Properties of the kaon-nucleon systems and the $\Lambda(1405)$ in the Skyrme model

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1. Introduction

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1. Kaon-nucleon system

- Strong attraction between the anti-kaon($\bar{K}$) and the nucleon(N)

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- $\bar{K}N$ bound state = $\Lambda(1405)$?

  → $\bar{K}N$ and $\pi\Sigma$ resonance = $\Lambda(1405)$

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- Few body nuclear system with $\bar{K}$

$\bar{K}N$ interaction is important
to investigate the few body systems with $\bar{K}$

2. Theoretical studies for the $\bar{K}N$ interaction

- Phenomenological approach

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- Chiral theory: based on a 4-point local interaction

  T. Hyodo and W. Weise, Phys. Rev. **C 77** (2008).

  K. Miyahara and T. Hyodo, Phys. Rev. C **93** (2016).

Introduction

1. Kaon-nucleon system

- Strong attraction between the anti-kaon($\bar{K}$) and the nucleon(N)

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- $\bar{K}N$ bound state = $\Lambda(1405)$?

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- Few body nuclear system with $\bar{K}$

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2. Theoretical studies for the $\bar{K}N$ interaction

- Phenomenological approach

Investigate the $\bar{K}N$ system and the $\Lambda(1405)$
in the Skyrme model,
where the nucleon is described as a soliton.

2. The Skyrme model

The Skyrme model 1

1. The Skyrme model

T. H. R. Skyrme, Proc. Roy. Soc. Lond. A **260**, 127 (1961).
J. K. Perring and T. H. R. Skyrme, Nucl. Phys. **31**, 550 (1962).

- Describe the interaction between mesons and baryons by mesons
- Baryon emerges as a soliton of meson fields

2. The SU(3) Skyrme Lagrangian

$$L = \frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2 + L_{SB} + L_{WZ}$$

Parameters: F_π, e, $M_\pi = 0$, $M_K \neq 0$

$$U = \exp\left[i\frac{2}{F_\pi}\lambda_a\phi_a\right]$$

λ_a: Gell-Mann matrices (a = 1, 2, … ,8)

$$\phi = \frac{1}{\sqrt{2}}\lambda_a\phi_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

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Parameters: F_π, e, $M_\pi = 0$

$$U = \exp\left[i \frac{2}{F_\pi} \phi\right], \quad \phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 & \pi^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 \end{pmatrix}$$

The Skyrme model 2

3. One nucleon solution in the hedgehog Ansatz

π has three degrees of freedom(π^0, π^+, π^-)

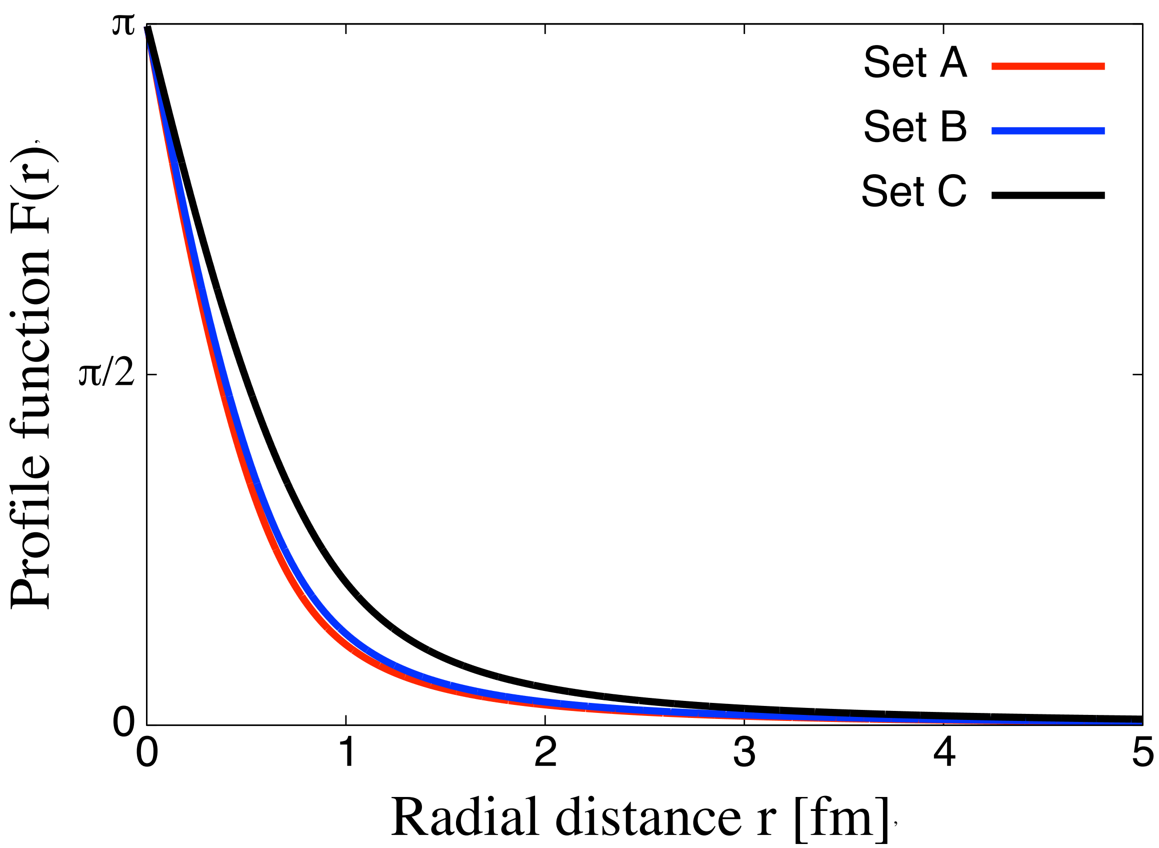
- two of these: the angles of the radial vector, θ, φ
- the rest: a function depending on r

⇒ a special configuration called the hedgehog Ansatz

Hedgehog ansatz: $U_H = \exp\left[i\boldsymbol{\tau}\cdot\hat{r}F(r)\right]$

minimize the mass of the soliton

with B.C.: $F(\infty) = 0,\ F(0) = \pi$



	F_π [MeV]	e	eF_π [MeV]
Set A	205	4.67	957
Set B	186	4.82	897
Set C	129	5.45	703

Set C: G. S. Adkins *et al.*, Nucl. Phys. B **228**, 552 (1983).

The Skyrme model 3

4. Collective quantization

The Hedgehog ansatz is classical solution

→ without spin or isospin

→ become a physical state by quantization

$$U_H(\boldsymbol{x}) \to U_H(t, \boldsymbol{x}) = A(t) \exp\left[i\tau_a R_{ab}(t) \hat{r}_b F(r)\right] A^\dagger(t)$$

$A(t)$: 2×2 isospin rotation matrix

$R_{ab}(t)$: 3×3 spatial rotation matrix

The baryon with $I = J$ from the symmetry which the hedgehog ansatz has

5. Nucleon energy

$$E = M_{sol} + \boxed{\frac{J(J+1)}{2\Lambda}}$$

the rotation energy

M_{sol}: soliton mass

J: spin or isospin

Λ: moment of inertia

3. Method

CK and EH approaches

1. The Callan-Klebanov (CK) bound state approach[1]

$$U_{CK} = A(t)\sqrt{U_\pi}U_K\sqrt{U_\pi}A^\dagger(t)$$

$A(t)$: Isospin rotation matrix

$$U_\pi = \begin{pmatrix} U_H & 0 \\ 0 & 1 \end{pmatrix}$$

(U_H: The Hedgehog soliton)

2. Our approach (EH approach)[2]

$$U_{EH} = A(t)\sqrt{U_\pi}A^\dagger(t)U_K A(t)\sqrt{U_\pi}A^\dagger(t)$$

$$U_K = \exp\left[i\frac{2}{F_\pi}\lambda_a K_a\right], \quad a = 4,5,6,7$$

$$\lambda_a K_a = \sqrt{2}\begin{pmatrix} 0_{2\times 2} & K \\ K^\dagger & 0 \end{pmatrix}$$

$$K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \quad K^\dagger = \begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix}$$

[1] C. G. Callan and I. Klebanov, Nucl. Phys. B **262** (1985).
C. G. Callan, K .Hornbostel and I. Klebanov, Phys. Lett. B **202** (1988).
[2] T. Ezoe and A. Hosaka Phys. Rev. D **94**, 034022 (2016).

