Improved mean-field approximations for inferring marginals and model parameters

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Outline of the talk

- Two different derivations of mean-field approximations
  - Plefka expansion
  - Cluster Variational Method
- How good these approximations are?
- How one can try to improve it in order to overcome the limitations due to:
  - Loops
  - Ergodicity breaking

Physics & Machine Learning

- This is a Physics talk!
- But useful for Machine Learning (I hope ;-)
- Common problem: compute quickly and accurately the free-energy

$$F(\boldsymbol{J},\boldsymbol{h}) = \log Z(\boldsymbol{J},\boldsymbol{h}) = \log \sum_{\{s_i\}} \exp\left(\sum J_{ij} s_i s_j + \sum_i h_i s_i\right)$$

and the marginals

$$m_i = \langle s_i \rangle \qquad C_{ij} = \langle s_i s_j \rangle - m_i m_j$$

Simplifications for this talk

- Ising variables $s_i = \pm 1$ ...can be extended to Potts
- Pairwise interactions

$$H(\boldsymbol{s}) = -\sum_{(ij)} J_{ij} s_i s_j - \sum_i h_i s_i$$

...can be extended to more general graphical models

- The measure is

$$P(\boldsymbol{s}) = \frac{1}{Z(\boldsymbol{J}, \boldsymbol{h})} \exp\left[\beta \sum_{(ij)} J_{ij} s_i s_j + \beta \sum_i h_i s_i\right]$$

Often $\beta = 1$

- No hidden variables

Inferring marginals is useful for machine learning

- Willing to maximize the log-likelihood

$$L(\boldsymbol{J},\boldsymbol{h}) = \sum_i h_i \langle s_i \rangle_{\text{data}} + \sum_{ij} J_{ij} \langle s_i s_j \rangle_{\text{data}} - \log Z(\boldsymbol{J},\boldsymbol{h})$$

with respect to J and h one gets

$$\begin{aligned} \langle s_i \rangle_{\text{data}} &= \partial_{h_i} F(\boldsymbol{J},\boldsymbol{h}) = m_i(\boldsymbol{J},\boldsymbol{h}) \\ \langle s_i s_j \rangle_{\text{data}} &= \partial_{J_{ij}} F(\boldsymbol{J},\boldsymbol{h}) = C_{ij}(\boldsymbol{J},\boldsymbol{h}) + m_i m_j \end{aligned}$$

- Most likely model parameters can be found matching empirical marginals with model marginals

Boltzmann machine learning

- Matching between empirical and model marginals is achieved via a learning

$$\delta h_i = \eta \left(\langle s_i \rangle_{\text{data}} - m_i \right)$$

$$\delta J_{ij} = \eta \left(\langle s_i s_j \rangle_{\text{data}} - (C_{ij} + m_i m_j) \right)$$

- Marginals can be computed by:
  - Monte Carlo -> exact but slow...
  - Mean-field approximations
    -> faster
    -> no learning, if analytic expressions for
    $m_i(\boldsymbol{J}, \boldsymbol{h}), C_{ij}(\boldsymbol{J}, \boldsymbol{h})$ can be inverted

Alternatively…

- Directly maximize an easy-to-compute approximation to $L(\boldsymbol{J}, \boldsymbol{h})$ (e.g. the pseudo-likelihood)
  -> see next talk by Aurelien Decelle
- If input data are not configurations but average values for the marginals $\langle s_i \rangle_{\mathrm{data}}, \langle s_i s_j \rangle_{\mathrm{data}}$ then:
  - No pseudo-likelihood maximization
  - Matching empirical and model marginals
  - The maximum entropy probability distribution has only fields and pairwise couplings

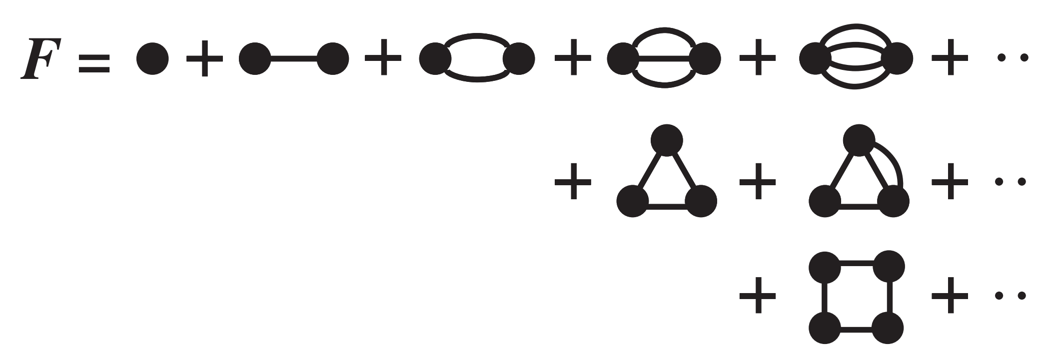
Free-energy mean-field expansions

- How to generalize most common mean-field approx?
- Plefka derives an expansion in the couplings intensity for the Gibbs free-energy with given magnetizations
- Cluster Variational Method (Kikuchi, Morita, An) is an expansion of the entropy in terms of correlations up to a given distance (maximal region size)
- Both expansions have the naive mean-field approximation as the first-order approximation
- Beyond naive MF and Bethe approximations the relation among the 2 expansions in not trivial (Yasuda Tanaka)

Plefka's expansion

$$
\begin{aligned}
\mathcal{F}_{\text{Plefka}}[\boldsymbol{m}_\Lambda]
&= \overbrace{\overbrace{-\sum_{i\in\Lambda} h_i m_i + \frac{1}{\beta}\sum_{i\in\Lambda}\left(\frac{1+m_i}{2}\ln\frac{1+m_i}{2} + \frac{1-m_i}{2}\ln\frac{1-m_i}{2}\right) - \sum_{(ij)\in\mathcal{B}} J_{ij}m_i m_j}^{\text{naiveMF}} - \frac{\beta}{2}\sum_{(ij)\in\mathcal{B}} {J_{ij}}^2(1-{m_i}^2)(1-{m_j}^2)}^{\text{TAP}} \\
&\quad - \frac{2\beta^2}{3}\sum_{(ij)\in\mathcal{B}} {J_{ij}}^3 m_i m_j(1-{m_i}^2)(1-{m_j}^2) - \beta^2\sum_{(ijk)\in\mathcal{T}} J_{ij}J_{jk}J_{ki}(1-{m_i}^2)(1-{m_j}^2)(1-{m_k}^2) \\
&\quad - \frac{\beta^3}{12}\sum_{(ij)\in\mathcal{B}} {J_{ij}}^4(1-{m_i}^2)(1-{m_j}^2)(15{m_i}^2{m_j}^2 - 3{m_i}^2 - 3{m_j}^2 - 1) \\
&\quad - 2\beta^3\sum_{(ijk)\in\mathcal{T}} J_{ij}J_{jk}J_{ki}(1-{m_i}^2)(1-{m_j}^2)(1-{m_k}^2)(J_{ij}m_i m_j + J_{jk}m_j m_k + J_{ki}m_k m_i) \\
&\quad - \beta^3\sum_{(ijkl)\in\mathcal{Q}} J_{ij}J_{jk}J_{kl}J_{li}(1-{m_i}^2)(1-{m_j}^2)(1-{m_k}^2)(1-{m_l}^2) + \mathcal{O}({J_{ij}}^5).
\end{aligned}
$$

diagrammatical representation



NaiveMF and TAP approximations

$$F_{\mathrm{nMF}} = \sum_i \left[H\left(\frac{1+m_i}{2}\right) + H\left(\frac{1-m_i}{2}\right)\right] + \sum_i h_i m_i + \sum_{i \neq j} J_{ij} m_i m_j \qquad H(x) \equiv -x\ln(x)$$

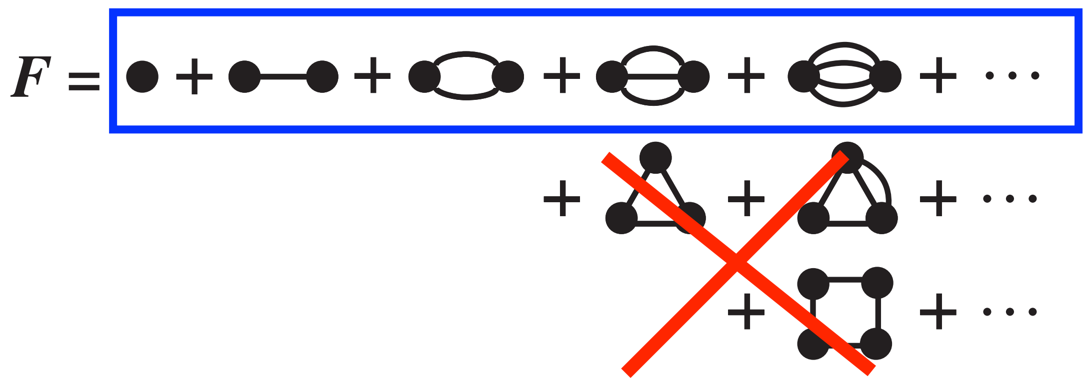
$$\frac{\partial F_{\mathrm{nMF}}}{\partial m_i} = \sum_j J_{ij} m_j + h_i - \mathrm{atanh}(m_i) = 0 \quad \Rightarrow \quad \boxed{m_i = \tanh\left[h_i + \sum_j J_{ij} m_j\right]}$$

$$F_{\mathrm{TAP}} = \sum_i \left[H\left(\frac{1+m_i}{2}\right) + H\left(\frac{1-m_i}{2}\right)\right] + \sum_i h_i m_i + \sum_{i \neq j} \left(J_{ij} m_i m_j + \tfrac{1}{2} J_{ij}^2 (1-m_i^2)(1-m_j^2)\right)$$

$$\boxed{m_i = \tanh\left[h_i + \sum_j J_{ij}\big(m_j - \underbrace{J_{ij}(1-m_j^2)m_i}_{\text{Onsager reaction term}}\big)\right]}$$

Bethe approximation

- On a tree



sum to Bethe approx

- Not easy to write the explicit expression for the Bethe approximation
- Very hard to go beyond the Bethe approximation…

Cluster Variational Method (CVM)

$$F = -\ln Z = \min_p \mathcal{F}(p) = \min_p \sum_{\boldsymbol{s}} [p(\boldsymbol{s})H(\boldsymbol{s}) + p(\boldsymbol{s}) \ln p(\boldsymbol{s})] \qquad \sum_{\boldsymbol{s}} p(\boldsymbol{s}) = 1$$

Pelizzola review

energy (easy)

entropy (hard) -> approximate by truncating the expansion in cumulants

$$F_{\mathrm{CVM}}(b, J, h) = E(b, J, h) - \frac{1}{\beta} S(b)$$

$$E(b, J, h) = -\sum_{(ij)} J_{ij} \mathrm{Tr}[s_i s_j b_{ij}] - \sum_i h_i \mathrm{Tr}[s_i b_i]$$

$$S(b) = -\sum_{r \in R} c_r \mathrm{Tr}[b_r \log b_r]$$

$$b_r = b_r(s_r)$$ local beliefs

$$s_r = \{r_i : i \in r\}$$

sum over regions

counting numbers (Moebius coeff.)

Cluster Variational Method (CVM)

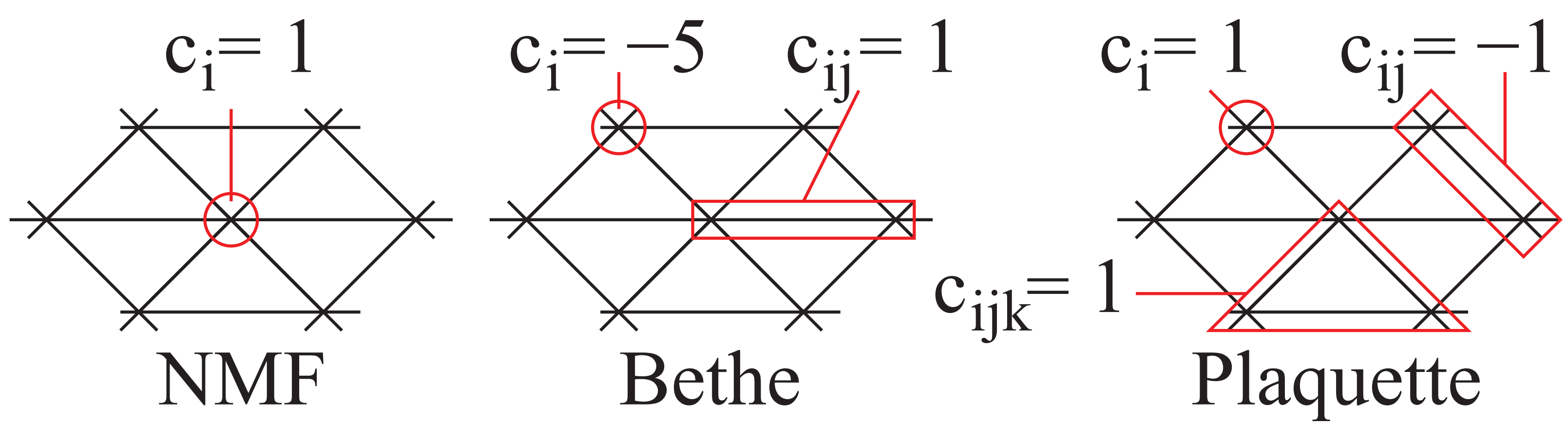
- Beliefs must be normalized and locally consistent

$$\sum_{s_r} b_r(s_r) = 1 \qquad \sum_{s_{r \setminus t}} b_r(s_r) = b_t(s_t)$$

- Local consistency is not global consistency!
- Beliefs are approximations to true marginals
- Beliefs can be parametrized by magnetizations and connected correlations, e.g.

