New and Old results on confinement problem from Lattice QCD

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"New Frontiers in QCD 2018 - Confinement, Phase Transition, Hadrons, and Hadron Interactions"

Plan

- Introduction. Dual superconductor scenario of confinement. Old results.
- New results for SU(2) group
- Deconfinement transition in QC_2D at T=0 and nonzero quark chemical potential
- New proposal for confinement mechanism

Computer simulations of the nonabelian gauge theories in lattice regularization is one of the most powerful nonperturbative methods which does not use uncontrolled approximations

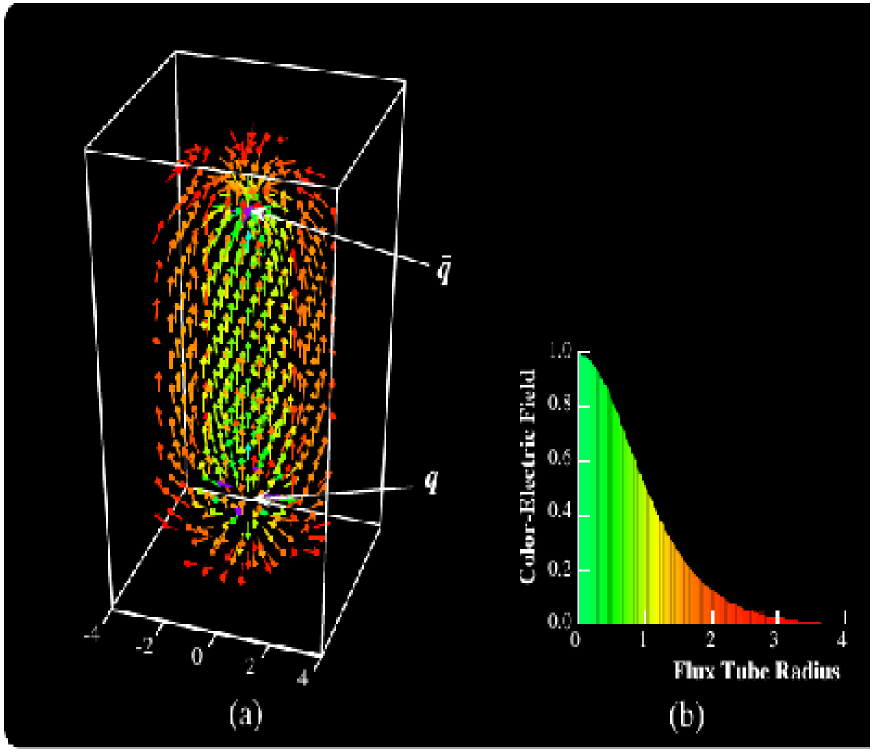
It allows to obtain numerically precise results for many hadronic observables.

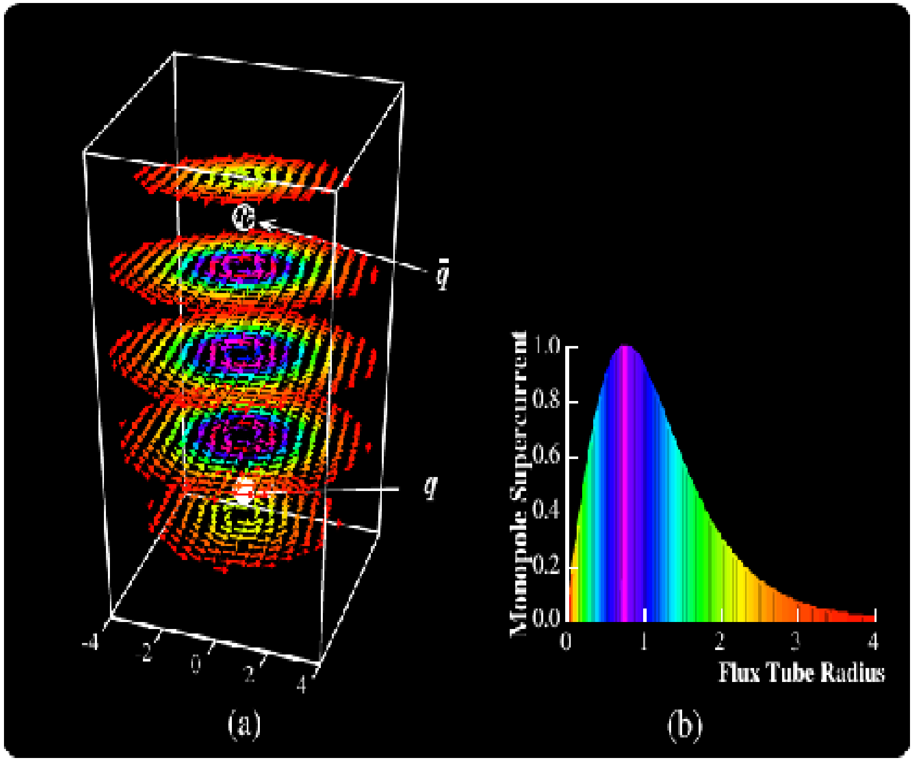
Apart from this the numerical sumulations are aimed at getting information which can be helpful for understanding the nature of the nonperturbative phenomena like confinement and chiral symmetry breaking

Dual superconductor scenario - one of the most popular ideas about nature of confinement t' Hooft '75, Mandelstam '76

A dual superconductor is a superconductor in which the roles of the electric and magnetic fields are exchanged.
Formation of the Abrikosov-Nilsen-Olesen string in a usual superconductor due to condensation of electric charges is dual to formation of the flux tube in QCD due to condensation of color-magnetic monopoles
Superconductor is described by Landau - Ginzburg model (Abelian Higgs model)
Dual superconductor – by dual Abelian Higgs model

It is yet unsolved task to rigorously prove that infrared QCD is dual to Abelian Higgs model





profile of the color-electric field(left) and profile of the magnetic currents (right) in DLG .

Koma, 2001

Lattice simulations demonstrated that

- in the confinement phase color-magnetic monopoles are condensed (percolation of magnetic currents)

- monopoles are not condensed in the deconfinement phase and the temperature of their condensation transition coincides with confinement-deconfinement phase transition temperature

- Abelian and monopole dominance for the string tension and other IR relevant quantities

- monopoles are interrelated with instantons/calorons/dyons

At present, there is no analytical proof of the existence of the condensate of abelian magnetic monopoles in gluodynamics and in chromodynamics.

However, in all theories allowing for an analytical proof of confinement, the latter is due to the condensation of monopoles.

These analytical examples are:

compact electrodynamics	Polyakov ’75
the 3D Georgi–Glashow model	Polyakov ’77
super-symmetric Yang–Mills theory	Seiberg and Witten ’94

Dirac monopole

$$\vec{A}(\vec{x}) = \frac{g_m}{4\pi} \frac{sin\theta}{r(1+cos\theta)} \vec{e}_\phi \,, \qquad \vec{e}_\phi = (-sin\phi, cos\phi, 0),$$

$$\vec{H}(\vec{x}) = \vec{\partial} \times \vec{A}(\vec{x}) + \vec{H}_{st}(\vec{x}) = \frac{g_m}{4\pi r^2} \frac{\vec{r}}{r} \,,$$

$$\vec{H}_{st}(\vec{x}) = g_m \vec{e}_z \int_{-\infty}^{0} dz' \delta\left(\vec{x} - \vec{R}(z')\right) \,, \qquad \vec{R}(z') = \{0, 0, z'\} \,.$$

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) + F_{\mu\nu,st}(x)$$

Lattice definitions for compact *U(1)*

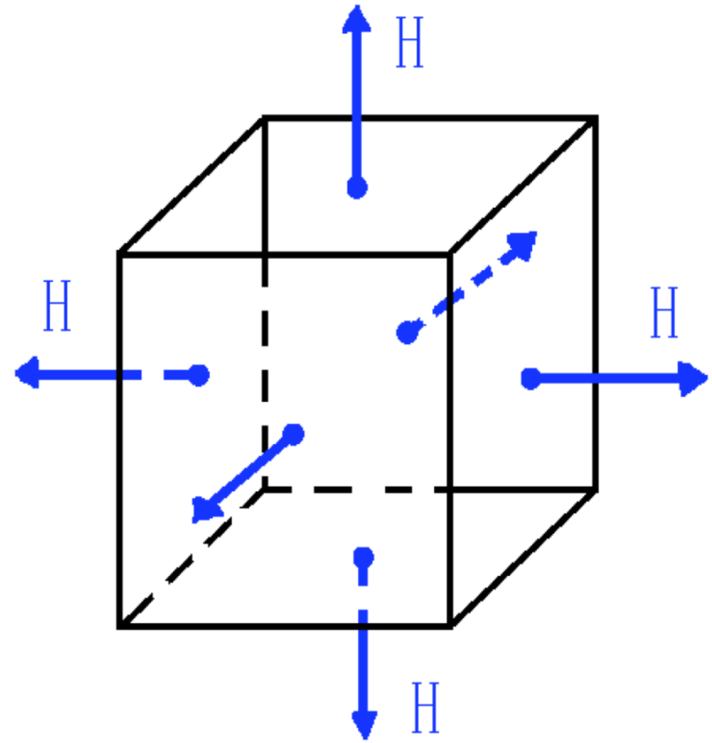
$$U_\mu(s) = exp(i\theta_\mu(s))\,, \quad \theta_\mu(s) \in [-\pi, \pi)$$

$$\theta_{\mu\nu}(s) = \partial_\mu \theta_\nu(s) - \partial_\nu \theta_\mu(s)$$

$$\overline{\theta}_{\mu\nu}(s) = \theta_{\mu\nu}(s) + 2\pi m_{\mu\nu}(s)\,,$$

$$-\pi \leq \overline{\theta}_{\mu\nu}(s) < \pi\,, \quad m_{\mu\nu} = 0, \pm 1, \pm 2$$

3-dimensional
image



4-D magnetic currents   (DeGrand and Toussaint, 1980)

$$k_\mu(s^*) = \frac{1}{4\pi}\varepsilon_{\mu\nu\rho\sigma}\partial_\nu\overline{\theta}_{\rho\sigma}(s) = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\partial_\nu m_{\rho\sigma}(s)$$

Conservation law:

$$\sum_{\mu} \partial_\mu k_\mu(s^*) = 0$$

s^* – site on a dual lattice

Magnetic currents k_μ form closed loops,
these loops are combined into clusters

t'Hooft-Polyakov monopole

$$\mathcal{L} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} (D_\mu \phi)^a (D_\mu \phi)^a + \frac{\lambda}{4} (\phi^a \phi^a - \mu^2)^2$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + e\epsilon^{abc} A^b_\mu A^c_\nu$$

topological origin:
non trivial homotopy π_2: a non trivial mapping of the sphere S^2 at spatial infinity onto $SU(2)/U(1)$

Global $SU(2)$ is broken down to $U(1)$ which direction is determined by scalar field direction at infinity.
$U(1)$ gauge invariant Abelian $F_{\mu\nu}$ can be defined

$$F_{\mu\nu} = \hat{\phi}^a F^a_{\mu\nu} - \frac{1}{e}\epsilon^{abc}\hat{\phi}^a D_\mu \hat{\phi}^b D_\nu \hat{\phi}^c$$

Magnetic field

$$eH_i = \frac{1}{r^2}\hat{x}_i + O(e^{-m_W r})$$

Then magnetic charge

$$g_m = \frac{4\pi}{e}$$

In the unitary gauge $\phi^1 = \phi^2 = 0$

$$e\vec{A}^3(\vec{x}) = -\frac{sin\theta}{r(1+cos\theta)}\vec{e}_\phi$$

i.e. form of Dirac monopole with charge $g_m = \frac{4\pi}{e}$

Without scalar field solution also exist.
$A_4^a(x)$ plays role of scalar field
In the unitary gauge

$$A_4^1 = A_4^2 = 0$$

$$g\vec{A}^3(\vec{x}) = -\frac{sin\theta}{r(1+cos\theta)}\vec{e}_\phi$$

Note that in this gauge it also satisfies Maximally Abelian gauge (MAG) condition:

$$\left(\partial_\mu \delta_{kl} + \epsilon_{k3l} A_\mu^3(x)\right) A_\mu^l(x) = 0\,, \quad k = 1,2$$

t'Hooft's idea: Partial gauge fixing

$$X(x) \to X'(x) = g(x)X(x)g^\dagger(x)\,, \quad X(x) = X_a(x)T_a$$

gauge fixing condition: $g(x) : X'(x)$ is diagonal
Gauge freedom is fixed up to $U(1)^{N_c-1}$ which is maximal Abelian subgroup or Cartan subgroup.
Gauge field has Abelian components $a^i_\mu(x) \equiv (A_\mu(x))_{ii}$

$$a^i_\mu(x) \to a^i_\mu(x) + \frac{1}{g}\partial_\mu \alpha_i$$

and off-diagonal components
$c^{ij}_\mu(x) \equiv (A_\mu(x))_{ij}\,, i \neq j$

$$c^{ij}_\mu(x) \to e^{i\left(\alpha_i(x)-\alpha_j(x)\right)} c^{ij}_\mu(x)$$

There is a singularity at locations where two or more eigenvalues are equal. In the vicinity of such singularity gauge field has a form of the t'Hooft - Polyakov monopole, i.e. it has a magnetic charge.

$$A^3_{sing} T_3 = -\frac{1}{g} \vec{e}_\phi \frac{1 + cos\theta}{rsin\theta} T_3$$

$$g_m = -\frac{4\pi}{g} T_3$$

Examples of $X(x)$: $F_{12}(x), L(x)$

Thus QCD becomes equivalent to theory with color magnetic monopoles, 'photons', and charged matter fields: off-diagonal gluons and quarks.

Maximally Abelian gauge

MA gauge condition

$$\left(\partial_\mu \delta_{kl} + \epsilon_{k3l} A_\mu^3(x)\right) A_\mu^l(x) = 0\,, \quad k = 1,2$$

solutions: extremums over gauge transformations of the functional

$$F[A] = \int d^4x \;\; \{(\,A_\mu^1)^2 + (\,A_\mu^2)^2\,\}$$

Abelian projection:

$$A_\mu^a\, T^a \;\rightarrow A_\mu^3\, T^3$$ (in observables)

Lattice formulation - by Kronfeld, Laursen, Schierholz, Wiese,1989

Abelian dominance hypothesis

Ezawa, Iwazaki '82

Physical observables, related to the infrared properties of the theory, can be computed with the help of the Abelian variables i.e.

$$< \mathcal{O} > = \frac{1}{Z} \int e^{-S} \mathcal{O}(U_\mu) \mathcal{D} U_\mu$$

and

$$< \mathcal{O} >^{Ab} = \frac{1}{Z} \int e^{-S} \mathcal{O}(u_\mu) \mathcal{D} U_\mu$$

give approximately equal values of the infrared physical quantities.

Example: $O = W(r,t)$; static potential is derived from the Wilson loop: $V(r) = \alpha/r + \sigma r$.