CK and EH approaches

Common property: Flavor SU(3) symmetry is broken

1. The Callan-Klebanov (CK) bound state approach[1]

$$U_{CK} = A(t)\sqrt{U_\pi}U_K\sqrt{U_\pi}A^\dagger(t)$$

- Introduce the kaon as fluctuations around the hedgehog soliton
- Form a bound state of the kaon and the hedgehog soliton
- Quantize the system to generate hyperons

2. Our approach (EH approach)[2]

$$U_{EH} = A(t)\sqrt{U_\pi}A^\dagger(t)U_K A(t)\sqrt{U_\pi}A^\dagger(t)$$

- Quantize the hedgehog soliton to generate the nucleon
- Introduce the kaon as fluctuations around the nucleon
- describe kaon-nucleon systems

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- Form a bound state of the kaon and the hedgehog soliton
- Quantize the system to generate hyperons
- Projection after variation

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$$U_{EH} = A(t)\sqrt{U_\pi}A^\dagger(t)U_K A(t)\sqrt{U_\pi}A^\dagger(t)$$

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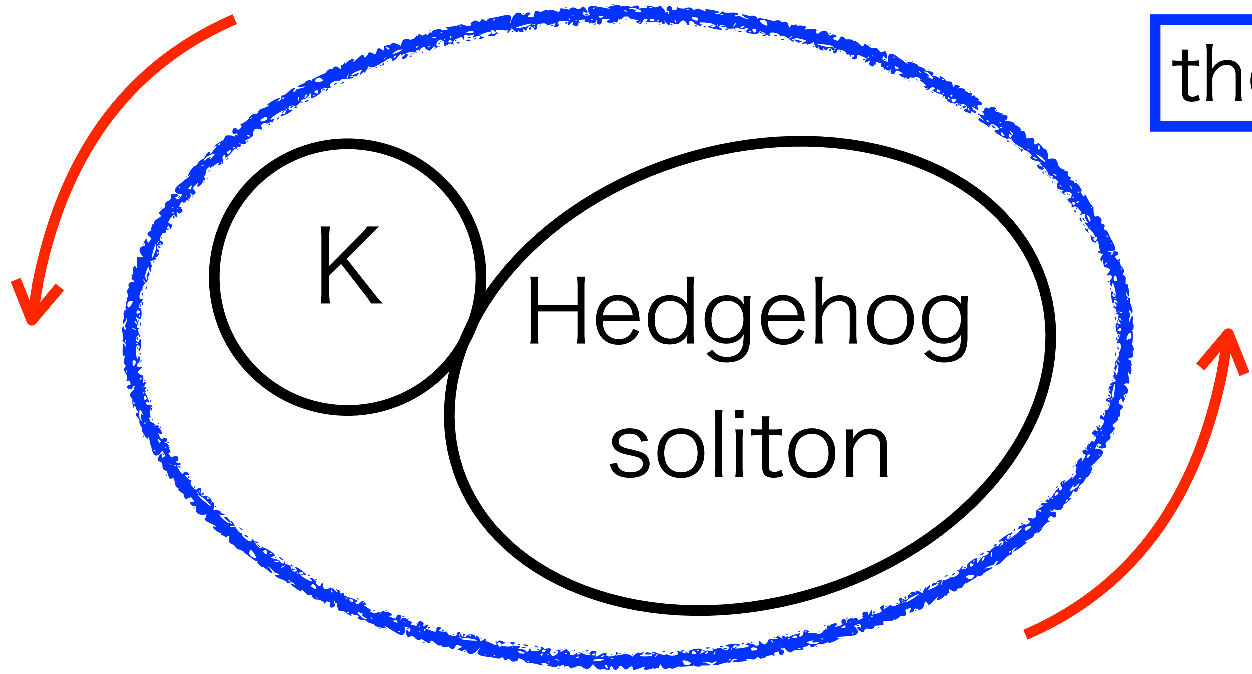
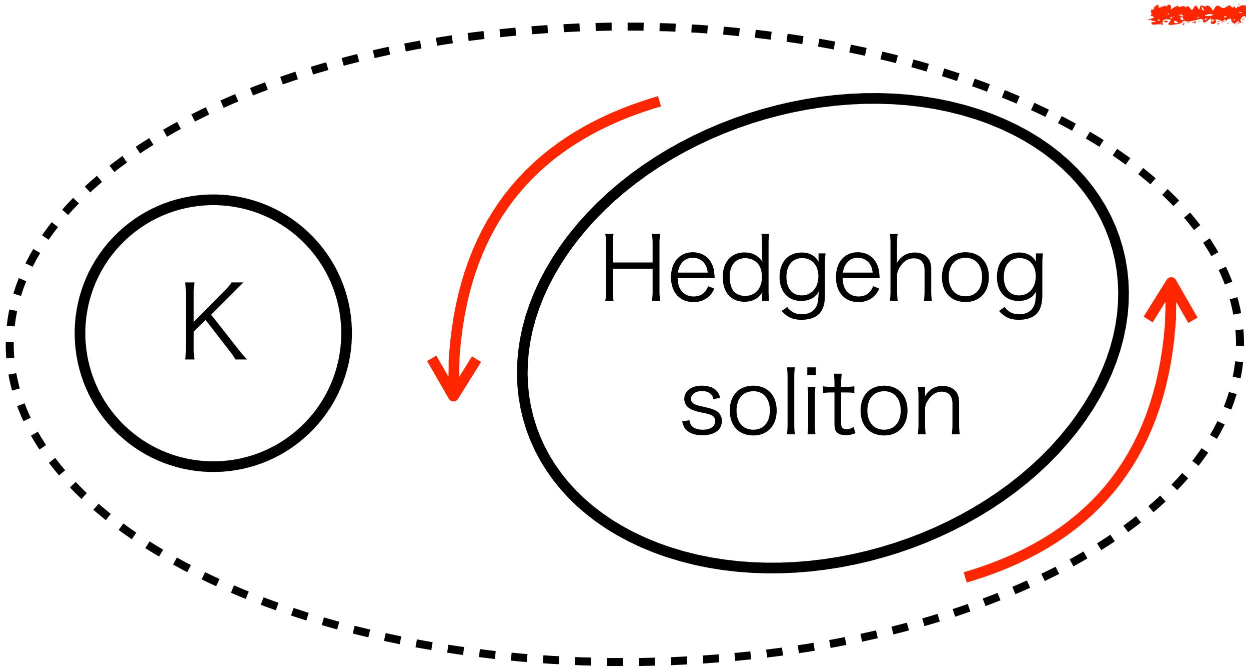
Difference in CK and EH

(1) Callan-Klebanov Ansatz $U_{CK} = A(t)\sqrt{U_\pi}U_K\sqrt{U_\pi}A^\dagger(t)$



the kaon around the hedgehog soliton

(2) Ezoe-Hosaka Ansatz $U_{EH} = A(t)\sqrt{U_\pi}A^\dagger(t)U_K A(t)\sqrt{U_\pi}A^\dagger(t)$



the kaon around
the “**rotating**” hedgehog soliton

Derivation 1

1. Substitute our ansatz for the Lagrangian

Ansatz

$$U_{EH} = A(t)\sqrt{U_\pi}A^\dagger(t)\,U_K\,A(t)\sqrt{U_\pi}A^\dagger(t)$$

$$U_\pi = \begin{pmatrix} U_H & 0 \\ 0 & 1 \end{pmatrix}$$

U_H: Hedgehog soliton (2×2 matrix)

$$U_K = \exp\left[i\frac{2}{F_\pi}\lambda_a K_a\right], \quad a = 4,5,6,7$$

$$\lambda_a K_a = \sqrt{2}\begin{pmatrix} 0_{2\times 2} & K \\ K^\dagger & 0 \end{pmatrix} \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$$

Lagrangian

$$L = \frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2 + L_{SB} + L_{WZ}$$

