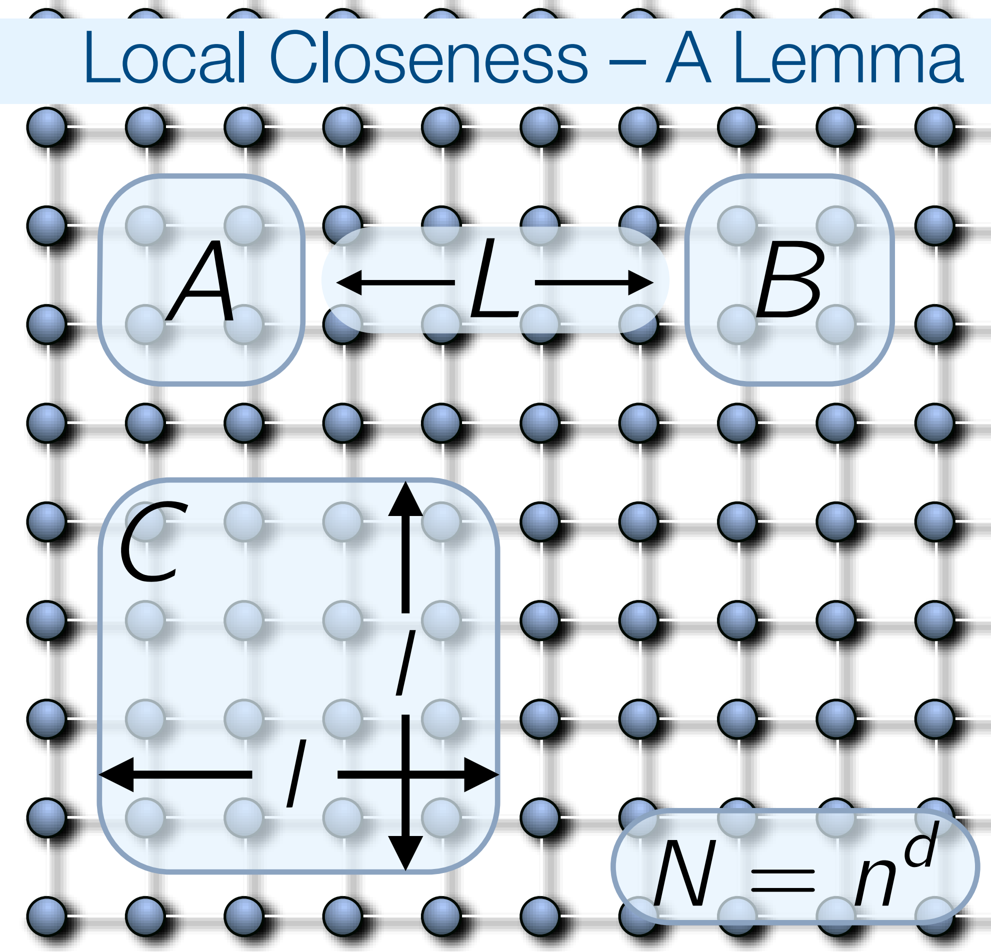

non-t.i.: $\mathbb{E}[\quad]$

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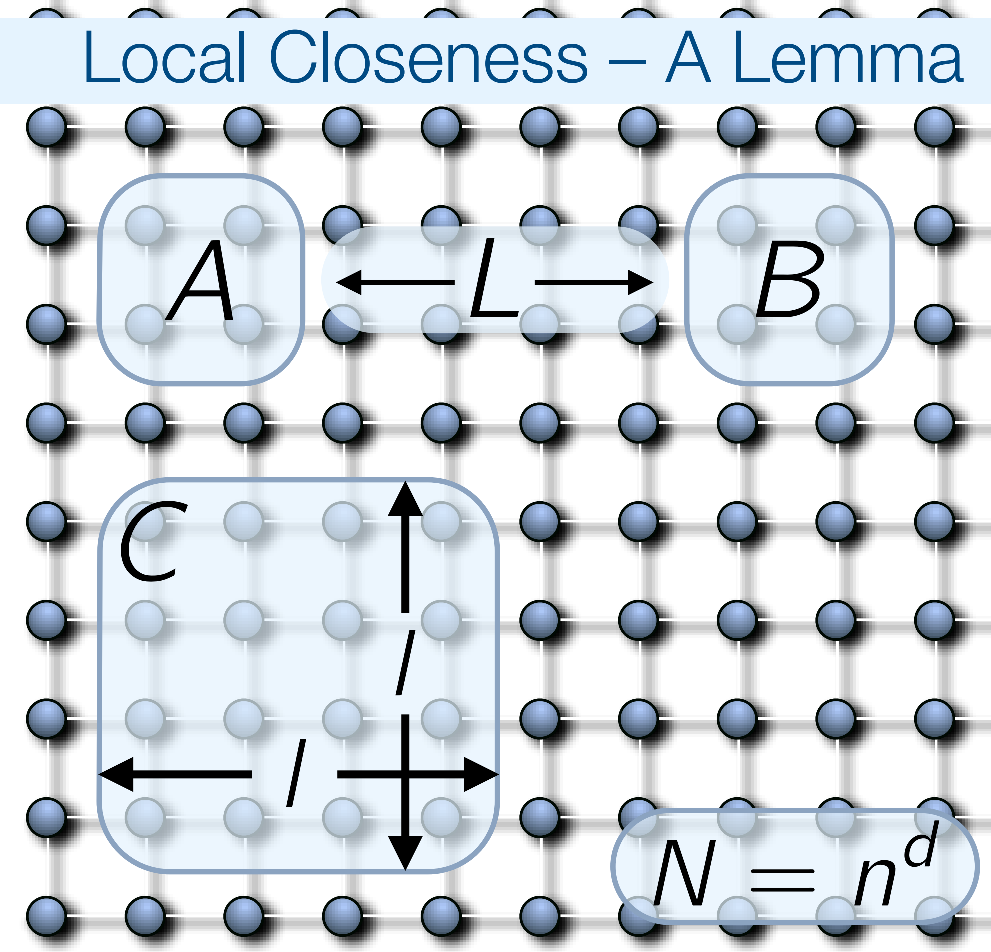

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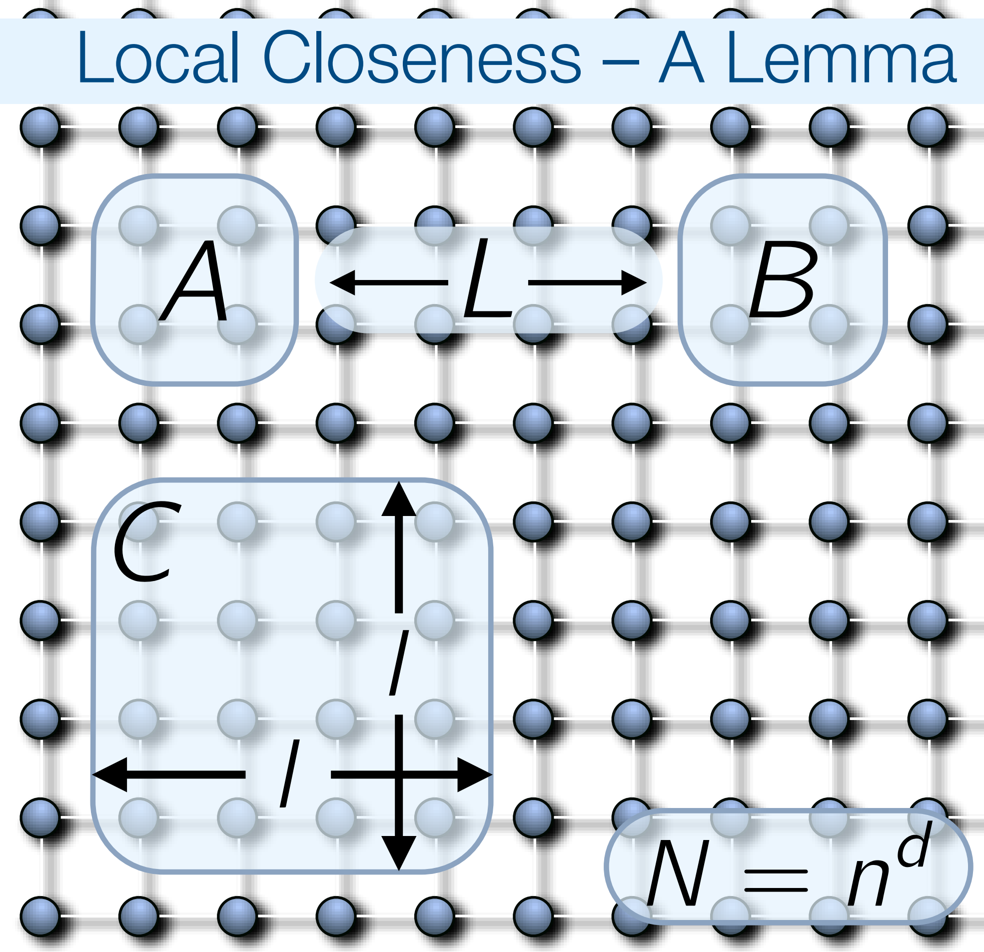

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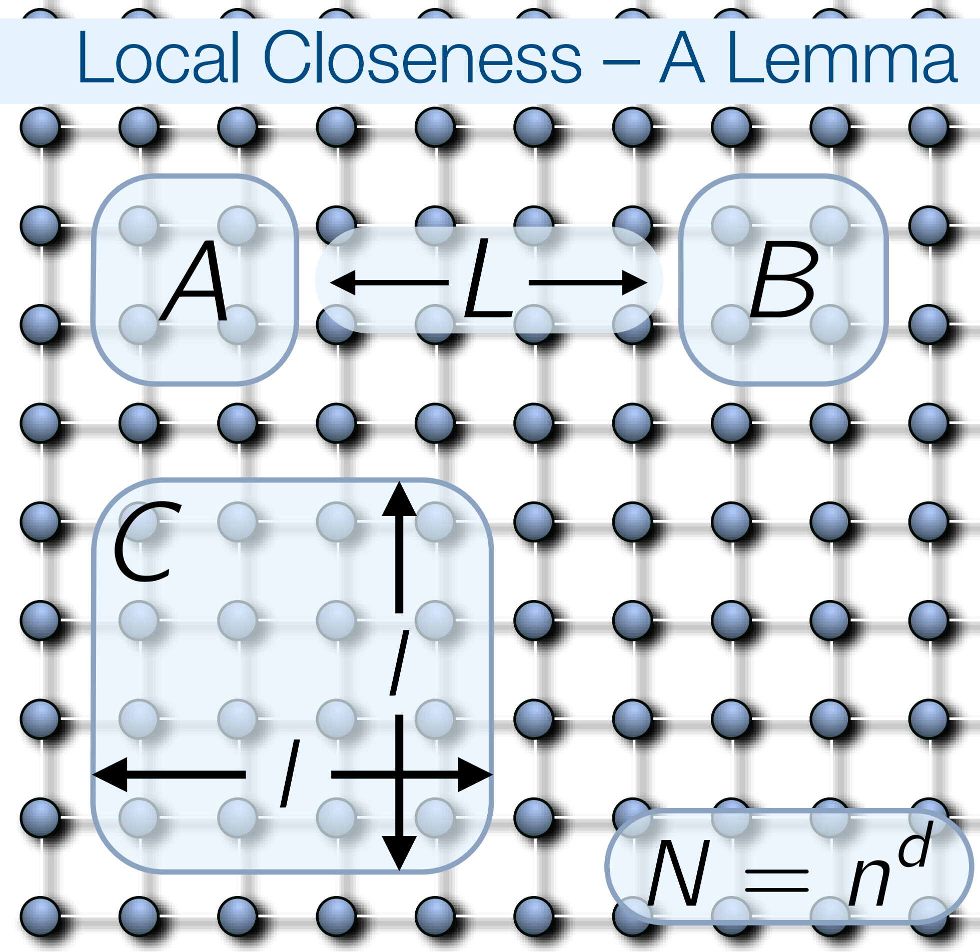

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- Quantum Substate Theorem Jain, Radhakrishnan, Sen (2009); Jain, Nayak (2011)

$$S^{2\sqrt{\epsilon}}(\hat{\varrho}\|\hat{\tau}) \le S^{2\sqrt{\epsilon}}_{\max}(\hat{\varrho}\|\hat{\tau}) \le \frac{S(\hat{\varrho}\|\hat{\tau})+1}{\epsilon} + \log\left(\frac{1}{1-\epsilon}\right)$$

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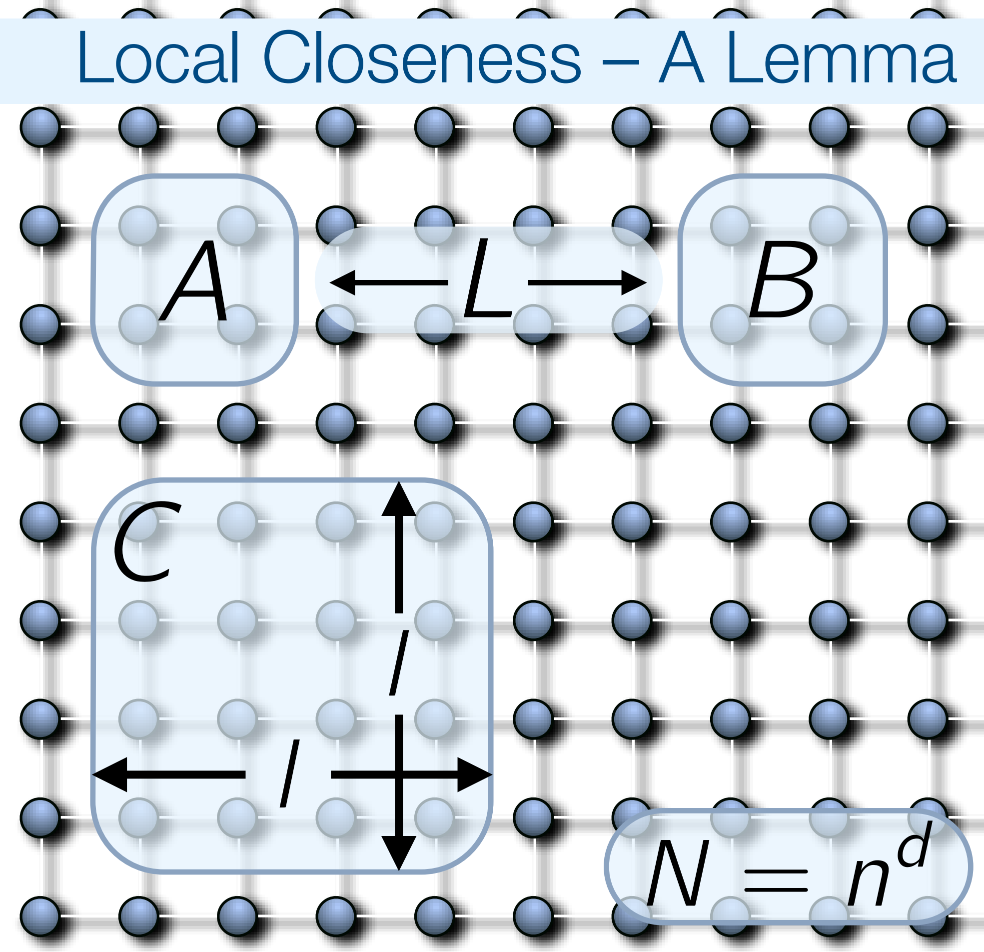