Abelian projection gives very good approximation for σ but not for α

Suzuki and Yotsuyanagi, 1990

Bonati, D'Elia and Di Giacomo, 2010

It was argued that MAG is a proper Abelian gauge to find gauge invariant monopoles since monopoles can be identified in this gauge by the Abelian flux, but this is not possible in other Abelian gauges.
In other words, the efficiency of the method to detect monopoles (DeGrand-Toussaint) depends on the choice of the gauge.
It was demonstrated for a class of gauges which interpolate between the Maximal Abelian gauge and the Landau gauge, how monopoles gradually escape detection.

Old results

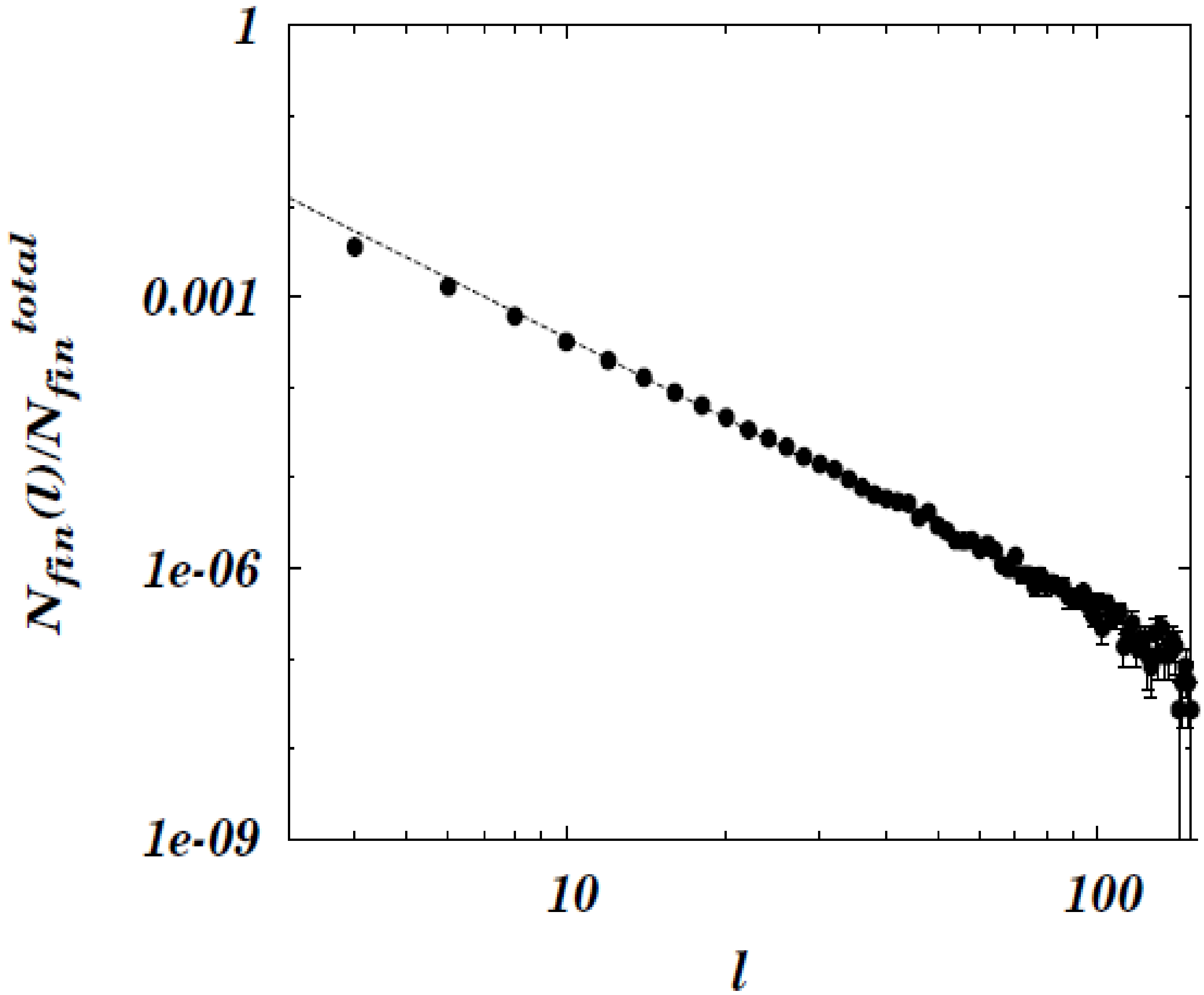
Trajectories of the Abelian monopoles form three different types of clusters:

- Large cluster (one per configuration percolating cluster, of infinite size on the infinite lattice)
magnetic currents from this cluster are called IR monopoles
VB, Mitryushkin and Mueller-Preussker, 1992; Hart and Teper, 1996

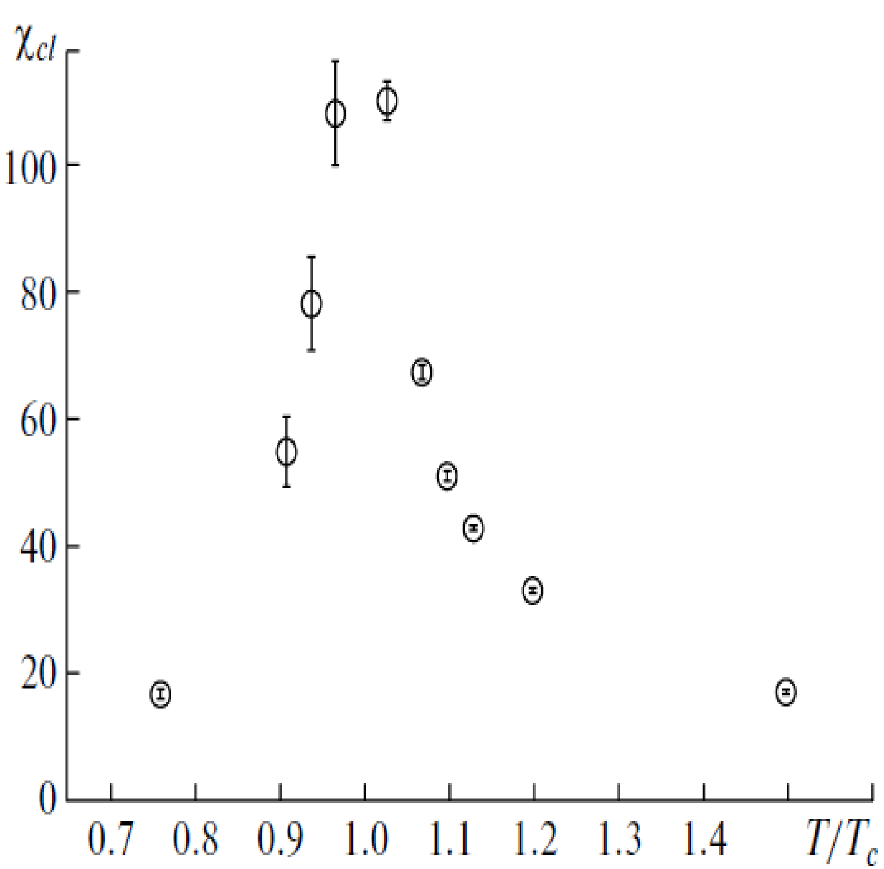
- Finite size clusters with distribution of length $N(L) = C/L^3$
Hart and Teper, 1996
Both observations are in accordance with percolation theory, $1/L^3$ dependence was also derived within the polymer approach to the field theory for free or Coulomb-like interacting scalar particles
Chernodub and Zakharov, 2003

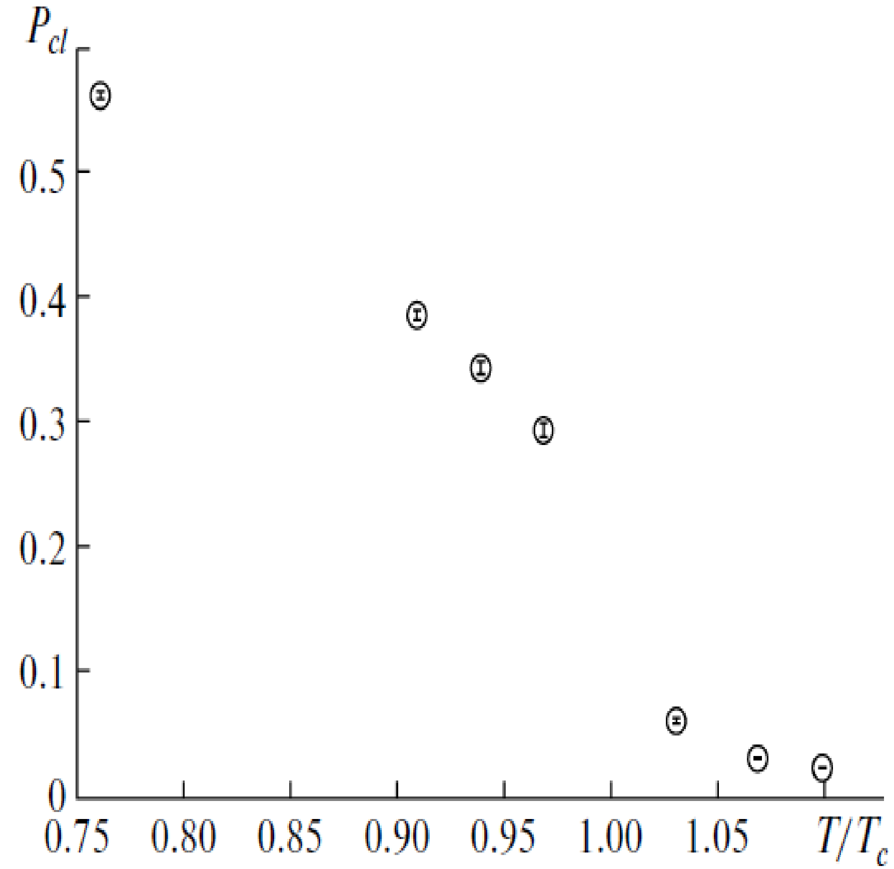
- Small clusters with length $L = O(a)$. These are UV monopoles

The length distribution of finite clusters in SU(2) gluodynamics. Lattice $32^4, a \approx 0.13$fm, VB, Boyko, Polikarpov, Zakharov, '03



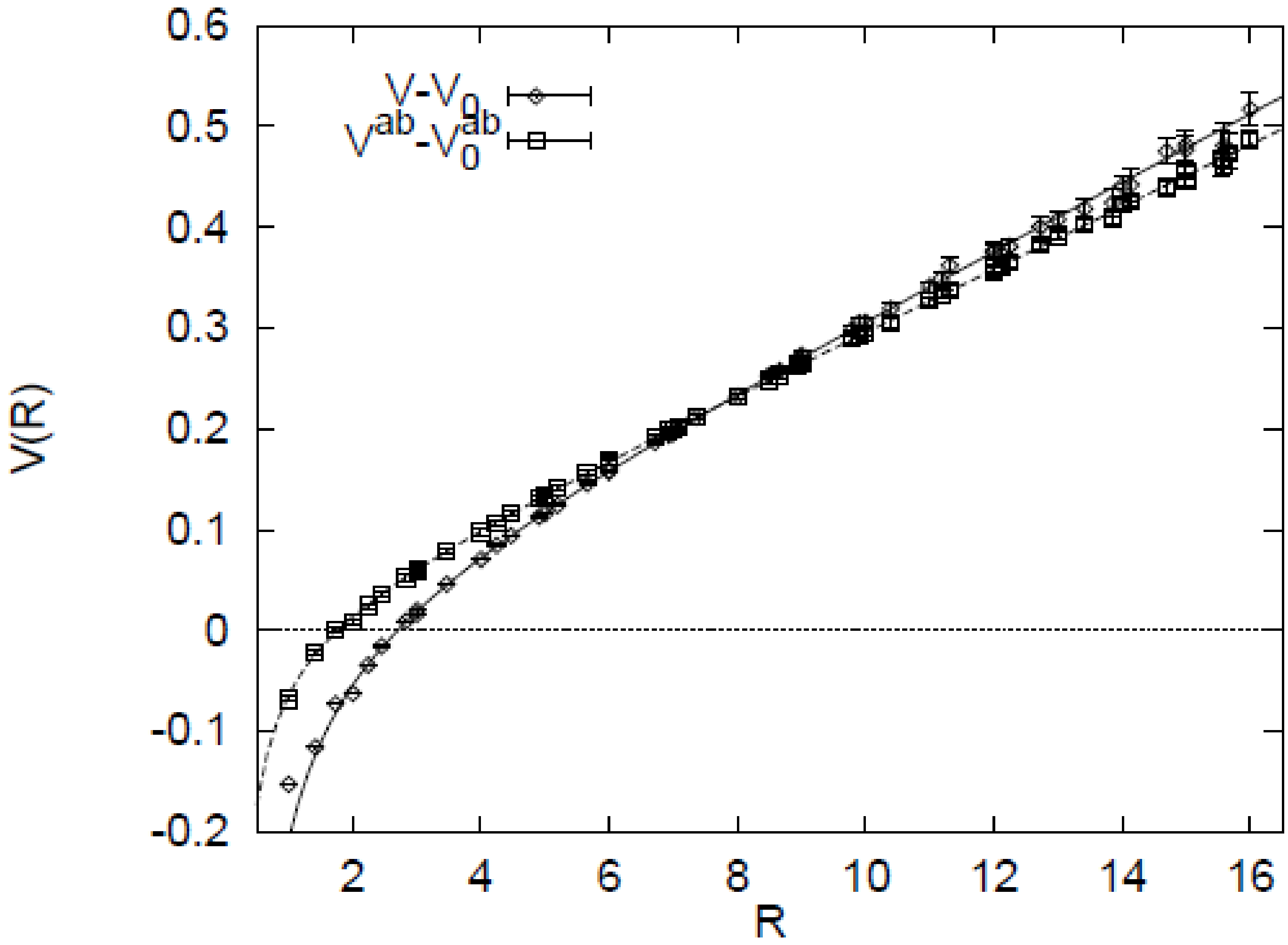
Percolation transition at T_c





Percolation susceptibility (left) and percolation probability (right) in SU(2) gluodynamics

Abelian and monopole dominance



Abelian and nonabelian static potentials. Bali, VB, Mueller-Preussker, Schilling, 1996