Obtained Lagrangian

2. Expand U_K up to second order of the kaon field K

$$L = L_{SU(2)} + L_{KN}$$

$$L_{SU(2)} = \frac{1}{16} {F_\pi}^2 \mathrm{tr}\left[\partial_\mu \tilde{U}^\dagger \partial^\mu \tilde{U}\right] + \frac{1}{32e^2}\mathrm{tr}\left[\partial_\mu \tilde{U}\tilde{U}^\dagger, \partial_\nu \tilde{U}\tilde{U}^\dagger\right]^2$$

$$\begin{aligned} L_{KN} \quad = \quad & \left(D_\mu K\right)^\dagger D^\mu K - K^\dagger a_\mu^\dagger a^\mu K - m_K^2 K^\dagger K \\ & + \frac{1}{(eF_\pi)^2}\left\{ -K^\dagger K \mathrm{tr}\left[\partial_\mu \tilde{U}\tilde{U}^\dagger, \partial_\nu \tilde{U}\tilde{U}^\dagger\right]^2 - 2\left(D_\mu K\right)^\dagger D_\nu K \mathrm{tr}\left(a^\mu a^\nu\right) \right. \\ & \qquad \left. - \frac{1}{2}\left(D_\mu K\right)^\dagger D^\mu K \mathrm{tr}\left(\partial_\nu \tilde{U}^\dagger \partial^\nu \tilde{U}\right) + 6\left(D_\nu K\right)^\dagger \left[a^\nu, a^\mu\right] D_\mu K \right\} \\ & + \frac{3i}{F_\pi^2} B^\mu \left[\left(D_\mu K\right)^\dagger K - K^\dagger \left(D_\mu K\right)\right] \end{aligned}$$

$$\tilde{U} = A(t) U_H A^\dagger(t), \quad \tilde{\xi} = A(t)\sqrt{U_H} A^\dagger(t) \qquad D_\mu K = \partial_\mu K + v_\mu K$$

$$v_\mu = \frac{1}{2}\left(\tilde{\xi}^\dagger \partial_\mu \tilde{\xi} + \tilde{\xi}\partial_\mu \tilde{\xi}^\dagger\right)$$

$$a_\mu = \frac{1}{2}\left(\tilde{\xi}^\dagger \partial_\mu \tilde{\xi} - \tilde{\xi}\partial_\mu \tilde{\xi}^\dagger\right)$$

$$B^\mu = -\frac{\varepsilon^{\mu\nu\alpha\beta}}{24\pi^2}\mathrm{tr}\left[\left(\tilde{U}^\dagger \partial_\nu \tilde{U}\right)\left(\tilde{U}^\dagger \partial_\alpha \tilde{U}\right)\left(\tilde{U}^\dagger \partial_\beta \tilde{U}\right)\right]$$

G. S. Adkins, C. R. Nappi and E. Witten, Nucl. Phys. B **228** (1983)

Derivation 2

3. Decompose the kaon filed

$$\begin{pmatrix} K^+ \\ K^0 \end{pmatrix} = \psi_I K(t, \boldsymbol{r}) \to \psi_I \; K(\boldsymbol{r})\, e^{-iEt}$$

Spatial wave function ($K(\boldsymbol{r})$)

Isospin wave function (ψ_I)

4. Expand the $K(r)$ by the spherical harmonics

$$K(\boldsymbol{r}) = \sum_{l,m} C_{lm\alpha} Y_{lm}(\theta, \phi)\, k_l^{\alpha}(r)$$

$Y_{lm}(\theta, \varphi)$: Spherical harmonics

l : orbital angular momentum

m : the 3rd component of l

α : the other quantum numbers

5. Take a variation with respect to the kaon radial function

⇒Obtain the equation of motion for the kaon around the nucleon

4. Kaon-Nucleon system

T. Ezoe. and A. Hosaka Phys. Rev. D **94**, 034022 (2016).
T. Ezoe and A. Hosaka, Phys. Rev. D **96**, 054002 (2017).

Result: E.o.M and potential

1. Equation of motion(E.o.M)

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2 h(r)\frac{dk_l^{\alpha}(r)}{dr}\right) - E^2 f(r) k_l^{\alpha}(r) + \left(m_K^2 + V(r)\right) k_l^{\alpha}(r) = 0$$: Klein-Gordon like

Result: E.o.M and potential

1. Equation of motion(E.o.M)

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2 h(r)\frac{dk_l^\alpha(r)}{dr}\right) - E^2 f(r)k_l^\alpha(r) + \left(m_K^2 + V(r)\right)k_l^\alpha(r) = 0$$: Klein-Gordon like

$$\longrightarrow -\frac{1}{m_K+E}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dk_l^\alpha(r)}{dr}\right) + U(r)k_l^\alpha(r) = \varepsilon k_l^\alpha(r)$$: Schrödinger like

$(E = m_K + \varepsilon)$

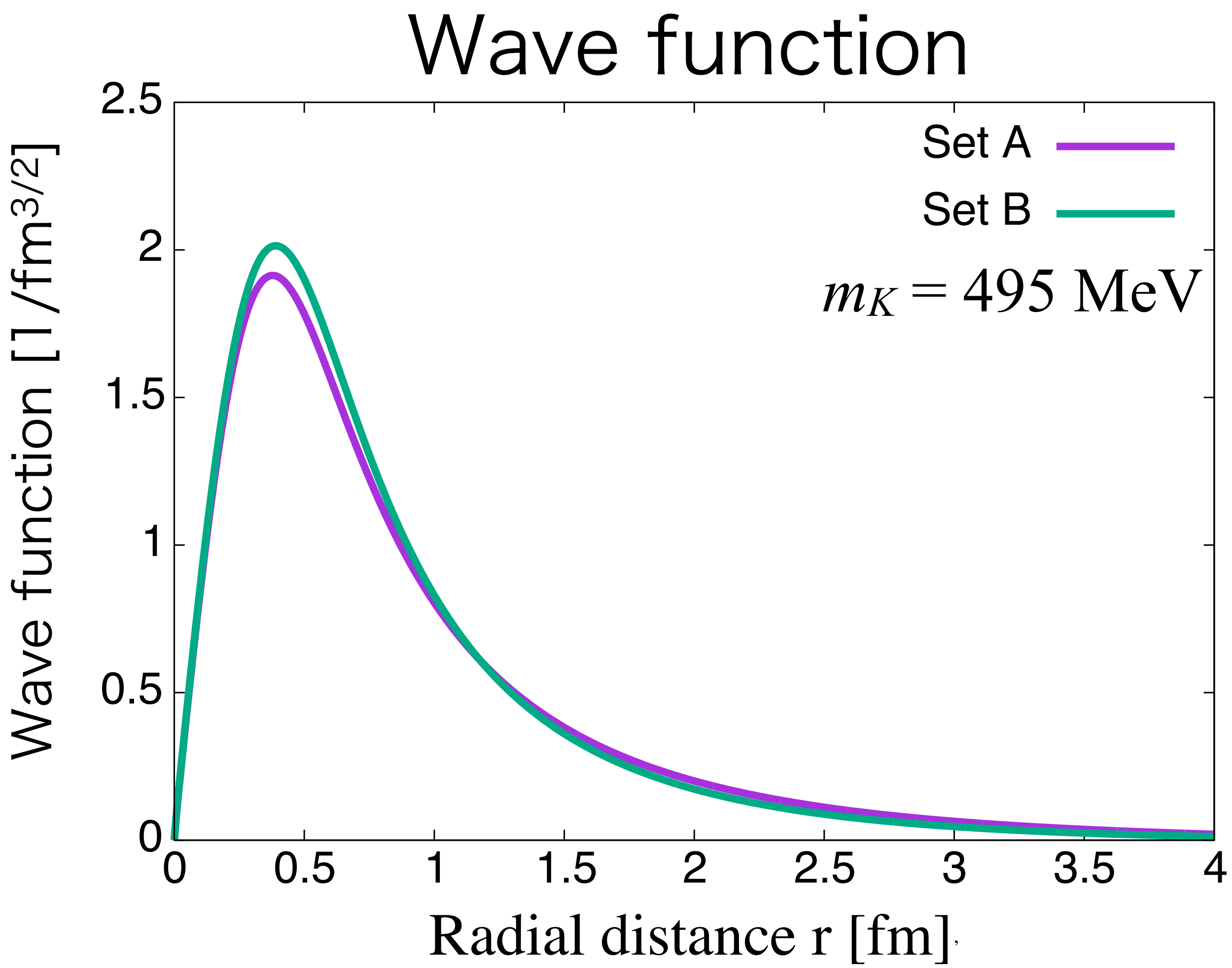
$$\begin{aligned} U(r) &= -\frac{1}{m_K+E}\left[\frac{h(r)-1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right) + \frac{dh(r)}{dr}\frac{d}{dr}\right] - \frac{(f(r)-1)E^2}{m_K+E} + \frac{V(r)}{m_K+E} \\ &= U_0^c(r) + U_\tau^c(r)\boldsymbol{\tau}^K\cdot\boldsymbol{\tau}^N + \left(U_0^{LS}(r) + U_\tau^{LS}(r)\boldsymbol{\tau}^K\cdot\boldsymbol{\tau}^N\right)\boldsymbol{L}\cdot\boldsymbol{S} \end{aligned}$$

