COLOR and NOISE

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Dynamics of heavy quarks and their bound states in a quark-gluon plasma

WORK IN PROGRESS !

Results presented are based on

A. Beraudo, JPB, C. Ratti, NPA 806 (2008) 312 [arXiv: 0712.4394]
A. Beraudo, JPB, P. Faccioli and G. Garberoglio, Nucl.Phys. A846 (2010) 104-142 [arXiv: 1005.1245]
JPB, D. de Boni, P. Faccioli and G. Garberoglio, Nucl.Phys. A946 (2016) 49-88 [arXiv: 15003.03857]

JPB, M. Escobedo-Espinosa, arXiv:1711.10812, 1803.07996

Similar effort by Y. Akamatsu and collaborators (cf. Akamatsu's talk next week)

Outline

Basic concepts

influence functional, density matrix, complex potential, etc, (QED)

Extension to QCD

why it is not "trivial"

Basic hamiltonian

(Ignore color here)

$$H = H_Q + H_1 + H_{\rm pl}$$

For a heavy quark-antiquark pair

$$H_Q = \frac{\boldsymbol{p}_1^2}{2M} + \frac{\boldsymbol{p}_2^2}{2M} + V(\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

Linear coupling to plasma gauge field

$$H_1 = g \int_{\boldsymbol{r}} n(\boldsymbol{r}) A_0(\boldsymbol{r})$$

Simple setting

Initial density matrix

$$\mathcal{D}(t_0) = \mathcal{D}_Q(t_0) \otimes \mathcal{D}_{\mathrm{pl}}(t_0)$$

$$\mathcal{D}_{\mathrm{pl}}(t_0) = \frac{1}{Z_{\mathrm{pl}}} \sum_m \mathrm{e}^{-\beta E_m}$$

Reduced density matrix

$$\mathcal{D}_Q = \mathrm{Tr}_{\mathrm{pl}} \mathcal{D}$$

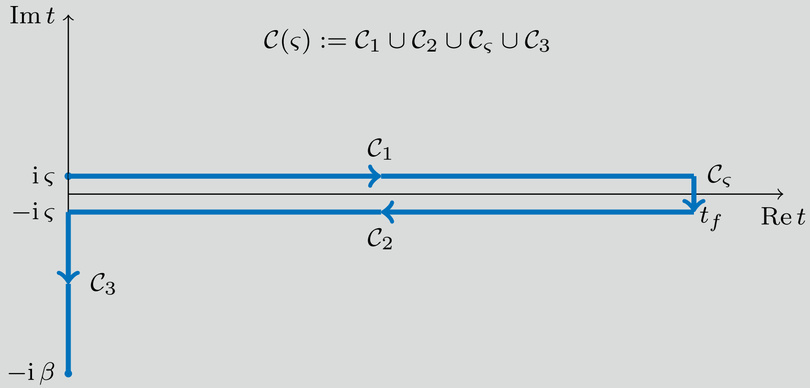
Basic question

$$P(\boldsymbol{X}_f, t_f | \boldsymbol{X}_i, t_i) = |\langle \boldsymbol{X}_f, t_f | \boldsymbol{X}_i, t_i \rangle|^2 = \langle \boldsymbol{X}_f | \mathcal{D}_Q(t_f) | \boldsymbol{X}_f \rangle$$

Path integral formulation

$$(Q_f, t_f|Q_i t_i) = \int_{x(t_i)=Q_i}^{x(t_f)=Q_f} [\mathcal{D}x(t)] \exp\left[i \int_{t_i}^{t_f} dt \left(\frac{1}{2} M\dot{x}^2 - V(x)\right)\right]$$

$$P(\boldsymbol{Q}_f, t_f|\boldsymbol{Q}_i, t_i) = \left|(\boldsymbol{Q}_f, t_f|\boldsymbol{Q}_i, t_i)\right|^2$$



$$P(Q_f, t_f|Q_i t_i) = \int_C [\mathcal{D}x(t)] \exp\left[i \int_C dt_C \left(\frac{1}{2} M\dot{x}^2 - V(x)\right)\right]$$

$$V(x) = gA_0(x)$$

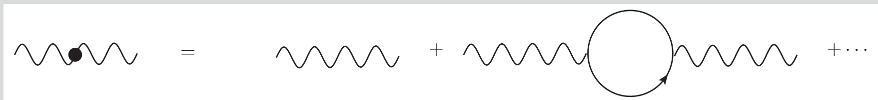
Path integral and influence functional

$$P(Q_f, t_f | Q_i, t_i) = \int_C DQ\, e^{iS_0[Q]}\, e^{i\Phi[Q]}$$

$$e^{i\Phi[\boldsymbol{Q}]} = \int \boldsymbol{D} A_0\, e^{-i \int_C d^4x\, g\rho(x) A_0(x)} e^{iS_2[A_0]}$$

'Integrate out' the plasma particles and keep the quadratic part of the resulting action (HTL approximation)

$$S_2[A_0] \;=\; -\frac{1}{2}\int_c dx \left(A_0(x) \nabla^2 A_0(x) \right) - i\ \mathrm{Tr}\ \ln\left[\, i\gamma^\mu \partial_\mu - m - e\gamma^0 A_0(x)\, \right]$$



