

Fluctuations and multiparticle correlations in heavy-ion collisions

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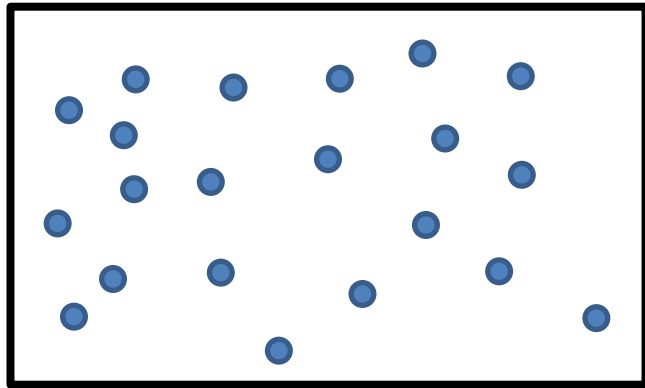
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Outline

- correlation, interaction
- cumulants and STAR data
- clusters
- two event classes
- long-range correlations
- rapidity repulsion
- summary

Poisson distribution



$$N = 10^{10}$$

$$p = 10^{-9}$$

$$\langle n \rangle = Np = 10$$



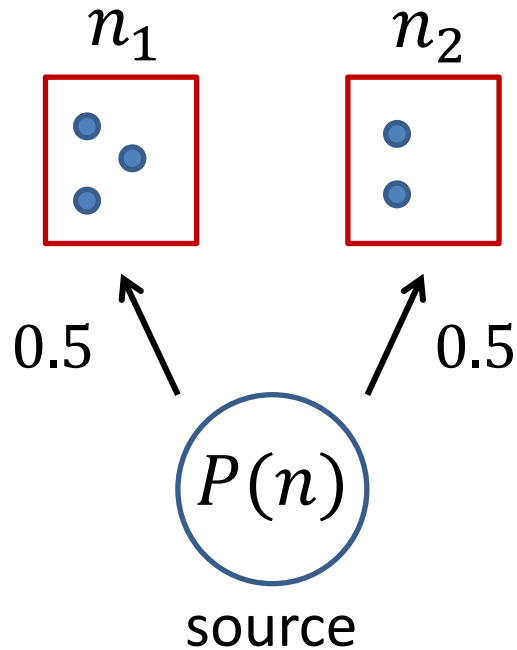
event # 1 ● ● ● ●

event # 2 ● ● ● ● ● ● ● ● ● ● ● ●

$P(n) = \text{Poisson}$ if $N \rightarrow \infty$, $p \rightarrow 0$, $Np = \langle n \rangle$

Such source (multiplicity distribution) is characterized by
All **factorial cumulants** $C_n = 0$, $n = 2, 3, \dots$ (“no correlations”)

In what sense “no correlations”?



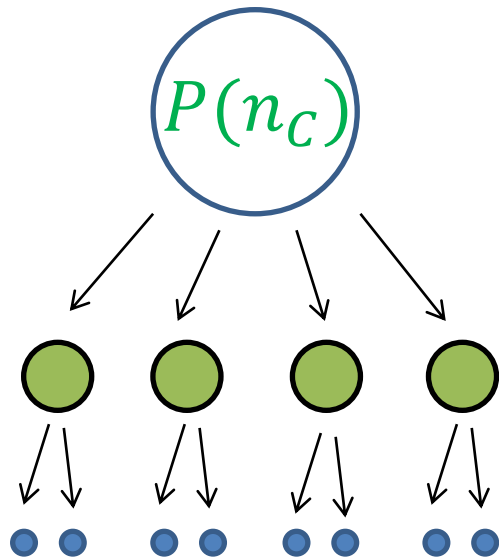
$$P(n_1, n_2) \stackrel{?}{=} P(n_1)P(n_2)$$

It is true for $P(n) = \text{Poisson}$ only
 fixed N
 finite N
 resonances
 volume fluctuation

$$P(n_1, n_2) = P(n) \frac{n!}{n_1! n_2!} \left(\frac{1}{2}\right)^{n_1} \left(\frac{1}{2}\right)^{n_2}$$

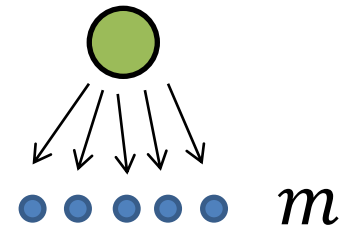
$$n = n_1 + n_2$$

Multiparticle correlations



Poisson

m particle cluster



$$C_2 \neq 0$$

$$C_k = 0, k > 2$$

$$C_{2,3,\dots,m} \neq 0$$

$$C_k = 0, k > m$$

factorial
cumulants

$$C_k = \frac{d^k}{dz^k} \ln \left(\sum_n P(n) z^n \right) \Big|_{z=1}$$

Two-particle correlation function

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \mathbf{C}_2(y_1, y_2)$$

Integrating both sides over some bin in rapidity

$$\langle n(n - 1) \rangle = \langle n \rangle^2 + \mathbf{C}_2$$

$$\langle n(n - 1) \rangle = \int \rho_2(y_1, y_2) dy_1 dy_2$$

$$\langle n \rangle = \int \rho(y) dy$$

factorial cumulant
(integrated correlation
function)

$$\mathbf{C}_2 = \int \mathbf{C}_2(y_1, y_2) dy_1 dy_2$$

Genuine three-particle correlation

$$\rho_3(y_1, y_2, y_3) = \rho(y_1)\rho(y_2)\rho(y_3) + \rho(y_1)\mathbf{C}_2(y_2, y_3) + \dots$$

three possibilities

$$+ \mathbf{C}_3(y_1, y_2, y_3)$$

Integrating both sides

$$\langle n(n-1)(n-2) \rangle = \langle n \rangle^3 + 3\langle n \rangle \mathbf{C}_2 + \mathbf{C}_3$$

factorial cumulant
(integrated correlation
function)

$$\mathbf{C}_3 = \int \mathbf{C}_3(y_1, y_2, y_3) dy_1 dy_2 dy_3$$

and analogously for higher-order correlation functions

Interaction

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \mathbf{C}_2(y_1, y_2)$$

$$\mathbf{C}_2 = \int \mathbf{C}_2(y_1, y_2) dy_1 dy_2$$

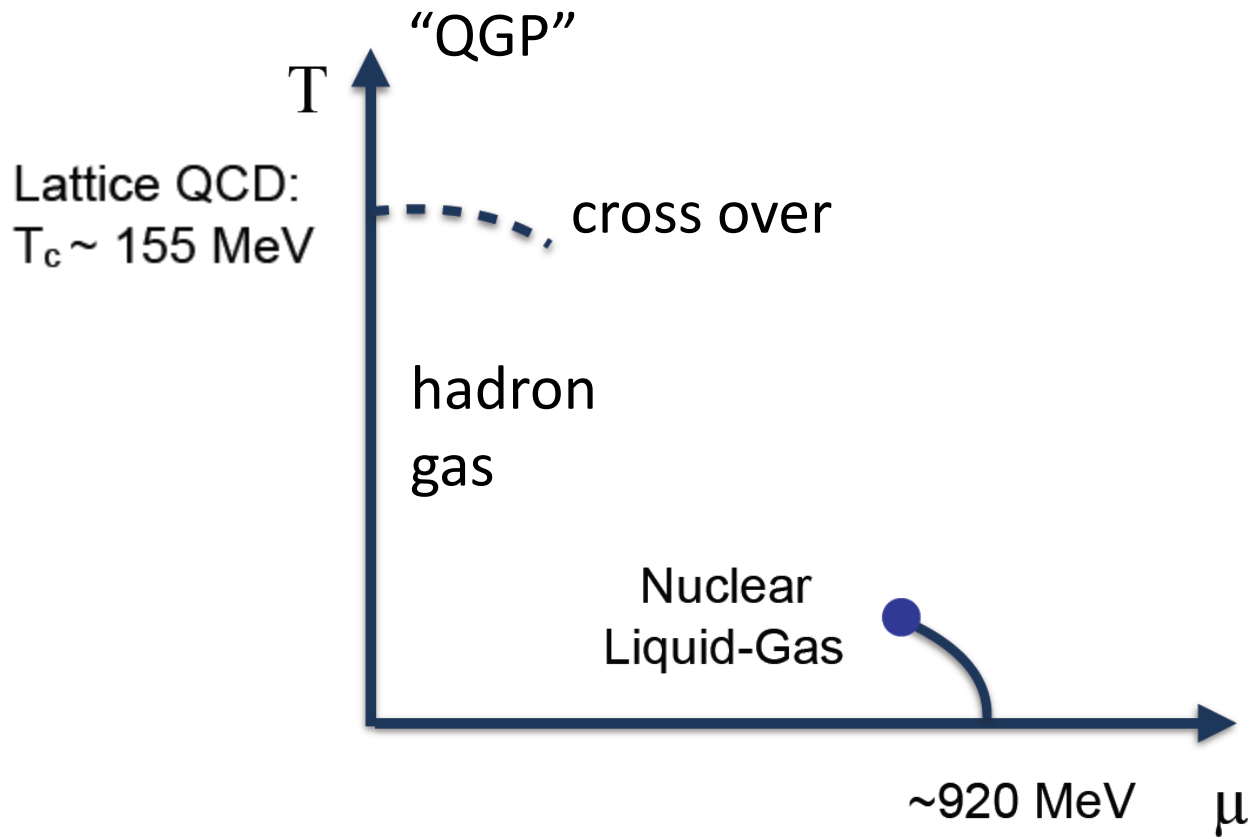
factorial cumulant
(integrated correlation
function)

For Poisson $\mathbf{C}_2 = 0$ but $\mathbf{C}_2(y_1, y_2)$ can have a non-trivial shape due to, e.g., interactions

For example (elliptic flow):

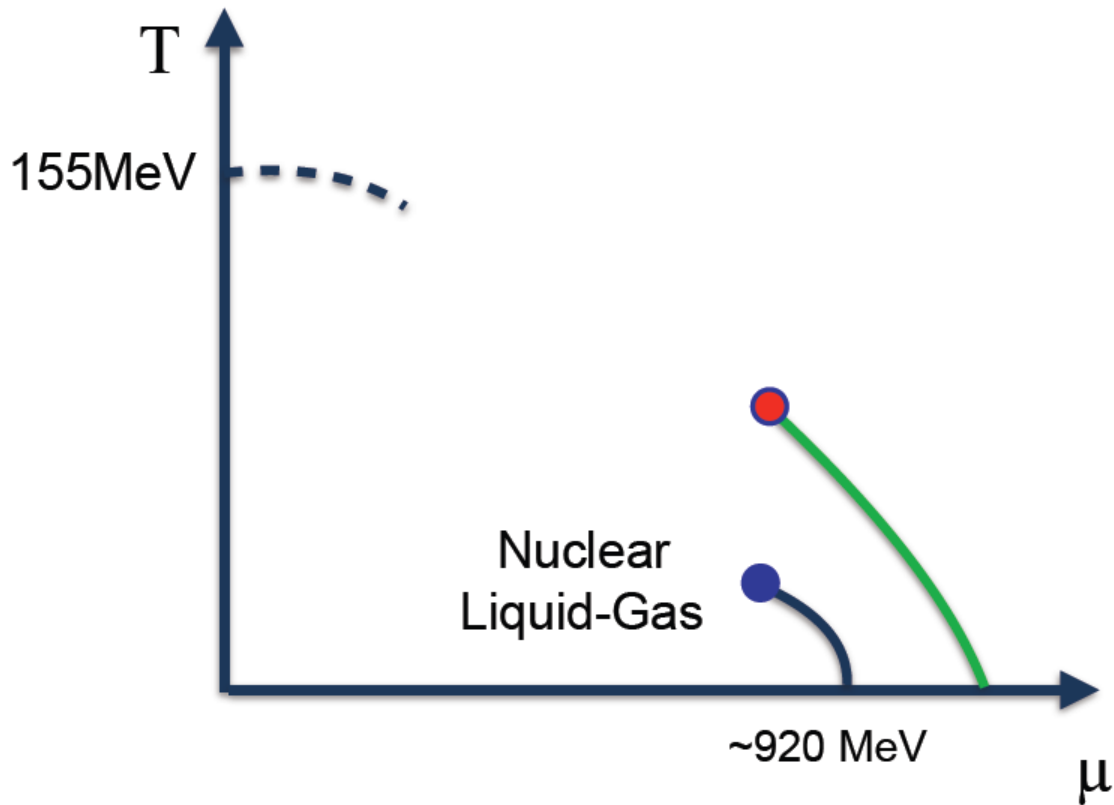
$$\mathbf{C}_2(\phi_1, \phi_2) \sim \cos(2\Delta\phi), \quad \Delta\phi = \phi_1 - \phi_2$$

What we know about the QCD phase diagram



The rest is everybody's guess.

Usual expectation based on various effective models



On the experimental side all we can do is to measure various fluctuation observables and hope to see some nontrivial energy or/and system-size dependence

see, e.g.,

Stephanov, Rajagopal, Shuryak, PRL (1998)

Stephanov, PRL (2009)

Skokov, Friman, Redlich, PRC (2011)

There are some intriguing results:

STAR, HADES

Higher order cumulants

Proton v_1 (STAR)

HBT radii (STAR)

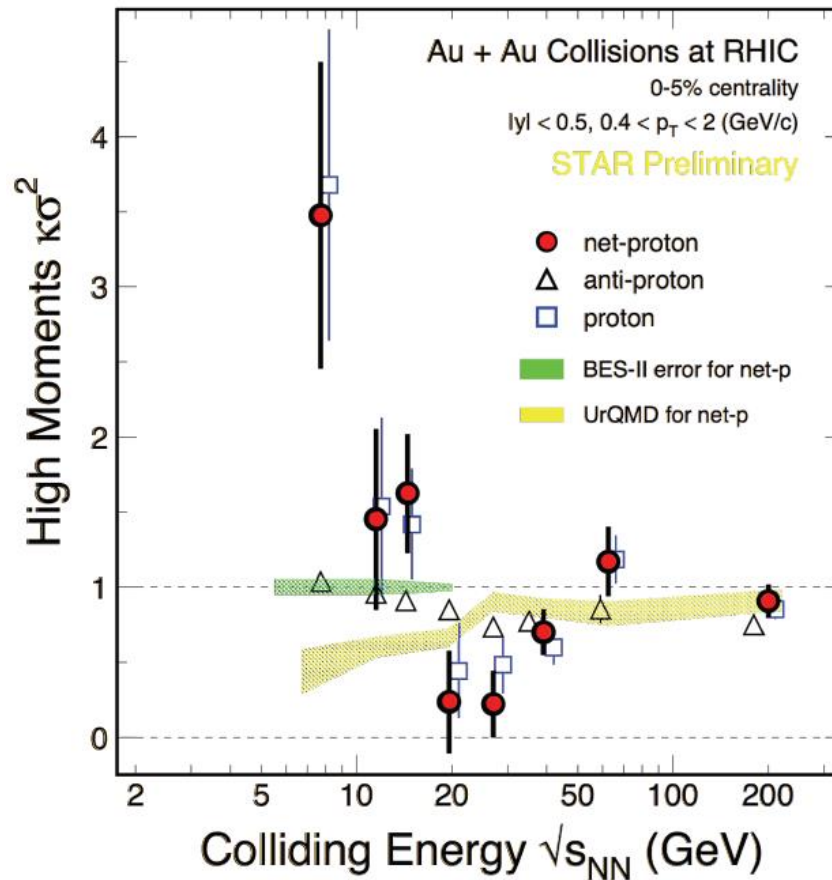
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Intermittency in the transverse momentum phase space