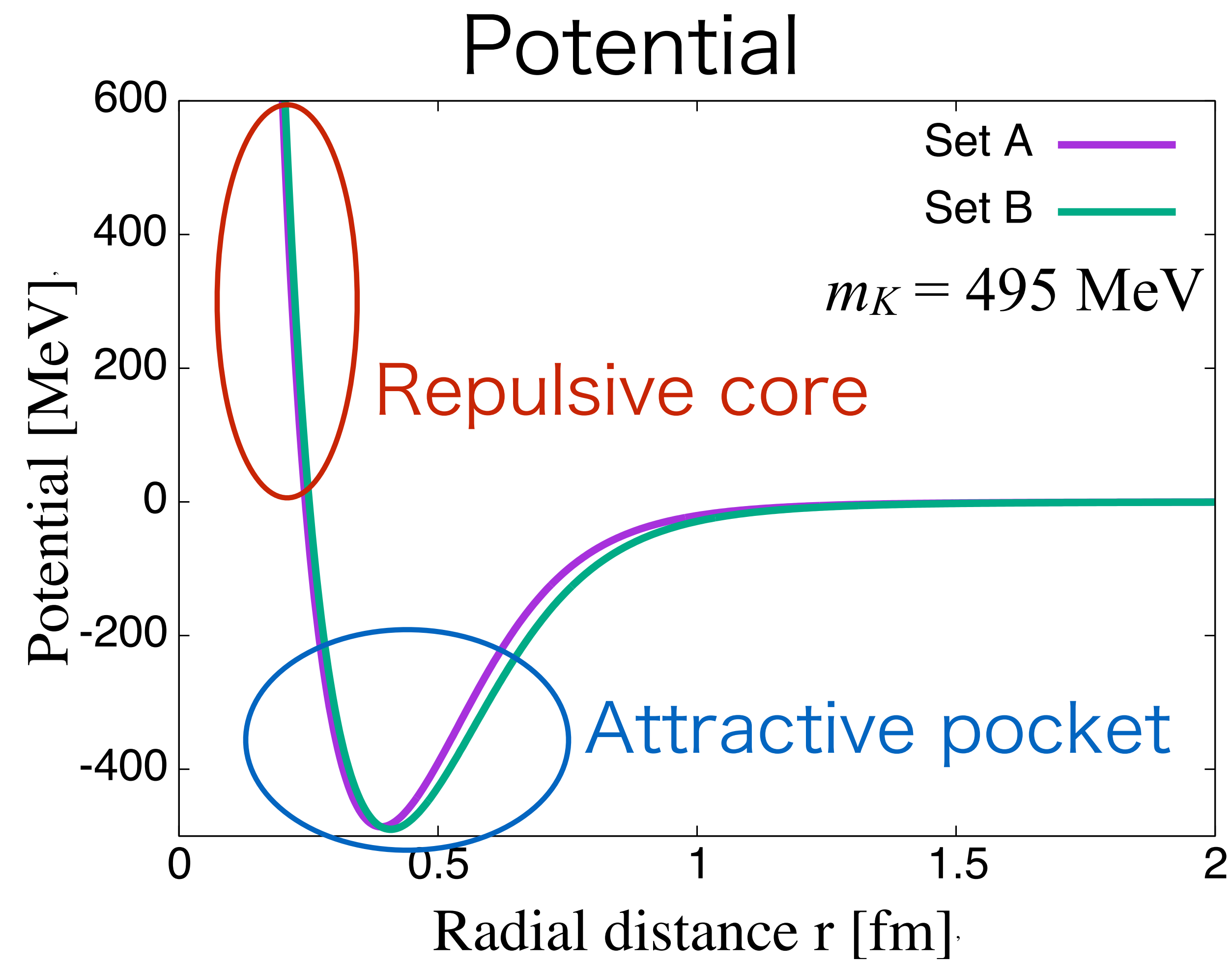
2. Properties of resulting potential U

1. Nonlocal and depend on the kaon energy
2. Contain isospin dependent and independent **central forces** and the similar **spin-orbit(LS) forces**
3. A repulsive component is proportional to $1/r^2$ at short distances

3. Equivalent local potential: $\tilde{U}(r) = \dfrac{U(r)k_l^\alpha(r)}{k_l^\alpha(r)}$

$\bar{K}N$ ($J^P = 1/2^-$, $I = 0$) Bound state





• Parameter sets and bound state properties

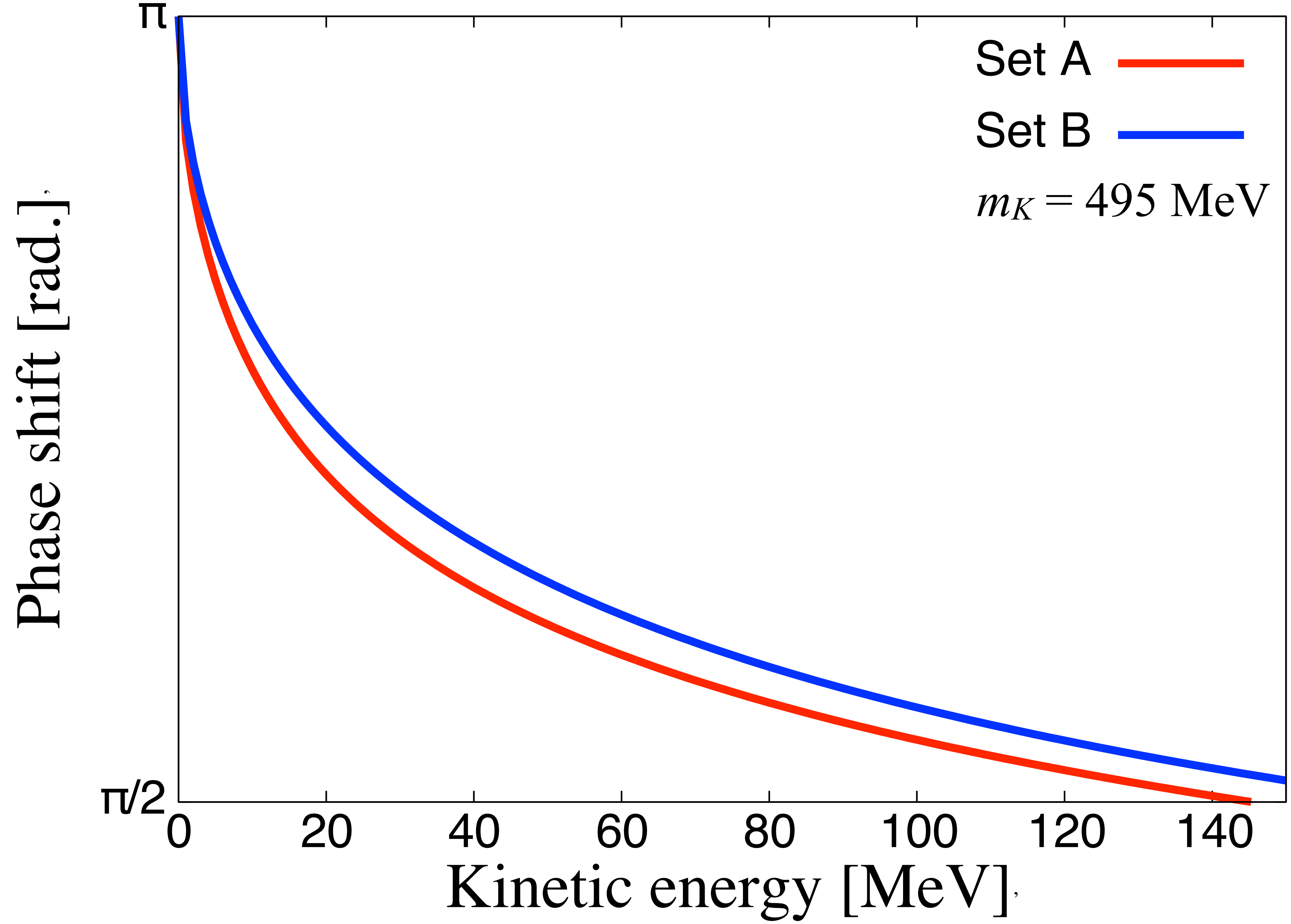
	F_π [MeV]	e	B.E. [MeV]	$\sqrt{\langle r_N^2 \rangle}$ [fm]	$\sqrt{\langle r_K^2 \rangle}$ [fm]
Parameter set A	205	4.67	19.9	0.43	1.30
Parameter set B	186	4.82	32.2	0.46	1.15

$$\langle r_N^2 \rangle = \int_0^\infty dr\ r^2 \rho_B(r), \quad \rho_B(r) = -\frac{2}{\pi} \sin^2 F F'$$

G. S. Adkins, C. R. Nappi and E. Witten, Nucl. Phys. B **228** (1983)

$$\langle r_K^2 \rangle = \int dV\ r^2 \left[Y_{00}(\hat{r})\, k_0^0(r) \right]^2 = \int_0^\infty dr\ r^4 k^2(r) \qquad Y_{00} = \frac{1}{\sqrt{4\pi}}$$

