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Phenomenology of a pseudoscalar glueball and charmed mesons

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Introduction

• Quantum Chromodynamics (QCD)

- Symmetries of the QCD Lagrangian.

  if all quark massless then we have chiral symmetry

$$U(N_f)_r \times U(N_f)_l = SU(N_f)_r \times SU(N_f)_l \times U(1)_V \times U(1)_A$$

- Spontaneous breaking of chiral symmetry by quark condensates.
- Explicit breaking of global chiral symmetry by quark masses and chiral anomaly.

• Effective chiral models of (QCD).

Motivation

- Decay of the pseudoscalar glueball into scalar and pseudoscalar mesons.
- Decay of the excited pseudoscalar glueball.
- Linear sigma model with vector and axial vector degree of freedom.
- Inclusion of the charmed mesons into linear sigma model
  (extended Linear Sigma Model - eLSM).
- Extension from low-energy to high-energy mesons.
- Study of the model for $T = \mu = 0$ (spectroscopy in vacuum).

Fields of the model

- **Mesons: quark-antiquark states ($q\bar{q}$)**

  (scalar, pseudoscalar, vector and axialvector quarkonia.)

Quantum number: J^{PC}

J^{PC} → Charge Conjugation

J → TotalSpin

P → Parity

- **Glueballs: The scalar and the pseudoscalar glueball**

- **Charm quarks ??**

Decays of the pseudoscalar glueball

A globally chirally invariant for three flavours

Interaction Lagrangian for the pseudoscalar glueball with scalar and pseudoscalar mesons

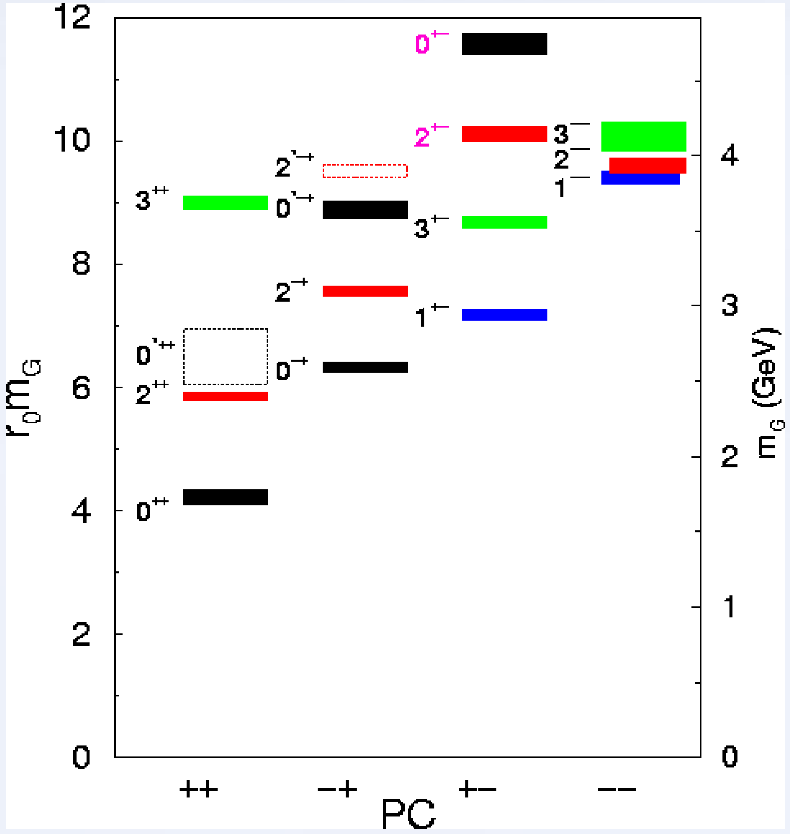
$$\mathcal{L}_{\tilde{G}}^{int} = i c_{\tilde{G}\Phi} \tilde{G} \left(\det\Phi - \det\Phi^{\dagger} \right)$$

where $c_{\tilde{G}\Phi}$ is a dimensionless coupling constant and Φ reads for three flavours, $N_f = 3$:

$$\Phi = (S^a + iP^a)\, t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_S^+ + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_S^0 + iK^0 \\ K_S^- + iK^- & \bar{K}_S^0 + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

Glueballs

Lattice QCD calculation



The pseudoscalar glueball $\breve{G} \equiv |gg\rangle$ at the border within light and heavy

$$M_{\tilde{G}} = 2.6 \ \text{GeV}, \ J^{PC} = 0^{-+}, \ I = 0.$$

The first excited pseudoscalar Glueball

$$M_{\tilde{G}'} = 3.7 \, \text{GeV} \ , \ J^{PC} = 0^{*-+}.$$

[C. Morningstar and M. J. Peardon, AIP Conf. Proc. 688, 220 (2004)

[arXiv:nucl-th/0309068]];

Two experiments related to our work:

1. PANDA experiment at FAIR facility.
   It will be capable to scan the mass region above 2.5 GeV.

2. BESIII experiment.
   The resonance X(2370) could be a pseudoscalar glueball with a mass 2.37 GeV.

Predictions for a pseudoscalar glueball

- Predict branching ratios for decays into three pseudoscalar mesons $\tilde{G} \to PPP$.