$$b_i(s_i) = \frac{1 + m_i s_i}{2} \qquad b_{ij}(s_i, s_j) = \frac{1 + m_i s_i + m_j s_j + (c_{ij} + m_i m_j) s_i s_j}{4}$$

Cluster Variational Method (CVM)



$$\sum_{r\in R:t\subset r} c_r = 1 \quad \forall t$$

$$P(\boldsymbol{s}) \approx \prod_i b_i(s_i)$$

$$P(\boldsymbol{s}) \approx \prod_{(ij)} b_{ij}(s_i, s_j) \prod_i b_i(s_i)^{-5} = \prod_{(ij)} \frac{b_{ij}(s_i, s_j)}{b_i(s_i) b_j(s_j)} \prod_i b_i(s_i)$$

Exact on trees

How to choose the regions

- Original CVM: all maximal regions and all their intersections (recursively until single site regions)
- Region-based free energy approximation (Yedidia et al.): choose regions at your will as long as each site (variable node) and interaction (factor node) has coefficient c=1
- More is better, but computationally ineffective
- Try to include all relevant correlations $\mathrm{diam}(r_{\max}) \approx \xi$
- Easy to derive Bethe and clear how to go beyond Bethe

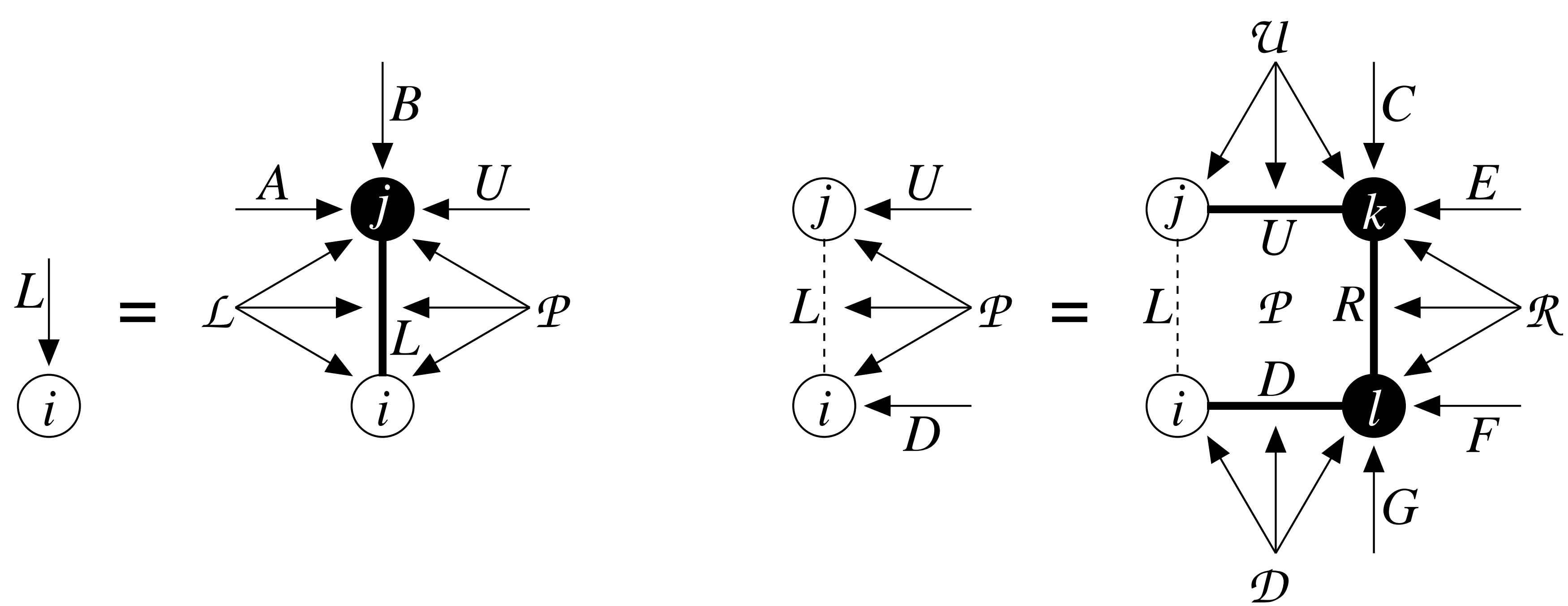
How to find the beliefs

- Introduce Lagrange multipliers (called messages) enforcing the consistency constraint for each pair of regions

$$m_{r \to t}(s_t) \quad \forall r, t \in R : t \subset r$$

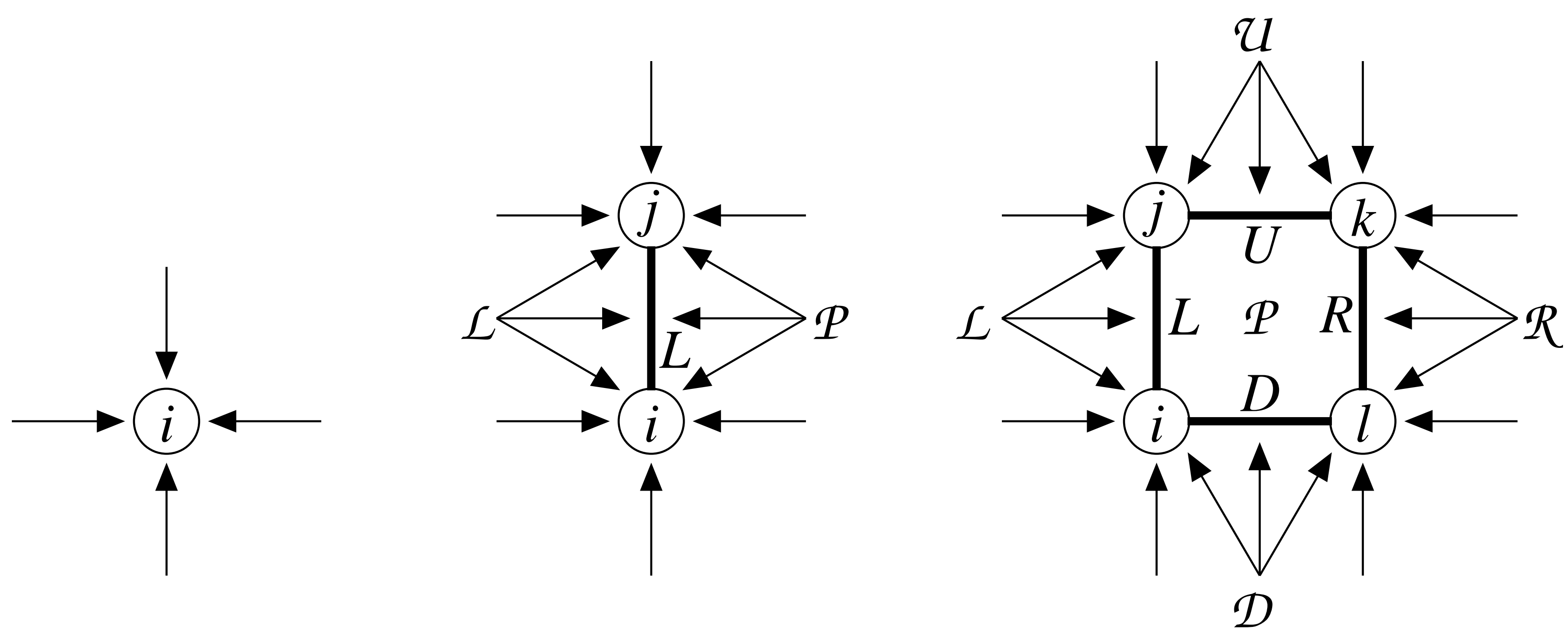
  These are generalized cavity probability distributions

- Solve equations for messages iteratively -> Belief Propagation (BP), Generalized Belief Propagation (GBP)



How to find the beliefs

- After converging to the fixed point of the message passing algorithm, compute beliefs from the messages



- N.B. several equivalent MPA and each may have several equivalent fixed point (gauge invariance).

Plefka's expansion vs. CVM

- Plefka's expansion has N parameters (the magnetizations) nMF, TAP, 3rd order, 4th order,...,Bethe.
- CVM may have much more parameters to optimize over E.g. on the 2D square lattice:
  - nMF -> N magnetizations
  - Bethe -> N magnetizations + 2N nn correlations
  - Plaquette -> N magnetizations + 2N nn correlations + 2N nnn correlations + 4N 3-spin corr. + N 4-spin corr.
- CVM much richer description, but hard to get analytical expressions to estimate model parameters

Bethe approximation

Independent pair approx.

- The two derivations are equivalent: correlations only depends on magnetizations at the fixed point

$$\frac{\partial F_{\text{Bethe}}}{\partial C_{ij}} = 0 \implies \quad J_{ij} = \frac{1}{4}\ln\left(\frac{((1+m_i)(1+m_j)+c_{ij})((1-m_i)(1-m_j)+c_{ij})}{((1+m_i)(1-m_j)-c_{ij})((1-m_i)(1+m_j)-c_{ij})}\right)$$

$$c_{ij}(m_i,m_j,t_{ij}) = \frac{1}{2t_{ij}}\left(1+t_{ij}^2-\sqrt{(1-t_{ij}^2)^2-4t_{ij}(m_i-t_{ij}m_j)(m_j-t_{ij}m_i)}\right)-m_im_j$$

$$f(m_1,m_2,t) = \frac{1-t^2-\sqrt{(1-t^2)^2-4t(m_1-m_2t)(m_2-m_1t)}}{2t(m_2-m_1t)}$$

$$\boxed{m_i = \tanh\left[h_i+\sum_j \operatorname{atanh}\Big(t_{ij}f(m_j,m_i,t_{ij})\Big)\right]}$$

Small couplings expansion leads to nMF, TAP, ...

Computing correlations by linear response

- Connected correlations are always null in nMF, TAP
  Even in Bethe between non-neighbours spins
- Non trivial (and better) correlations can be obtained via linear response (Kappen Rodriguez, 1998)

$$\chi_{ij} = \frac{\partial m_i}{\partial h_j}$$

$$(\chi^{-1}_{\mathrm{nMF}})_{ij} = \frac{\delta_{ij}}{1-m_i^2} - J_{ij} \ ,$$

$$(\chi^{-1}_{\mathrm{TAP}})_{ij} = \left[\frac{1}{1-m_i^2} + \sum_k J_{ik}^2(1-m_k^2)\right]\delta_{ij} - \left(J_{ij} + 2J_{ij}^2 m_i m_j\right)$$

$$(\chi^{-1}_{\mathrm{BA}})_{ij} = \left[\frac{1}{1-m_i^2} - \sum_k \frac{t_{ik} f_2(m_k, m_i, t_{ik})}{1 - t_{ik}^2 f(m_k, m_i, t_{ik})^2}\right]\delta_{ij} - \frac{t_{ij} f_1(m_j, m_i, t_{ij})}{1 - t_{ij}^2 f(m_j, m_i, t_{ij})^2}$$

$$f_1(m_1, m_2, t) \equiv \partial f(m_1, m_2, t)/\partial m_1 \qquad f_2(m_1, m_2, t) \equiv \partial f(m_1, m_2, t)/\partial m_2$$

Estimating model parameters via MFA

- Assume $\chi = C$!
- Estimate couplings from matching only off-diagonal elements of χ^{-1} and C^{-1}_{data}

$$J_{ij}^{\mathrm{nMF}} = -(C^{-1})_{ij} \qquad J_{ij}^{\mathrm{TAP}} = \frac{\sqrt{1 - 8m_i m_j (C^{-1})_{ij}} - 1}{4m_i m_j}$$

$$J_{ij}^{\mathrm{BA}} = -\mathrm{atanh}\Bigg[\frac{1}{2(C^{-1})_{ij}}\sqrt{1 + 4(1-m_i^2)(1-m_j^2)(C^{-1})^2_{ij}} - m_i m_j - \frac{1}{2(C^{-1})_{ij}}\sqrt{\left(\sqrt{1 + 4(1-m_i^2)(1-m_j^2)(C^{-1})^2_{ij}} - 2m_i m_j (C^{-1})_{ij}\right)^2 - 4(C^{-1})^2_{ij}}\Bigg]$$

- Easier than computing marginals: no need to run MPA!