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- Quantum Substate Theorem Jain, Radhakrishnan, Sen (2009); Jain, Nayak (2011)
- Lemma Datta, Renner (2009); Brandão, Plenio (2010); Brandão, Horodecki (2012)

$$S_{\max}(\hat{\varrho}\|\hat{\pi}) \leq \lambda$$
$$\kappa = 2^\lambda \|\hat{\tau} - \hat{\pi}\|_{\mathrm{tr}}$$
$$\longrightarrow \quad S_{\max}^{\sqrt{8\kappa}}(\hat{\varrho}\|\hat{\tau}) \leq \lambda + \log\left(\tfrac{1}{1-\kappa}\right)$$

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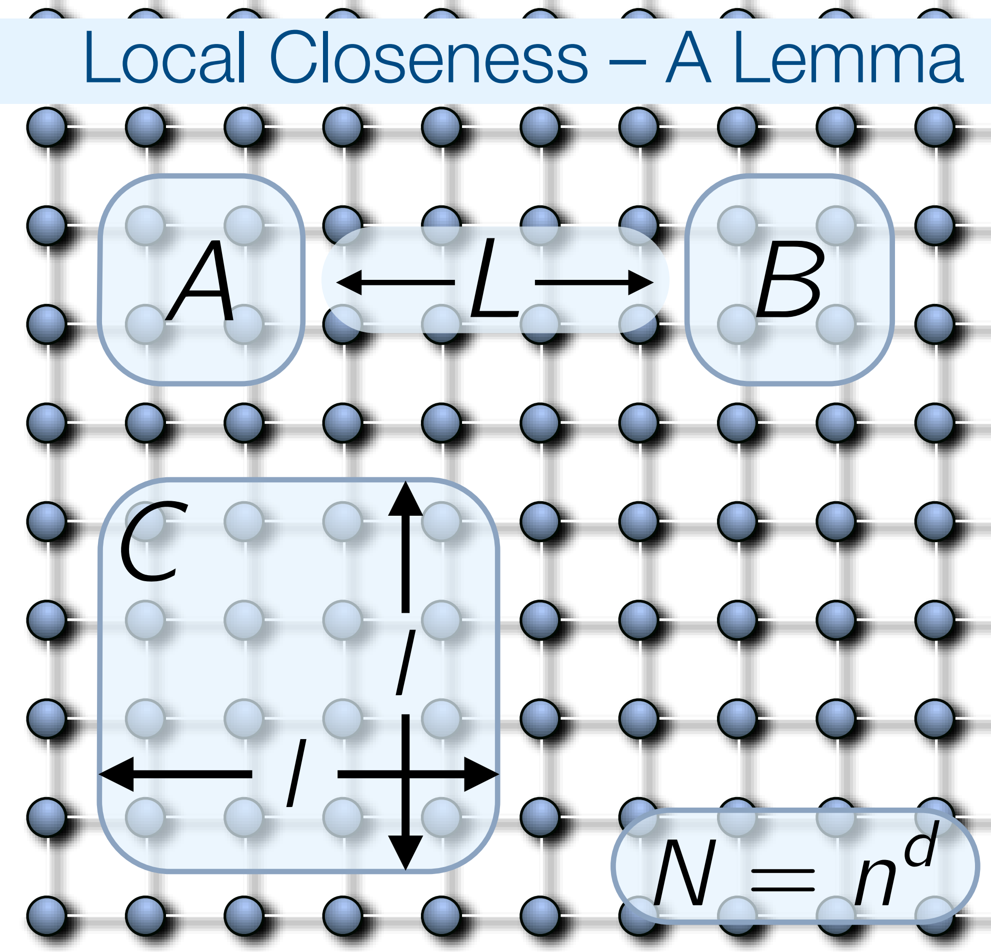

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$$\|\hat{\tau}_{C_1\cdots C_M} - \hat{\tau}_{C_1} \otimes \cdots \otimes \hat{\tau}_{C_1}\| \leq \sum_{j=2}^{M} \mathrm{cov}(\hat{A}_1 \cdots \hat{A}_{j-1}, \hat{A}_j)$$

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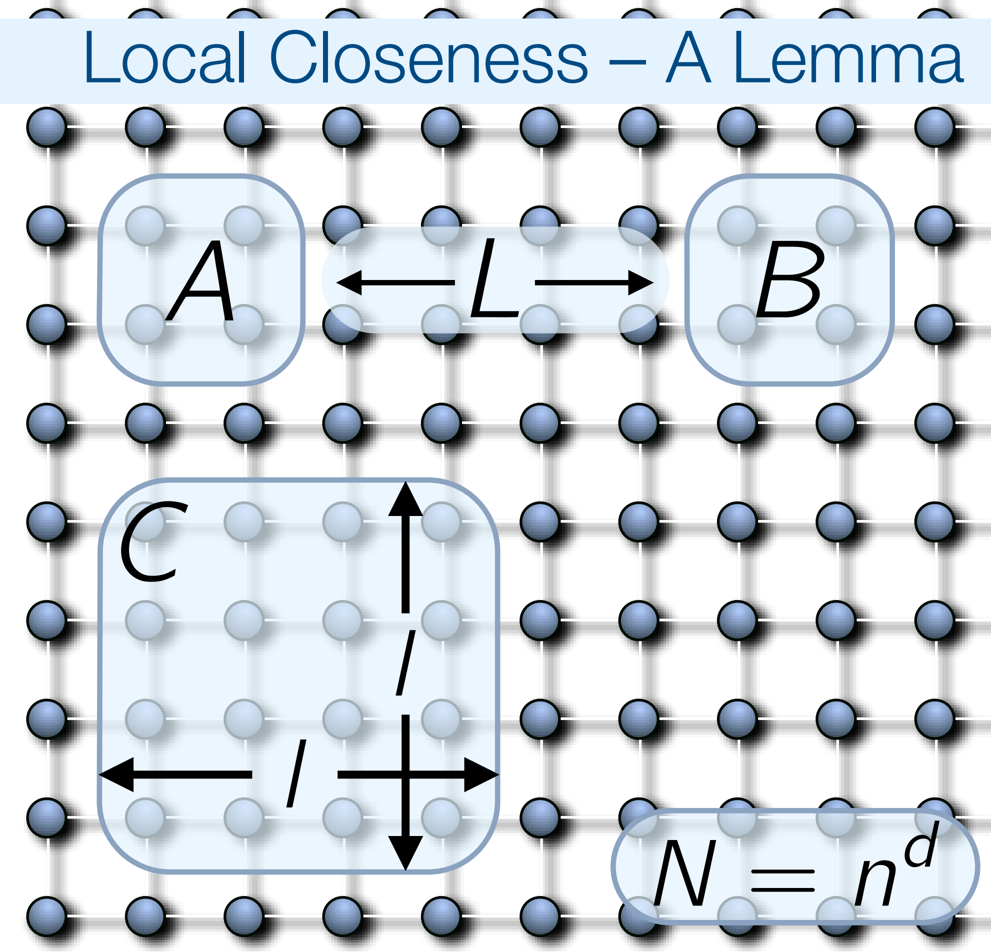

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- Quantum Substate Theorem Jain, Radhakrishnan, Sen (2009); Jain, Nayak (2011)
- Lemma Datta, Renner (2009); Brandão, Plenio (2010); Brandão, Horodecki (2012)
- Pinsker's inequality $\|\hat{\varrho} - \hat{\tau}\|_{\mathrm{tr}} \leq \ln(4) S(\hat{\varrho}\|\hat{\tau})$
- Super-additivity $\sum_{j=1}^{M} S(\hat{\varrho}_{C_j}\|\hat{\tau}_{C_j}) \leq S(\hat{\varrho}\|\hat{\tau}_{C_1} \otimes \cdots \otimes \hat{\tau}_{C_M})$

Equivalence of Statistical Mechanical Ensembles for Non-Critical Quantum Systems
Brandão, Cramer, arxiv:1502.03263

$$X = \sum_{i=1}^{N} X_i$$

X_i : “weakly correlated”

Central Limit Theorem:

$$\mathbb{P}[X \leq x] = F(x) \xrightarrow{N\to\infty} G(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{x} \mathrm{d}y \, \mathrm{e}^{-\frac{(y-\mu)^2}{2\sigma^2}}$$

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$\mu = \langle X \rangle, \;\; \sigma^2 = \left\langle (X-\mu)^2 \right\rangle$

$$\hat{H} = \sum_{i\in\Lambda} \hat{H}_i = \sum_k E_k |k\rangle\langle k| \quad \text{local}$$

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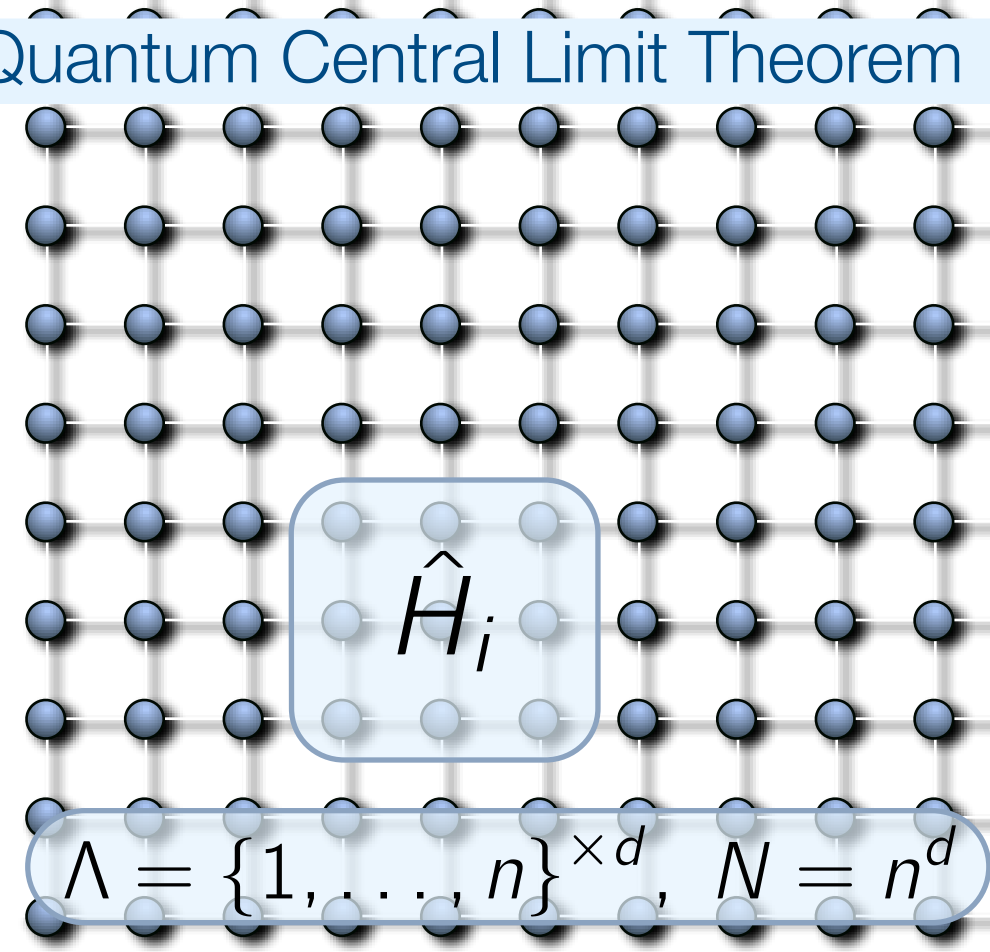