Strongly intensive variables

Preliminary STAR data

X.Luo, N.Xu, 1701.02105



my notation

$$K_4/K_2$$

cumulants

$$K_i = \frac{d^i}{dt^i} \ln \left(\sum_n P(n) e^{tn} \right) \Big|_{t=0}$$

Factorial cumulants vs cumulants

factorial
cumulant

$$C_i = \frac{d^i}{dz^i} \ln \left(\sum_n P(n) z^n \right) \Big|_{z=1}$$

cumulant

$$K_i = \frac{d^i}{dt^i} \ln \left(\sum_n P(n) e^{tn} \right) \Big|_{t=0}$$

cumulants naturally appear
in statistical physics

Poisson:

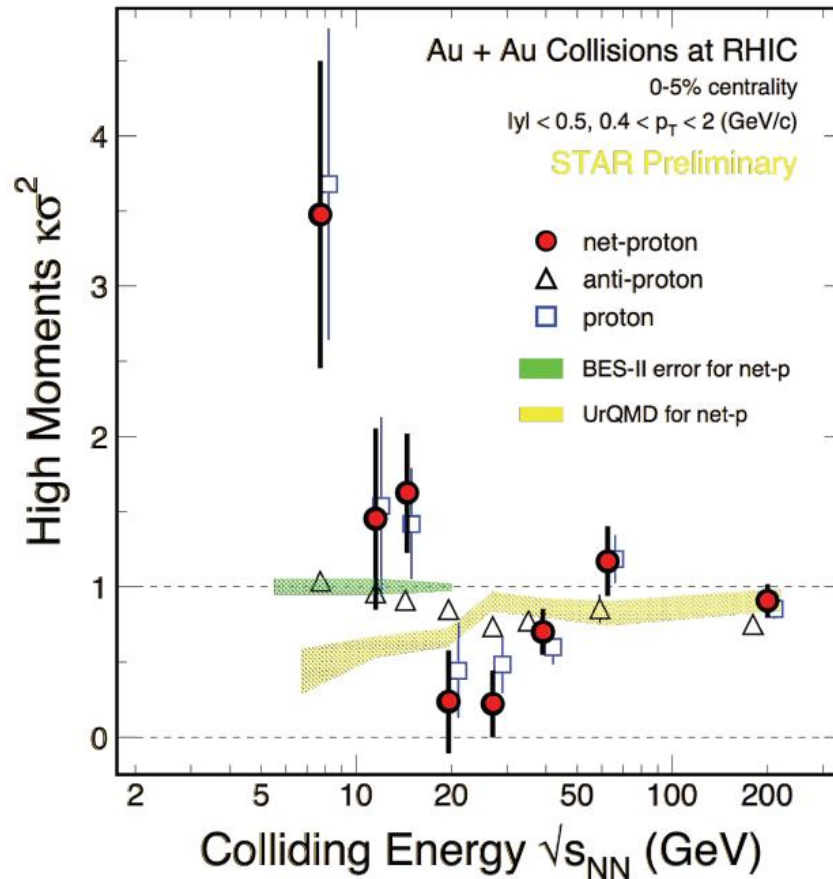
$$C_i = 0$$

$$K_i = \langle n \rangle$$

$$Z = \sum_i e^{-\beta(E_i - \mu N_i)}$$

Preliminary STAR data

X.Luo, N.Xu, 1701.02105



my notation

$$K_4/K_2$$

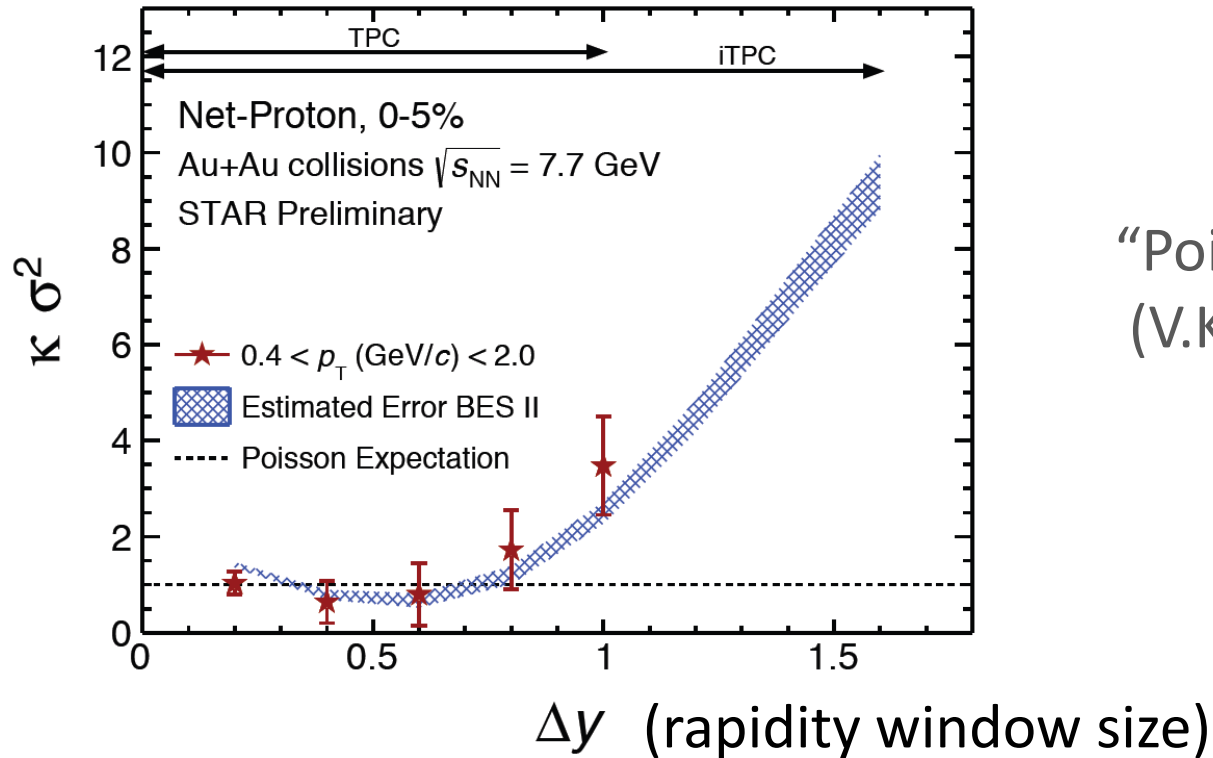
Is proton signal at 7.7 GeV large?

Is antiproton signal at 7.7 GeV small?

Can we and how to directly compare different energies?

Preliminary STAR data at 7.7 GeV

X.Luo, N.Xu, 1701.02105



“Poissonizer” ?
(V.Koch)

$$-(\Delta y)/2 < y < (\Delta y)/2$$

Is this dependence expected?

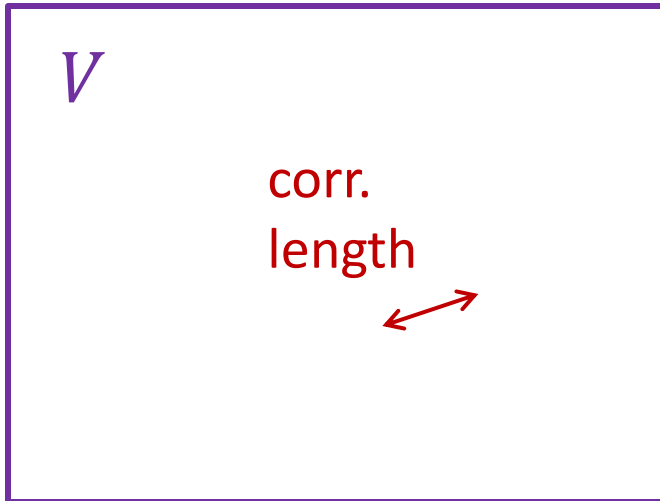
Is it somehow related to the QCD phase diagram?

General remarks:

“Cumulant ratios do not depend on volume” but depend on volume fluctuation

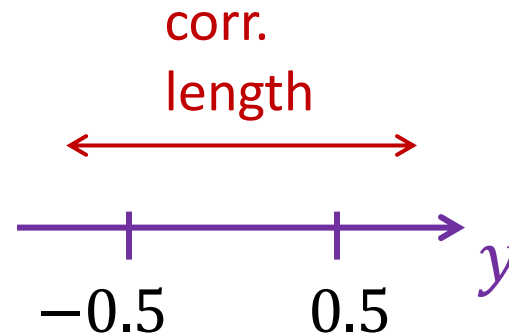
It is true if a correlation length is much smaller than the system size

real coordinate space



Here this condition is satisfied

momentum rapidity space



Correlation length is usually larger than one unit of rapidity.

Cumulant ratios are expected to depend on acceptance in rapidity

Cumulants are not optimal

$$K_2 = \langle (\delta N)^2 \rangle \quad \delta N = N - \langle N \rangle \quad N - \text{number of protons}$$

$$K_3 = \langle (\delta N)^3 \rangle$$

we neglect anti-protons,
good at low energies

$$K_4 = \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2$$

$$K_i = \langle N \rangle + \textit{physics}[2, \dots, i]$$

physics = two-, three-, n -particle
factorial cumulants

for Poisson distribution $K_i = \langle N \rangle$, (*physics* = 0)

We have

$$K_2 = \langle N \rangle + \mathbf{C}_2$$

$$K_3 = \langle N \rangle + 3\mathbf{C}_2 + \mathbf{C}_3$$

$$K_4 = \langle N \rangle + 7\mathbf{C}_2 + 6\mathbf{C}_3 + \mathbf{C}_4$$

cumulants mix integ.
correlation functions
of different orders

$$K_5 = \langle N \rangle + 15C_2 + 25C_3 + 10C_4 + C_5$$

$$K_6 = \langle N \rangle + 31C_2 + 90C_3 + 65C_4 + 15C_5 + C_6$$

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \mathbf{C}_2(y_1, y_2)$$

$$\mathbf{C}_2 = \int \mathbf{C}_2(y_1, y_2) dy_1 dy_2 \quad \text{factorial cumulant}$$

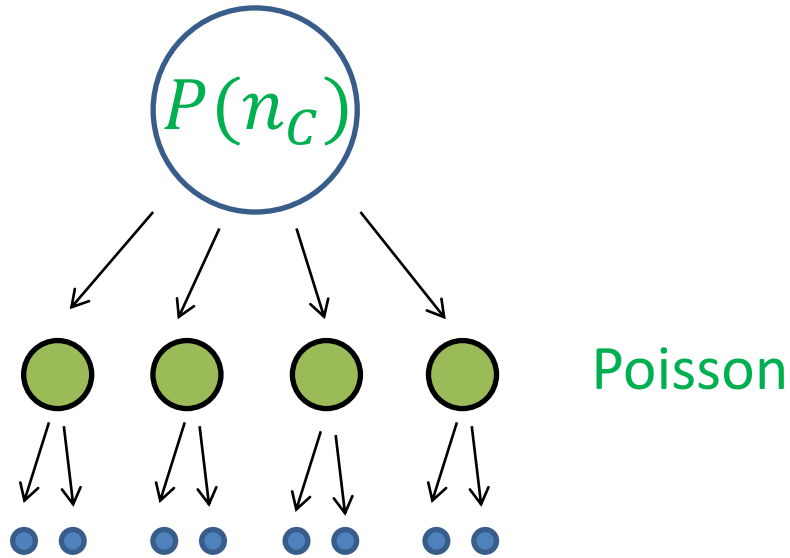
See, e.g.,

B. Ling, M. Stephanov, PRC 93 (2016) 034915

AB, V. Koch, N. Strodthoff, PRC 95 (2017) 054906

AB, V.Koch, D.Oliinychenko, J.Steinheimer, 1804.04463 (K_5 and K_6)

Suppose we have a system with two-particle clusters only



In this case all information is contained in $\langle n \rangle$ and K_2 .
No point to measure $K_{3,4,\dots}$