$\bar{K}N$ ($J^P = 1/2^-$, $I = 0$) scattering tate

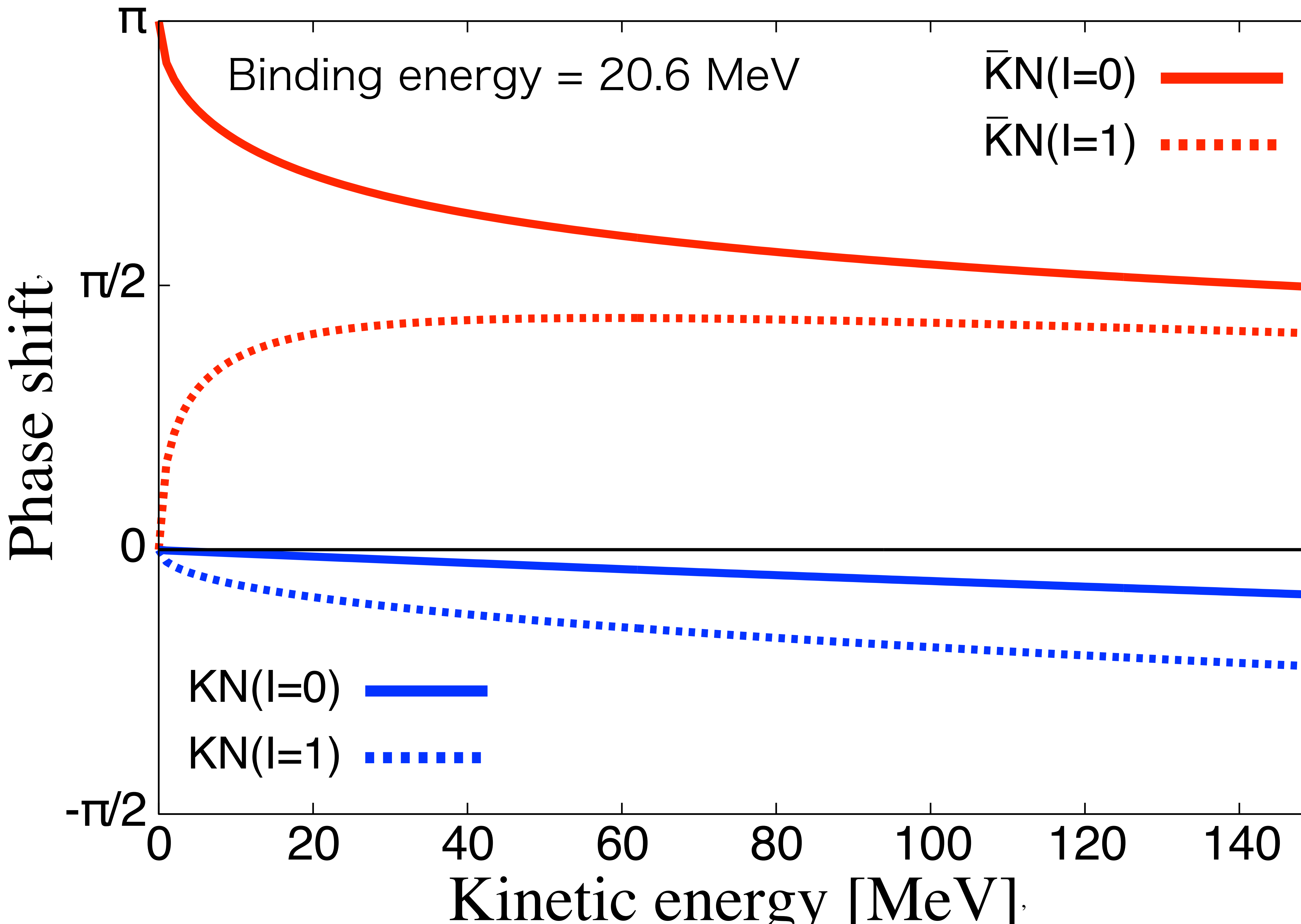


cf) Bound state properties

	F_π [MeV]	e	B.E. [MeV]	$\sqrt{\langle r_N^2 \rangle}$ [fm]	$\sqrt{\langle r_K^2 \rangle}$ [fm]
Parameter set A	205	4.67	19.9	0.43	1.30
Parameter set B	186	4.82	32.2	0.46	1.15

KN and $\bar{K}$N scattering states ($J^P = 1/2^-$)

Parameter set: $m_K = 495$ MeV, $F_\pi = 205$ MeV, $e = 4.67$



5. $\Lambda(1405)$: $\bar{K}N$-$\pi\Sigma$ coupling

$\bar{K}N$-$\pi\Sigma$ coupling

• Ansatzes

$$\begin{cases} U_{CK} = A(t)\sqrt{U_\pi}U_K\sqrt{U_\pi}A^\dagger(t) & \text{[1]} \\ U_{EH} = A(t)\sqrt{U_\pi}A^\dagger(t)U_K A(t)\sqrt{U_\pi}A^\dagger(t) & \text{[2]} \end{cases}$$

$A(t)$: Isospin rotation matrix

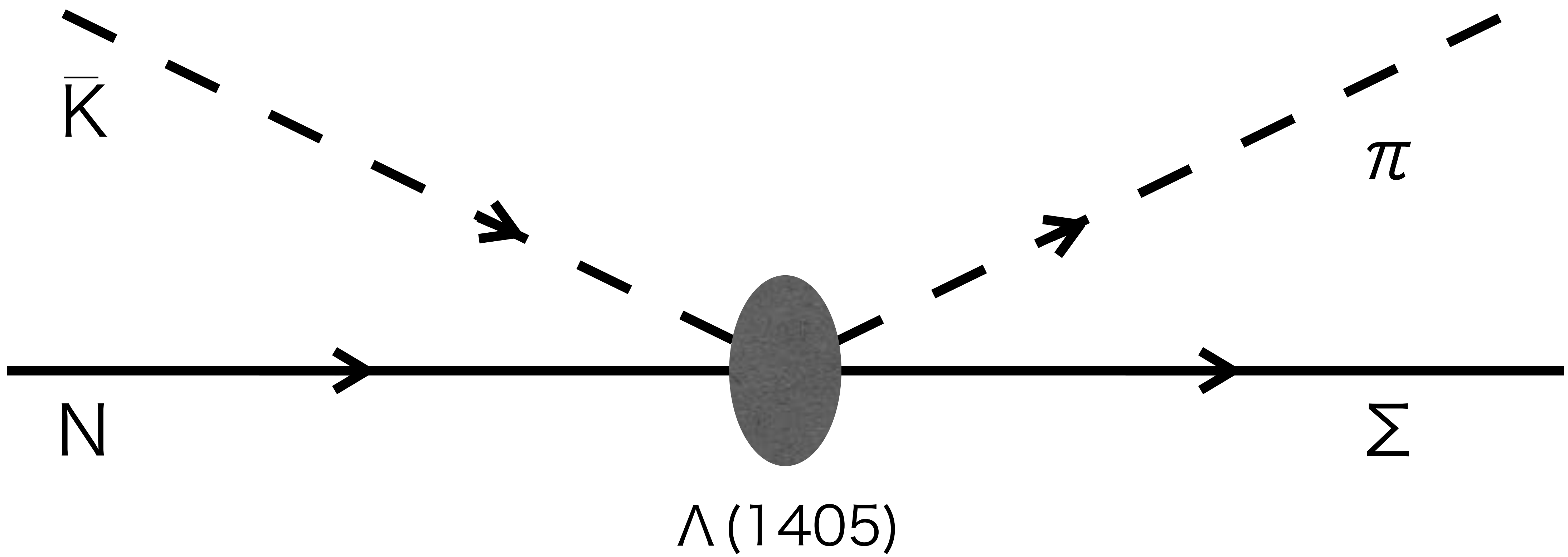
• Under the isospin rotation

$$\begin{cases} K \to K_{CK} = A(t)K \\ K \to K_{EH} = K \end{cases}$$

Physical kaon

Unphysical kaon (s-quark)

• $\bar{K}N$-$\pi\Sigma$ coupling



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$\bar{K}N$-$\pi\Sigma$ coupling

• Ansatzes

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$A(t)$: Isospin rotation matrix

