Schwinger's formula and the axial Ward identity for chirality production

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Outline

1. Background
   - Motivation: Chiral Magnetic Effect
   - Schwinger Mechanism
   - Axial Ward Identity Expectation Values
2. In-Out Vacuum States
   - In-Out Green's Function
   - In-Out Psuedoscalar and Axial Ward Identity VEV
3. In-In Vacuum States
   - Inequivalent Vacuum States
   - Solution: In-In Vacuum States
   - In-In Green's Function
4. Chirality Generation from the Schwinger Mechanism
   - In-In Psuedoscalar and Axial Ward Identity VEV
   - In-In Chirality Density

Table of Contents

Electromagnetic current generated in the direction of magnetic field due to a net **chirality**.[1]

But how to generate chirality without insertion by hand?

→ The Schwinger mechanism.

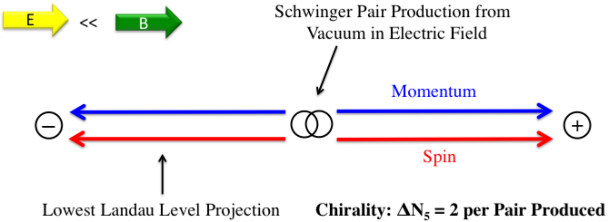
[1] K. Fukushima, D. Kharzeev, and H. Warringa, *PRD* 78, 074033 (2008).
K. Fukushima, *PTEP* No. 193, 2012.

- Instability of the QFT vacuum under an external electric field: **Schwinger pair production**[2]
- Imaginary part of effective action under constant electric field

$$\text{Num. of pairs} = 1 - |\langle \Omega_{out} | \Omega_{in} \rangle|^2$$

$$\sim \exp\left(-\frac{\pi m^2}{eE}\right)$$

Polarization of produced particles spins' under a strong magnetic field sets up a net chirality.



[2] J. Schwinger, *Phys. Rev.* 82, 664 (1951).

The **axial Ward identity**,

$$\partial_\mu j^{\mu 5} = 2im\bar{\Psi}\gamma^5\Psi + \frac{e^2}{2\pi^2}\vec{E}\cdot\vec{B},$$

for $j^{\mu 5} = \bar{\Psi}\gamma^\mu\gamma^5\Psi$ and fermion mass, m, is exact and well-known at the **operator** level.

- Yet, it's **vacuum expectation value (VEV)** behavior requires elucidation.
- Particularily the case when

$$\langle\Omega_{in}| \neq \langle\Omega_{out}|$$

  such as for the **Schwinger mechanism** in a non-trivial topology!

Table of Contents

Schwinger's well-known In-Out Green's function for homogeneous fields is

$$G(x,y) = i\frac{\langle\Omega_{out}|\, T\{\Psi(x)\bar{\Psi}(y)\}\,|\Omega_{in}\rangle}{\langle\Omega_{out}|\Omega_{in}\rangle}$$
$$= (-\not{D} + im)\int_0^\infty dTK(x,y,T)$$

with kernel in proper time, T,

$$K(x,y,T) = \langle x|\exp(-i(\hat{\not{D}}^2 + m^2)T)\,|y\rangle\,.$$

- Kernel may be cast into worldline path integral representation.
- Kernel is solved in **homogeneous parallel electric and magnetic fields**.

In-Out Psuedoscalar and Axial Anomaly VEVs:

$$\frac{\langle\Omega_{out}|\,\bar{\Psi}\gamma^5\Psi\,|\Omega_{in}\rangle}{\langle\Omega_{out}|\Omega_{in}\rangle} = i\,\mathrm{tr}\,\gamma^5 G(x,x)$$

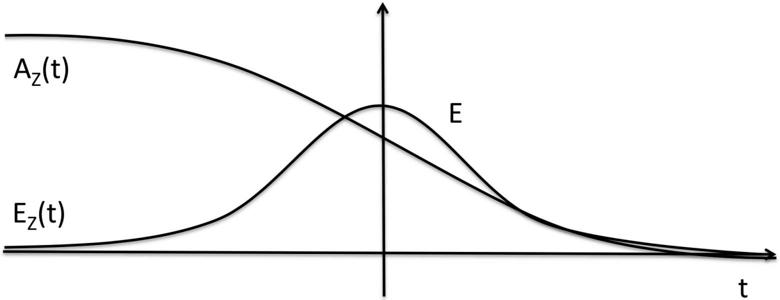
$$= \frac{ie^2 EB}{4m\pi^2}$$

$$\frac{\langle\Omega_{out}|\,\partial_\mu j^{\mu 5}\,|\Omega_{in}\rangle}{\langle\Omega_{out}|\Omega_{in}\rangle} = 0$$

- **Conservation of chiral current!?** No effects from Schwinger pair production.
- While a valid calculation, it's interpretation as a true mean expectation value is commonly misused.
- Also, naïve treatment of massless limit would lead to an incorrect result. Only after calculation can the massless limit be taken.

