RUB

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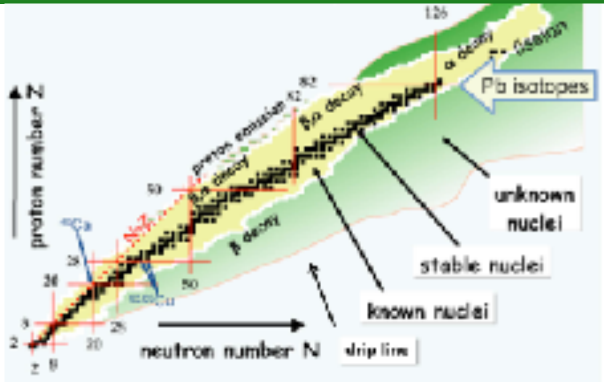
Chiral EFT for nuclear forces: State of the art and future perspectives

Bundesministerium für Bildung und Forschung
DFG Deutsche Forschungsgemeinschaft
erc European Research Council

Why (precision) nuclear physics?



After discovery of Higgs boson,

the strong sector remains the only poorly understood part of the SM!

Interesting topic on its own. Some current frontiers:

— the nuclear chart and limits of stability FAIR, GANIL, ISOLDE,…

— EoS for nuclear matter (gravitational waves from n-star mergers) LIGO/Virgo,…

— hypernuclei (neutron stars) JLab, JSI/FAIR, J-PARC, MAMI,…

But also highly relevant for searches for BSM physics, e.g.:

— direct Dark Matter searches (WIMP-nucleus scattering)

— searches for 0νββ decays

— searches for nucleon/nuclear EDMs

— proton radius puzzle (complementary experiments with light nuclei…)

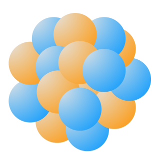
→ need a reliable approach to nuclear structure with quantified uncertainties!

EFTs for nuclear physics:

exploit scale separation in a system of interest

A = 0,1: Chiral perturbation theory

A > 1: Halo-EFT ($Q \ll (\Delta E_{core}\, m)^{1/2}$), pionless EFT ($Q \ll M_\pi$),
chiral EFT ($Q \sim M_\pi \ll M_\rho$)



A >> 1: In-medium chiral EFT; EFTs using collective DOFs
(e.g. to describe deformed nuclei)

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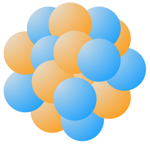


A >> 1: In-medium chiral EFT; EFTs using collective DOFs (e.g. to describe deformed nuclei)

From QCD to nuclei

QCD → **lattice**

symmetries (especially the chiral symmetry);
lost of information (LECs)

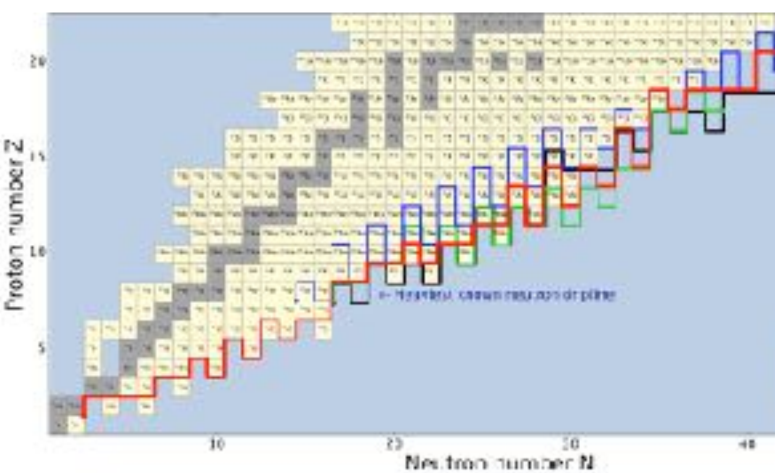
effective chiral Lagrangian $\mathcal{L}_{\rm eff}(\pi, N)$

integrate out $|\vec{p}\,| \sim \sqrt{M_\pi m_N}$ (but retain $|\vec{p}\,| \sim M_\pi$):
Chiral Perturbation Theory

nuclear forces and currents

ab initio many-body methods:
lattice, FY, NCSM,…

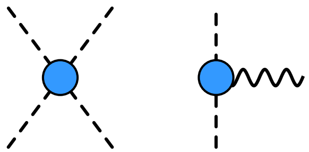
nuclear structure and dynamics



Chiral Effective Field Theory

GB dynamics

Weinberg, Gasser, Leutwyler, ...

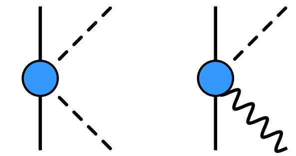


Chiral Perturbation Theory

$$Q = \frac{\text{momenta of particles or } M_\pi \sim 140 \text{ MeV}}{\text{breakdown scale } \Lambda_b}$$

πN dynamics

Bernard-Kaiser-Meißner et al.



Effective Lagrangian:

$$\begin{aligned}
\mathcal{L}_\pi &= \frac{F^2}{4}\mathrm{Tr}(\nabla^\mu U \nabla_\mu U^\dagger + \chi_+) \; + \; \ldots \, , \\
\mathcal{L}_{\pi N} &= \bar{N}(iv\cdot D + g_A \, u \cdot S)N \; + \; \ldots \, , \\
\mathcal{L}_{NN} &= -\frac{1}{2}C_S(\bar{N}N)^2 + 2C_T(\bar{N}SN)^2 \; + \; \ldots
\end{aligned}$$

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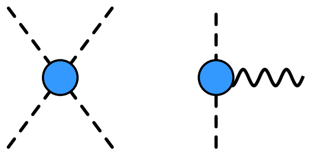

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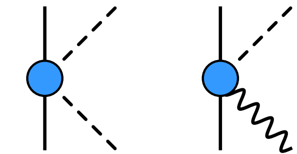

Effective Lagrangian:

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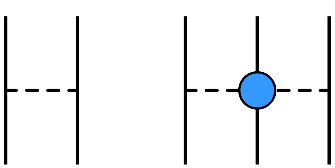
Nuclear forces

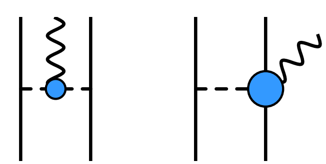
Weinberg, van Kolck, Kaiser, EGM, ...



Auxilliary quantities (not observable),
unitary ambiguity, renormalizability, ...

Nuclear currents

Park et al, Bochum-Bonn, JLab-Pisa



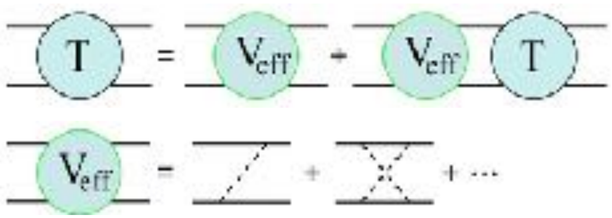
Nuclear chiral EFT

Chiral EFT for nuclear systems:

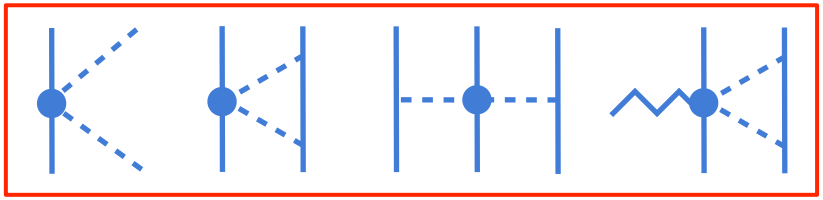
expansion for nuclear forces & currents + resummation (Schrödinger equation)

Weinberg, van Kolck, Kaiser, EE, Glöckle, Meißner, Entem, Machleidt, Krebs, ...

$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\ldots}_{\text{derived in ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$



- systematically improvable
- unified approach for $\pi\pi$, πN, NN
- consistent many-body forces and currents
- error estimations

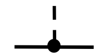
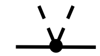


Notice:

- derivation of nuclear forces is much more involved than just calculation of Feynman diagrams; have to deal with non-uniqueness and renormalizability…
- nonperturbative treatment of chiral nuclear forces in the Schrödinger equation requires the introduction of a finite cutoff [alternatively, use semi-relativistic approach, EE, Gegelia, et al. '12…'15]

Method of UT for nuclear forces

EE, Glöckle, Meißner, NPA 637 (1998) 107;   EE, PLB 639 (2006) 456

- Begin with the $\mathcal{L}_{\rm eff}\,[\pi,N]$ without external fields
- Canonical formalism: $\mathcal{L}_{\rm eff}\,[\pi,N] \longrightarrow H[\pi,N] =$  +  + …

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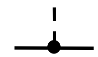
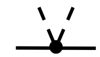
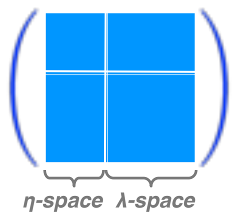
- Canonical formalism: $\mathcal{L}_{\mathrm{eff}}\,[\pi,\mathrm{N}] \longrightarrow H[\pi,\mathrm{N}] =$  +  + …
- Apply UT in Fock space to decouple purely nucleonic states [model space] from the rest

$$H \;\to\; \tilde{H} \;=\; U^\dagger$$ 

$$U \;=\; \begin{pmatrix} \tilde{H}_{\mathrm{nucl}} & 0 \\ 0 & \tilde{H}_{\mathrm{rest}} \end{pmatrix}$$

Using Okubo's minimal parametrization of U in terms of $A = \lambda A \eta$ leads to the

decoupling equation: $\boxed{\lambda(H - [A,\; H] - AHA)\eta = 0}$

which is solved perturbatively employing the chiral expansion

Method of UT for nuclear forces

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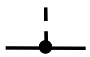
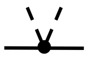
- Canonical formalism: ***L*$_{\rm eff}$** [π,N] ⟶ *H*[π,N] =  +  + …
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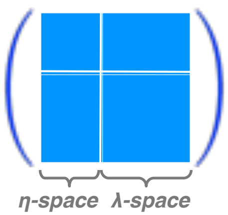
$$H \;\to\; \tilde{H} \;=\; U^{\dagger}$$ 