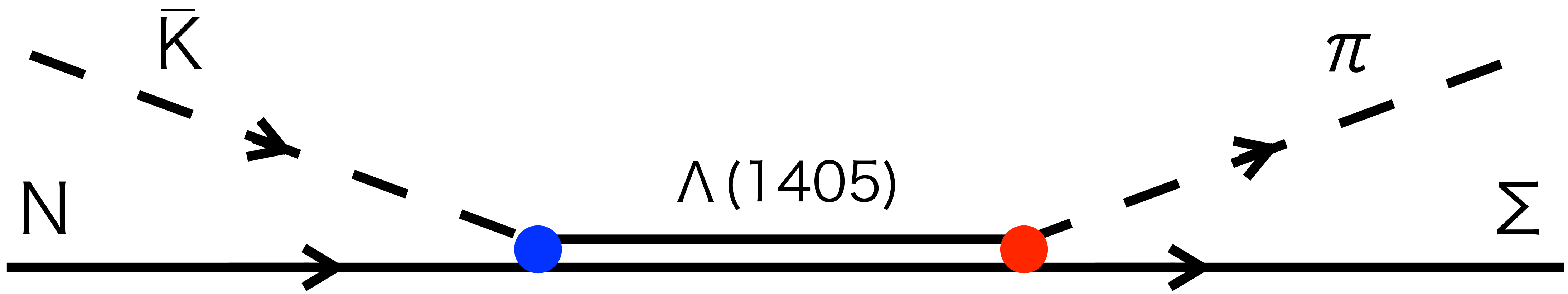
• Under the isospin rotation

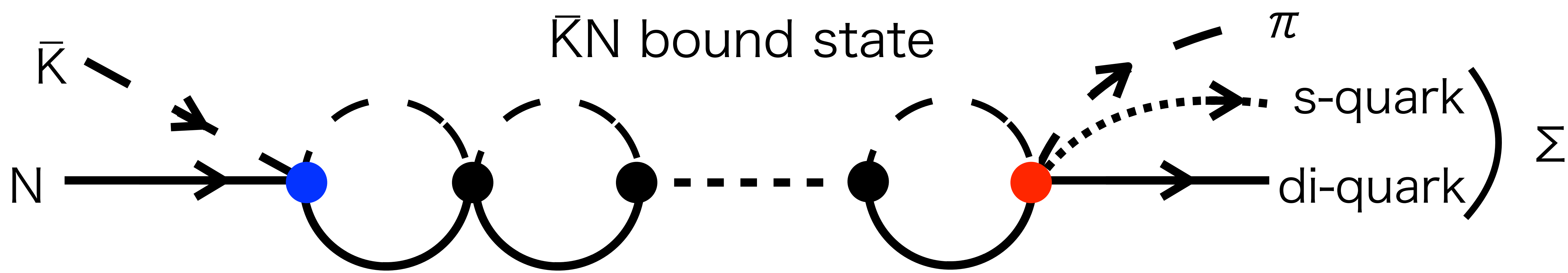
$$\begin{cases} K \to K_{CK} = A(t)K \\ K \to K_{EH} = K \end{cases}$$

Physical kaon ($K_{EH} = K$)

Unphysical kaon (s-quark) ($A(t)K$)

• $\bar{K}N$-$\pi\Sigma$ coupling





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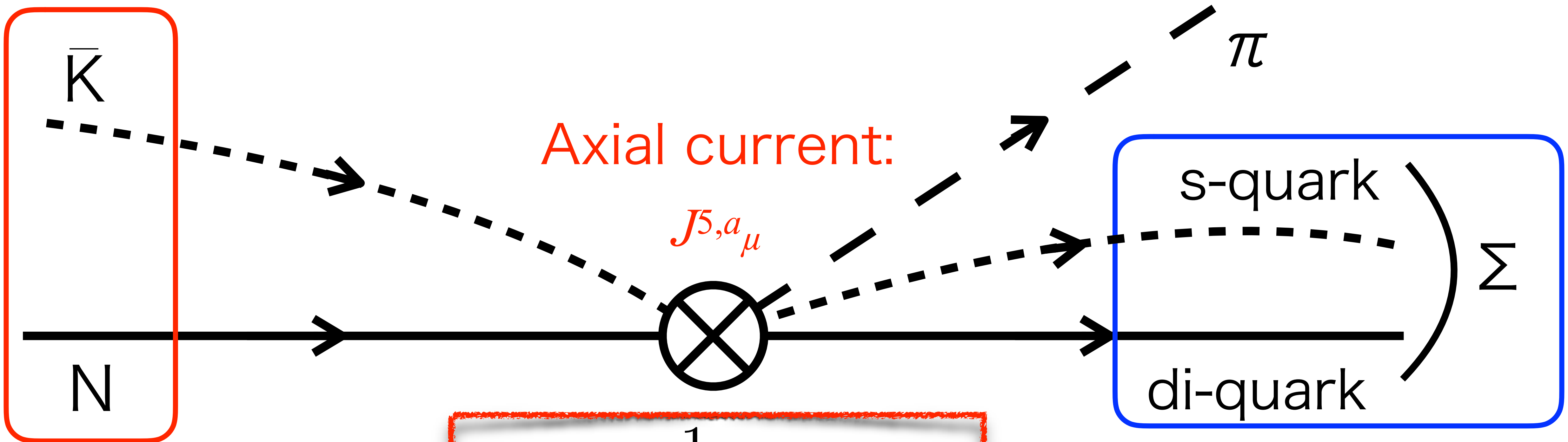


$$\mathcal{L}_{int} = \frac{1}{F_\pi} \partial^\mu \pi^a J^{5,a}_\mu$$

- K̄-annihilation
- Weak coupling limit
  (the EH approach)

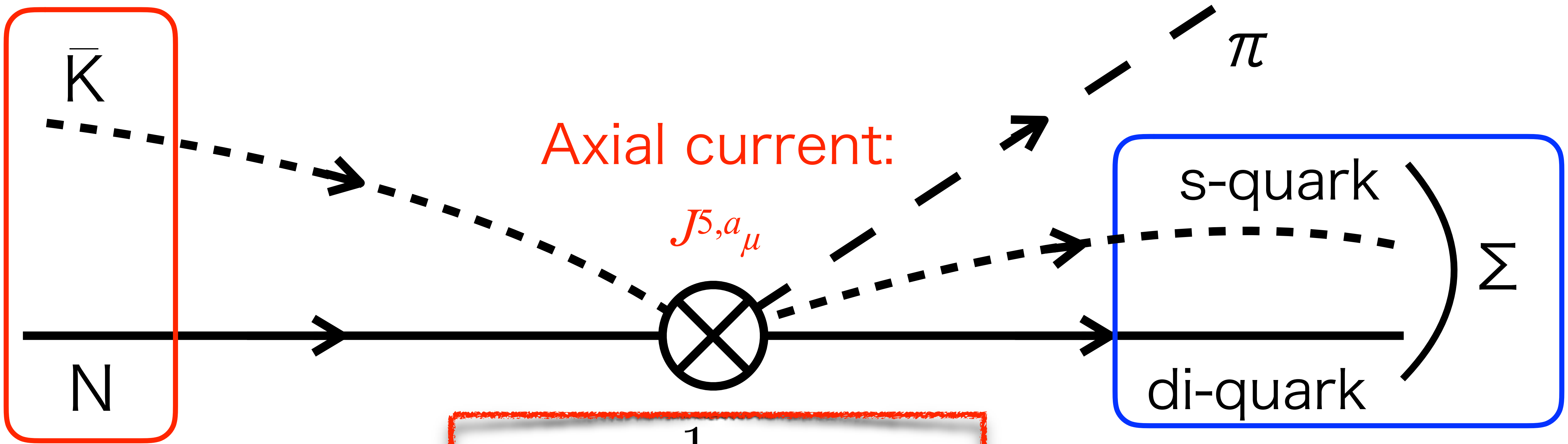
- s-creation
- Strong coupling limit
  (the CK approach)

・Lagrangian

$$\mathcal{L} \propto \partial^\mu \pi^a J^{5,a}_\mu \longrightarrow \langle \pi\Sigma | \mathcal{L} | \bar{K}N \rangle \propto \langle \pi | \partial^\mu \pi^a | 0 \rangle \langle \Sigma | J^{5,a}_\mu | \bar{K}N \rangle$$

$(a = -1, 0, 1)$

$\bar{K}N$-$\pi\Sigma$ coupling



$$\mathcal{L}_{int} = \frac{1}{F_\pi}\partial^\mu \pi^a J^{5,a}_\mu$$

- $\bar{K}$-annihilation
- Weak coupling limit (the EH approach)

- s-creation
- Strong coupling limit (the CK approach)

- **<u>Lagrangian</u>**

$$\mathcal{L} \propto \partial^\mu \pi^a J^{5,a}_\mu \longrightarrow \langle \pi\Sigma | \mathcal{L} | \bar{K}N \rangle \propto \langle \pi | \partial^\mu \pi^a | 0 \rangle \langle \Sigma | J^{5,a}_\mu | \bar{K}N \rangle$$