Quantity	Case (i): $M_{\tilde{G}} = 2.6$ GeV	Case (ii): $M_{\tilde{G}} = 2.37$ GeV
$\Gamma_{\tilde{G}\to KK\eta}/\Gamma_{\tilde{G}}^{tot}$	0.049	0.043
$\Gamma_{\tilde{G}\to KK\eta'}/\Gamma_{\tilde{G}}^{tot}$	0.019	0.011
$\Gamma_{\tilde{G}\to \eta\eta\eta}/\Gamma_{\tilde{G}}^{tot}$	0.016	0.013
$\Gamma_{\tilde{G}\to \eta\eta\eta'}/\Gamma_{\tilde{G}}^{tot}$	0.0017	0.00082
$\Gamma_{\tilde{G}\to \eta\eta'\eta'}/\Gamma_{\tilde{G}}^{tot}$	0.00013	0
$\Gamma_{\tilde{G}\to KK\pi}/\Gamma_{\tilde{G}}^{tot}$	0.47	0.47
$\Gamma_{\tilde{G}\to \eta\pi\pi}/\Gamma_{\tilde{G}}^{tot}$	0.16	0.17
$\Gamma_{\tilde{G}\to \eta'\pi\pi}/\Gamma_{\tilde{G}}^{tot}$	0.095	0.090

The decay of the pseudoscalar glueball into three pions vanishes:

$$\Gamma_{\tilde{G}\to\pi\pi\pi} = 0$$

W. I. Eshraim; S. Janowski; F. Giacosa; D. H. Rischke. Phys.Rev. D87 (2013) 054036 [arXiv: 1208.6474 [hep-ph].

Predict branching ratios for decays into a scalar and a pseudoscalar meson

$$\tilde{G} \to PS$$

Quantity	Case (i): $M_{\tilde{G}} = 2.6$ GeV	Case (ii): $M_{\tilde{G}} = 2.37$ GeV
$\Gamma_{\tilde{G}\to KK_S}/\Gamma^{tot}_{\tilde{G}}$	0.060	0.070
$\Gamma_{\tilde{G}\to a_0\pi}/\Gamma^{tot}_{\tilde{G}}$	0.083	0.10
$\Gamma_{\tilde{G}\to \eta\sigma_N}/\Gamma^{tot}_{\tilde{G}}$	0.0000026	0.0000030
$\Gamma_{\tilde{G}\to \eta'\sigma_N}/\Gamma^{tot}_{\tilde{G}}$	0.039	0.026
$\Gamma_{\tilde{G}\to \eta\sigma_S}/\Gamma^{tot}_{\tilde{G}}$	0.012 (0.015)	0.0094 (0.017)
$\Gamma_{\tilde{G}\to \eta'\sigma_S}/\Gamma^{tot}_{\tilde{G}}$	0 (0.0082)	0 (0)

Could be measured by 

whereas

$$K_s = K_0^*(1430), a_0 = a_0(1450), \sigma_N \approx f_0(1370), \sigma_S \approx f_0(1710)$$

The full width of the pseudoscalar glueball is expected to be small

W. I. Eshraim; S. Janowski; F. Giacosa; D. H. Rischke. Phys.Rev. D87 (2013) 054036 [arXiv: 1208.6474 [hep-ph]; W.I. Eshraim and S. Janowski, PoS ConfinementX 118, (2012) [arXiv:1301.3345 [hep-ph]]; W.I. Eshraim and S. Janowski, J. Phys.Conf. Ser. 426, 012018 (2013) [arXiv:1211.7323 [hep-ph]].

Charmed mesons in the extended Linear Sigma Model

Charmed mesons in the model

- The number of fields in the model

$$4N_f^2 + 2 \text{ fields}$$

- For $N_f = 4$ there are *66* fields: *64* quark-antiquark fields + one pseudoscalar glueball $\tilde{G}$ +one scalar glueball G

Quantum numbers (J^{PC})

Pseudoscalar mesons: 0^{-+}	Scalar mesons: 0^{++}	Vector mesons: 1^{--}	Axial-vector mesons: 1^{++}
D^0	$D_0^{*0} \Rightarrow D_0^*(2400)^0$	$D^{*0} \Rightarrow D^*(2007)^0$	$D_1^0 \Rightarrow D_1(2420)^0$
$D^\pm$	$D_0^{*\pm} \Rightarrow D_0^*(2400)^\pm$	$D^{*\pm} \Rightarrow D^*(2010)^\pm$	$D_1^\pm \Rightarrow D_1(2420)^\pm$
$D_s^\pm$	$D_{S0}^{*\pm} \Rightarrow D_{S0}^*(2317)^\pm$	$D_S^\pm$	$D_{S1}^\pm \Rightarrow D_{S1}(2536)^\pm$
$\eta_C \Rightarrow \eta_C(1S)$	$\chi_{c0} \Rightarrow \chi_{c0}(1P)$	$J/\psi \Rightarrow J/\psi(1S)$	$\chi_{c1} \Rightarrow \chi_{c1}(1P)$

W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph]]; W. I. Eshraim, F. Giacosa, and D. H. Rischke , Eur. Phys. J. A 51 (2015) 112 [arXiv:1405.5861 [hep-ph]]

Including charm degree of freedom

1) Pseudoscalar fields:

$$P=\frac{1}{\sqrt{2}}\begin{pmatrix}\frac{1}{\sqrt{2}}(\eta_N+\pi^0) & \pi^+ & K^+ & D^0\\ \pi^- & \frac{1}{\sqrt{2}}(\eta_N-\pi^0) & K^0 & D^-\\ K^- & \overline{K}^0 & \eta_S & D_S^-\\ \overline{D}^0 & D^+ & D_S^+ & \eta_c\end{pmatrix}$$

2) Scalar fields:

$\overline{n}n \propto \overline{u}u+\overline{d}d$

$$S=\frac{1}{\sqrt{2}}\begin{pmatrix}\frac{1}{\sqrt{2}}(\sigma_N+a_0^0) & a_0^+ & K_0^{*+} & D_0^{*0}\\ a_0^- & \frac{1}{\sqrt{2}}(\sigma_N-a_0^0) & K_0^{*0} & D_0^{*-}\\ K_0^{*-} & \overline{K}_0^{*0} & \sigma_S & D_{S0}^{*-}\\ \overline{D}_0^{*0} & D_0^{*+} & D_{S0}^{*+} & \chi_{c0}\end{pmatrix}$$

$\overline{s}s$

$\overline{c}c$

The multiplet of the scalar and pseudoscalar quark-antiquark states: $\Phi = S + iP$

3) Vector fields:

$$V^\mu=\frac{1}{\sqrt{2}}\begin{pmatrix}\frac{1}{\sqrt{2}}(\omega_N+\rho^0) & \rho^+ & K^*(892)^+ & D^{*0}\\ \rho^- & \frac{1}{\sqrt{2}}(\omega_N-\rho^0) & K^*(892)^0 & D^{*-}\\ K^*(892)^- & \overline{K}^*(892)^0 & \omega_S & D_S^{*-}\\ \overline{D}^{*0} & D^{*+} & D_S^{*+} & J/\psi\end{pmatrix}^\mu$$

4) Axial vector fields:

$$A^\mu=\frac{1}{\sqrt{2}}\begin{pmatrix}\frac{1}{\sqrt{2}}(f_{1,N}+a_1^0) & a_1^+ & K_1^+ & D_1^0\\ a_1^- & \frac{1}{\sqrt{2}}(f_{1,N}-a_1^0) & K_1^0 & D_1^-\\ K_1^- & \bar{K}_1^0 & f_{1,S} & D_{S1}^-\\ \bar{D}_1^0 & D_1^+ & D_{S1}^+ & \chi_{c,1}\end{pmatrix}^\mu$$

The left-handed matrix: $L^\mu = V^\mu + A^\mu$ and the right-handed matrix: $R^\mu = V^\mu - A^\mu$

W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph]];
W. I. Eshraim, F. Giacosa, and D. H. Rischke , Eur. Phys. J. A 51 (2015) 112 [arXiv:1405.5861 [hep-ph]].

Linear Sigma Model Lagrangian with (axial-)vector mesons

$$\mathcal{L} = \mathcal{L}_{dil} + \mathrm{Tr}[(D^\mu\Phi)^\dagger(D^\mu\Phi)] - m_0^2\left(\frac{G}{G_0}\right)^2 \mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\,\mathrm{Tr}(\Phi^\dagger\Phi)^2$$
$$+ \mathrm{Tr}\left\{\left[\left(\frac{G}{G_0}\right)^2\frac{m_1^2}{2} + \Delta\right][(L^\mu)^2 + (R^\mu)^2]\right\} + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] - 2\,\mathrm{Tr}[\varepsilon\Phi^\dagger\Phi]$$
$$- \frac{1}{4}\mathrm{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + c(det\Phi - det\Phi^\dagger)^2 + i\tilde{c}\,\tilde{G}\,(\det\Phi - \det\Phi^\dagger) + i\frac{g_2}{2}\{\mathrm{Tr}(L_{\mu\nu}[L^\mu, L^\nu])$$
$$+ \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)Tr[(L^\mu)^2 + (R^\mu)^2] + h_2\,\mathrm{Tr}[(\Phi R^\mu)^2 + (L^\mu\Phi)^2] + 2h_3\mathrm{Tr}(\Phi R_\mu\Phi^\dagger L^\mu) + \mathrm{Tr}(R_{\mu\nu}[R^\mu, R^\nu])\} + \ldots,$$

where $\mathcal{L}_{dil}$ is the dilaton Lagrangian,

$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda^2}\left(G^4\ln\frac{G^2}{\Lambda^2} - \frac{G^4}{4}\right)$$

D.Parganlija, P.Kovacs, G.Wolf, F.Giacosa and D.H. Rischke, Phys. Rev. D 87 (2013) 014011 [arXiv:1208.0585 [hep-ph]];
W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph]].

Spontaneous Symmetry Breaking (SSB)

Shifting the fields

$$G \to G + G_0 \,,\quad \sigma_N \to \sigma_N + \phi_N \,,\quad \sigma_S \to \sigma_S + \phi_S$$

where,

$$\phi_N = Z_\pi f_\pi \qquad \phi_S = \frac{2Z_k f_k - \phi_N}{\sqrt{2}}$$

D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, Phys. Rev. D 87 (2013) 014011 [arXiv:1208.0585 [hep-ph].

For $N_f = 4$ new shift

$$\chi_{C0} \to \chi_{C0} + \phi_C$$

W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph]];
W. I. Eshraim, F. Giacosa, and D. H. Rischke , Eur. Phys. J. A 51 (2015) 112 [arXiv:1405.5861 [hep-ph]].

where

$$\phi_C = \frac{2Z_D f_D - \phi_N}{\sqrt{2}} = \sqrt{2} Z_{D_s} f_{D_s} - \phi_S = \frac{Z_{\eta_C} f_{\eta_C}}{\sqrt{2}}$$

There are 29 eqs. of square masses of mesons with 15 unknown parameters.

Parameters

The values of the $N_f = 3$ parameters :

Parameter	Value	Parameter	Value
m_1^2	0.413×10^6 MeV^2	m_0^2	-0.918×10^6 MeV^2
$\phi_C^2 c/2$	$450 \cdot 10^{-6}$ MeV^{-2}	δ_S	0.151×10^6MeV^2
g_1	5.84	h_1	0
h_2	9.88	h_3	3.87
ϕ_N	164.6 MeV	ϕ_S	126.2 MeV
λ_1	0	λ_2	68.3

[D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, Phys. Rev. D 87 (2013) 014011 [arXiv:1208.0585 [hep-ph]].