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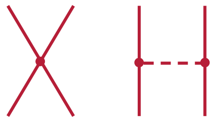
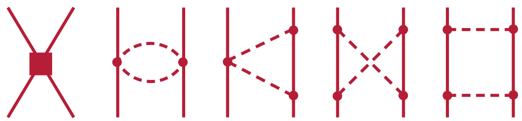
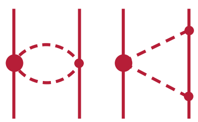
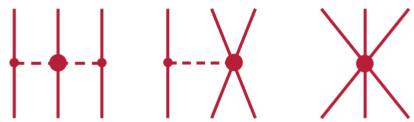
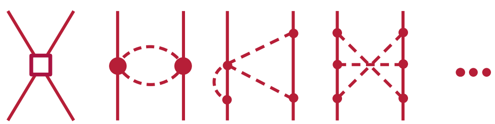
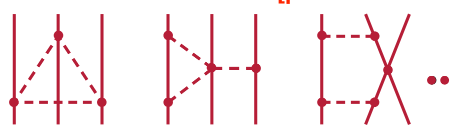
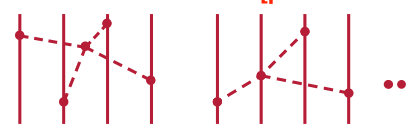
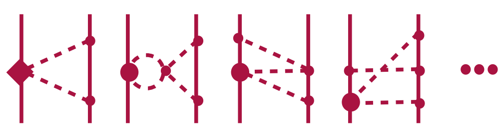
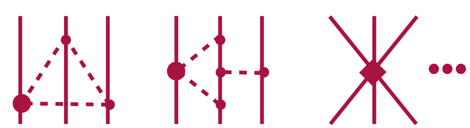
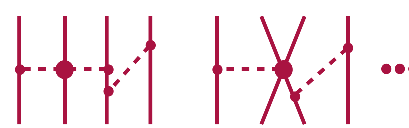
- Apply all possible additional UTs on the η-subspace consistent with a given chiral order [6 angles α_i for static N^{3}LO contributions]
- Renormalizability of the potentials [all 1/(d-4) poles must be canceled by the c.t. from ***L*$_{\rm eff}$**]

⟶ fixes some of the α_i and leads to unique (static) expressions

For more details see: EE, *Nuclear Forces from Chiral Effective Field Theory: A Primer*, arXiv:1001.3229[nucl-th]

Chiral expansion of nuclear forces [W-counting]

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)	 Weinberg '90	—	—
NLO (Q^2)	 Ordonez, van Kolck '92	—	—
N^2LO (Q^3)	 Ordonez, van Kolck '92	 van Kolck '94; EE et al. '02	—
N^3LO (Q^4)	 Kaiser '00 - '02	[parameter-free]  Bernard, EE, Krebs, Meißner,'08, '11	[parameter-free]  EE '06
N^4LO (Q^5)	 Entem, Kaiser, Machleidt, Nosyk '15 EE, Krebs, Meißner '15	 Girlanda, Kievsky, Viviani '11 Krebs, Gasparyan, EE '12,'13 (short-range loop contrib. still missing)	 still have to be worked out

— A similar program is being pursued for in chiral EFT with explicit $\Delta(1232)$ DOF

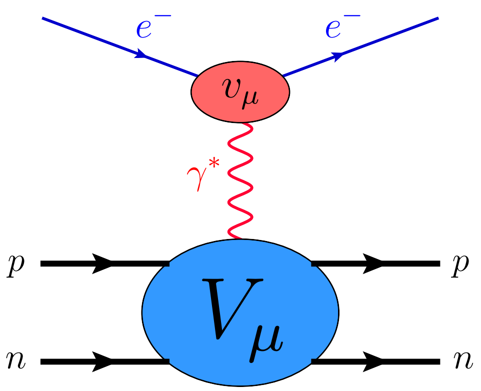
Current operators

- Switch on external sources $s,\ p,\ r_\mu,\ l_\mu$ and consider ***local*** chiral rotations:

$$r_\mu \;\to\; r'_\mu = R\, r_\mu R^\dagger + iR\,\partial_\mu R^\dagger\,, \qquad l_\mu \;\to\; l'_\mu = L\, l_\mu L^\dagger + iL\,\partial_\mu L^\dagger\,,$$
$$s + i\,p \;\to\; s' + i\,p' = R(s + i\,p)L^\dagger\,, \qquad s - i\,p \;\to\; s' - i\,p' = L(s - i\,p)R^\dagger$$

- Decouple π's to get (nonlocal) nuclear $H_{\rm eff}[a, v, s, p]$ (MUT) & get currents via

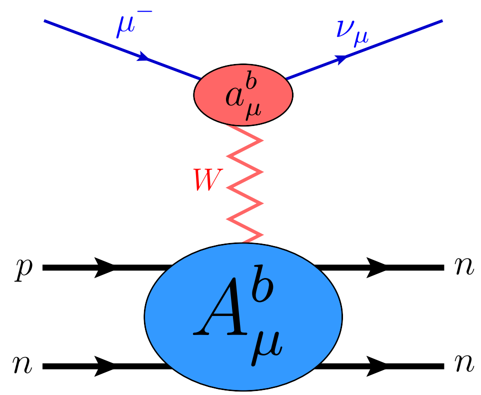
$$V_\mu^a(\vec x\,) = \frac{\delta H_{\rm eff}}{\delta v_a^\mu(\vec x, t)}, \quad A_\mu^a(\vec x\,) = \frac{\delta H_{\rm eff}}{\delta a_a^\mu(\vec x, t)}$$ calculated at $a = v = p = 0,\ s = m_q$.



Park, Min, Rho '95

Pastore et al. (TOPT) '08 — '11: not renormalized...

Kölling, EE, Krebs, Meißner (MUT) '09,'12;
Krebs et al., in preparation: complete (1 loop) & renormalized



Park, Min, Rho '93

Baroni et al. (TOPT) '16: incomplete...

Krebs, EE, Meißner (MUT) '17: complete (1 loop) & renormalized

— about 250 topologies

— 2-loop/1-loop/tree for 1N/2N/3N operators

Also derived scalar currents, Krebs et al., in preparation

Continuity equations

Krebs, EE, Meißner, Annals Phys. 378 (17) 317

Unexpected result: the continuity equation $\vec{k} \cdot \vec{j} \neq [H_{\rm str}, \rho]$! Why?

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Naive (MUT): $\underbrace{H_{\rm eff}[h]}_{a,\, v,\, s,\, p} = \eta \, U^\dagger_{\rm str} \, H_{\pi N}[h] \, U_{\rm str} \, \eta$ ← cannot renormalize $V_\mu^a, \; A_\mu^a \ldots$

determined in the strong sector ($a = v = p = 0, \; s = m_q$)

Solution: employ a more general class of UT's, namely

$$H_{\rm eff}[h, \dot h] = U^\dagger_\eta[h] \, \eta \, U^\dagger_{\rm str} \, H_{\pi N}[h] \, U_{\rm str} \, \eta \, \underbrace{U_\eta[h]}_{\text{subject to the constraint } U_\eta[0,0,m_q,0] = \eta} + i \left(\frac{\partial}{\partial t} U^\dagger_\eta[h] \right) U_\eta[h]$$

← *induce k_0-dependence in the currents (off-shell effect…)*

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Continuity equations = manifestations of the chiral symmetry, $h(x) \xrightarrow{\text{SU(2)}_L \times \text{SU(2)}_R} h'(x)$: $H_{\rm eff}[h, \dot h]$ and $H_{\rm eff}[h', \dot h']$ should be unitary equivalent, i.e. there exists such $U(t)$ that

$$H_{\rm eff}[h', \dot h'] = U^\dagger(t)\, H_{\rm eff}[h, \dot h]\, U(t) + i\left(\frac{\partial}{\partial t} U_\eta^\dagger(t)\right) U_\eta(t)$$