Gaussian integration yields

$$\Phi[\boldsymbol{Q}] = \frac{g^2}{2} \iint_C d^4x d^4y\ \rho(x) \Delta_c(x-y) \rho(y)$$

$$\Delta(x-y) \equiv i\langle\, T_C\, [A_0(x) A_0(y)\,]\rangle$$

Quark antiquark correlator

$$G^{>}(t,\boldsymbol{r}_1;t,\boldsymbol{r}_2|0,\boldsymbol{r}_1';0,\boldsymbol{r}_2') = \frac{1}{Z}\mathrm{Tr}\left\{\mathrm{e}^{-\beta H} J_Q(t;\boldsymbol{r}_1,\boldsymbol{r}_2) J_Q^{\dagger}(0;\boldsymbol{r}_1',\boldsymbol{r}_2')\right\}$$

Large time behaviour $(t\, m_D \gg 1)$ and large mass limit:

$$\overline{G}(t, r_1 - r_2) \underset{t\to\infty}{\sim} \exp[-iV_{\mathrm{eff}}(r_1 - r_2)t]$$

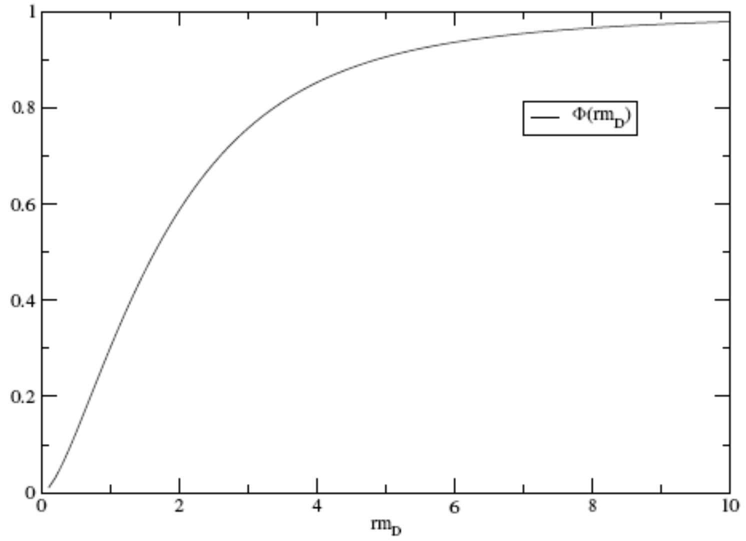
$$V_{\mathrm{eff}}(\boldsymbol{r}) = V(\boldsymbol{r}) + iW(\boldsymbol{r})$$

$$\begin{aligned}
V_{\mathrm{eff}}(r_1 - r_2) &\equiv g^2 \int \frac{dq}{(2\pi)^3}\left(1 - e^{iq\cdot(r_1 - r_2)}\right) D_{00}(\omega = 0, q) \\
&= g^2 \int \frac{dq}{(2\pi)^3}\left(1 - e^{iq\cdot(r_1 - r_2)}\right)\left[\frac{1}{q^2 + m_D^2} - i\frac{\pi m_D^2\, T}{|q|(q^2 + m_D^2)^2}\right] \\
&= -\frac{g^2}{4\pi}\left[m_D + \frac{e^{-m_D r}}{r}\right] - i\frac{g^2 T}{4\pi}\phi(m_D r)
\end{aligned}$$

(*first obtained by M. Laine et al hep-ph/ 0611300)

The imaginary part of the effective potential

$$\Gamma(\boldsymbol{r}) = W(\boldsymbol{r}) - W(0) = 2\phi(m_D r)$$

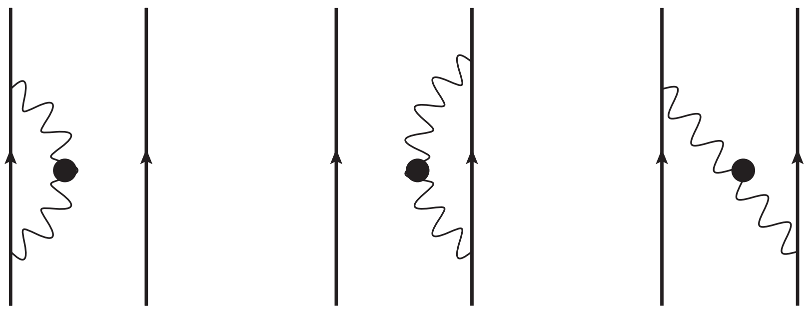


At large distance the imaginary part is twice the 'damping rate' of the heavy quark

For one heavy quark

$$\partial_t \langle \boldsymbol{r} | \mathcal{D}_Q | \boldsymbol{r}' \rangle = \cdots - \Gamma(\boldsymbol{r} - \boldsymbol{r}') \langle \boldsymbol{r} | \mathcal{D}_Q | \boldsymbol{r}' \rangle$$

At short distance, interference produces cancellation: a small dipole does not "see" the electric field fluctuations.



Approximations:

Low frequency response of the plasma

Semi-classical expansion

→ Langevin equation

$$\frac{M}{2}\ddot{\boldsymbol{r}}^i = -\gamma_{ij}\boldsymbol{v}^j - \boldsymbol{\nabla}^i V(\boldsymbol{r}) + \xi^i(\boldsymbol{r},t)$$