$(a = -1, 0, 1)$

Axial current and matrix element

1. Axial current

$$L = \frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2 + L_{SB} + L_{WZ}$$

Axial transformation:

$$U \to \exp\left(i\boldsymbol{\theta}\cdot\frac{\boldsymbol{T}}{2}\right)U\exp\left(i\boldsymbol{\theta}\cdot\frac{\boldsymbol{T}}{2}\right)$$

$$\begin{aligned} J_\mu^{5,a} &= J_\mu^{5,a,(kin)} + J_\mu^{5,a,(Skyrme)} + J_\mu^{5,a,(WZ)} \\ &\equiv i\mathrm{tr}\left[T^a\left(R_\mu + L_\mu\right)\right] + \frac{i}{16e^2}\mathrm{tr}\left[T^a\left\{\left[R^\nu,\left[R_\nu, R^\mu\right]\right] - \left[L^\nu,\left[L_\nu, L^\mu\right]\right]\right\}\right] \\ &\quad + \frac{N_c}{96\pi^2}\epsilon_{\mu\nu\alpha\beta}\mathrm{tr}\left[T^a\left(R^\nu R^\alpha R^\beta + L^\nu L^\alpha L^\beta\right)\right] \end{aligned}$$

$$R_\mu = U\partial_\mu U^\dagger, \quad L_\mu = U^\dagger\partial_\mu U$$

→ $J_\mu^{5,a} = J_\mu^{(0),5,a} + J_\mu^{(2),5,a}$

→ $\langle\Sigma^0|\, J_{\mu=0}^{(2),5,a=3}\, |\bar{K}N\rangle$

for $\Lambda(1405) \to \pi^0\Sigma^0$ in the non-relativistic limit

The axial current has $O(N_c^0)$ and $O(1/N_c)$ components

Axial current and matrix element

2. Leading components for the axial current

$$J_{\mu=0}^{(2),5,a=3} = i\frac{F_\pi^2}{16}\left(\frac{2\sqrt{2}}{F_K}\right)^2 \mathrm{tr}\left[\tau^3 X_0^{(kin)}\right] - \frac{i}{16e^2}\left(\frac{2\sqrt{2}}{F_K}\right)^2 \mathrm{tr}\left[\tau^3 X_0^{(Skyrme)}\right] - \frac{N_c}{96\pi^2}\left(\frac{2\sqrt{2}}{F_K}\right)^2 \epsilon^{ijk}\mathrm{tr}\left[\tau^3 X_{0ijk}^{(WZ)}\right]$$

$$X_0^{(kin)} = \frac{1}{2}\left\{\tilde{\xi}K\dot{K}^\dagger\tilde{\xi}^\dagger - \tilde{\xi}\dot{K}K^\dagger\tilde{\xi}\tilde{\xi}^\dagger\right\} - \left\{\tilde{\xi}\leftrightarrow\tilde{\xi}^\dagger\right\}$$

$$\begin{aligned} X_0^{(Skyrme)} = &\left(\frac{1}{2}\left[\tilde{R}^i,\left[\tilde{R}_i,\tilde{\xi}\dot{K}K^\dagger\tilde{\xi}^\dagger - \tilde{\xi}K\dot{K}^\dagger\tilde{\xi}^\dagger\right]\right]\right. \\ &-\tilde{R}^i\tilde{U}\partial_i\left(\tilde{\xi}^\dagger K\right)\dot{K}^\dagger\tilde{\xi}^\dagger + 2\tilde{U}\partial^i\left(\tilde{\xi}^\dagger K\right)\dot{K}^\dagger\tilde{\xi}^\dagger\tilde{R}_i \\ &\left.+\tilde{\xi}\dot{K}\left\{K^\dagger\tilde{\xi}\partial^i\tilde{U} - \partial^i\left(K^\dagger\tilde{\xi}^\dagger\right)\right\}\tilde{R}_i - 2\tilde{R}^i\tilde{\xi}\dot{K}\left\{K^\dagger\tilde{\xi}\partial_i\tilde{U}^\dagger - \partial_i\left(K^\dagger\tilde{\xi}^\dagger\right)\right\}\right) \\ &-\left(\tilde{\xi}\leftrightarrow\tilde{\xi}^\dagger\right) \end{aligned}$$

$$\begin{aligned} X_{0ijk}^{(WZ)} = &\left(\frac{1}{2}\left[\tilde{R}_i\tilde{R}_j\left\{\tilde{U}\partial_k\left(\tilde{\xi}^\dagger K\right)K^\dagger\tilde{\xi}^\dagger - \tilde{\xi}K\partial_k\left(K^\dagger\tilde{\xi}\right)\tilde{U}^\dagger\right\}\right.\right. \\ &+\tilde{R}_i\left\{\tilde{U}\partial_j\left(\tilde{\xi}^\dagger K\right)K^\dagger\tilde{\xi}^\dagger - \tilde{\xi}K\partial_j\left(K^\dagger\tilde{\xi}\right)\tilde{U}^\dagger\right\}\tilde{R}_k + \left.\left\{\tilde{U}\partial_i\left(\tilde{\xi}^\dagger K\right)K^\dagger\tilde{\xi}^\dagger - \tilde{\xi}K\partial_i\left(K^\dagger\tilde{\xi}\right)\tilde{U}^\dagger\right\}\tilde{R}_j\tilde{R}_k\right] \\ &-\left[\tilde{U}\partial_i\left(\tilde{\xi}^\dagger K\right)\left\{K^\dagger\tilde{\xi}\partial_j\tilde{U}^\dagger - \partial_j\left(K^\dagger\tilde{\xi}^\dagger\right)\right\}\tilde{R}_k + \tilde{R}_i\tilde{U}\partial_j\left(\tilde{\xi}^\dagger K\right)\left.\left\{K^\dagger\tilde{\xi}\partial_k\tilde{U}^\dagger - \partial_k\left(K^\dagger\tilde{\xi}^\dagger\right)\right\}\right]\right) \\ &+\left(\xi\leftrightarrow\xi^\dagger\right) \end{aligned}$$

$\tilde{U} = A(t)\,U_H A^\dagger(t)$, $\tilde{\xi} = A(t)\sqrt{U_H}A^\dagger(t)$, $\tilde{R}_i = \tilde{U}\partial_i\tilde{U}^\dagger$: 2×2 matrix

Axial current and matrix element

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$$-\frac{N_c}{96\pi^2}\left(\frac{2\sqrt{2}}{F_K}\right)^2 \epsilon^{ijk}\mathrm{tr}\left[\tau^3 X_{0ijk}^{(WZ)}\right]$$

$$X_0^{(kin)} = \frac{1}{2}\left\{\tilde{\xi}K\dot{K}^\dagger\tilde{\xi}^\dagger - \tilde{\xi}\dot{K}K^\dagger\tilde{\xi}\tilde{\xi}^\dagger\right\} - \left\{\tilde{\xi}\leftrightarrow\tilde{\xi}^\dagger\right\}$$

$$X_0^{(Skyrme)} = \Big(\frac{1}{2}\left[\tilde{R}^i,\left[\tilde{R}_i,\tilde{\xi}\dot{K}K^\dagger\tilde{\xi}^\dagger - \tilde{\xi}K\dot{K}^\dagger\tilde{\xi}^\dagger\right]\right]$$
$$-\tilde{R}^i\tilde{U}\partial_i\left(\tilde{\xi}^\dagger K\right)\dot{K}^\dagger\tilde{\xi}^\dagger + 2\tilde{U}\partial^i\left(\tilde{\xi}^\dagger K\right)\dot{K}^\dagger\tilde{\xi}^\dagger\tilde{R}_i$$
$$+\tilde{\xi}\dot{K}\left\{K^\dagger\tilde{\xi}\partial^i\tilde{U} - \partial^i\left(K^\dagger\tilde{\xi}^\dagger\right)\right\}\tilde{R}_i - 2\tilde{R}^i\tilde{\xi}\dot{K}\left\{K^\dagger\tilde{\xi}\partial_i\tilde{U}^\dagger - \partial_i\left(K^\dagger\tilde{\xi}^\dagger\right)\right\}\Big)$$
$$-\left(\tilde{\xi}\leftrightarrow\tilde{\xi}^\dagger\right)$$

