Thermodynamics near the first order phase transition point of SU(3) gauge theory

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WHOT-QCD Collaboration

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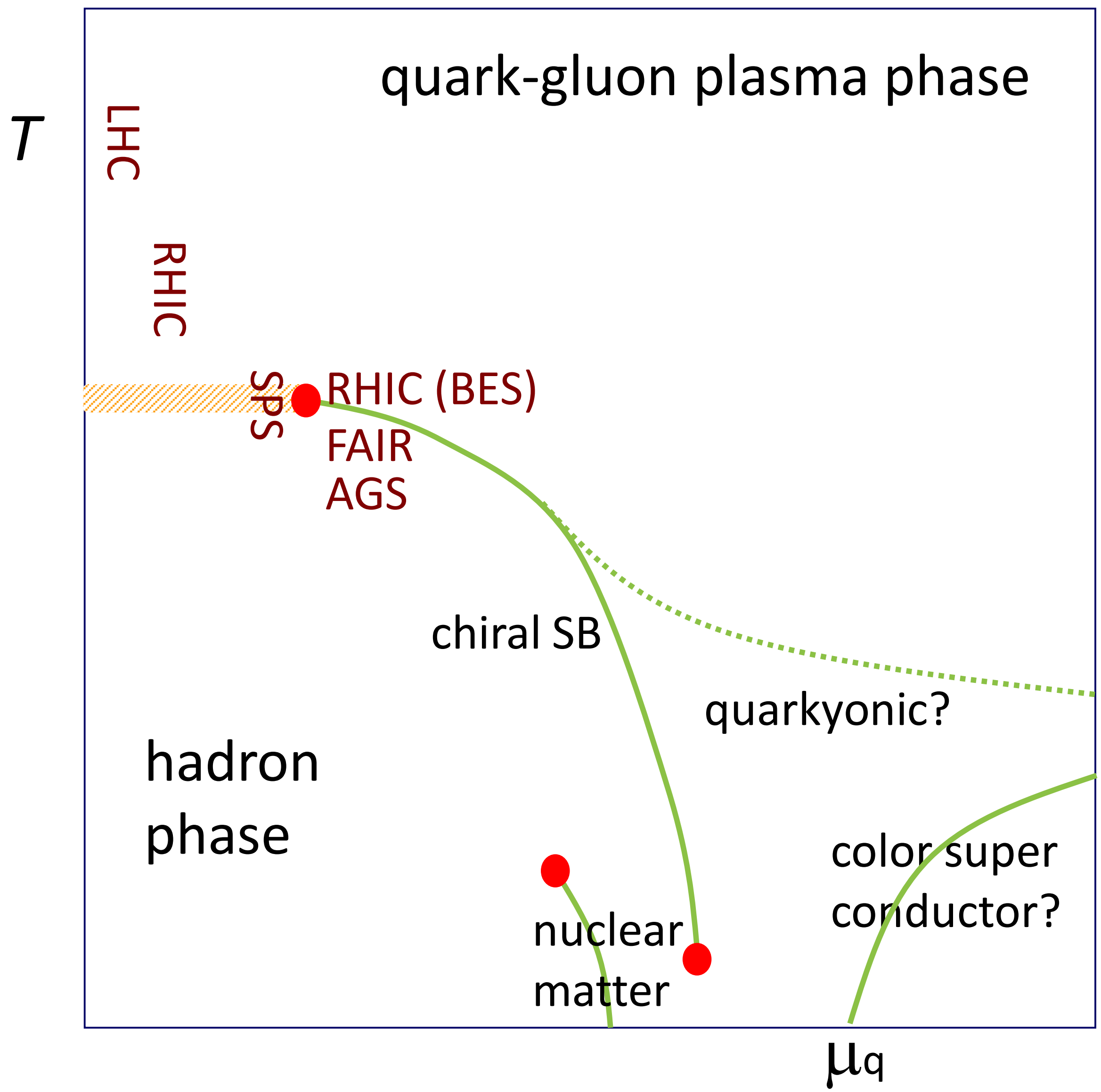
Phase structure of QCD at high temperature & density

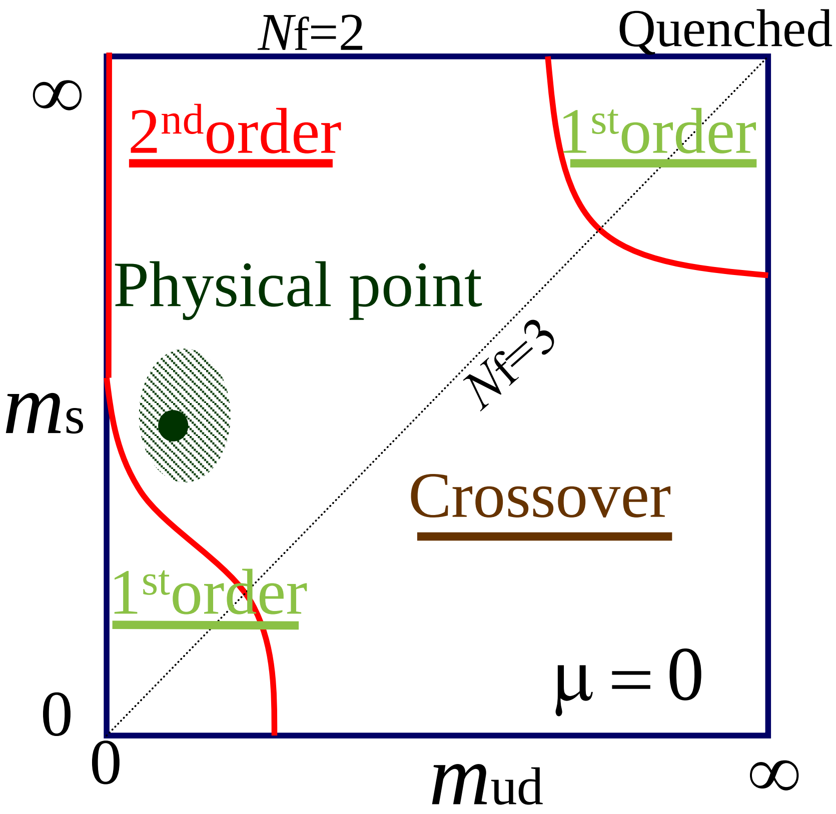
- Phase transition lines
- Critical point
- First order transition at high density

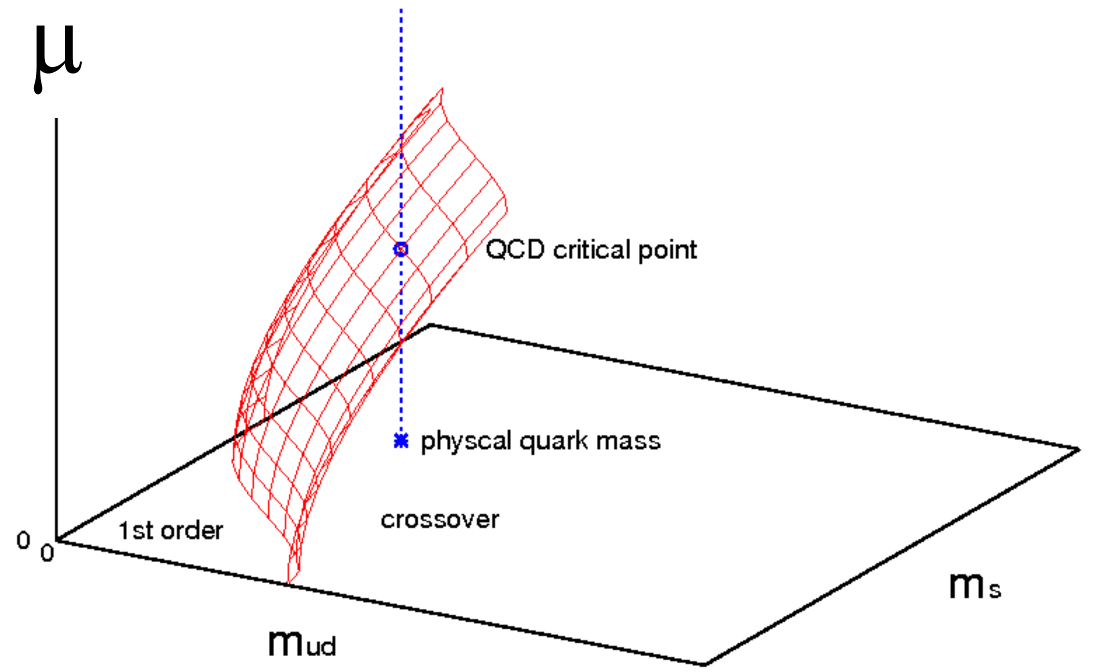


Lattice QCD Simulations

- Direct simulation:
  Impossible at $\mu \neq 0$.

Quark Mass dependence of QCD phase transition





- The determination of the boundary of 1st order region: important.
- On the line of physical mass, the crossover at low density

➡ 1st order transition at high density.

Contents of this talk

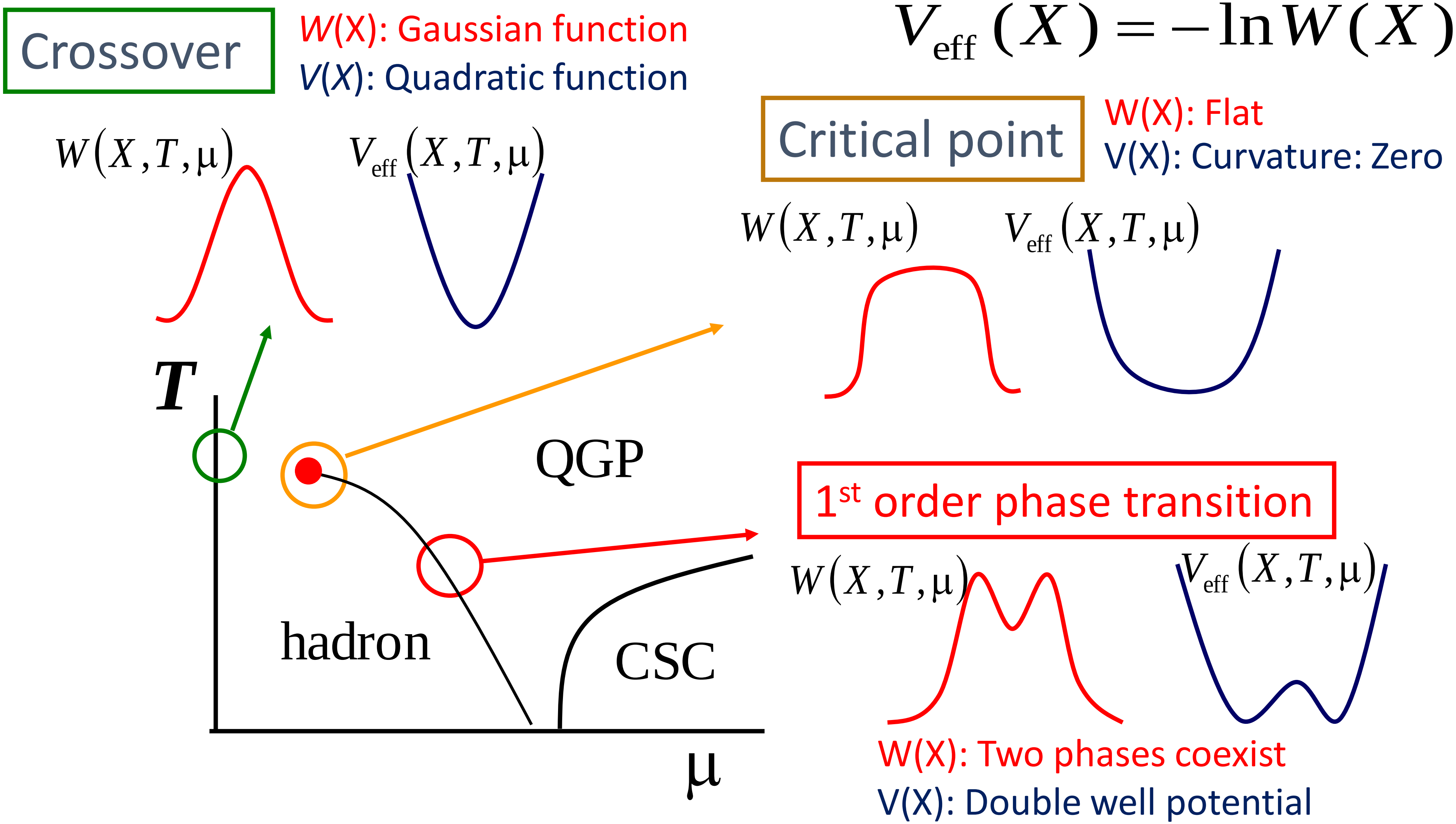
- First order phase transition in the SU(3) gauge theory (Quenched QCD) and heavy quark region of QCD
- Domain wall between two phases at 1st order transition
- Latent heat and pressure gap
  - Derivative method
  - Gradient flow method
- Endpoint of the 1st order phase transition at finite quark mass and chemical potential.

Distribution function & the effective potential

$$W(X;m,T,\mu) \equiv \frac{1}{Z}\int DU\,\delta\left(X-\hat{X}\right)\left(\det M(m,\mu)\right)^{N_f} e^{-S_g} \quad \text{(Histogram)}$$

X: order parameters, total quark number, average plaquette, etc.

$$V_{\rm eff}(X) = -\ln W(X)$$



Histogram method (Reweighting method)

- **Monte-Carlo method** (S_g: gauge action, M: qaurk matrix)
  - Generate configurations with the probability of the Boltzmann weight.

$$\langle O \rangle_{(m,T,\mu)} = \frac{1}{Z}\int DU\, O\left(\det M(m,\mu)\right)^{N_f} e^{-S_g} \approx \frac{1}{N_{\mathrm{conf.}}}\sum_{\{\mathrm{conf.}\}} O$$

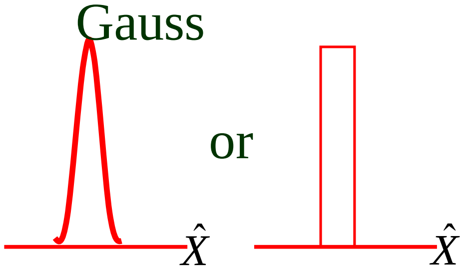
- Distribution function (Histogram)

  X: order parameters, total quark number, average plaquette etc.

$$W(X;m,T,\mu) \equiv \frac{1}{Z}\int DU\, \delta\left(X\text{-}\hat{X}\right)\left(\det M(m,\mu)\right)^{N_f} e^{-S_g}$$

$$W(X) \approx \frac{1}{N_{\mathrm{conf.}}}\sum_{\{\mathrm{conf.}\}} \delta\left(X\text{-}\hat{X}\right)$$

$$\delta\left(\hat{X}\right) \approx$$



  - Expectation values

$$\langle O[X] \rangle_{(m,T,\mu)} = \frac{1}{Z}\int dX\, O[X]\, W(X,m,T,\mu), \quad Z(m,T,\mu) = \int dX\, W(X,m,T,\mu)$$

Reweighting method for the plaquette distribution function

$$W(P,\beta,m,\mu) \equiv \int DU\,\delta(\hat{P}-P)\prod_{f=1}^{N_f}\det M(m_f,\mu_f)\,e^{6N_{\rm site}\beta\hat{P}} \qquad (\beta = 6/g^2)$$

$S_g = -6N_{\rm site}\beta\hat{P}$ plaquette P (1x1 Wilson loop for the standard action)

$$R(P,\beta,\beta_0 m,m_0,\mu) \equiv W(P,\beta,m,\mu)/W(P,\beta_0,m_0,0)$$ (Reweight factor)