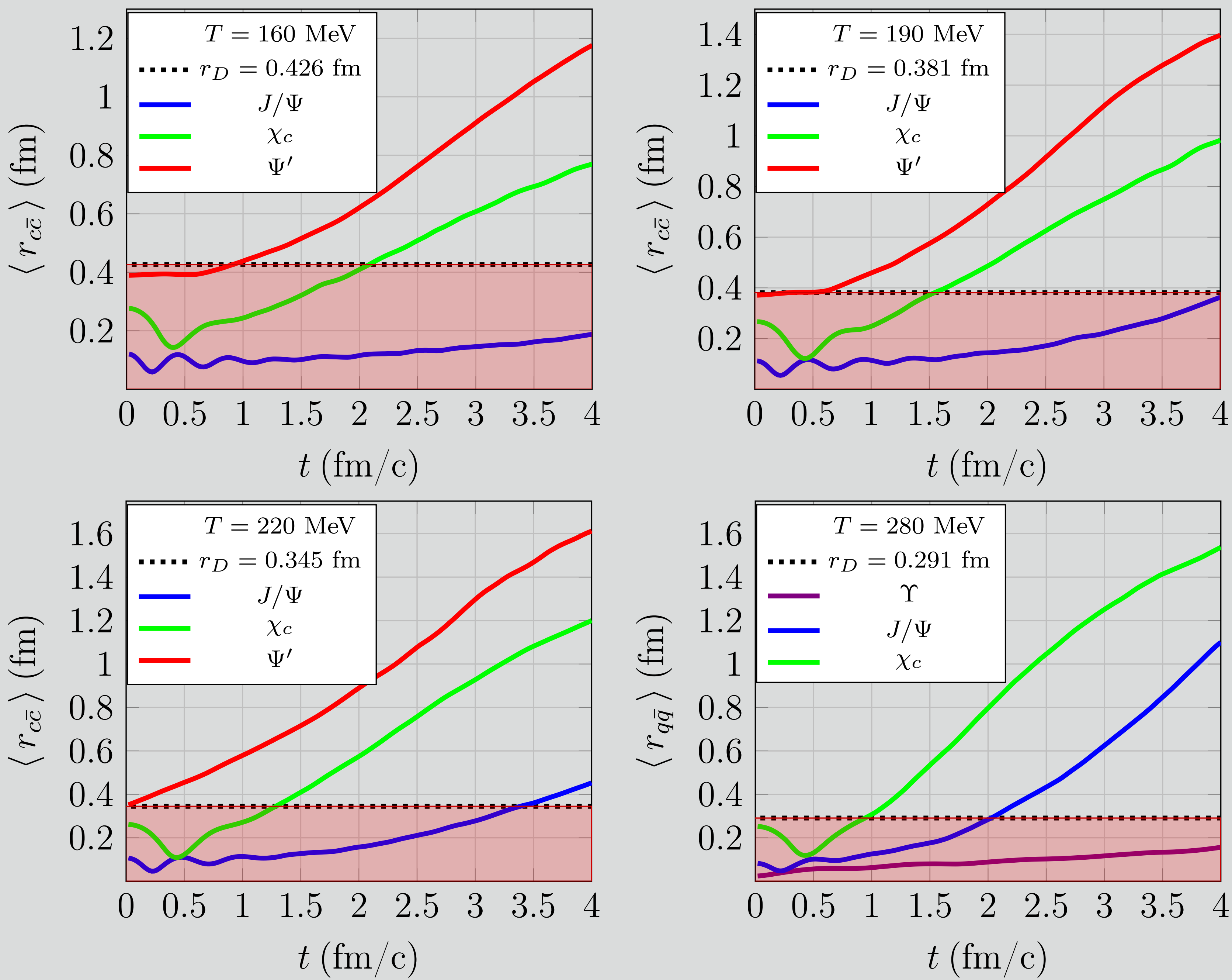
$$\gamma_{ij}(\boldsymbol{r}) = \frac{1}{2T}\eta_{ij}(\boldsymbol{r}) \qquad \langle \xi^i(\boldsymbol{r},t)\xi^i(\boldsymbol{r},t')\rangle = \eta_{ij}(\boldsymbol{r})\delta(t-t')$$