Estimating model parameters via MFA

- Independent Pair (IP) approximation

$$J_{ij}^{\mathrm{IP}} = \frac{1}{4}\ln\left(\frac{\Big((1+m_i)(1+m_j)+C_{ij}\Big)\Big((1-m_i)(1-m_j)+C_{ij}\Big)}{\Big((1+m_i)(1-m_j)-C_{ij}\Big)\Big((1-m_i)(1+m_j)-C_{ij}\Big)}\right)$$

- Sessak-Monasson (SK) small correlation expansion

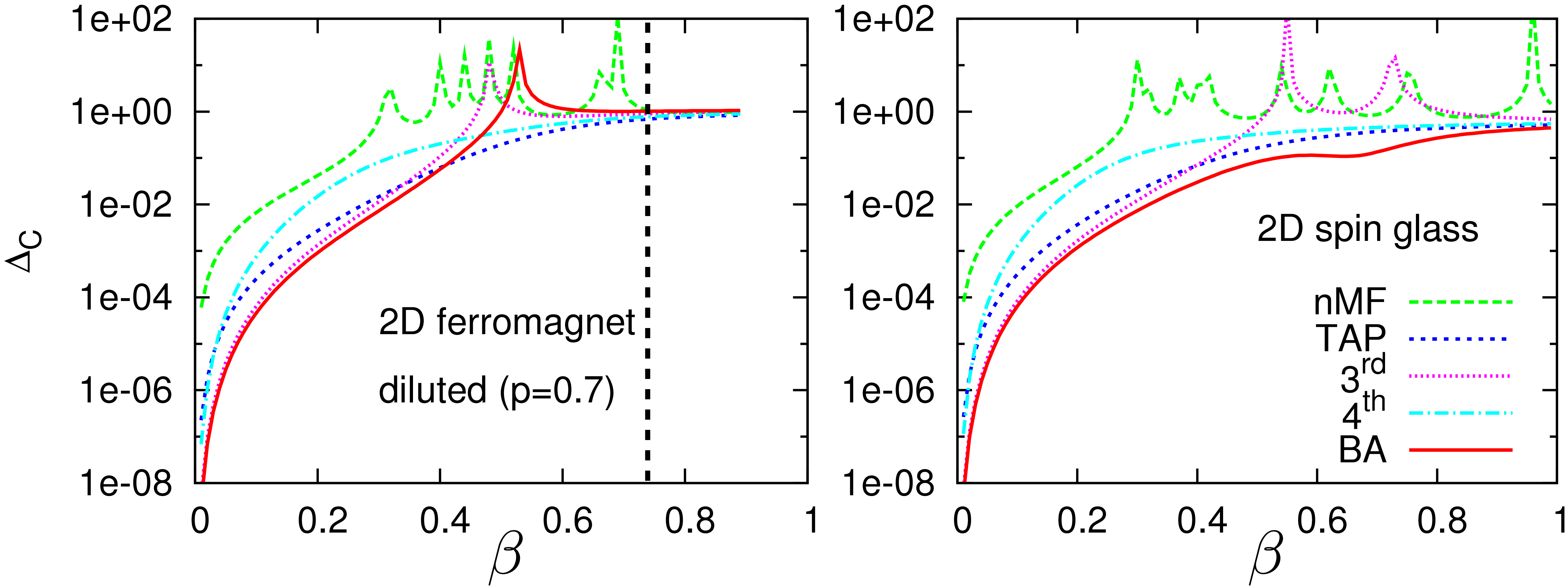
$$J_{ij}^{\mathrm{SM}} = -(C^{-1})_{ij} + J_{ij}^{\mathrm{IP}} - \frac{C_{ij}}{(1-m_i^2)(1-m_j^2)-(C_{ij})^2}$$

- Fields estimates from self-consistency equation
  E.g. for nMF

$$m_i = \tanh[\beta(h_i + \sum_j J_{ij} m_j)] \implies h_i = \frac{\mathrm{atanh}(m_i)}{\beta} - \sum_j J_{ij}^{\mathrm{nMF}} m_j$$

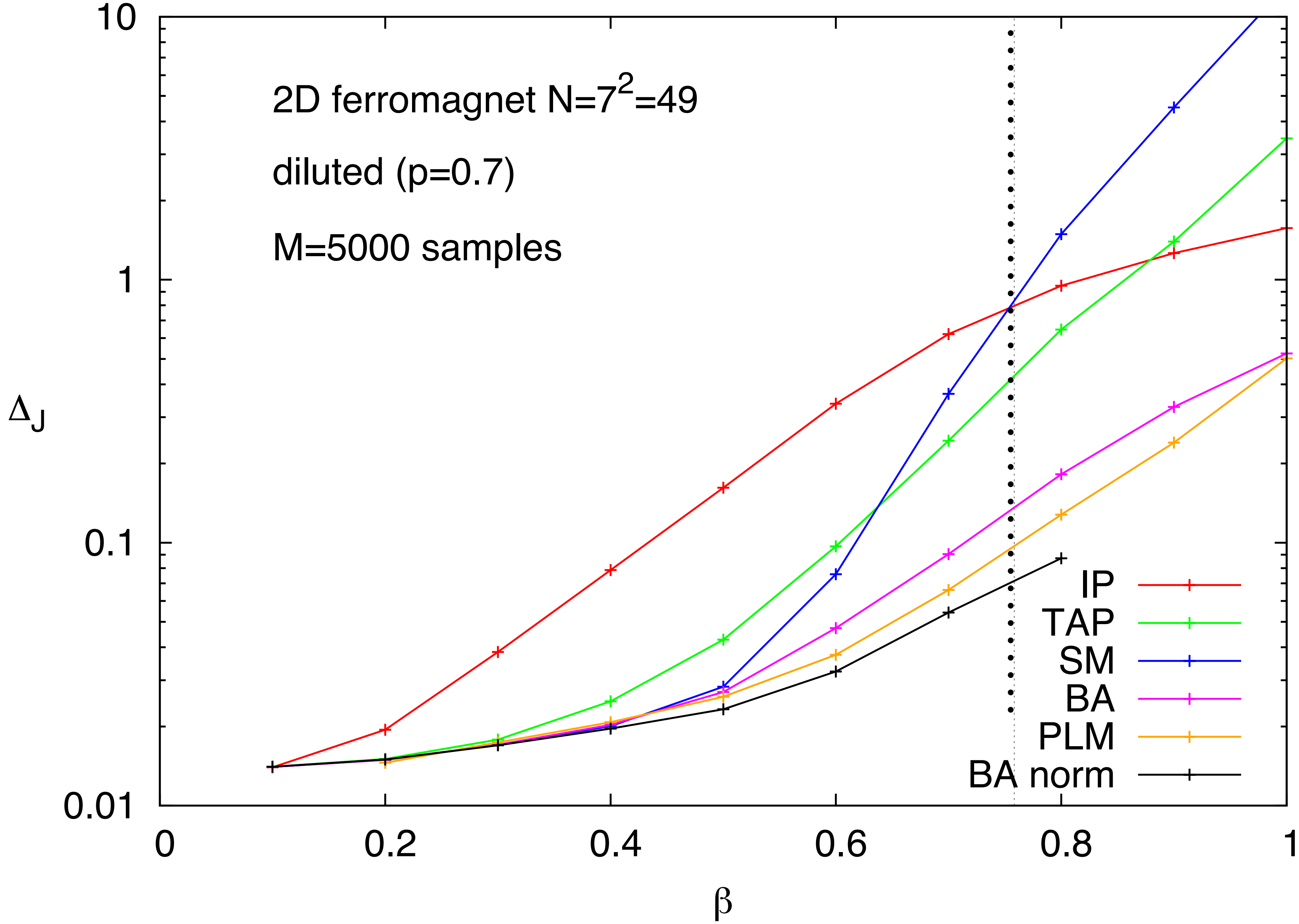
How good are MFA?

- Estimating marginals $\Delta_C = \sqrt{\frac{1}{N^2}\sum_{i,j}(\chi_{ij} - C_{ij}^{\mathrm{true}})^2}$



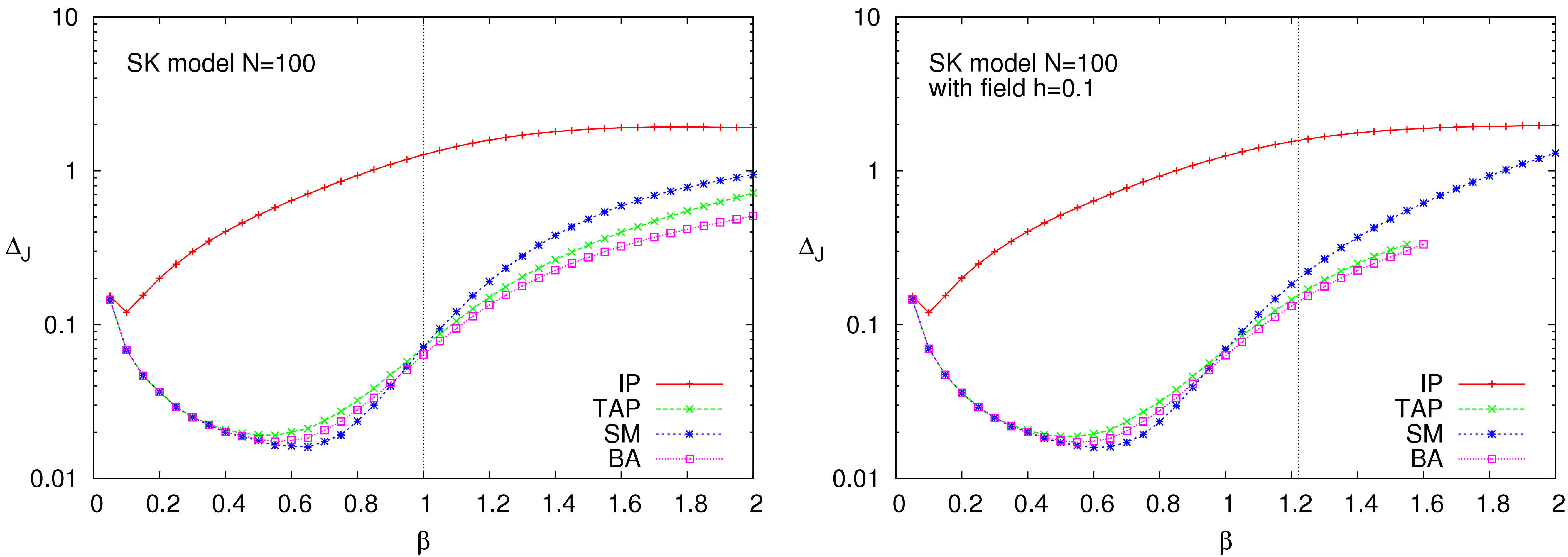
How good are MFA?

- Estimating model parameters



How good are MFA?

- Problems with strongly frustrated models in a field



- Due to negative discriminants in coupling estimates

How good are MFA?

$$\begin{pmatrix} 1 & & C_{ij} \\ & \ddots & \\ C_{ij} & & 1 \end{pmatrix} \stackrel{?}{=} \begin{pmatrix} \chi_{11} & & \chi_{ij} \\ & \ddots & \\ \chi_{ij} & & \chi_{NN} \end{pmatrix}$$

- The linear response correlation matrix (the only we can compute in MFA) has “wrong” element on the diagonal.
- It is different from the true correlation matrix.
- In the Bethe approximation the 2 estimates of nearest neighbor correlations (χ_{ij} and C_{ij}) are different.
  And linear response estimate is generally better.
- This is due to loops ignored in the MFA

Limitations of MFA

- Ergodicity breaking
  - MFA assume that a single state exists, and that correlations decay fast with distance
  - If many states exist correlations no longer decay, and MF estimates become poor
- Presence of loops
  - Even in presence of a single state, the loops may change a lot the correlations with respect to Susceptibility Propagation estimate, obtained assuming a loopless graph

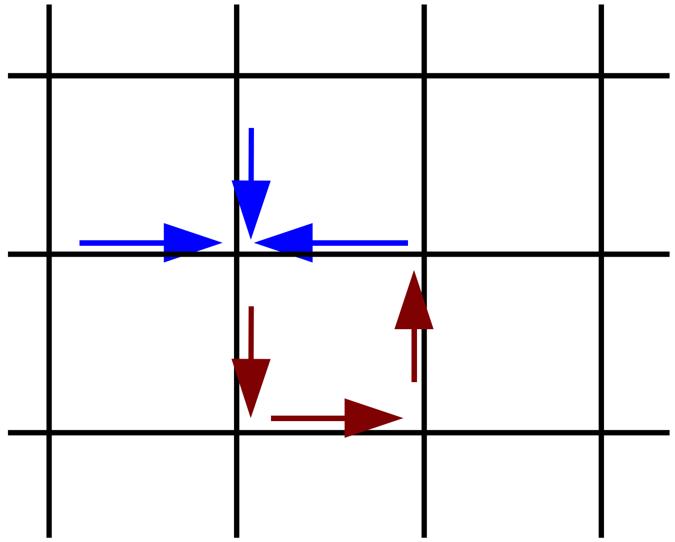
MFA fail because of loops

- E.g. Bethe approximation in the high temperature phase
- Since m=0, at the CVM free-energy minimum

$$\langle \sigma_i \sigma_j \rangle_c^{\mathrm{BA}} = c_{ij}^* = \tanh(\beta J_{ij}) < \langle \sigma_i \sigma_j \rangle_c^{\mathrm{exact}}$$

- Linear response (Susceptibility Propagation)

$$\chi_{ii} = 1 + \sum_{j \in \partial i} u_{j \to i, i} \neq 1$$



- So in general for a ferromagnet in the high T phase holds

$$C_{ij} < C_{ij}^{\mathrm{true}} < \chi_{ij}$$

Adding loops to Bethe ?