$$C_2 = 2\langle n_C \rangle \quad C_{3,4,\dots} = 0$$

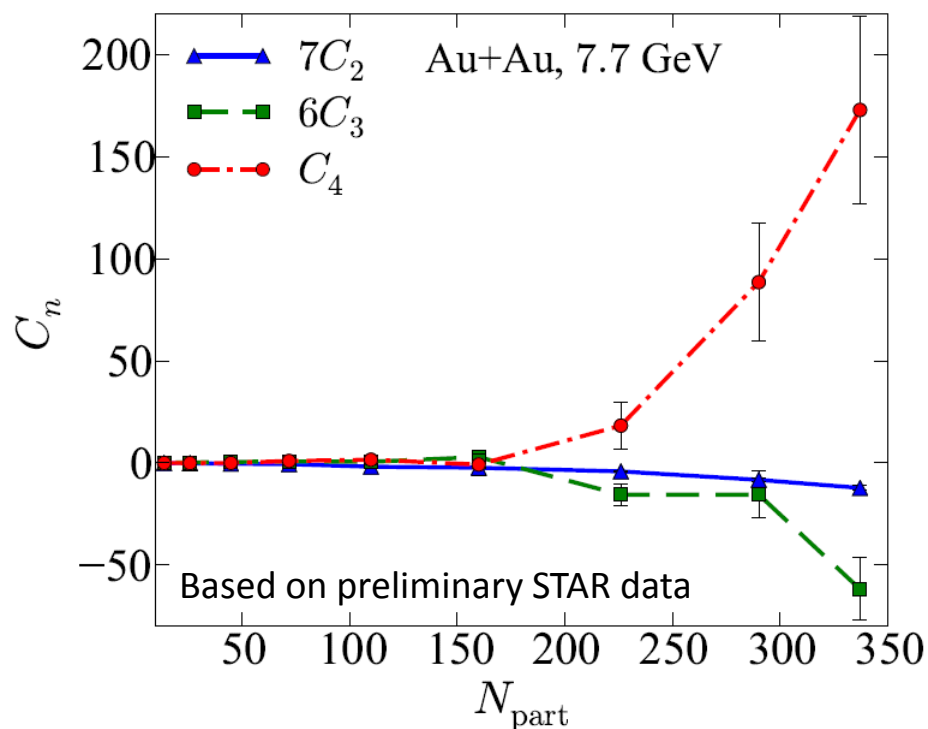
$$K_i = 2^i \langle n_C \rangle$$

and for example: $\frac{K_4}{K_2} = 4$

looks nontrivial
but no new
information

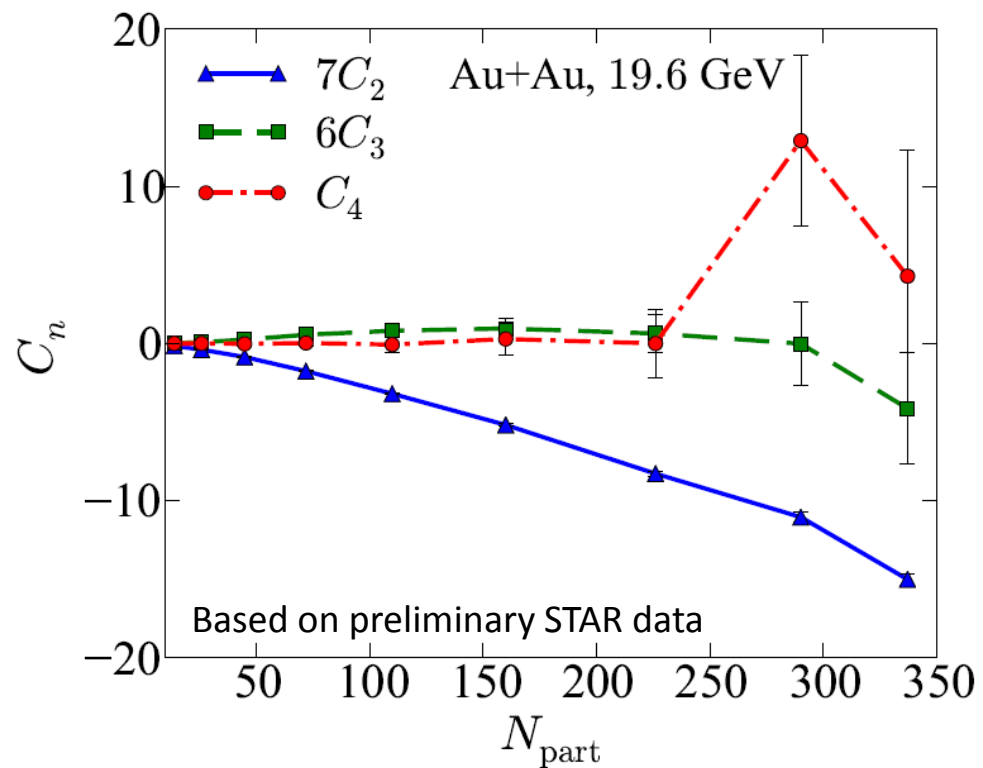
Using preliminary STAR data we obtain C_n

central signal at **7.7 GeV** is driven by large 4-particle correlations



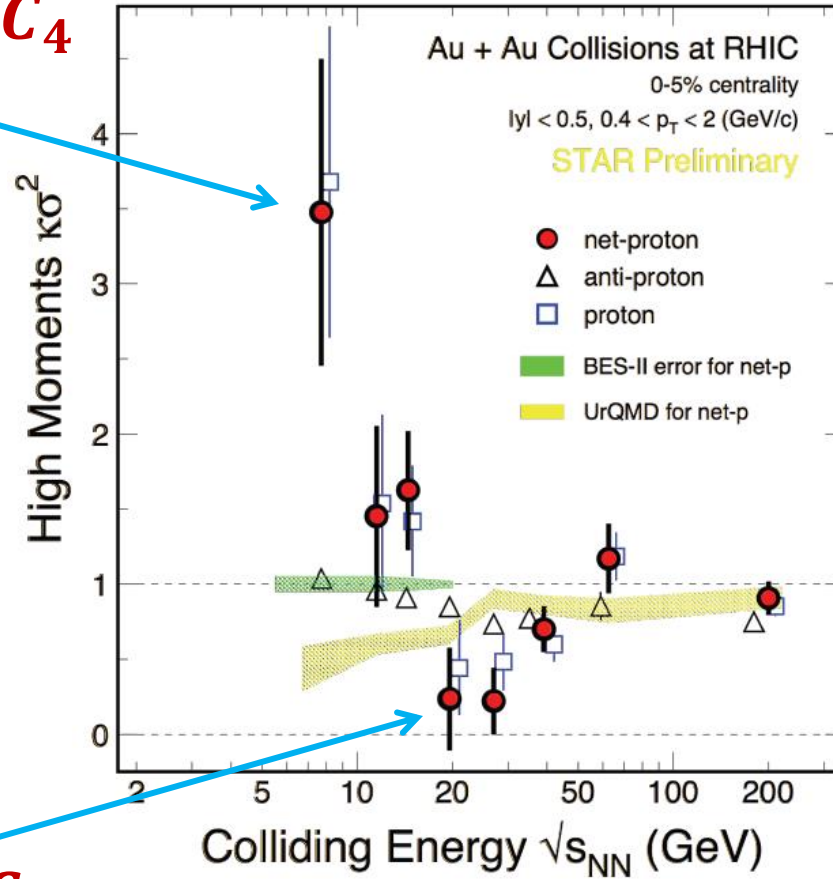
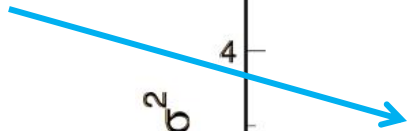
$$C_4(7.7) \sim 170$$

central signal at **19.6 GeV** is driven by 2-particle correlations

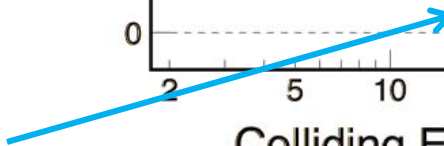


C_4 and $6C_3$ cancelation in most central coll.

here we see C_4

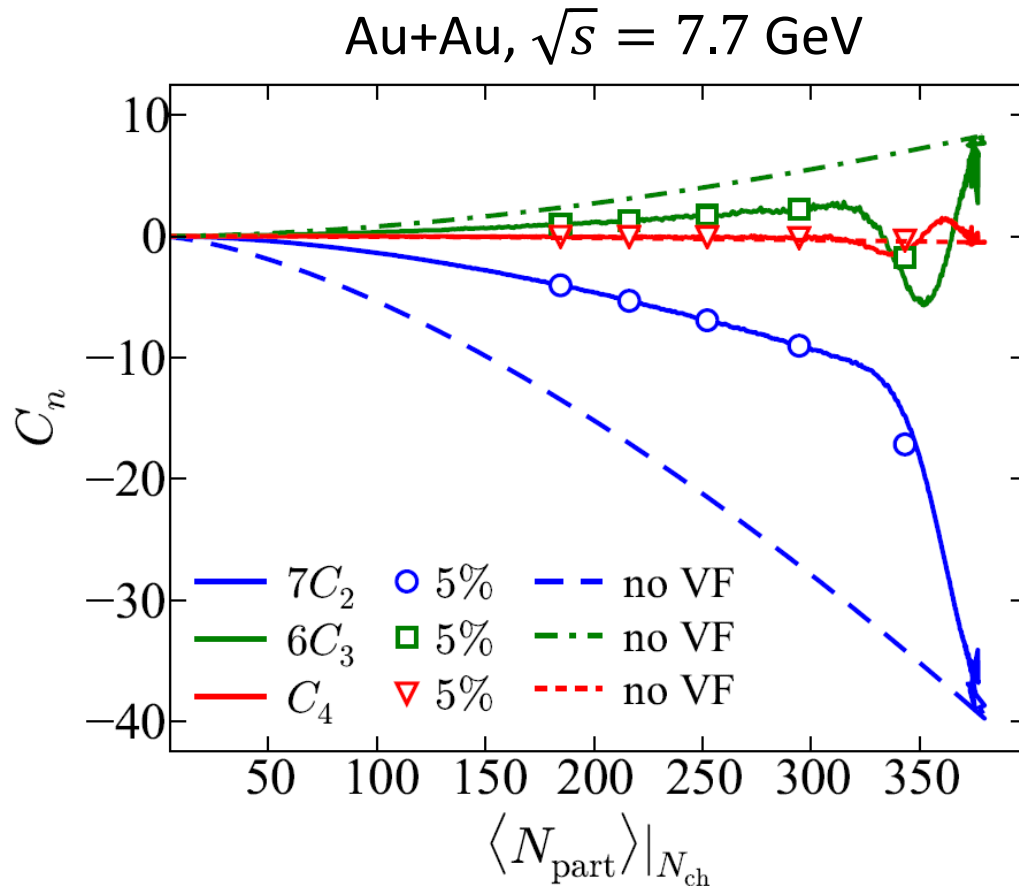


and here C_2



Baryon conservation + volume fluctuation (minimal model)

- independent baryon stopping (baryon conservation by construction)
- N_{part} fluctuations (volume fluctuation - VF)



STAR

$$C_4 \sim 170$$

$$6C_3 \sim -60$$

$$7C_2 \sim -15$$

we follow the STAR way (centrality etc.) as closely as possible

AB, V. Koch, V. Skokov, EPJC 77 (2017) 288

See also, P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 960 (2017) 114

Volume fluctuation + baryon conservation seems to be important for C_2 but irrelevant for C_3 and C_4 (7.7 GeV).

C_4 observed by STAR is larger by almost **three orders of magnitude** than the minimal model.

To explain C_4 we need a strong source of multi-proton correlations.

Proton clusters?

Let's put the STAR numbers in perspective.

Suppose that we have **clusters** (distributed according to Poisson) decaying always to 4 protons

$$C_k = \langle N_{cl} \rangle \cdot 4! / (4 - k)!$$

↑
mean number
of clusters

$$C_4 = \langle N_{cl} \rangle \cdot 24$$

for 5-proton clusters:

$$C_k = \langle N_{cl} \rangle \cdot 5! / (5 - k)!$$

$$C_4 = \langle N_{cl} \rangle \cdot 120$$

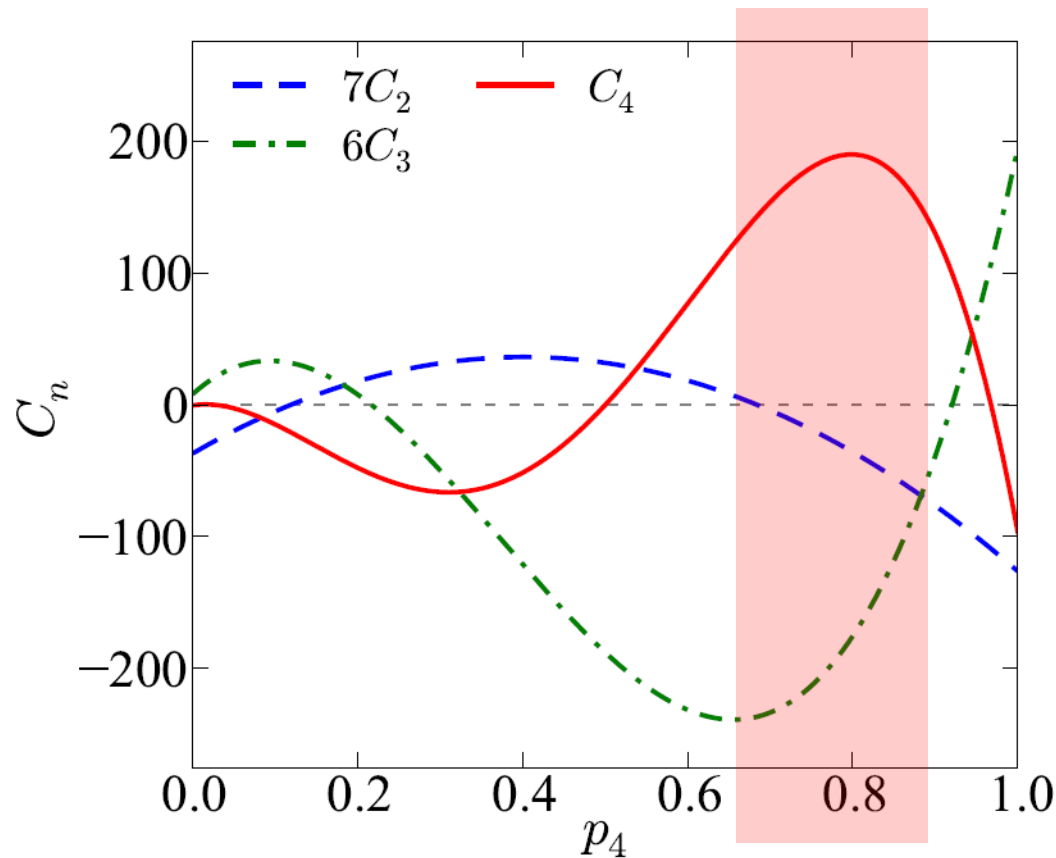
and $\langle N_{cl} \rangle \sim 1$

To obtain $C_4 \approx 170$ we need $\langle N_{cl} \rangle \sim 7$, it means 28 protons.
STAR sees on average 40 protons in central collisions.

In this model $C_2 > 0$ and $C_3 > 0$ contrary to the STAR data

Toy model:

- 16 protons stop in quartets with probability p_4
- remaining protons stop independently with some small probability $p_1 \sim 0.1$



qualitatively
consistent
with STAR

STAR

$$C_4 \sim 170$$

$$6C_3 \sim -60$$

$$7C_2 \sim -15$$

We obviously need more serious cluster model.
See, e.g., E.Shuryak, J.M. Torres-Rincon, 1805.04444

Can we describe the STAR data at 7.7 GeV with *ordinary* multiplicity distributions?