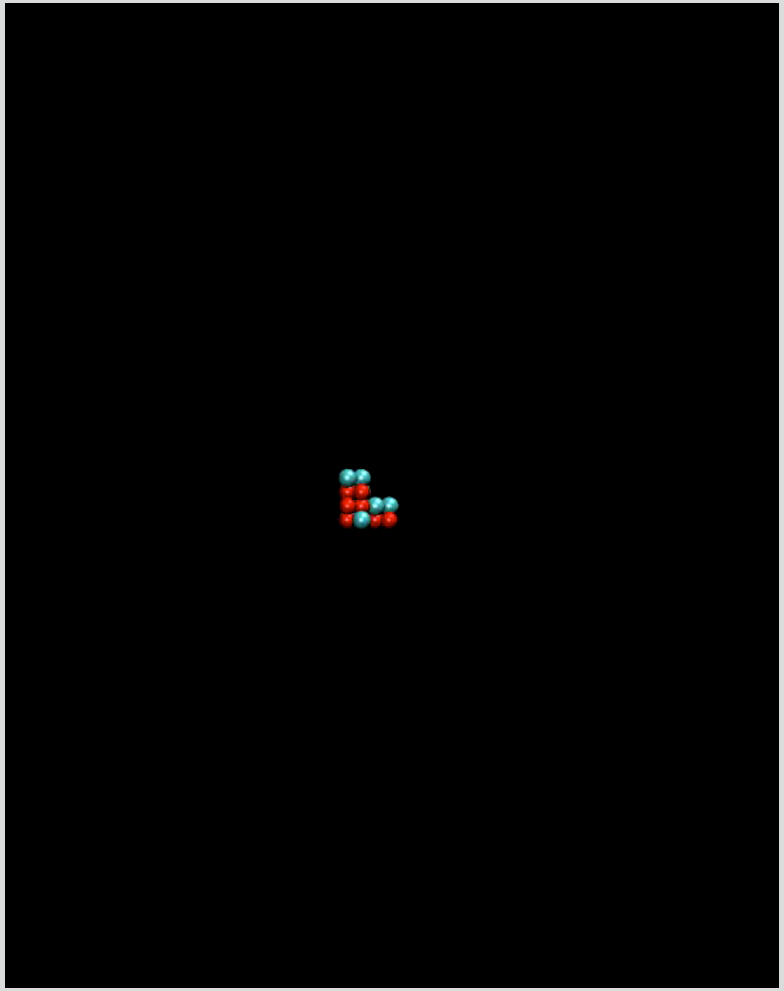
Non trivial noise

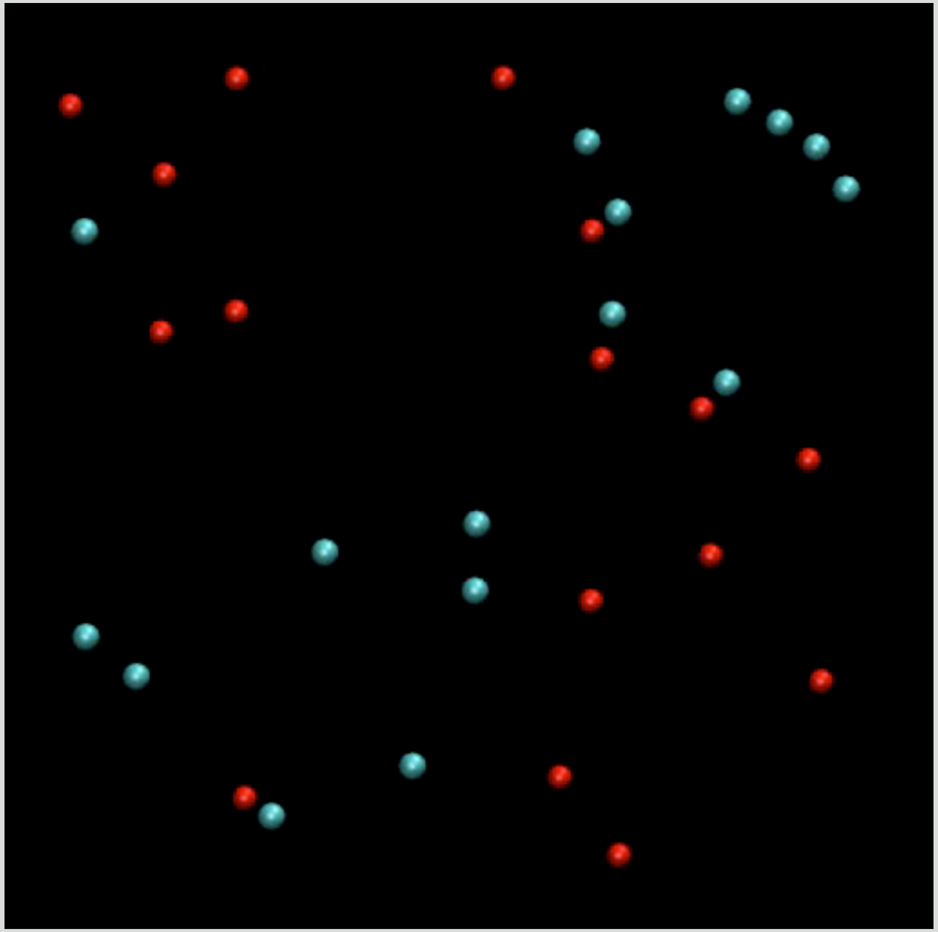
Isotropic plasma

$$\eta_{ij}(\boldsymbol{r}) = \delta_{ij}\eta(\boldsymbol{r}) \qquad \eta(\boldsymbol{r}) = \frac{1}{6}\left(\nabla^2 W(0) + \nabla^2 W(\boldsymbol{r})\right)$$

Sequential suppression







Extension to QCD

Hamiltonian for a quark-antiquark pair

$$H = H_Q + H_1 + H_{\rm pl}$$

$$H_Q = H_{\rm s,o} = -\frac{\Delta_{\boldsymbol{r}}}{M} - \frac{\Delta_{\boldsymbol{R}}}{4M} + V_{\rm s,o}(\boldsymbol{r})$$

$$V_{\rm s}(\boldsymbol{r}) = -\frac{C_F \alpha_s}{r} \qquad V_0(\boldsymbol{r}) = \frac{\alpha_s}{2N_c r}$$

(singlet) **(octet)**

$$H_1 = -g \int_{\boldsymbol{x}} a_0^A(\boldsymbol{x}) n^A(\boldsymbol{x})$$

$$n^A(\boldsymbol{x}) = \delta(\boldsymbol{x} - \hat{\boldsymbol{r}})\, T^A \otimes \mathbb{I} - \mathbb{I} \otimes \delta(\boldsymbol{x} - \hat{\boldsymbol{r}})\, \tilde{T}^A$$

Equation of motion for the pair density matrix within the same approximations as in QED

$$\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} + i[H_Q, \mathcal{D}_Q(t)] \approx -\frac{i}{2}\int_{\mathbf{x}\mathbf{x}'} V(\mathbf{x}-\mathbf{x}')[n^a_{\mathbf{x}} n^a_{\mathbf{x}'}, \mathcal{D}_Q]$$
$$+ \frac{1}{2}\int_{\mathbf{x}\mathbf{x}'} W(\mathbf{x}-\mathbf{x}')\left(\{n^a_{\mathbf{x}} n^a_{\mathbf{x}'}, \mathcal{D}_Q\} - 2 n^a_{\mathbf{x}} \mathcal{D}_Q n^a_{\mathbf{x}'}\right)$$
$$+ \frac{i}{4T}\int_{\mathbf{x}\mathbf{x}'} W(\mathbf{x}-\mathbf{x}')\left([n^a_{\mathbf{x}}, \dot{n}^a_{\mathbf{x}'} \mathcal{D}_Q] + [n^a_{\mathbf{x}}, \mathcal{D}_Q \dot{n}^a_{\mathbf{x}'}]\right)$$