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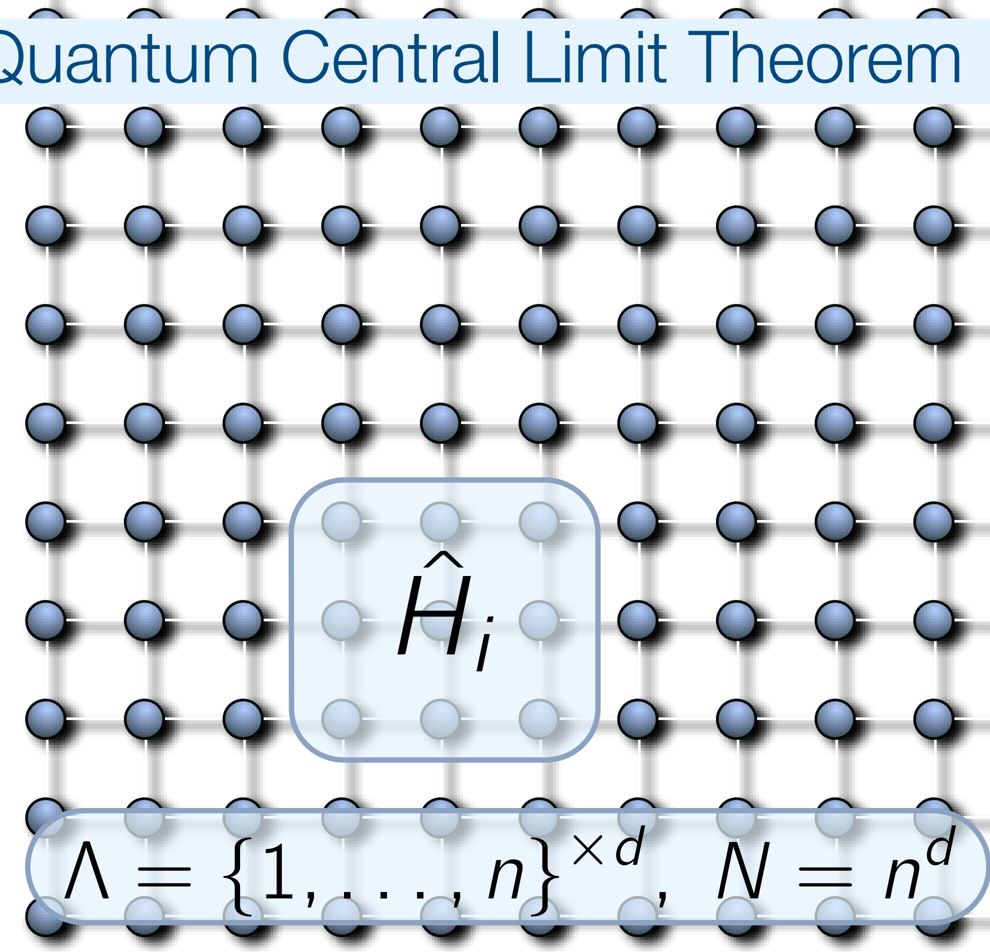

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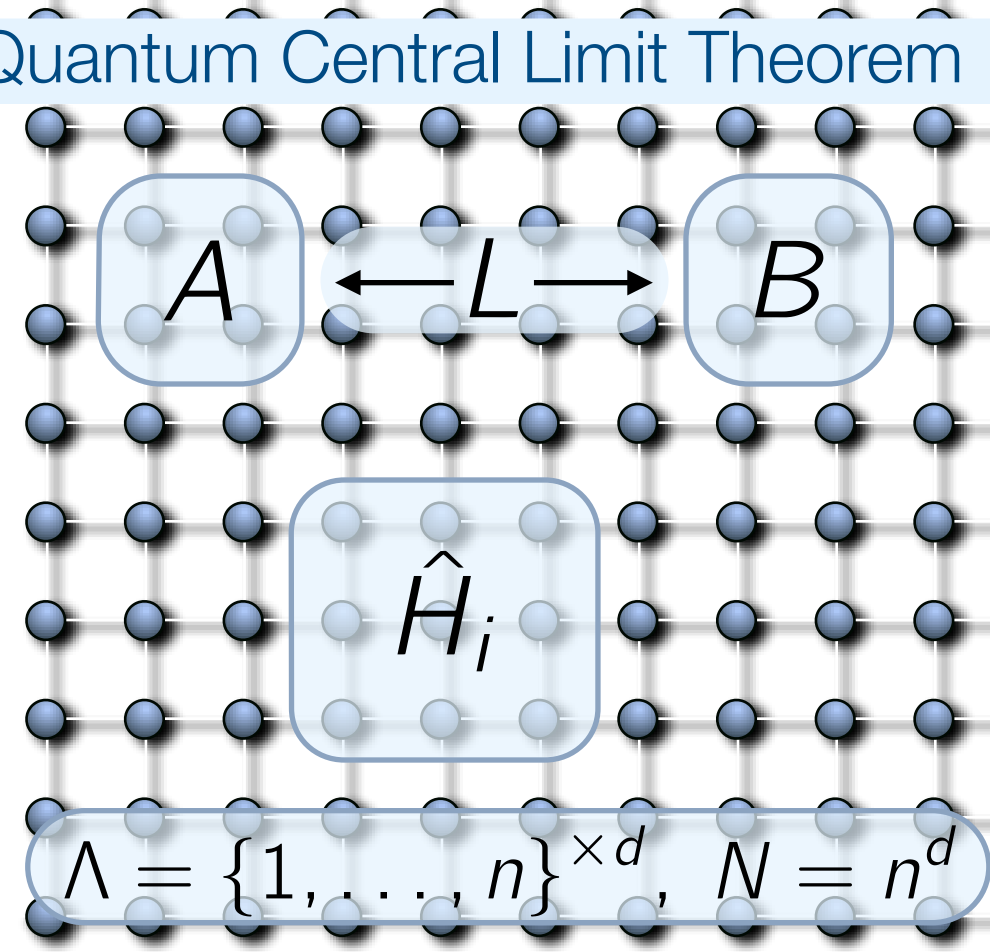

$$\hat{\varrho} : \frac{|\langle \hat{A}\hat{B}\rangle - \langle \hat{A}\rangle\langle \hat{B}\rangle|}{\|\hat{A}\|\|\hat{B}\|} \leq N^z e^{-L/\xi}$$

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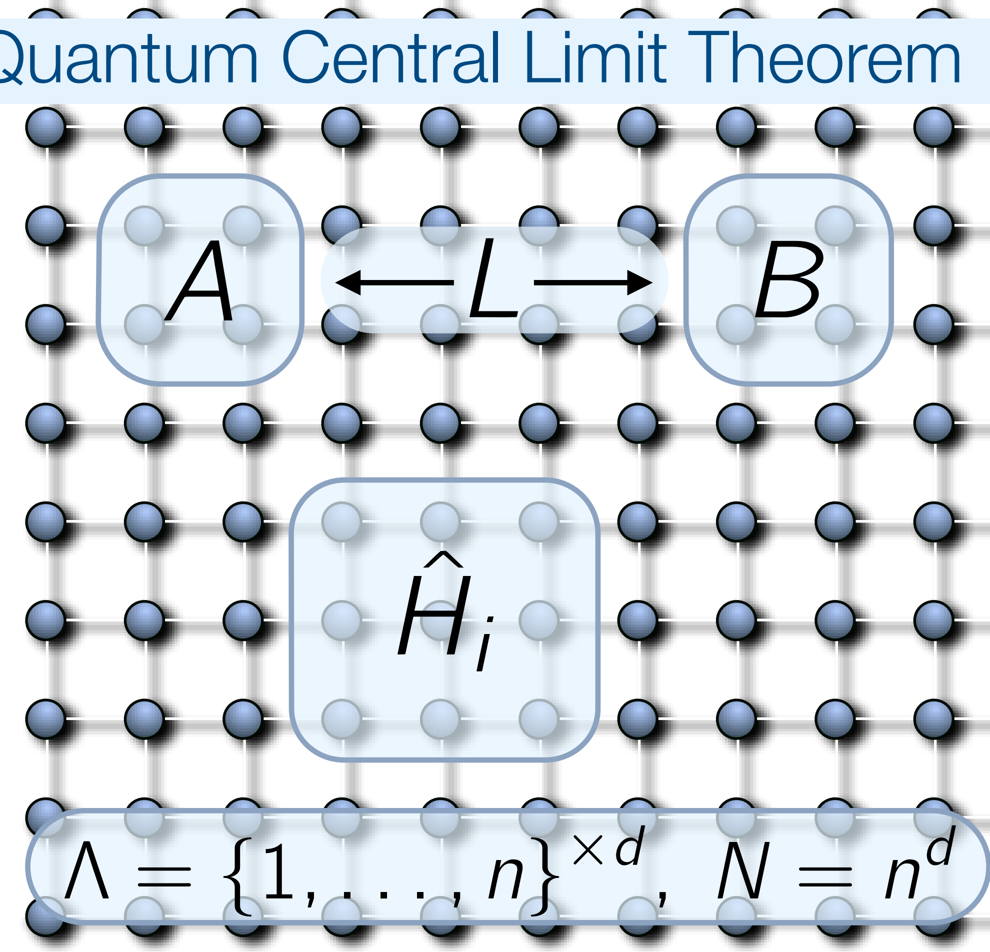

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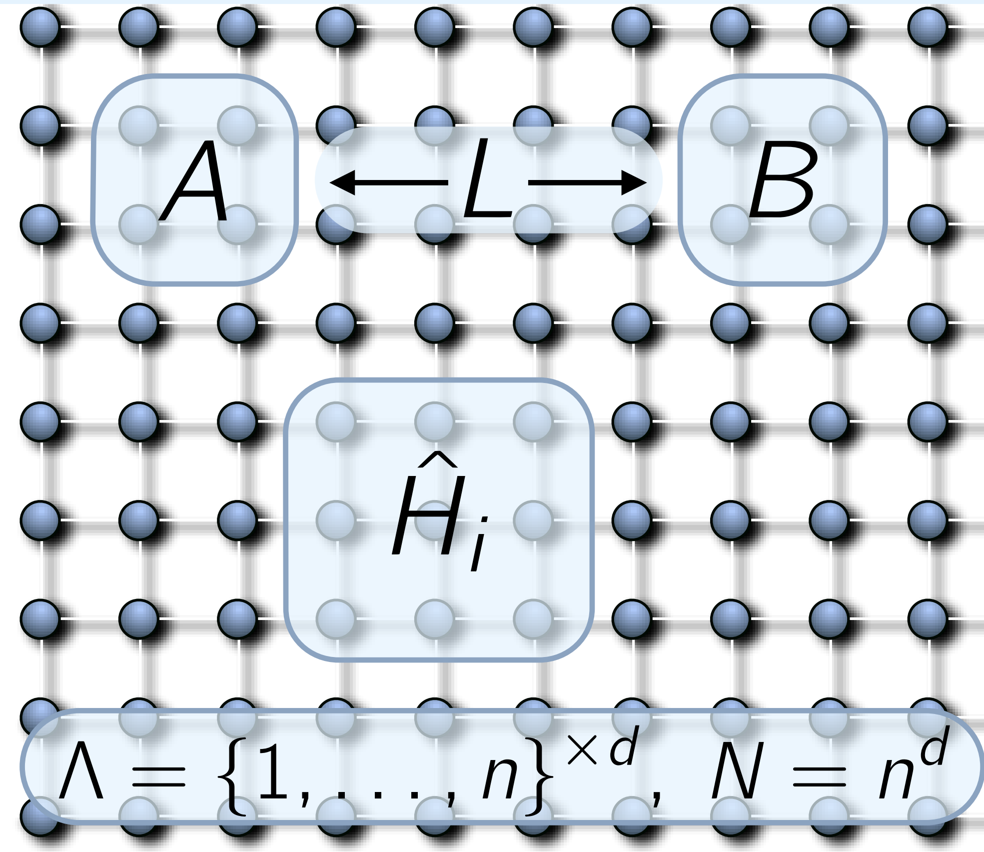

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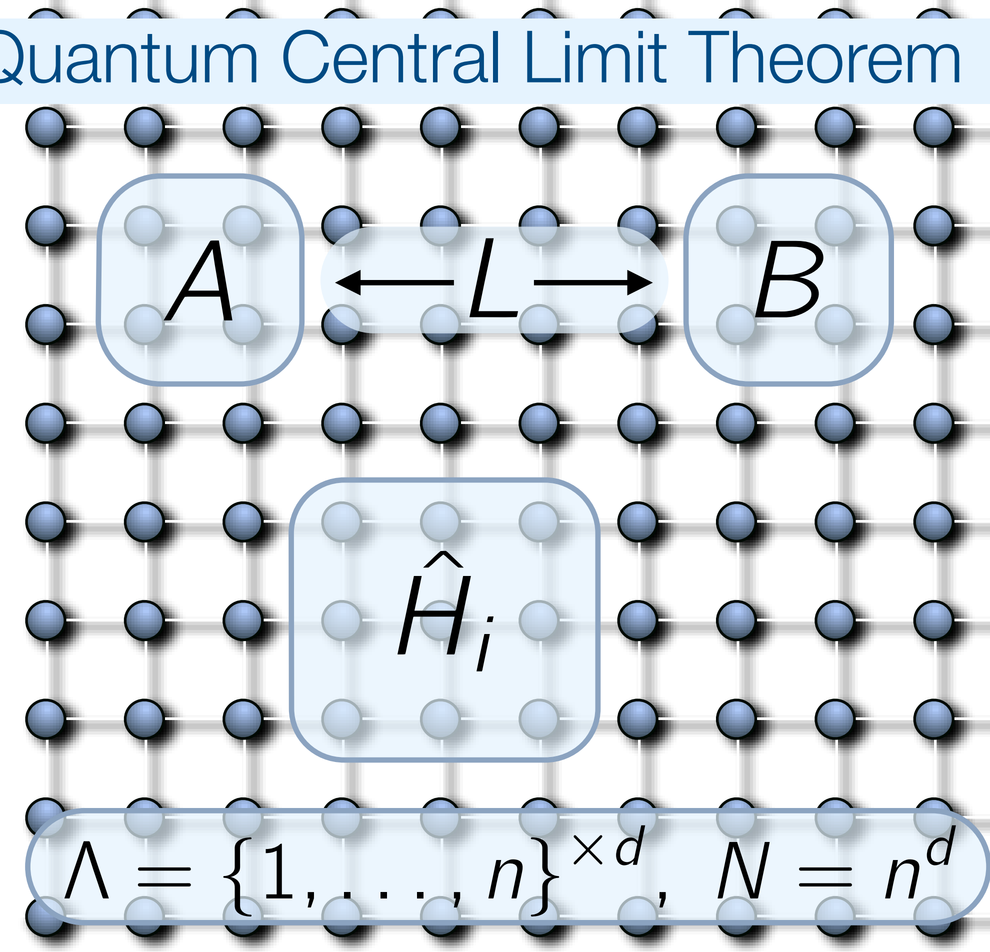

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Central Limit Theorem:

$$\sum_{E_k \leq x} \langle k|\hat{\varrho}|k\rangle = F(x)$$

relation to density of states: $\hat{\varrho} \propto \mathbb{1}$

$$\left|\left\{k : E - \Delta E < E_k \leq E\right\}\right|$$

$$\propto F(E) - F(E - \Delta E)$$

Berry—Esseen: $\sup_x |F(x) - G(x)| \leq C \frac{\ldots}{\sqrt{N}}$

Cramer, Brandão, Guta, in prep. (2015)