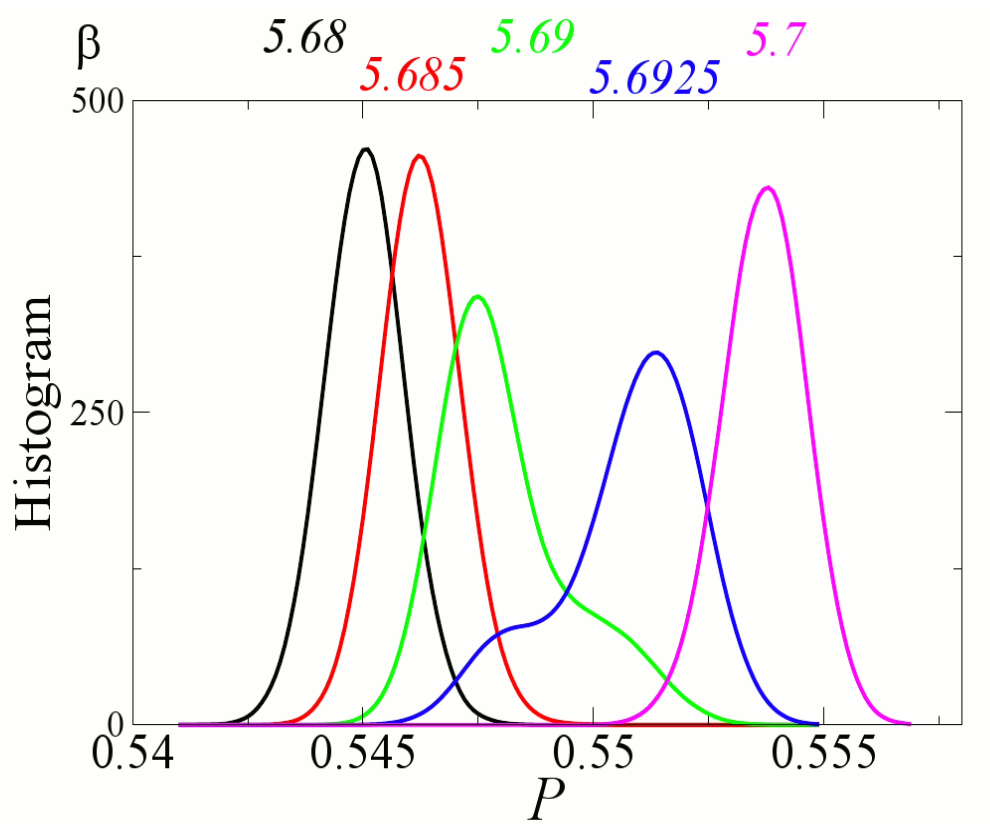
$$R(P) = \frac{\left\langle \delta(\hat{P}-P)\,e^{6N_{\rm site}(\beta-\beta_0)\hat{P}}\prod_f \frac{\det M(m_f,\mu_f)}{\det M(m_0,0)} \right\rangle_{(\beta_0,\mu=0)}}{\left\langle \delta(\hat{P}-P)\right\rangle_{(\beta_0,\mu=0)}} \equiv \left\langle e^{6N_{\rm site}(\beta-\beta_0)\hat{P}}\prod_f \frac{\det M(m_f,\mu_f)}{\det M(m_0,0)} \right\rangle_{P:{\rm fixed}}$$

Effective potential:

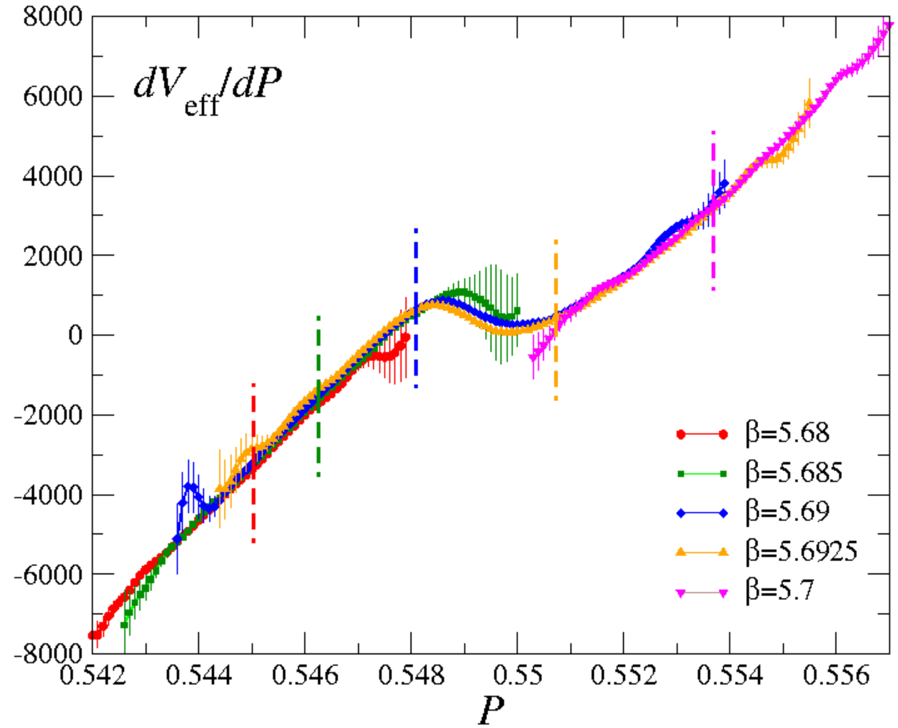
$$V_{\rm eff}(P,\beta,m,\mu) = -\ln\left[W(P,\beta,m,\mu)\right] = V_{\rm eff}(P,\beta_0,m_0,0) - \ln R(P,\beta,\beta_0 m,m_0,\mu)$$

Distribution function in quenched simulations

Plaquette histogram at $K=1/m_q=0$.



Derivative of V_{eff} at β=5.69



$$V_{\mathrm{eff}}(\beta_2) = V_{\mathrm{eff}}(\beta_1) - 6N_{\mathrm{site}}(\beta_2 - \beta_1)P$$

$$\frac{dV_{\mathrm{eff}}}{dP}(\beta_2) = \frac{dV_{\mathrm{eff}}}{dP}(\beta_1) - 6N_{\mathrm{site}}(\beta_2 - \beta_1)$$

First order phase transition

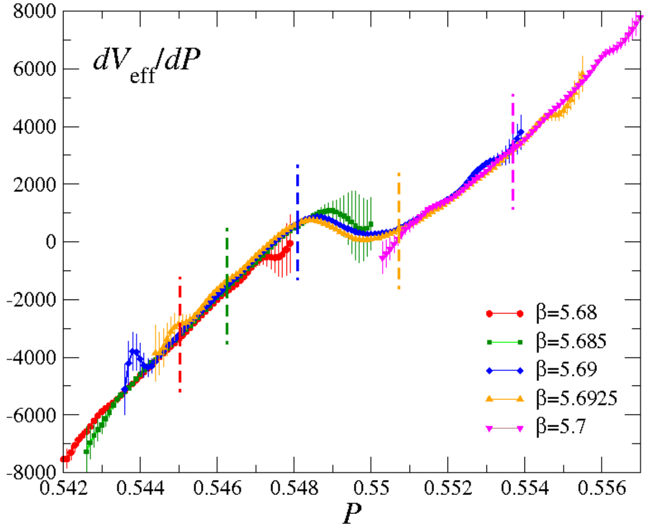
$dV_{\mathrm{eff}}/dP = 0$ at the peak position of $V_{\mathrm{eff}}(P)$.

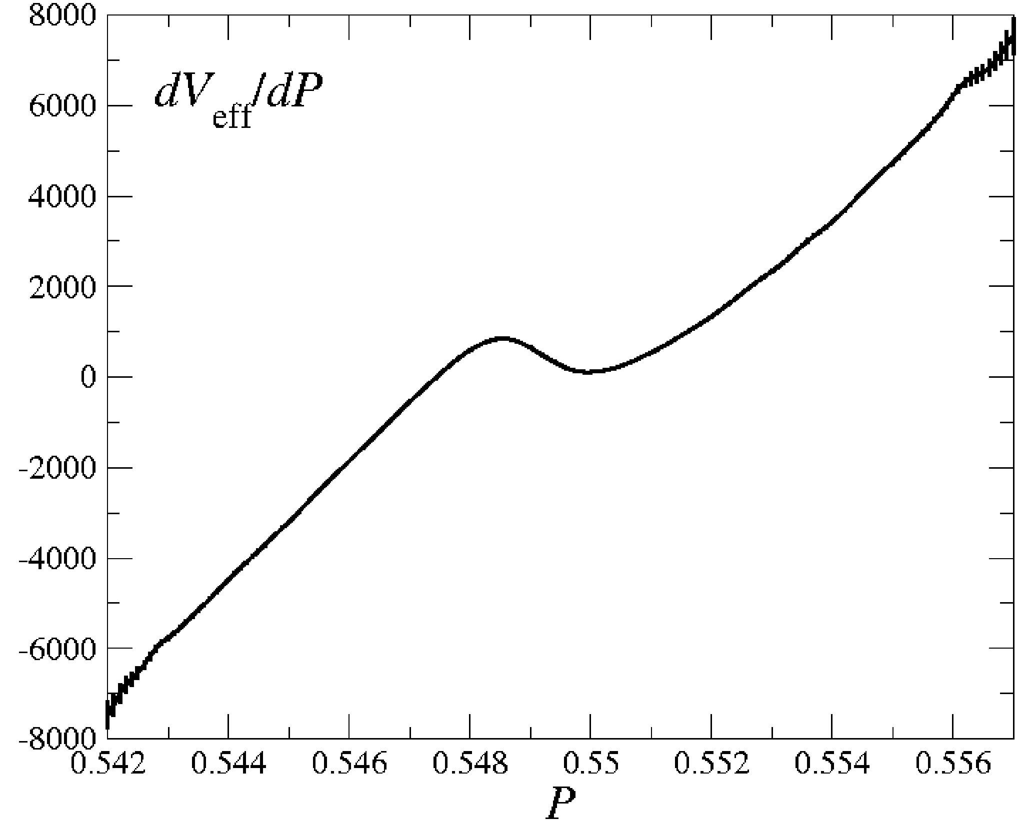
In this case, the curvature of V_{eff} is independent of β.

$$N_{\mathrm{site}} = 24^3 \times 4$$

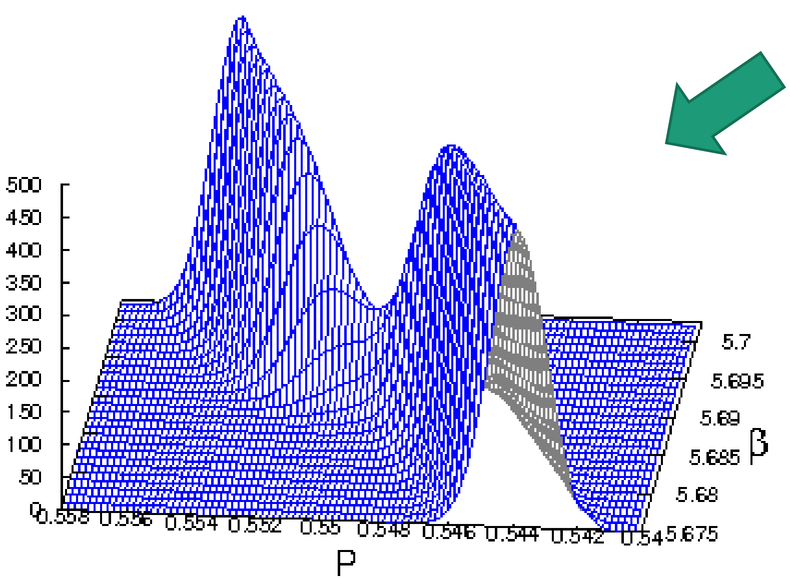
Distribution function in a quenched simulation

Derivative of the plaquette effective potential





Plaquette distribution function

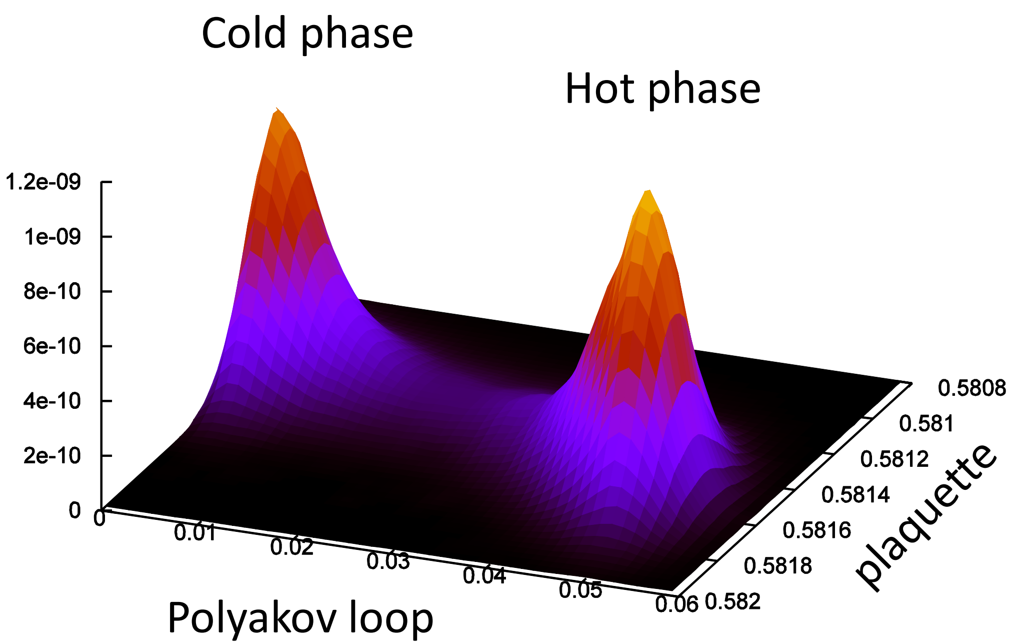


multi-point reweighting method

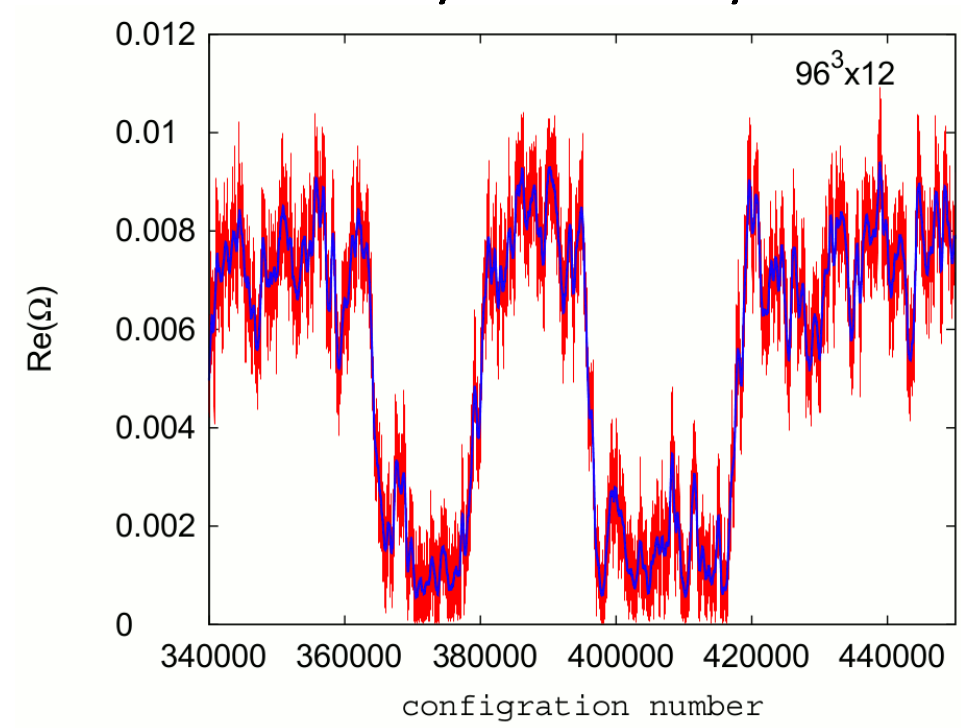
- Adopting β, average with the weight of N_{conf}
- Ferrenberg-Swendsen, Phys.Rev.Lett. 63, 1195 (1989); S.E., Phys. Rev. D78, 074507 (2008); WHOT-QCD, Phys.Rev.D89, 034507(2014)

Histogram and Time history

Histogram



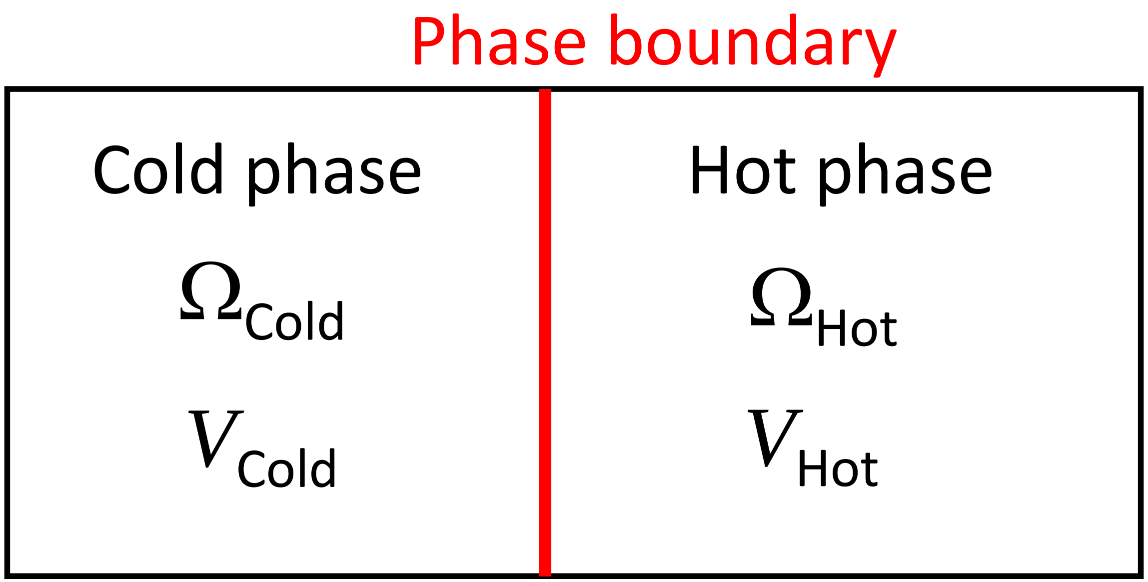
Time history of the Polyakov loop



- Polyakov loop: order parameter of the confinement
- Two peaks in the histogram.
- Flip-flops between two phases.
- Mixed configurations are rare.