Quark-antiquark pair in the large mass limit

Singlet-octet representation

$$\mathcal{D}_Q(t) \quad = \quad D_\mathrm{s}(t)|\mathrm{s}\rangle\langle\mathrm{s}| + \mathrm{D_o(t)}\sum_\mathrm{C}|\mathrm{o^C}\rangle\langle\mathrm{o^C}|$$

$$\frac{\mathrm{d}D_\mathrm{s}}{\mathrm{d}t} = -2C_F\Gamma(\boldsymbol{r})(D_\mathrm{s} - D_\mathrm{o})$$

$$\frac{\mathrm{d}D_\mathrm{o}}{\mathrm{d}t} = -\frac{1}{N_c}\Gamma(\boldsymbol{r})(D_\mathrm{o} - D_\mathrm{s})$$

Alternative representation

$$D_0 = \frac{1}{N_c^2}(D_s + (N_c^2 - 1)D_o) \qquad D_8 = \frac{2}{N_c}(D_s - D_o)$$

$$\frac{\partial D_0}{\partial t} = 0$$

('unpolarized' or maximum entropy state)

$$\frac{\partial D_8}{\partial t} = -N_c\Gamma(\boldsymbol{r})D_8$$

Langevin equation with a random color force

$$\partial_t D_0' + \frac{2\mathbf{p}\cdot\nabla}{M} D_0' - \frac{C_F}{4}\mathcal{H}_{ij}(0)\Delta_p^{ij} D_0' - \frac{2C_F F^i(\mathbf{r})F^j(\mathbf{r})}{N_c^2\Gamma(\mathbf{r})}\Delta_p^{ij} D_0' - \frac{C_F}{2MT}\mathcal{H}_{ij}(0)\nabla_p^i(p^j D_0') = 0$$

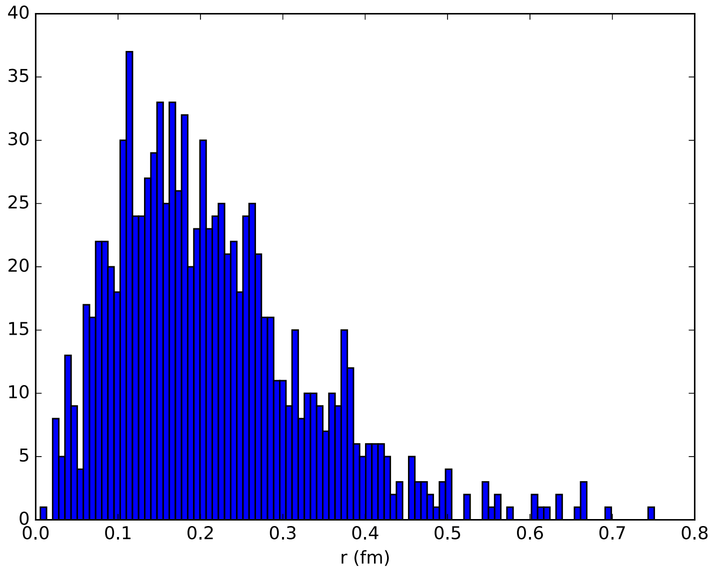
New random color force

$$\mathcal{H}_{ij}(\boldsymbol{y}) \equiv \frac{\partial^2 W(\boldsymbol{y})}{\partial y_i \partial y_j}$$

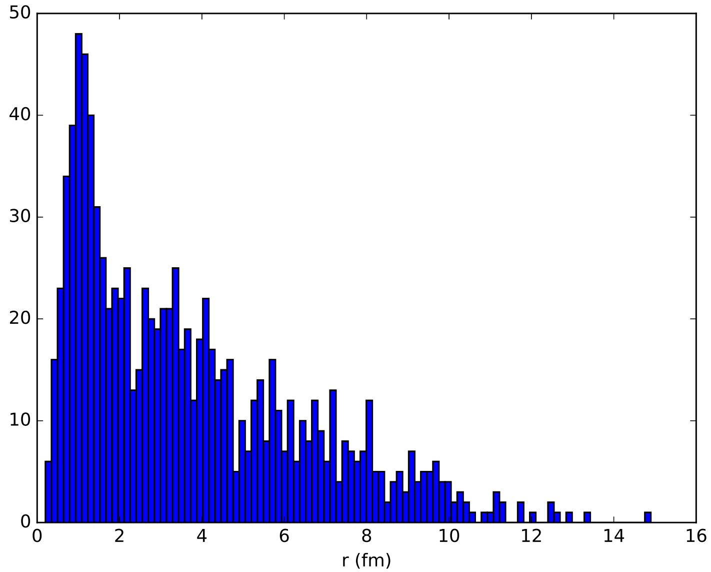
Heavy quarkonium
Histogram of distances

[1711.10812]