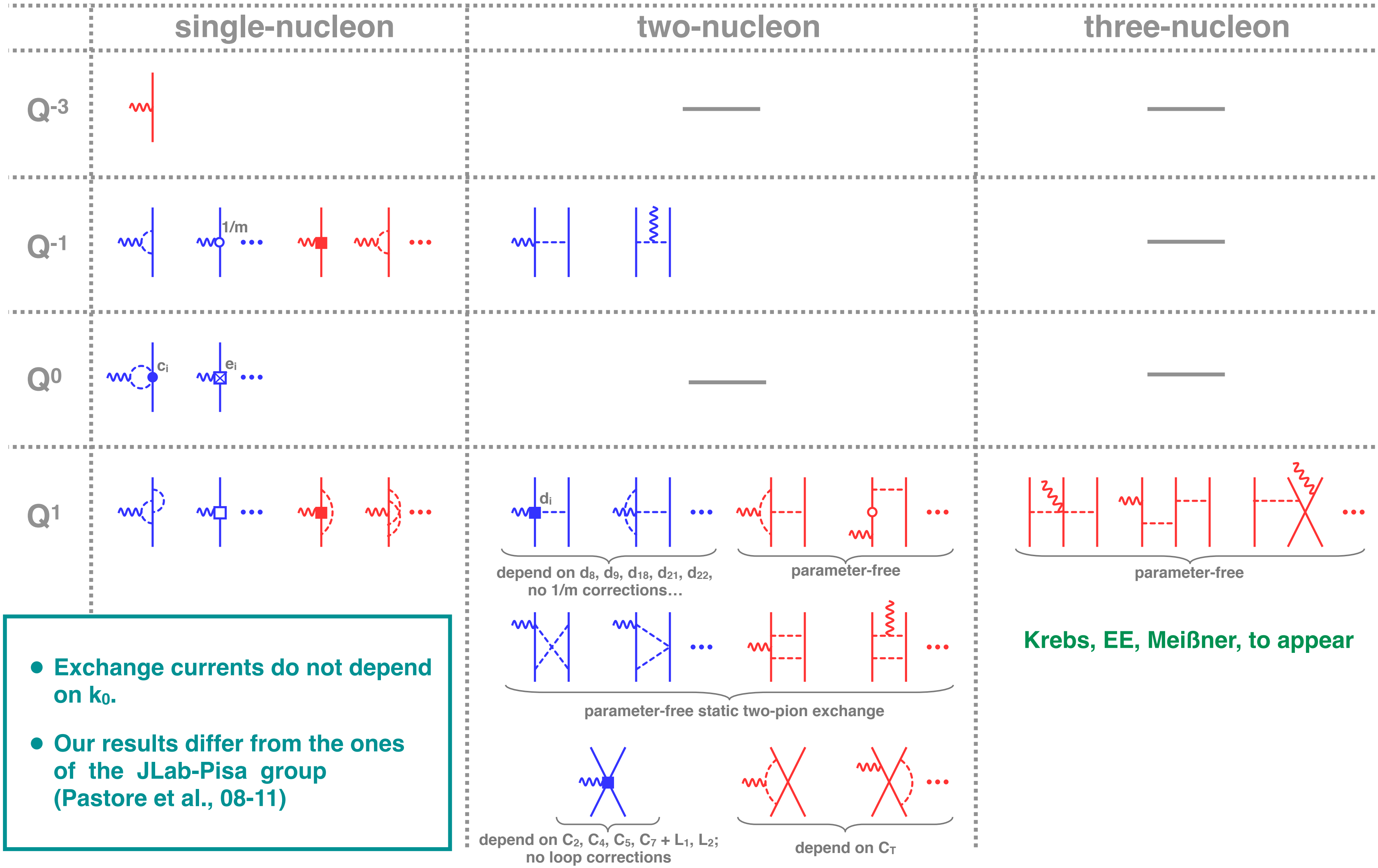
This implies the relations for currents $V_\mu^i(k) := \frac{\delta H_{\rm eff}}{\delta v_i^\mu(k)}$, $A_\mu^i(k) := \frac{\delta H_{\rm eff}}{\delta a_i^\mu(k)}$, $P^i(k) := \frac{\delta H_{\rm eff}}{\delta p_i(k)}$:

$$\boxed{\begin{aligned} \vec{k} \cdot \vec{A}^i(\vec{k}, 0) &= \left[H_{\rm str},\ A_0^i(\vec{k}, 0) - \frac{\partial}{\partial k_0}\left(\vec{k} \cdot \vec{A}^i(k) + [H_{\rm str},\ A_0^i(k)] + i m_q P^i(k)\right)\right] + i m_q P^i(\vec{k}, 0) \\ \vec{k} \cdot \vec{V}^i(\vec{k}, 0) &= \left[H_{\rm str},\ V_0^i(\vec{k}, 0) - \frac{\partial}{\partial k_0}\left(\vec{k} \cdot \vec{V}^i(k) + [H_{\rm str},\ V_0^i(k)]\right)\right] \end{aligned}}$$

Electromagnetic currents

Kölling, EE, Krebs, Meißner, PRC 80 (09) 045502;
PRC 86 (12) 047001

Chiral expansion of the electromagnetic current and charge operators

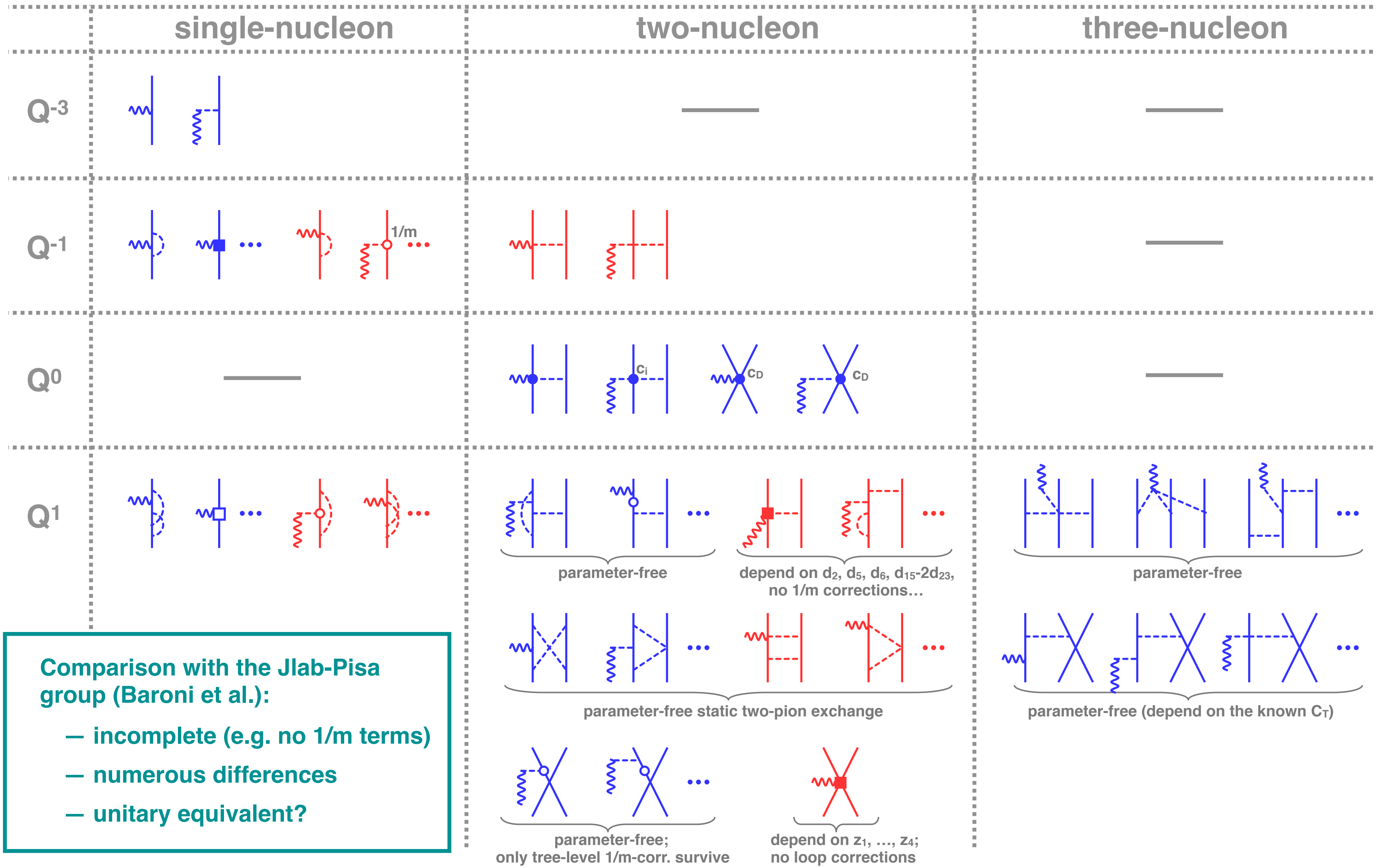


- Exchange currents do not depend on k_0.
- Our results differ from the ones of the JLab-Pisa group (Pastore et al., 08-11)

Exchange axial currents

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

Chiral expansion of the axial current and charge operators



Comparison with the Jlab-Pisa group (Baroni et al.):

— incomplete (e.g. no 1/m terms)

— numerous differences

— unitary equivalent?

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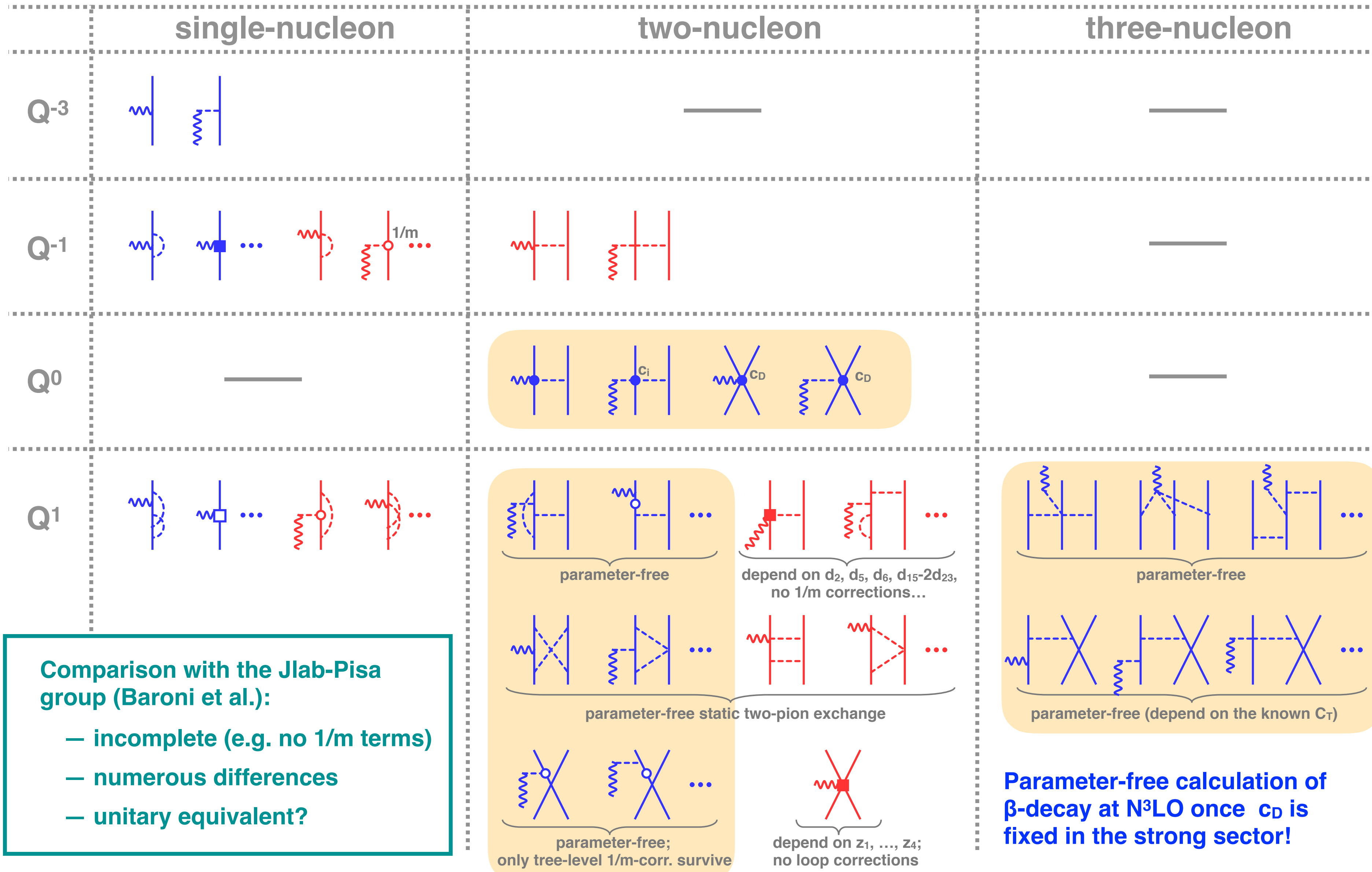

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Parameter-free calculation of β-decay at N^3LO once c_D is fixed in the strong sector!