Two phases coexists at the 1st order PT

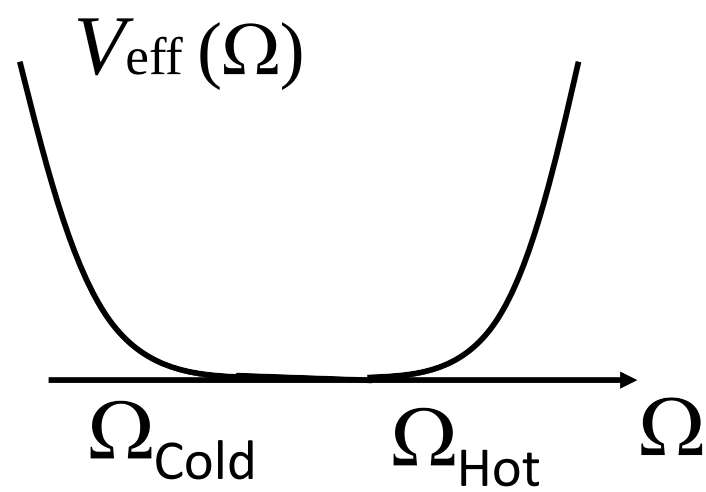
- Order parameter: Polyakov loop Ω (spatial average)



$$\Omega = \frac{V_{\text{Cold}}\Omega_{\text{Cold}} + V_{\text{Hot}}\Omega_{\text{Hot}}}{V}$$

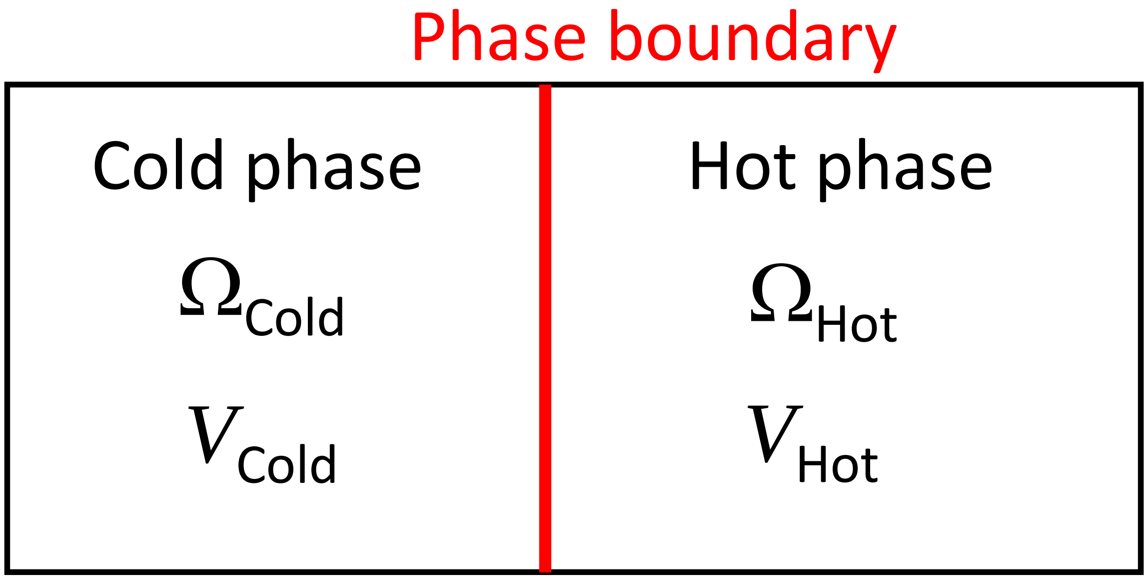
V: Volume

If no energy loss at the phase boundary, V_{eff} must be flat.



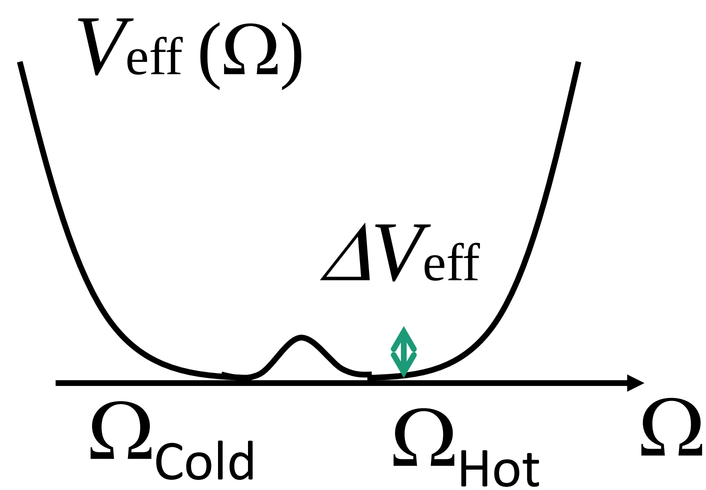
Two phases coexists at the 1st order PT

- Order parameter: Polyakov loop Ω (spatial average)



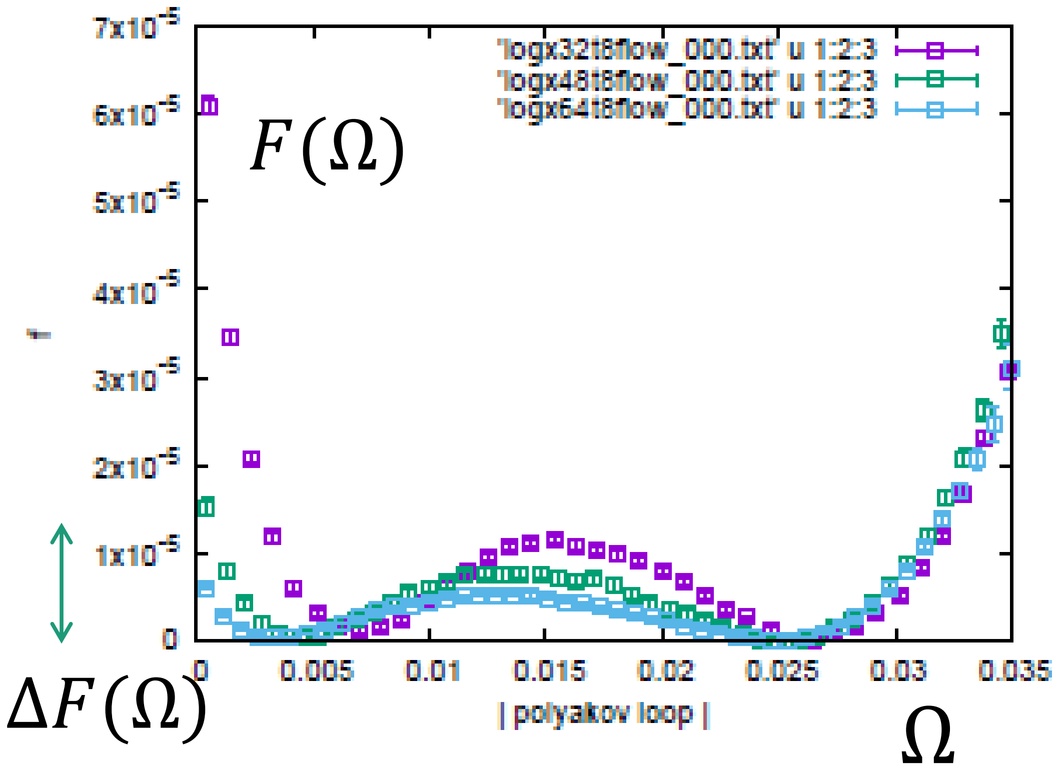
$$\Omega = \frac{V_{\text{Cold}}\Omega_{\text{Cold}} + V_{\text{Hot}}\Omega_{\text{Hot}}}{V}$$

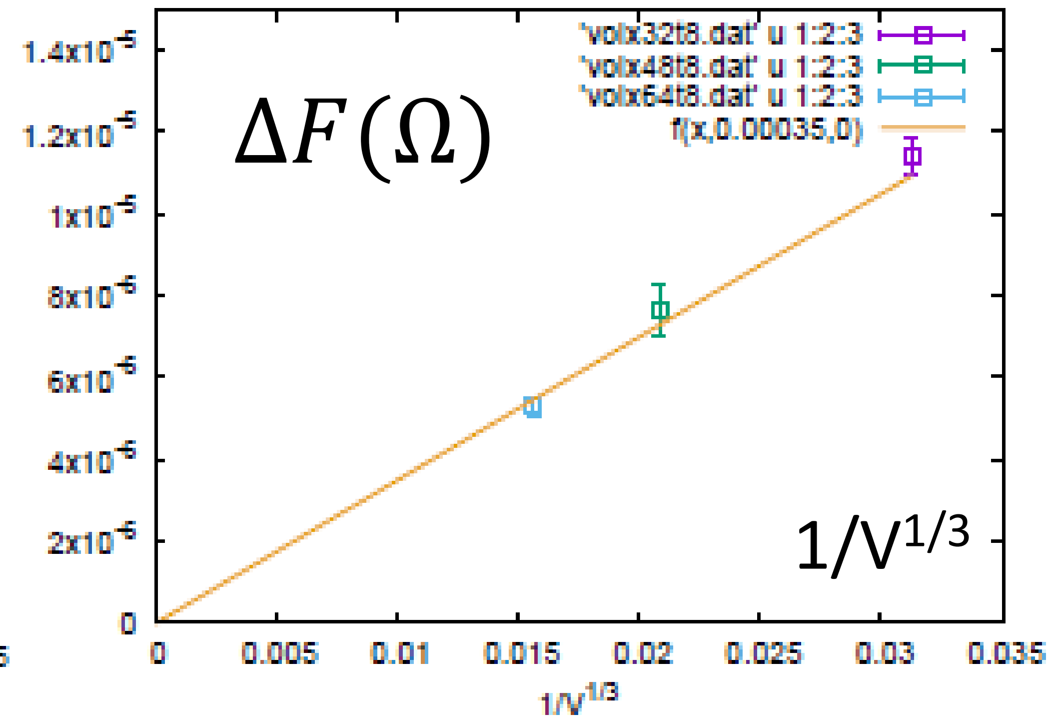
The energy loss ΔV at the phase boundary is in proportion to $V^{2/3}$



$$\Delta F(\Omega) = \frac{\Delta V_{\text{eff}}(\Omega)}{V} \sim {}^{1}\!/_{V^{1/3}}$$

Energy loss at the domain wall





$32^3 \times 8$ lattice
$48^3 \times 8$ lattice
$64^3 \times 8$ lattice

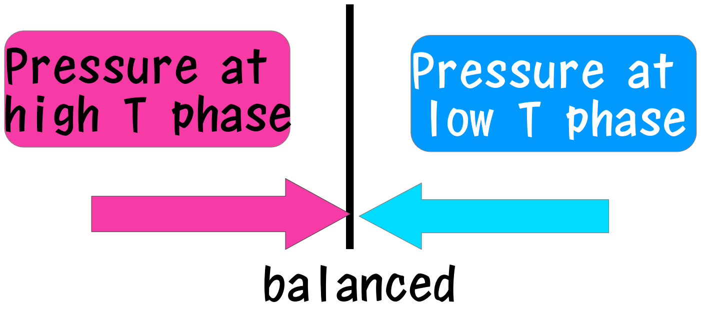
- $\Delta F(\Omega)$ is in proportion to $1/V^{1/3}$.
- It suggests a domain wall between two phases exsists at the 1st order phase transition.

Latent heat and pressure gap

Whot-QCD, Phys. Rev. D94, 014506 (2016) + α

- The latent heat (energy gap) the most basic quantity.
- The gap of pressure must vanish.

  Reliability of the calculation can be confirmed.

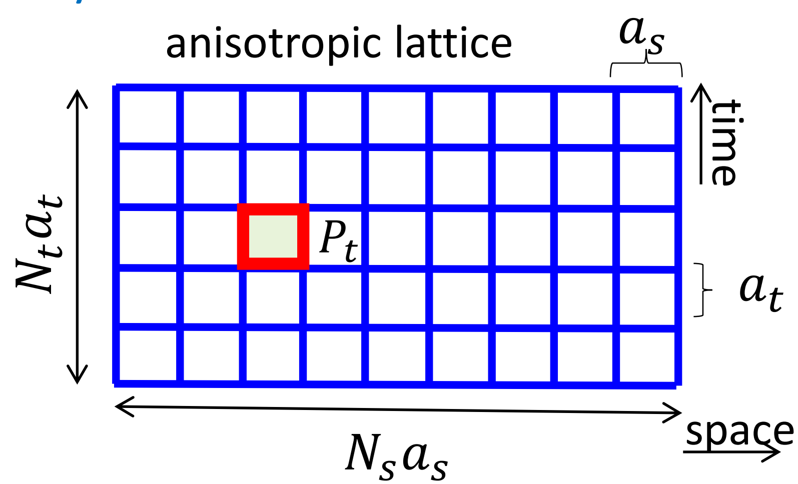


- We study the equation of state at the first order phase transition of SU(3) gauge theory.
- Gaps of energy density and pressure are measured using the derivative method.
  - Continuum extrapolation is performed.
- We tested the gradient flow method for the calculation of EoS.

[H. Suzuki, 2013]

Thermodynamic quantities by the derivative method

[F. Karsch, Nucl. Phys. B205 (1982) 285]



energy density

$$\epsilon = -\frac{1}{V}\frac{\partial lnZ}{\partial T^{-1}}\bigg|_V$$

pressure

$$p = T\frac{\partial lnZ}{\partial V}\bigg|_T$$

temperature $\frac{1}{T} = N_t a_t$ volume $V = (N_s a_s)^3$

$$Z = \int DU\, \mathrm{e}^{-S}$$

For the SU(3) gauge theory, $S = -3N_{\rm site}(\beta_s P_s + \beta_t P_t)$

($P_{s(t)}$ space-like (time-like) plaquette)

$$\epsilon = -\frac{3N_t^4 T^4}{\xi^3}\left\{\left(a_t\frac{\partial\beta_s}{\partial a_t} - \xi\frac{\partial\beta_s}{\partial\xi}\right)(\langle P_s\rangle - \langle P\rangle_0) - \left(a_t\frac{\partial\beta_t}{\partial a_t} - \xi\frac{\partial\beta_t}{\partial\xi}\right)(\langle P_t\rangle - \langle P\rangle_0)\right\}$$

$$p = \frac{N_t^4 T^4}{\xi^3}\left\{\frac{\partial\beta_s}{\partial\xi}(\langle P_s\rangle - \langle P\rangle_0) + \frac{\partial\beta_t}{\partial\xi}(\langle P_t\rangle - \langle P\rangle_0)\right\}$$

Independent variables: a_t, $\xi = \frac{a_s}{a_t}$

$\langle P\rangle_0$: The expectation value at $T = 0$

For $\xi = 1$, the gap of the energy density

$$\frac{\Delta\epsilon}{T^4} = -3N_t^4\left\{\left(a_t\frac{\partial\beta_s}{\partial a_t} - \frac{\partial\beta_s}{\partial\xi}\right)(\langle P_s\rangle_{\rm hot} - \langle P_s\rangle_{\rm cold}) + \left(a_t\frac{\partial\beta_t}{\partial a_t} - \frac{\partial\beta_t}{\partial\xi}\right)(\langle P_t\rangle_{\rm hot} - \langle P_t\rangle_{\rm cold})\right\}$$

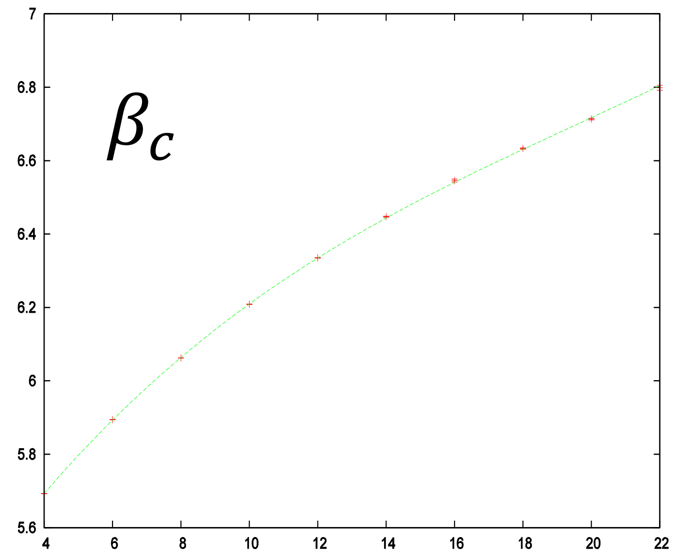
$$a_t\frac{\partial\beta_s}{\partial a_t}, a_t\frac{\partial\beta_t}{\partial a_t}, \frac{\partial\beta_s}{\partial\xi}, \frac{\partial\beta_t}{\partial\xi}$$

These 4 coefficients must be determined.