Model with **two event classes**

$$P(N) = (1 - \alpha)P_{(a)}(N) + \alpha P_{(b)}(N)$$



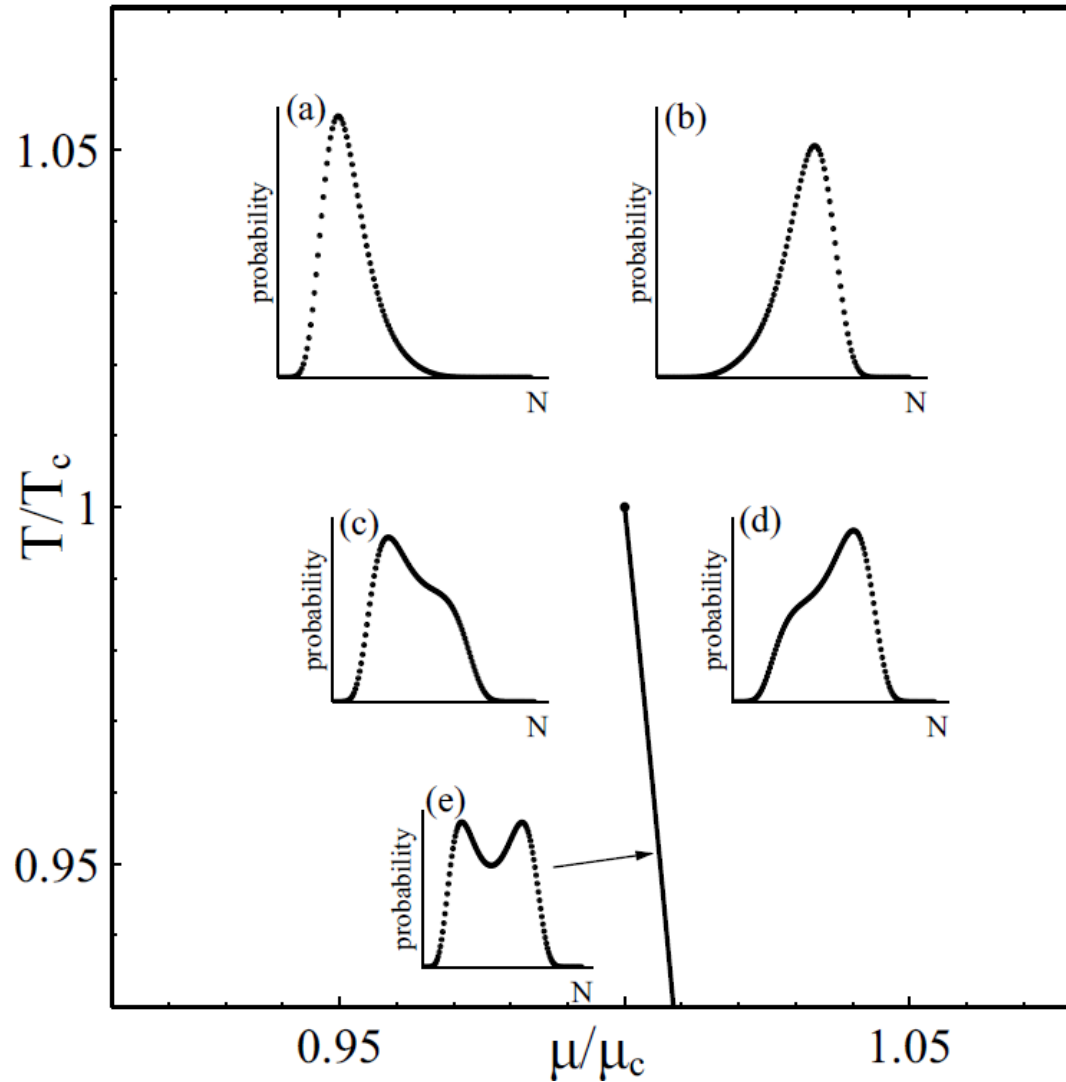
Poisson,
binomial,
etc..



Poisson,
binomial,
etc.

That is, with probability $1 - \alpha$ we have $P_{(a)}(N)$ and with probability α we have $P_{(b)}(N)$

A finite volume van der Waals model



$$C_2 = \alpha(1 - \alpha)\bar{N}^2 \approx \alpha\bar{N}^2,$$

$$C_3 = -\alpha(1 - \alpha)(1 - 2\alpha)\bar{N}^3 \approx -\alpha\bar{N}^3,$$

$$C_4 = \alpha(1 - \alpha)(1 - 6\alpha + 6\alpha^2)\bar{N}^4 \approx \alpha\bar{N}^4,$$

$$\bar{N} = \langle N_{(a)} \rangle - \langle N_{(b)} \rangle$$

$$\frac{C_6}{C_5} \approx \frac{C_5}{C_4} \approx \frac{C_4}{C_3} = -17 \pm 6$$

parameter-free
prediction at 7.7 GeV ($\alpha \ll 1$)

$$K_5/K_2 \approx -34$$

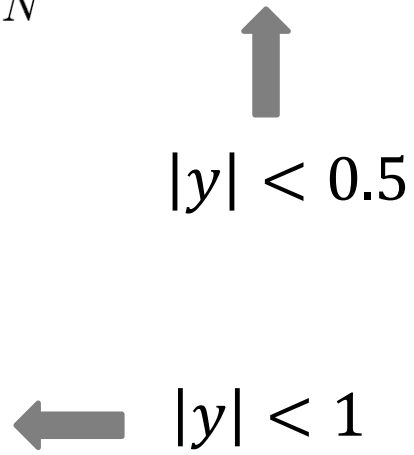
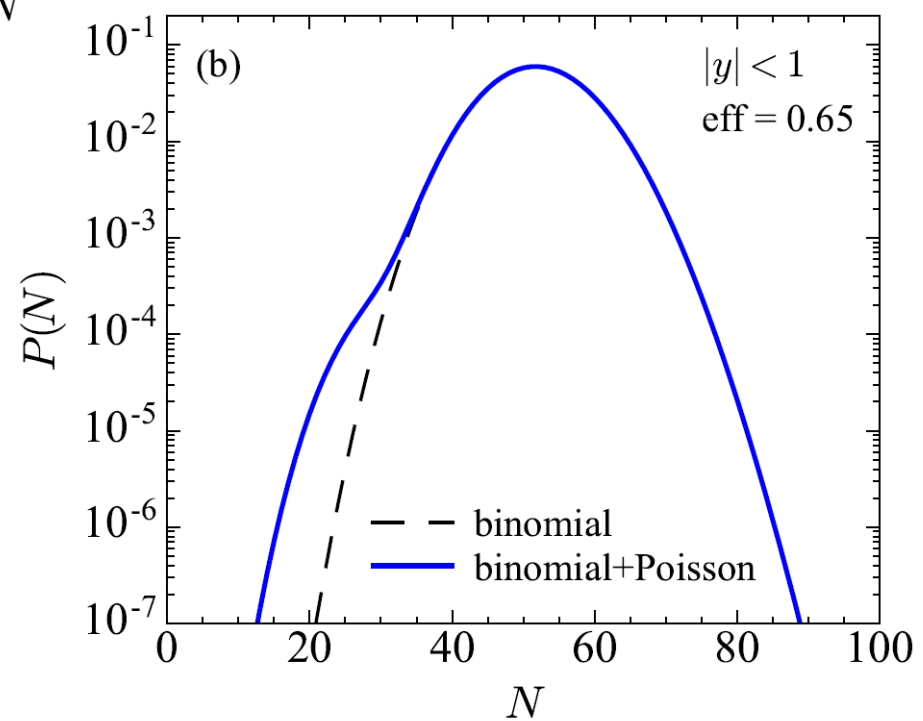
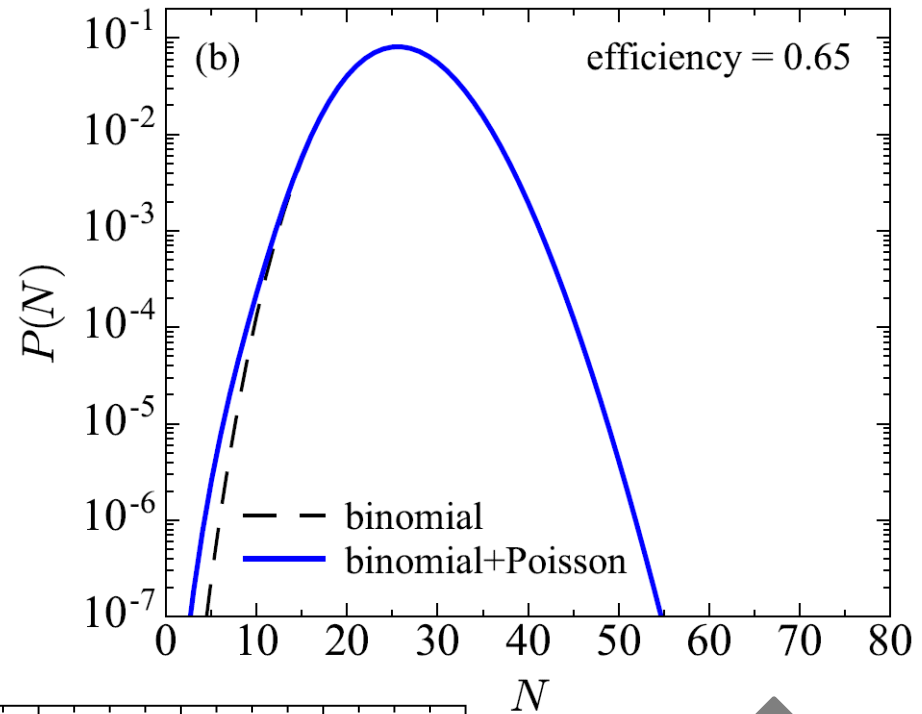
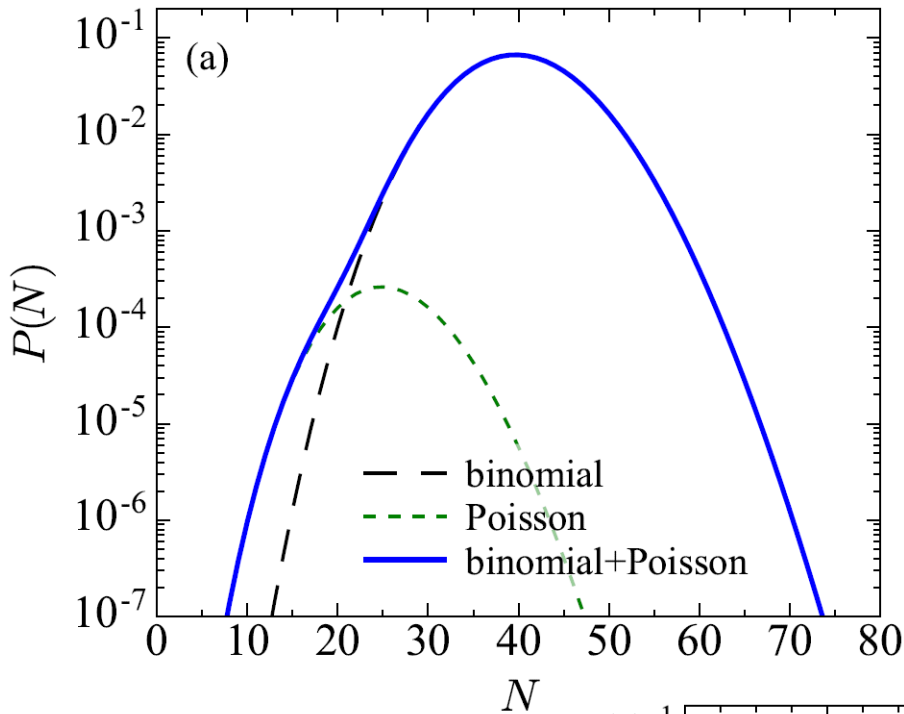
$$K_6/K_2 \approx 312$$