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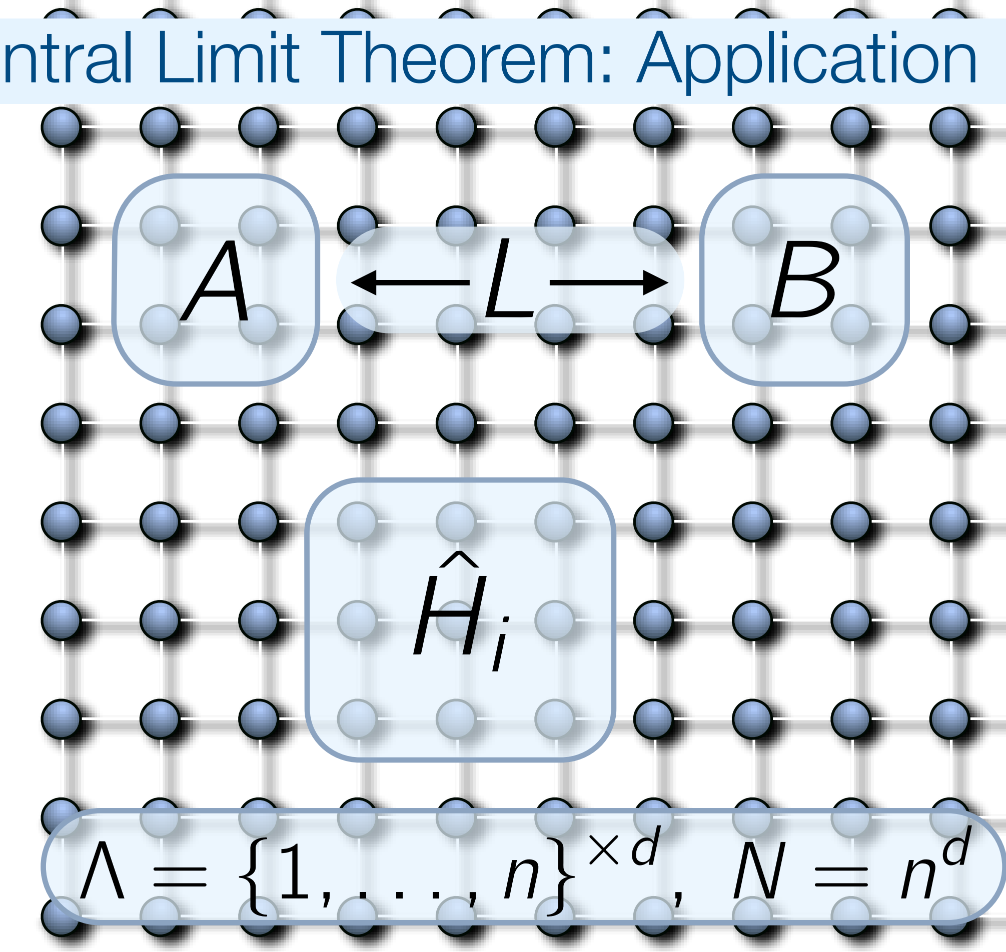
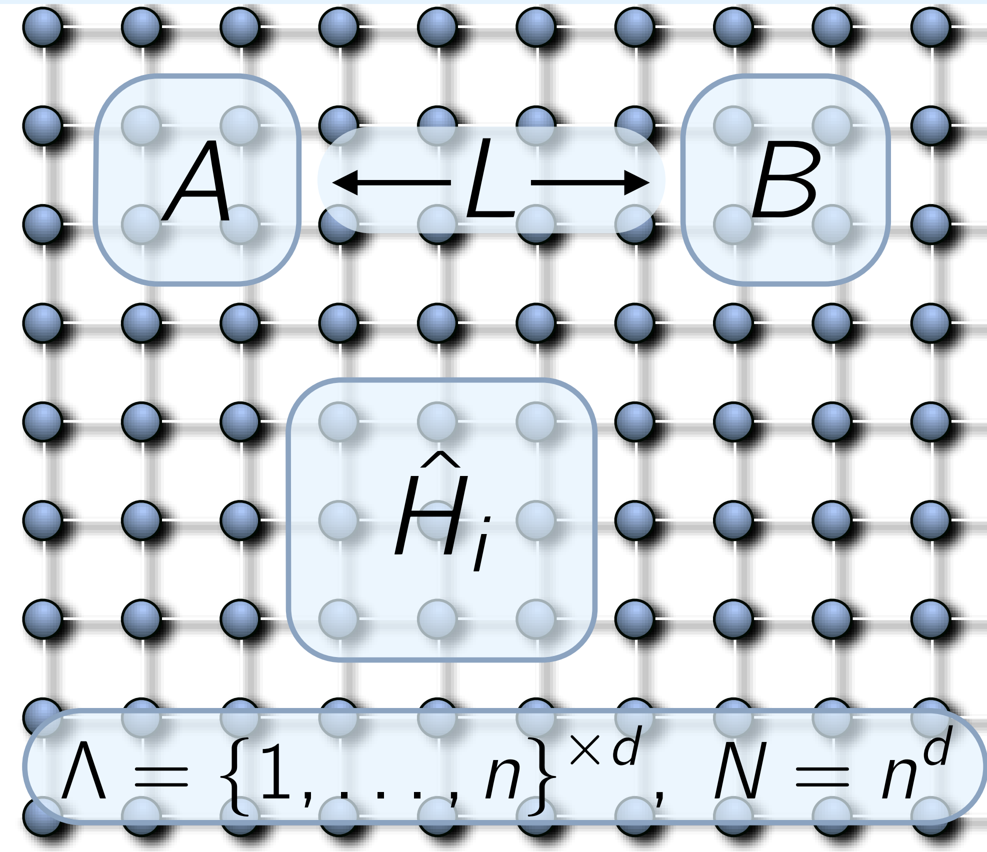
$d = 1:$ Araki (1969)

$d > 1,\ T > T_c:$ Kliesch, Gogolin, Kastoryano, Riera, Eisert (2014)

canonical state $\hat{\varrho}_T = \mathrm{e}^{-\hat{H}/T}/Z$





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canonical state $\hat{\varrho}_T = \mathrm{e}^{-\hat{H}/T}/Z$

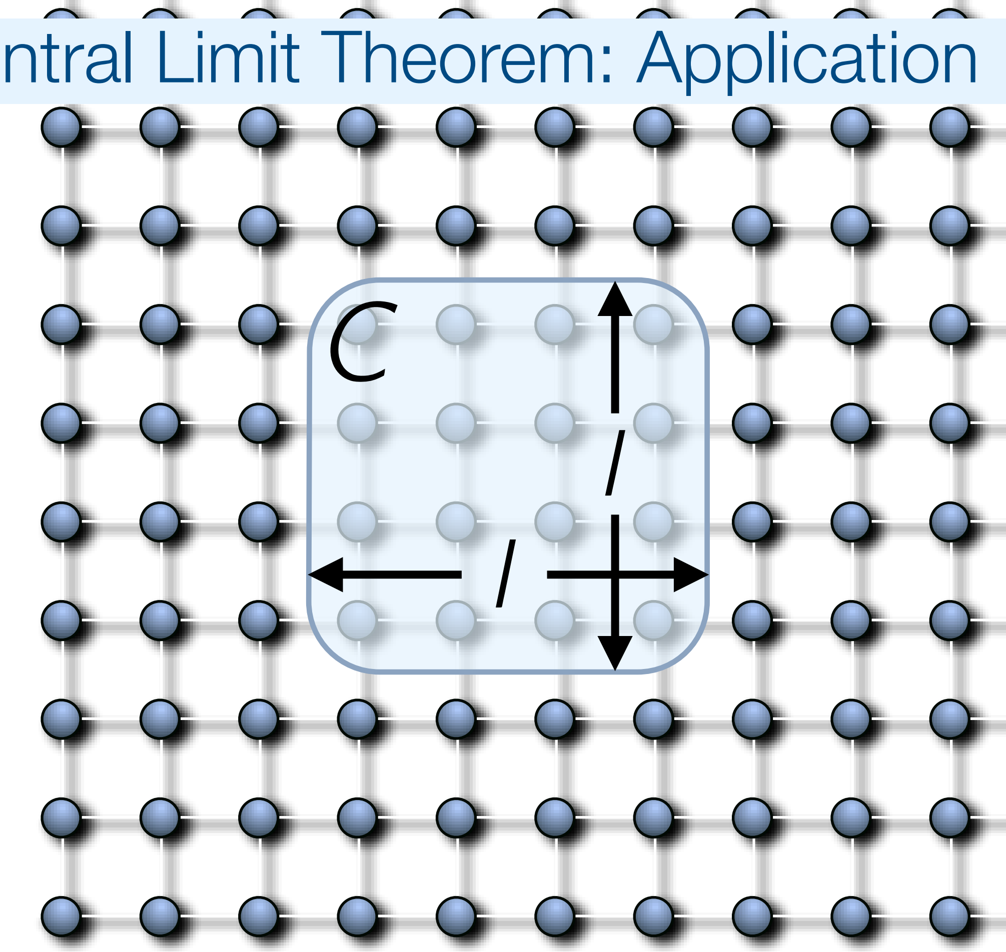
with energy density $u(T) = \frac{\mathrm{tr}[\hat{H}\hat{\varrho}_T]}{N} \quad (= \frac{\mu}{N})$

specific heat capacity $c(T) = \frac{\partial}{\partial T} u(T) \quad (= \frac{\sigma^2}{NT^2})$

canonical state $\hat{\tau} = \mathrm{e}^{-\hat{H}/T}/Z$

for which states $\hat{\varrho}$ (and which l) is

$$\|\hat{\varrho}_C - \hat{\tau}_C\|_{\mathrm{tr}} \leq \epsilon \text{ ?}$$



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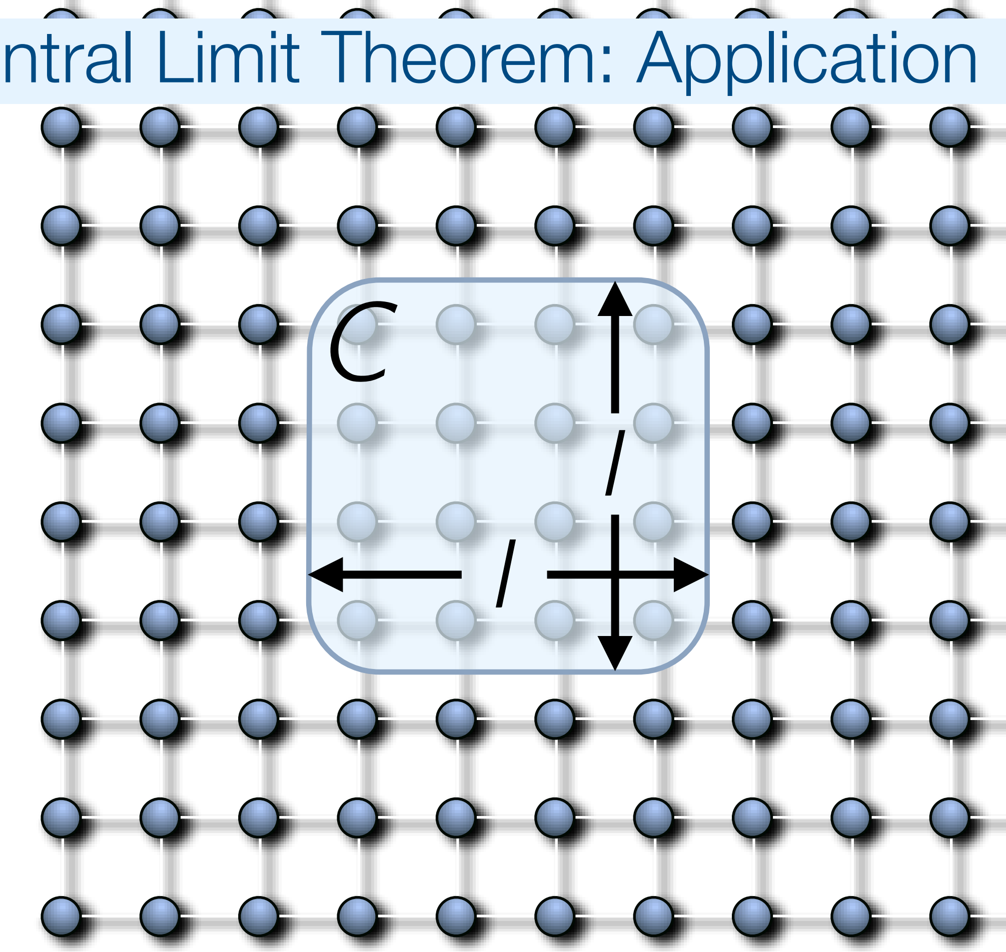
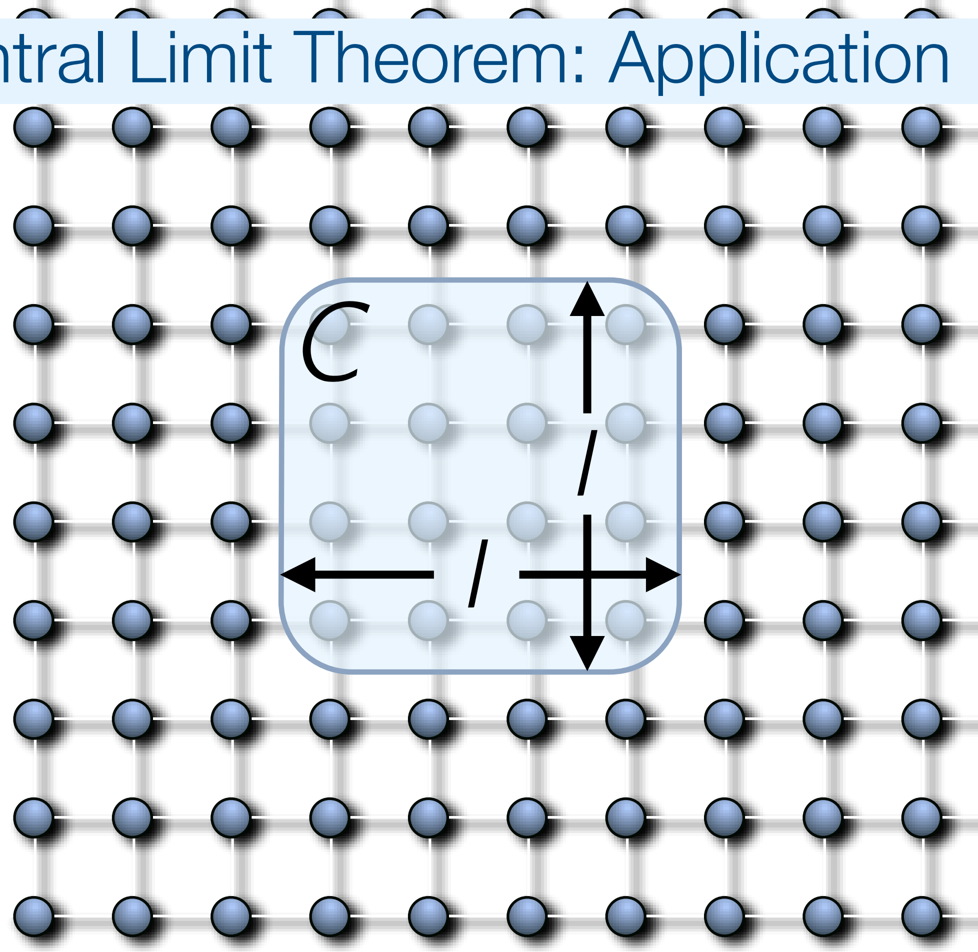
which states $\hat{\varrho}$ are locally thermal?