Applications 1:
A new generation of chiral NN potentials

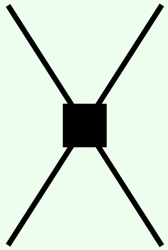
— semi-local, coordinate-space-regularized up to N^4LO
EE, Krebs, Meißner, EPJA 51 (2015) 53; PRL 115 (2015) 122301

— semi-local, momentum-space-regularized up to N^4LO^+
Reinert, Krebs, EE, EPJA 54 (2018) 88

— nonlocal, momentum-space-regularized up to N^4LO^+
Entem, Machleidt, Nosyk, PRC 96 (2017) 024004

The long and short of nuclear forces

- Short-range interactions have to be tuned to experimental data. In the isospin limit, one has according to NDA:



LO [Q^0]:	2 operators (S-waves)
NLO [Q^2]:	+ 7 operators (S-, P-waves and ε_1)
N^{2}LO [Q^3]:	no new terms
N^{3}LO [Q^4]:	+ **12 operators** (S-, P-, D-waves and ε_1, ε_2)
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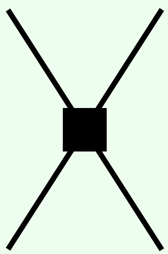

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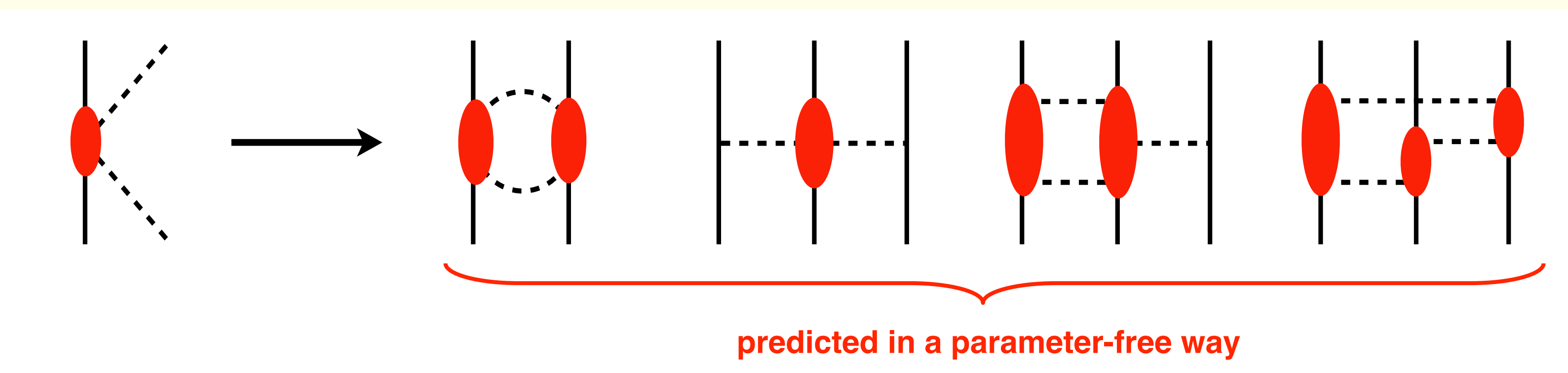
- The long-range part of nuclear forces and currents is completely determined by the chiral symmetry of QCD + experimental information on πN scattering



Pion-nucleon scattering

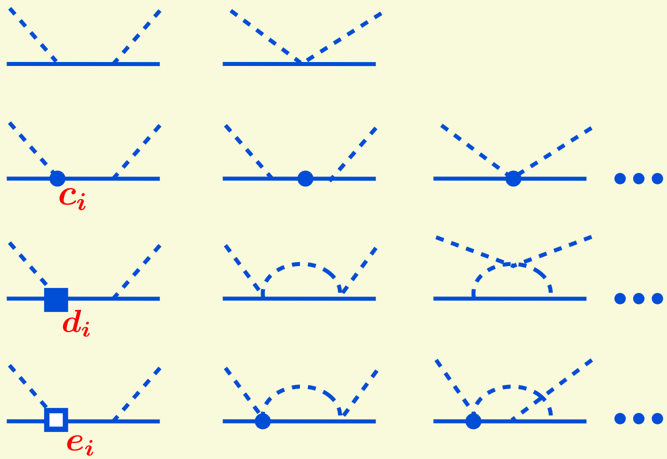
Chiral expansion of the pion-nucleon scattering amplitude up to Q^4

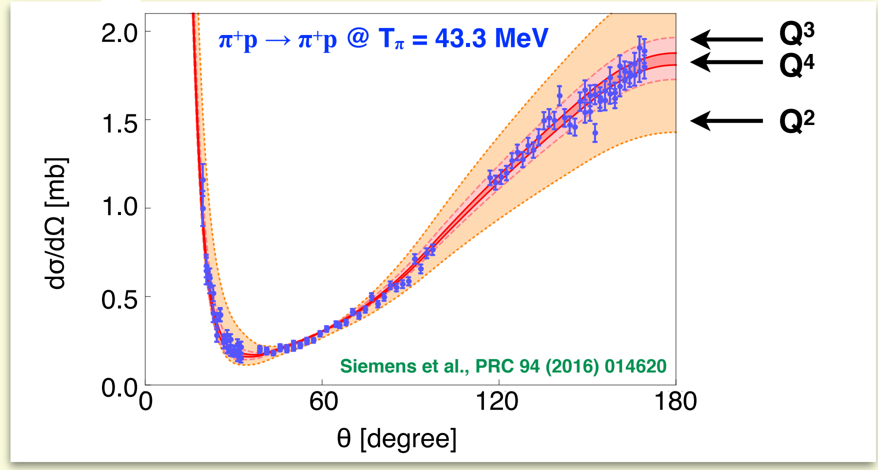
Order Q:

Order Q^2:

Order Q^3:

Order Q^4:





HB ChPT with and without Δ(1232) DOF

Fettes, Meißner '98, '00; Krebs, Gasparyan, EE '12

Covariant baryon ChPT using the IR framework

Becher, Leutwyler '00; Hoferichter et al. '10

Covariant baryon ChPT using the EOMS scheme

Alarcon, Camalich, Oller '13; Chen, Yao, Zheng'13

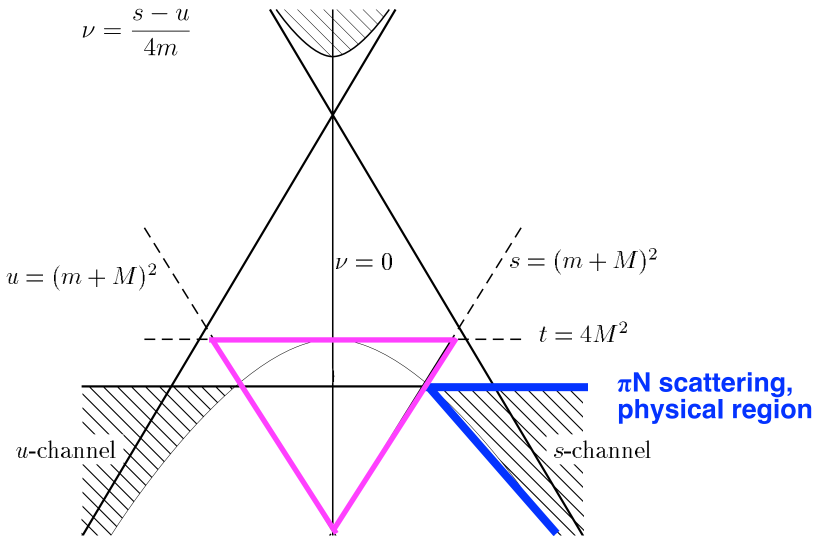
Covariant baryon ChPT using the EOMS scheme with explicit Δ(1232) DOF

Yao, Siemens, Bernard, EE, Gasparyan, Gegelia, Krebs, Meißner '16; Siemens, Bernard, EE, Gasparyan, Krebs, Meißner '16,'17

Roy-Steiner equation analysis

Dietsche et al., JHEP 1206 (12) 043; Hoferichter et al., Phys. Rept. 625 (16) 1; Hoferichter, Ruiz de Elvira, Kubis, Meißner, PRL 115 (2015) 092301

Determination of πN LECs



Matching ChPT to πN Roy-Steiner equations

Hoferichter, Ruiz de Elvira, Kubis, Meißner, PRL 115 (2015) 092301

- χ expansion of the πN amplitude expected to converge best within the Mandelstam triangle