Determination of the anisotropy coefficients at $\xi = a_s/a_t = 1$

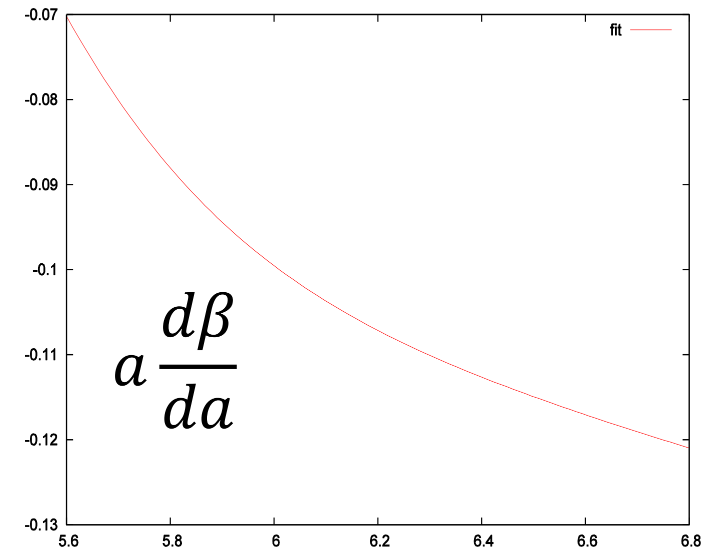
Isotropic lattice ($\beta = \beta_s = \beta_t$): $$\left(a_t \frac{\partial \beta_s}{\partial a_t}\right)_{\xi=1} = \left(a_t \frac{\partial \beta_t}{\partial a_t}\right)_{\xi=1} = a\frac{d\beta}{da}$$

$a\frac{d\beta}{da}$ is determined by the data of the critical b ($\beta_c(N_t)$)



$$\frac{1}{T_c} = N_t a_t$$

$$a\frac{d\beta}{da} = -N_t \frac{d\beta}{dN_t}$$



Data: Francis, kaczmarek, Laine, Neuhaus, Ohno, Phys. Rev. D 91, 096002 (2015) and our data for $N_t = 4\sim22$

String tension is independent of $\xi = \frac{a_s}{a_t}$

$$\left(\frac{\partial \beta_s}{\partial \xi} + \frac{\partial \beta_t}{\partial \xi}\right)_{a_t:\mathbf{fixed},\xi=1} = \frac{3}{2} a \frac{d\beta}{da}$$

[F. Karsch, Nucl. Phys. B205 (1982) 285]

Ratio of the anisotropy coefficients

The slope of the phase transition line in the (β_s, β_t) plane: r_t

[Ejiri, Iwasaki, Kanaya, Phys.Rev.D 58,094505 (1998)]

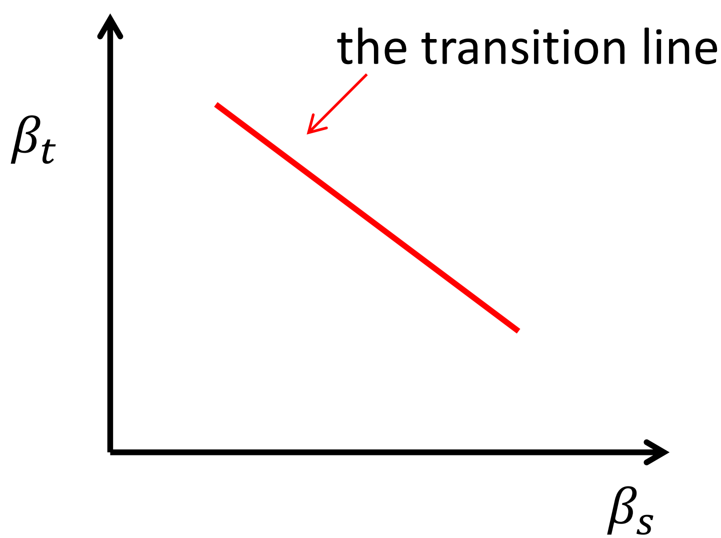
Along the phase transition line, a_t is constant

because $\frac{1}{T_c} = N_t a_t$.

When one changes

$(\beta_s, \beta_t) \rightarrow (\beta_s + d\beta_s, \beta_t + d\beta_t)$,

$$\mathrm{d}a_t = \frac{\partial a_t}{\partial \beta_t} d\beta_s + \frac{\partial a_t}{\partial \beta_t} d\beta_t = 0$$



The slope of the transition line

$$r_t = \frac{d\beta_s}{d\beta_t} = -\frac{\left(\frac{\partial a_t}{\partial \beta_t}\right)_{\xi=1}}{\left(\frac{\partial a_t}{\partial \beta_s}\right)_{\xi=1}} = \frac{\left(\frac{\partial \beta_s}{\partial \xi}\right)_{\xi=1}}{\left(\frac{\partial \beta_t}{\partial \xi}\right)_{\xi=1}}$$

$$\begin{pmatrix} \frac{\partial \beta_s}{\partial a_t} & \frac{\partial \beta_t}{\partial a_t} \\ \frac{\partial \beta_s}{\partial \xi} & \frac{\partial \beta_t}{\partial \xi} \end{pmatrix} = \frac{1}{\left(\frac{\partial \xi}{\partial \beta_t}\right)\left(\frac{\partial a_t}{\partial \beta_s}\right) - \left(\frac{\partial \xi}{\partial \beta_s}\right)\left(\frac{\partial a_s}{\partial \beta_t}\right)} \begin{pmatrix} \frac{\partial \xi}{\partial \beta_t} & -\frac{\partial \xi}{\partial \beta_s} \\ -\frac{\partial a_t}{\partial \beta_t} & \frac{\partial a_t}{\partial \beta_s} \end{pmatrix}$$

Using the reweighting method,

(β_s, β_t)-dependence of

the Polyakov loop susceptibility is measured.

Anisotropy coefficients

$$\left(\frac{\partial \beta_s}{\partial \xi} + \frac{\partial \beta_t}{\partial \xi}\right)_{a_t:\mathbf{fixed},\xi=1} = \frac{3}{2} a \frac{d\beta}{da}$$

$$r_t = \frac{\left(\frac{\partial \beta_s}{\partial \xi}\right)_{\xi=1}}{\left(\frac{\partial \beta_t}{\partial \xi}\right)_{\xi=1}}$$

$$\Rightarrow \quad \left(\frac{\partial \beta_s}{\partial \xi}\right)_{\xi=1} = \frac{3r_t}{2(1+r_t)} a \frac{d\beta}{da} \qquad \left(\frac{\partial \beta_t}{\partial \xi}\right)_{\xi=1} = \frac{3}{2(1+r_t)} a \frac{d\beta}{da}$$

Conventional combinations of the energy density and pressure

$$\frac{\Delta(\epsilon + p)}{T^4} = 3N_t^4 a \frac{d\beta}{da} \frac{r_t - 1}{r_t + 1} \{(\langle P_s \rangle_{\mathrm{hot}} - \langle P_s \rangle_{\mathrm{cold}}) - (\langle P_t \rangle_{\mathrm{hot}} - \langle P_t \rangle_{\mathrm{cold}})\}$$

$$\frac{\Delta(\epsilon - 3p)}{T^4} = 3N_t^4 a \frac{d\beta}{da} \{(\langle P_s \rangle_{\mathrm{hot}} - \langle P_s \rangle_{\mathrm{cold}}) - (\langle P_t \rangle_{\mathrm{hot}} - \langle P_t \rangle_{\mathrm{cold}})\}$$

Measurement of the slope of the transition line r_t

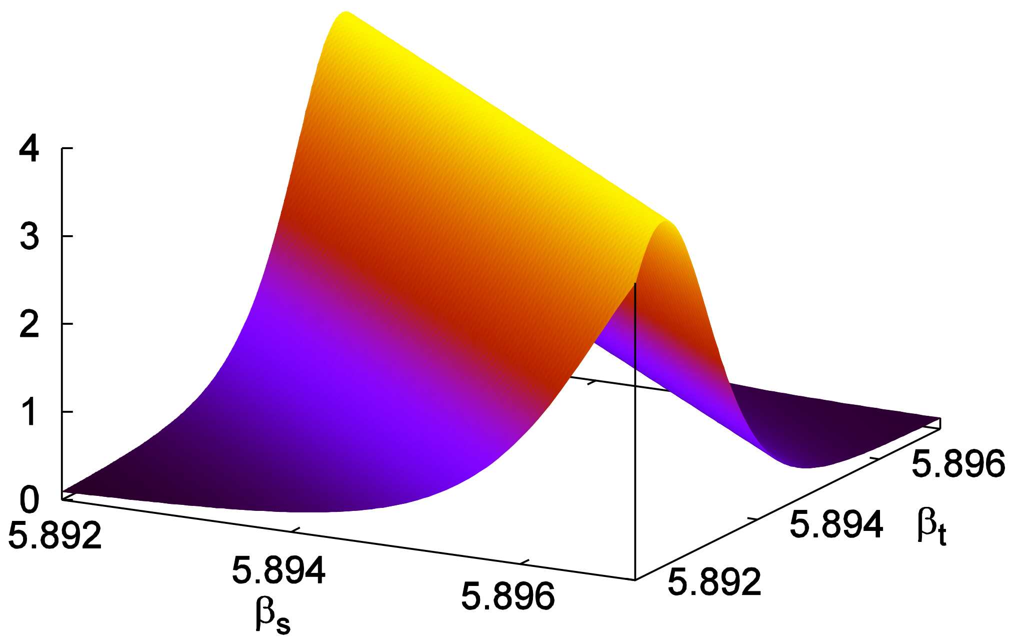
$$r_t = \frac{d\beta_s}{d\beta_t} = \frac{\left(\frac{\partial \beta_s}{\partial \xi}\right)_{\xi=1}}{\left(\frac{\partial \beta_t}{\partial \xi}\right)_{\xi=1}}$$

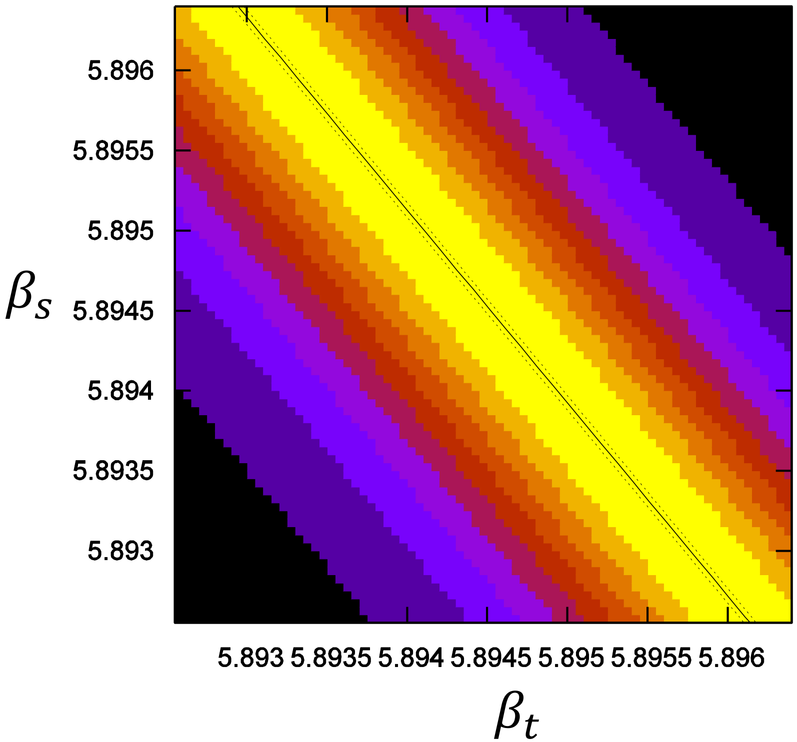
We used the reweighting method.
The slope r_t can be determined with sufficient accuracy.

Order parameter: Polyakov loop $\Omega(x,t)$

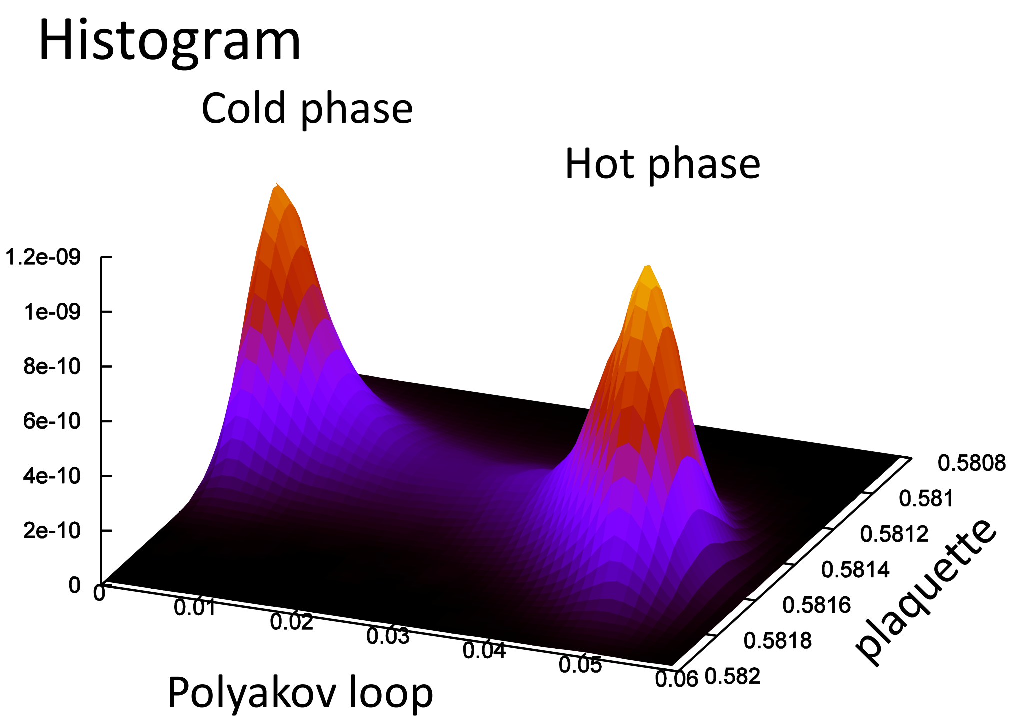
Transition point:
Peak position of Polyakov loop susceptibility

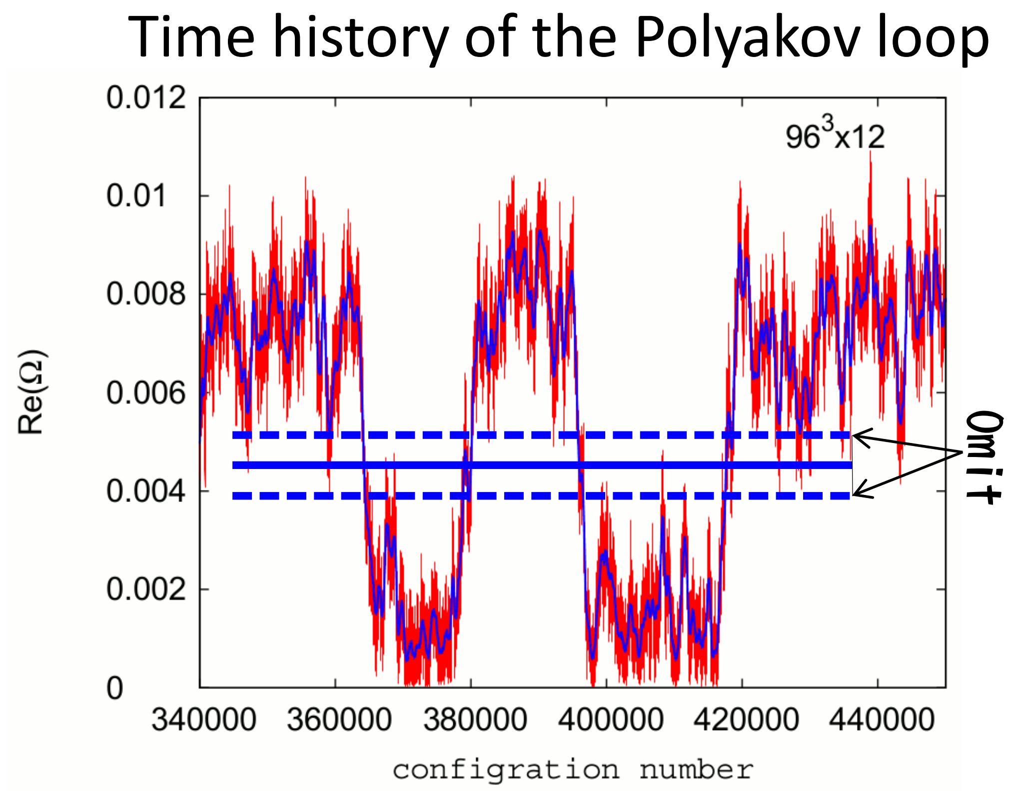
$$\chi_\Omega(\beta_s,\beta_t) = N_s^3\left(\langle\Omega^2\rangle_{(\beta_s,\beta_t)} - \langle\Omega\rangle^2_{(\beta_s,\beta_t)}\right)$$





Separation of the hot and cold phases





- We identify the phase by the Polyakov loop.
- Two peaks in the histogram.
- Flip-flops between two phases.
- Mixed configurations are rare. (We omit mixed configurations.)