$t = 0$



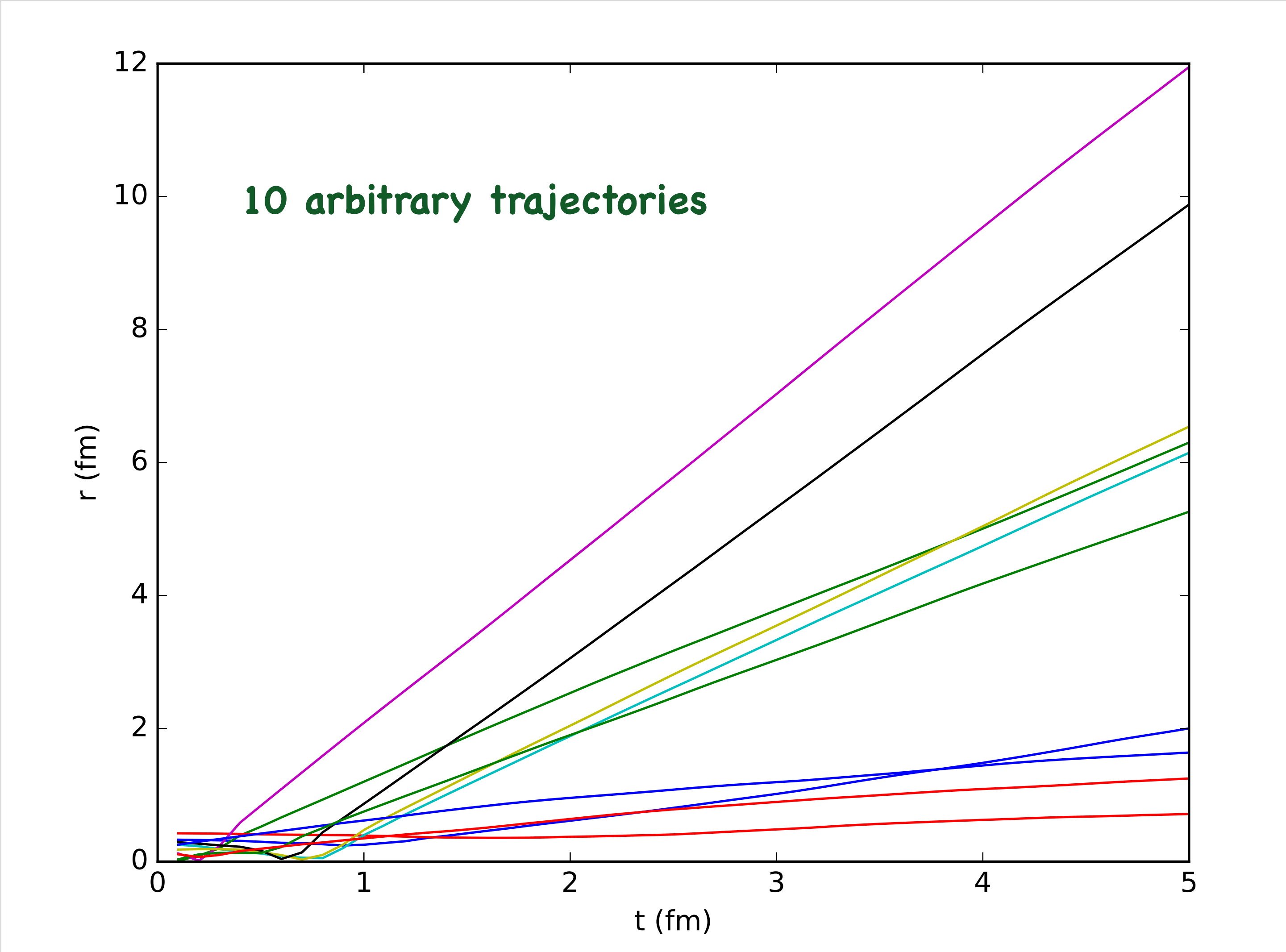
$t = 5fm$



$M = 4881$ MeV $\qquad T = 350$ MeV

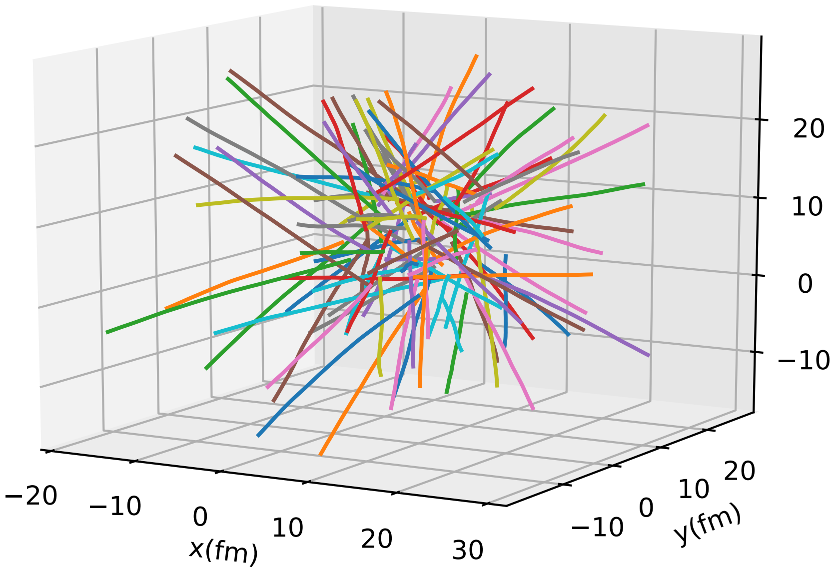
TOO MUCH SUPPRESSION !