One can decompose the Abelian vector potential into monopole and photon parts

$$A_\mu^{mon}(x) = 2\pi \sum_{y,\nu} D(x-y)\partial_\nu m_{\mu\nu}(x)$$

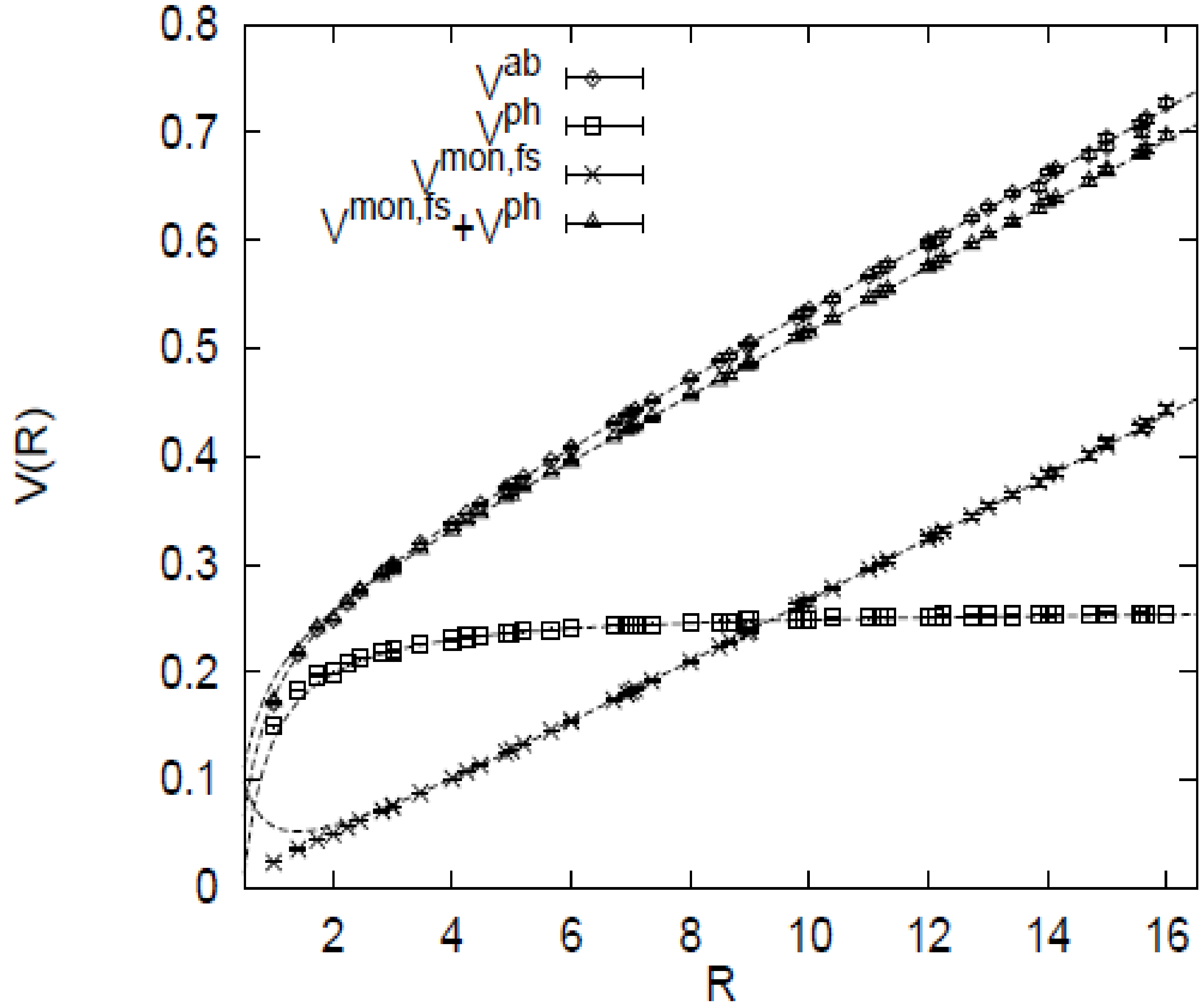
$$A_\mu^{phot}(x) = A_\mu(x) - A_\mu^{mon}(x)$$

$$u_\mu^{mon}(x) = exp(iA_\mu^{mon}(x))$$

$$u_\mu^{ph}(x) = exp(iA_\mu^{ph}(x))$$

$$U_\mu^{mod}(x) = U_\mu(x) u_\mu^{mon,\dagger}(x)$$

U_μ^{mod} - nonabelian gauge field with monopoles removed (modified)



Abelian static potential in comparison with 'monopole' and 'photon' static potentials

Results in $SU(2)$:

$$\sigma^{ab}/\sigma = 0.92(4)$$

$$\sigma^{mon}/\sigma^{ab} = 0.95(2)$$

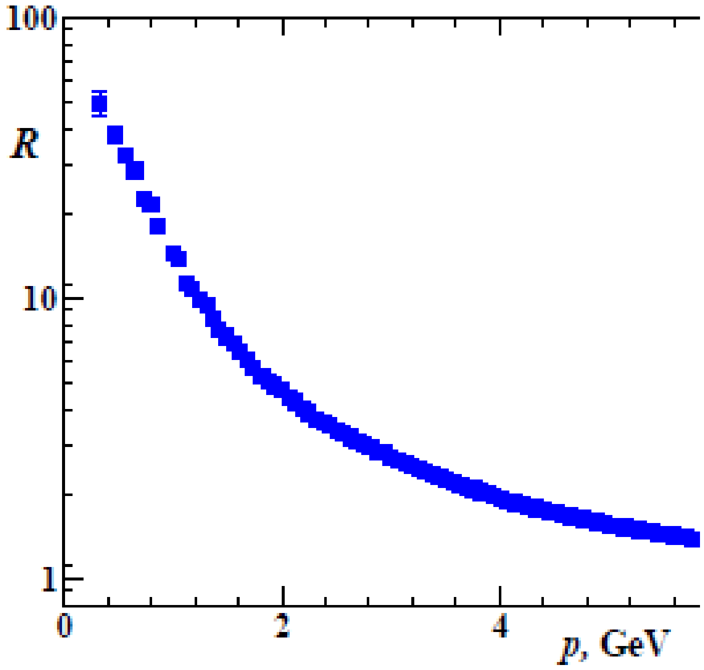
$$\sigma^{ab,2}/\sigma^{ab} = 2.23(5)$$

(it is 8/3 in SU(2)

σ^{ab}/σ was computed in the limit of infinite cutoff
σ^{ab}/σ was computed for improved lattice action and universality of the Abelian dominance had been demonstrated
VB, Ilgenfritz, Mueller-Preussker, 2005

Dominance of the diagonal gluon propagator in IR had been found Amemiya and Suganuma, 1999 (in coordinate space)

VB, Chernodub, Gubarev, Morozov and Polikarpov, 2003 (in momentum space)



$$R = \frac{D_{diag}(p_{min})}{D_{offdiag}(p_{min})} = 50(5), \quad p_{min} = 325 \text{ MeV}$$

Ratio of diagonal to offdiagonal transverse propagators

Properties of superconductors are often described in terms of a penetration depth λ and a correlation length ξ , which are equal to the inverse vector and Higgs masses.

They were computed on the lattice from the Abelian flux tube properties.

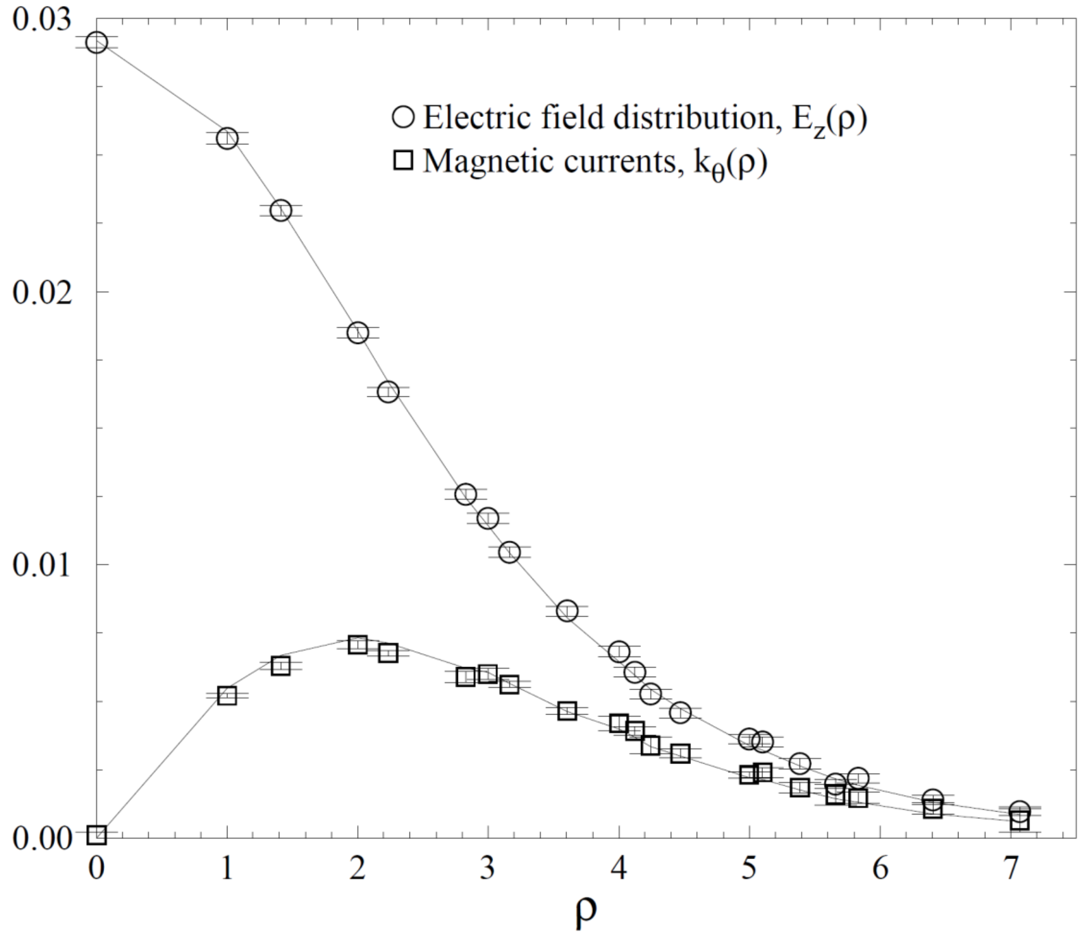
V. Singh, D. A. Browne, R. W. Haymaker, 1993

C. Schlichter, G. S. Bali, K. Schilling, 1998

F. V. Gubarev, M. Ilgenfritz, **M. I. Polikarpov**, T. Suzuki, 1999

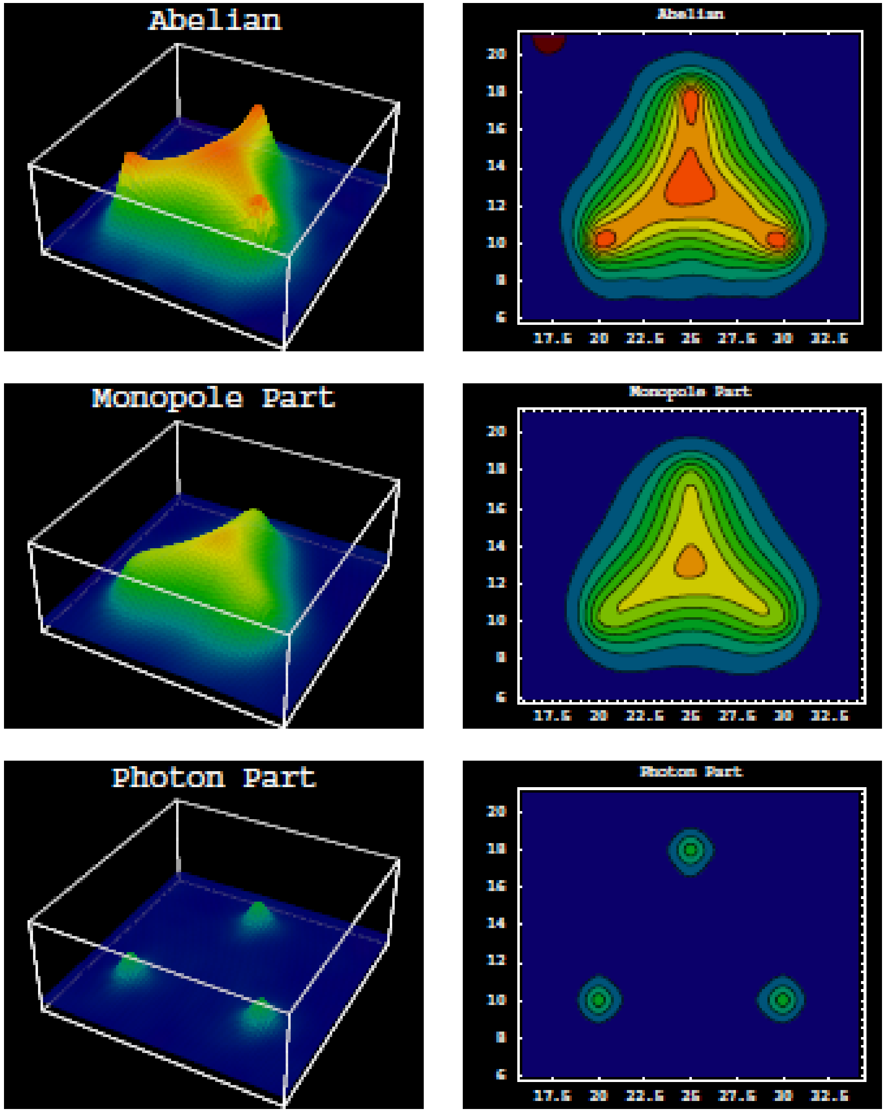
The classical equations of motion for the Abelian Higgs model were numerically solved

$$\mathcal{L}_{AHM} = \frac{1}{4g^2} F_{\mu\nu}^2(B) + \frac{1}{2}|(\partial_\mu - iB_\mu)\Phi|^2 + \lambda(|\Phi|^2 - \eta^2)^2 \ ,$$



The lattice data for distribution of the electric flux and magnetic currents were nicely fitted by the classical equations of motion of the dual Abelian Higgs model. It was found that the mass of the vector boson is equal to the mass of the monopole (Higgs particle) within numerical errors. The effective dual Abelian Higgs Model appears to lie on the border between type-I and type-II superconductivity. The classical string tension (energy per unit length of the Abrikosov vortex) is 94% of the full non-Abelian string tension.