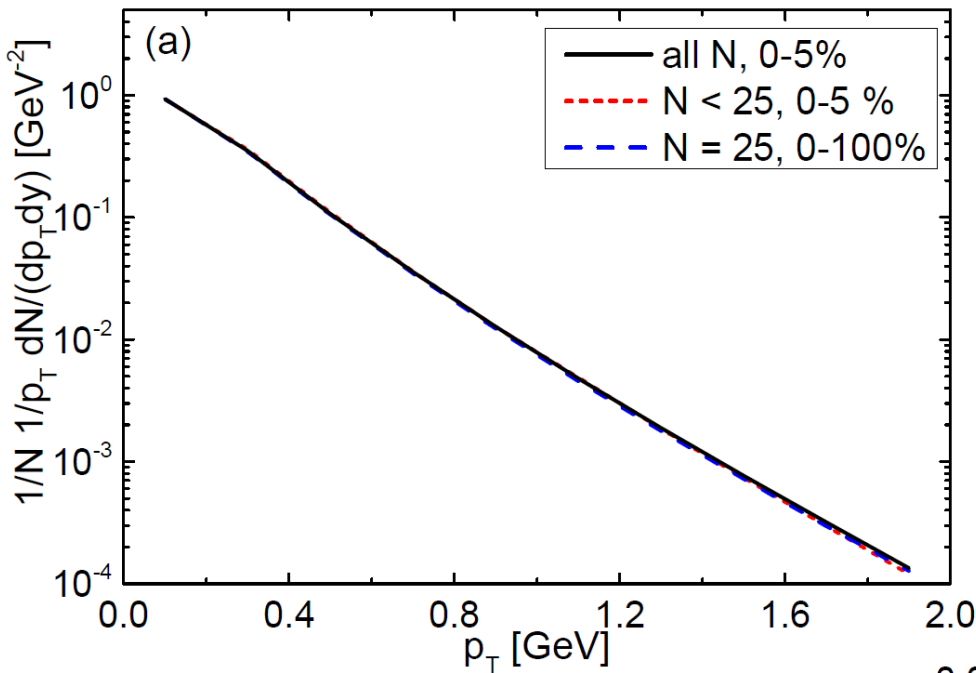
assuming $C_4 = 170$

We can describe the data with $\alpha \approx 0.0033$

$$\langle N_{(a)} \rangle \approx 40, \langle N_{(b)} \rangle \approx 25$$

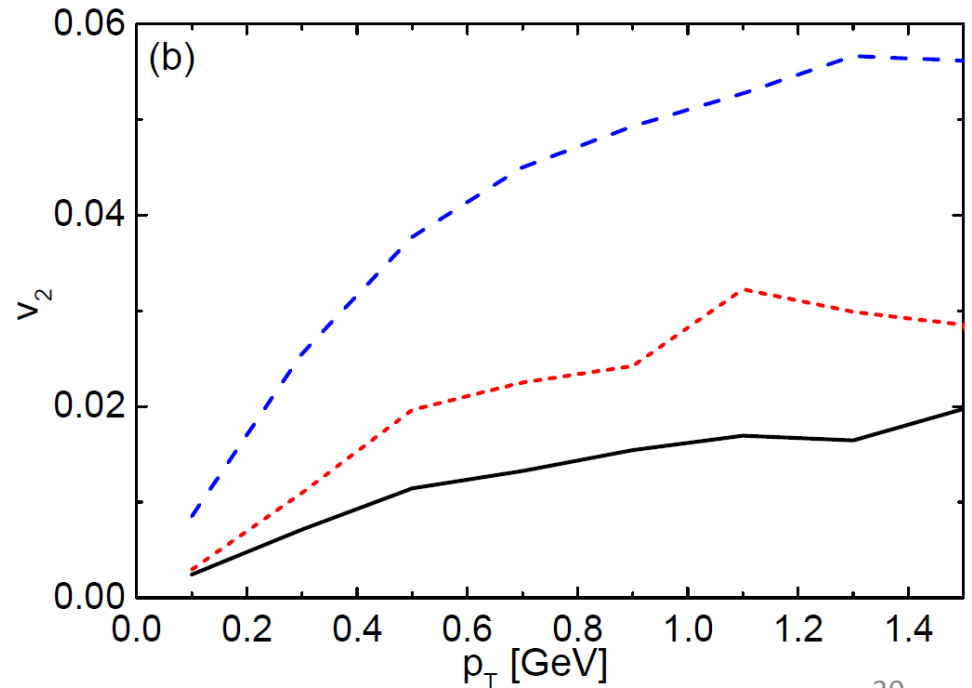
Now we can plot $P(N)$





Cuts in the number of protons
for central collisions

UrQMD

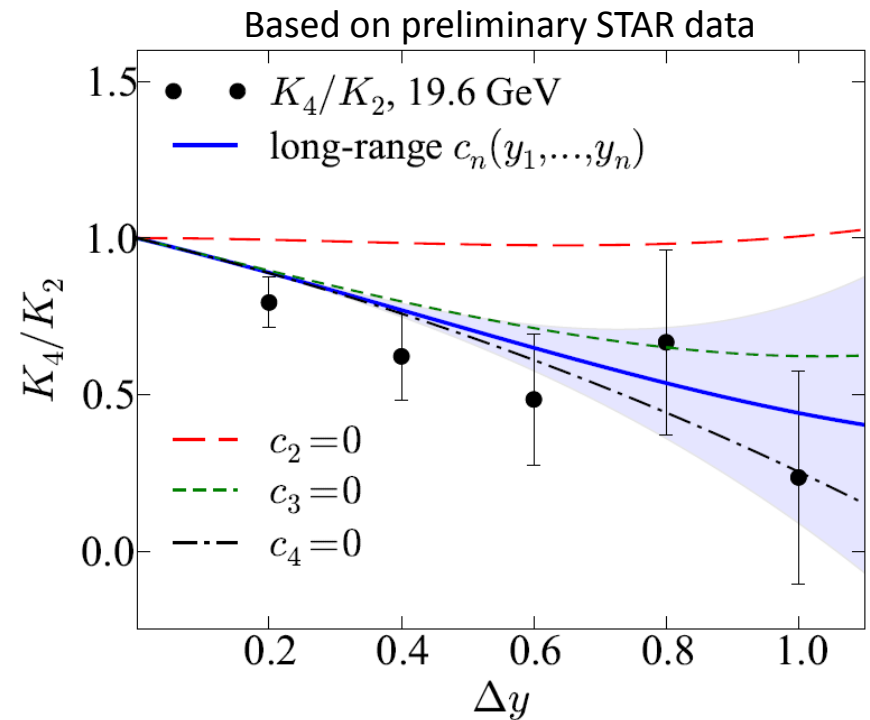
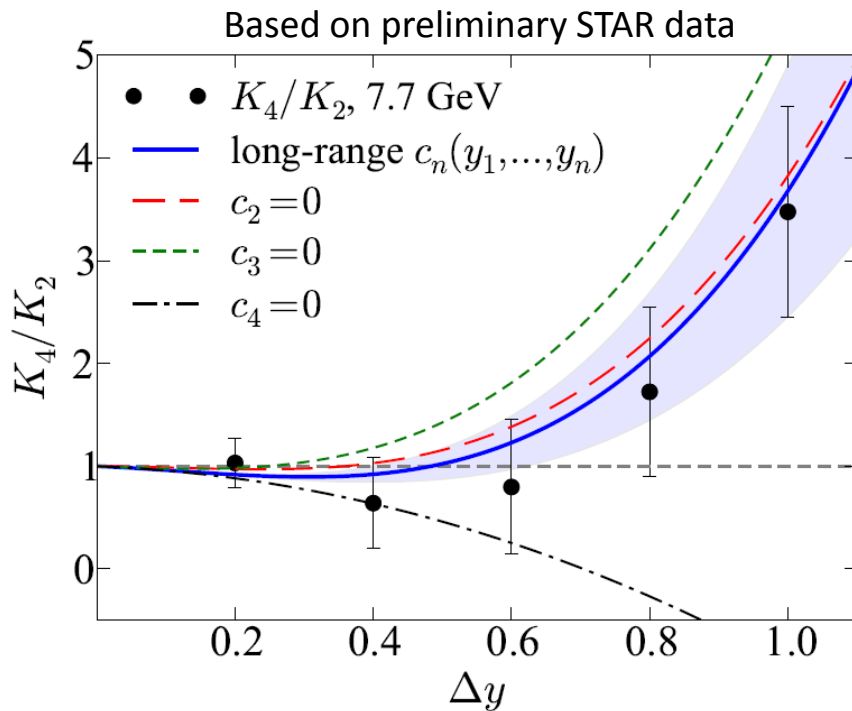


Rapidity dependence consistent with long-range correlations

$$c_n(y_1, \dots, y_n) = \frac{C_n(y_1, \dots, y_n)}{\rho(y_1) \cdots \rho(y_n)}$$

if $c_n(y_1, \dots, y_n) = \text{const}$

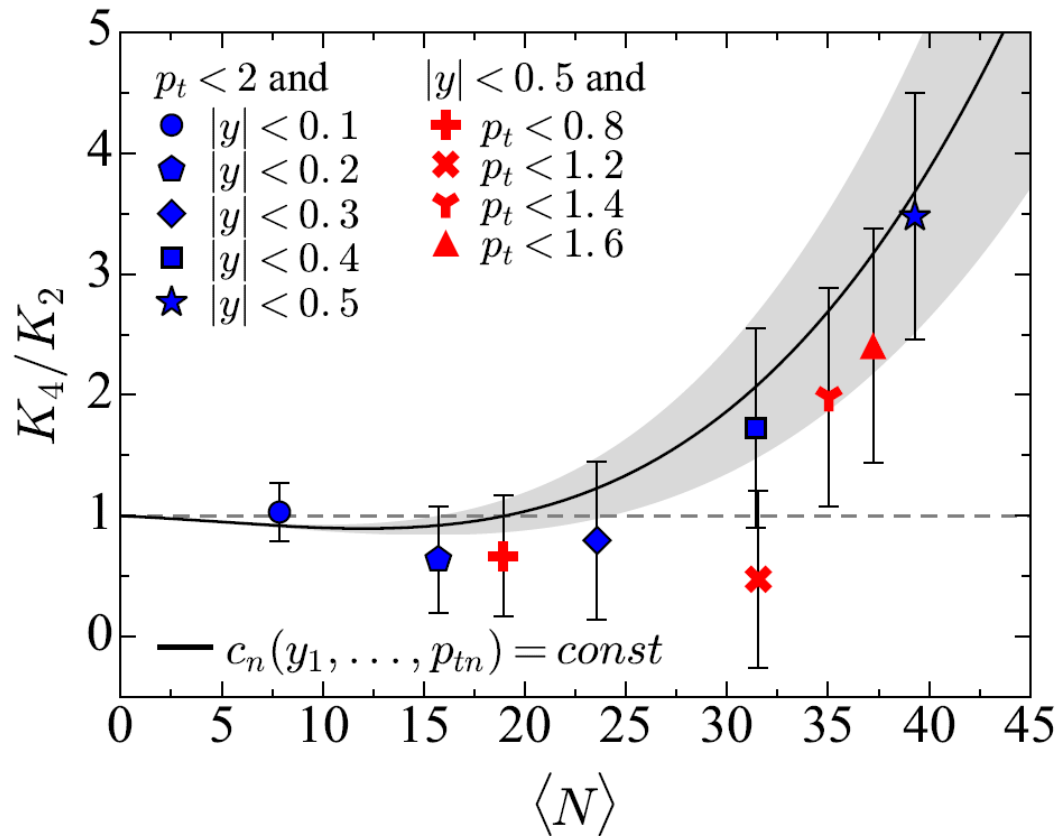
$$C_n \sim \langle N \rangle^n \sim (\Delta y)^n$$



Constant correlation

$$c_n(y_1, p_{t1}, \dots, y_n, p_{tn}) = \text{const}$$

physics independent on rapidity
and transverse momentum



Acceptance: missing link between models and data

Cumulant ratios strongly depend on acceptance in rapidity (as actually expected) and in transverse momentum.

Comparison with models which do not have experimental acceptance is questionable (should be done with extra caution).

For small enough $\langle N \rangle$ things look like Poisson but this is actually misleading.

Repulsive vs attractive rapidity correlations

$$c_2(y_1, y_2) = c_2^0 + \underline{\gamma_2} (y_1 - y_2)^2$$

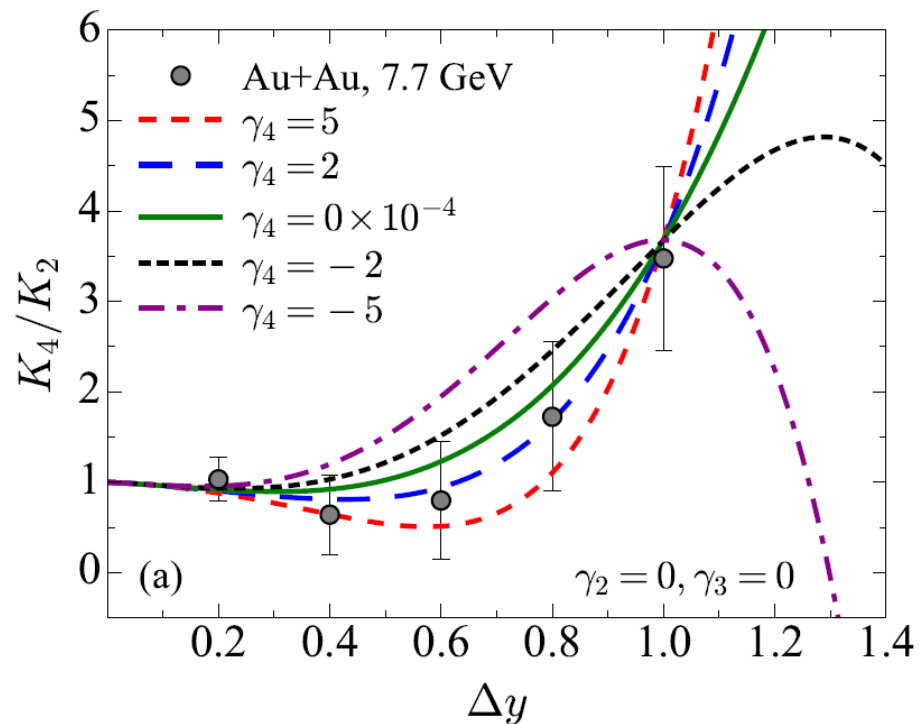
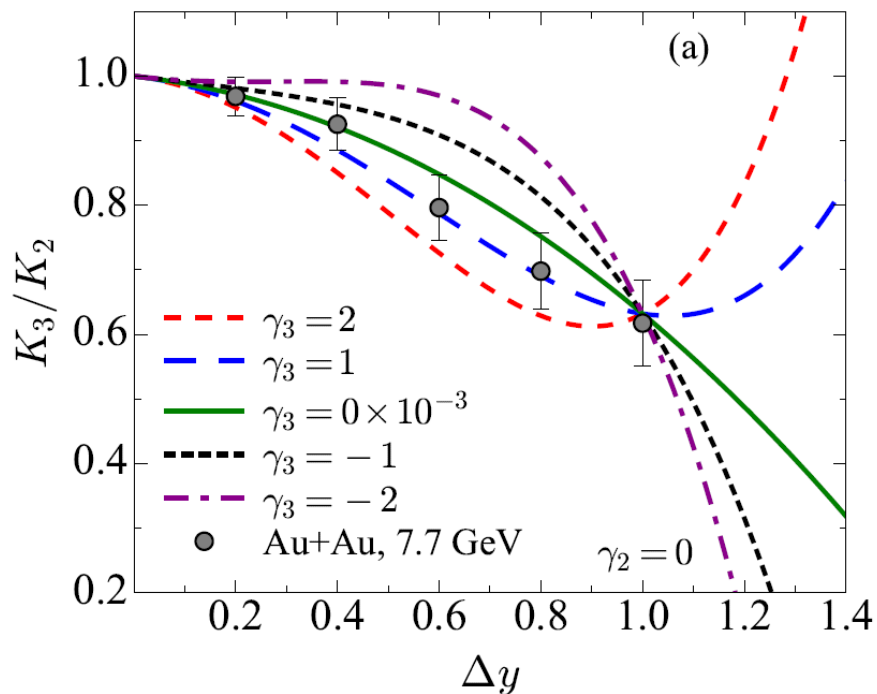
$$c_3(y_1, y_2, y_3) = c_3^0 + \underline{\gamma_3} \frac{1}{3} \left[(y_1 - y_2)^2 + (y_1 - y_3)^2 + (y_2 - y_3)^2 \right]$$

$$c_4(y_1, y_2, y_3, y_4) = c_4^0 + \underline{\gamma_4} \frac{1}{6} \left[(y_1 - y_2)^2 + (y_1 - y_3)^2 + (y_1 - y_4)^2 \right. \\ \left. + (y_2 - y_3)^2 + (y_2 - y_4)^2 + (y_3 - y_4)^2 \right]$$

$\gamma_n > 0$ - rapidity “repulsion”

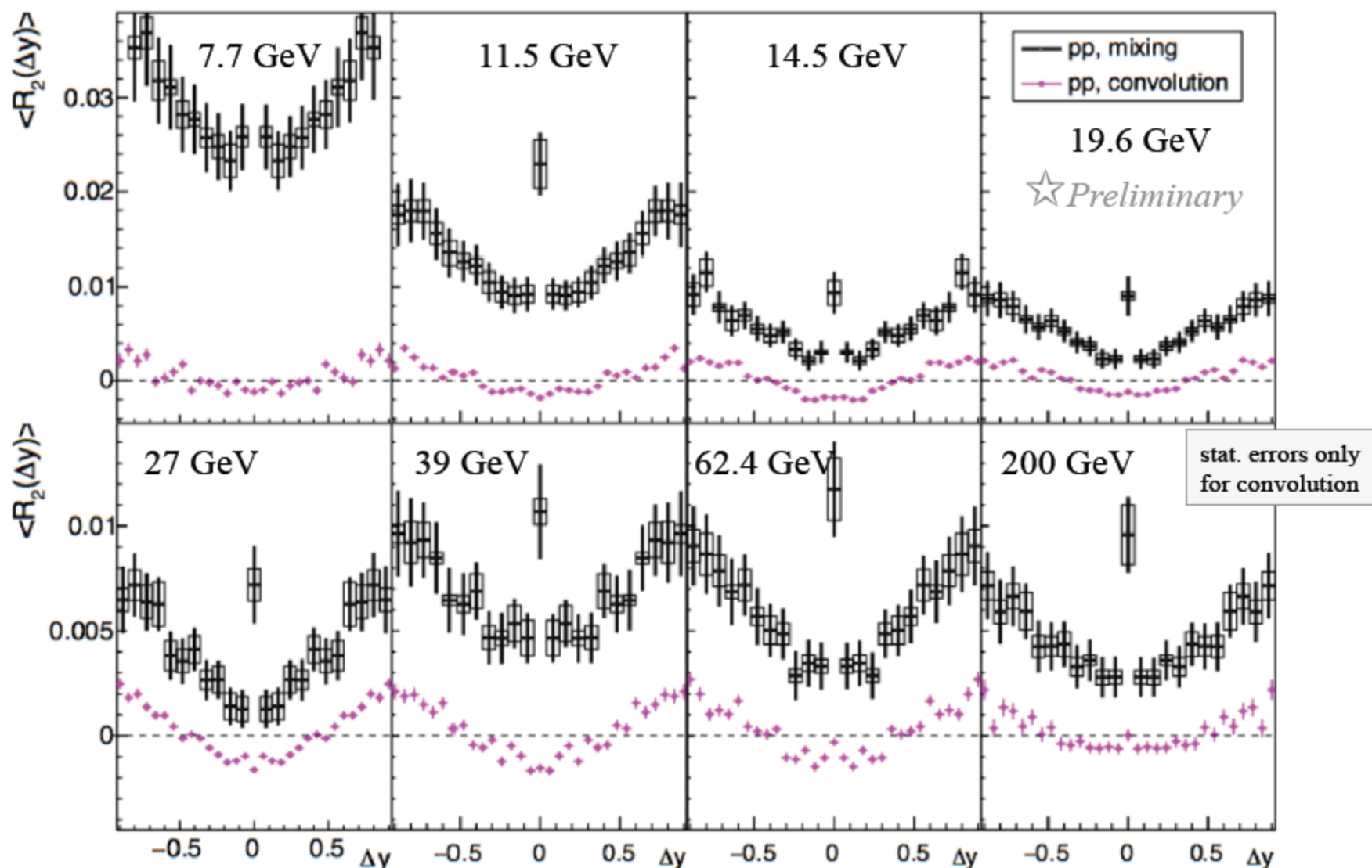
$\gamma_n < 0$ - rapidity “attraction”

It seems that rapidity repulsion ($\gamma_{3,4} > 0$) is favored



$\gamma_{3,4} < 0$ (attraction) seems to be excluded

Presence of proton clusters would naively result in $\gamma_{2,3,4} < 0 \dots$



Better control of finite multiplicity effects from convolution
 LS proton anticorrelation for $\Delta y \sim 0$. Weak beam energy dependence.