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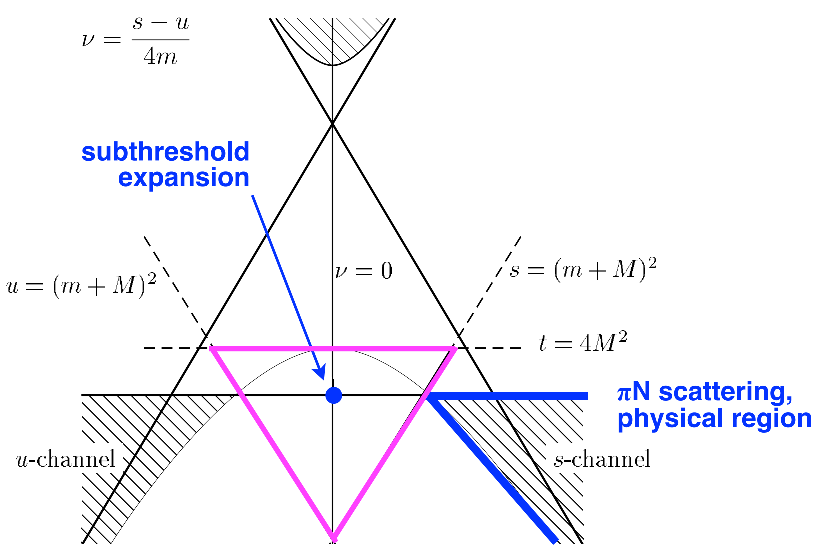

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$$\bar{X} = \sum_{m,n} x_{mn}\, \nu^{2m+k} t^n, \qquad X = \{A^\pm,\, B^\pm\}$$

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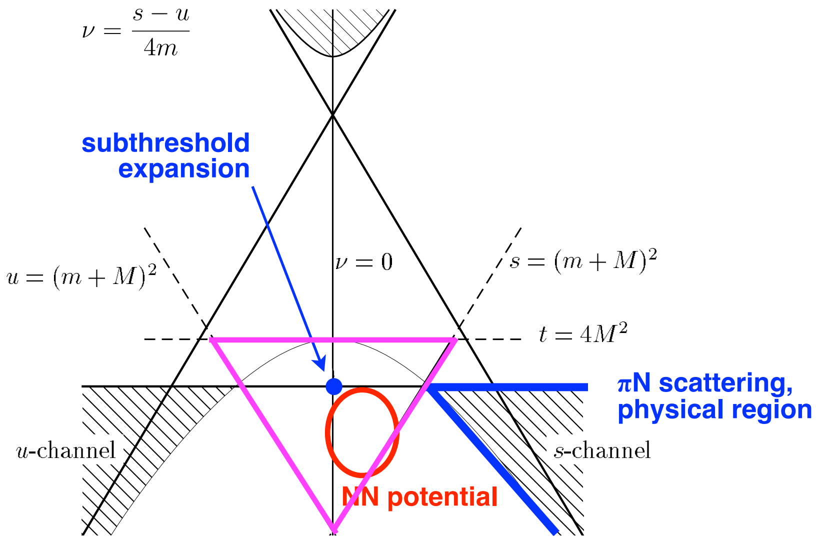

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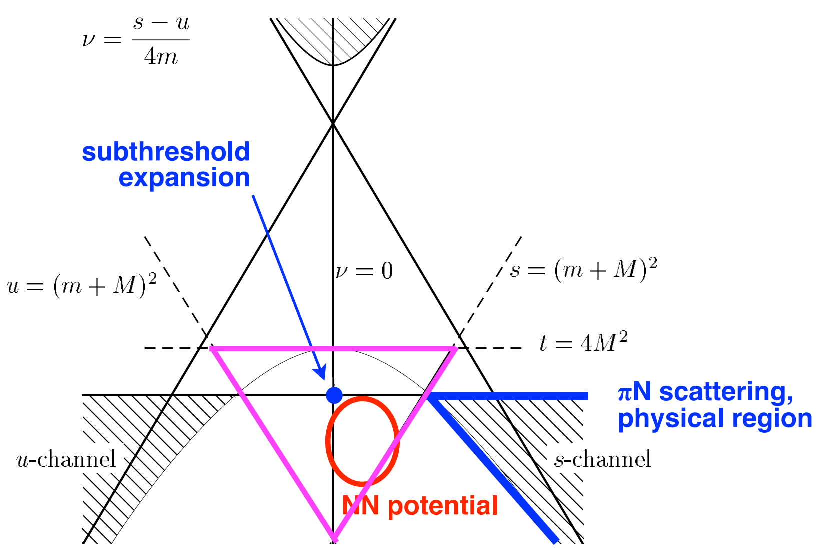

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Relevant LECs (in GeV⁻ⁿ) extracted from πN scattering

	c_1	c_2	c_3	c_4	$\bar{d}_1+\bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14}-\bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{17}$	
$[Q^4]_{\mathrm{HB,NN}}$, GW PWA	-1.13	3.69	-5.51	3.71	5.57	-5.35	0.02	-10.26	1.75	-0.58	Krebs, Gasparyan, EE, PRC85 (12) 054006
$[Q^4]_{\mathrm{HB,NN}}$, KH PWA	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-0.37	
$[Q^4]_{\mathrm{HB,NN}}$, Roy-Steiner	-1.10	3.57	-5.54	4.17	6.18	-8.91	0.86	-12.18	1.18	-0.18	Hoferichter et al., PRL 115 (15) 092301
$[Q^4]_{\mathrm{covariant}}$, data	-0.82	3.56	-4.59	3.44	5.43	-4.58	-0.40	-9.94	-0.63	-0.90	Siemens et al., PRC94 (16) 014620

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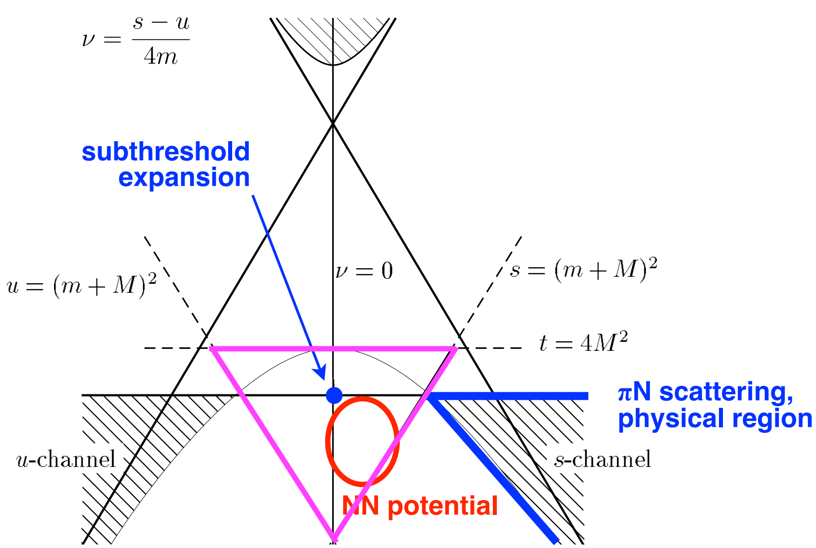

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— Some LECs show sizable correlations (especially c_1 and c_3)…

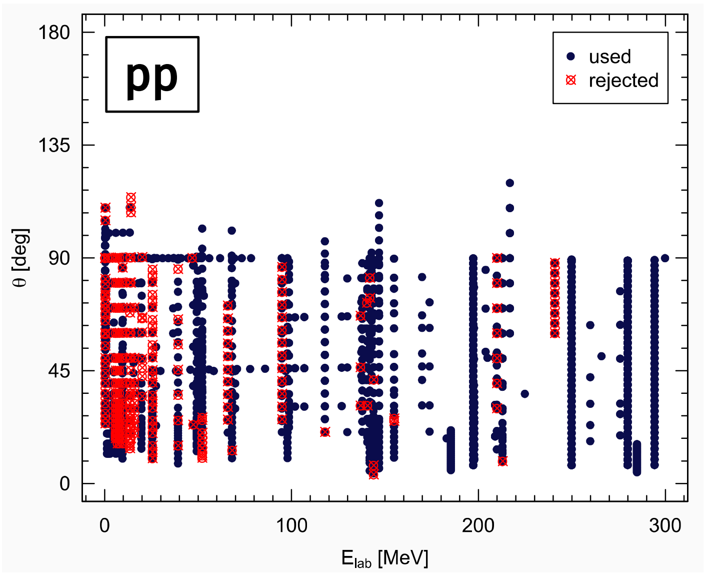
— RKE N^{4}LO [Reinert, Krebs, EE, EPJA 54 (2018) 88]: Q^4 fit to RS and Q^4 fit to KH PWA

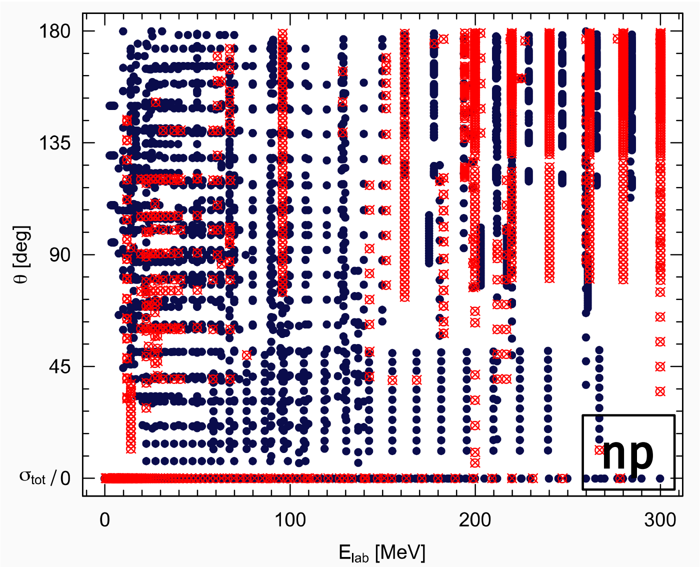
With the LECs taken from πN, the long-range NN force is completely fixed (parameter-free)