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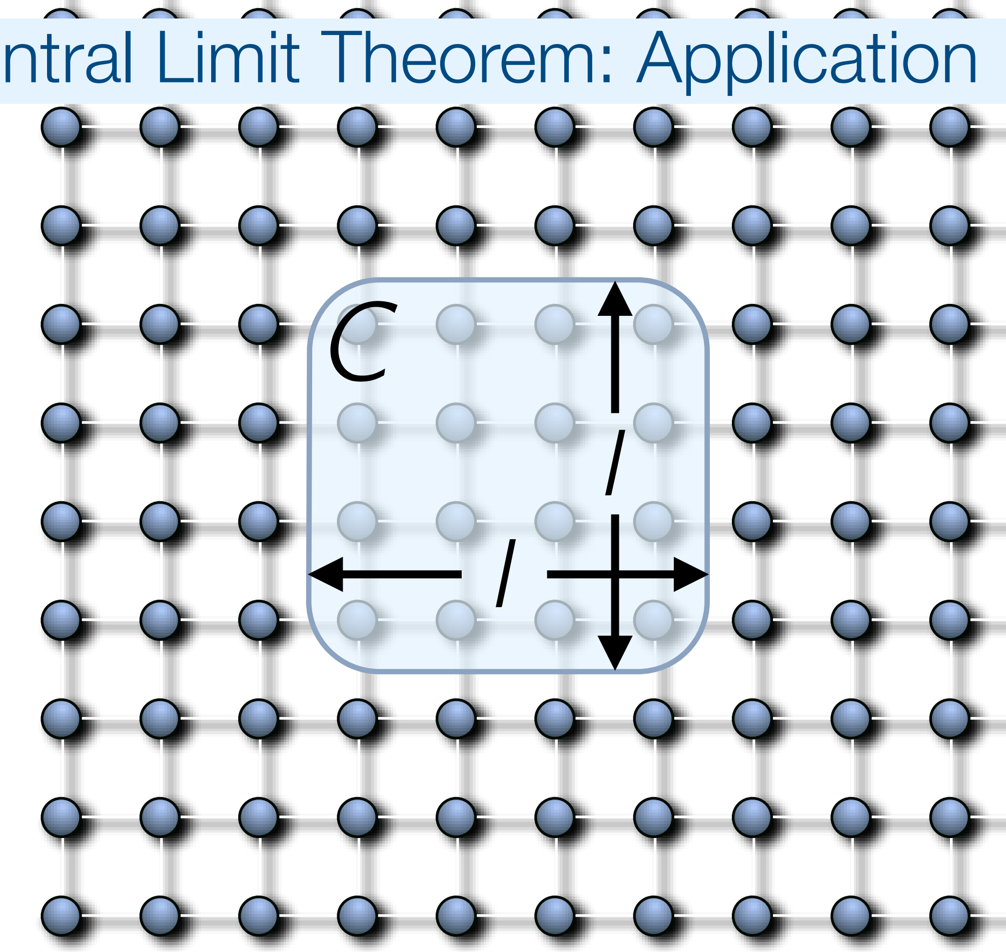
for microcanonical states $\hat{\varrho} = \frac{\mathbb{1}_{M_\delta}}{|M_\delta|}$

this question goes back to Boltzmann and Gibbs

previous work:

- Thermodynamical functions
  [Lebowitz, Lieb (1969); Lima (1971/72); Touchette (2009)]
- States [Mueller, Adlam, Masanes, Wiebe (2013)]
- Popescu, Short, Winter (2005); Riera, Gogolin, Eisert (2011)

thermodynamical limit, t.i.



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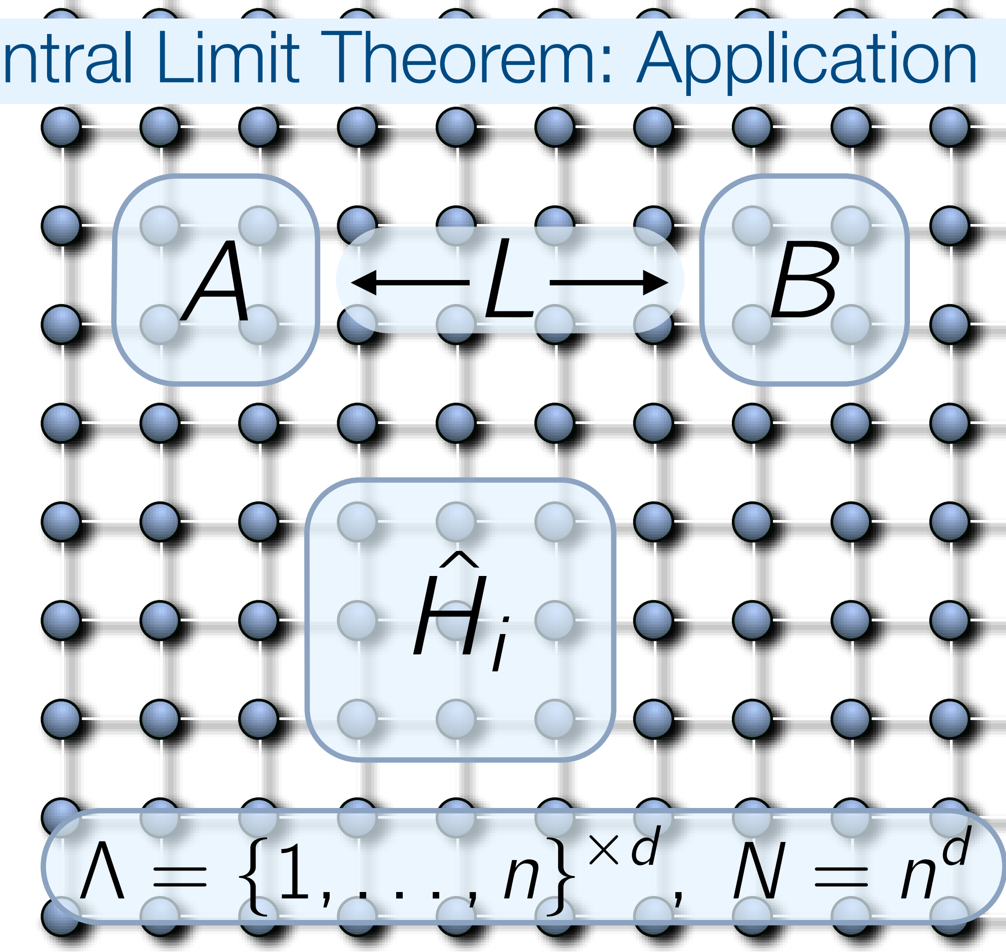
this question goes back to Boltzmann and Gibbs

here:

- Finite size, explicit bounds
- Not necessarily translational invariant
- More general than microcanonical

$$\hat{H} = \sum_{i \in \Lambda} \hat{H}_i = \sum_k E_k |k\rangle\langle k| \quad \text{local}$$



$$\hat{\varrho}_T : \; \frac{|\langle \hat{A}\hat{B}\rangle - \langle \hat{A}\rangle\langle \hat{B}\rangle|}{\|\hat{A}\|\|\hat{B}\|} \leq N^z e^{-L/\xi}$$

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$d > 1, \; T > T_c$: Kliesch, Gogolin, Kastoryano, Riera, Eisert (2014)

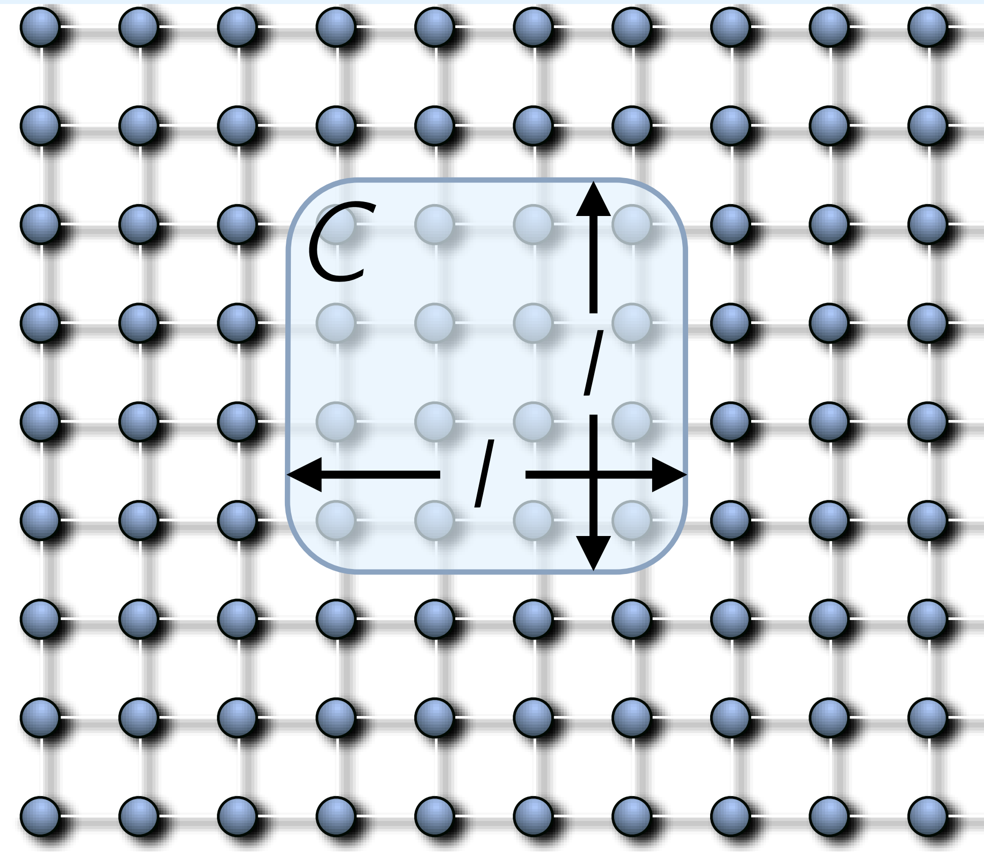
canonical state $\hat{\varrho}_T = e^{-\hat{H}/T}/Z$

$\hat{\varrho}$: state on microcanonical subspace

$$M_\delta = \{|k\rangle : |E_k - N u(T)| \leq \delta\sqrt{N}\}, \quad \frac{\log^{2d}(N)}{\sqrt{N}} \lesssim \delta \lesssim 1$$

quantum Berry—Esseen

$$S(\hat{\varrho}\|\hat{\varrho}_T) \lesssim \log(|M_\delta|) - S(\hat{\varrho}) + \log^{2d}(N)$$



canonical state $\hat{\tau} = \mathrm{e}^{-\hat{H}/T}/Z$

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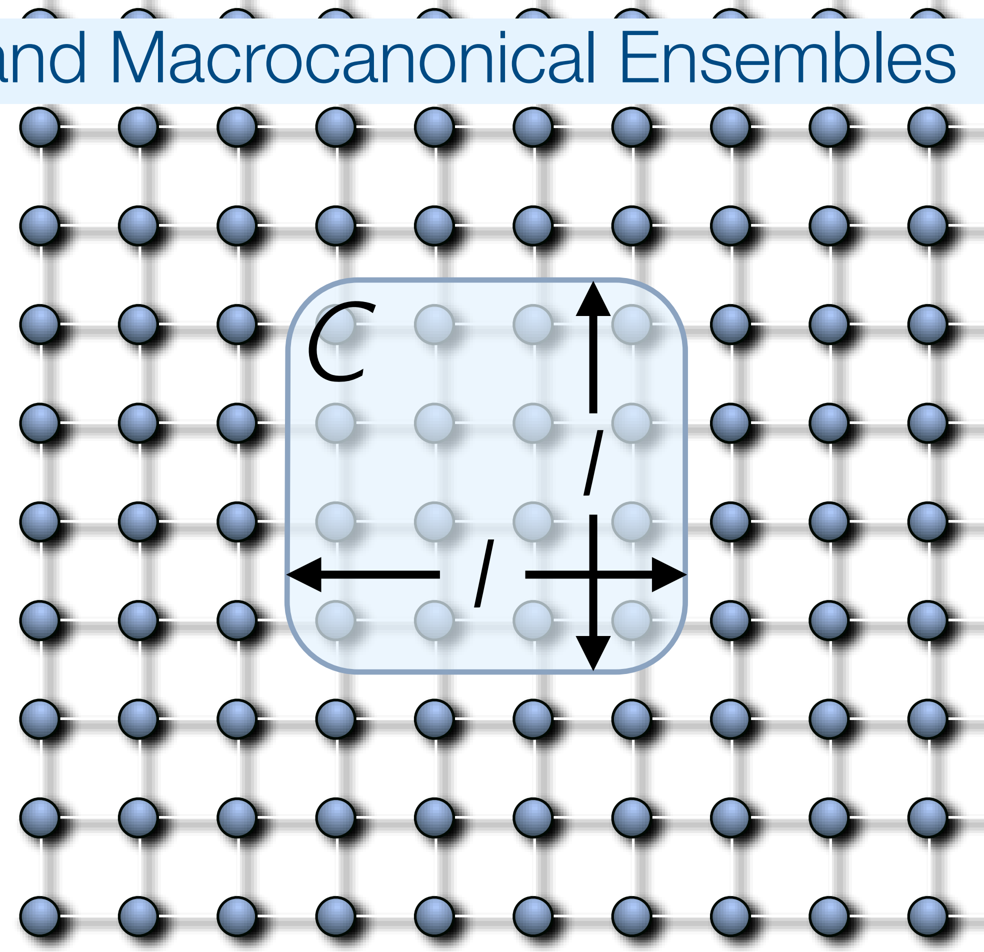
with

$$M_\delta = \{|k\rangle : |E_k - N u(T)| \leq \delta \sqrt{N}\}, \; \frac{\log^{2d}(N)}{\sqrt{N}} \lesssim \delta \lesssim 1$$

and l such that $l^d \lesssim \frac{(\epsilon^2 N)^{\frac{1}{d+1}}}{\ln(N)}$



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$\delta = 0$: Eigenstate Thermalization