Conclusions:

Four-proton factorial cumulant (int. correlation function) at 7.7 GeV is surprisingly large. Three orders of magnitude larger than the minimal model.

Volume fluctuation and baryon conservation seem to be irrelevant for C_3 and C_4 . C_2 (and K_2) is likely contaminated by background.

Proton clusters?

Two event classes? Bumpy structure of $P(N)$.

Parameter-free predictions.

Evidence of long-range proton correlation in rapidity and transverse momentum. Perhaps the first evidence of multiproton repulsion in rapidity.

Backup

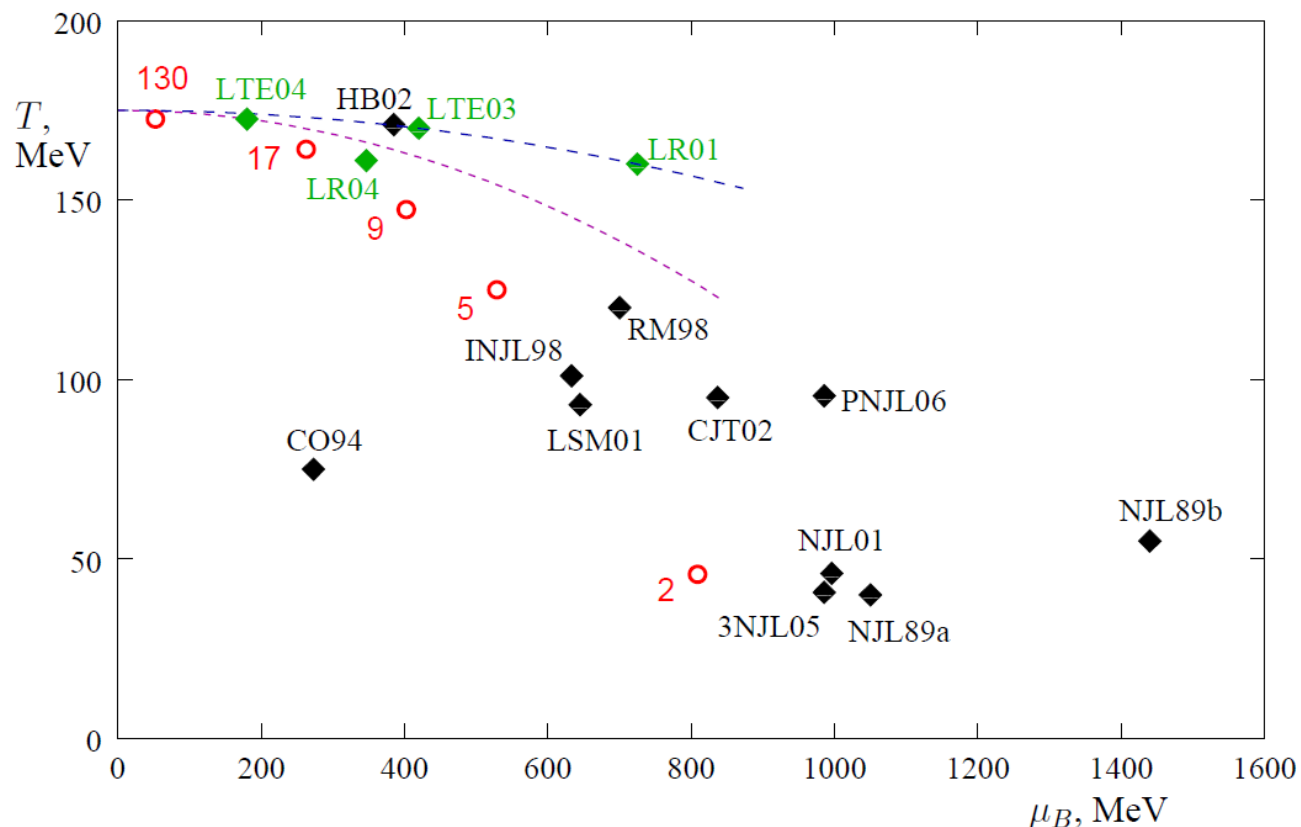


Figure 4: Comparison of predictions for the location of the QCD critical point on the phase diagram. Black points are model predictions: NJLa89, NJLb89 – [12], CO94 – [13, 14], INJL98 – [15], RM98 – [16], LSM01, NJL01 – [17], HB02 – [18], CJT02 – [19], 3NJL05 – [20], PNJL06 – [21]. Green points are lattice predictions: LR01, LR04 – [22], LTE03 – [23], LTE04 – [24]. The two dashed lines are parabolas with slopes corresponding to lattice predictions of the slope $dT/d\mu_B^2$ of the transition line at $\mu_B = 0$ [23, 25]. The red circles are locations of the freezeout points for heavy ion collisions at corresponding center of mass energies per nucleon (indicated by labels in GeV) – Section 5.

So are factorial cumulants “easy”?

Factorial cumulants measure deviations from Poisson

Consider a source giving always **one particle**

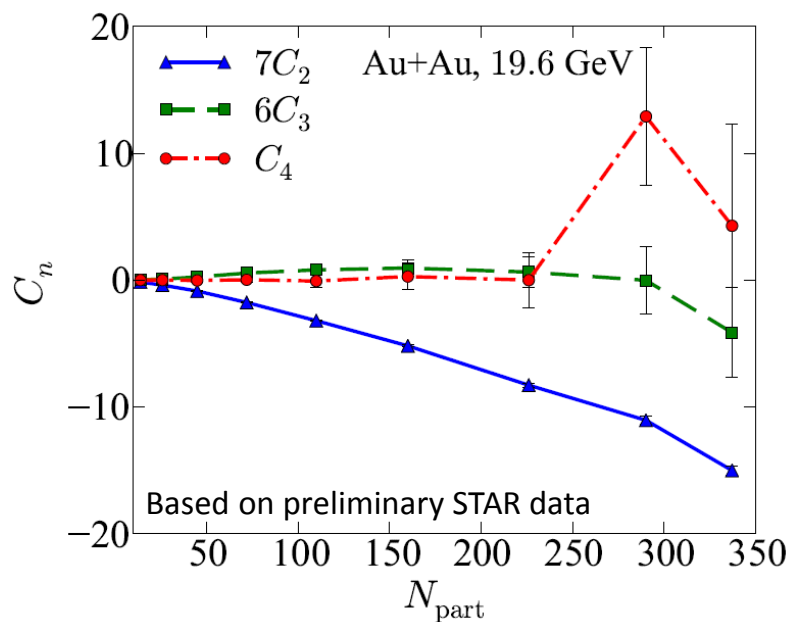
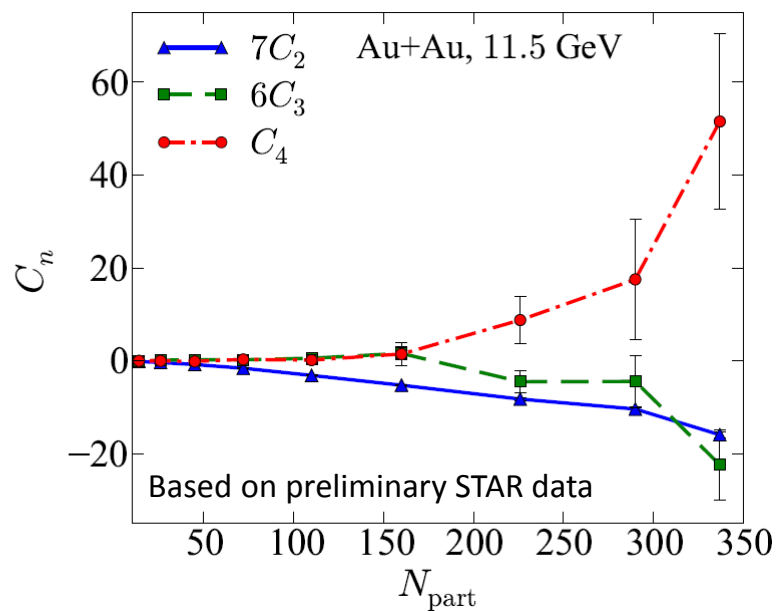
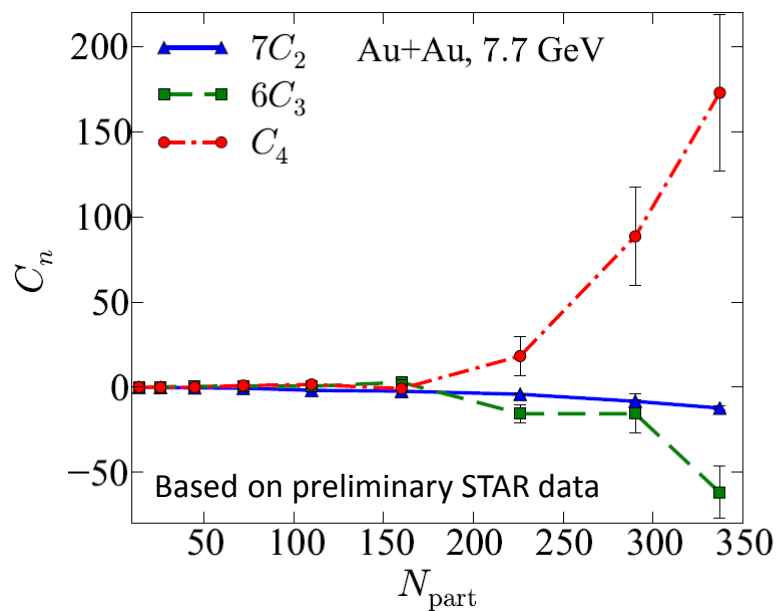
$$\begin{array}{l} \textcircled{P(n)} \quad P(n) = 1 \quad \text{for } n = 1 \\ \quad \quad \quad = 0 \quad \text{for } n > 1 \end{array}$$