- Several attempts
  - Loop calculus (Chertkov Chernyak)
  - BP + correlations between neighbors (Montanari Rizzo, Mooij Kappen, Rizzo Wammenhove Kappen, Ohzeki)
- All require in some sense the convergence of BP, but loops make BP stop converging...
- None is able to make predictions in a frustrated model with many loops at low enough temperatures

Make MFA & LR consistent

- Choose your preferred MFA free-energy

$$F_{\mathrm{MFA}}(\{m_i\}, \{C_{ij}\}, \ldots)$$

- Enforce consistency with linear response estimates

$$\boxed{\chi_{ii} = 1 - m_i^2 \qquad \chi_{ij} = C_{ij}}$$

free energy
minimum
curvature

free energy
minimum
location

General framework for MFA + LR

$$\boxed{F_\lambda = F_{\mathrm{MFA}}(\{m_i\}, \{C_{ij}\}, \ldots) + \sum_i \lambda_i m_i^2 + \sum_{i<j} \lambda_{ij} C_{ij}}$$

Your preferred MFA

can be set to zero to recover known approx. or used to satisfy

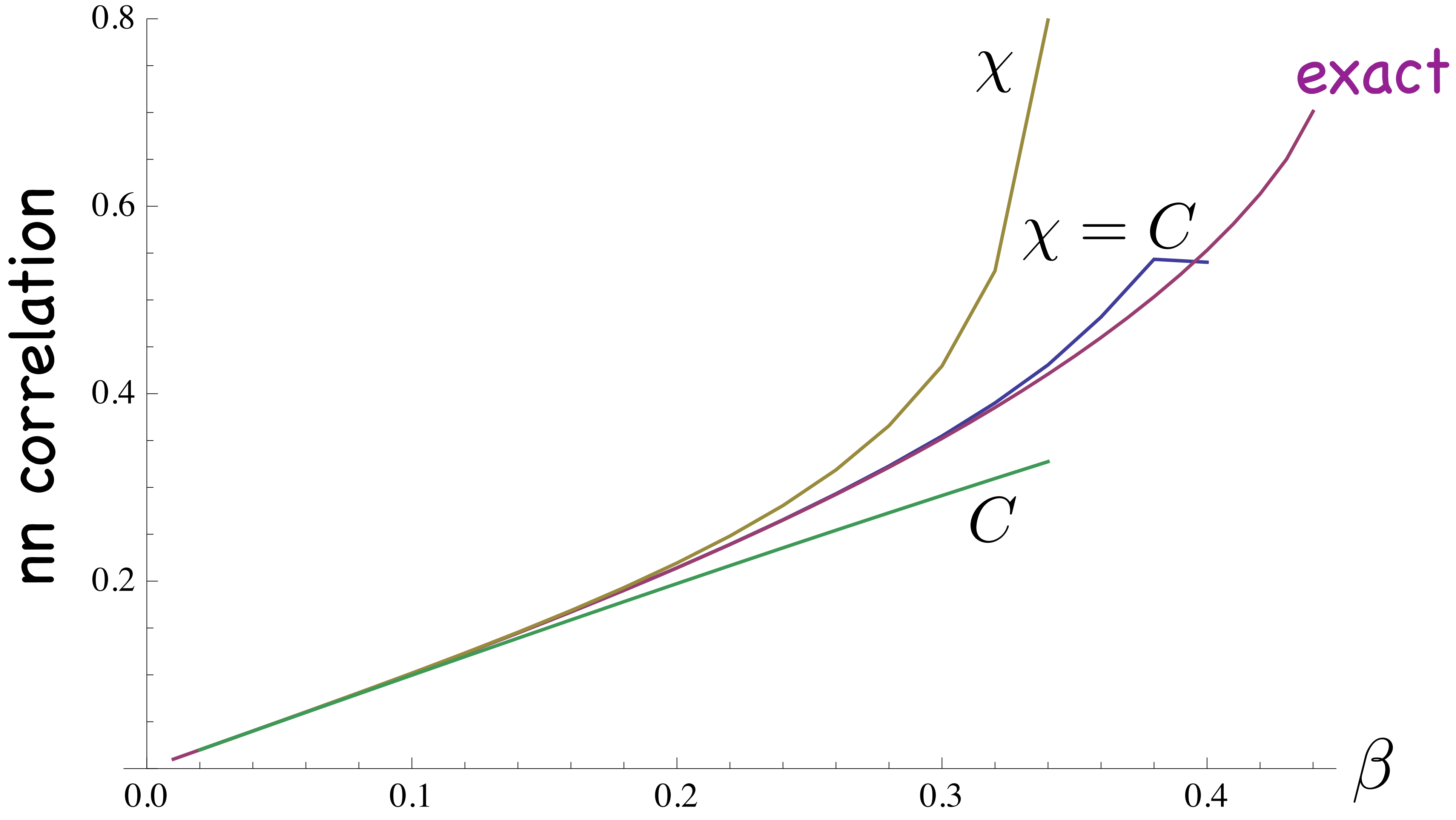
$\chi_{ii} = 1 - m_i^2 \quad \chi_{ij} = C_{ij}$

Other proposals for fixing χ_{ii}

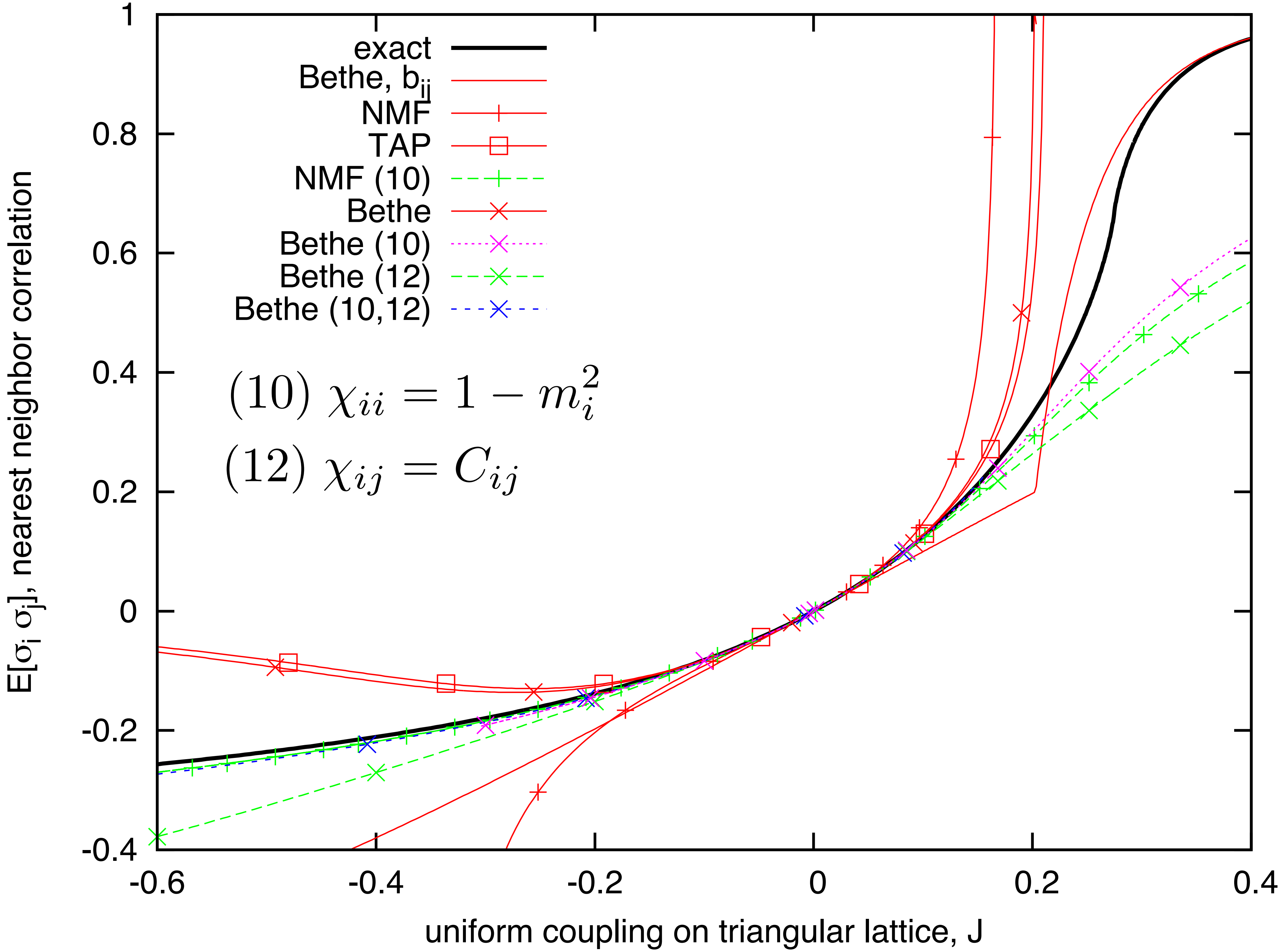
- Kappen Rodriguez (1998)
  MF + self-couplings J_{ii}
- Opper Winter (2001)
  Adaptive TAP = TAP + λ_i
- FRT (2012)
  Bethe with normalized correlations $\widehat{\chi}_{ij} \equiv \dfrac{\chi_{ij}}{\sqrt{\chi_{ii}\chi_{jj}}}$
  useful for the inverse pb.
- Yasuda Tanaka (2013)
  I-SuscP = Bethe + λ_i

Bethe + linear response

- Ising model on a 2D square lattice

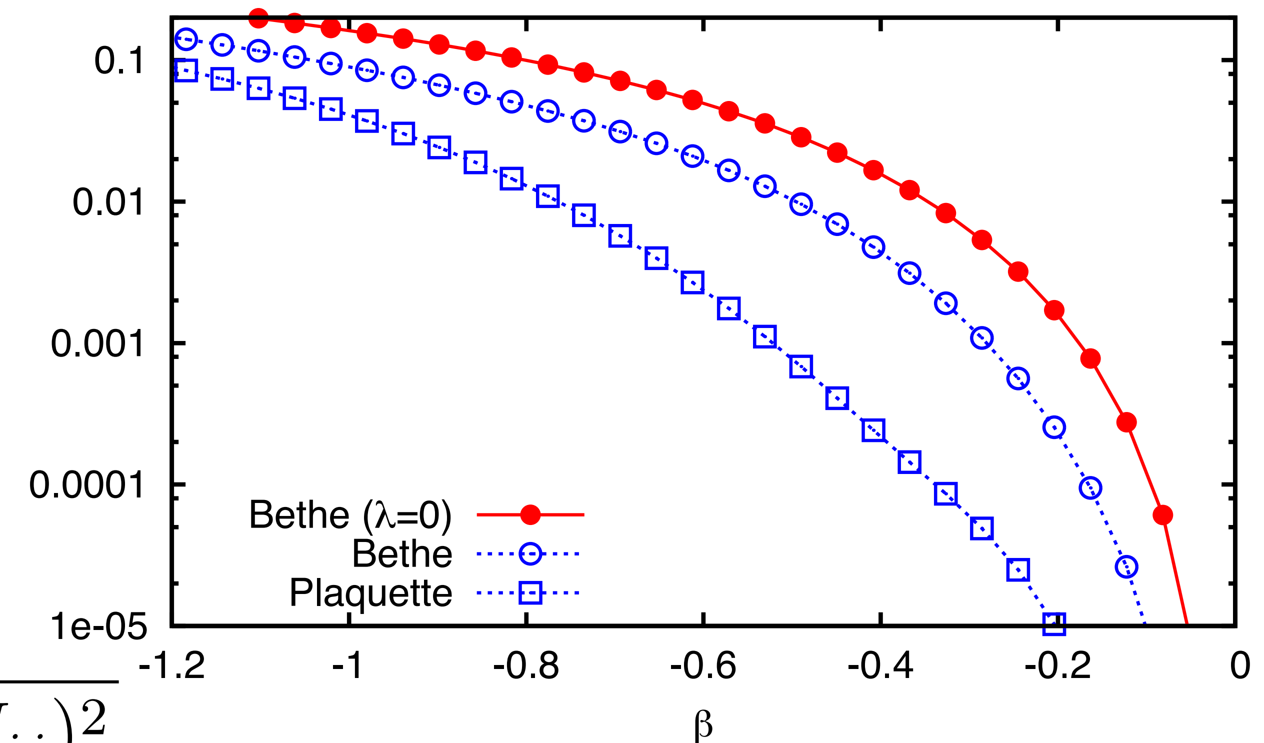


MFA + LR: estimating marginals



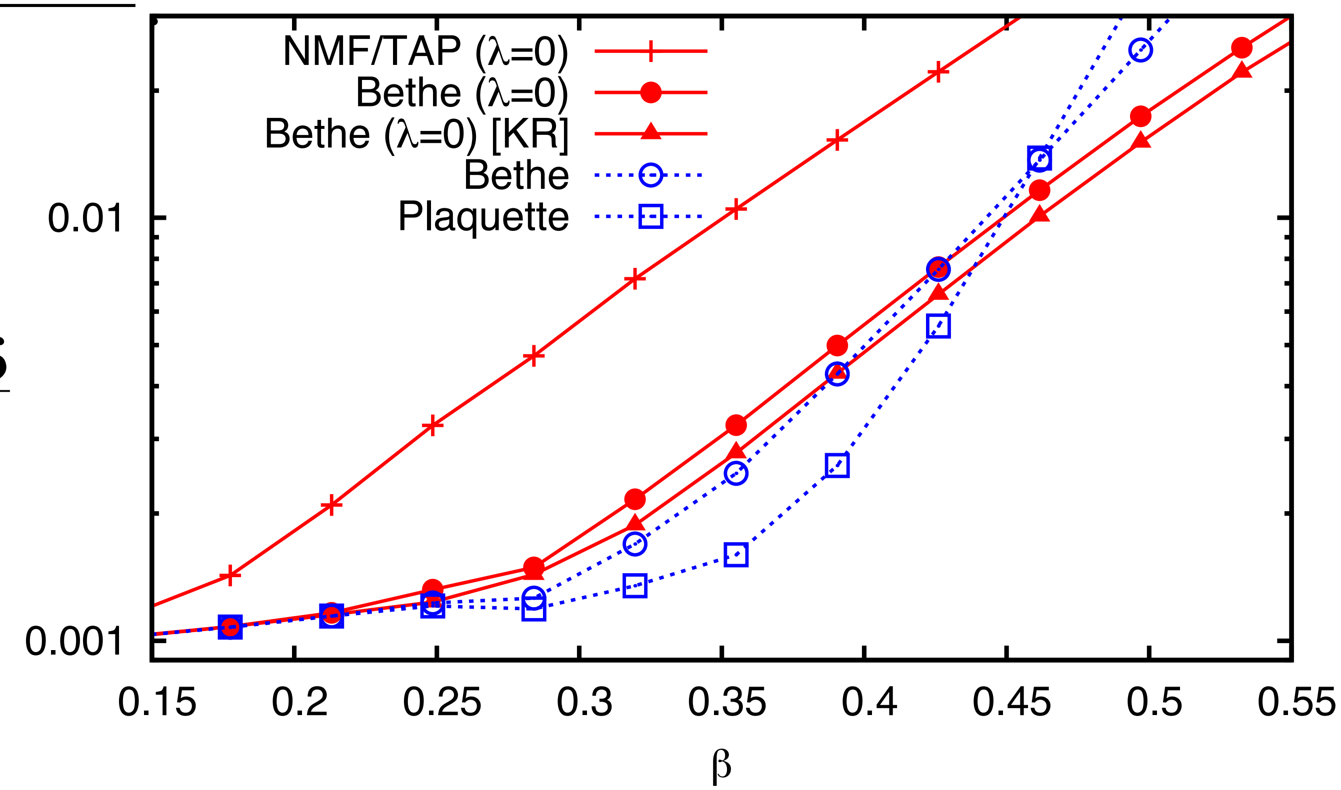
MFA + LR: estimating model param.