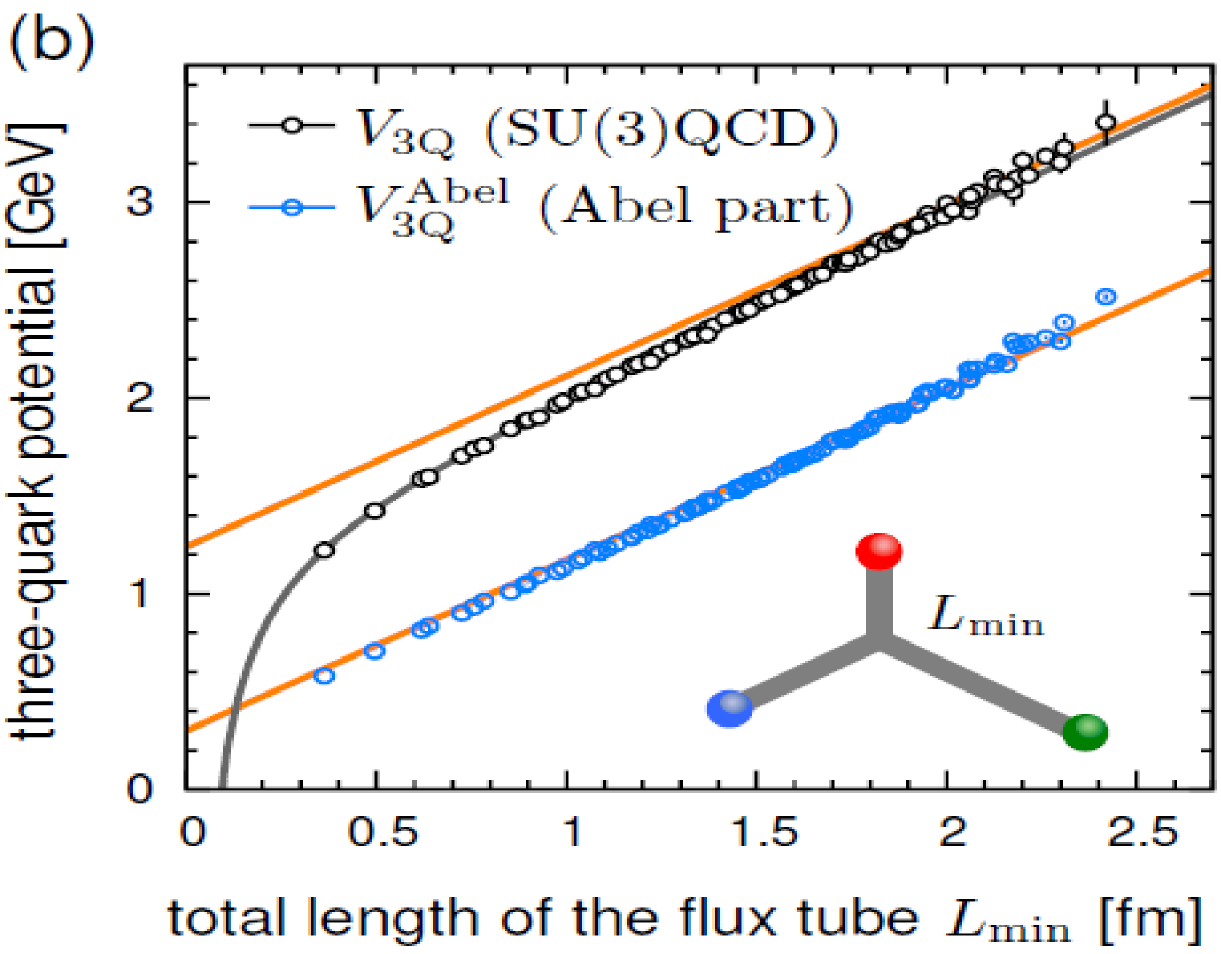
Ichie, VB, Streuer, Schierholz, 2003, 3Q system



Abelian action density in three-quark system (static baryon) in lattice QCD

Suganuma, Sakumichi, 2015

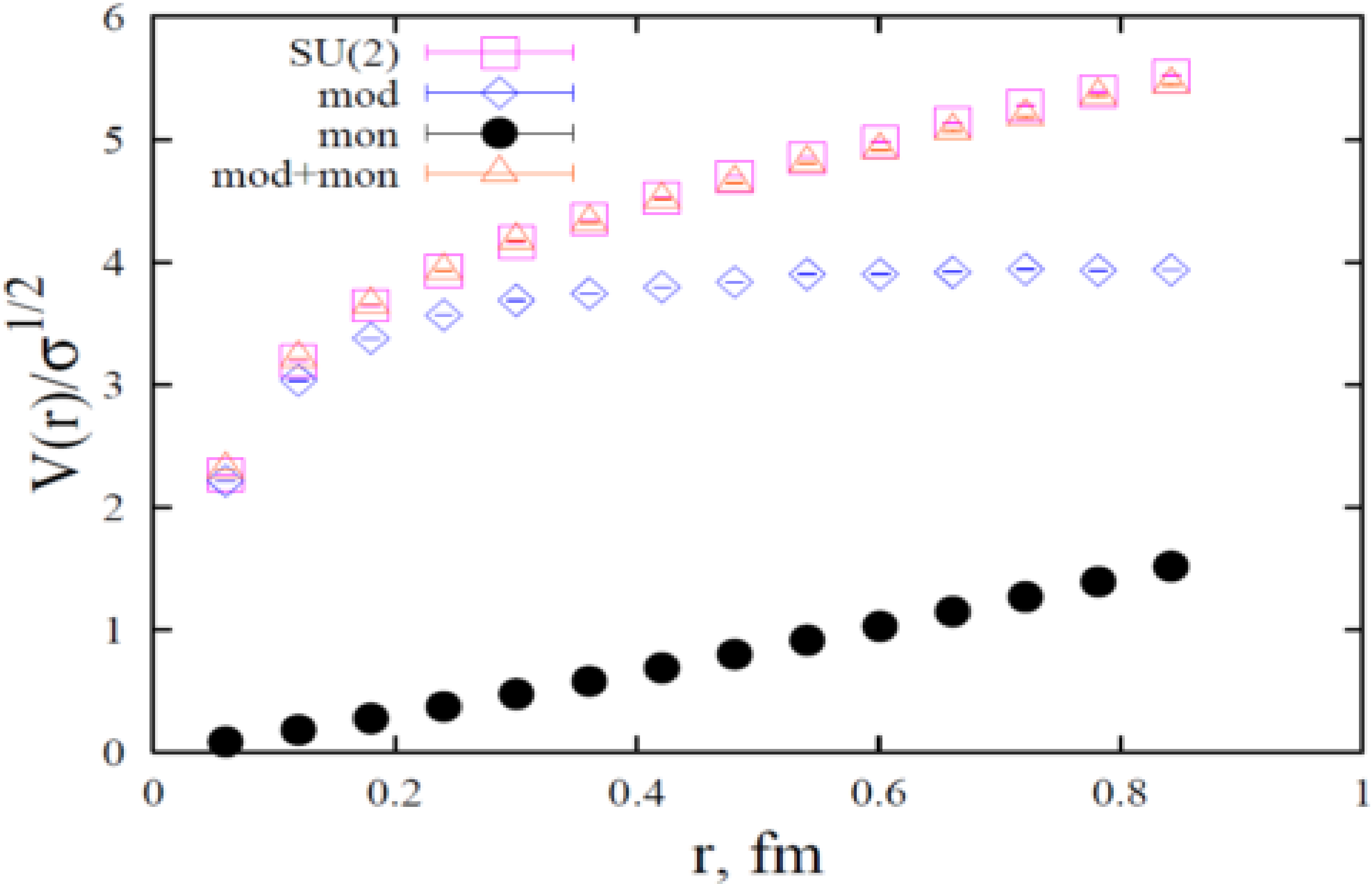
$$\sigma_{3Q} \approx \sigma_{3Q}^{abel}$$

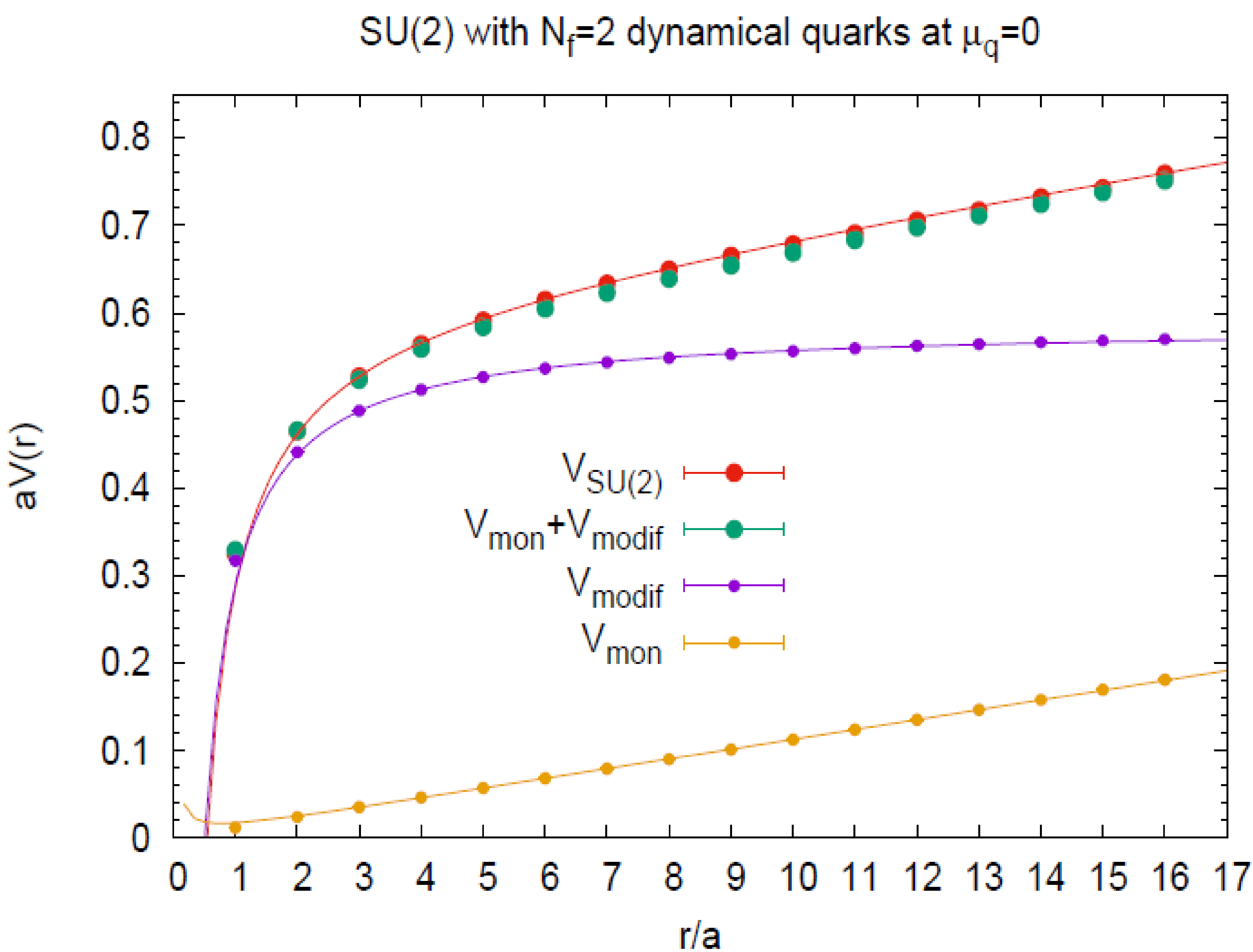


$V^{mon} + V^{mod}$ approximates the nonabelian static potential with high accuracy at all distances.

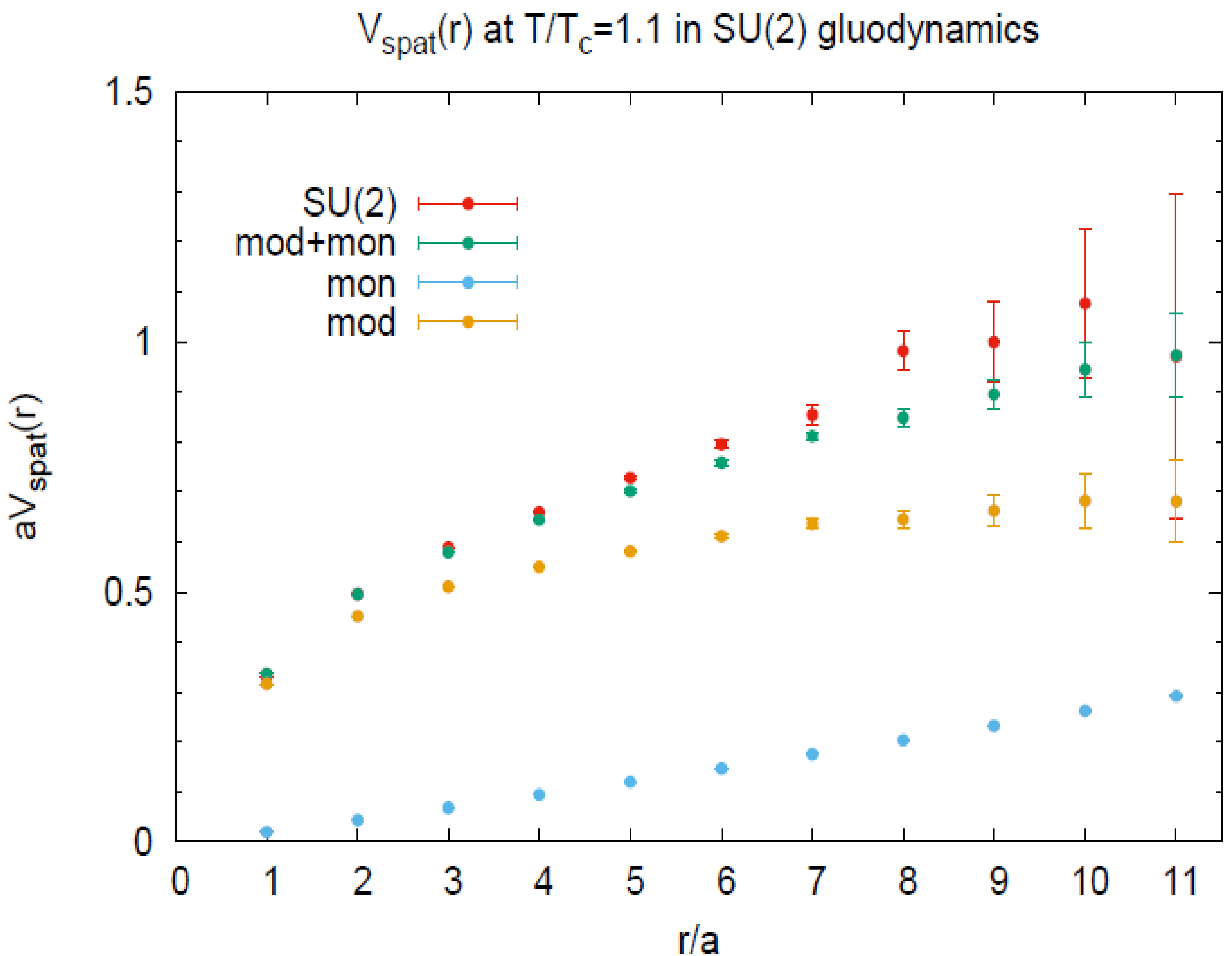
SU(2) gluodanamics, $24^4, a = 0.08$fm

VB, Polikarpov, Schierholz, Suzuki, Syritsyn





SU(2) with N_f=2 dynamical quarks at μ_q=0
aV(r)
r/a
$V_{SU(2)}$
$V_{mon}+V_{modif}$
V_{modif}
V_{mon}
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17