NN data analysis

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

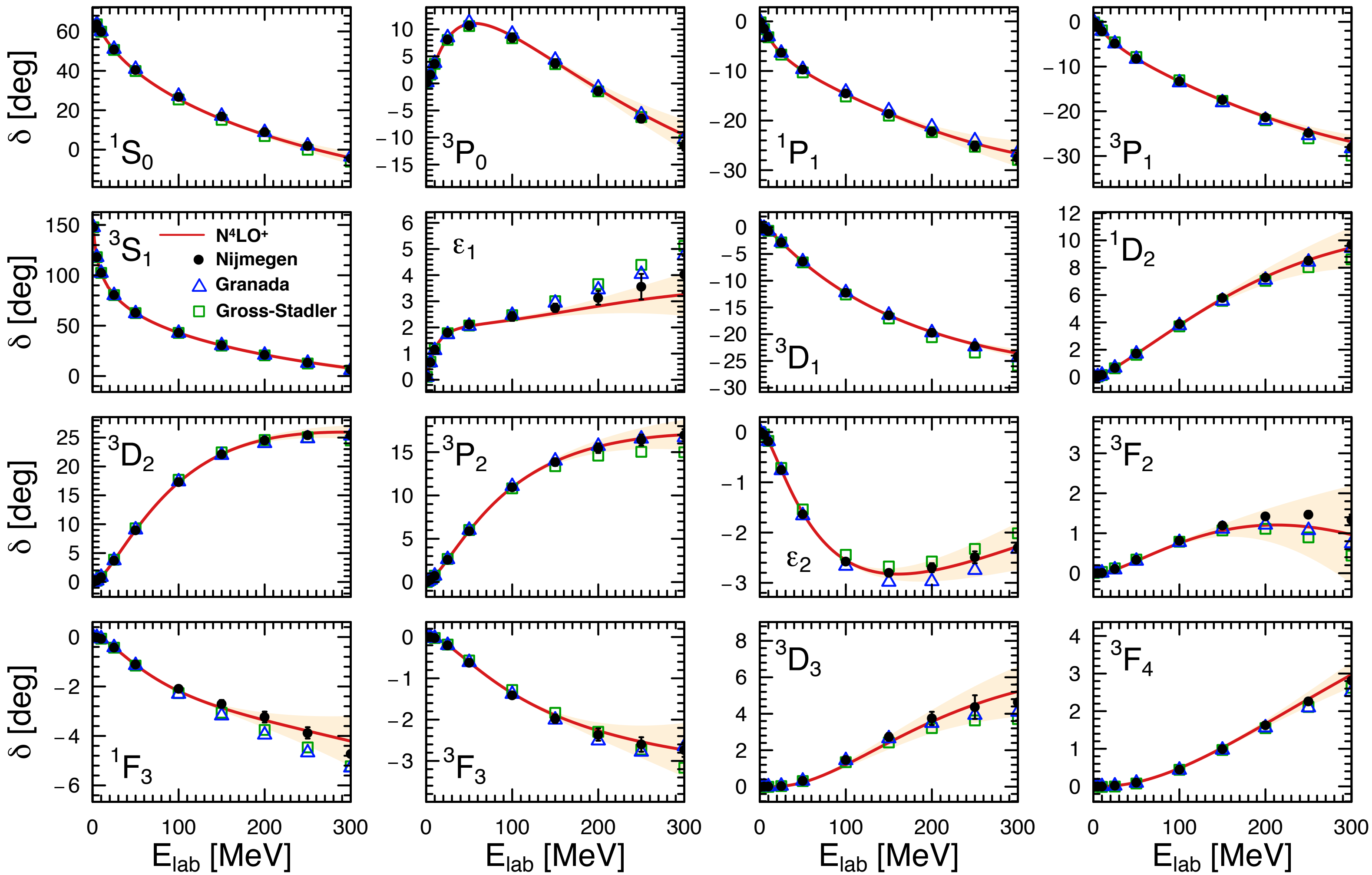
- Use local/nonlocal regulator for long-range/short-range contributions
- To fix NN contact interactions, use scattering data together with $B_d = 2.224575(9)$ MeV and $b_{np} = 3.7405(9)$ fm.
- Since 1950-es, about 3000 proton-proton + 5000 neutron-proton scattering data below 350 MeV have been measured.
- However, certain data are mutually incompatible within errors and have to be rejected.
  2013 Granada database [Navarro-Perez et al., PRC 88 (2013) 064002], rejection rate: 31% np, 11% pp:
  2158 proton-proton + 2697 neutron-proton data below $E_{lab} = 300$ MeV





State-of-the-art NN potentials

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88



— N⁴LO⁺ yields currently the best description of the 2013 Granada database ($E_{lab} < 300$ MeV)

— 40% less parameters (27+1) compared to high-precision potentials

— Clear evidence of the parameter-free chiral 2π exchange

Error analysis

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

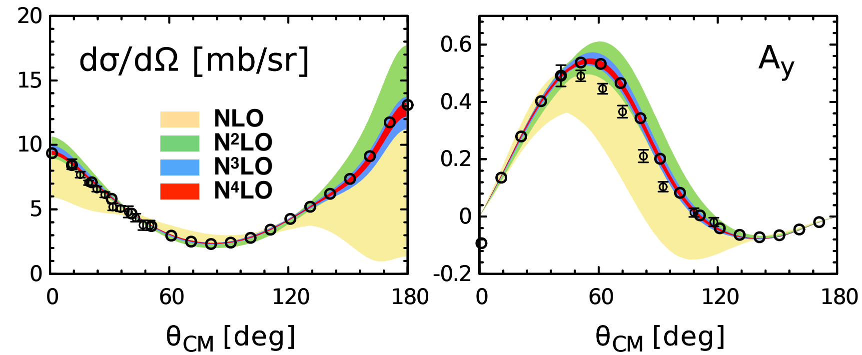
1. Truncation error

EE, Krebs, Meißner, EPJA 51 (2015) 53

$$X^{(i)}(p) = X^{(0)} + \underbrace{\Delta X^{(2)}}_{\sim Q^2 X^{(0)}} + \ldots + \underbrace{\Delta X^{(i)}}_{\sim Q^i X^{(0)}}$$

Expansion parameter: $Q = \max \left\{ \frac{p}{\underbrace{\Lambda_b}_{\simeq\, 600\ \mathrm{MeV}}}, \frac{M_\pi}{\Lambda_b} \right\}$

proton-neutron scattering at E_{lab}=143 MeV



Use the explicitly calculated $\Delta X^{(i)}$ to estimate the uncertainty $\delta X^{(i)}$ at order Q^i:

$$\left\{ \delta X^{(0)} = Q^2 |X^{(0)}|, \;\; \delta X^{(i)} = \max_{2 \le j \le i} \left(Q^{i+1} |X^{(0)}|, \, Q^{i+1-j} |\Delta X^{(j)}| \right) \right\} \;\; \wedge \;\; \delta X^{(i)} \ge \max_{j,k} \left(|X^{(j \ge i)} - X^{(k \ge i)}| \right)$$

Has been validated/extended within a Bayesian approach BUQEYE Collaboration, Furnstahl et al.,'15 - '18

2. Statistical uncertainties

Estimated in the standard way using the covariance matrix (quadratic approximation)

3. Uncertainties due to πN LECs $c_{1,2,3,4}$, $d_{1,2,3,5,14,15}$ and $e_{14,17}$

Estimated using 2 sets of πN LECs (Roy-Steiner equation analysis & KH PWA)

4. Choice of E_{max} in the fits

Uncertainty estimated at N^{4}LO/N^{4}LO$^+$ by performing fits with E_{max} = 220…300 MeV

Error analysis

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

In most cases, the uncertainty is dominated by truncation errors. At N^{4}LO and at very low energies, other sources of errors become comparable (especially πN LECs…).

Example: deuteron asymptotic normalizations (relevant for nuclear astrophysics)

Our determination:

statistical error, truncation error, πN LECs, variation of E_{max}

$$A_S = 0.8847^{(+3)}_{(-3)}(3)(5)(1)\ \mathrm{fm}^{-1/2}$$

$$\eta \equiv \frac{A_D}{A_S} = 0.0255^{(+1)}_{(-1)}(1)(4)(1)$$

Exp: $A_S = 0.8781(44)\ \mathrm{fm}^{-1/2}$, $\eta = 0.0256(4)$

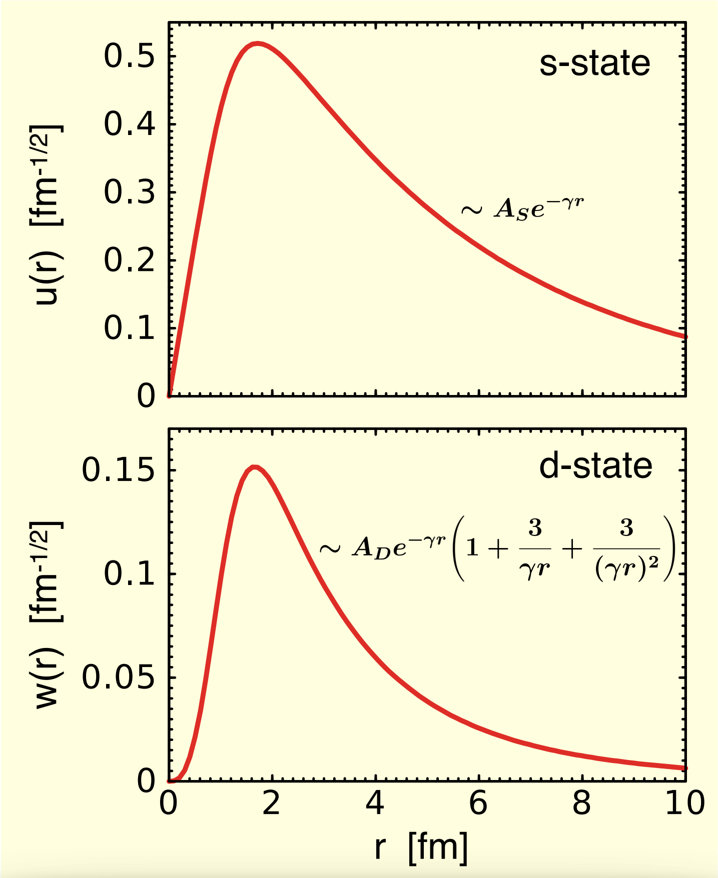
Borbely et al. '85 Rodning, Knutson '90

Nijmegen PWA [errors are „educated guesses"] Stoks et al. '95

$$A_S = 0.8845(8)\ \mathrm{fm}^{-1/2}, \quad \eta = 0.0256(4)$$

Granada PWA [errors purely statistical] Navarro Perez et al. '13

$$A_S = 0.8829(4)\ \mathrm{fm}^{-1/2}, \quad \eta = 0.0249(1)$$



Applications 2: Beyond the 2N system

— LENPIC Collaboration —

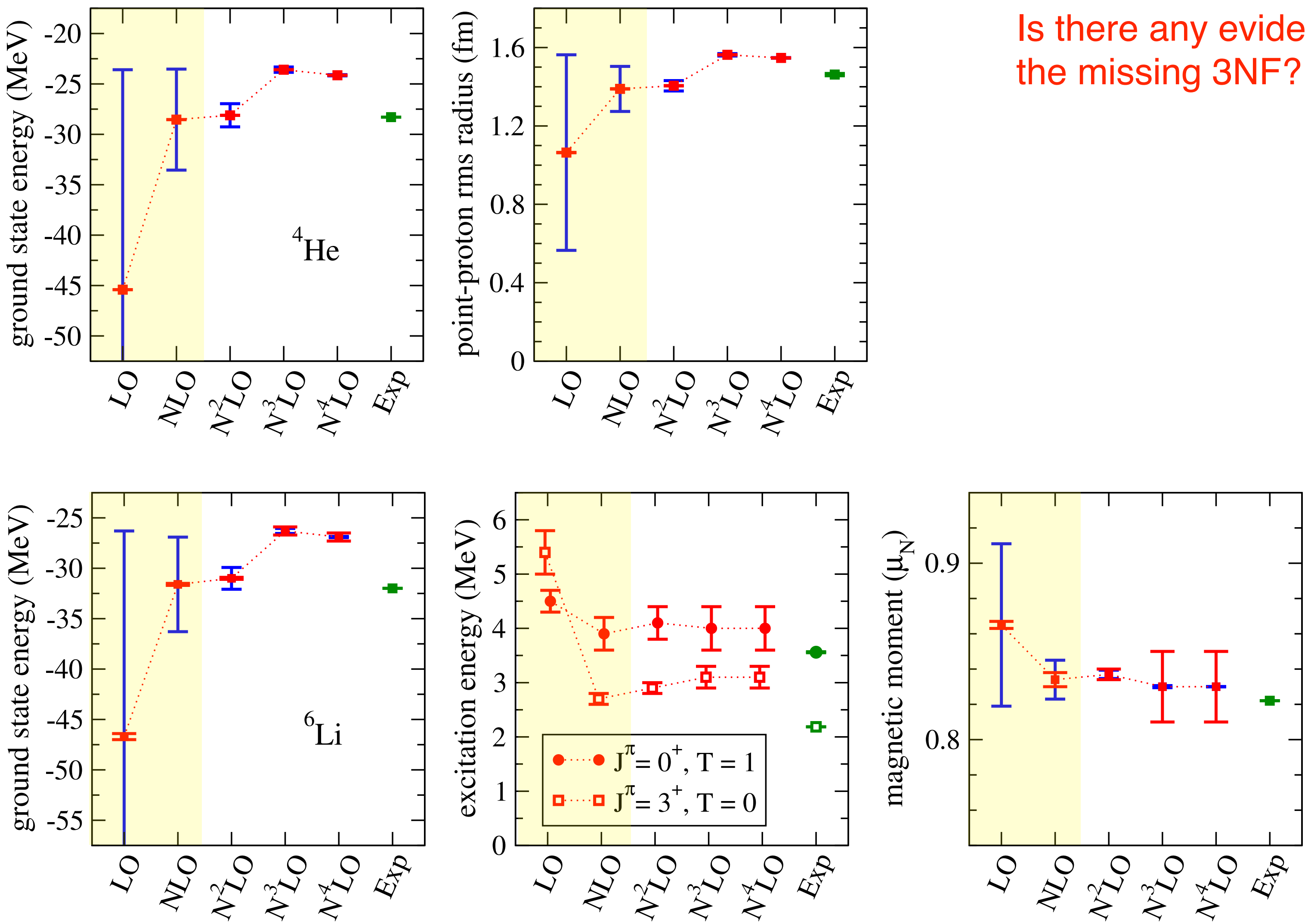
Goal: precision tests of chiral nuclear forces & currents in light nuclei

Strategy: go to high orders, do not compromise the πN LECs, no fine tuning to heavy nuclei, careful error analysis

LENPIC: Low **E**nergy **N**uclear **P**hysics **I**nternational **C**ollaboration

Light nuclei based on 2NF alone

LENPIC Collaboration (Maris et al.), EPJ Web of Conf. 113 (2016) 04015



Is there any evidence of the missing 3NF?

LENPIC: Low Energy Nuclear Physics International Collaboration

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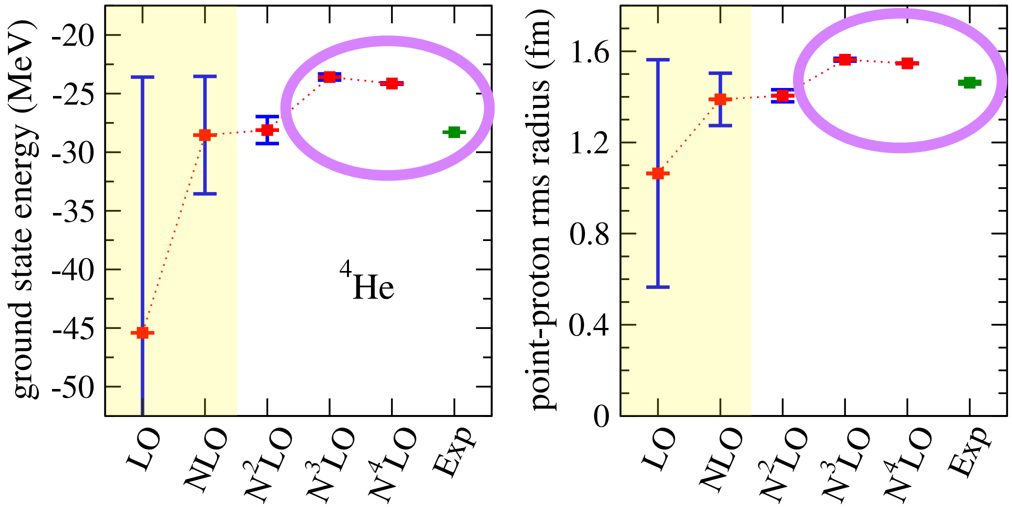
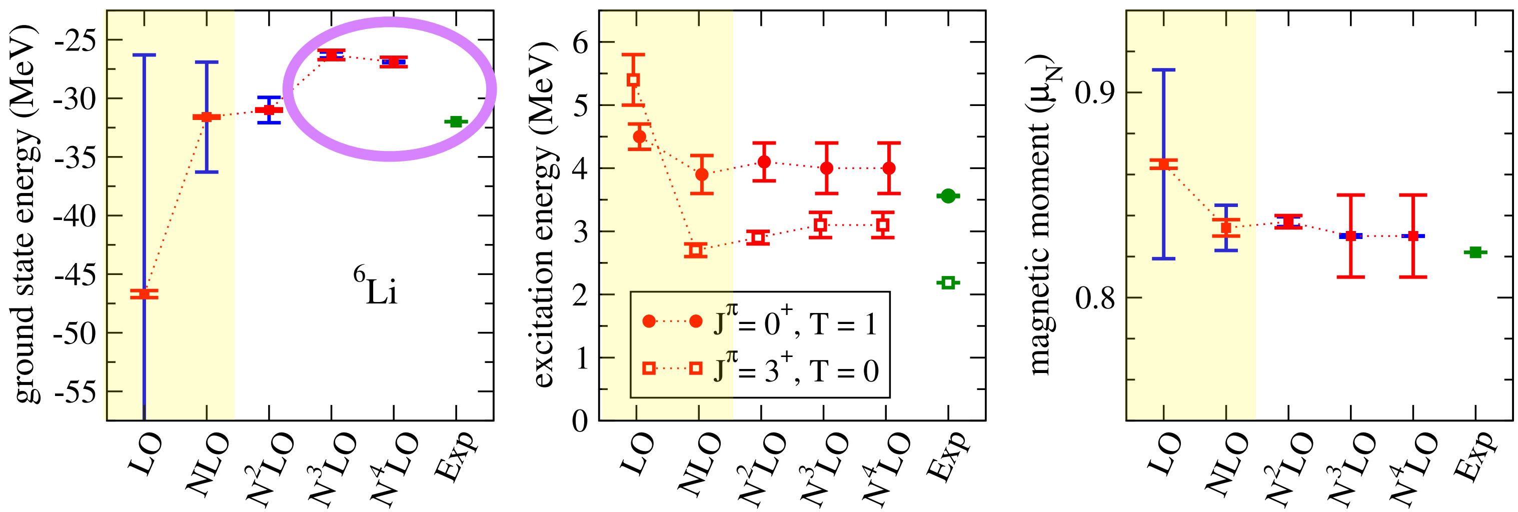


Is there any evidence of the missing 3NF?