⇒ χ^2 /d.o.f = 1.23

The new three parameters for $N_f = 4$ are $\phi_C, \delta_C, \varepsilon_C$.

By fit with χ^2 /d.o.f = 1 :

$$\phi_C = (176 \pm 28)\ \text{MeV},\ \ \delta_C = (3.91 \pm 0.36) \times 10^6\ \text{MeV}^2,\ \varepsilon_C = (2.23 \pm 0.71) \times 10^6\ \text{MeV}^2\ .$$

W. I. Eshraim, F. Giacosa, and D. H. Rischke , Eur. Phys. J. A 51 (2015) 112 [arXiv:1405.5861 [hep-ph]].

Results

Masses of light mesons:

Observable	our Value [MeV]	Experimental Value [MeV]
$m_{f_{1N}}$	1186	1281.8 ± 0.6
m_{a_1}	1185	1230 ± 40
$m_{f_{1S}}$	1372	1426.4 ± 0.9
m_{K^*}	885	891.66 ± 0.26
m_{K_1}	1281	1272 ± 7
m_{σ_1}	1362	(1200-1500)-i(150-250)
m_{a_0}	1363	1474 ± 19
m_{σ_2}	1531	1720 ± 60
m_{w_N}	783	782.65 ± 0.12
m_{w_S}	975	1019.46 ± 0.020
m_ρ	783	775.5 ± 38.8
m_η	509	547.853 ± 0.024
m_π	141	139.57018 ± 0.00035
$m_{\eta'}$	962	957.78 ± 0.06
$m_{K_0^*}$	1449	1425 ± 50
m_K	485	493.677 ± 0.016

W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph];
D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, Phys. Rev. D 87 (2013) 014011 [arXiv:1208.0585 [hep-ph].

Masses of (open and hidden) charmed mesons:

Resonance	Quark content	J^P	Our Value [MeV]	Experimental Value [MeV]
D^0	$u\bar{c}, \bar{u}c$	0^-	1981 ± 73	1864.86 ± 0.13
$D_S^{\pm}$	$s\bar{c}, \bar{s}c$	0^-	2004 ± 74	1968.50 ± 0.32
$\eta_c(1S)$	$c\bar{c}$	0^-	2673 ± 118	2983.7 ± 0.7
$D_0^*(2400)^0$	$u\bar{c}, \bar{u}c$	0^+	2414 ± 77	2318 ± 29
$D_{S0}^*(2317)^{\pm}$	$s\bar{c}, \bar{s}c$	0^+	2467 ± 76	2317.8 ± 0.6
$\chi_{c0}(1P)$	$c\bar{c}$	0^+	3144 ± 128	3414.75 ± 0.31
$D^*(2007)^0$	$u\bar{c}, \bar{u}c$	1^-	2168 ± 70	2006.99 ± 0.15
D_s^*	$s\bar{c}, \bar{s}c$	1^-	2203 ± 69	2112.3 ± 0.5
$J/\psi(1S)$	$c\bar{c}$	1^-	2947 ± 109	3096.916 ± 0.011
$D_1(2420)^0$	$u\bar{c}, \bar{u}c$	1^+	2429 ± 63	2421.4 ± 0.6
$D_{S1}(2536)^{\pm}$	$s\bar{c}, \bar{s}c$	1^+	2480 ± 63	2535.12 ± 0.13
$\chi_{c1}(1P)$	$c\bar{c}$	1^+	3239 ± 101	3510.66 ± 0.07

W. I. Eshraim, F. Giacosa, and D. H. Rischke , Eur. Phys. J. A 51 (2015) 112 [arXiv:1405.5861 [hep-ph]].
W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph].

Mass difference and decay constants

The mass difference of the squared charmed (axial-)vector mesons:

mass difference	theoretical value MeV^2	experimental value MeV^2
$m^2_{D_1} - m^2_{D^*}$	$(1.2 \pm 0.6) \times 10^6$	1.82×10^6
$m^2_{\chi_{C1}} - m^2_{J/\psi}$	$(1.8 \pm 1.3) \times 10^6$	2.73×10^6
$m^2_{D_{S1}} - m^2_{D^*_S}$	$(1.2 \pm 0.6) \times 10^6$	1.97×10^6

Weak decay constant of D, D_S, and f_{η_C} .

$$f_D = (254 \pm 17)\ \text{MeV}\ ,\ f_{D_S} = (261 \pm 17)\ \text{MeV}\ ,\ f_{\eta_C} = (314 \pm 39)\ \text{MeV}.$$

[Exp. value = 206.7 ± 8.9] *MeV*, [Exp. value = 260.5 ± 5.4] *MeV*, [Exp. value = 335 ± 75] *MeV*

W. I. Eshraim, F. Giacosa, and D. H. Rischke , Eur. Phys. J. A 51 (2015) 112 [arXiv:1405.5861 [hep-ph]].

Decay widths of open charmed mesons:

Decay Channel	Theoretical result [MeV]	Experimental result [MeV]
$D_0^*(2400)^0 \to D\pi = D^+\pi^- + D^0\pi^0$	139^{+243}_{-114}	$D^+\pi^-$ seen; full width $\Gamma = 267 \pm 40$
$D_0^*(2400)^+ \to D\pi = D^+\pi^0 + D^0\pi^+$	51^{+182}_{-51}	$D^+\pi^0$ seen; full width: $\Gamma = 283 \pm 24 \pm 34$
$D^*(2007)^0 \to D^0\pi^0$	0.025 ± 0.003	seen; < 1.3
$D^*(2007)^0 \to D^+\pi^-$	0	not seen
$D^*(2010)^+ \to D^+\pi^0$	$0.018^{+0.002}_{-0.003}$	0.029 ± 0.008
$D^*(2010)^+ \to D^0\pi^+$	$0.038^{+0.005}_{-0.004}$	0.065 ± 0.017
$D_1(2420)^0 \to D^*\pi = D^{*+}\pi^- + D^{*0}\pi^0$	65^{+51}_{-37}	$D^{*+}\pi^-$ seen; full width: $\Gamma = 27.4 \pm 2.5$
$D_1(2420)^0 \to D^0\pi\pi = D^0\pi^+\pi^- + D^0\pi^0\pi^0$	0.59 ± 0.02	seen
$D_1(2420)^0 \to D^+\pi^-\pi^0$	$0.21^{+0.01}_{-0.015}$	seen
$D_1(2420)^0 \to D^+\pi^-$	0	not seen; $\Gamma(D^+\pi^-)/\Gamma(D^{*+}\pi^-) < 0.24$
$D_1(2420)^+ \to D^*\pi = D^{*+}\pi^0 + D^{*0}\pi^+$	65^{+51}_{-36}	$D^{*0}\pi^+$ seen; full width: $\Gamma = 25 \pm 6$
$D_1(2420)^+ \to D^+\pi\pi = D^+\pi^+\pi^- + D^+\pi^0\pi^0$	0.56 ± 0.02	seen
$D_1(2420)^+ \to D^0\pi^0\pi^+$	0.22 ± 0.01	seen
$D_1(2420)^+ \to D^0\pi^+$	0	not seen; $\Gamma(D^0\pi^+)/\Gamma(D^{*0}\pi^+) < 0.18$
$D_{S1}(2536)^+ \to D^*K = D^{*0}K^+ + D^{*+}K^0$	25^{+22}_{-15}	seen; full width $\Gamma = 0.92 \pm 0.03 \pm 0.04$
$D_{S1}(2536)^+ \to D^+K^0$	0	not seen
$D_{S1}(2536)^+ \to +D^0K^+$	0	not seen

W. I. Eshraim, F. Giacosa, and D. H. Rischke , Eur. Phys. J. A 51 (2015) 112 [arXiv:1405.5861 [hep-ph]].

Decay widths of hidden charmed mesons:

- The decay widths of charmonium state depend on the parameters λ_1 and h_1.

Using fit including the decay widths of charmonium state χ_{C0}, we obtain

$$\lambda_1 = -0.16 \quad \text{and} \quad h_1 = 0.046.$$

W. I. Eshraim, EPJ Web Conf. 126 (2016) 04017.

Mixing matrix of the three scalar fields (σ_N, σ_S, G)

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.94 & -0.17 & 0.29 \\ 0.21 & 0.97 & -0.12 \\ -0.26 & 0.18 & 0.95 \end{pmatrix} \begin{pmatrix} \sigma_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2} \\ \sigma_S \equiv \bar{s}s \\ G \equiv gg \end{pmatrix}$$

where G is a scalar glueball. **S. Janowski, F. Giacosa and D. H. Rischk, Phys. Rev. D90 (2014) 114005 .**

Decay widths of hidden charmed mesons:

1) Decay widths of (axial-)vector charmonium states: $\Gamma_{J/\psi} = 0$ and $\Gamma_{\chi_{c1}} = 0$

2) Decay widths of a pseudoscalar charmonium state (η_C):

Decay Channel	theoretical result [MeV]	Experimental result [MeV]
$\Gamma_{\eta_c \to \overline{K}_0^* K}$	0.01	-
$\Gamma_{\eta_c \to a_0 \pi}$	0.01	-
$\Gamma_{\eta_c \to f_0(1370)\eta}$	0.00018	-
$\Gamma_{\eta_c \to f_0(1500)\eta}$	0.006	-
$\Gamma_{\eta_c \to f_0(1710)\eta}$	0.000032	-
$\Gamma_{\eta_c \to f_0(1370)\eta'}$	0.027	-
$\Gamma_{\eta_c \to f_0(1500)\eta'}$	0.024	-
$\Gamma_{\eta_c \to f_0(1710)\eta'}$	0.0006	-
$\Gamma_{\eta_c \to \eta\eta\eta}$	0.052	-
$\Gamma_{\eta_c \to \eta'\eta'\eta'}$	0.0023	-
$\Gamma_{\eta_c \to \eta'\eta\eta}$	0.44	-
$\Gamma_{\eta_c \to \eta'\eta'\eta}$	0.0034	
$\Gamma_{\eta_c \to \eta K \overline{K}}$	0.15	0.32±0.17
$\Gamma_{\eta_c \to \eta' K K}$	0.41	
$\Gamma_{\eta_c \to \eta\pi\pi}$	0.12	0.54±0.18
$\Gamma_{\eta_c \to \eta'\pi\pi}$	0.08	1.3±0.0.6
$\Gamma_{\eta_c \to K K \pi}$	0.095	-

W. I. Eshraim, EPJ Web Conf. 126 (2016) 04017.

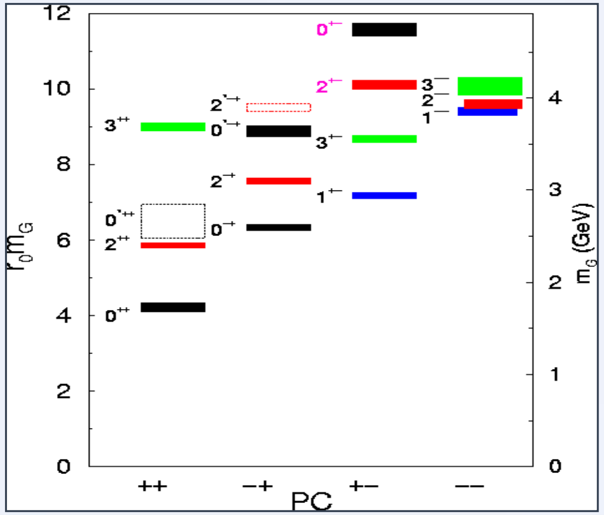
Decay width of η_C into a pseudoscalar glueball