- <u>Inferring the couplings</u> of a 2D triangular diluted antiferromagnet from correlations (infinite statistics)



$$\Delta_J = \sqrt{\frac{\sum_{i<j}(J'_{ij} - J_{ij})^2}{\sum_{i<j} J_{ij}^2}}$$

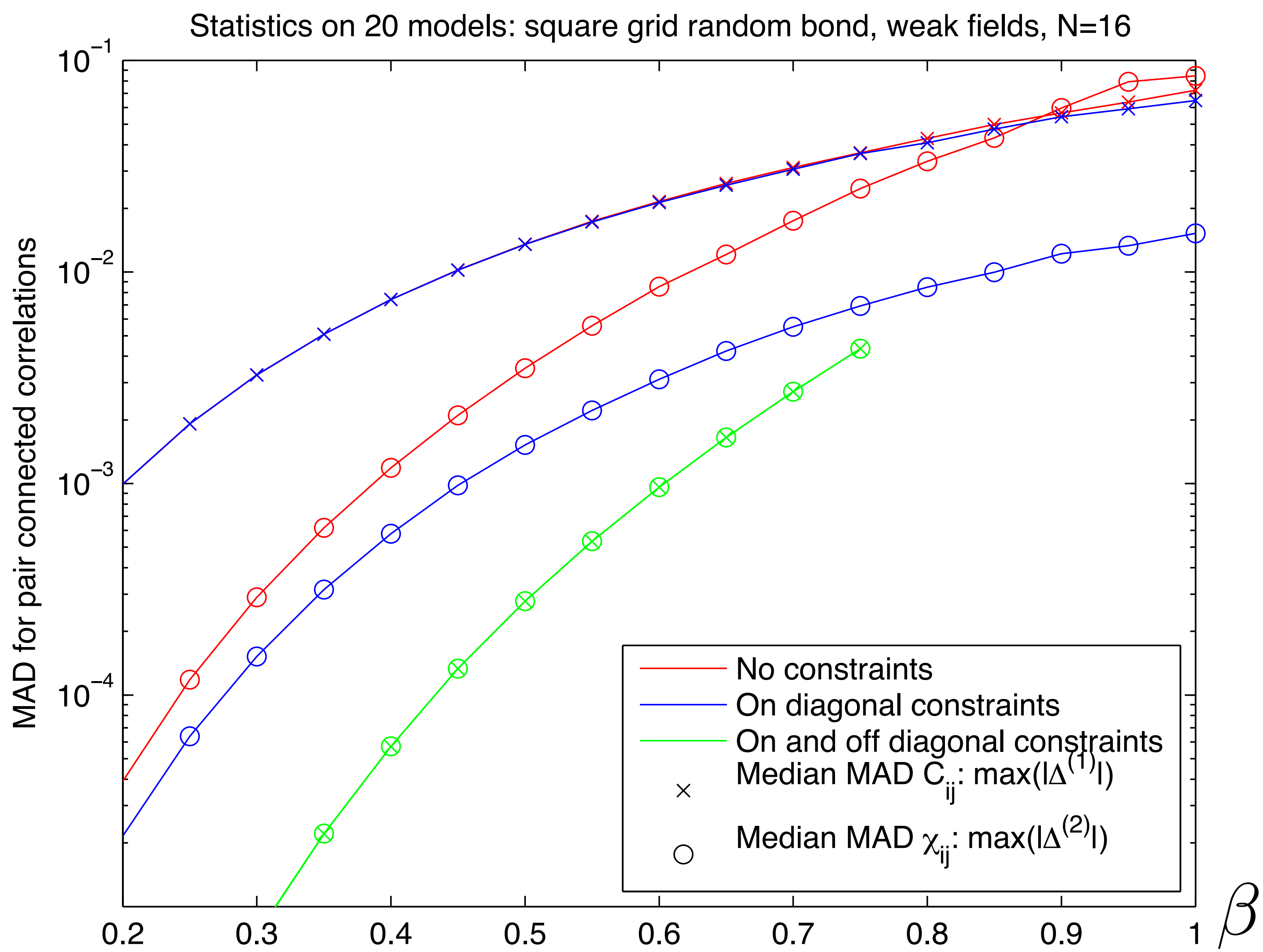
- <u>Inferring the couplings</u> of a 2D diluted Ising model (finite stat.)



MFA + LR: disordered models

- 2D spin glass in a random field

$J_{ij} \in [-1, 1],\ h_i \in [-0.25, 0.25]$



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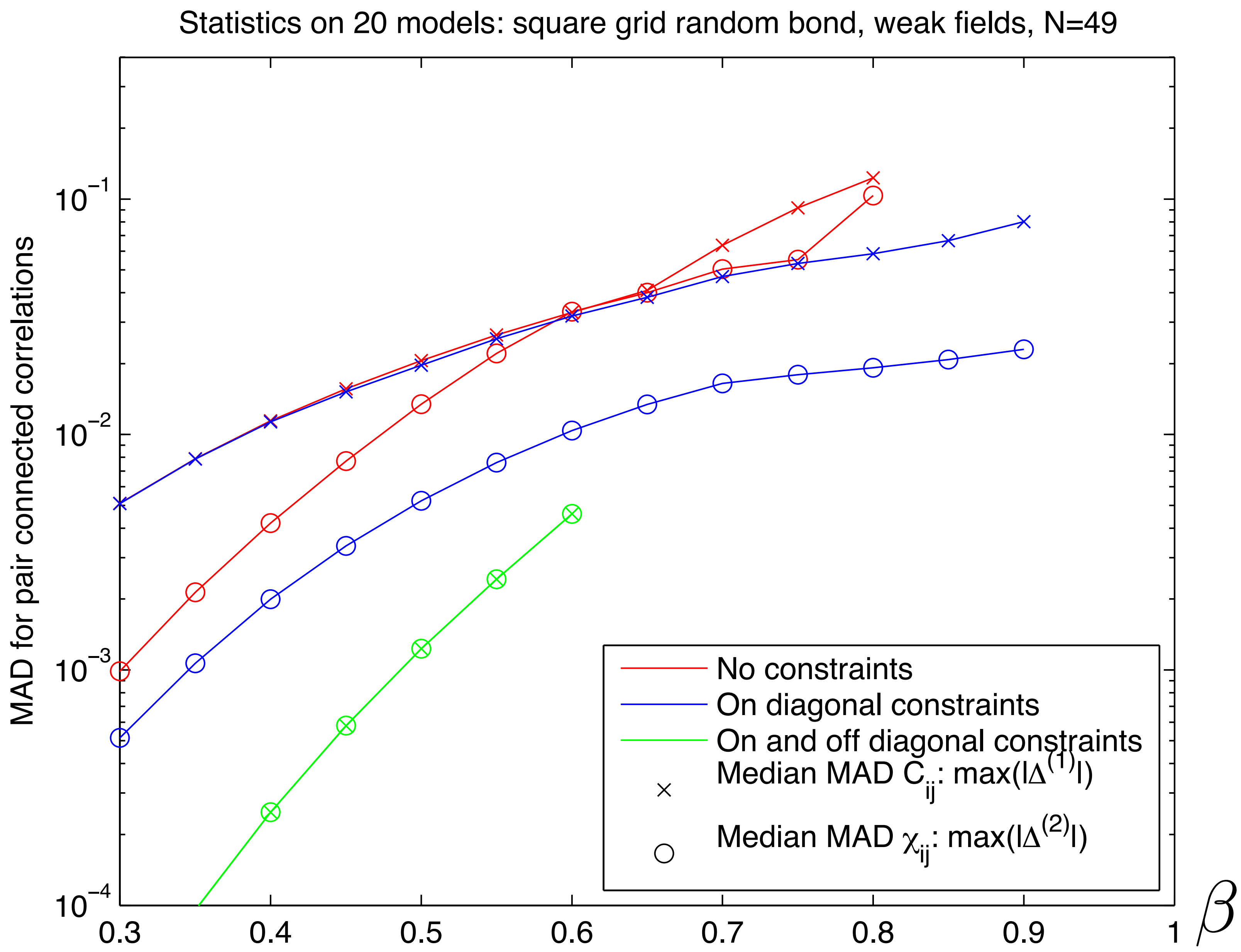
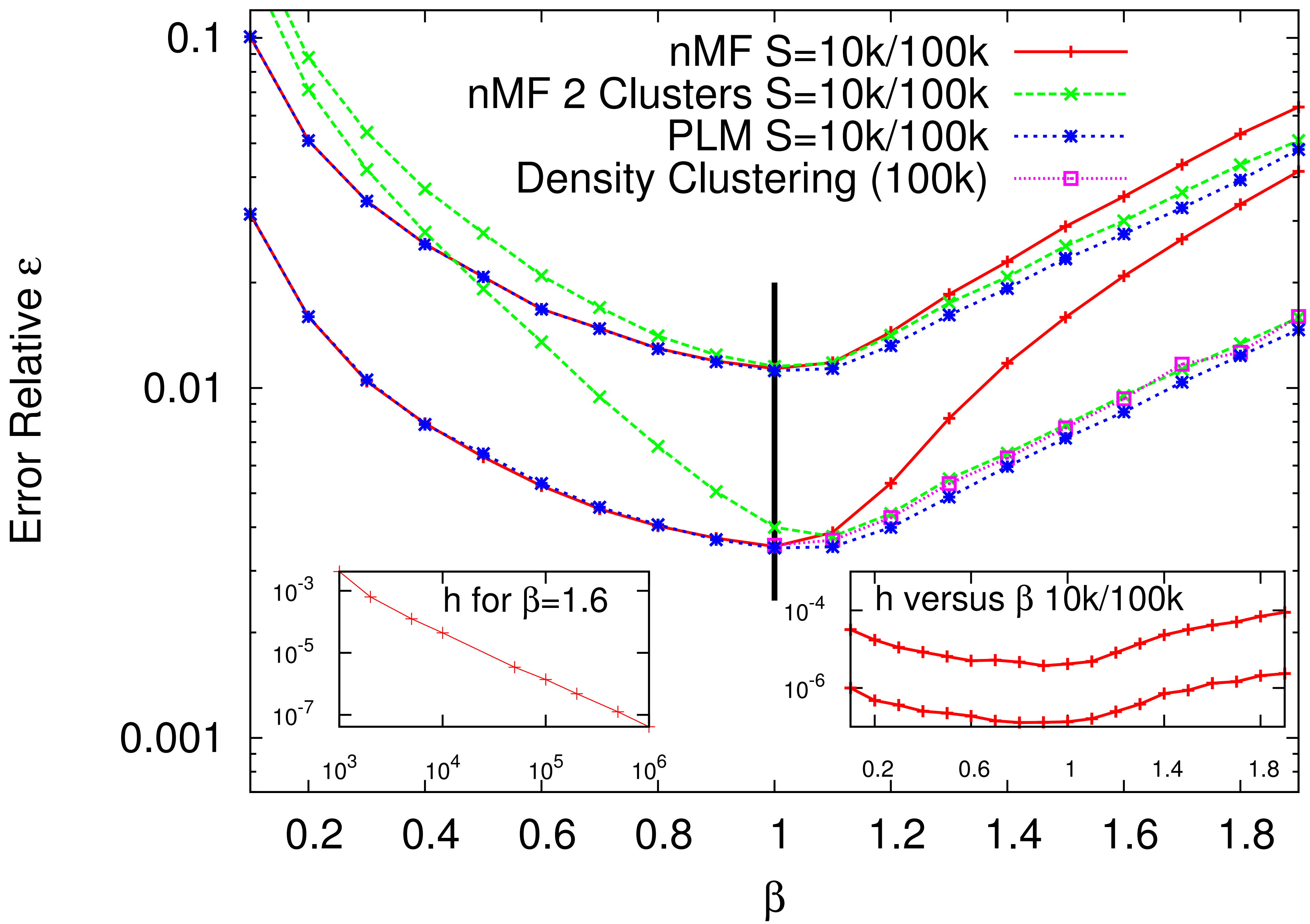