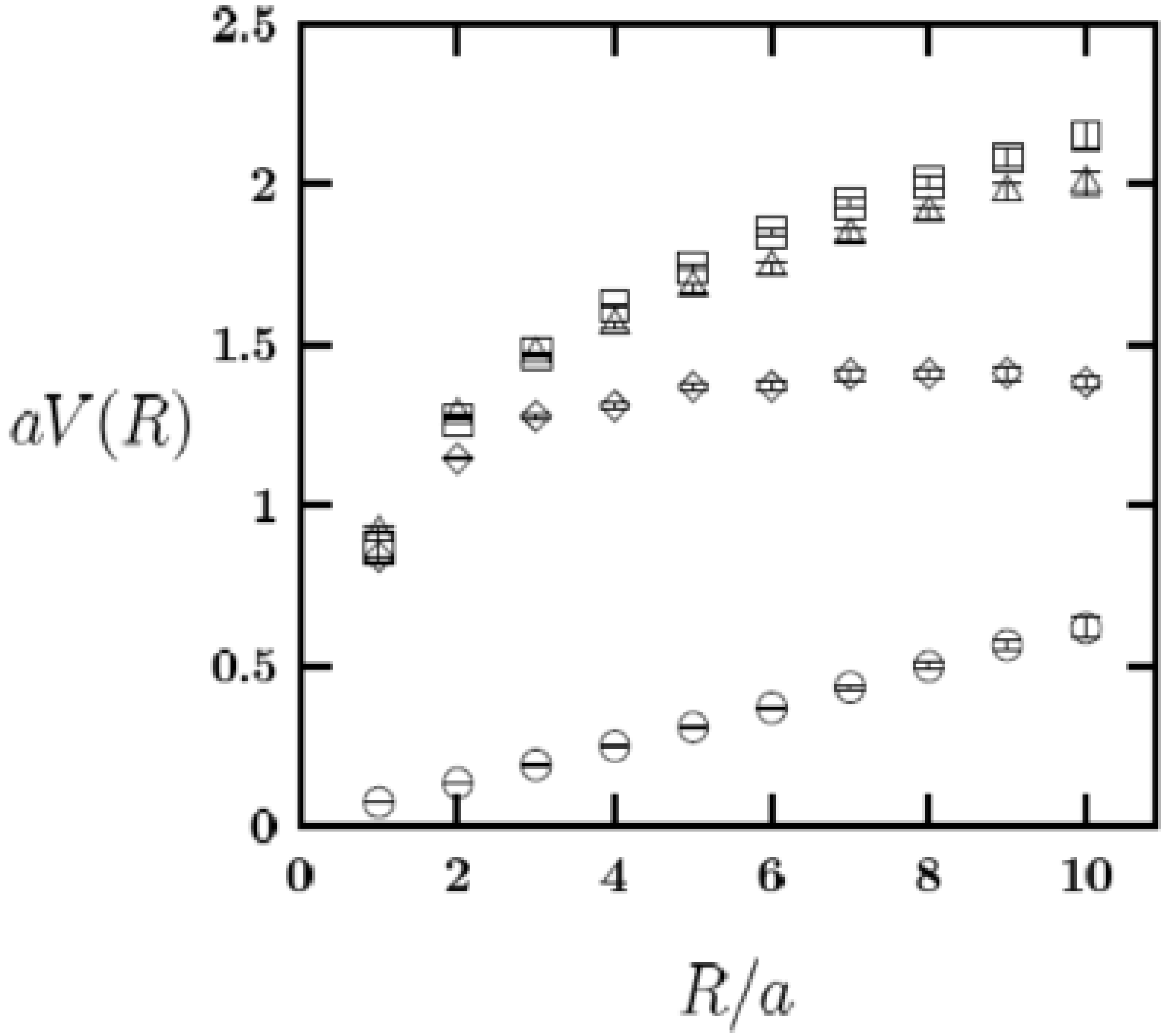
$V_{spat}(r)$ at T/T_c=1.1 in SU(2) gluodynamics
SU(2)
mod+mon
mon
mod
$aV_{spat}(r)$
r/a
0
0.5
1
1.5
0
1
2
3
4
5
6
7
8
9
10
11

Known problem:
in the adjoint representation

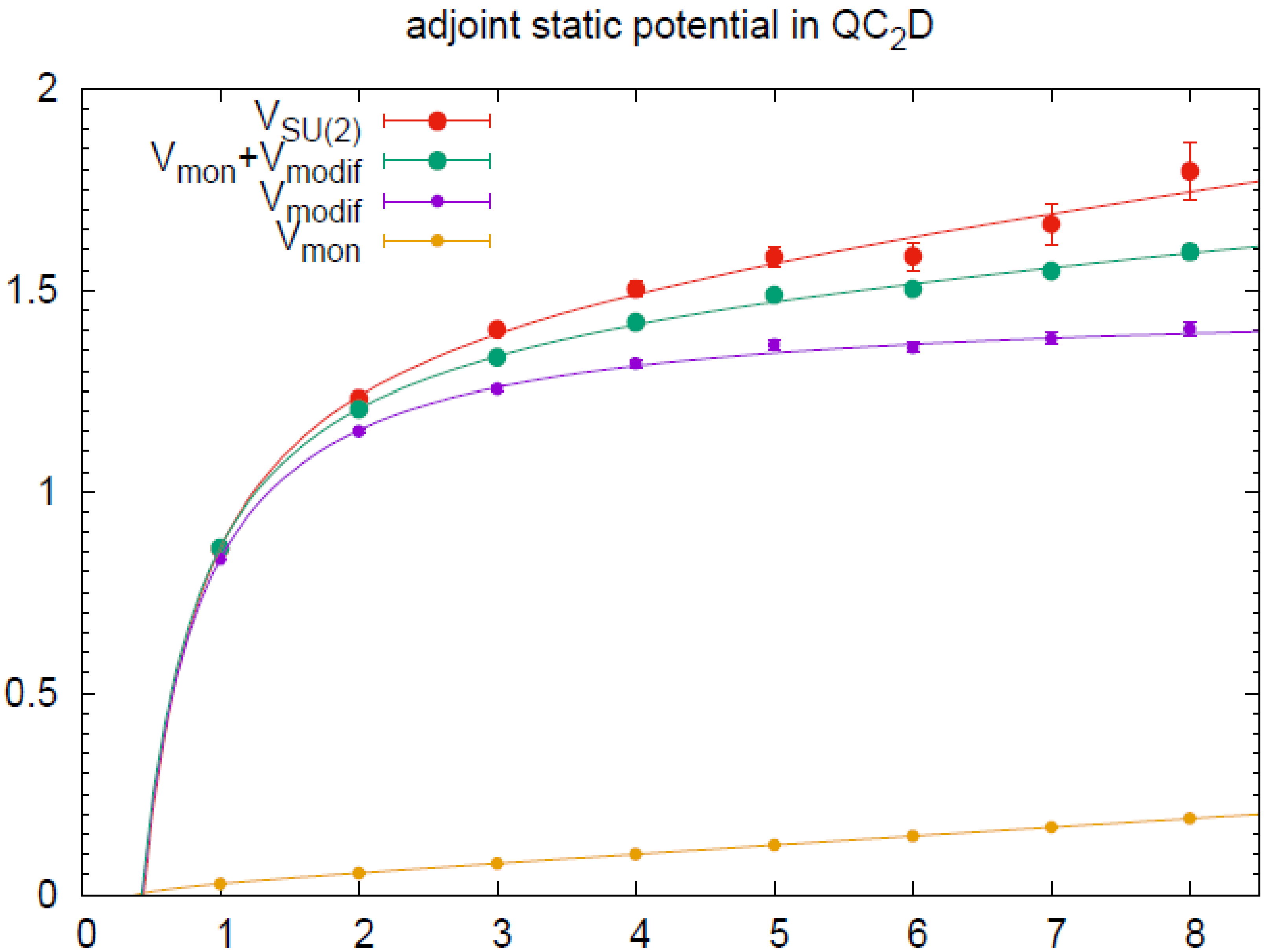
$$W(C)_{adj} \to 1 + W_{abel,2} + W_{abel,-2} = 1 + 2cos(\phi(C))$$

The abelian projected string tension $\sigma_{abel,adj} = 0$.
This is correct result, since asymptotic string tension $\sigma_{adj} = 0$
But this does not agree with the Casimir scaling at intermediate distances

Two conclusions:
off-diagonal gluons become relevant
abelian projection procedure should be modified

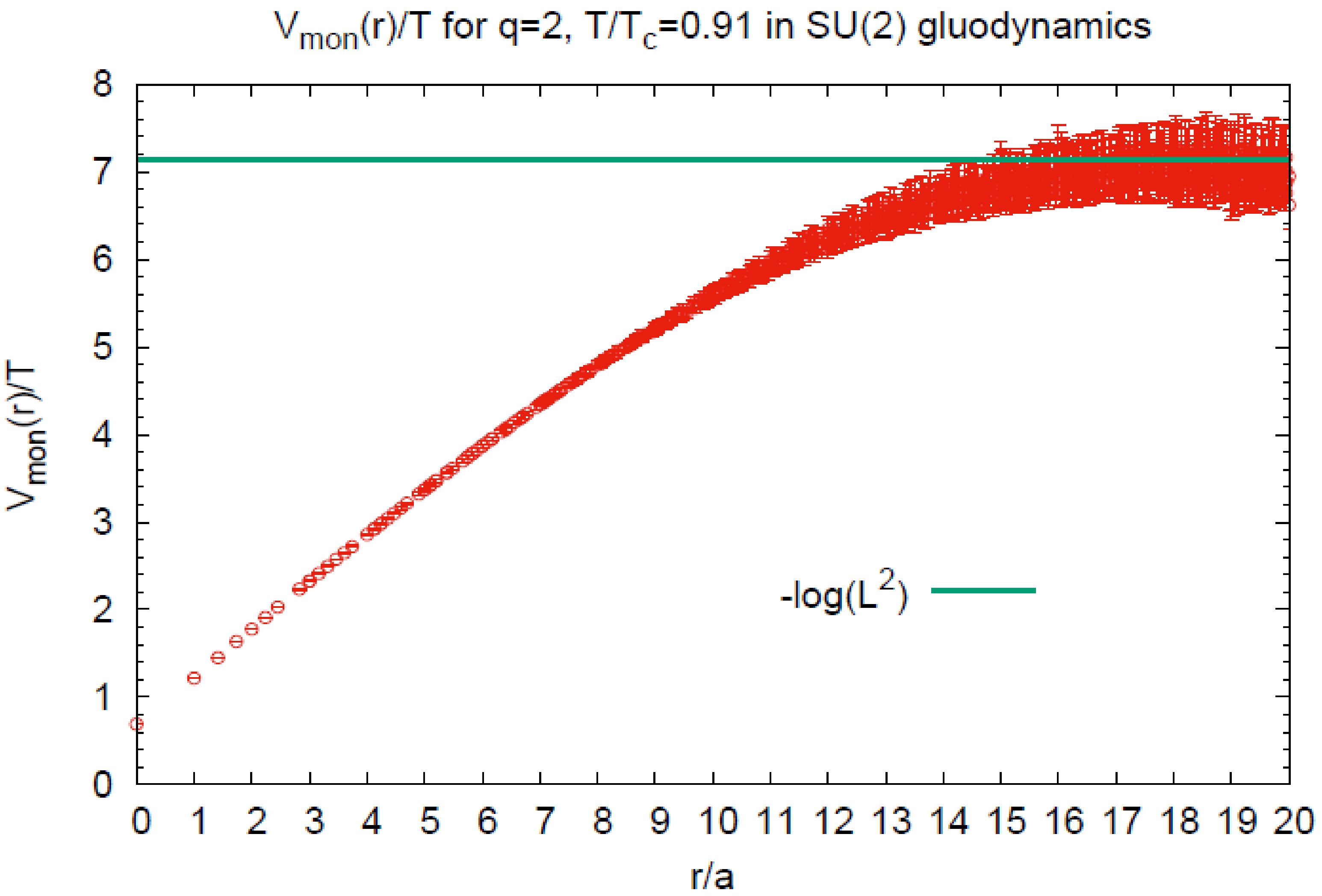


$V^{mon,2}(R)$, $V^{mod,adj}(R)$ and their sum in comparison with $V^{adj}(R)$



adjoint static potential in QC_2D
$V_{SU(2)}$
$V_{mon}+V_{modif}$
V_{modif}
V_{mon}
2
1.5
1
0.5
0
0
1
2
3
4
5
6
7
8

Screening of the q=2 monopole potential



Conclusions I

- DS scenario of confinement is supported by numerical evidence obtained in lattice simulations.

  There is no theoretical understanding of this ‘phenomenology’. Hope for future

- Decomposition of the static potential

$V(r) = V^{mon}(r) + V^{mod}(r)$ is one of such observations

- Solution to adjoint potential problem
- First observation of the screening of q=2 monopole potential

Deconfinement transition in QC_2D at T=0 and nonzero quark chemical potential

This part of my talk is based on

JHEP 03 (2018) 161

Braguta V., VB, Ilgenfritz M., Kotov A., Molochkov A., Nikolaev A.

I also use slides from the talk given by

Nikolaev A. at XQCD 2018

for SU(2) gauge group

$$\det\left[M(\mu_q)\right] = \det\left[(\tau_2 C\gamma_5)^{-1} M(\mu_q)(\tau_2 C\gamma_5)\right] = \\ = \det\left[M(\mu_q^*)\right]^*, \text{ where } C = \gamma_2\gamma_4$$

At real μ_q in QC_2D

$\det\left[M(\mu_q)\right]$ is real, $\det\left[M^\dagger(\mu_q)M(\mu_q)\right] > 0$ at $m_q \neq 0$.

Similarities

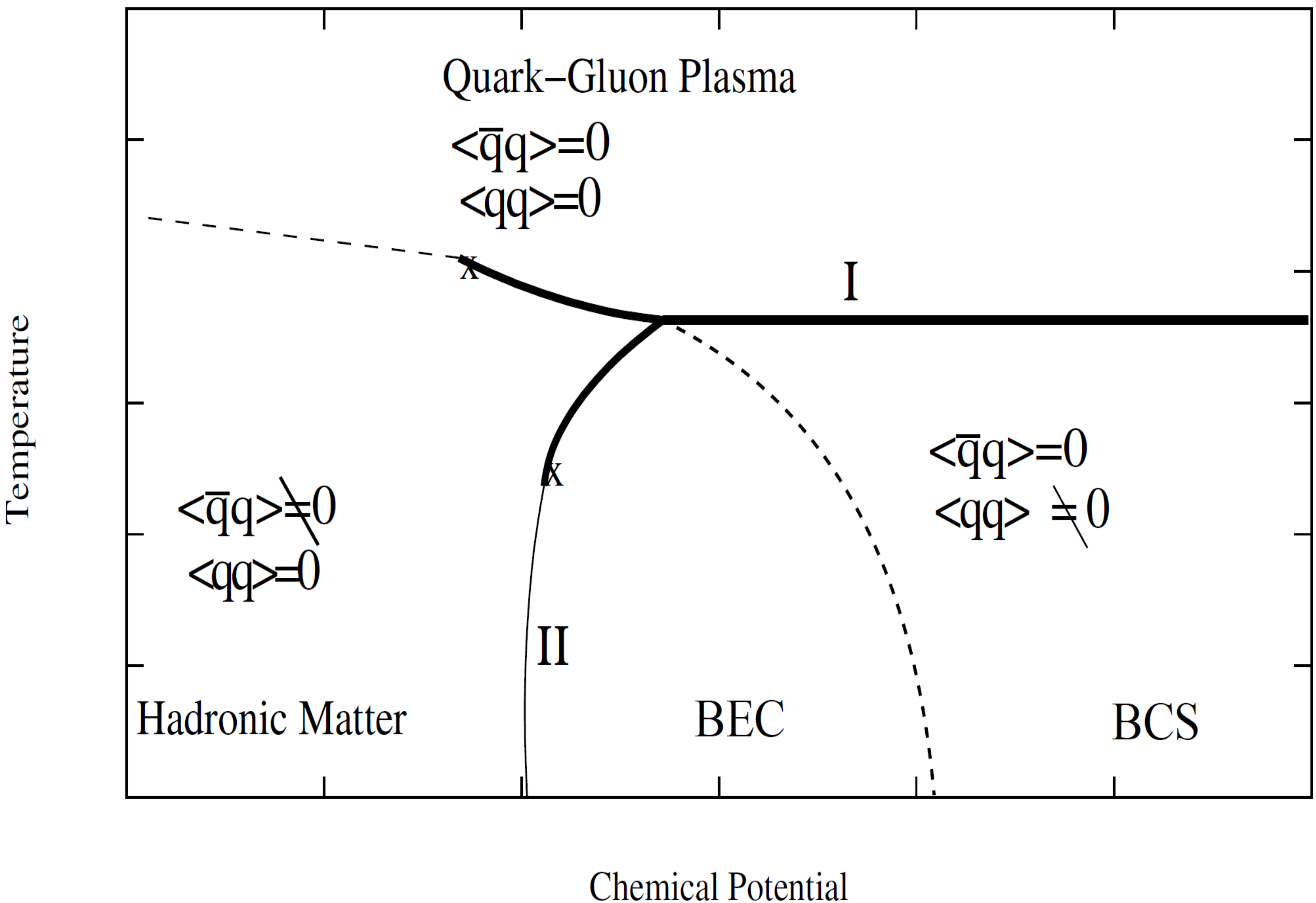
- Phase transitions: confinement/deconfinement, chiral symmetry restoration
- Some observables (normalized) are nearly equal in both theories:

  Topological susceptibility [B. Lucini *et. al.*, Nucl. Phys. B715 (2005) 461]:
  $\chi^{1/4}/\sqrt{\sigma} = 0.3928(40)\ (SU(2)), \quad \chi^{1/4}/\sqrt{\sigma} = 0.4001(35)\ (SU(3))$

  Critical temperature [B. Lucini *et. al.*, Phys. Lett. B712 (2012) 279]:
  $T_c/\sqrt{\sigma} = 0.7092(36)\ (SU(2)), \quad T_c/\sqrt{\sigma} = 0.6462(30)\ (SU(3))$

  Shear viscosity:
  $\eta/s = 0.134(57)\ (SU(2))$ [N.Yu. Astrakhantsev *et. al.*, JHEP 1509 (2015) 082]
  $\eta/s = 0.102(56)\ (SU(3))$ [H.B. Meyer, PRD 76 (2007) 101701]

- Mass spectrum (T. DeGrand, Y. Liu, PRD 94, 034506 (2016))
- Thermodynamical properties (M. Caselle *et. al.* JHEP 1205 (2012) 135)



Quark−Gluon Plasma
$\langle\bar{q}q\rangle=0$
$\langle qq\rangle=0$
I
x
x
$\langle\bar{q}q\rangle\neq0$
$\langle qq\rangle=0$
$\langle\bar{q}q\rangle=0$
$\langle qq\rangle\neq0$
II
Hadronic Matter
BEC
BCS
Temperature
Chemical Potential

SU(2) with $\mu_q > 0$ was first studied by A. Nakamura, Phys. Lett. B149, 391 (1984).

Later work:

J.B. Kogut *et. al.*, Nucl. Phys. **B582** (2000) 477–513

J.B. Kogut, D. Toublan, D.K. Sinclair, Nucl. Phys. **B642** (2002) 181–209

S. Cotter, P. Giudice, S. Hands, J.-I. Skullerud, PRD **87**, 034507 (2013)

T. Boz, S. Cotter, L. Fister, D. Mehta ,J.-I. Skullerud, EPJ **A49** (2013)

V.V. Braguta *et. al.*, PRD **94**, 114510 (2016) (our previous study)

We study $N_f = 2$ of rooted staggered fermions:

$$Z = \int DU\, det\left[M^\dagger(\mu_q)M(\mu_q) + \lambda^2\right]^{\frac{1}{4}} e^{-S_G^{impr.}[U]},$$

where $S_G^{impr.}[U]$ is the tree-level improved gauge action and

$$M_{xy}(\mu_q) = m_q a\delta_{xy} + \frac{1}{2}\sum_{\mu=1}^{4}\eta_\mu(x)\left[U_{x,\mu}\delta_{x+\hat{\mu},y}e^{\mu_q a\delta_{\mu,4}} - \right.$$

$$\left. -U^\dagger_{x-\hat{\mu},\mu}\delta_{x-\hat{\mu},y}e^{-\mu_q a\delta_{\mu,4}}\right].$$

Simulations settings:

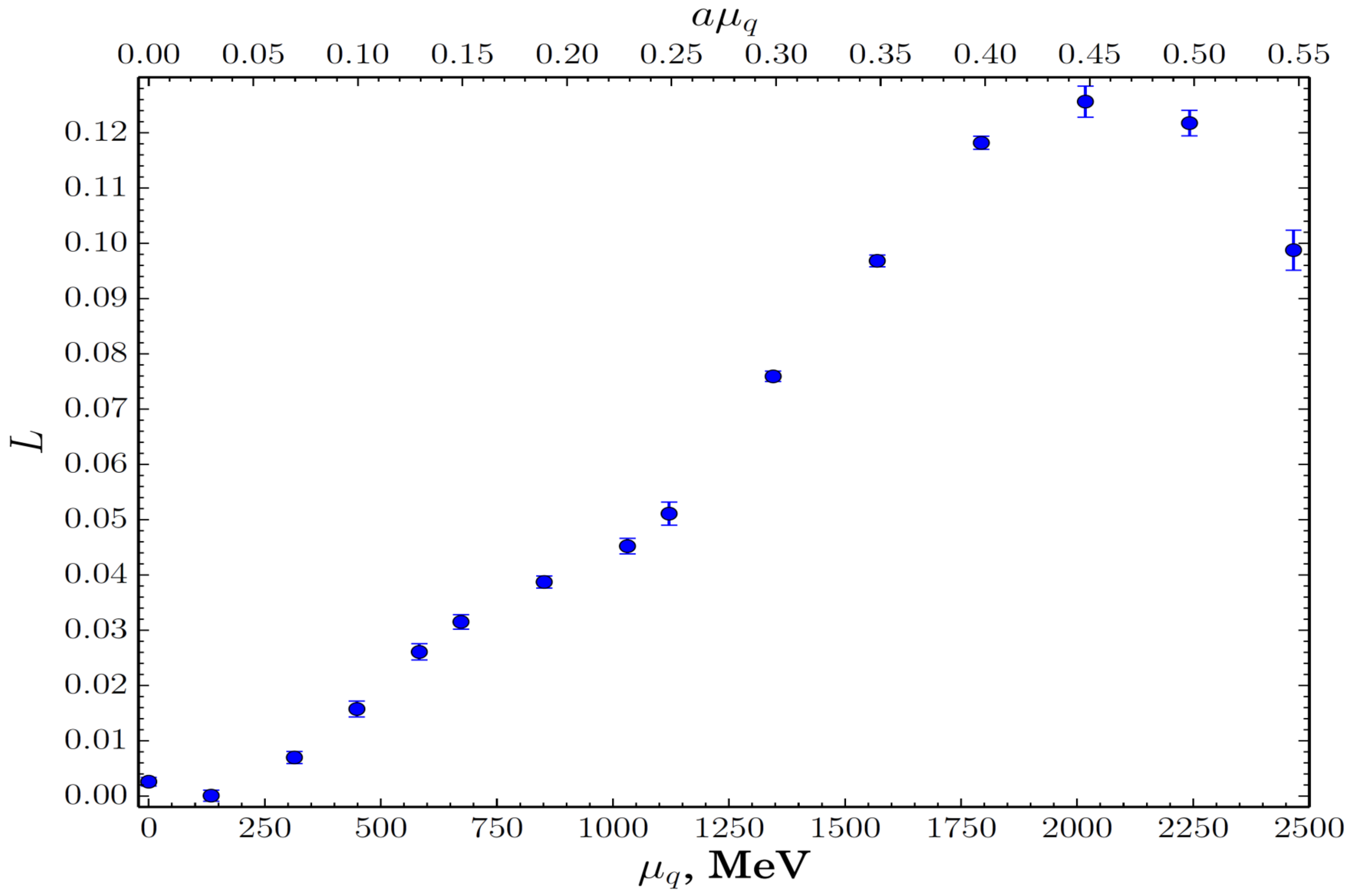
Lattice: 32^4 (T$= 0$)

$\beta = 1.8$, $a = 0.044(1)$ fm (Sommer parameter), $L_s \approx 1.4$ fm

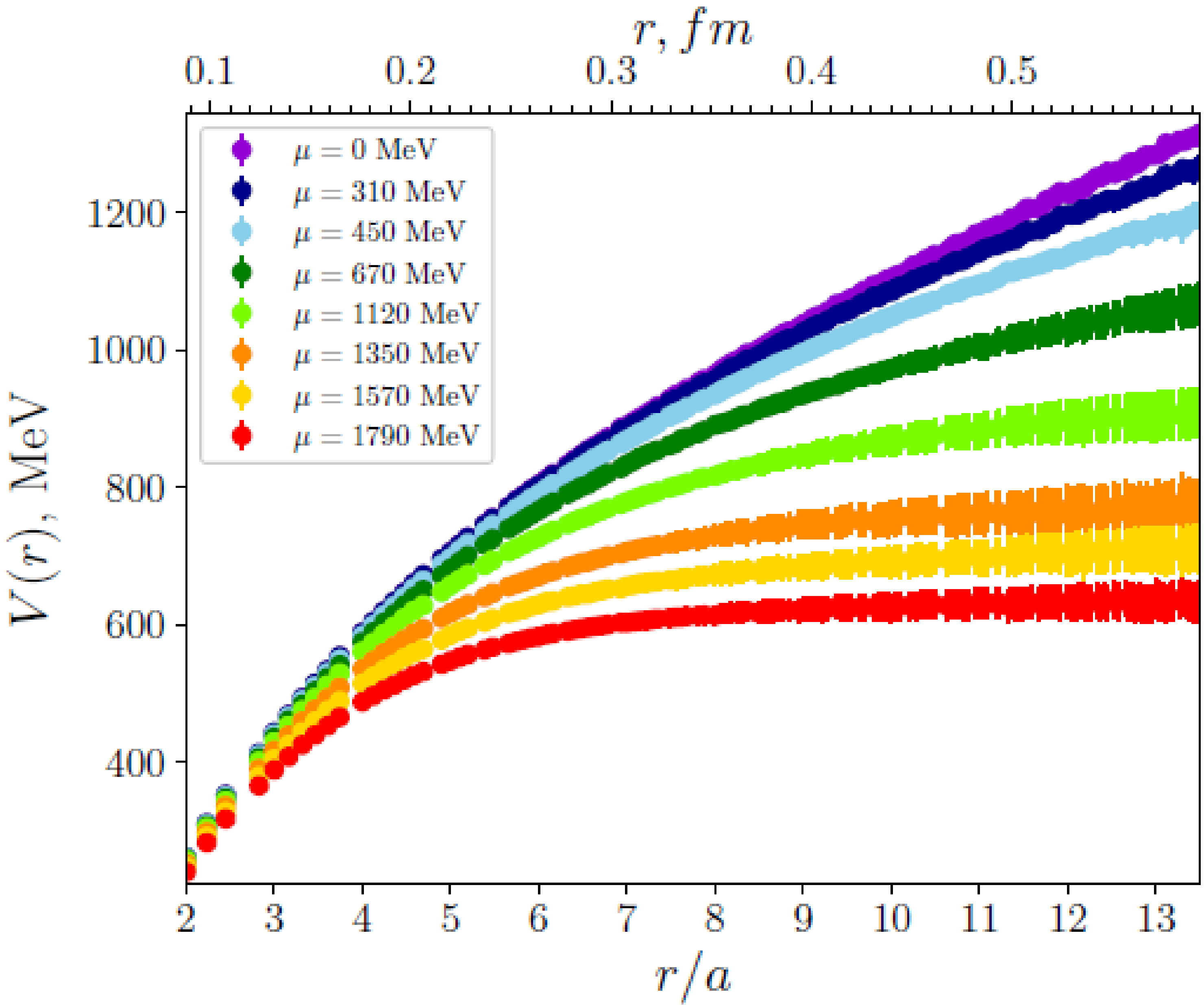
$ma = 0.0075$, $M_\pi = 740(40)$ MeV; $M_\pi L_s \approx 5$, $M_\pi / M_\rho \approx 0.55$

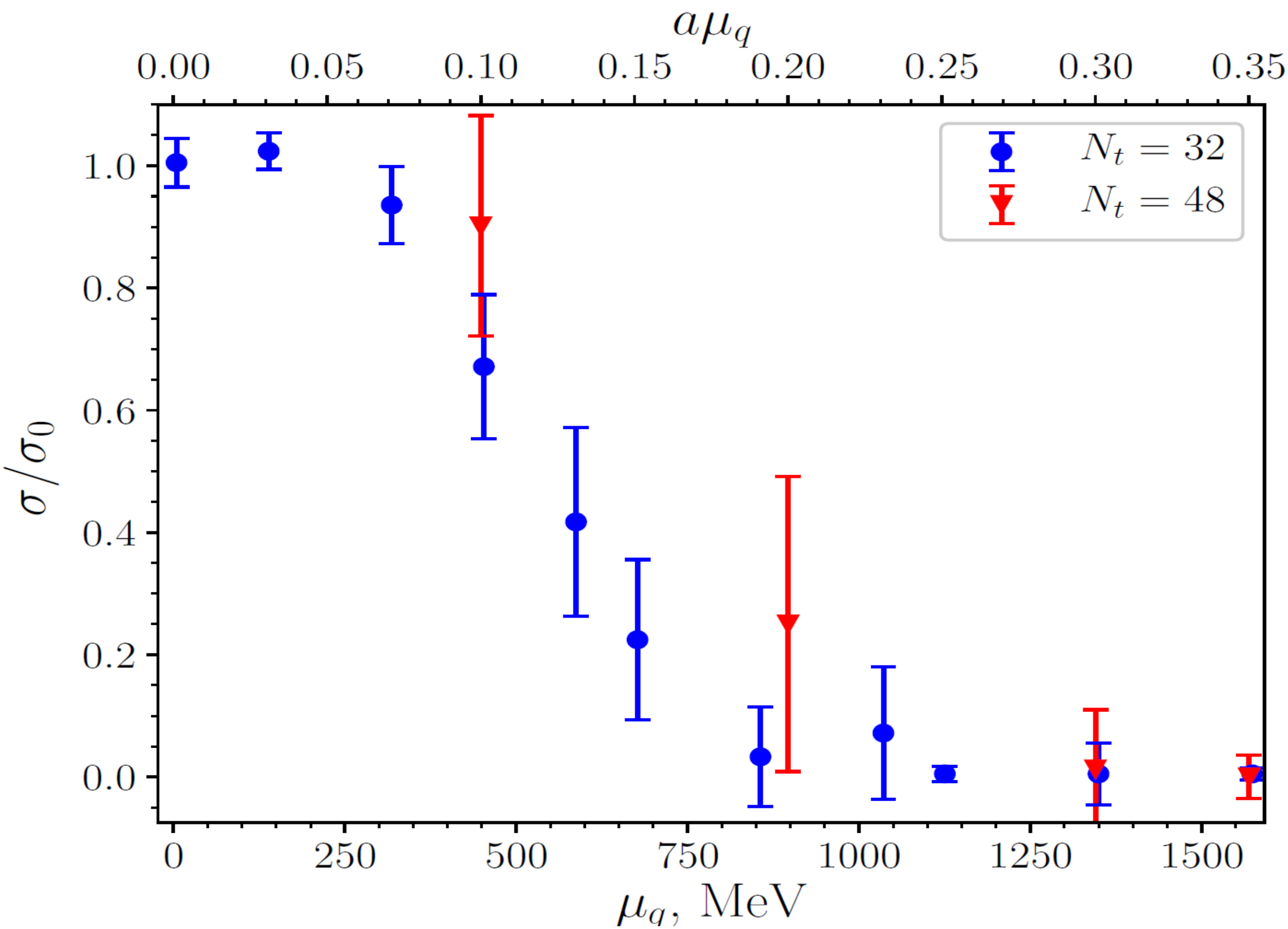
Fixed $\lambda = 0.00075$, $\lambda^2 << (ma)^2$