Vanishing pressure gap $\Delta p = 0$

$$\frac{\Delta p}{T^4} = N_t^4 \left\{ \frac{\partial \beta_s}{\partial \xi} \left(\langle P_s \rangle_{\text{hot}} - \langle P_s \rangle_{\text{cold}} \right) + \frac{\partial \beta_t}{\partial \xi} \left(\langle P_t \rangle_{\text{hot}} - \langle P_t \rangle_{\text{cold}} \right) \right\} = 0$$

Condition for $\Delta p = 0$

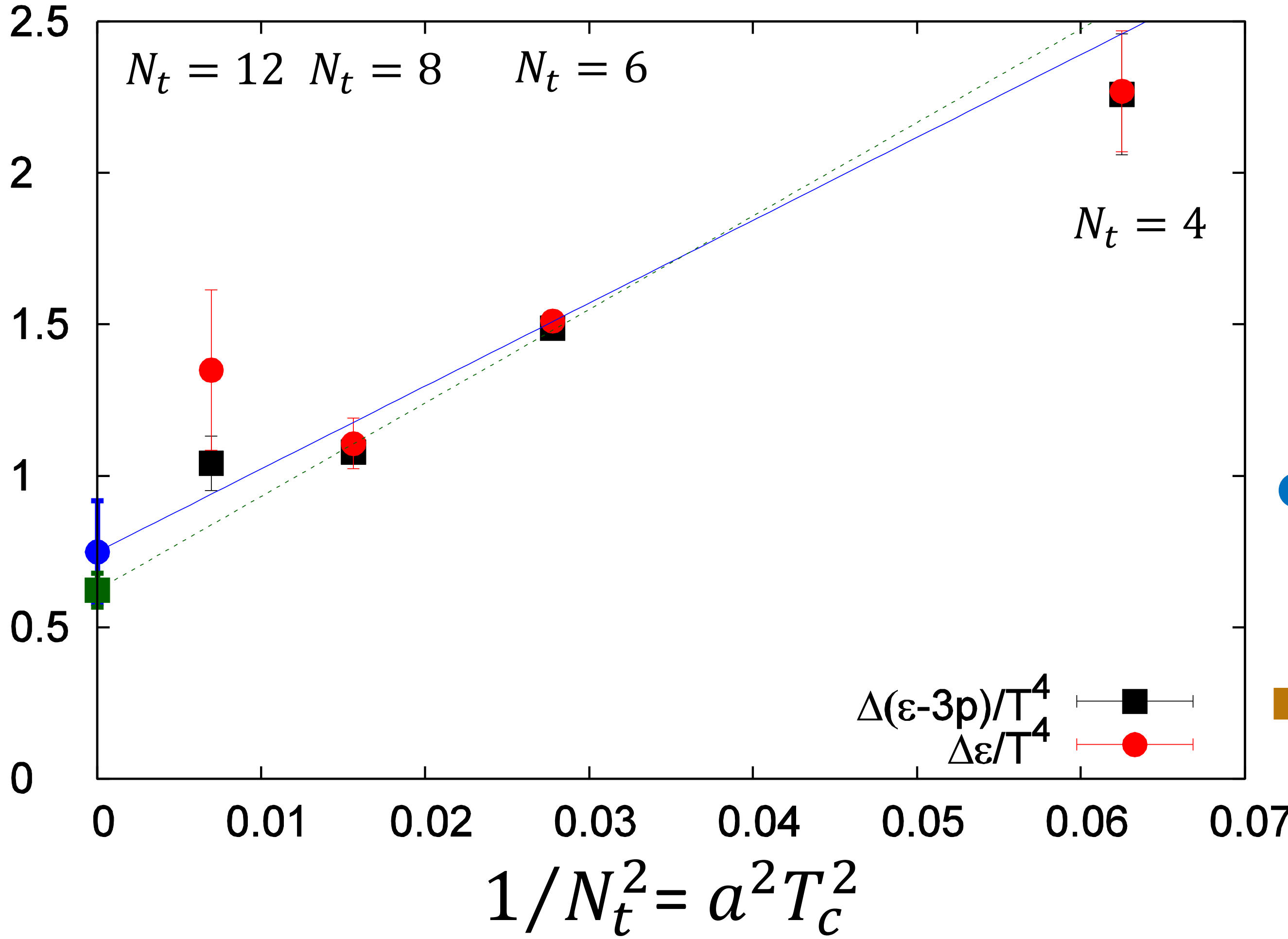
$$\frac{\frac{\partial \beta_s}{\partial \xi}}{\frac{\partial \beta_t}{\partial \xi}} = r_t = -\frac{\langle P_t \rangle_{\text{hot}} - \langle P_t \rangle_{\text{cold}}}{\langle P_s \rangle_{\text{hot}} - \langle P_s \rangle_{\text{cold}}}$$

lattice	r_t	$\frac{\langle P_t \rangle_{hot} - \langle P_t \rangle_{cold}}{\langle P_s \rangle_{hot} - \langle P_s \rangle_{cold}}$
$48^3 \times 6$	-1.2020(39)	1.216(50)
$64^3 \times 6$	-1.2022(52)	1.2053(38)
$48^3 \times 8$	-1.209(33)	1.204(14)
$64^3 \times 8$	-1.255(37)	1.2344(66)
$64^3 \times 12$	-1.16(61)	1.324(84)
$96^3 \times 12$	-1.204(53)	1.283(53)

The pressure gap is zero on each finite lattice.

Continuum extrapolation of the latent heat

Fit the data at $N_t = 6, 8, 12$ with a linear function of $1/N_t^2$ assuming $O(a^2)$ error.



continuum limit

$$\frac{\Delta\epsilon}{T^4} = 0.75 \pm 0.17$$

$$\frac{\Delta\epsilon - 3\Delta p}{T^4} = 0.623 \pm 0.056$$

Whot-QCD, Phys. Rev. D94, 014506 (2016)

EoS by the Gradient Flow (H. Suzuki, 2013)

[H. Suzuki, Prog. Theor. Exp. Phys. 2014, 083B03 (2013); FlowQCD, Phys. Rev. D90, 011501(2014)]

- Gradient flow: smearing by a kind of diffusion equation
- Smeared field strength: $F_{\mu\nu} \xrightarrow{\text{Gradient Flow}} G_{\mu\nu}$
- Dim. 4 operators:

$$E(t,x) = \frac{1}{4}\delta_{\mu\nu} G_{\rho\sigma}(t,x) G_{\rho\sigma}(t,x)$$

$$U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x) G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu} G_{\rho\sigma}(t,x) G_{\rho\sigma}(t,x)$$

- Energy momentum tensor

$$T^R_{\mu\nu} = \lim_{t\to 0}\left\{\frac{1}{\alpha_U(t)} U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}\left[E(t,x) - \langle E(t,x)\rangle_0\right]\right\}$$

$$\alpha_U(t) = g^2[1 + 2b_0 s_1 g^2 + \cdots] \qquad \alpha_E(t) = \frac{1}{2b_0}[1 + 2b_0 s_2 g^2 + \cdots]$$

g: running coupling constant with $\overline{\mathrm{MS}}$ scheme based on the 4-loop beta function.

EoS by the Gradient Flow (H. Suzuki, 2013)

[H. Suzuki, Prog. Theor. Exp. Phys. 2014, 083B03 (2013); FlowQCD, Phys. Rev. D90, 011501(2014)]

- Energy density and Pressure

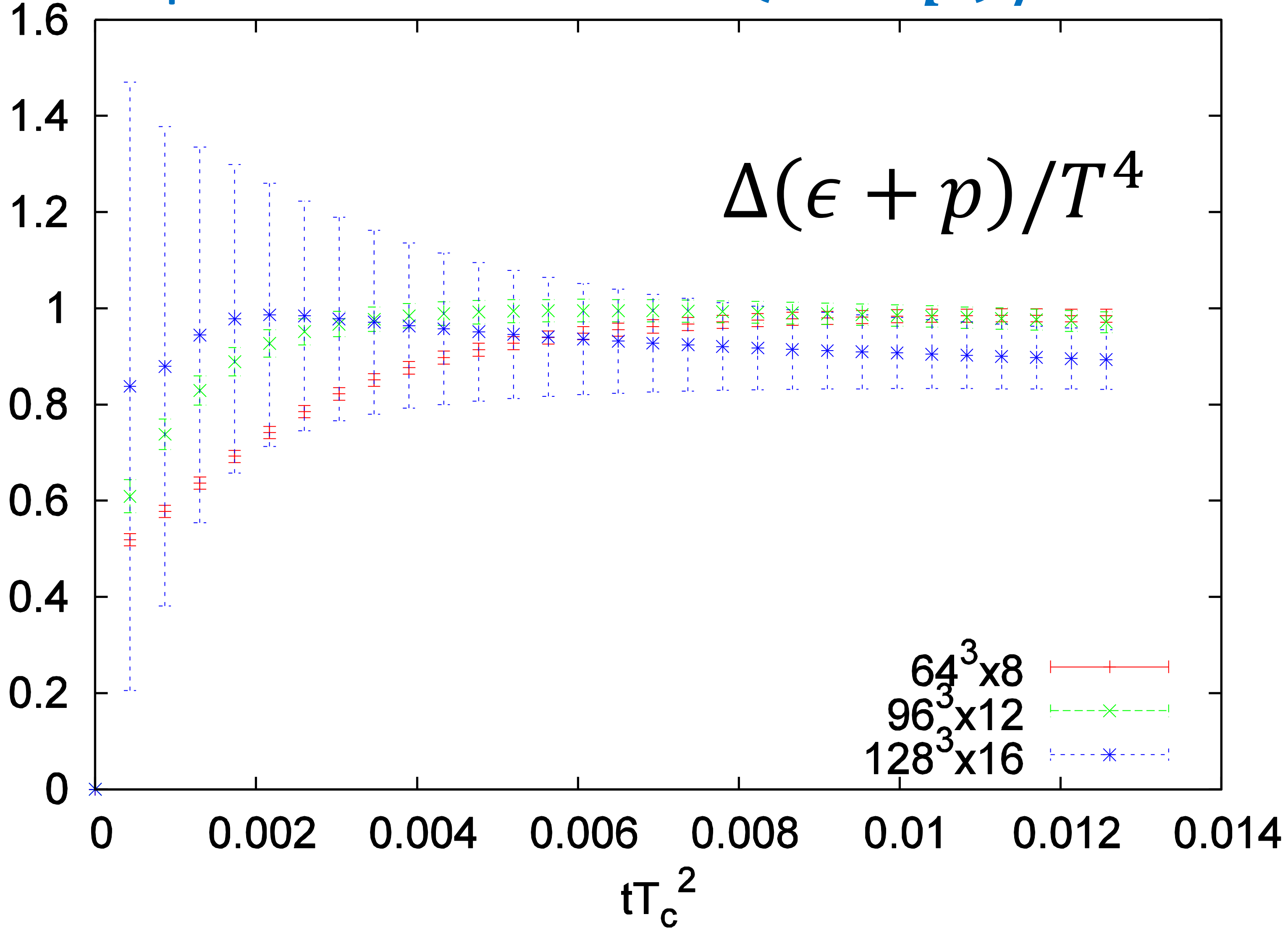
$$\epsilon = \langle T_{00} \rangle \qquad\qquad p = \frac{1}{3}\sum_i \langle T_{ii} \rangle$$

- Take the continuum limit for fixed flow time and volume

- Take the t=0 limit to remove unwanted higher dimension operators.

- Take the volume infinity limit

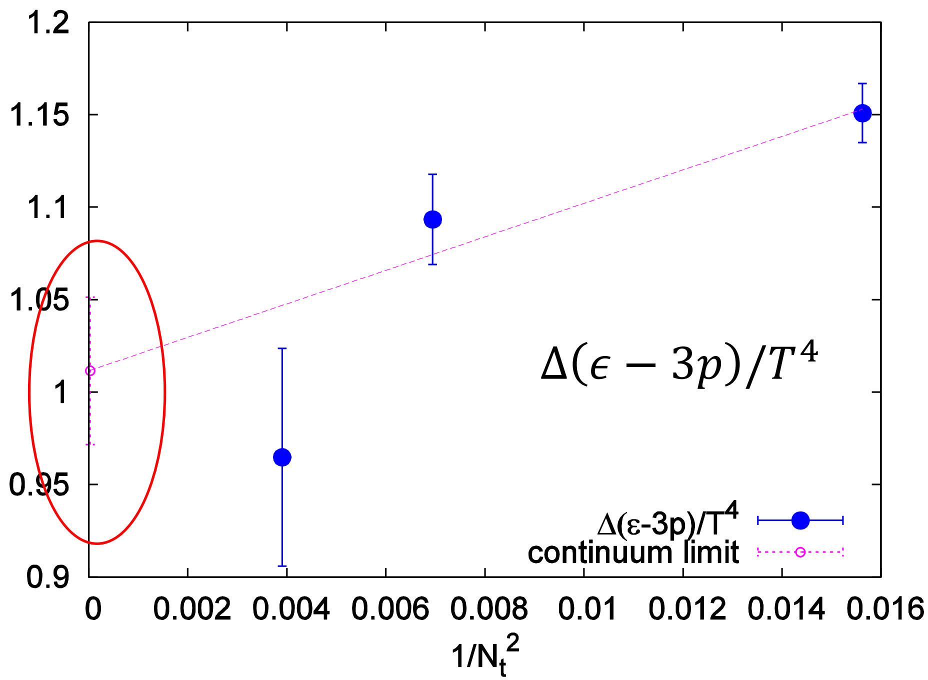
T-dependence of $\Delta(\epsilon + p)/T^4$