Ergodicity breaking & MFA

- **Estimate model parameters** in a phase with many states
  - Pseudo-likelihood based methods are rather insensitive to ergodicity breaking (see Aurelien's talk)
  - However also MFA can be used if data are properly clustered
  - Each cluster of data returns comparable estimates for couplings and fields

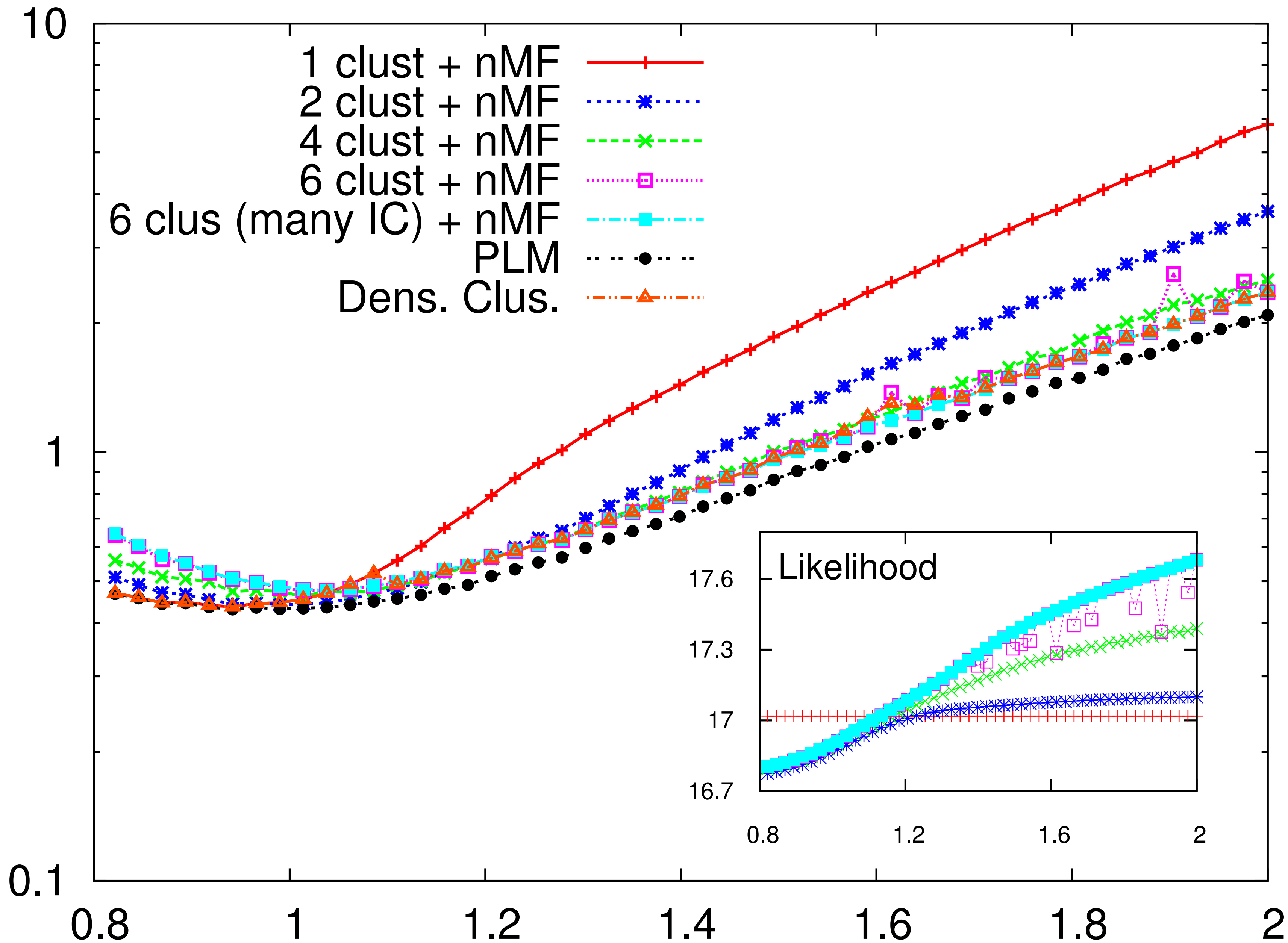
Ergodicity breaking & MFA

Curie-Weiss model N=100



Ergodicity breaking & MFA

Hopfield model P=3 (6 minima)

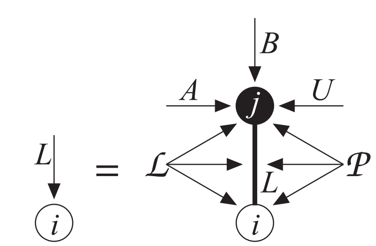


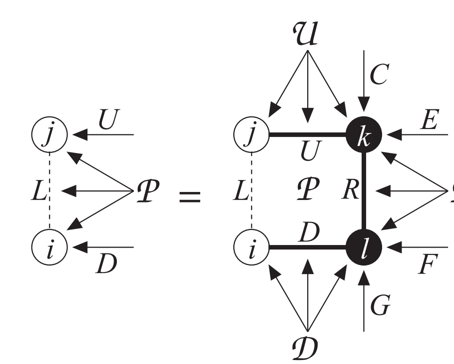
Ergodicity breaking & MFA

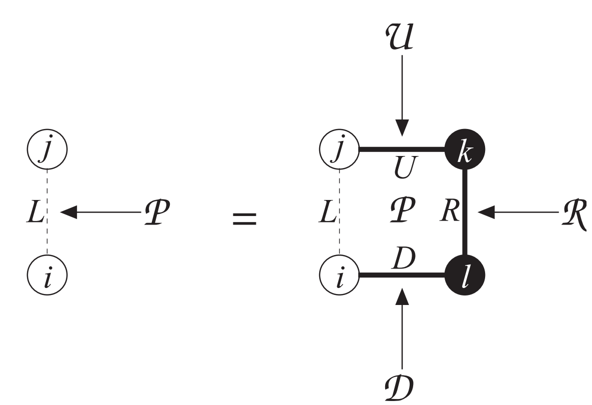
- The problem of **estimating marginals** is much harder in presence of ergodicity breaking
- What happens when we use MFA in a disordered model with many states? (relevant for <u>multimodal models</u>)
- On random graphs: replica symmetry breaking (RSB) and Survey Propagation with Parisi parameter m
  a.k.a. SP(m) is ok to describe 1RSB solutions
  With a little effort one can obtain 2RSB solutions...
- On finite dimensional lattices our understanding is still very limited :-(

CVM on finite dimensional spin glasses

- Edwards-Anderson (EA) model in d=2 with symmetric couplings ($J_{ij} = \pm 1$) is the most difficult situation
- For d=2 the EA model has no phase transition (in the thermodynamical limit!) but low temperature physics is still dominated by <u>many different local minima</u> in the free-energy
- Algorithms to optimize CVM free-energy:
  - BP
  - plaquette GBP (parent-to-child), HAK (2-ways)
  - Double loop
  - MPA on the dual lattice (m=0)

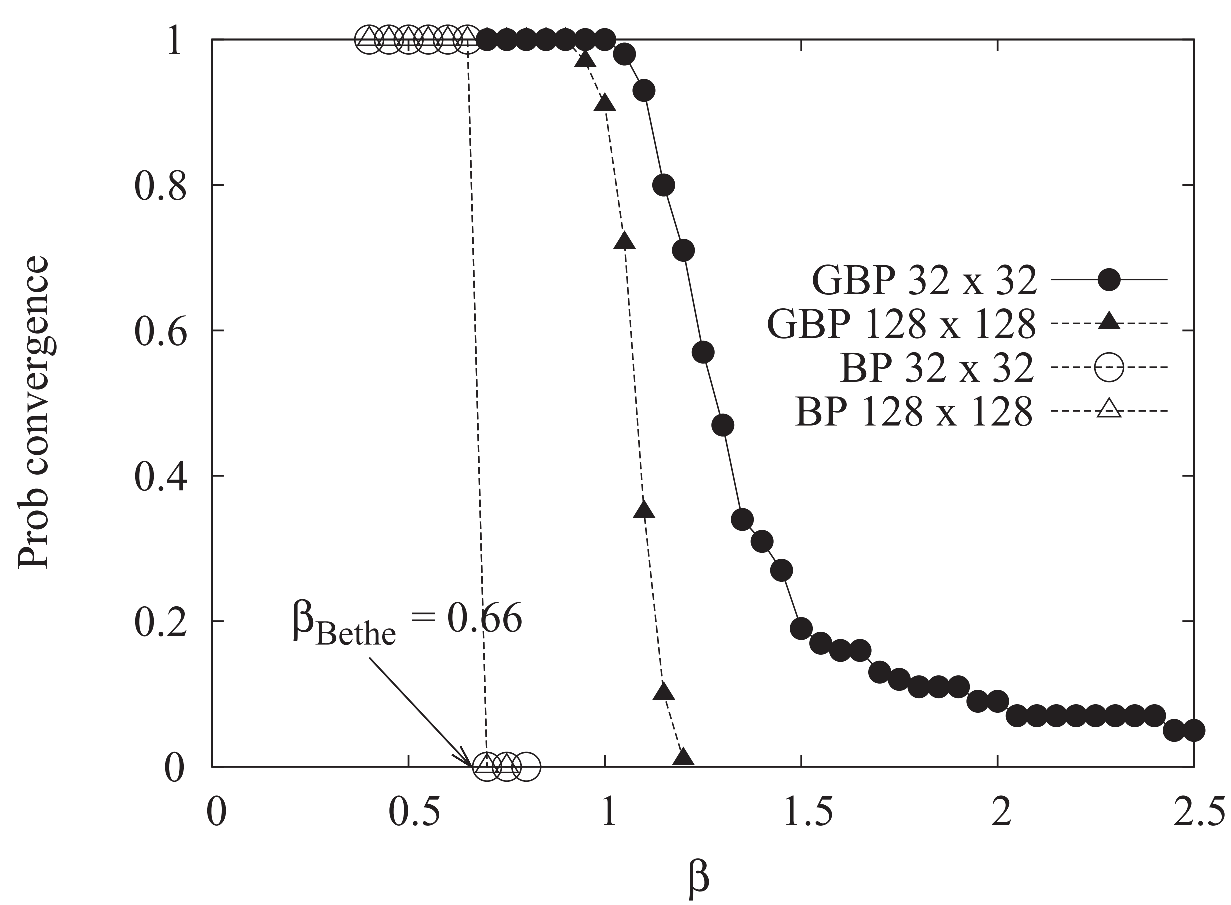






MPA convergence on 2d spin glasses

- Double loop, HAK and MPA on the dual converge at any temperature
- BP and GBP only for high enough temperatures