BESIII: $m_{\tilde{G}} = 2370\ MeV$

$$\Gamma_{\eta_{C \to \tilde{G}\pi\pi}} = \begin{cases} = 0.16\ MeV \\ = 0.124\ MeV \end{cases}$$

Could be measured by

Lattice QCD calculations:



$m_{\tilde{G}} = 2600\ MeV$

W. I. Eshraim, EPJ Web Conf. 126 (2016) 04017.

Decay widths of a pseudoscalar charmonium state (χ_{c0}):

Decay Channel	theoretical result [MeV]	Experimental result [MeV]
$\Gamma_{\chi_{c0}\to a_0 a_0}$	0.004	-
$\Gamma_{\chi_{c0}\to k_1 \overline{K}_1}$	0.005	-
$\Gamma_{\chi_{c0}\to \eta\eta}$	0.022	0.031±0.0039
$\Gamma_{\chi_{c0}\to \eta'\eta'}$	0.02	0.02±0.0035
$\Gamma_{\chi_{c0}\to \eta\eta'}$	0.004	<0.0024
$\Gamma_{\chi_{c0}\to K^* K_0^*}$	0.00007	-
$\Gamma_{\chi_{c0}\to \rho\rho}$	0.01	-
$\Gamma_{\chi_{c0}\to f_0(1370) f_0(1370)}$	0.005	<0.003
$\Gamma_{\chi_{c0}\to f_0(1500) f_0(1500)}$	0.004	<0.0005
$\Gamma_{\chi_{c0}\to f_0(1370) f_0(1500)}$	0.000004	<0.001
$\Gamma_{\chi_{c0}\to f_0(1370) f_0(1710)}$	0.0003	0.0069±0.004
$\Gamma_{\chi_{c0}\to f_0(1500) f_0(1710)}$	0.00004	<0.0007
$\Gamma_{\chi_{c0}\to K_0^* K\eta}$	0.008	-
$\Gamma_{\chi_{c0}\to K_0^* K\eta'}$	0.004	-
$\Gamma_{\chi_{c0}\to f_0(1370)\eta\eta}$	0.0004	-
$\Gamma_{\chi_{c0}\to f_0(1500)\eta\eta}$	0.003	-
$\Gamma_{\chi_{c0}\to f_0(1370)\eta'\eta'}$	0.0027	-
$\Gamma_{\chi_{c0}\to f_0(1370)\eta\eta'}$	0.000089	-
$\Gamma_{\chi_{c0}\to f_0(1500)\eta\eta'}$	0.011	-
$\Gamma_{\chi_{c0}\to f_0(1710)\eta\eta}$	0.00008	-
$\Gamma_{\chi_{c0}\to f_0(1710)\eta\eta'}$	0.00003	-

Decay Channel	theoretical result [MeV]	Experimental result [MeV]
$\Gamma_{\chi_{c0}\to \overline{K}_0^{*0} K_0^{*0}}$	0.01	0.01±0.0047
$\Gamma_{\chi_{c0}\to K^- K^+}$	0.059	0.061±0.007
$\Gamma_{\chi_{c0}\to \pi\pi}$	0.089	0.088±0.0092
$\Gamma_{\chi_{c0}\to \overline{K}^{*0} K^{*0}}$	0.0175	0.017±0.0072
$\Gamma_{\chi_{c0}\to ww}$	0.01	0.0099±0.0017
$\Gamma_{\chi_{c0}\to \phi\phi}$	0.004	0.0081±0.0013
$\Gamma_{\chi_{c0}\to k_1^+ K^-}$	0.005	0.063±0.0233

W. I. Eshraim, EPJ Web Conf. 126 (2016) 04017.

Decay modes of the excited pseudoscalar glueball

Interaction Lagrangain for the excited pseudoscalar glueball

with a pseudoscalar glueball and the ordinary scalar and pseudoscalar mesons

$$\mathcal{L}^{int}_{\tilde{G}\tilde{G}'} = c_{\tilde{G}\tilde{G}'}\tilde{G}\tilde{G}'\, Tr\left(\Phi^{\dagger}\Phi\right)$$

with a scalar glueball and the pseudo(scalar) mesons

$$\mathcal{L}^{int}_{\tilde{G}G} = ic_{\tilde{G}G\Phi}\tilde{G}G\left(\det\Phi - \det\Phi^{\dagger}\right)$$

with scalar and pseudoscalar mesons

$$\mathcal{L}^{int}_{\tilde{G}\Phi} = ic_{\tilde{G}\Phi}\tilde{G}\left(\det\Phi - \det\Phi^{\dagger}\right)$$

Walaa I. Eshraim, Stefan Schramm, Phys.Rev. D95 (2017) 014028 [arXiv:1606.02207 [hep-ph]].