$$C_k = \frac{d^k}{dz^k} \ln(z) \Big|_{z=1}$$

$$C_2 = -1, \quad C_3 = 2, \quad C_4 = -6, \dots, \quad C_9 = 40320$$

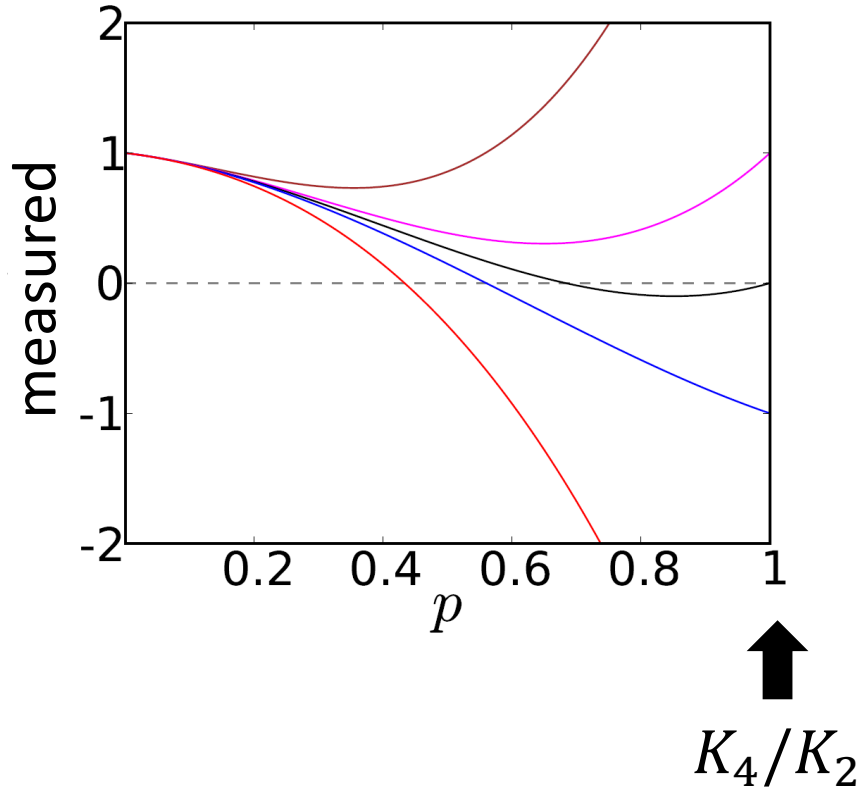
$$C_k = (-1)^{k-1} (k-1)!$$

Comparison of 7.7, 11.5 and 19.6 GeV



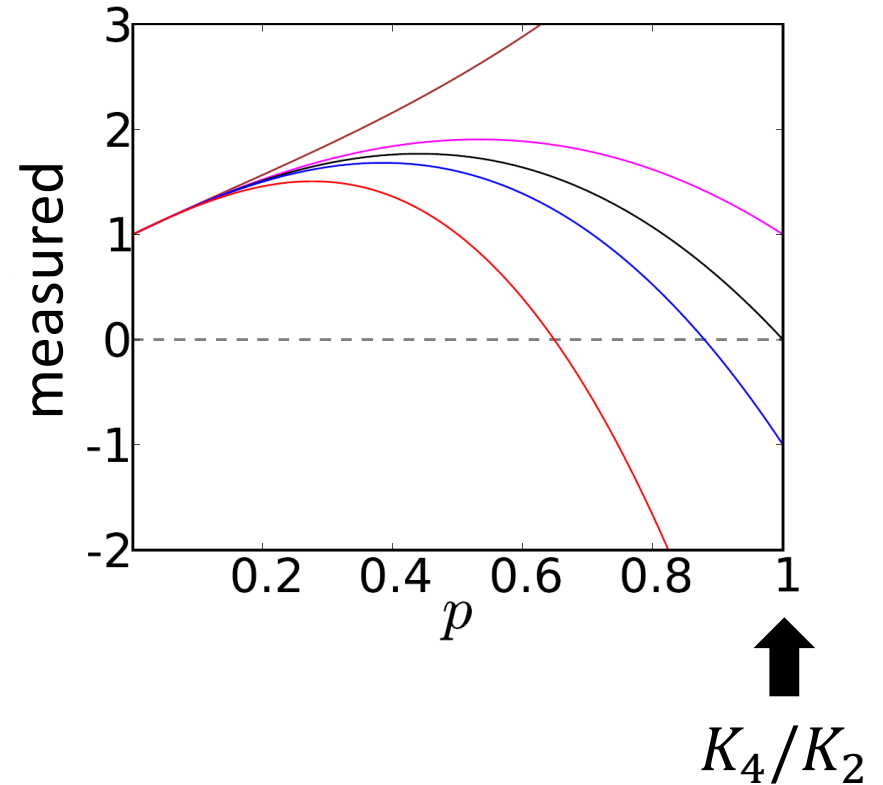
Efficiency

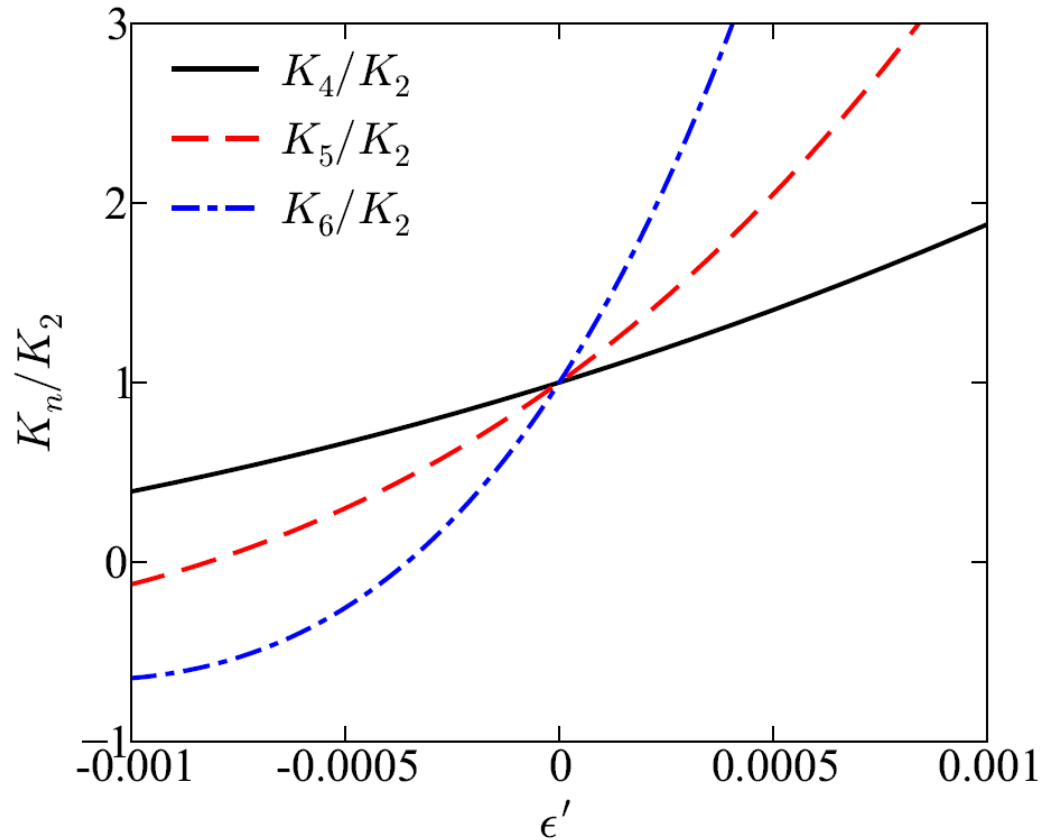
multiplicity distr. narrower than Poisson



$$\frac{K_4}{K_2} = 5, 1, 0, -1, -5$$

multiplicity distr. broader than Poisson





Large corrections for small ϵ'

$$P(N) = \frac{\langle N \rangle^N}{N!} e^{-\langle N \rangle},$$

$$\epsilon(N) = \epsilon_0 + \epsilon'(N - \langle N \rangle)$$

Mixed integrated correlation functions

$$C_2^{(2,0)} = -F_{1,0}^2 + F_{2,0}$$

$$F_{i,k} \equiv \left\langle \frac{N!}{(N-i)!} \frac{\bar{N}!}{(\bar{N}-k)!} \right\rangle$$

$$C_2^{(1,1)} = -F_{0,1}F_{1,0} + F_{1,1}$$

$$C_3^{(3,0)} = 2F_{1,0}^3 - 3F_{1,0}F_{2,0} + F_{3,0}$$

$$C_3^{(2,1)} = 2F_{0,1}F_{1,0}^2 - 2F_{1,0}F_{1,1} - F_{0,1}F_{2,0} + F_{2,1}$$

Cumulants

$$K_2 = \langle N \rangle + \langle \bar{N} \rangle + C_2^{(2,0)} + C_2^{(0,2)} - 2C_2^{(1,1)}$$

$$K_3 = \langle N \rangle - \langle \bar{N} \rangle + 3C_2^{(2,0)} - 3C_2^{(0,2)} + C_3^{(3,0)} - C_3^{(0,3)} - 3C_3^{(2,1)} + 3C_3^{(1,2)}$$

For $C_4^{(i,k)}$ and K_4 see the appendix of

AB, V. Koch, N. Strodthoff , PRC 95 (2017) 054906

First model (AMPT) calculations by

Yufu Lin, Lizhu Chen, Zhiming Li, PRC 96 (2017) 044906

Mixed correlation functions and cumulants

$$C_2^{(2,0)} = -F_{1,0}^2 + F_{2,0}$$

$$C_2^{(1,1)} = -F_{0,1}F_{1,0} + F_{1,1}$$

$$C_3^{(3,0)} = 2F_{1,0}^3 - 3F_{1,0}F_{2,0} + F_{3,0}$$

$$C_3^{(2,1)} = 2F_{0,1}F_{1,0}^2 - 2F_{1,0}F_{1,1} - F_{0,1}F_{2,0} + F_{2,1}$$

$$C_4^{(4,0)} = -6F_{1,0}^4 + 12F_{1,0}^2F_{2,0} - 3F_{2,0}^2 - 4F_{1,0}F_{3,0} + F_{4,0}$$

$$C_4^{(3,1)} = -6F_{0,1}F_{1,0}^3 + 6F_{1,0}^2F_{1,1} + 6F_{0,1}F_{1,0}F_{2,0} - 3F_{1,1}F_{2,0} - 3F_{1,0}F_{2,1} - F_{0,1}F_{3,0} + F_{3,1}$$

$$C_4^{(2,2)} = (-6F_{0,1}^2 + 2F_{0,2})F_{1,0}^2 + 8F_{0,1}F_{1,0}F_{1,1} - 2F_{1,1}^2 - 2F_{1,0}F_{1,2} + (2F_{0,1}^2 - F_{0,2})F_{2,0} - 2F_{0,1}F_{2,1} + F_{2,2}$$

$$K_2 = \langle N \rangle + \langle \bar{N} \rangle + C_2^{(2,0)} + C_2^{(0,2)} - 2C_2^{(1,1)}$$

$$K_3 = \langle N \rangle - \langle \bar{N} \rangle + 3C_2^{(2,0)} - 3C_2^{(0,2)} + C_3^{(3,0)} - C_3^{(0,3)} - 3C_3^{(2,1)} + 3C_3^{(1,2)}$$

$$K_4 = \langle N \rangle + \langle \bar{N} \rangle + 7C_2^{(2,0)} + 7C_2^{(0,2)} - 2C_2^{(1,1)} + 6C_3^{(3,0)} + 6C_3^{(0,3)} - 6C_3^{(2,1)} - 6C_3^{(1,2)} + C_4^{(4,0)} + C_4^{(0,4)} - 4C_4^{(3,1)} - 4C_4^{(1,3)} + 6C_4^{(2,2)}$$

$$C_{n+m}^{(n,m)} = \frac{C_{n+m}^{(n,m)}}{\langle N \rangle^n \langle \bar{N} \rangle^m}$$

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \mathbf{C}_2(y_1, y_2)$$

correlation
function

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2)[1 + \mathbf{c}_2(y_1, y_2)]$$

reduced correlation
function

e.g., does not depend
on binomial efficiency

“coupling”

$$\mathbf{c}_2 = \frac{\int \rho(y_1)\rho(y_2)\mathbf{c}_2(y_1, y_2)dy_1dy_2}{\int \rho(y_1)\rho(y_2)dy_1dy_2} = \frac{\mathbf{C}_2}{\langle N \rangle^2}$$

and the second order cumulant

$$K_2 = \langle N \rangle + \underbrace{\langle N \rangle^2 \mathbf{c}_2}_{\mathbf{C}_2}$$

We obtain

$$c_2 = \frac{\int \rho(y_1)\rho(y_2)c_2(y_1, y_2)dy_1dy_2}{\int \rho(y_1)\rho(y_2)dy_1dy_2}$$

$$K_2 = \langle N \rangle + \langle N \rangle^2 c_2$$

$$K_3 = \langle N \rangle + 3\langle N \rangle^2 c_2 + \langle N \rangle^3 c_3$$

$$K_4 = \langle N \rangle + 7\langle N \rangle^2 c_2 + 6\langle N \rangle^3 c_3 + \langle N \rangle^4 c_4$$

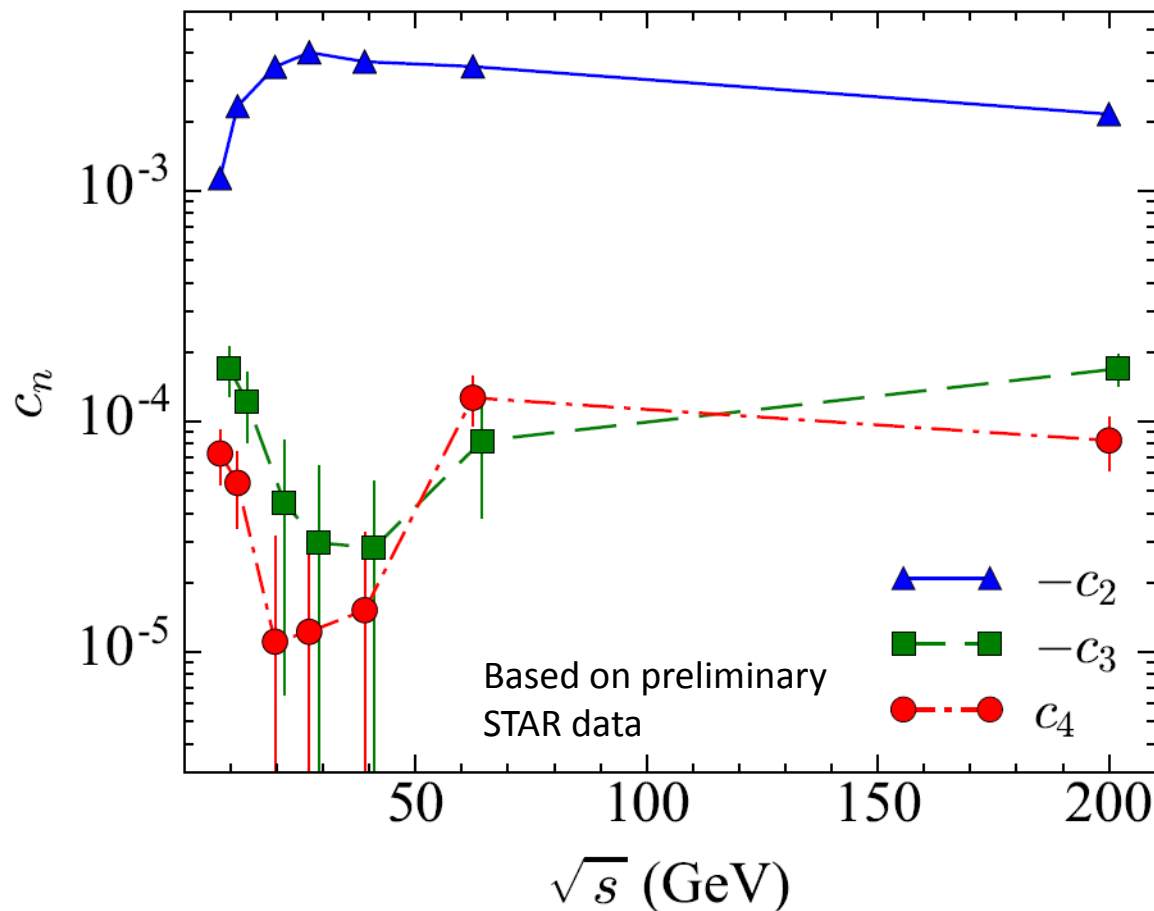
For $c_n(y_1, \dots, y_n) = \text{const}$, K_n strongly depends on rapidity window size since $\langle N \rangle \sim \Delta y$