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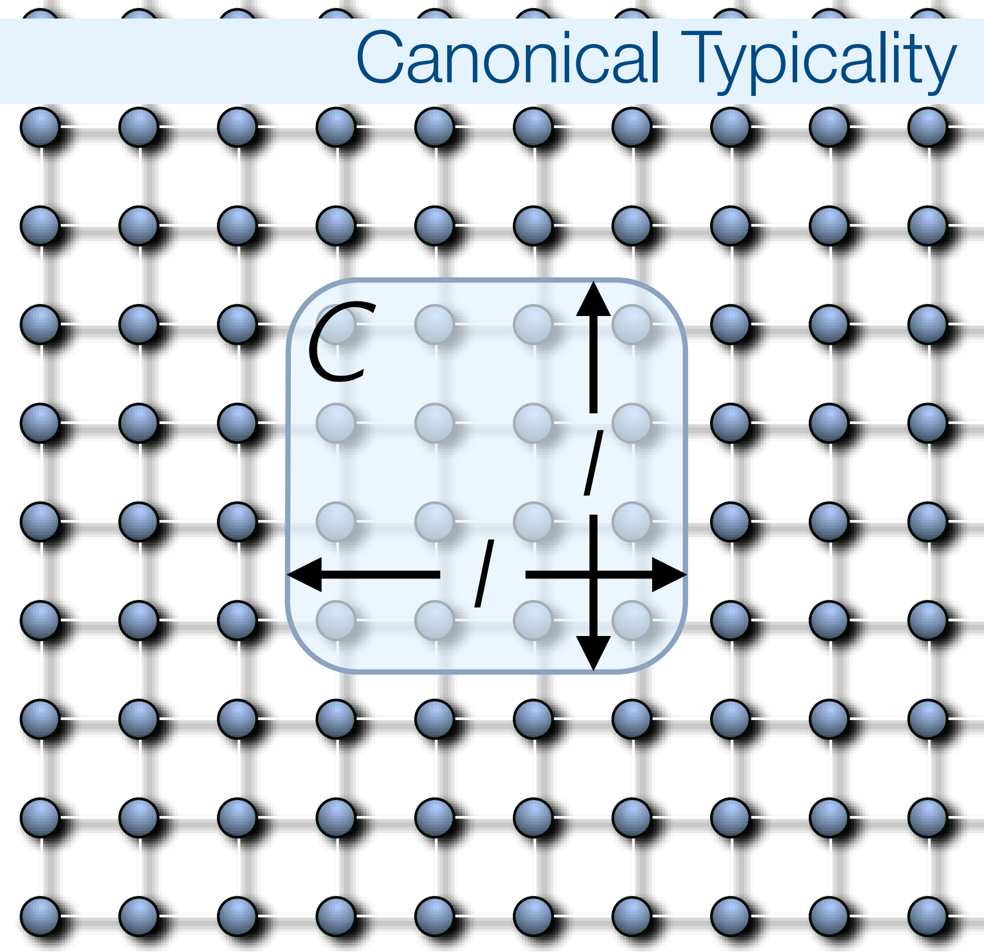

pure states $\hat{\varrho}$ drawn from the subspace spanned by M_δ:

$$\mathbb{P}\left[\|\hat{\varrho}_c - (\mathrm{m.c.})_C\|_{\mathrm{tr}} \leq \sqrt{\epsilon} + 2^{l^d}/\sqrt{|M_\delta|}\right] \geq 1 - 2\mathrm{e}^{-|M_\delta|\epsilon}$$

Popescu, Short, Winter (2005)

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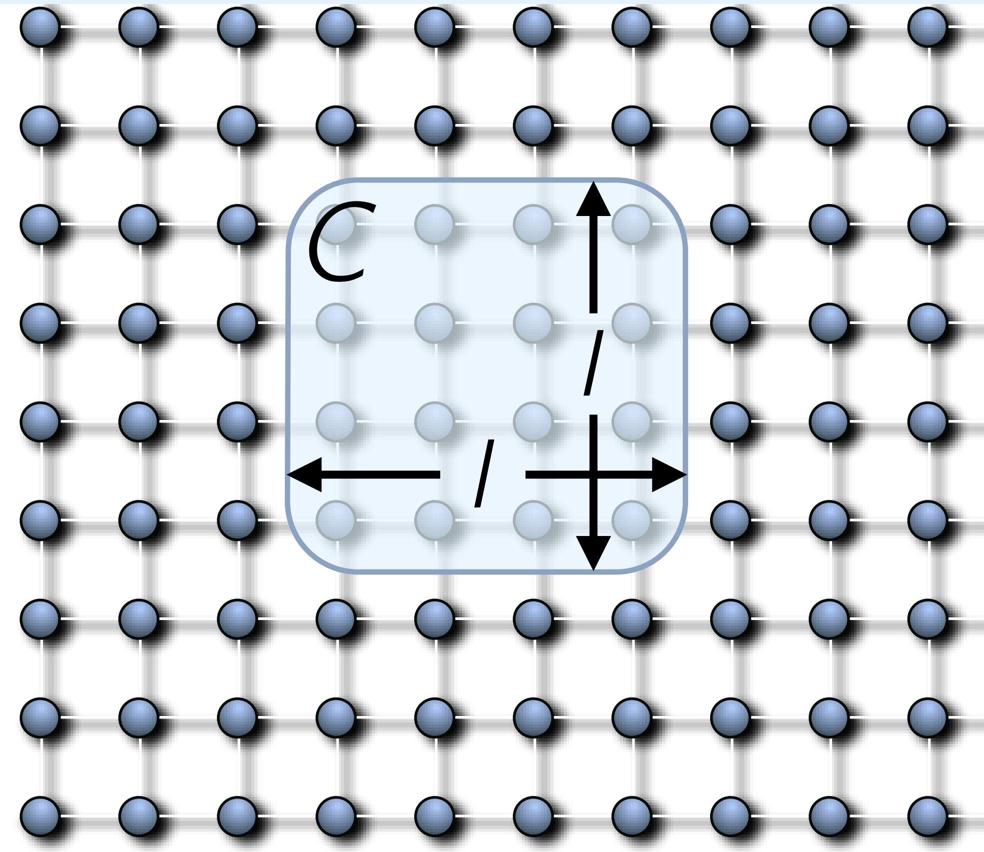

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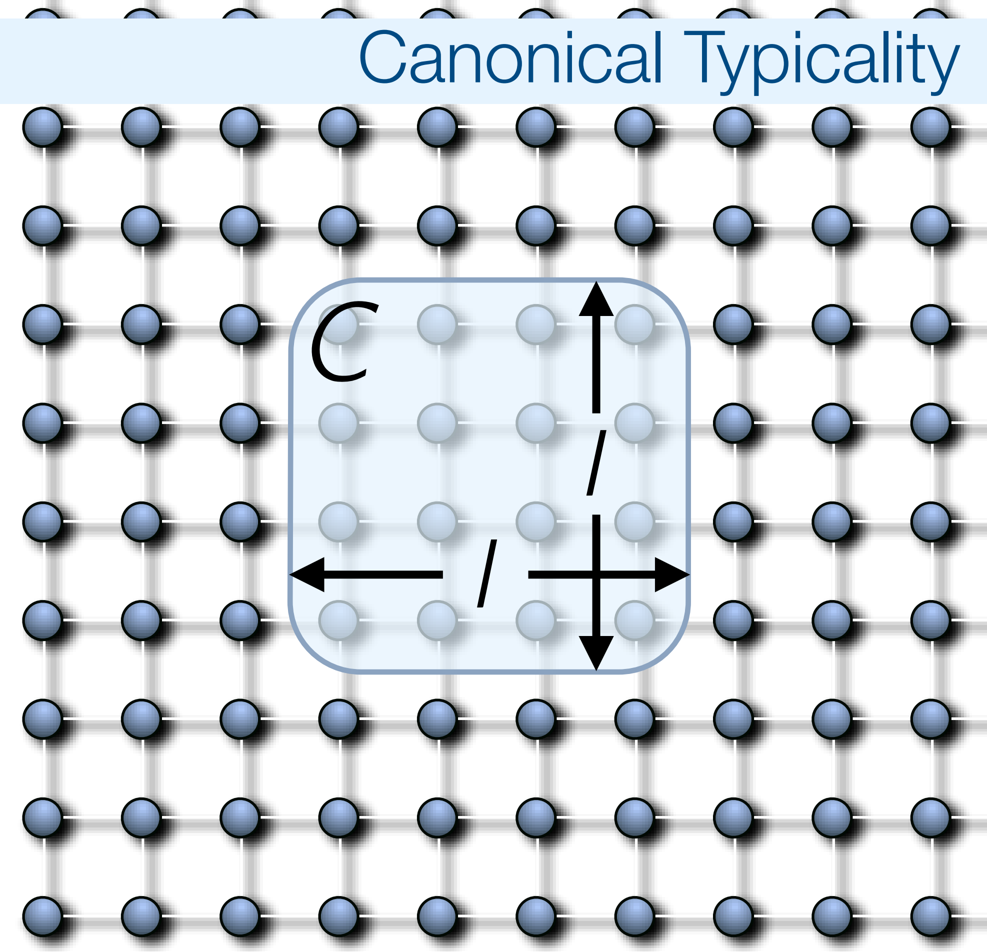
Popescu, Short, Winter (2005)

QBE

$$|M_\delta| \geq \exp\left[S(\hat{\tau}) - \log^{2d}(N)\sqrt{N}\right]$$



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Popescu, Short, Winter (2005)

$$\underset{\text{QBE}}{\geq} 1 - 2\exp\left[-\epsilon\exp\left(S(\hat{\tau}) - \log^{2d}(N)\sqrt{N}\right)\right] =: p$$

canonical state $\hat{\tau} = \mathrm{e}^{-\hat{H}/T}/Z$

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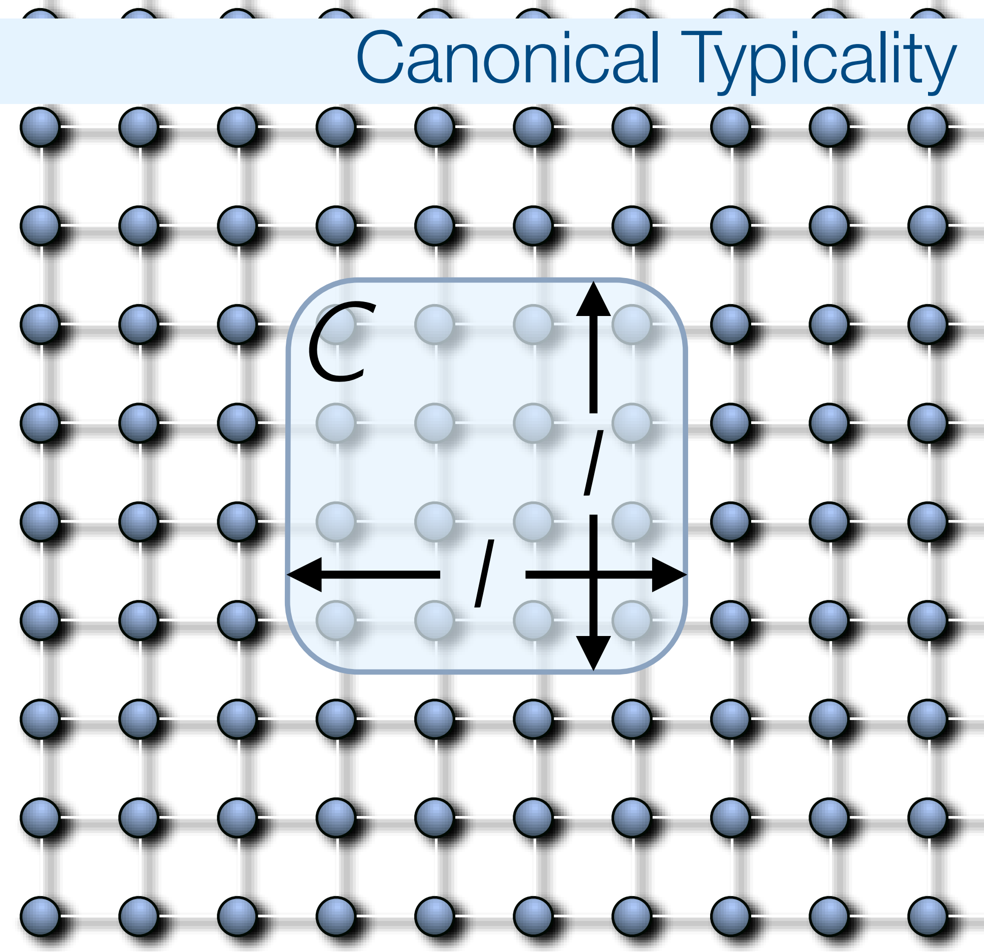
$$\|\hat{\varrho}_C - \hat{\tau}_C\|_{\mathrm{tr}} \leq \epsilon \;?$$