Deviations from the data are consistent with the estimated size of the 3N forces

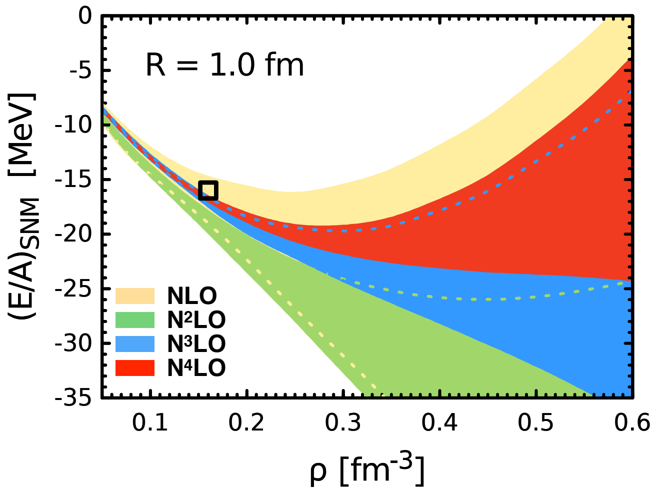


LENPIC: Low Energy Nuclear Physics International Collaboration

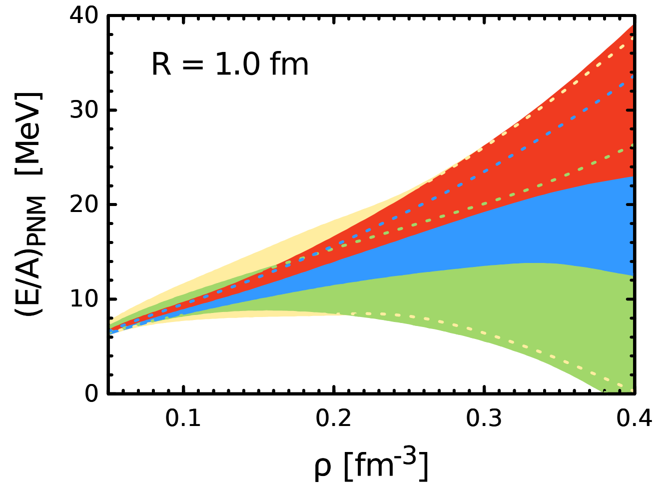
Brueckner-Hartree-Fock based on 2NF alone

Jinniu Hu, Ying Zhang, EE, Ulf-G. Meißner, Jie Meng, PRC 96 (2017) 034307

Symmetric nuclear matter



Pure neutron matter

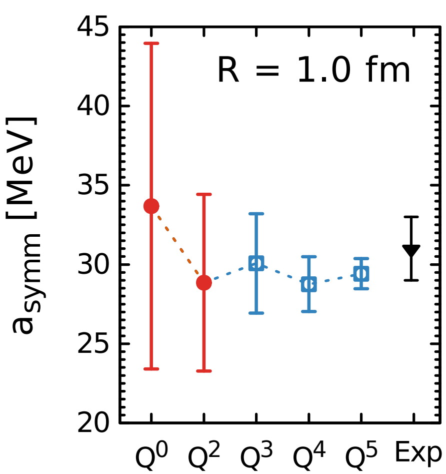


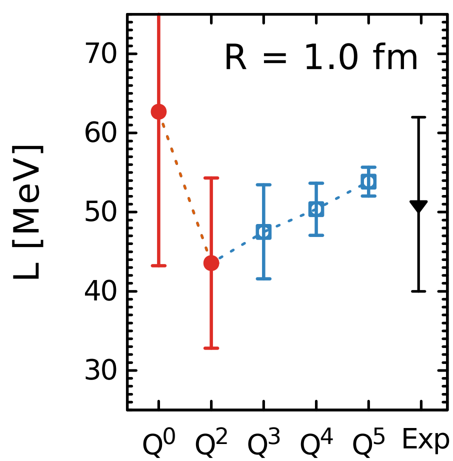
Achievable accuracy at N^{4}LO at ρ_0:

± 0.3 MeV for SNM,
± 0.7 MeV for PNM,

semi-quantitative up to ~ $2\rho_0$ …

Symmetry energy and the slope parameter at the saturation density

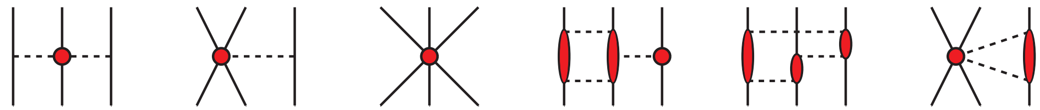




$$a_{\rm symm}(\rho) = \left(\frac{E}{A}\right)_{\rm PNM} - \left(\frac{E}{A}\right)_{\rm SNM}$$

$$L = 3\rho \frac{\partial (E/A)_{\rm SNM}}{\partial \rho}$$

Three-nucleon forces



N^2LO: tree-level graphs, 2 new LECs
van Kolck ’94; EE et al ’02

N^3LO: leading 1 loop, parameter-free
Ishikawa, Robilotta ’08; Bernard, EE, Krebs, Meißner ’08, ’11

N^4LO: full 1 loop, almost completely worked out, several new LECs
Girlanda, Kievski, Viviani ’11; Krebs, Gasparyan, EE ’12,’13; EE, Gasparyan, Krebs, Schat ’14

Three-nucleon forces

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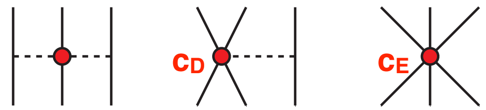

Determination of the LECs c_D, c_E: Triton BE & pd elastic cross section minimum @70 MeV

LENPIC: Low Energy Nuclear Physics International Collaboration

Three-nucleon forces

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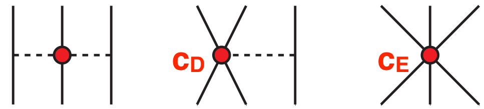

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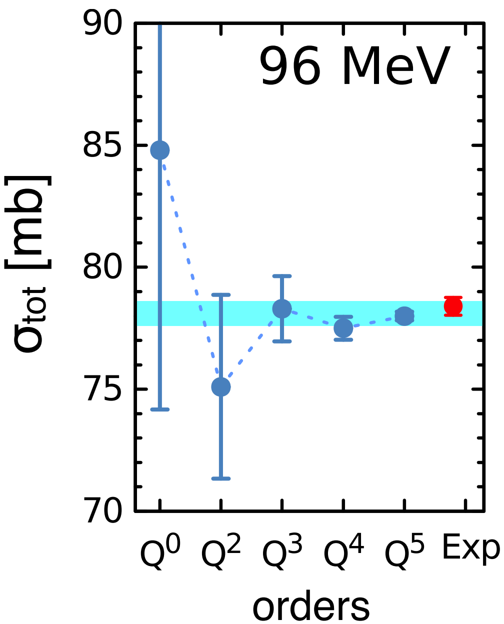
np total cross section



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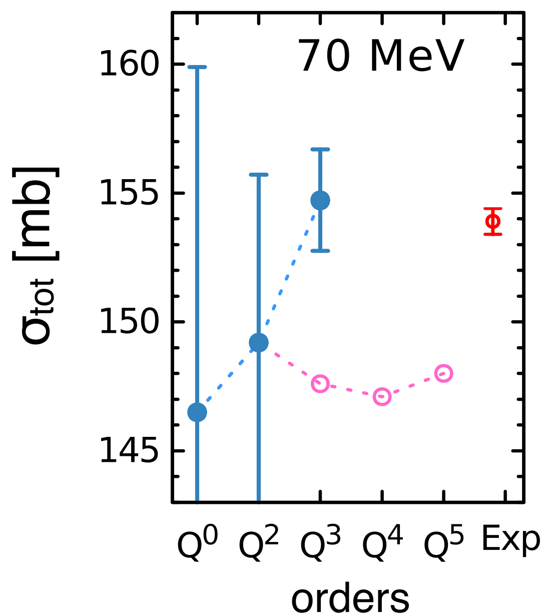

LENPIC, preliminary

LENPIC: Low Energy Nuclear Physics International Collaboration

Radii of heavy nuclei: A smoking gun?

- Preliminary results indicate that radii of heavier nuclei are underestimated (~ 15% for ^{16}O)

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- Expected to be correlated with the results for ^{2}H radius, but effects seem to increase with A.

	r_D, ^{2}H (fm)	r_p, ^{3}H (fm)	r_p, ^{4}He (fm)
AV18/AV18+UIX	**1.967** (−0.4%)	**1.584** (−1%)	**1.44** (−2%)

from: Lazauskas, Carbonell, PRC 70 (2004) 044002

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NNLO$_{sat}$: r_D = 1.978 fm (+0.13%)
Ekström et al., PRC91 (2015) 051301

ΔNNLO(450): r_D = 1.982 fm (+0.3%)
Ekström et al., PRC97 (2018) 024332

However, NN data seem to prefer smaller r_D:

	RKE N^{4}LO$^+$	Granada PWA (δ-shell)	Nijm I	Nijm II	Reid93	CD-Bonn	Exp.
r_D, ^{2}H (fm)	**1.965 . . . 1.968**	**1.965**	**1.967**	**1.968**	**1.969**	**1.966**	**1.975**

LENPIC: Low Energy Nuclear Physics International Collaboration

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- Work in progress with Hebeler, Roth et al. towards understanding the systematics…

LENPIC: **L**ow **E**nergy **N**uclear **P**hysics **I**nternational **C**ollaboration

Outlook

Next steps (work in progress)

- Regularization of many-body forces & currents becomes nontrivial @N^3LO (χ symmetry…)
  High-derivative method (cutoff in the Lagrangian) **Slavnov, NPB 31 (1971) 301**
- N^3LO analysis of Nd scattering, light and medium-mass nuclei; parameter-free calculation of the ^3H β-decay; μ-decay & MuSun experiment @PSI; e.m. reactions, …

Frontiers & challenges for the (near) future

Precision physics beyond the 2N system: challenge the theory

- Understanding the issue with the radii of heavier nuclei
- Extending the analysis of the 3NF to N^4LO will require partial wave analysis of Nd scattering to fix short-range LECs. Long-standing puzzles in 3N continuum (A_y, SST, …)!
- Pushing ab initio methods to heavier nuclei and reactions

Chiral EFT as a tool to deal with nuclear effects when looking at physics of/beyond the SM (parity violation, EDM, 0νββ, proton charge radius,…)

EFT for lattice QCD (extrapolations), **lattice QCD for EFT** (m_q dependence, „data", …)