Polyakov loop L



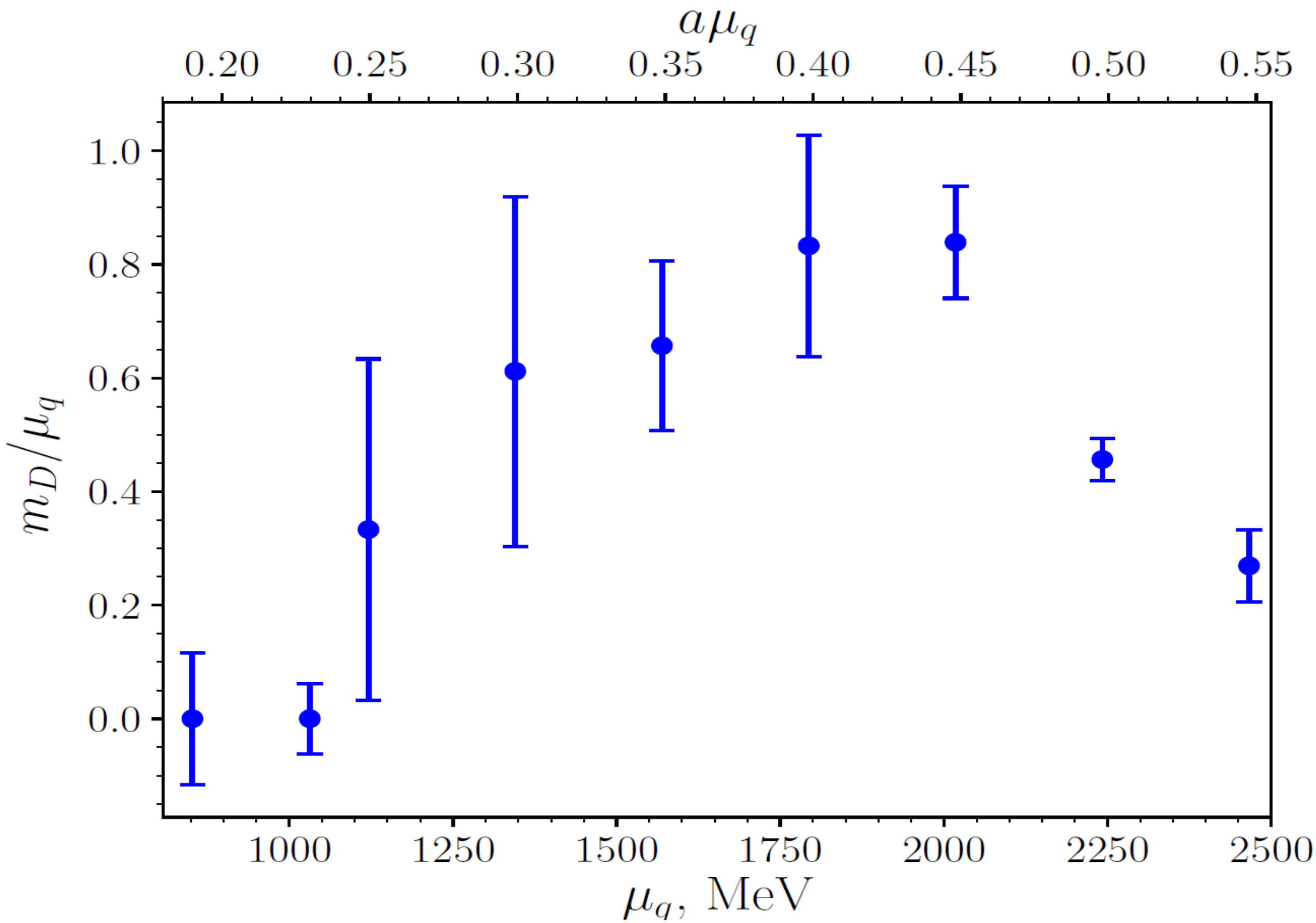
Static quarks potential





Deconfinement at $\mu_q > 900 - 1100$ MeV

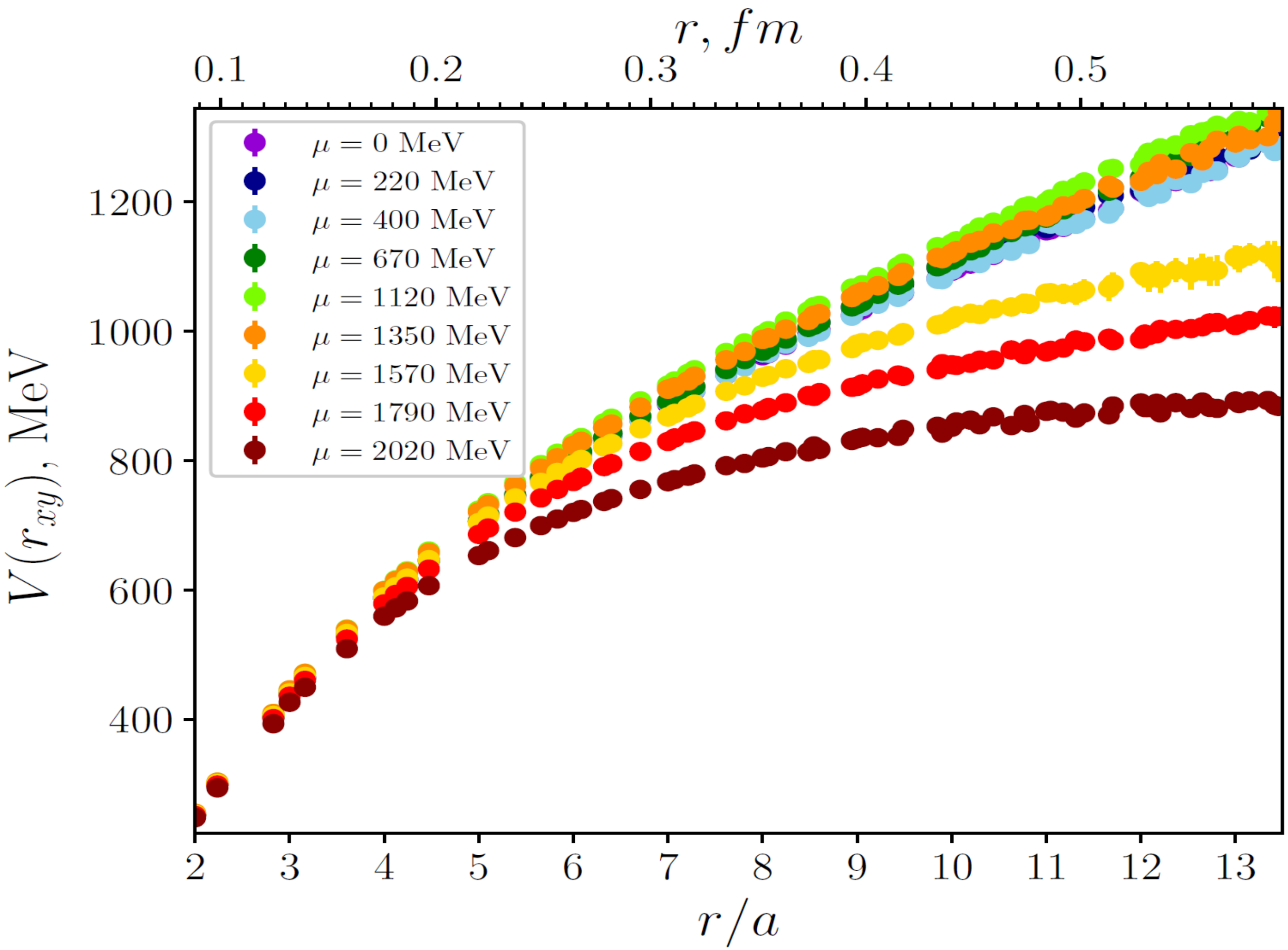
Good fit of V(r) by the Cornell potential at $\mu_q \leqslant 1100$ MeV



$V(r) = A - B\, e^{-m_D r}/r$, good fit at $\mu_q \geqslant 850$ MeV

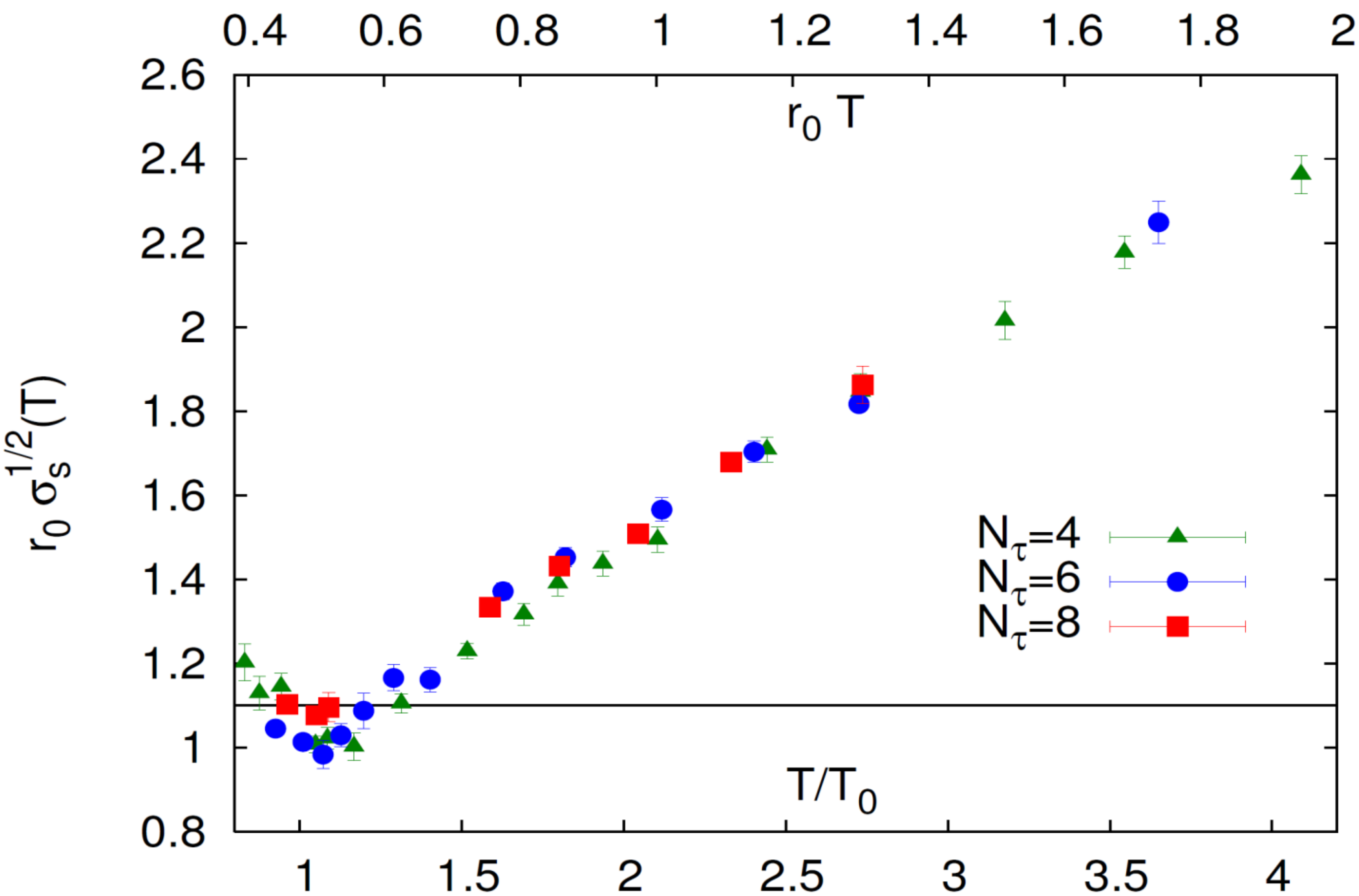
Debye mass rises with chemical potential