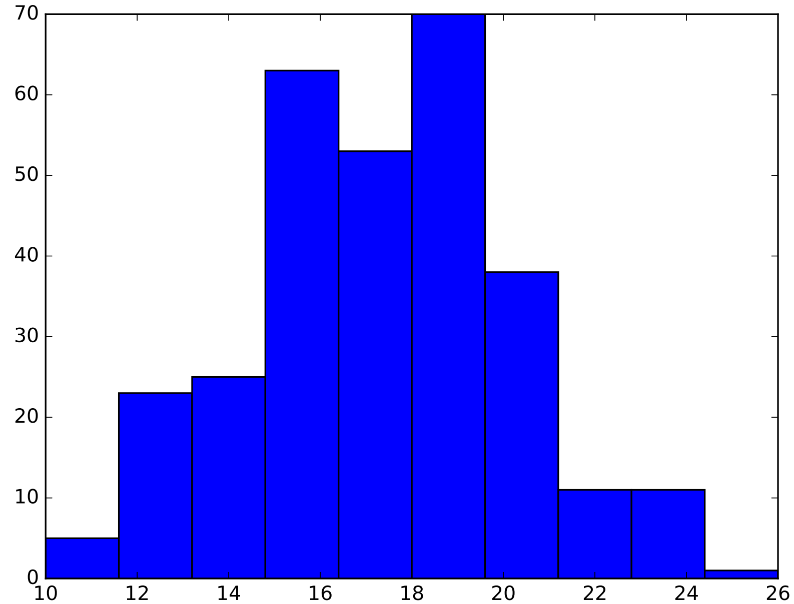
The color random force can produce unphysical kicks



Simulating 50 pairs

(after tuning parameters to avoid unphysical kicks)





A fair fraction of the pairs remain "bound" after t=5fm/c (recombination)

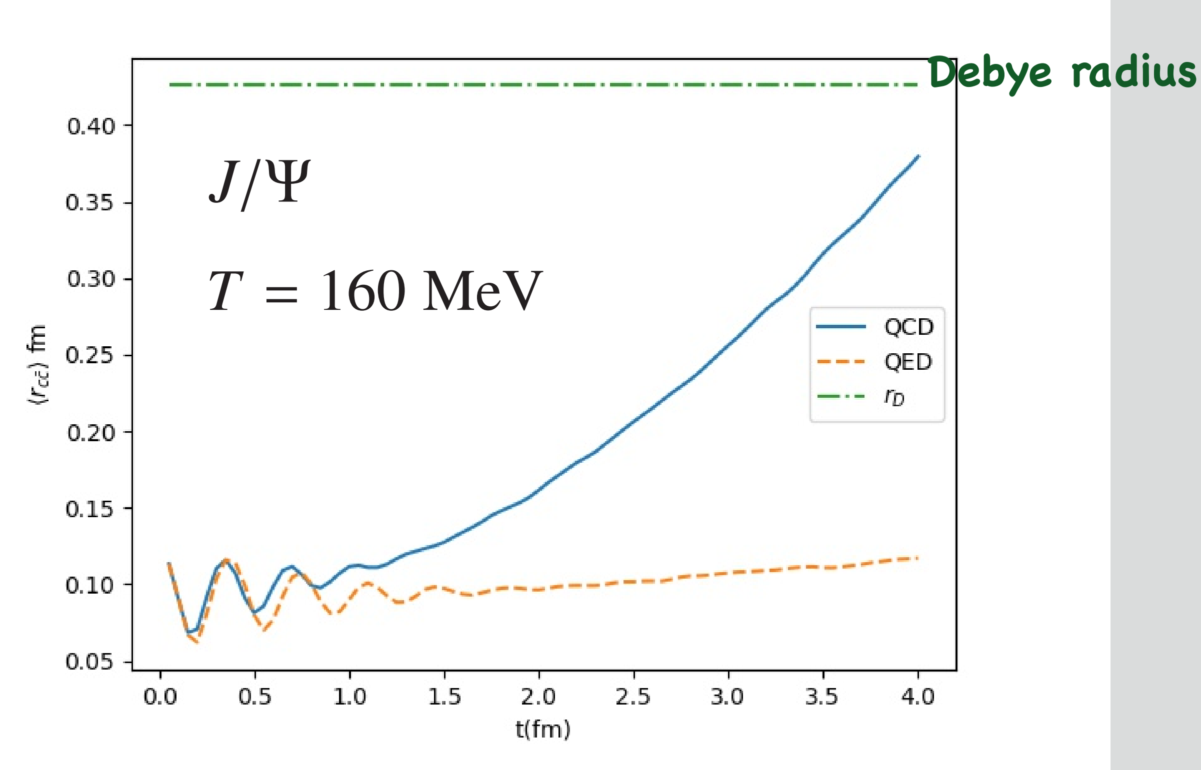
Alternative option

$$\left[\partial_t + \frac{2\mathbf{p}\cdot\nabla_\mathbf{r}}{M} + C_F \mathbf{F}(\mathbf{r})\cdot\nabla_\mathbf{p}\right] P_s = -2C_F\Gamma(\mathbf{r})\left(P_s - \frac{P_o}{N_c^2-1}\right)$$

$$\left[\partial_t + \frac{2\mathbf{p}\cdot\nabla_\mathbf{r}}{M} - \frac{1}{2N_c}\mathbf{F}(\mathbf{r})\cdot\nabla_\mathbf{p}\right] P_o = -\frac{1}{N_c}\Gamma(\mathbf{r})(P_o - (N_c^2-1)P_s)$$