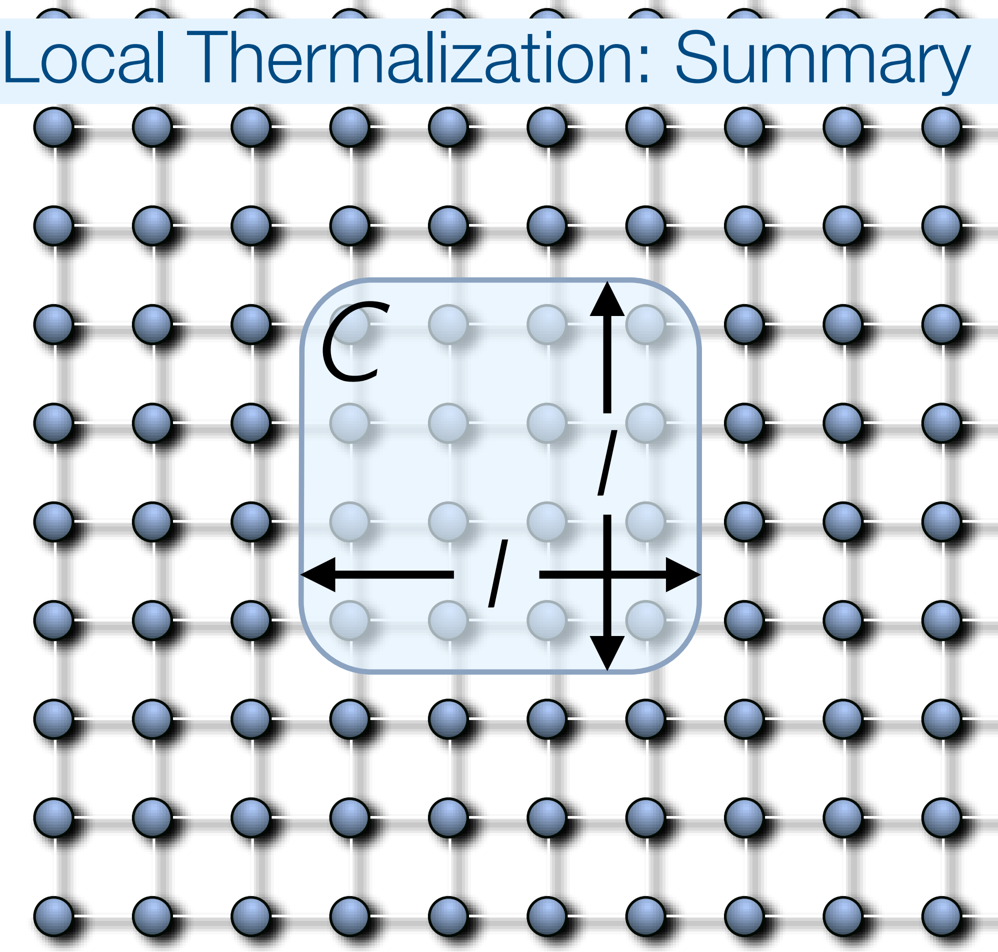
which states $\hat{\varrho}$ are locally thermal?

pure states $\hat{\varrho}$ drawn from the subspace spanned by M_δ:

$\hat{\tau}, M_\delta, \delta, l$ as before → with probability at least p

$$\|\hat{\varrho}_C - \hat{\tau}_C\|_{\mathrm{tr}} \leq \epsilon + 2^{l^d} \exp\left[-\left(S(\hat{\tau}) - \log^{2d}(N)\sqrt{N}\right)\right]$$

cf. Riera, Gogolin, Eisert (2011); Mueller, Adlam, Masanes, Wiebe (2013)



canonical state $\hat{\tau} = \mathrm{e}^{-\hat{H}/T}/Z$

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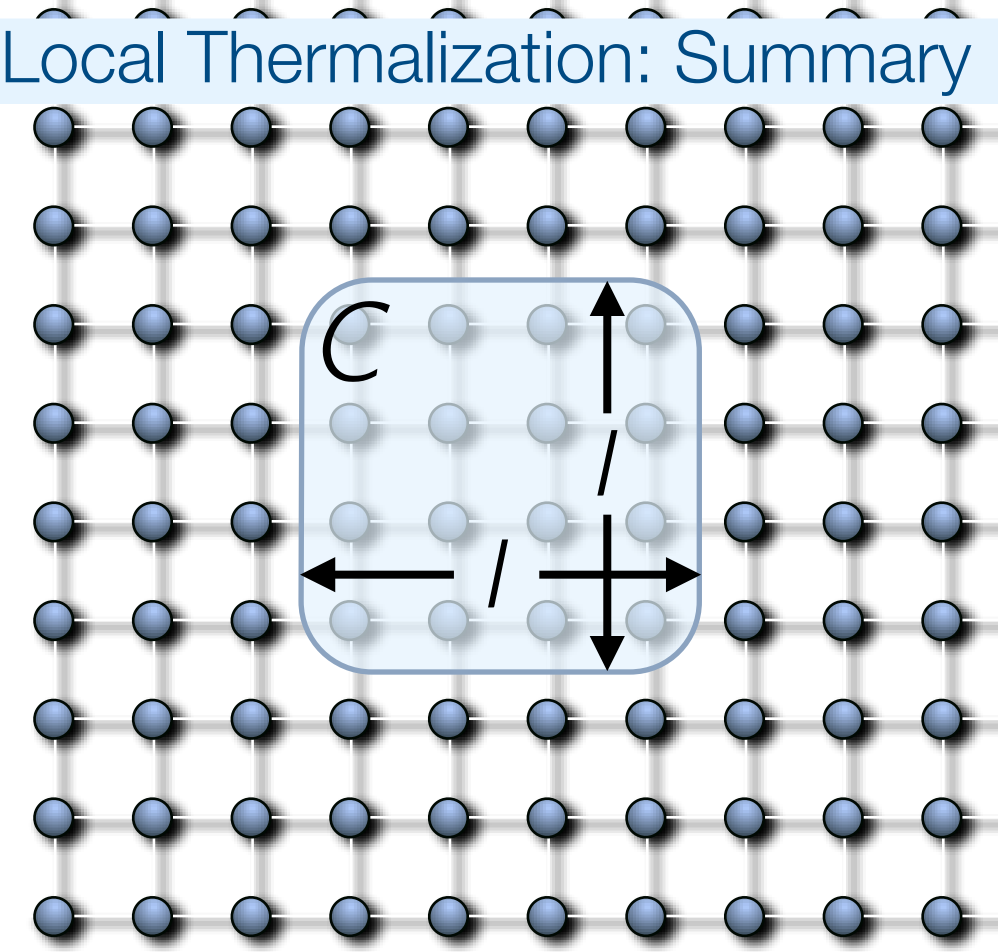
which states $\hat{\varrho}$ are locally thermal?

$\hat{\tau}$, l as before then those

- with small free energy $F_T(\hat{\varrho}) \lesssim F_T(\hat{\tau}) + \frac{T\epsilon^2(\epsilon^2 N)^{\frac{1}{d+1}}}{\ln(N)}$

$F_T(\hat{\varrho}) = \mathrm{tr}[\hat{H}\hat{\varrho}] - TS(\hat{\varrho})$



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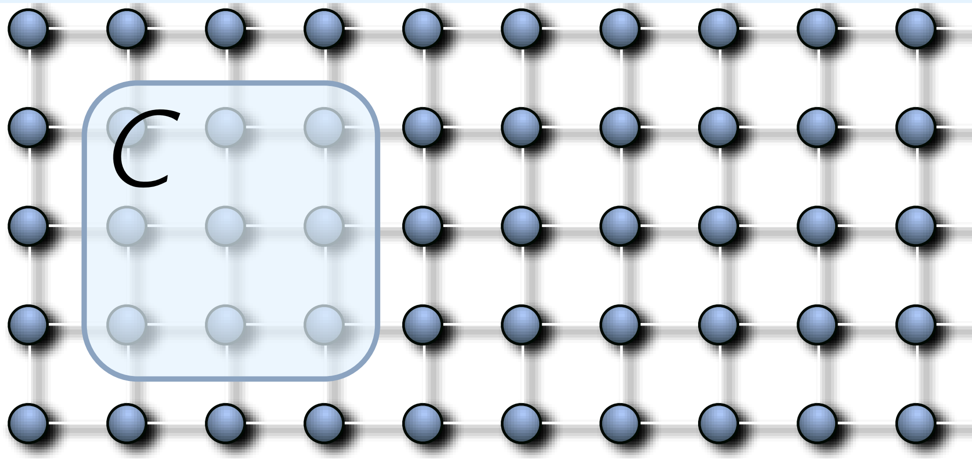
which states $\hat{\varrho}$ are locally thermal?

$\hat{\tau}$, M_δ, δ, l as before then those

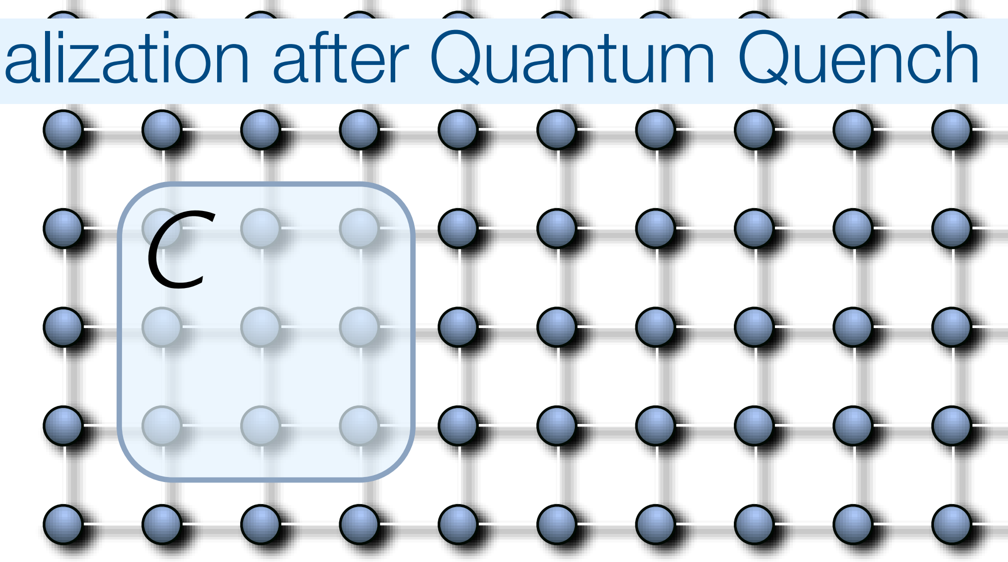
- with small free energy $F_T(\hat{\varrho}) \lesssim F_T(\hat{\tau}) + \frac{T\epsilon^2(\epsilon^2 N)^{\frac{1}{d+1}}}{\ln(N)}$
- in microcanonical subspace
  with large entropy $S(\hat{\varrho}) \geq \log(|M_\delta|) - \frac{\epsilon^2(\epsilon^2 N)^{\frac{1}{d+1}}}{\ln(N)}$
- “almost all” pure states in this subspace



$$\hat{\varrho}(t) = \mathrm{e}^{-\mathrm{i}t\hat{H}}\hat{\varrho}_0\mathrm{e}^{\mathrm{i}t\hat{H}}$$

$$\hat{H} = \sum_k E_k |k\rangle\langle k|$$

$$\hat{\omega} = \lim_{T\to\infty} \frac{1}{T}\int_0^T \mathrm{d}t\, \hat{\varrho}(t)$$



$$\hat{\varrho}(t) = e^{-it\hat{H}}\hat{\varrho}_0 e^{it\hat{H}}$$

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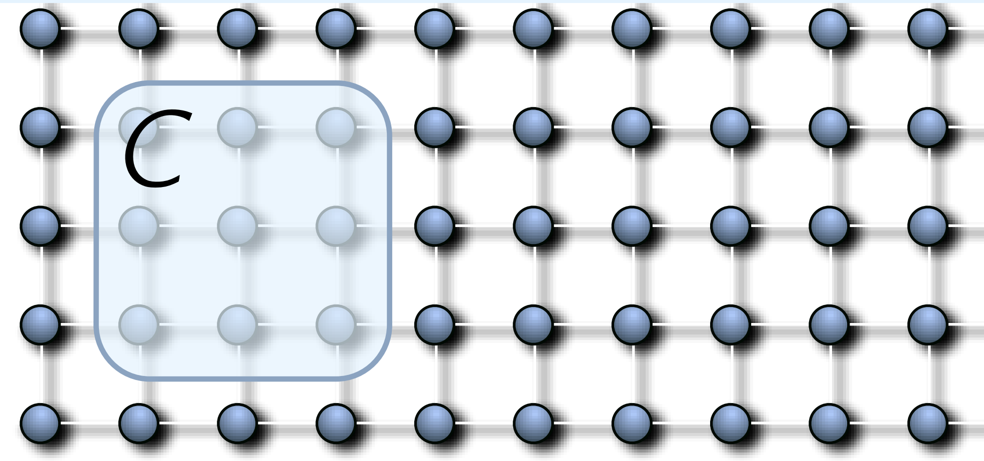
$$\lim_{T\to\infty} \frac{1}{T}\int_0^T dt\, \|\hat{\varrho}_C(t) - \hat{\omega}_C\|_{\mathrm{tr}} \leq 2^{|C|}\sqrt{\mathrm{tr}[\hat{\omega}^2]}$$

Linden, Popescu, Short, Winter (2008)