Results

Branching ratios for the decay of the excited pseudoscalar glueball into the pseudoscalar glueball

Quantity	The theoretical result
$\Gamma_{\tilde{G}\to\tilde{G}' KK}/\Gamma^{tot}_{\tilde{G}}$	0.0277
$\Gamma_{\tilde{G}\to\tilde{G}' \pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.9697
$\Gamma_{\tilde{G}\to\tilde{G}' \eta\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.0026
$\Gamma_{\tilde{G}\to\tilde{G}' \eta\eta}/\Gamma^{tot}_{\tilde{G}}$	0.000012

The branching ratio for the decay of the excited pseudoscalar glueball into charmonium state

$$\Gamma_{\tilde{G}\to\eta_C\pi\pi}/\Gamma^{tot}_{\tilde{G}_3} = 0.001$$

Could be measured by BESIII and PANDA!

Walaa I. Eshraim, Stefan Schramm, Phys.Rev. D95 (2017) 014028 [arXiv:1606.02207 [hep-ph]].

Branching ratios for the decays of the excited pseudoscalar glueball into *PS* and scalar-isoscalar states as well as η and $\eta^{'}$

Case (i):$\mathcal{L}^{int}_{\tilde{G}G}$	The theoretical result	Case (ii):$\mathcal{L}^{int}_{\tilde{G}\Phi}$	The theoretical result
$\Gamma_{\tilde{G}\to a_0\pi}/\Gamma^{tot}_{\tilde{G}_2}$	0.0325	$\Gamma_{\tilde{G}\to a_0\pi}/\Gamma^{tot}_{\tilde{G}_3}$	0.0313
$\Gamma_{\tilde{G}\to KK_S}/\Gamma^{tot}_{\tilde{G}_2}$	0.032	$\Gamma_{\tilde{G}\to KK_S}/\Gamma^{tot}_{\tilde{G}_3}$	0.001
$\Gamma_{\tilde{G}\to\eta f_0(1370)}/\Gamma^{tot}_{\tilde{G}_2}$	0.00004	$\Gamma_{\tilde{G}\to\eta f_0(1370)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0014
$\Gamma_{\tilde{G}\to\eta' f_0(1370)}/\Gamma^{tot}_{\tilde{G}_2}$	0.048	$\Gamma_{\tilde{G}\to\eta' f_0(1370)}/\Gamma^{tot}_{\tilde{G}_3}$	0.031
$\Gamma_{\tilde{G}\to\eta f_0(1500)}/\Gamma^{tot}_{\tilde{G}_2}$	0.0068	$\Gamma_{\tilde{G}\to\eta f_0(1500)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0067
$\Gamma_{\tilde{G}\to\eta' f_0(1500)}/\Gamma^{tot}_{\tilde{G}_2}$	0.0219	$\Gamma_{\tilde{G}\to\eta' f_0(1500)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0214
$\Gamma_{\tilde{G}\to\eta f_0(1710)}/\Gamma^{tot}_{\tilde{G}_2}$	0.0008	$\Gamma_{\tilde{G}\to\eta f_0(1710)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0007
$\Gamma_{\tilde{G}\to\eta' f_0(1710)}/\Gamma^{tot}_{\tilde{G}_2}$	0.001	$\Gamma_{\tilde{G}\to\eta' f_0(1710)}/\Gamma^{tot}_{\tilde{G}_3}$	0.001

Could be measured by BESIII and PANDA!

Walaa I. Eshraim, Stefan Schramm, Phys.Rev. D95 (2017) 014028 [arXiv:1606.02207 [hep-ph]].

Branching ratios for the decays of the excited pseudoscalar glueball into scalar-isoscalar states and (pseudo)scalar mesons