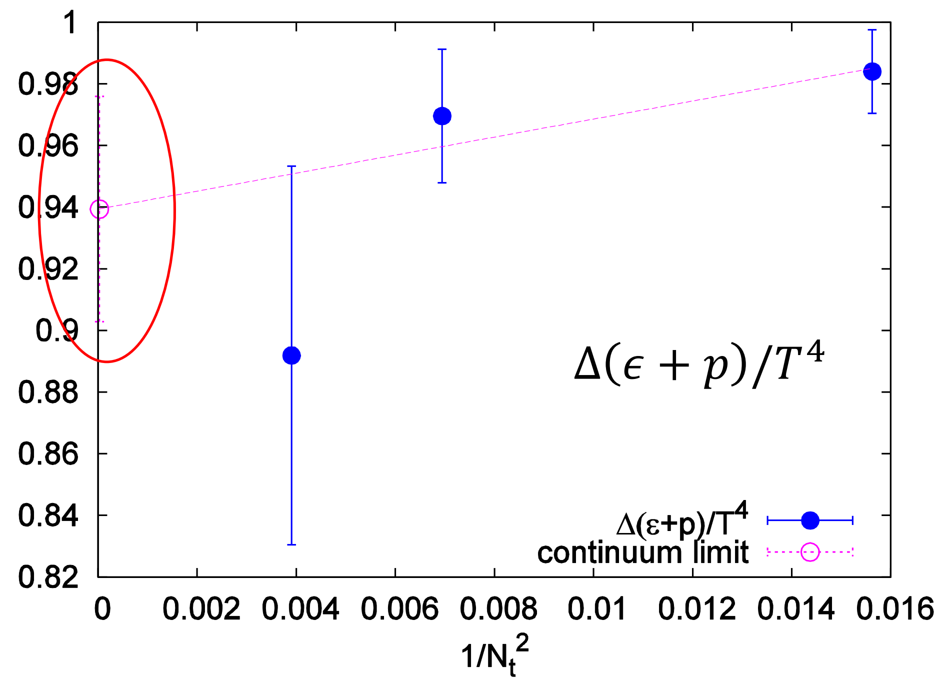


- Lattice discretization error is reduced as t increases.
- Statistical error is also reduced.
- Continuum extrapolation form the data on the same volume

$$(N_s/N_t = V^{\frac{1}{3}}T_c)$$

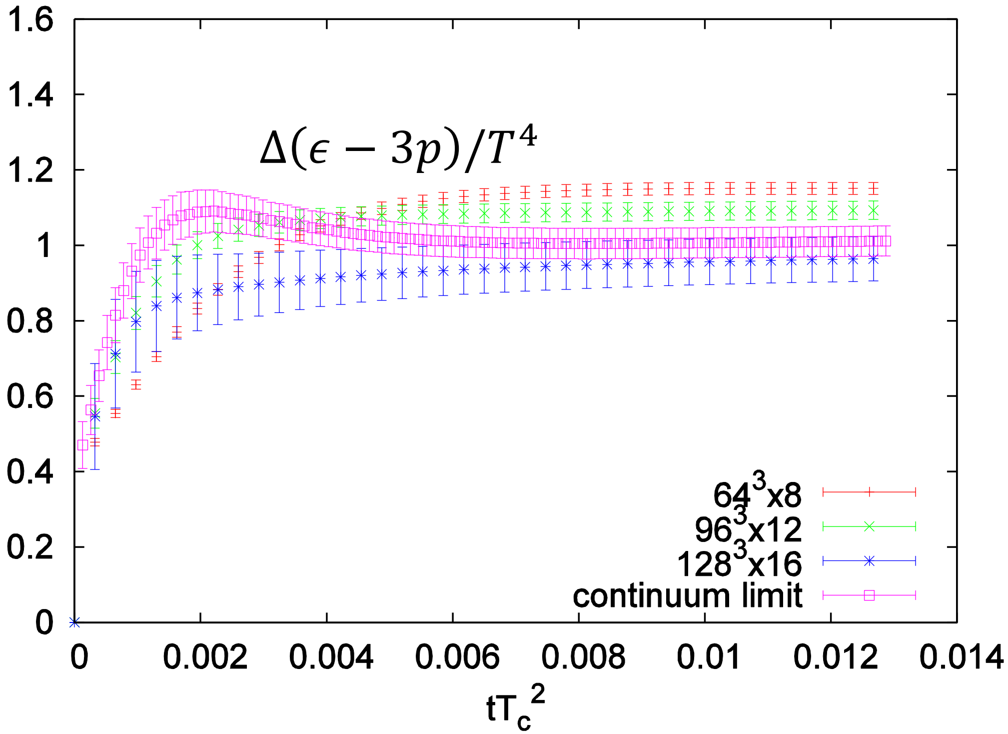
Continuum extrapolation

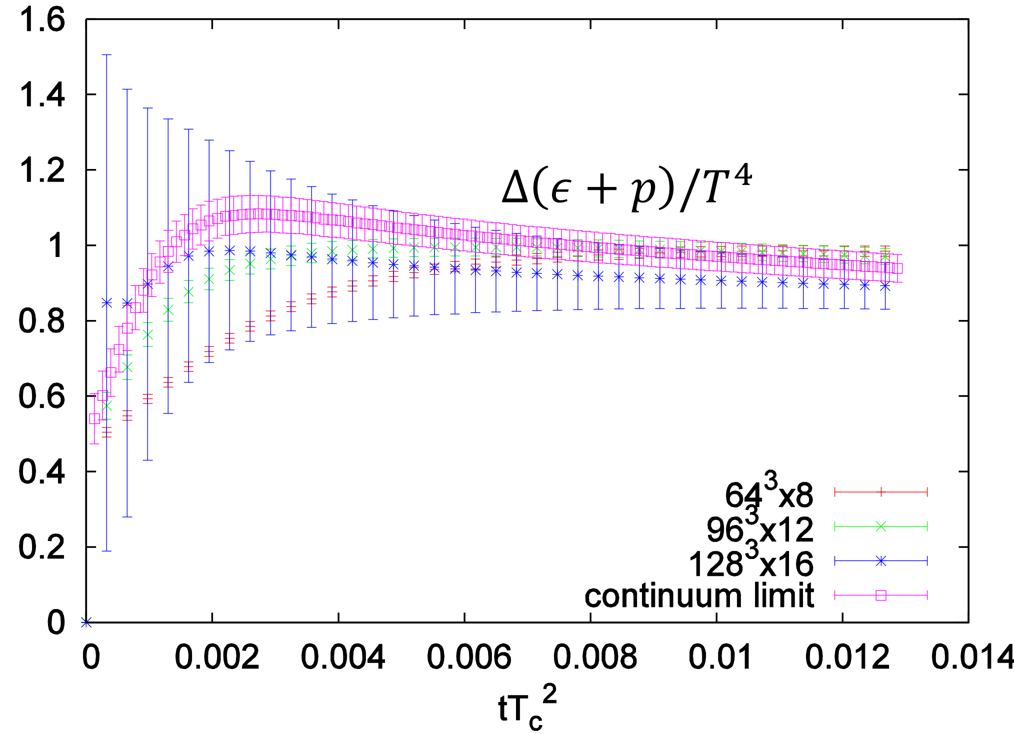




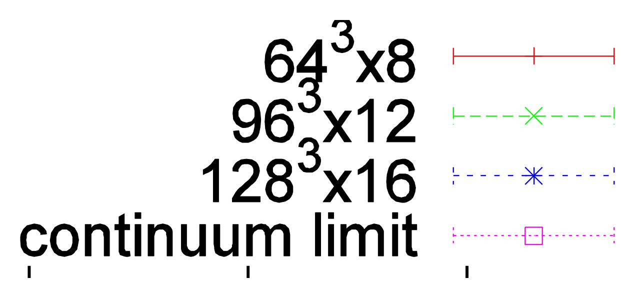
- Continuum extrapolation for $N_s/N_t = V^{\frac{1}{3}}T_c$ = 8. tT_c^2=0.013
- Horizontal axis is $1/N_t^2 \propto a_t^2$.

T-dependence, Continuum limit



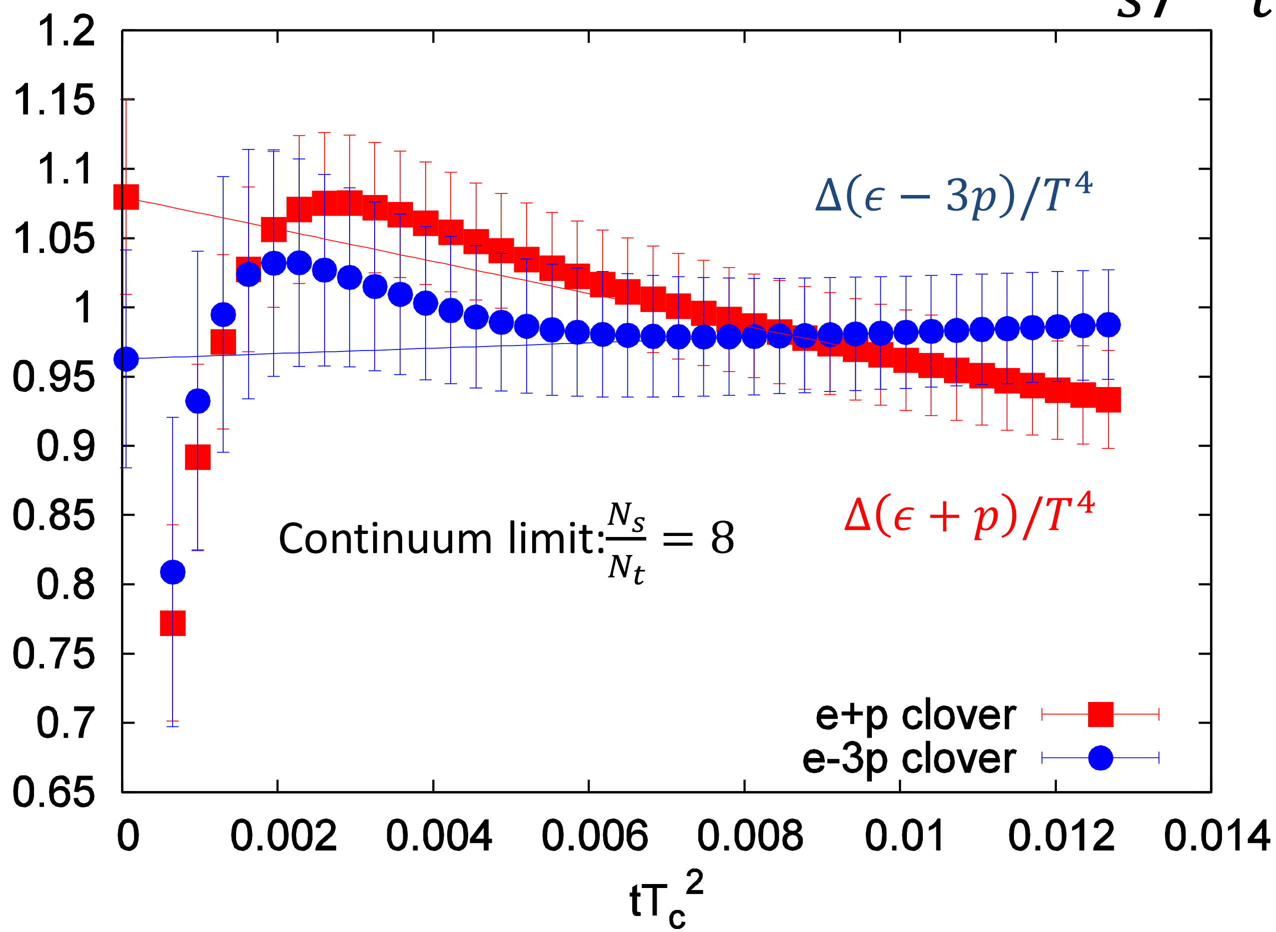


- The same spatial volume ($N_s/N_t = V^{\frac{1}{3}}T_c$)
- We take the t=0 limit with t>0.008.
- magenta: continuum limit



$\Delta p = 0.$ in the continuum limit

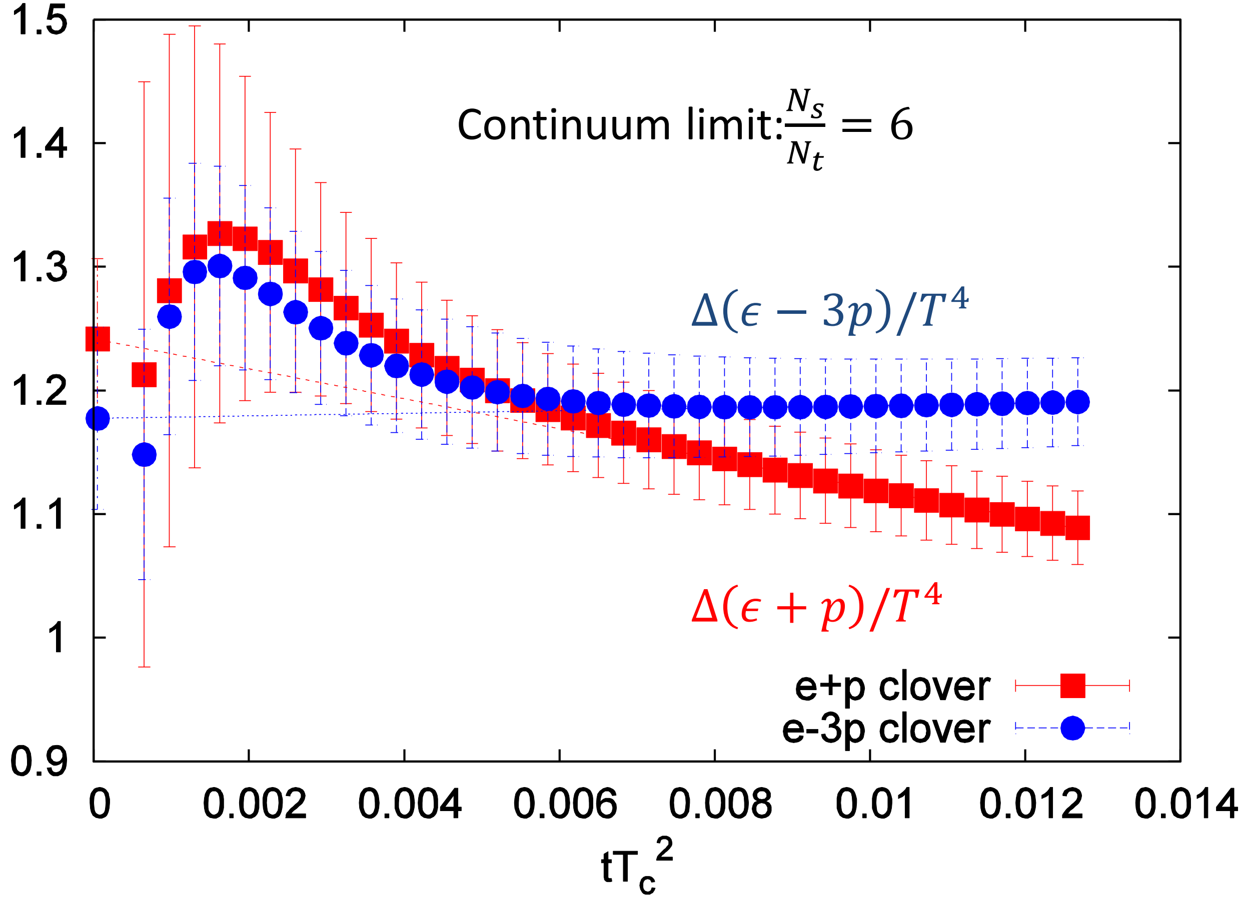
$N_s/N_t = V^{\frac{1}{3}}T_c = 8$



- These two results are consistent within the errors.
- This results suggests $\Delta p \to 0.$