$$P_o = (N_c^2-1)D_o$$

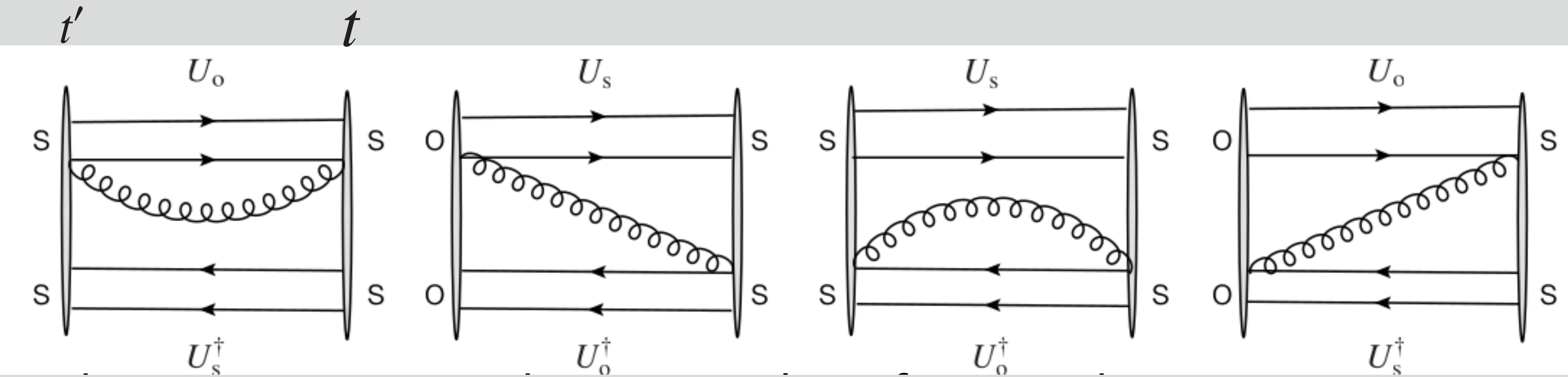
Treat the right hand side as a collision term in a Boltzmann eq.



Something missing....

A more precise evolution equation for the density matrix

[1803.07996]



$$(t' - t = \tau)$$

$$\frac{\mathrm{d}\mathcal{D}}{\mathrm{d}t} + i[H_Q, \mathcal{D}_Q(t)] =$$
$$-\int_{\mathbf{x}\mathbf{x}'}\int_0^{t-t_0} \mathrm{d}\tau\, [n^A_{\mathbf{x}}, U_Q(\tau) n^A_{\mathbf{x}'} \mathcal{D}_Q(t-\tau) U^\dagger_Q(\tau)]\, \Delta^>(\tau; \mathbf{x}-\mathbf{x}'))$$
$$-\int_{\mathbf{x}\mathbf{x}'}\int_0^{t-t_0} \mathrm{d}\tau\, [U_Q(\tau)\mathcal{D}_Q(t-\tau) n^A_{\mathbf{x}'} U^\dagger_Q(\tau), n^A_{\mathbf{x}}]\, \Delta^<(\tau; \mathbf{x}-\mathbf{x}'),$$

Before, we assumed

$$U_Q(\tau) \simeq 1 - iH_Q\tau$$

Combination of rate equation and Langevin equation

- There is no gap in the octet-octet transitions, so these can be treated with a Langevin equation (as in QED)
- Singlet to octet transitions can be treated with a rate equation

Illustration. Toy model with a single (singlet) bound state.

$$\frac{\mathrm{d}p^{\mathrm{s}}}{\mathrm{d}t} = g^2 C_F \int_{\mathbf{p}} \left(p^{\mathrm{o}}_{\mathbf{p}} - p^{\mathrm{s}} \mathrm{e}^{-\frac{E^{\mathrm{o}}_{\mathbf{p}} - E^{\mathrm{s}}}{T}} \right) \int_{\mathbf{q}} \Delta^{>}(\omega^{\mathrm{o}}_{\mathbf{p}} - E^{\mathrm{s}}, \mathbf{q}) |\langle \mathrm{s} | \mathcal{S}_{\mathbf{q}\cdot\hat{\boldsymbol{r}}} | \mathrm{o}, \mathbf{p} \rangle|^2$$

$$\frac{\partial p^{\mathrm{o}}_{\mathbf{p}}}{\partial t} - \gamma \nabla(\mathbf{p} p^{\mathrm{o}}_{\mathbf{p}}) - \frac{T\gamma M}{2} \Delta^2 p^{\mathrm{o}}_{\mathbf{p}} = - \frac{g^2}{2N_c} \frac{1}{\Omega} \left(p^{\mathrm{o}}_{\mathbf{p}} - p^{\mathrm{s}} \mathrm{e}^{-\frac{E^{\mathrm{o}}_{\mathbf{p}} - E^{\mathrm{s}}}{T}} \right) \int_{\mathbf{q}} \Delta^{>}(\omega^{\mathrm{o}}_{\mathbf{p}} - E^{\mathrm{s}}, \mathbf{q}) |\langle \mathrm{s} | \mathcal{S}_{\mathbf{q}\cdot\hat{\boldsymbol{r}}} | \mathrm{o}, \mathbf{p} \rangle|^2$$

	$\Omega = 1\,\mathrm{fm}^3$			$\Omega = 100\,\mathrm{fm}^3$		
	$5\,\mathrm{fm/c}$	$100\,\mathrm{fm/c}$	eq.	$5\,\mathrm{fm/c}$	$100\,\mathrm{fm/c}$	eq.
$T = 200\,\mathrm{MeV}$	0.86	0.136	0.0814	0.85	0.0438	0.00089
$T = 400\,\mathrm{MeV}$	0.39	0.0515	0.0175	0.36	0.0002	0.00018

Summary

- In QED, semi-classical approximation and low frequency response of plasma provide a consistent framework

- In QCD, singlet-octet transitions complicate the story (color dynamics cannot be treated semi-classically).

- Still a consistent approach can be obtained, mixing Langevin (classical dynamics) and rate equations (the imaginary potential is energy dependent).

- Note that screening and collision rates are NOT independent.