Spatial static potential

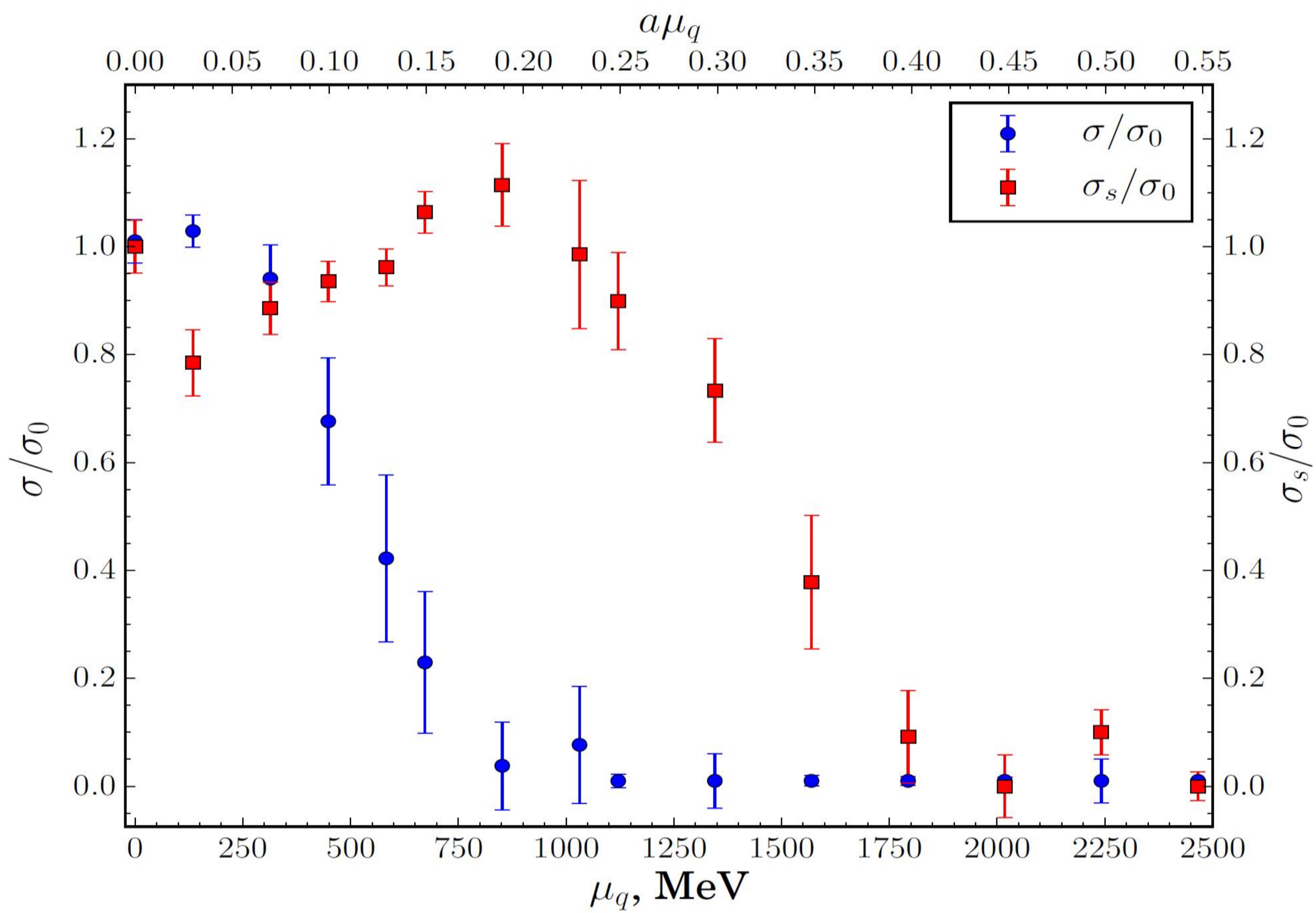


Different behavior compared to zero μ_q and finite T case

$\sigma_s \; at \; \mu = 0, T > 0$



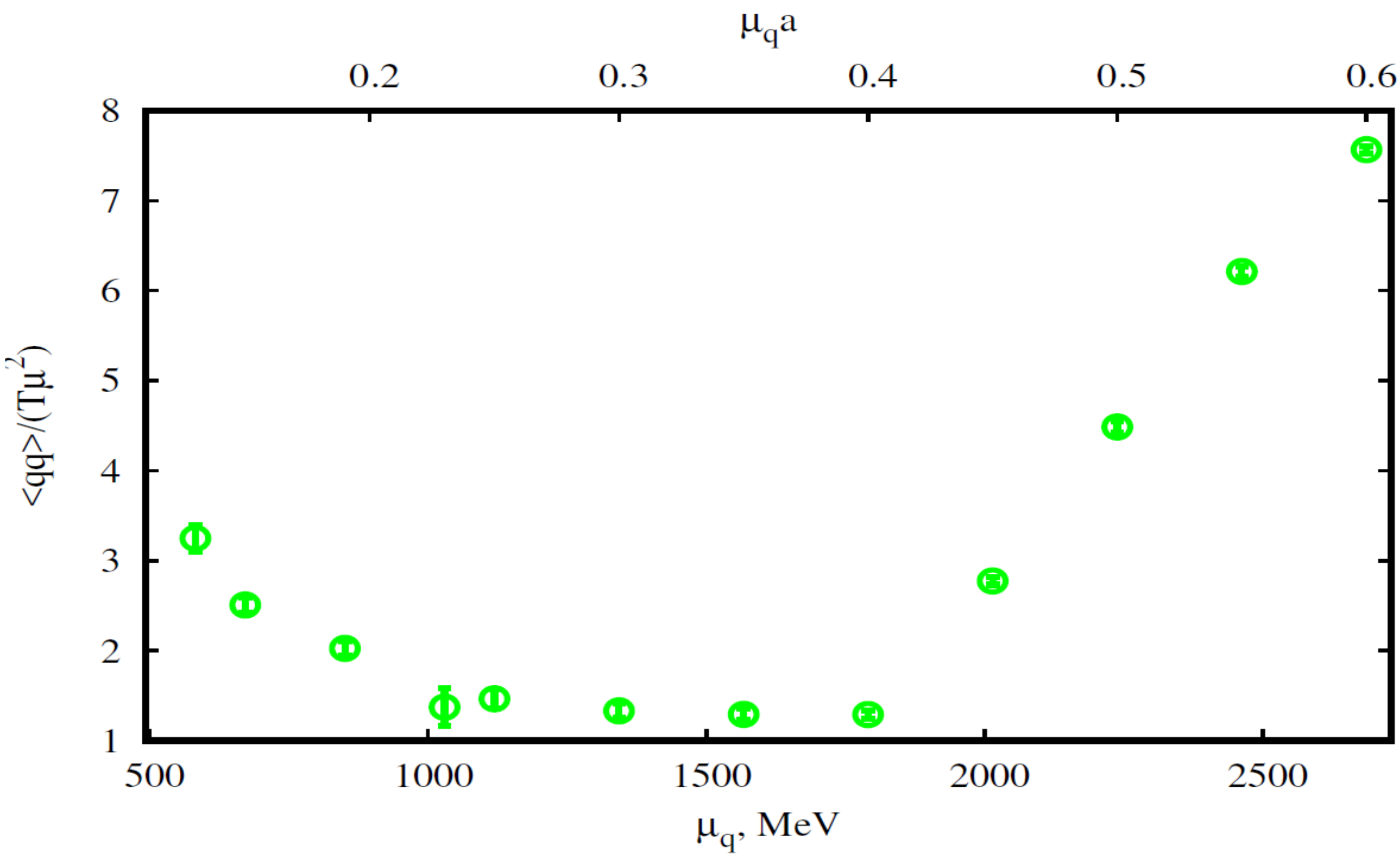
The Spatial String Tension and Dimensional Reduction in QCD, M. Cheng et al., PRD **78** (2008) 034506



σ goes to zero around $\mu_q = 1000$ MeV

σ_s goes to zero around $\mu_q = 2000$ MeV

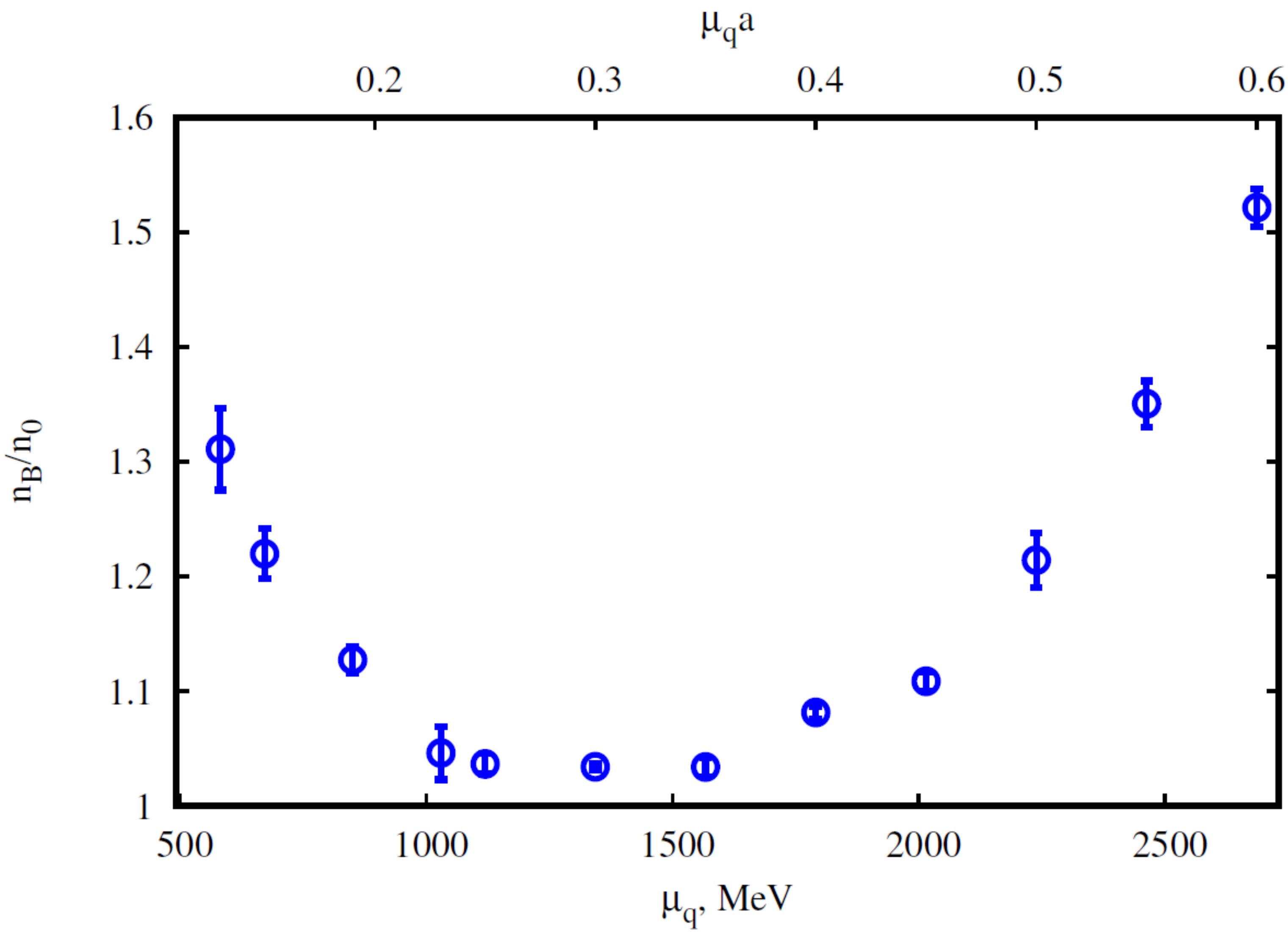
Diquark condensate

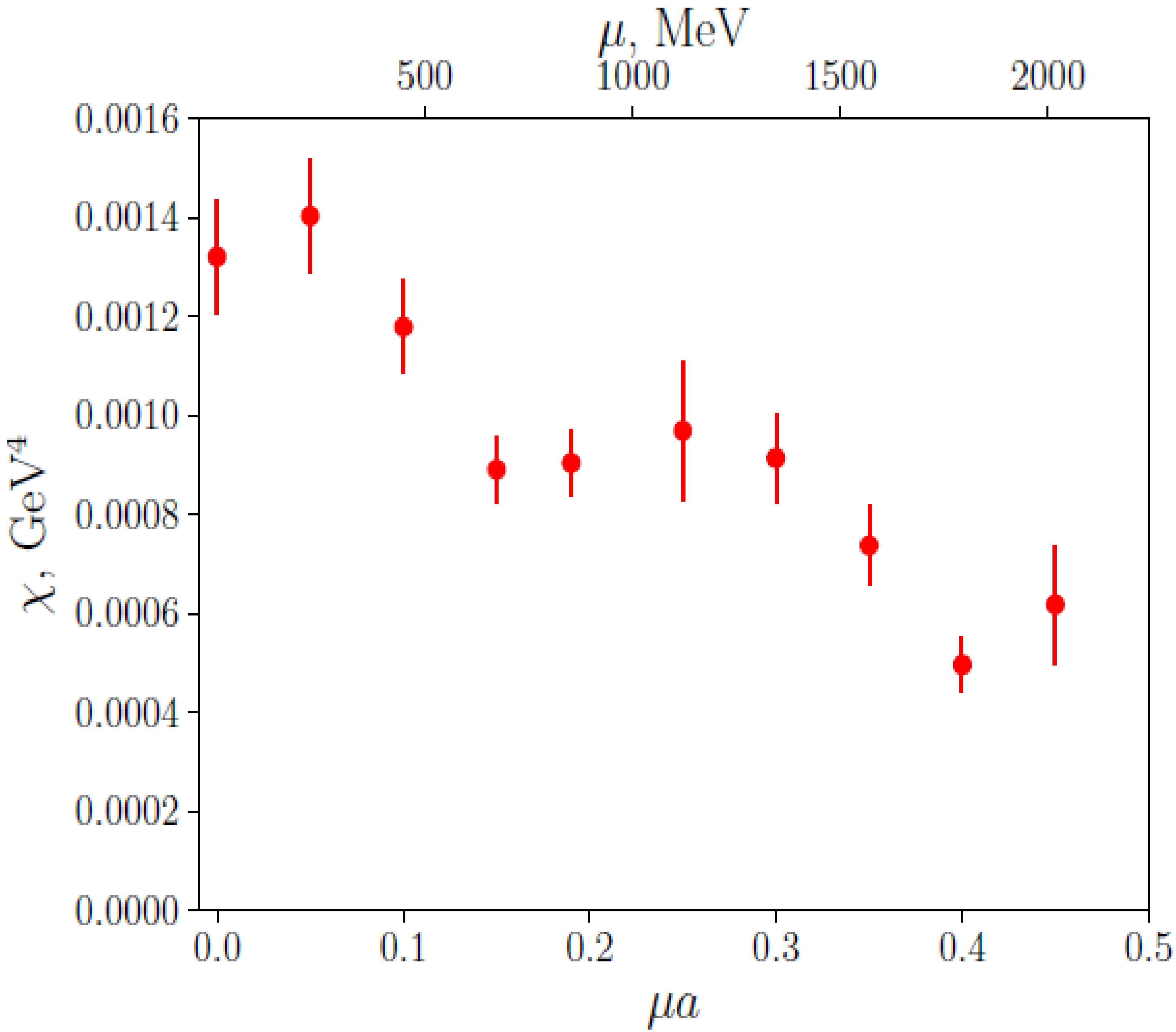


BCS phase for $\mu_q \geqslant 1000$ MeV

Diquark condensate rises for $\mu_q > 1800$ MeV

Number density





μ, MeV
500
1000
1500
2000
0.0016
0.0014
0.0012
0.0010
0.0008
0.0006
0.0004
0.0002
0.0000
χ, GeV^4
0.0
0.1
0.2
0.3
0.4
0.5
μa

Conclusions II

- Clear observation of transition to deconfinement at $T = 0$

  at μ_{qc}, determined by σ , between 850 and

  1000 MeV

- σ_s starts to decrease at $\mu_q \approx 1000$ MeV and

  becomes zero at about 2000 MeV

- Thus deconfinement at large density is different from deconfinement at large temperature

- There is no nonperturbative magnetic sector

New proposal for confinement mechanism

T. Suzuki arXiv:1402.1294

Suzuki, Ishiguro, VB, Phys.Rev. D97 (2018) no.3, 034501

Suzuki, Phys.Rev. D97 (2018) no.3, 034509

- confinement is due to violation of the non-Abelian Bianchi identities (VNABI)
- VNABI $J_\mu(x)$ are equal to Abelian-like monopole currents k_μ defined by the violation of the Abelian-like Bianchi identities.
- VNABI satisfies covariant conservation law $D_\mu J_\mu = 0$ and Abelian-like conservation law $\partial_\mu J_\mu = 0$
- There are $N^2 - 1$ conserved magnetic charges in SU(N) QCD.
- The charge of each component of VNABI is assumed to satisfy the Dirac quantization condition.

Each color component of the non-Abelian electric field E^a is squeezed by the corresponding color component of the solenoidal current J_μ^a.

No Abelian gauge fixing, no breaking global SU(n).

Numerical results in SU(2) gluodynamics were obtained supporting the gauge invariance of $< k_\mu^2 >$

Thus giving support to the proposal.