At 7.7 GeV, $K_4/K_2 \sim \langle N \rangle^3 \sim (\Delta y)^3$

btw, K_n is strongly efficiency dependent through $\langle N \rangle$

Couplings' point of view and global baryon conservation

AB, VK, preliminary



Global baryon conservation

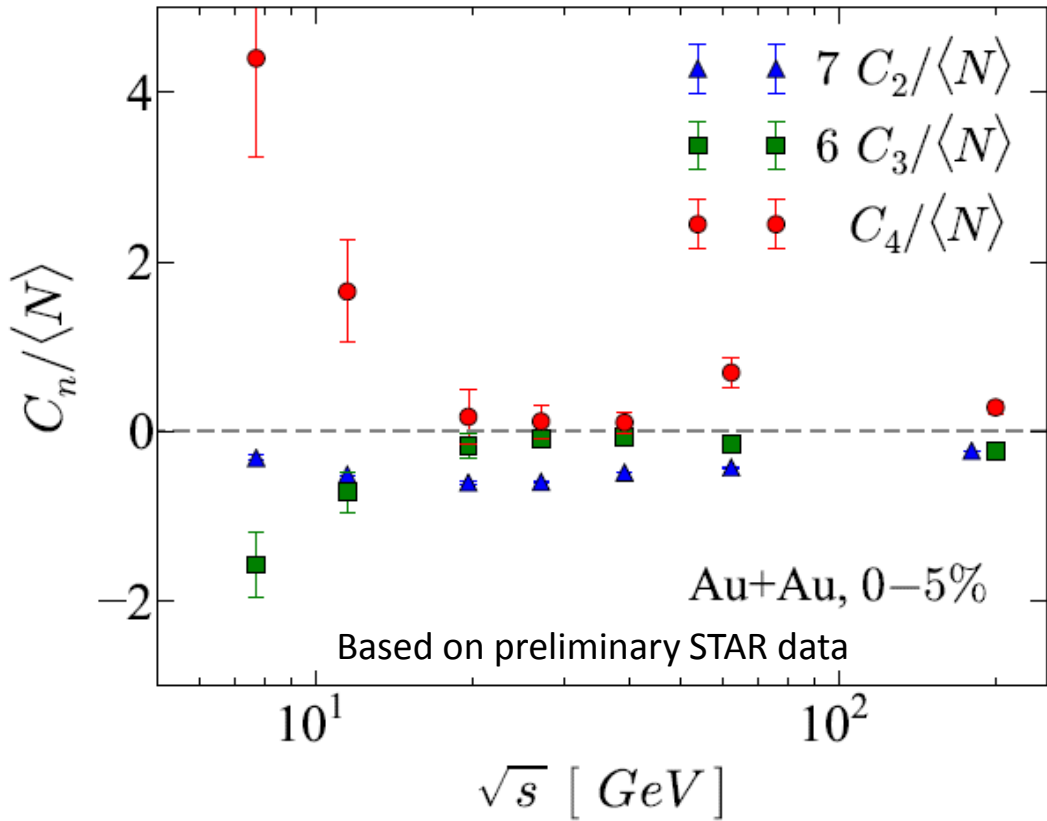
$$-c_2 = 1/B \approx 2 \cdot 10^{-3}$$

$$-c_3 = -2/B^2 \approx -10^{-5}$$

$$c_4 = -6/B^3 \approx -10^{-7}$$

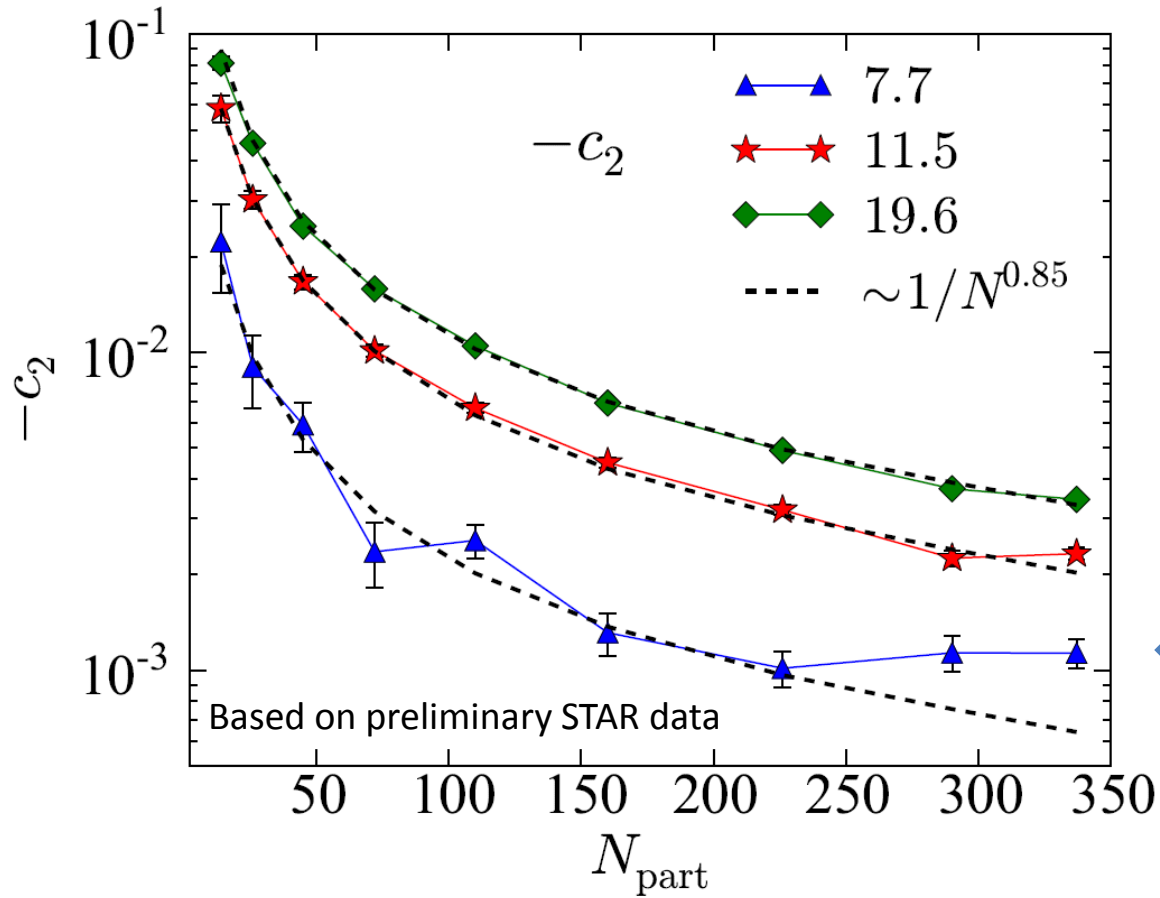
$$B \approx 400$$

c_n — integrated reduced correlation function (coupling)



C_4 at 62 GeV !

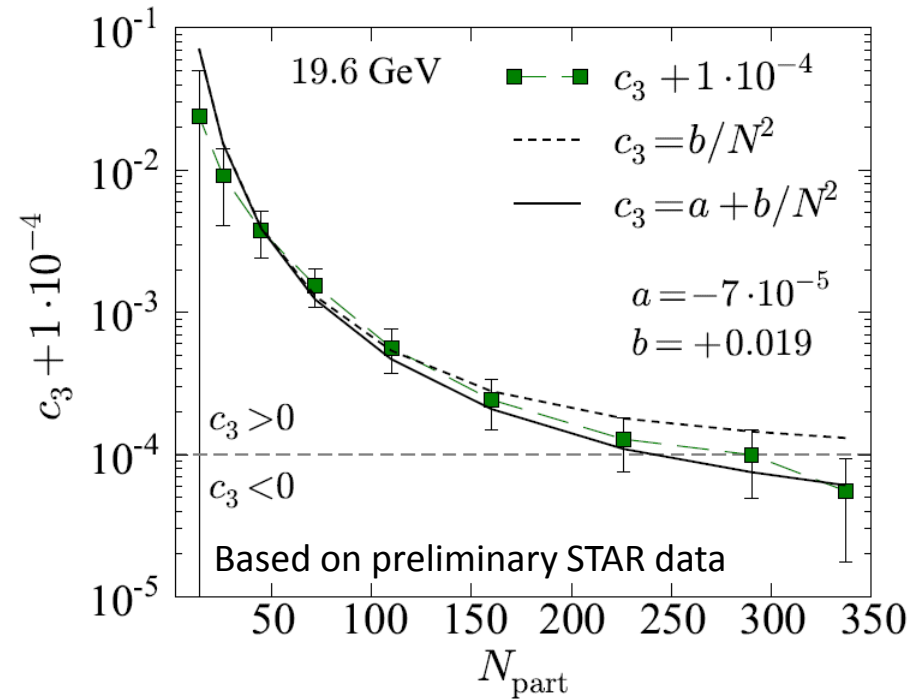
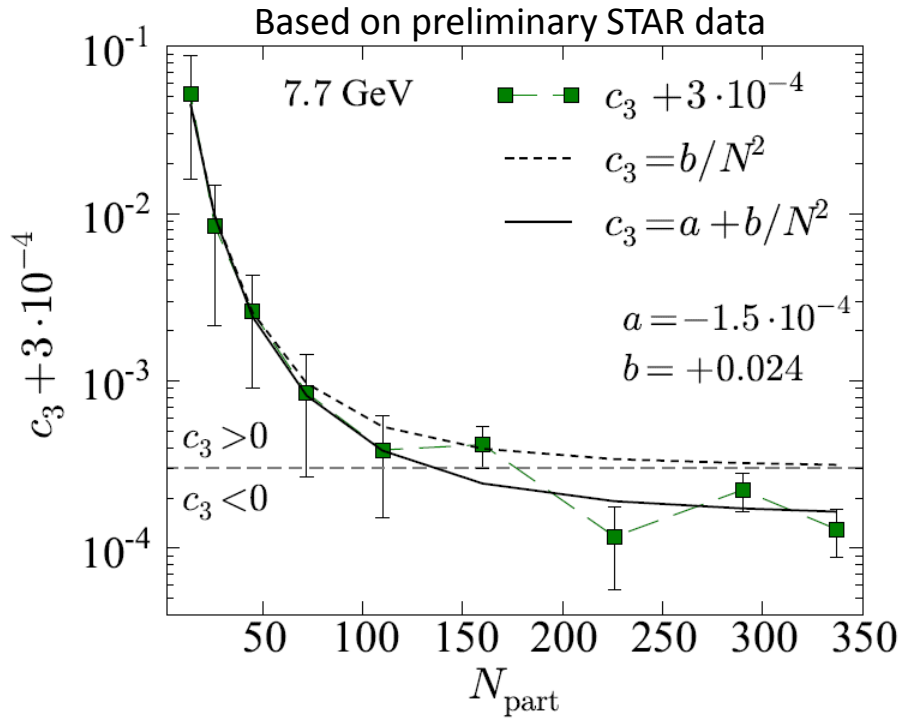
results for c_2



central 7 GeV points are somehow special

Using preliminary STAR data we obtain c_3

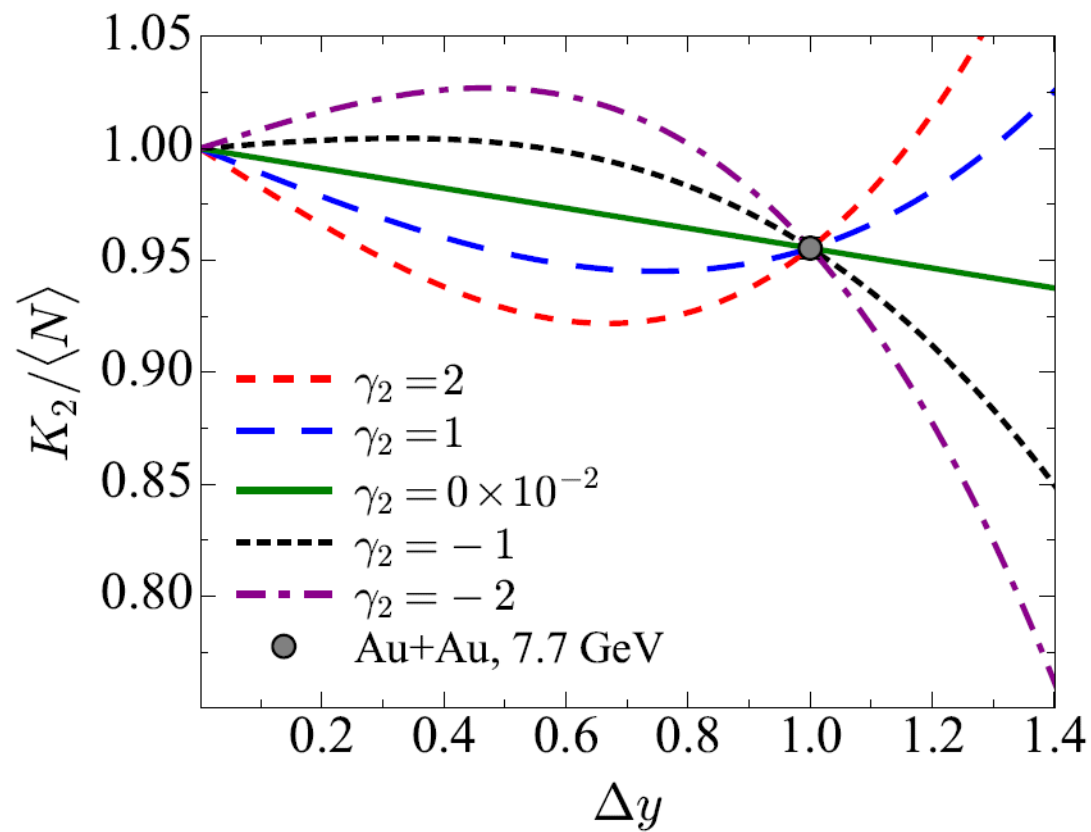
AB, V. Koch, N. Strodthoff,
PRC 95 (2017) 054906



At 7.7 GeV we see $1/N^2$ for small N_{part} then c_3 changes sign and stays roughly constant...

Similar story for c_4

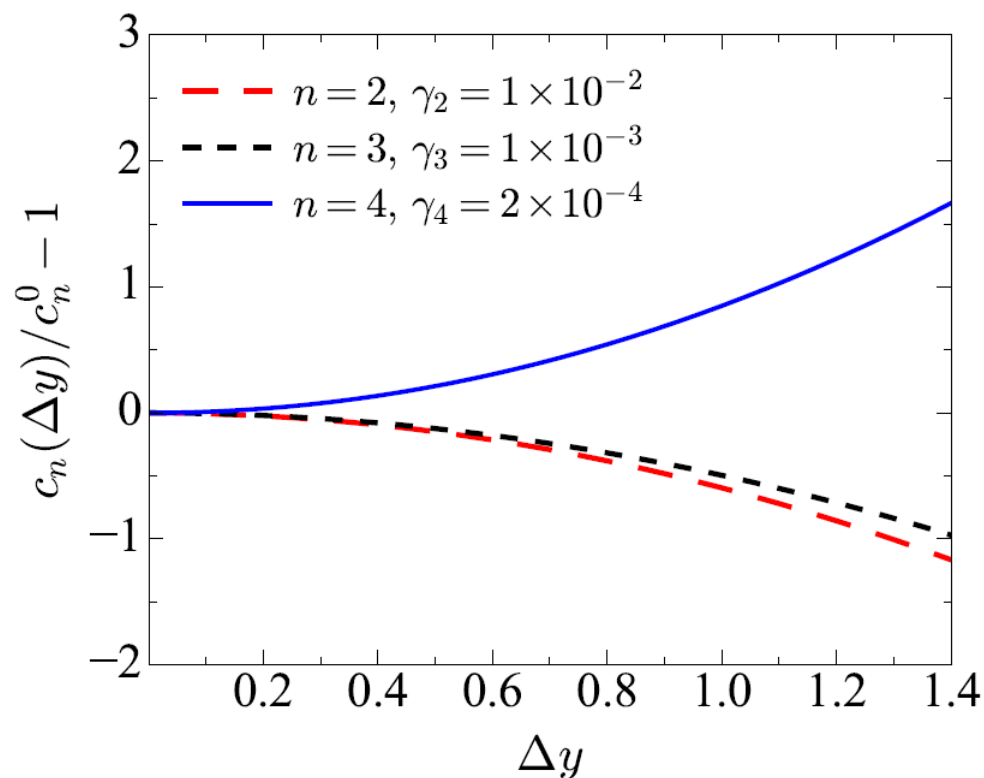
γ_2 is well visible in $K_2/\langle N \rangle$



We should study the integrated reduced correlation function

$$c_n(\Delta y) = \frac{C_n}{\langle N \rangle^n} = c_n^0 + \gamma_n \frac{1}{6} (\Delta y)^2$$

$$c_2 = \frac{\int \rho(y_1)\rho(y_2)c_2(y_1, y_2)dy_1dy_2}{\int \rho(y_1)\rho(y_2)dy_1dy_2}$$



Full acceptance

$$N_{(b)}$$

$$N_{(a)}$$

$$N_{(a)} + N_{(b)} = B = \text{const.}$$

baryon conservation

$$K_{2,(a)} = K_{2,(b)}$$

$$K_{3,(a)} = -K_{3,(b)}$$

$$K_{4,(a)} = K_{4,(b)}$$

$$K_{5,(a)} = -K_{5,(b)}$$

$$\frac{K_4}{K_2} \rightarrow 1, \quad \frac{K_3}{K_2} \rightarrow -1 \quad \text{for full acceptance}$$