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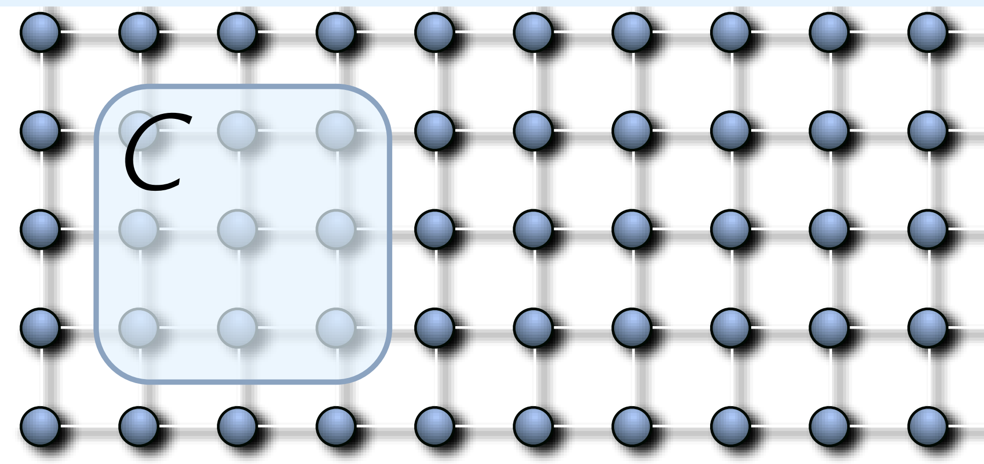
$$\hat{H} = \sum_k E_k |k\rangle\langle k|$$

$$\hat{\omega} = \lim_{T\to\infty}\frac{1}{T}\int_0^T \mathrm{d}t\,\hat{\varrho}(t) \underset{\text{non-degen. energy gaps}}{=} \sum_k \langle k|\hat{\varrho}_0|k\rangle |k\rangle\langle k|$$

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T \mathrm{d}t\,\|\hat{\varrho}_C(t) - \hat{\omega}_C\|_{\mathrm{tr}} \le 2^{|C|}\sqrt{\mathrm{tr}[\hat{\omega}^2]}$$

fraction of times for which $\|\hat{\varrho}_C(t) - \hat{\omega}_C\|_{\mathrm{tr}} \le \epsilon$ is at least $1 - 2^{|C|}\sqrt{\mathrm{tr}[\hat{\omega}^2]}/\epsilon$

Linden, Popescu, Short, Winter (2008)



$$\hat{\varrho}(t) = \mathrm{e}^{-\mathrm{i}t\hat{H}}\hat{\varrho}_0\mathrm{e}^{\mathrm{i}t\hat{H}}$$

$$\hat{H} = \sum_k E_k|k\rangle\langle k|$$

$$\hat{\omega} = \lim_{T\to\infty}\frac{1}{T}\int_0^T \mathrm{d}t\,\hat{\varrho}(t) = \sum_k \langle k|\hat{\varrho}_0|k\rangle|k\rangle\langle k|$$

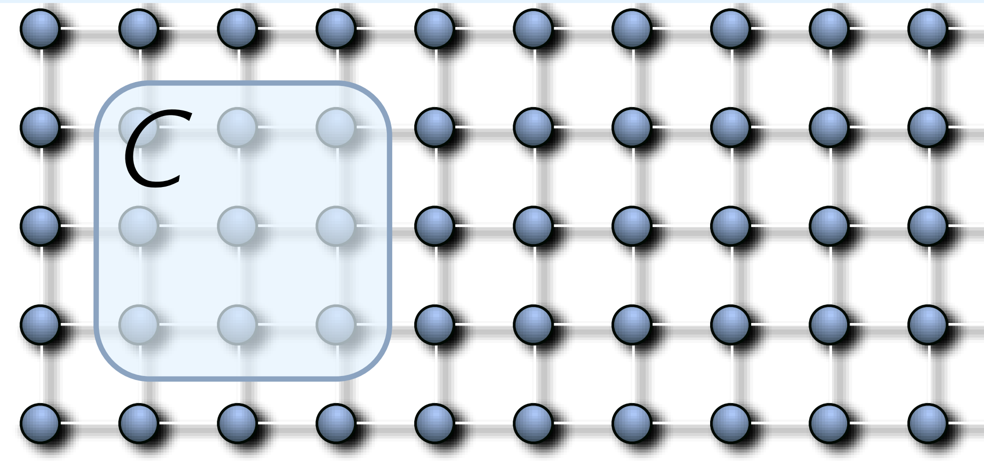
non-degen. energy gaps

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T \mathrm{d}t\,\|\hat{\varrho}_C(t)-\hat{\omega}_C\|_{\mathrm{tr}} \le 2^{|C|}\sqrt{\mathrm{tr}[\hat{\omega}^2]}$$

- Geometry irrelevant
- Even “global” observables
- Also “local” quenches

fraction of times for which $\|\hat{\varrho}_C(t)-\hat{\omega}_C\|_{\mathrm{tr}} \le \epsilon$ is at least $1-2^{|C|}\sqrt{\mathrm{tr}[\hat{\omega}^2]}/\epsilon$

Linden, Popescu, Short, Winter (2008)



$$\hat{\varrho}(t) = e^{-it\hat{H}}\hat{\varrho}_0 e^{it\hat{H}}$$

$$\hat{H} = \sum_k E_k |k\rangle\langle k|$$

$$\hat{\omega} = \lim_{T\to\infty} \frac{1}{T}\int_0^T dt\, \hat{\varrho}(t) = \sum_k \langle k|\hat{\varrho}_0|k\rangle |k\rangle\langle k|$$

non-degen. energy gaps

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T dt\, \|\hat{\varrho}_C(t) - \hat{\omega}_C\|_{tr} \leq 2^{|C|}\sqrt{tr[\hat{\omega}^2]}$$

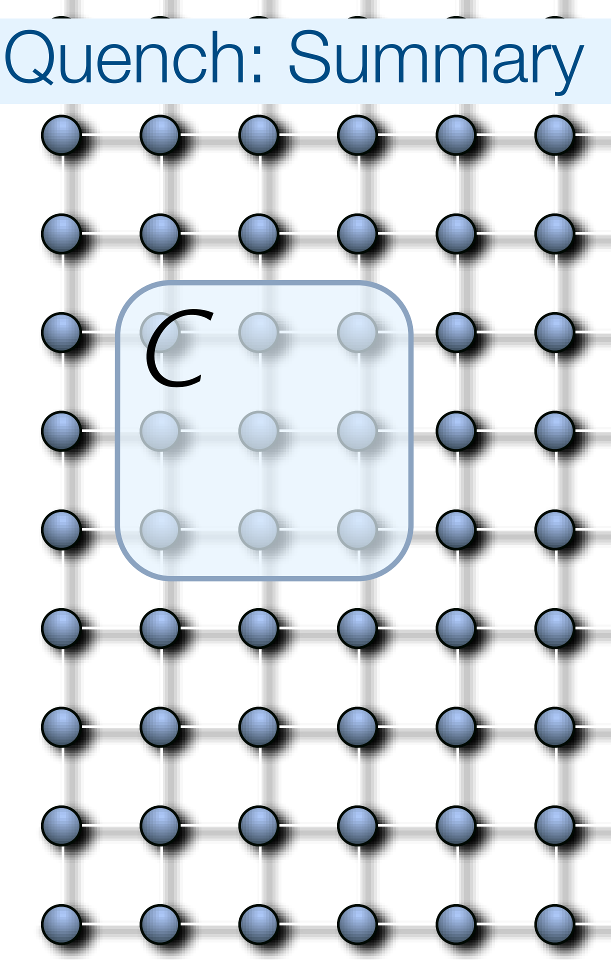
- Purity?
- Thermal?
- Time scale?

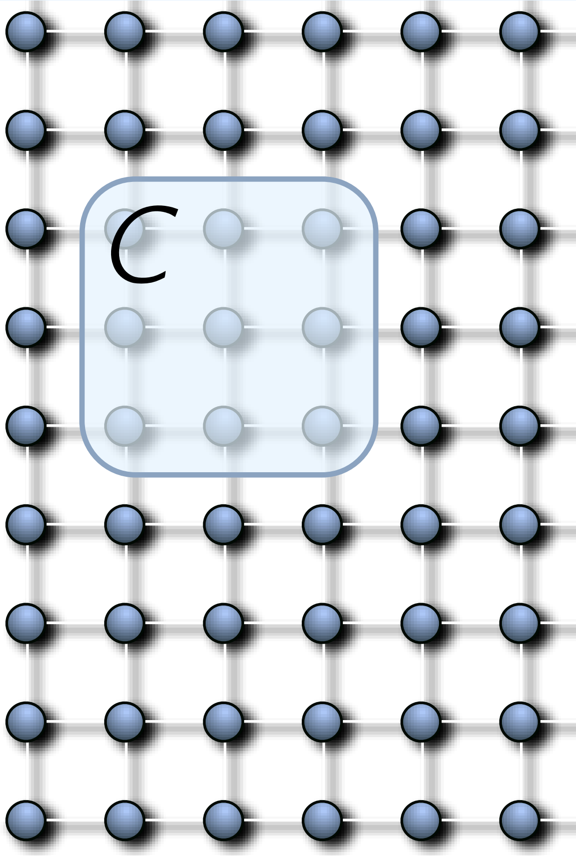
fraction of times for which $\|\hat{\varrho}_C(t) - \hat{\omega}_C\|_{tr} \leq \epsilon$ is at least $1 - 2^{|C|}\sqrt{tr[\hat{\omega}^2]}/\epsilon$

Linden, Popescu, Short, Winter (2008)

QBE → Purity

local Hamiltonian, sufficiently weakly correlated initial state: $\mathrm{tr}[\hat{\omega}^2] \lesssim \frac{\ln^{2d}(N)}{\sqrt{N}}$





QBE

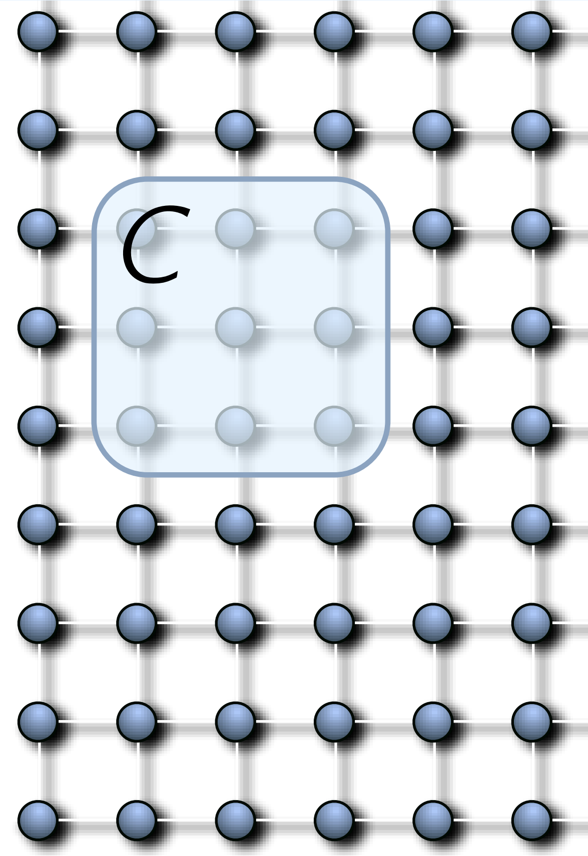
Purity

local Hamiltonian, sufficiently weakly correlated initial state: $\mathrm{tr}[\hat{\omega}^2] \lesssim \frac{\ln^{2d}(N)}{\sqrt{N}}$

Thermalization

integrable: no thermalization
(instead generalized Gibbs ensemble)

Local Thermalization after Quantum Quench: Summary



QBE

Purity

local Hamiltonian, sufficiently weakly correlated initial state: $\mathrm{tr}[\hat{\omega}^2] \lesssim \frac{\ln^{2d}(N)}{\sqrt{N}}$

Thermalization

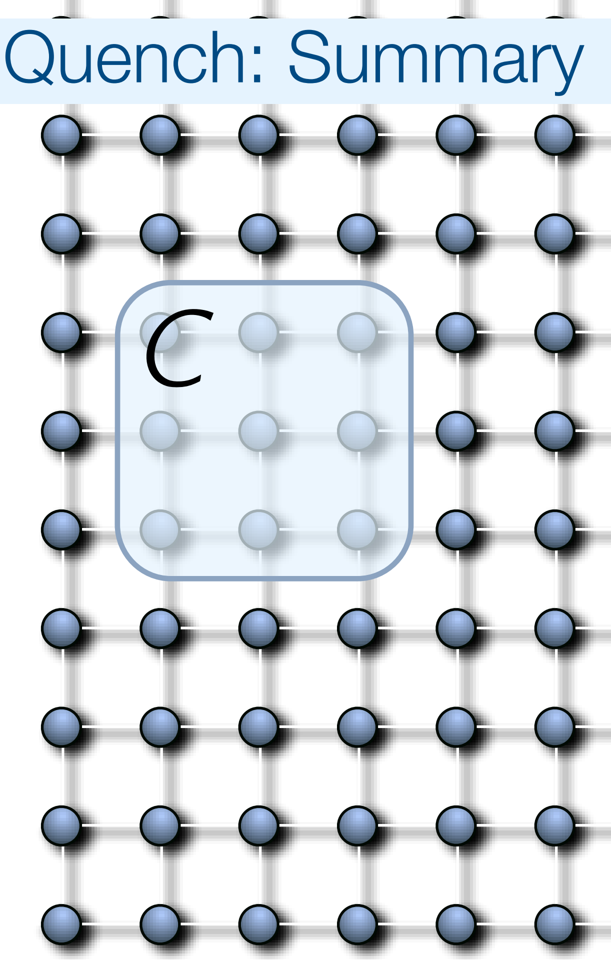
integrable: no thermalization (instead generalized Gibbs ensemble)

QBE

most Hamiltonians that are unitarily equivalent to a local Hamiltonian lead to fast thermalization*

Cramer, *Thermalization under randomized local Hamiltonians* (2012)

*the subsystem spends most of the times in $[0, N^{\frac{1}{5d}-\frac{1}{2}}]$ close to the maximally mixed state



Purity

QBE →

local Hamiltonian, sufficiently weakly correlated initial state: $\mathrm{tr}[\hat{\omega}^2] \lesssim \frac{\ln^{2d}(N)}{\sqrt{N}}$

Thermalization

integrable: no thermalization (instead generalized Gibbs ensemble)

QBE →

most Hamiltonians that are unitarily equivalent to a local Hamiltonian lead to fast thermalization*

Cramer, *Thermalization under randomized local Hamiltonians* (2012)

transl. inv., thermodynamic limit: entropic condition on initial state implies thermalization

Mueller, Adlam, Masanes, Wiebe, *Thermalization and canonical typicality in translation-invariant quantum lattice systems* (2013)

QBE → non-t.i., finite size

*the subsystem spends most of the times in $[0, N^{\frac{1}{5d}-\frac{1}{2}}]$ close to the maximally mixed state