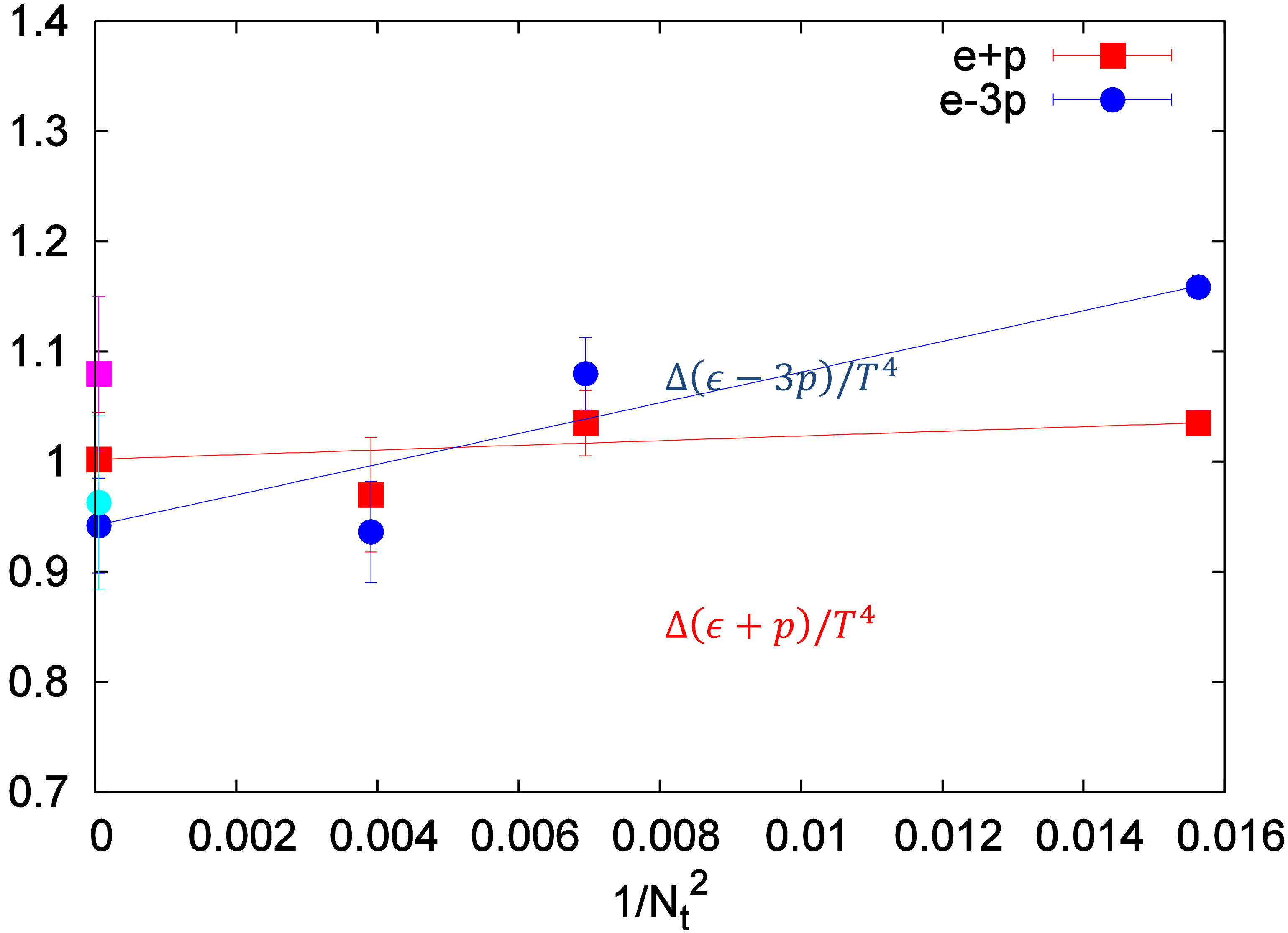
$\Delta p = 0.$ in the continuum limit

$$N_s/N_t = V^{\frac{1}{3}}T_c = 6$$



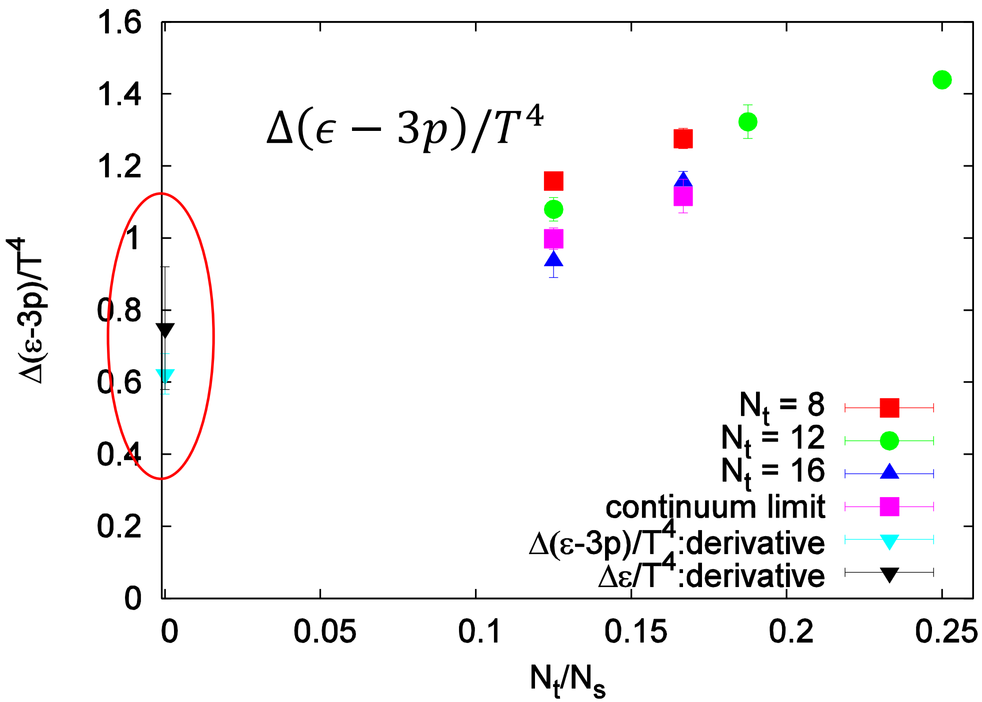
- These two results are consistent within the errors.
- This results suggests $\Delta p \to 0.$

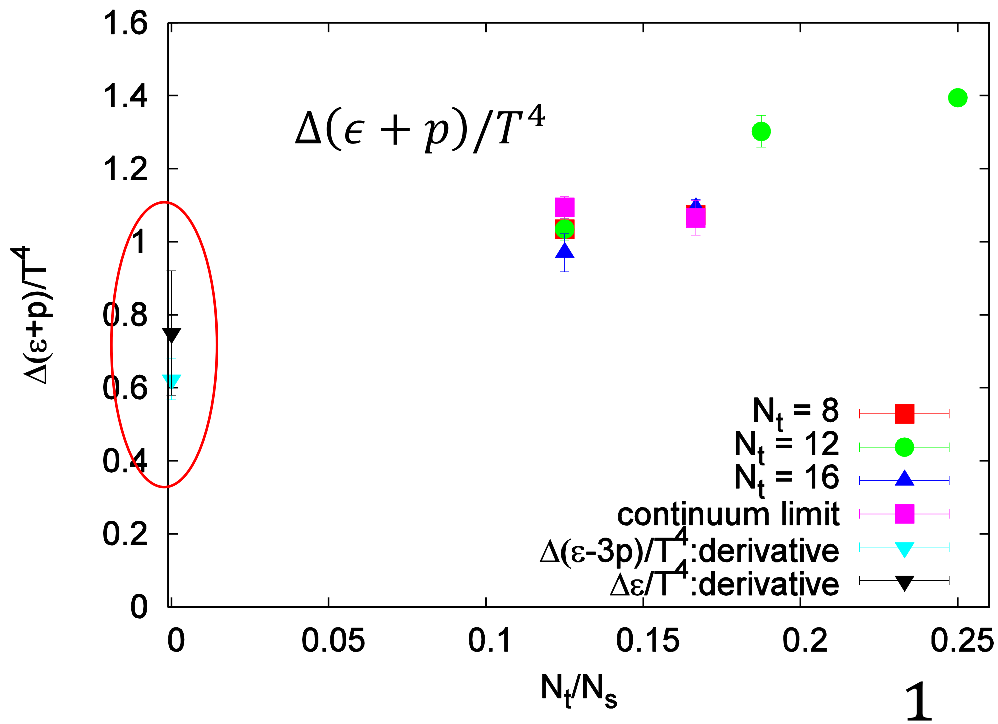
Continuum limit and t=0 limit



- t=0 limit with keeping finite lattice spacing. Δp=0 for Nt=12 and 16.
- Continuum limit after t=0 limit and t=0 after continuum limit are consistent.

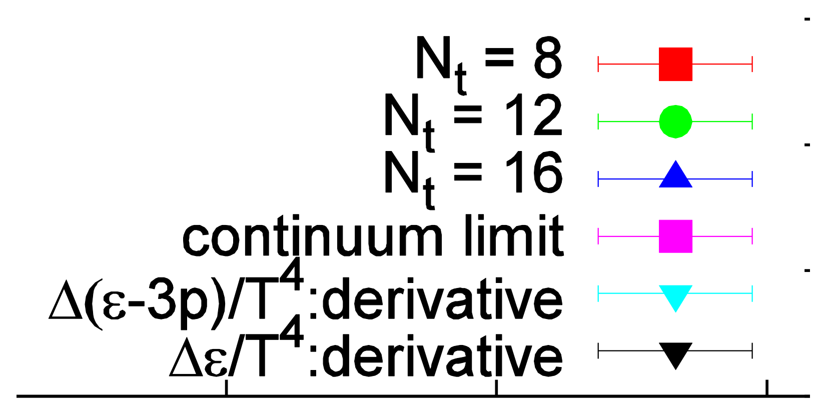
Volume dependence of latent heat





$$N_s/N_t = V^{\frac{1}{3}} T_c$$

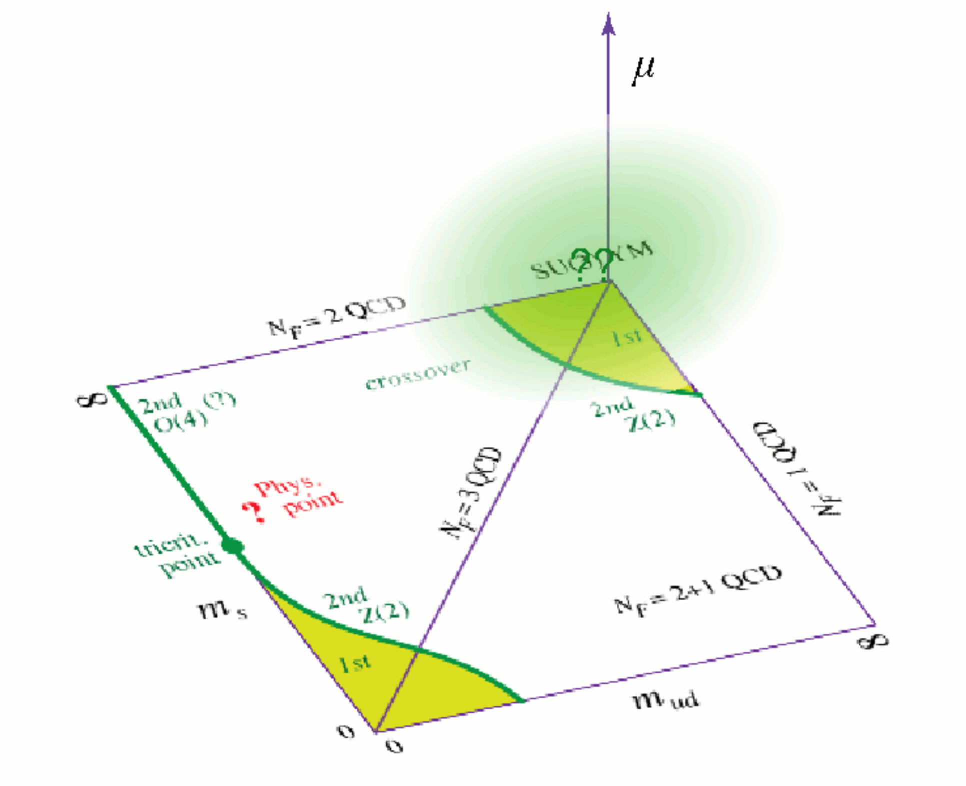
Results by the derivative method



- The volume dependence is visible.
- The results by the gradient flow method approaches that by the derivative method as the volume increases.

Endpoint of the first order Phase Transition

(WHOT-QCD Collab., Phys.Rev.D84, 054502(2011); Phys.Rev.D89, 034507(2014))



- We study the properties of $W(X)$ in the heavy quark region.
- Performing quenched simulations + Reweighting.
- We find the critical surface.
- Standard Wilson quark action + plaquette gauge action, $S_g = -6N_{\rm site}\beta P$

Hopping parameter expansion

κ～1/(quark mass)

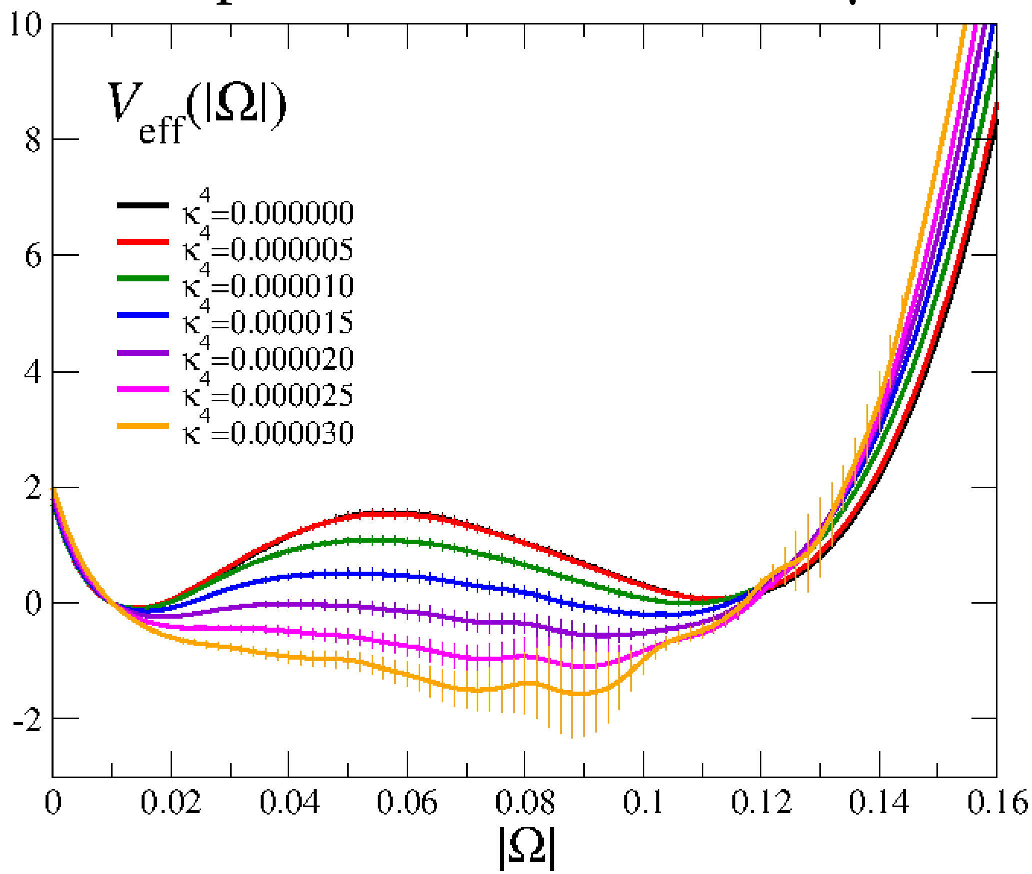
$$N_{\rm f}\ln\left(\frac{\det M(\kappa,\mu)}{\det M(0,0)}\right) = N_{\rm f}\left(288N_{\rm site}\kappa^4 P + 12\cdot 2^{N_t} N_s^3 \kappa^{N_t}\left(\cosh(\mu/T)\Omega_R + i\sinh(\mu/T)\Omega_I\right) + \cdots\right)$$

phase

P: plaquette, $\Omega=\Omega_R+i\Omega_I$: Polyakov loop $\det M(0,0)=1$

Order of the phase transition
Polyakov loop distribution (2-flavor)

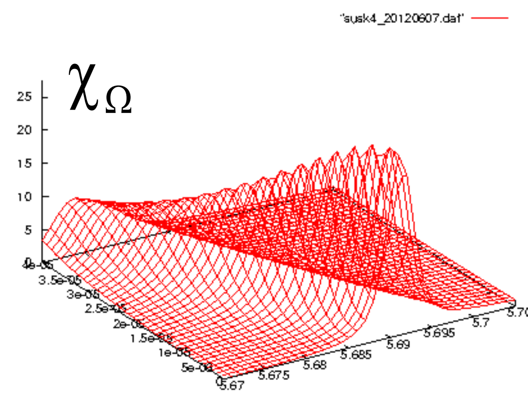
Effective potential of $|\Omega|$
on the pseudo-critical line at $\mu=0$



$$\text{Critical point}: \kappa^4 \approx 2.0\times10^{-5}$$

Phys.Rev.D89, 034507(2014)

- The pseudo-critical line is determined by χ_Ω peak.