$$X_{0ijk}^{(WZ)} = \Big(\frac{1}{2}\Big[\tilde{R}_i\tilde{R}_j\left\{\tilde{U}\partial_k\left(\tilde{\xi}^\dagger K\right)K^\dagger\tilde{\xi}^\dagger - \tilde{\xi}K\partial_k\left(K^\dagger\tilde{\xi}\right)\tilde{U}^\dagger\right\}$$
$$+\tilde{R}_i\left\{\tilde{U}\partial_j\left(\tilde{\xi}^\dagger K\right)K^\dagger\tilde{\xi}^\dagger - \tilde{\xi}K\partial_j\left(K^\dagger\tilde{\xi}\right)\tilde{U}^\dagger\right\}\tilde{R}_k + \left\{\tilde{U}\partial_i\left(\tilde{\xi}^\dagger K\right)K^\dagger\tilde{\xi}^\dagger - \tilde{\xi}K\partial_i\left(K^\dagger\tilde{\xi}\right)\tilde{U}^\dagger\right\}\tilde{R}_j\tilde{R}_k\Big]$$
$$-\Big[\tilde{U}\partial_i\left(\tilde{\xi}^\dagger K\right)\left\{K^\dagger\tilde{\xi}\partial_j\tilde{U}^\dagger - \partial_j\left(K^\dagger\tilde{\xi}^\dagger\right)\right\}\tilde{R}_k + \tilde{R}_i\tilde{U}\partial_j\left(\tilde{\xi}^\dagger K\right)\left\{K^\dagger\tilde{\xi}\partial_k\tilde{U}^\dagger - \partial_k\left(K^\dagger\tilde{\xi}^\dagger\right)\right\}\Big]\Big)$$
$$+\left(\xi\leftrightarrow\xi^\dagger\right)$$

$\tilde{U} = A(t)\,U_H A^\dagger(t)$, $\tilde{\xi} = A(t)\sqrt{U_H}A^\dagger(t)$, $\tilde{R}_i = \tilde{U}\partial_i\tilde{U}^\dagger$: 2×2 matrix

Axial current and matrix element

3. Leading component of the matrix element

$$\langle \Sigma^0 | \, J^{(2),5,a=3}_{\mu=0} \, | \bar{K} N \rangle \sim \langle \Sigma^0 | \, \mathrm{tr} \left[\tau^3 M_1 K K^\dagger M_2 \right] | \bar{K} N \rangle$$
(M_1, M_2: 2×2 matrices)

$$= \langle sd | \, \mathrm{tr} \left[\tau^3 M_1 K K^\dagger M_2 \right] | \bar{K} N \rangle$$
(d: di-quark with $I = J = 1$)

	Kaon
K	Annihilation
$K^\dagger$	Creation

Axial current and matrix element

3. Leading component of the matrix element

$$\langle \Sigma^0 | \, J^{(2),5,a=3}_{\mu=0} \, | \bar{K} N \rangle \sim \langle \Sigma^0 | \, \mathrm{tr} \left[\tau^3 M_1 K K^\dagger M_2 \right] | \bar{K} N \rangle$$ (M_1, M_2: 2×2 matrices)

$$= \langle sd | \, \mathrm{tr} \left[\tau^3 M_1 K K^\dagger M_2 \right] | \bar{K} N \rangle$$ (d: di-quark with $I = J = 1$)

	Kaon	Anti-Kaon	s-quark	$\bar{s}$-quark
K	Annihilation	Creation	Creation	Annihilation
$K^\dagger$	Creation	Annihilation	Annihilation	Creation

Axial current and matrix element

3. Leading component of the matrix element

$$\langle \Sigma^0 | J^{(2),5,a=3}_{\mu=0} | \bar{K} N \rangle \sim \langle \Sigma^0 | \mathrm{tr}\left[\tau^3 M_1 K K^\dagger M_2\right] | \bar{K} N \rangle$$ (M_1, M_2: 2×2 matrices)

$$= \langle sd | \mathrm{tr}\left[\tau^3 M_1 K K^\dagger M_2\right] | \bar{K} N \rangle$$ (d: di-quark with $I = J = 1$)

	Kaon	Anti-Kaon	s-quark	$\bar{s}$-quark
K	Annihilation	Creation	Creation	Annihilation
$K^\dagger$	Creation	Annihilation	Annihilation	Creation

K : s-quark creation operator in the CK approach: $K \to K_{CK} = \boldsymbol{A(t)}K$

$K^\dagger$: K̄-annihilation operator in the EH approach: $K^\dagger \to K_{EH} = K^\dagger$

Axial current and matrix element

3. Leading component of the matrix element

$$\langle\Sigma^0|\, J^{(2),5,a=3}_{\mu=0}\, |\bar{K}N\rangle \sim \langle\Sigma^0|\, \mathrm{tr}\left[\tau^3 M_1 K K^\dagger M_2\right] |\bar{K}N\rangle$$ (M_1, M_2: 2×2 matrices)

$$= \langle sd|\, \mathrm{tr}\left[\tau^3 M_1 K K^\dagger M_2\right] |\bar{K}N\rangle$$ (d: di-quark with $I=J=1$)

	Kaon	Anti-Kaon	s-quark	$\bar{s}$-quark
K	Annihilation	Creation	Creation	Annihilation
$K^\dagger$	Creation	Annihilation	Annihilation	Creation

K : s-quark creation operator in the CK approach: $K \to K_{CK} = \boldsymbol{A(t)}K$

$K^\dagger$: K̄-annihilation operator in the EH approach: $K^\dagger \to K_{EH} = K^\dagger$

4. Initial and final states

- Initial state

$$|\Lambda\,(1405)\rangle = |\bar{K}N\,(I=0)\rangle = \sqrt{\frac{1}{2}}\,|pK^-\rangle + \sqrt{\frac{1}{2}}\,|n\bar{K}^0\rangle$$

- Final state (d: di-quark with $I=J=1$)

$$|\Sigma^0\,(J_3=1/2)\rangle = |d\,(J=1)\,s\,(J=1/2)\rangle = \sqrt{\frac{2}{3}}\,|d\,(J_3=1)\,s_\downarrow\rangle + \sqrt{\frac{1}{3}}\,|d\,(J_3=0)\,s_\uparrow\rangle$$

Axial current and matrix element

5. Nucleon and di-quark states in the isospin space

・Isospin rotator A(t)

$$\begin{aligned} A &= a_0 + i\tau_i a_i \\ &= i\pi \begin{pmatrix} -|n\uparrow\rangle & -|n\downarrow\rangle \\ |p\uparrow\rangle & |p\downarrow\rangle \end{pmatrix} \end{aligned} \qquad i = 1,2,3, \quad \sum_{\mu=0}^{3} {a_\mu}^2 = 1$$

・Nucleon ($I = J = 1/2$)[3]

$$\begin{aligned} |p\uparrow\rangle &= \frac{1}{\pi}(a_1 + i a_2) \\ |p\downarrow\rangle &= -\frac{i}{\pi}(a_0 - i a_3) \\ |n\uparrow\rangle &= \frac{i}{\pi}(a_0 + i a_3) \\ |n\downarrow\rangle &= -\frac{1}{\pi}(a_1 - i a_2) \end{aligned}$$

・Di-quark ($I = J = 1$)

$$\begin{aligned} |d_{I_3=0,J_3=0}\rangle &= \sqrt{\frac{3}{2}}\frac{i}{\pi}\left({a_0}^2 - {a_1}^2 - {a_2}^2 + {a_3}^2\right) \\ |d_{I_3=0,J_3=1}\rangle &= \frac{\sqrt{3}}{\pi}(a_1 + i a_2)(a_0 + i a_3) \end{aligned}$$

$$\langle \Sigma^0 | J^{(2),5,a=3}_{\mu=0} | \bar{K} N \rangle \sim \langle sd | \operatorname{tr}\left[\tau^3 M_1 A K_{CK} K^\dagger_{EH} M_2\right] | \bar{K} N\rangle$$

$$\to \int d\mu(A)\, A^4$$

[3] G. S. Adkins, C. R. Nappi and E. Witten, Nucl. Phys. B **228**, 552 (1983).

6. Summary

Summary

Construct a new method to investigate
1. the kaon-nucleon systems
2. the $\bar{\mathrm{K}}$N-$\pi\Sigma$ coupling
in the Skyrme model

Results (kaon-nucleon system)

1. Properties of the obtained potential
   a. nonlocal and depends on the kaon energy
   b. contain **central and LS terms**
      with and without isospin dependence
   c. **repulsion proportional to $1/r^2$** for small r
2. $\bar{\mathrm{K}}$N$(I=0)$ bound states exist with B.E. of order ten MeV
3. Phases as functions of energy are qualitatively consistent with the experiments

Future work

・derive the coupling constant for the $\bar{\mathrm{K}}$N-$\pi\Sigma$ vertex

Thank you for you attention!!