Case (i): $\mathcal{L}^{int}_{\tilde{G}G}$	The theoretical result	Case (ii): $\mathcal{L}^{int}_{\tilde{G}\Phi}$	The theoretical result
$\Gamma_{\tilde{G}\to\eta\pi\pi}/\Gamma^{tot}_{\tilde{G}_2}$	0.095	$\Gamma_{\tilde{G}\to\eta\pi\pi}/\Gamma^{tot}_{\tilde{G}_3}$	0.1376
$\Gamma_{\tilde{G}\to\eta'\pi\pi}/\Gamma^{tot}_{\tilde{G}_2}$	0.111	$\Gamma_{\tilde{G}\to\eta'\pi\pi}/\Gamma^{tot}_{\tilde{G}_3}$	0.1069
$\Gamma_{\tilde{G}\to a_0 K K_S}/\Gamma^{tot}_{\tilde{G}_2}$	0.0026	$\Gamma_{\tilde{G}\to a_0 K K_S}/\Gamma^{tot}_{\tilde{G}_3}$	0.0025
$\Gamma_{\tilde{G}\to\eta a_0 a_0}/\Gamma^{tot}_{\tilde{G}_2}$	0.0001	$\Gamma_{\tilde{G}\to\eta a_0 a_0}/\Gamma^{tot}_{\tilde{G}_3}$	0.0001
$\Gamma_{\tilde{G}\to a_0\pi f_0(1370)}/\Gamma^{tot}_{\tilde{G}_2}$	0.0003	$\Gamma_{\tilde{G}\to a_0\pi f_0(1370)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0003
$\Gamma_{\tilde{G}\to a_0\pi f_0(1500)}/\Gamma^{tot}_{\tilde{G}_2}$	0.0034	$\Gamma_{\tilde{G}\to a_0\pi f_0(1500)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0032
$\Gamma_{\tilde{G}\to a_0\pi f_0(1710)}/\Gamma^{tot}_{\tilde{G}_2}$	0.0001	$\Gamma_{\tilde{G}\to a_0\pi f_0(1710)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0001
$\Gamma_{\tilde{G}\to\eta f_0^2(1370)}/\Gamma^{tot}_{\tilde{G}_2}$	0.0003	$\Gamma_{\tilde{G}\to\eta f_0^2(1370)}/\Gamma^{tot}_{\tilde{G}_3}$	0.001
$\Gamma_{\tilde{G}\to\eta' f_0^2(1370)}/\Gamma^{tot}_{\tilde{G}_2}$	0.03×10^{-6}	$\Gamma_{\tilde{G}\to\eta' f_0^2(1370)}/\Gamma^{tot}_{\tilde{G}_3}$	0.006×10^{-6}
$\Gamma_{\tilde{G}\to\eta f_0^2(1500)}/\Gamma^{tot}_{\tilde{G}_2}$	0.00004	$\Gamma_{\tilde{G}\to\eta f_0^2(1500)}/\Gamma^{tot}_{\tilde{G}_3}$	0.00001
$\Gamma_{\tilde{G}\to\eta f_0(1370) f_0(1500)}/\Gamma^{tot}_{\tilde{G}_2}$	0.00003	$\Gamma_{\tilde{G}\to\eta f_0(1370) f_0(1500)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0001
$\Gamma_{\tilde{G}\to\eta f_0(1370) f_0(1710)}/\Gamma^{tot}_{\tilde{G}_2}$	3.798×10^{-6}	$\Gamma_{\tilde{G}\to\eta f_0(1370) f_0(1710)}/\Gamma^{tot}_{\tilde{G}_3}$	7.25×10^{-6}
$\Gamma_{\tilde{G}\to K K_S f_0(1370)}/\Gamma^{tot}_{\tilde{G}_2}$	0.0025	$\Gamma_{\tilde{G}\to K K_S f_0(1370)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0025
$\Gamma_{\tilde{G}\to K K_S f_0(1500)}/\Gamma^{tot}_{\tilde{G}_2}$	0.00013	$\Gamma_{\tilde{G}\to K K_S f_0(1500)}/\Gamma^{tot}_{\tilde{G}_3}$	0.00013
$\Gamma_{\tilde{G}\to K K_S f_0(1710)}/\Gamma^{tot}_{\tilde{G}_2}$	6.2×10^{-6}	$\Gamma_{\tilde{G}\to K K_S f_0(1710)}/\Gamma^{tot}_{\tilde{G}_3}$	4.75×10^{-6}
$\Gamma_{\tilde{G}\to K K\eta}/\Gamma^{tot}_{\tilde{G}_2}$	0.0668	$\Gamma_{\tilde{G}\to K K\eta}/\Gamma^{tot}_{\tilde{G}_3}$	0.0643
$\Gamma_{\tilde{G}\to K K\eta'}/\Gamma^{tot}_{\tilde{G}_2}$	0.045	$\Gamma_{\tilde{G}\to K K\eta'}/\Gamma^{tot}_{\tilde{G}_3}$	0.044
$\Gamma_{\tilde{G}\to K_S K_S\eta}/\Gamma^{tot}_{\tilde{G}_2}$	0.0002	$\Gamma_{\tilde{G}\to K_S K_S\eta}/\Gamma^{tot}_{\tilde{G}_3}$	0.0002
$\Gamma_{\tilde{G}\to\eta^3}/\Gamma^{tot}_{\tilde{G}_2}$	0.024	$\Gamma_{\tilde{G}\to\eta^3}/\Gamma^{tot}_{\tilde{G}_3}$	0.0233
$\Gamma_{\tilde{G}\to\eta'^3}/\Gamma^{tot}_{\tilde{G}_2}$	0.0048	$\Gamma_{\tilde{G}\to\eta'^3}/\Gamma^{tot}_{\tilde{G}_3}$	0.0046
$\Gamma_{\tilde{G}\to\eta'\eta^2}/\Gamma^{tot}_{\tilde{G}_2}$	0.005	$\Gamma_{\tilde{G}\to\eta'\eta^2}/\Gamma^{tot}_{\tilde{G}_3}$	0.0048
$\Gamma_{\tilde{G}\to\eta'^2\eta}/\Gamma^{tot}_{\tilde{G}_2}$	0.0035	$\Gamma_{\tilde{G}\to\eta'^2\eta}/\Gamma^{tot}_{\tilde{G}_3}$	0.0034
$\Gamma_{\tilde{G}\to K K\pi}/\Gamma^{tot}_{\tilde{G}_2}$	0.489	$\Gamma_{\tilde{G}\to K K\pi}/\Gamma^{tot}_{\tilde{G}_3}$	0.471
$\Gamma_{\tilde{G}\to K_S K_S\pi}/\Gamma^{tot}_{\tilde{G}_2}$	0.002	$\Gamma_{\tilde{G}\to K_S K_S\pi}/\Gamma^{tot}_{\tilde{G}_3}$	0.0057

Walaa I. Eshraim, Stefan Schramm,
Phys.Rev. D95 (2017) 014028
[arXiv:1606.02207 [hep-ph]].

Could be measured by BESIII and PANDA!

Conclusions

1. In the case of $N_f = 3$: Decay of a pseudoscalar glueball with a mass above of 2 GeV.

2. Linear sigma model with $N_f = 4$ and vector and axial-vector mesons.

3. Masses of (open and hidden) charmed mesons.

4. Decay widths of (open and hidden) charmed mesons.

5. Decay widths of the first excited pseudoscalar glueball in cases of $N_f = 3$ and $N_f = 4$.

Outlook

1. Study of the chirally symmetric model with vector and axial-vector mesons in the case of isospin breaking for $N_f = 3$ at zero temperature and extending the model by the light scalar mesons.

2. Decay of a charmed axial-vector and pseudovector mesons into a vector and a pseudoscalar meson by using a relativistic quantum field theoretical model.

3. Study of the light tetraquark nonet and its extension to $N_f = 4$.

4. Study the scattering of glueballs.