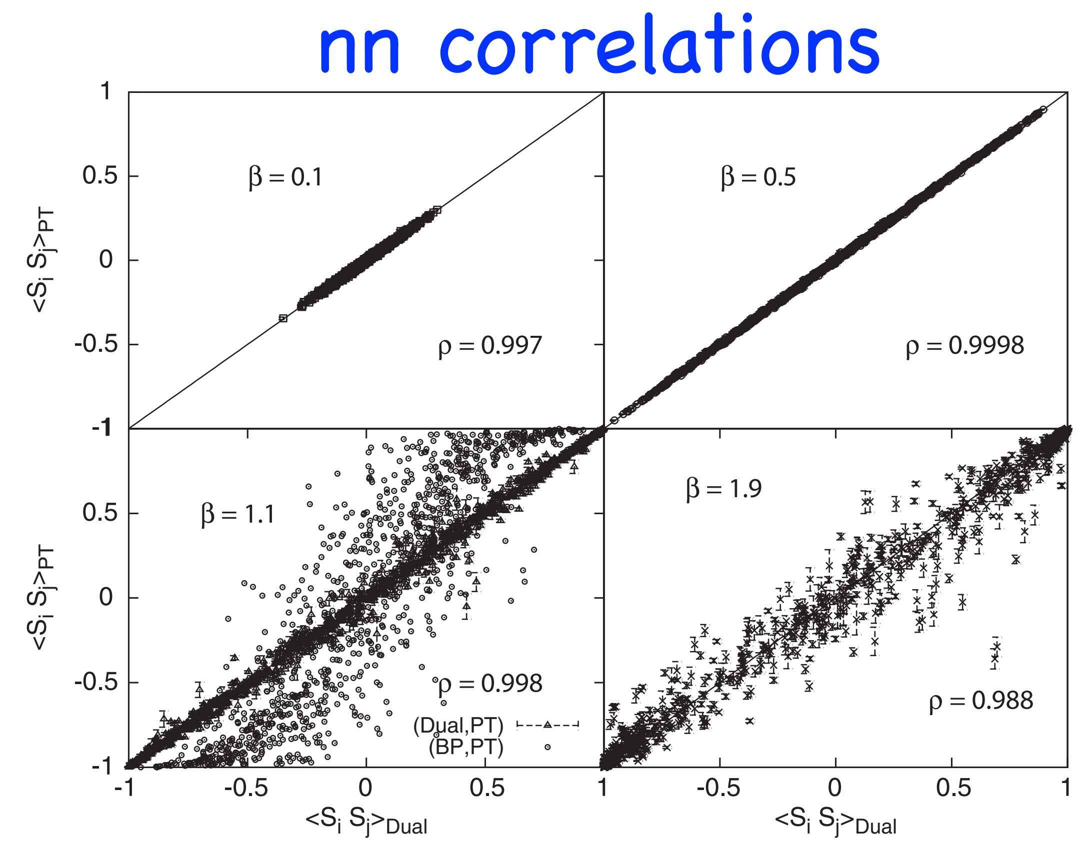


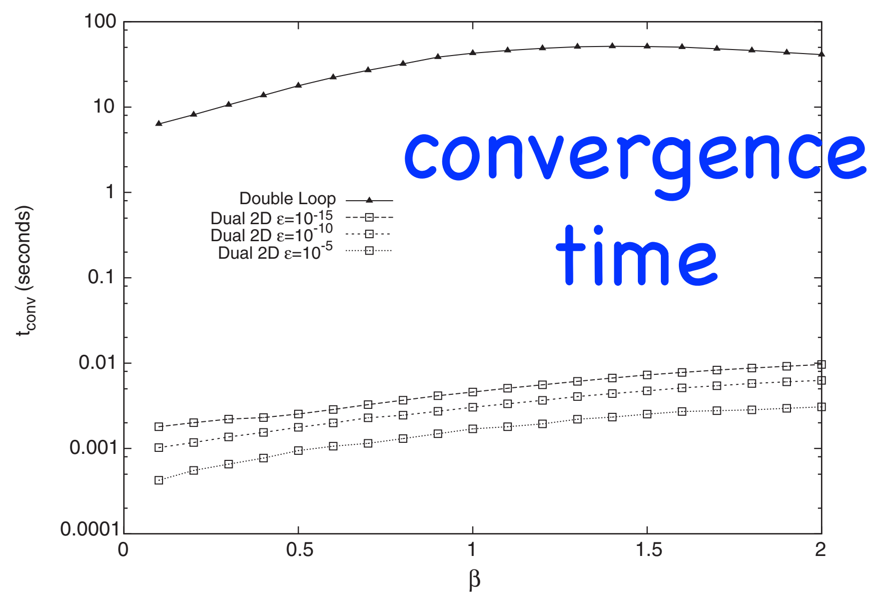
Tricks for making GBP converge

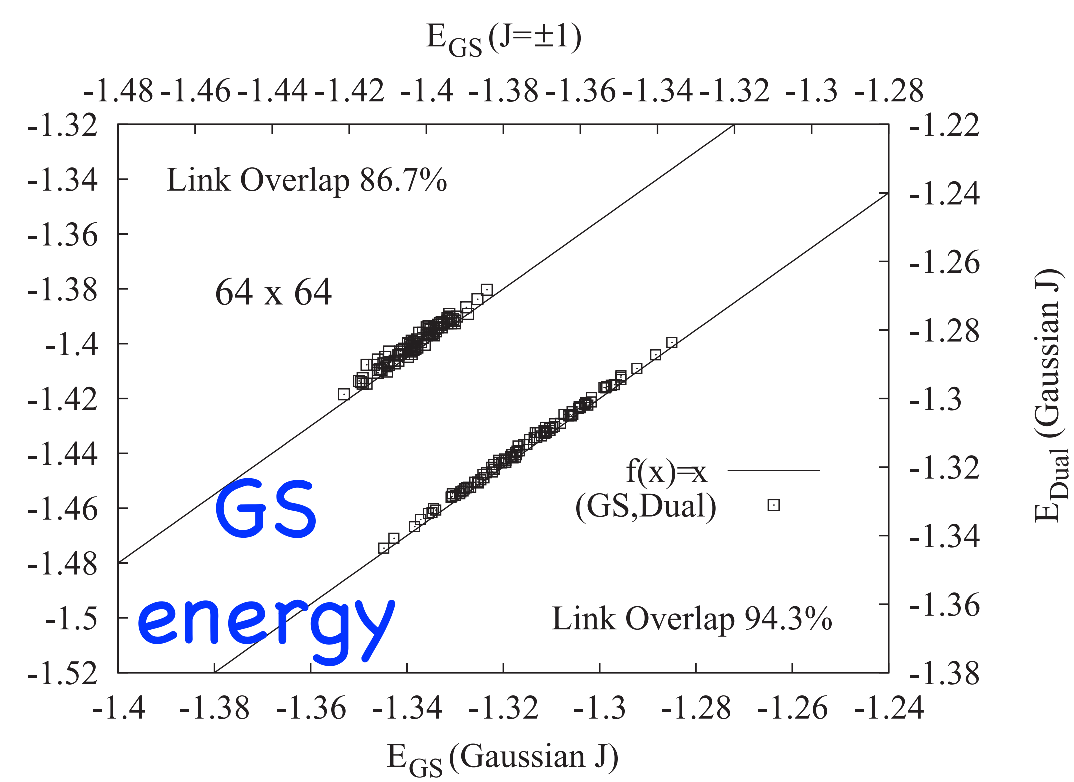
BP critical point on RRG of degree 4 is $\operatorname{atanh}(1/\sqrt{3}) \simeq 0.658$

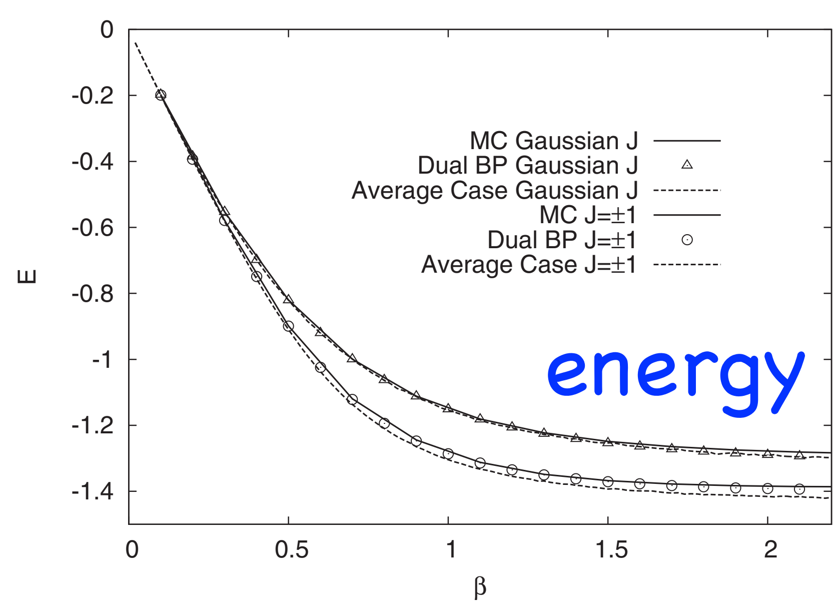
BP on the dual lattice

- It is very fast and accurate enough to find a very good approximation to ground states





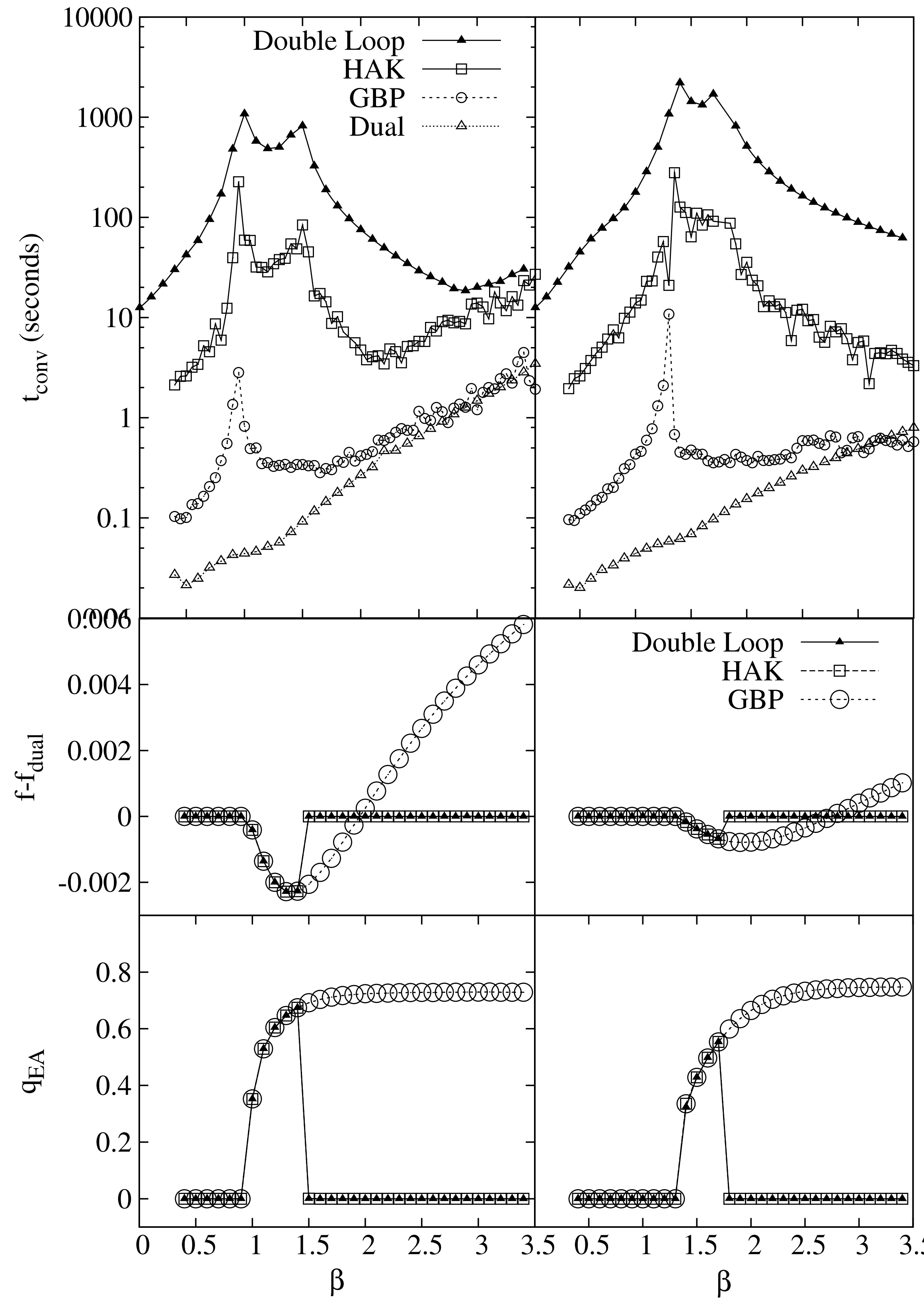




Multiple minima of CVM free-energy

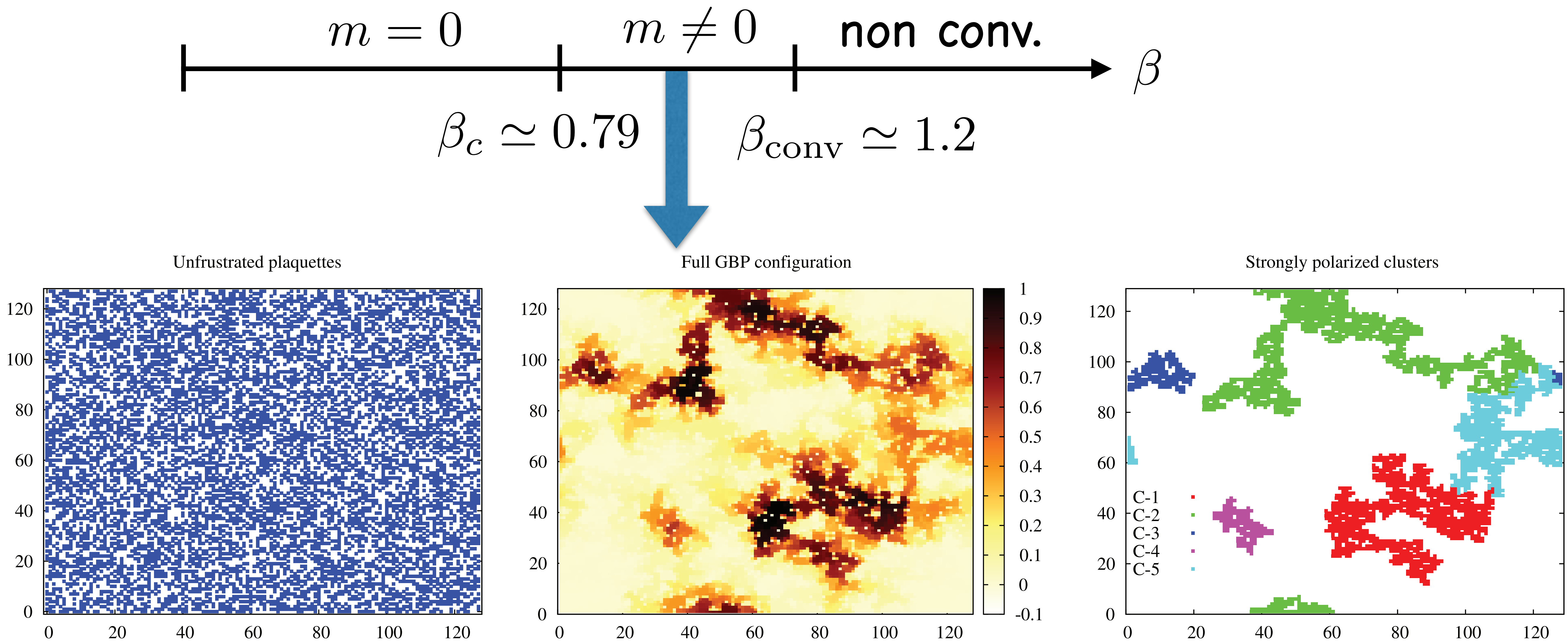
- 2 samples
- 4 algorithms
- convergence time
- free-energy
- order parameter