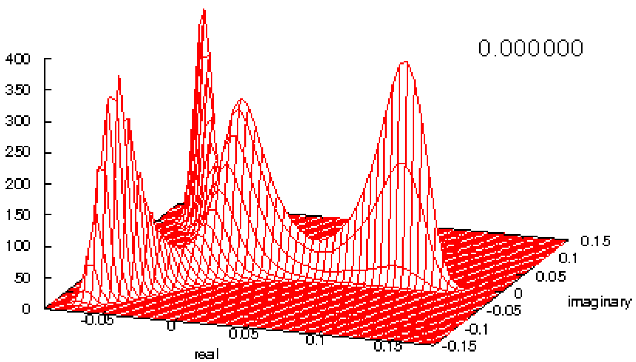
- Double-well at small κ
  - First order transition
- Single-well at large κ
  - Crossover

$\kappa \sim 1/(\text{quark mass})$

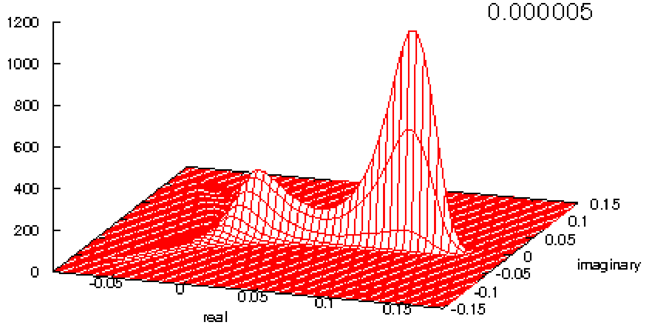
Polyakov loop distribution in the complex plane

(2-flavor, μ=0)

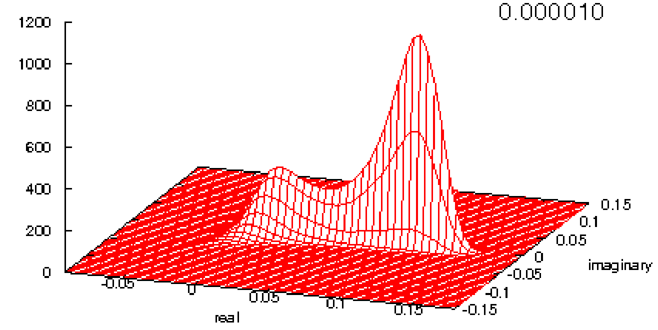
$\kappa^4 = 0.0$ Z(3) symmetric



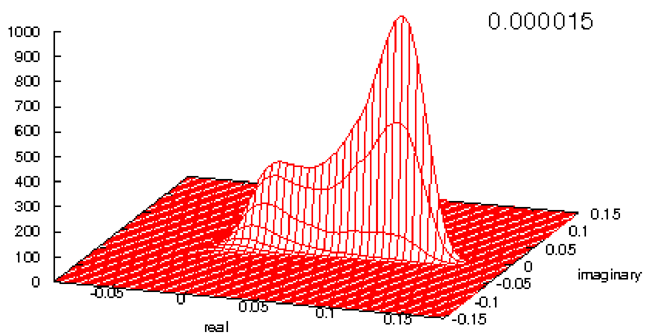
$\kappa^4 = 5.0\times10^{-6}$



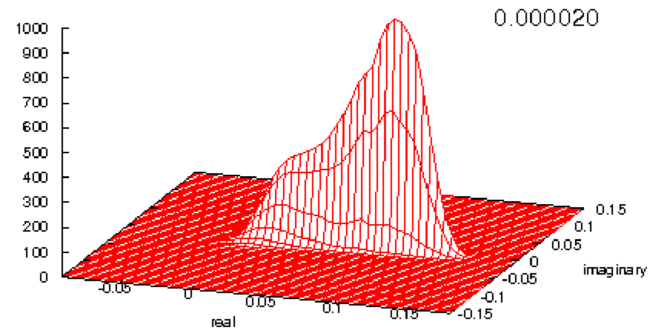
$\kappa^4 = 1.0\times10^{-5}$



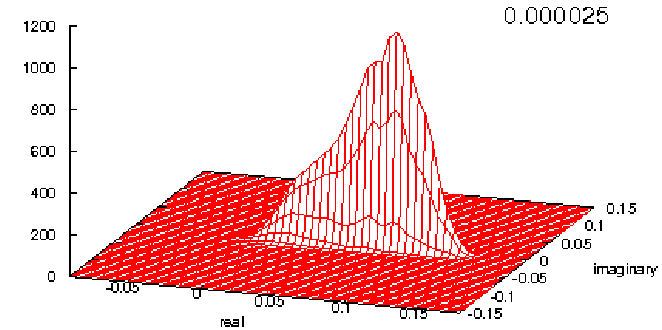
$\kappa^4 = 1.5\times10^{-5}$



$\kappa^4 = 2.0\times10^{-5}$



$\kappa^4 = 2.5\times10^{-5}$

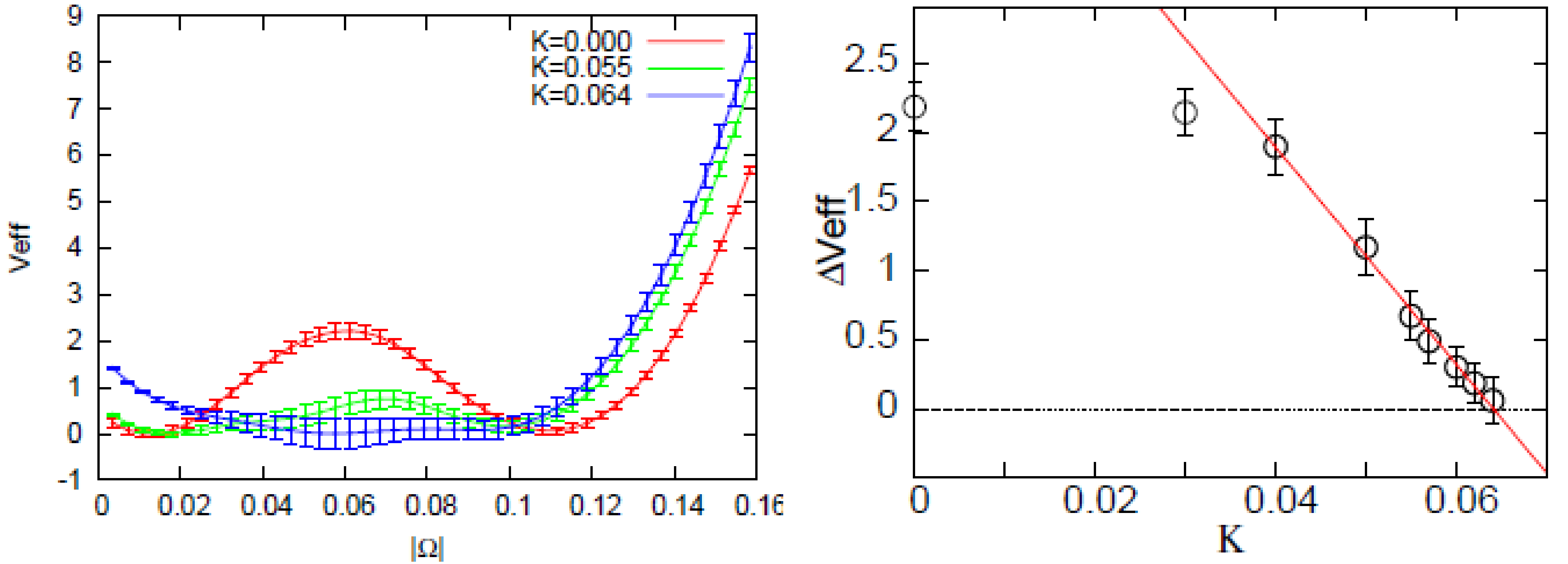


critical point

- at β_{pc} measured by the Polyakov loop susceptibility.

Determination of Critical K

- Effective potential on 32^3x4 lattice



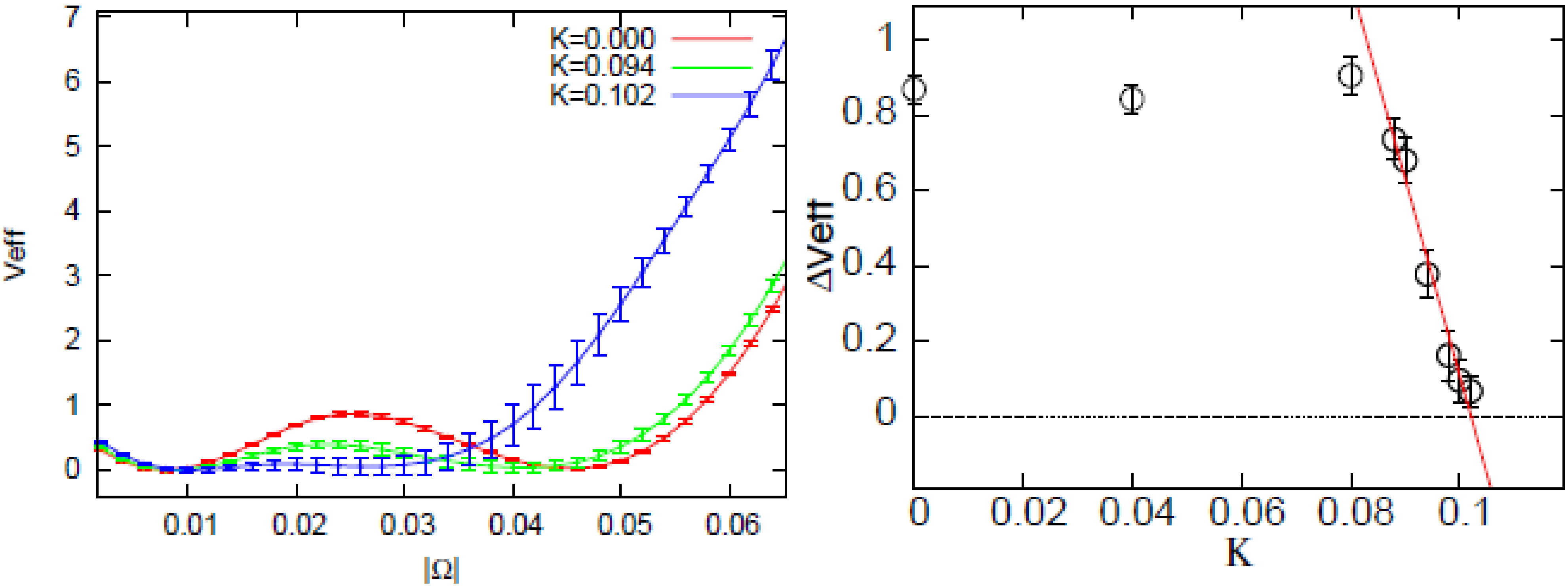
$$K_{ct,4} = 0.0641(18)$$

- The result on 24^3x4 lattice is Kct=0.658(3)($^{+4}_{-11}$)
- No spatial volume dependence.

[Phys.Rev.D84, 054502(2011)]

Determination of Critical K

- Effective potential on 48^3x6 lattice



$$K_{ct,6} = 0.1021(11)$$

Meson mass at the critical point (2-flavor)

m_{PS} : Pseudo scalar meson mass (T=0)

Two method

1.Reweighting using the detM by the hopping parameter expansion.

2. Full QCD simulations (T=0) at the critical point.

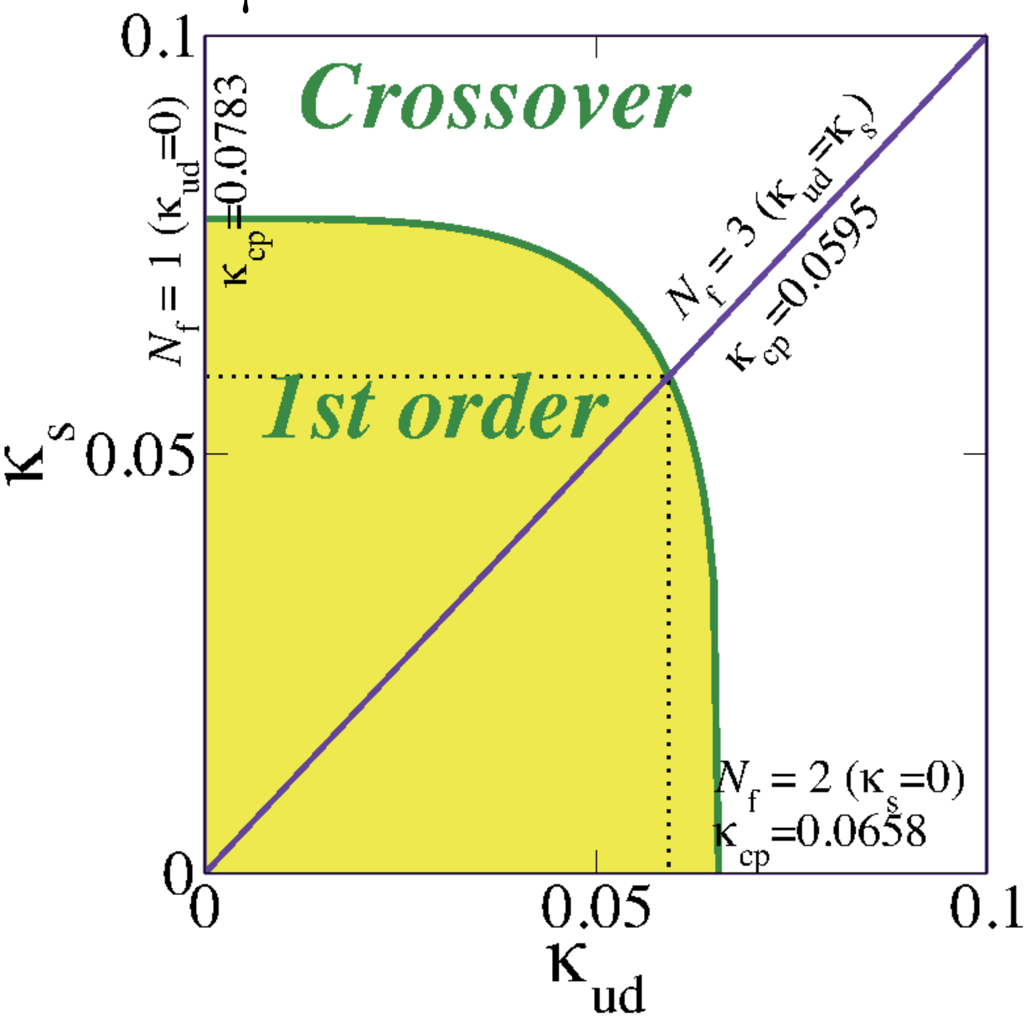
	N_t	K_{cp}	$m_{\mathrm{PS}}a$	m_{PS}/T_c
Reweighting	4	0.0641(18)	3.9332(24)	15.73(25)
	6	0.1021(11)	2.5048(23)	15.02(23)
full QCD	4	0.0641(18)	3.9354(22)	15.74(25)
	6	0.1021(11)	2.5059(15)	15.03(23)

- The results by the two methods are consistent.
- Nt (lattice spacing) dependence in m_{PS}/T_c is small.

Critical surface in the heavy quark region of (2+1)-flavor QCD [Phys.Rev.D84, 054502(2011); Phys.Rev.D89, 034507(2014)]

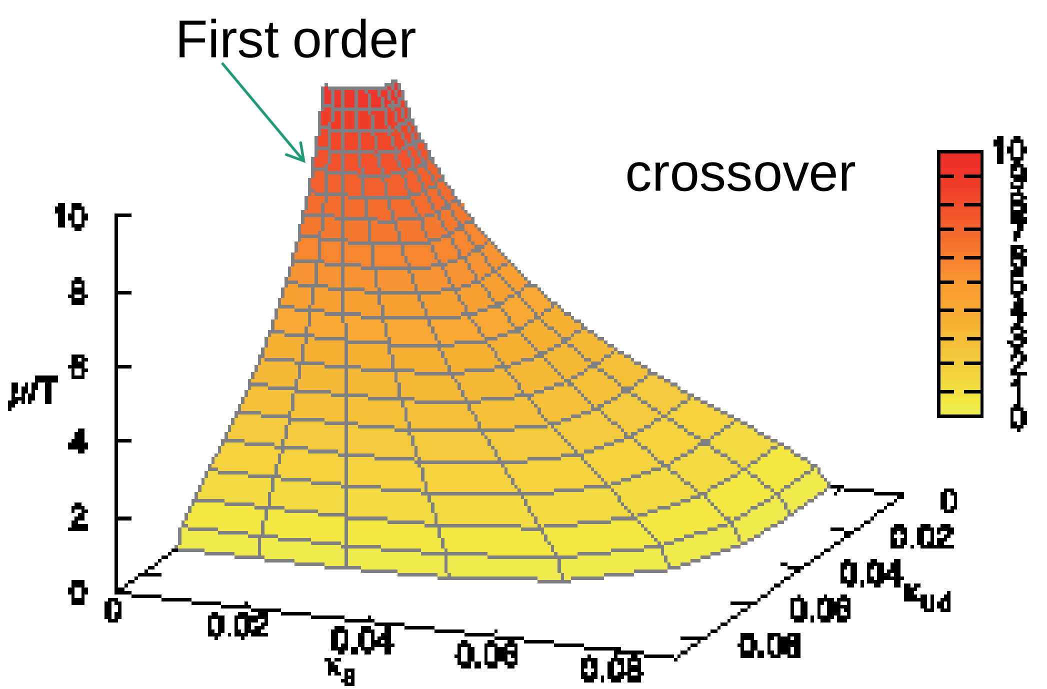
$\left(24^3 \times 4 \text{ lattice}\right)$

$\mu = 0$



$$\frac{m_{PS}}{T_c} \approx 15. \quad \text{at } \kappa_{cp} \text{ for 2-flavor}$$

Critical surface at finite density



Summary

- We studied the first order phase transition in the SU(3) gauge theory (Quenched QCD) and heavy quark region of QCD
- The simulation results suggest that two phases are separated by a domain wall at 1st order transition.
- We measured the latent heat and pressure gap by the derivative method and gradient flow method.
- The pressure gap vanishes at the transition point.
- The endpoint of the 1st order phase transition at finite quark mass and chemical potential are measured.
- The spatial volume dependence and lattice spacing dependence in the location of the endpoint seems to be small.