Table of Contents

Inequivalent asymptotic $t \to \pm\infty$ **states**, even for Sauter (pulsed) electric field.



$E(t) = E\cosh^{-2}(t)\hat{z}$ and $A_z = -E\tanh(t) + E$

$$\langle\Omega_{in}| \neq \langle\Omega_{out}|$$

Causal In-Out Green's function, G, for *mean* expected values is commonly misused.

- **Solution is provided by using similar, such as in-in, vacuum states**.
- An in-in precription is the same as a **Keldysh-Schwinger (KS), or real-time**, formalism.
- Any system which produces Schwinger pair production applies.
- However, direct application of the KS formalism to expectation values is challenging and few exact results are known...

Look for application of the worldline proper time formalism!

Fradkin, et. al.[3] have demonstrated that the In-In Green's function, $(\langle \Omega_{in} | \Omega_{in} \rangle = 1)$,

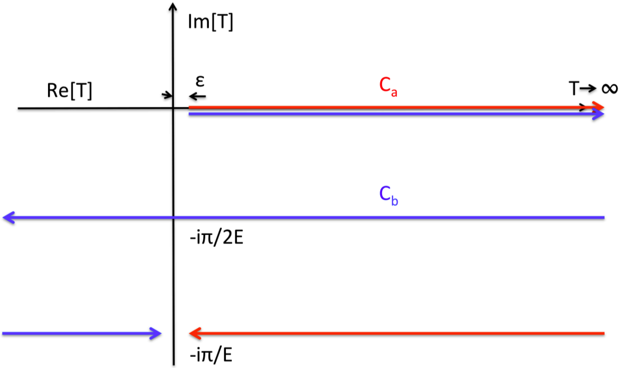
$$G_{in}(x, y) = i \langle \Omega_{in} | \, T\{\Psi(x)\bar{\Psi}(y)\} \, | \Omega_{in} \rangle \, ,$$

is expressible entirely in terms of the the worldline kernel, where $z = x - y$:

$$\begin{aligned} G_{in}(x, y) &= (-\not{D} + im) \\ &\quad \times \left[\theta(z_3) \int_{c_a} dT + \theta(-z_3) \int_{c_b} dT \right] K(x, y, T). \end{aligned}$$

[3] E. Fradkin, G. Gitman, and S. Shvartsman, *Quantum Electrodynamics in Unstable Vacuum* 1991.

In-In Green's Function
In-In Vacuum States



- KS-like contours in proper time.
- Return to In-Out Green's function in the absense of electric fields.

Table of Contents

In-In Pseudoscalar and Axial Anomaly VEVs:

$$\begin{aligned}\langle\Omega_{in}|\,\bar{\Psi}\gamma^5\Psi\,|\Omega_{in}\rangle &= i\,\mathrm{tr}\,\gamma^5 G_{in}(x,x)\\ &= \frac{ie^2EB}{4m\pi^2}\left[1-\exp\left(-\frac{m^2\pi}{eE}\right)\right]\\ \langle\Omega_{in}|\,\partial_\mu j^{\mu 5}\,|\Omega_{in}\rangle &= \frac{e^2EB}{2\pi^2}\exp\left(-\frac{m^2\pi}{eE}\right)\end{aligned}$$

- **Chirality is spontaneously generated from the vacuum through the Schwinger mechanism!** And only through the Schwinger mechanism.
- And moreover, we can see the **mass effects for the one-loop contributions to the axial-Ward identity**; the axial Ward identity is one-loop exact.
- The axial Ward identity is exponentially suppresed by the quadratic mass.

- **Formulation is exact**–for homogeneous parallel electric and magnetic fields.
- All Landau levels are kept, but **only the lowest Landau level contributes to the anomaly relation**.
  - We can understand this intuitively in that if a particle anti-particle pair is produced in parallel fields then higher Landau levels will not necessarily have their spin aligned with the magnetic field and cannot contribute to a net chirality.
- The electromagnetic current In-In VEV is also predictive of the CME, where the chirality is generated from the Schwinger mechanism.

Also, in an independent calculation, the **chirality density** may be found as

$$\langle\Omega_{in}|\,\bar{\Psi}\gamma^0\gamma^5\Psi\,|\Omega_{in}\rangle = \frac{e^2 EBt}{2\pi}\exp\left(-\frac{m^2\pi}{eE}\right),$$

confirming the above results.

Conclusions

- *How to generate chirality?* $\rightarrow$ The Schwinger mechanism.
- But what is the *Axial Ward identity VEV*, since $\langle \Omega_{in}| \neq \langle \Omega_{out}|$?
- **Calculate VEVs with both $\langle \Omega_{in}|$ states!**
- Get chirality production with mass effects, which come from the Schwinger mechanism.

Thank you for your time and attention!