$$q_{\mathrm{EA}} = \frac{1}{N}\sum_i m_i^2$$

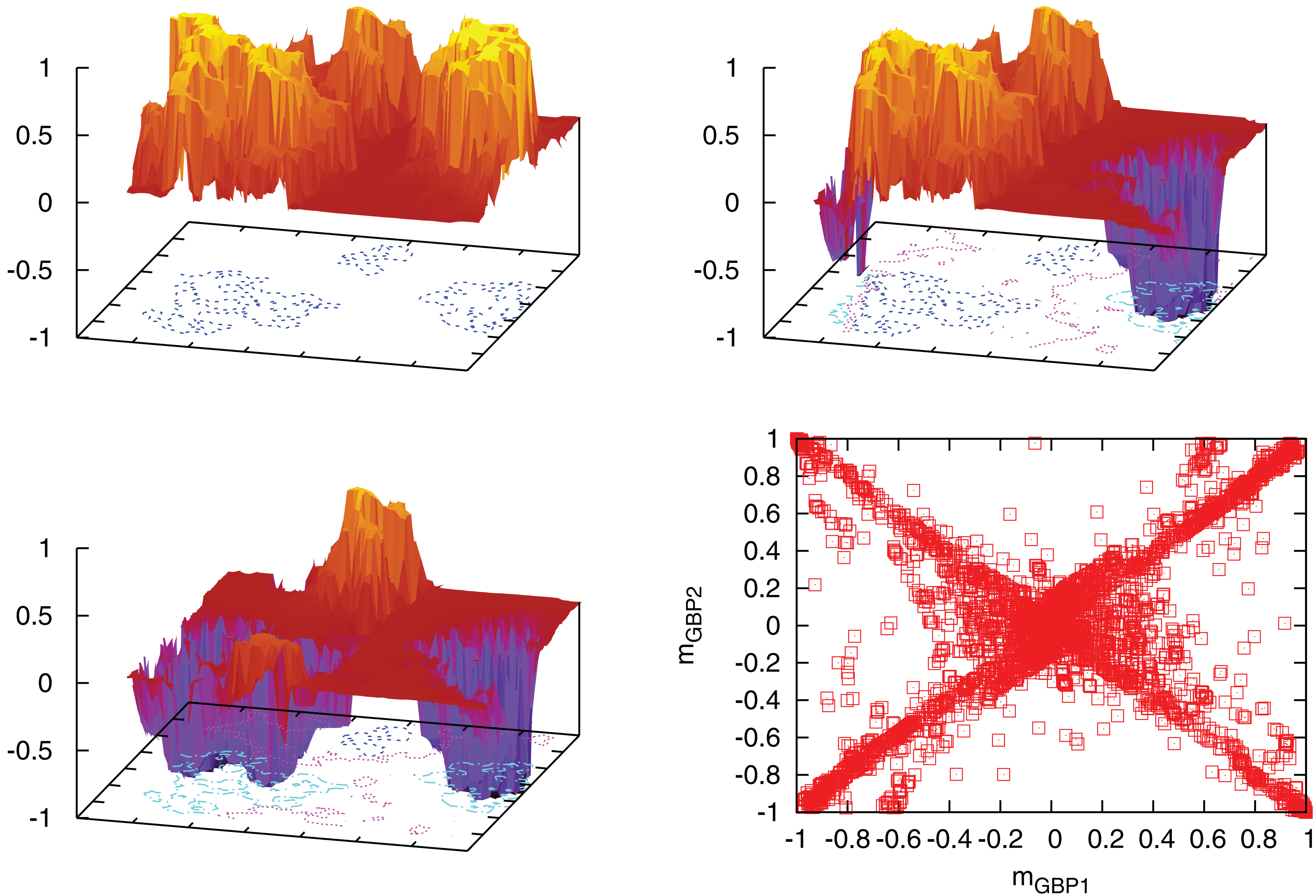


Multiple minima of CVM free-energy

- Running GBP on samples of the 2d EA model the general scenario is the following



Multiple minima of CVM free-energy

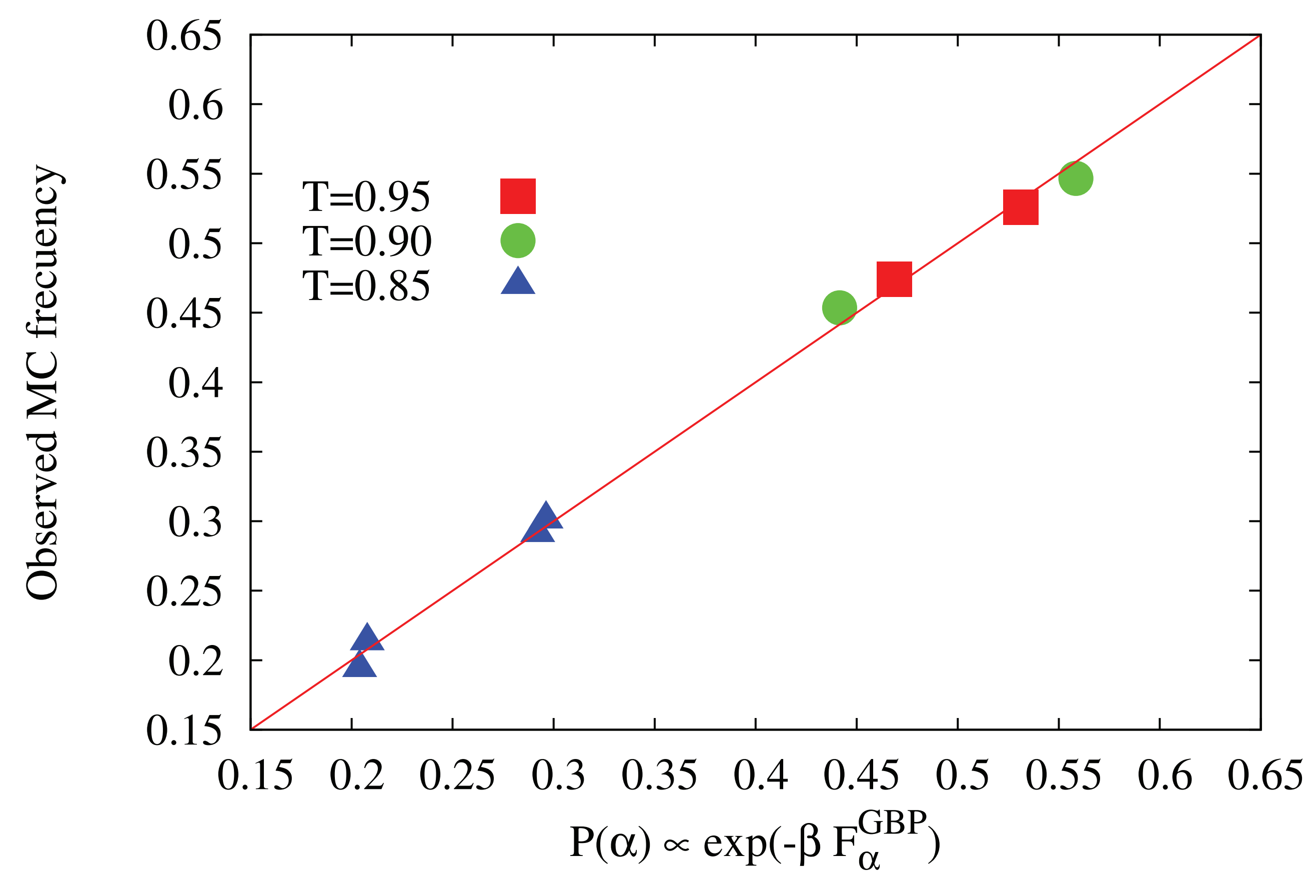


CVM vs Monte Carlo

- Do have the many CVM free-energy minima a physical meaning and role?
- Comparison with Monte Carlo dynamics
- Time spent close to a CVM free-energy minimum is equal to the CVM approximated weight of such a state

$$T_\alpha \propto w_\alpha = \frac{\exp(-\beta F_\alpha)}{\sum_{\alpha'=1}^{n} \exp(-\beta F_{\alpha'})}$$

time s.t. α was the closest state



CVM vs Monte Carlo

- Approximating the overlap distribution from CVM states (free-energy minima)

$$P(q) = \sum_{\alpha,\beta} w_\alpha w_\beta P_{\alpha\beta}(q)$$

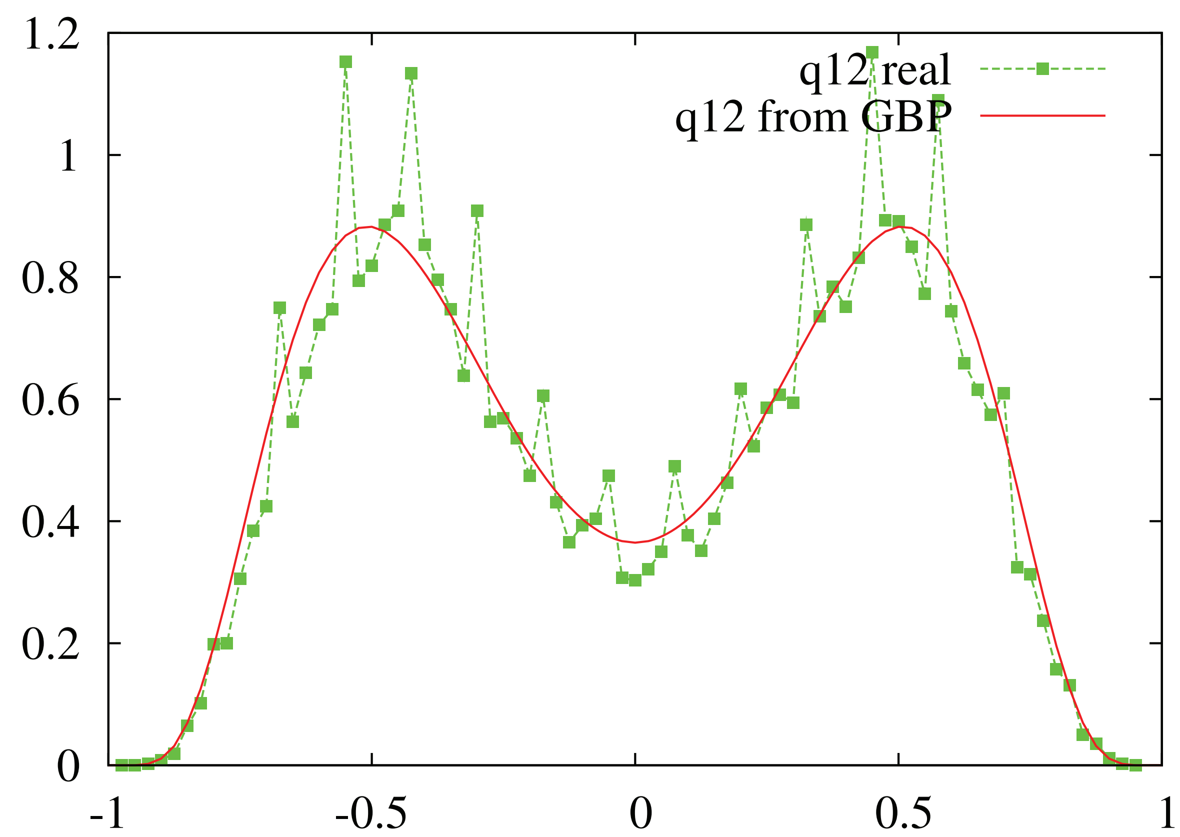
mean

$$q_{\alpha\beta} = \left\langle \frac{1}{N} \sum_i s_i^\alpha s_i^\beta \right\rangle = \frac{1}{N} \sum_i m_i^\alpha m_i^\beta$$

var

$$\sigma^2_{\alpha\beta} = \frac{1}{N^2} \sum_{ij} \left(C_{ij}^\alpha C_{ij}^\beta + m_i^\alpha m_j^\alpha C_{ij}^\beta + m_i^\beta m_j^\beta C_{ij}^\alpha \right)$$

estimation of would require generalized SuscProp...



Summary & open problems

- General framework for making MFA consistent with linear response
  - Recover several previous approximation (apparently unrelated): e.g. adaptive-TAP and SM approx.
  - Improves inference, but has some limitations (to be overcome...)
- GBP can converge to many non-trivial fixed point with physical relevance
  - Still missing a broad-purpose region-based algorithm that can deal with the many free-energy minima...

Figures in this talk are from

- *Inference algorithm for finite-dimensional spin glasses: Belief Propagation on the dual lattice*
  A. Lage-Castellanos, R. Mulet, F. Ricci-Tersenghi and T. Rizzo
  Phys. Rev. E 84, 046706 (2011)
- *Characterizing and Improving Generalized Belief Propagation Algorithms on the 2D Edwards-Anderson Model*
  E. Dominguez, A. Lage-Castellanos, R. Mulet, F. Ricci-Tersenghi and T. Rizzo
  J. Stat. Mech. P12007 (2011)
- *The Bethe approximation for solving the inverse Ising problem: a comparison with other inference methods*
  F. Ricci-Tersenghi
  J. Stat. Mech. P08015 (2012)
- *Mean-field method with correlations determined by linear response*
  J. Raymond and F. Ricci-Tersenghi
  Phys. Rev. E 87, 052111 (2013)
- *Message passing and Monte Carlo algorithms: Connecting fixed points with metastable states,*
  A. Lage-Castellanos, R. Mulet and F. Ricci-Tersenghi,
  Europhys. Lett. 107, 57011 (2014)
- *Solving the inverse Ising problem by mean-field methods in a clustered phase space with many states*
  Aurélien Decelle, Federico Ricci-Tersenghi,
  arxiv:1501.03034 (2015)
- *Correction of variational methods with pairwise linear response identities*
  J. Raymond and F. Ricci-Tersenghi,
  in preparation (2015)