AXIAL U(1) SYMMETRY IN LATTICE QCD AT HIGH TEMPERATURE

$$\langle \partial_\mu J_5^\mu \rangle = \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \langle F^{\mu\nu} F^{\rho\sigma} \rangle \to 0?$$

HIDENORI FUKAYA (OSAKA UNIV.)

FOR JLQCD COLLABORATION

PRD96, NO.3, 034509 (2017),

PRD93, NO.3, 034507 (2016), AND SOME UPDATES

DO YOU THINK AXIAL U(1) ANOMALY CAN DISAPPEAR (AT FINITE T) ?

Typical answer is

No, kidding!

And he/she tries to teach me

1. "Anomaly is EXPLICIT breaking of the theory,"
2. "Anomalous Ward-Takahashi identity HOLDS AT ANY ENERGY (or temperature),"
3. "You don't understand QFT…"

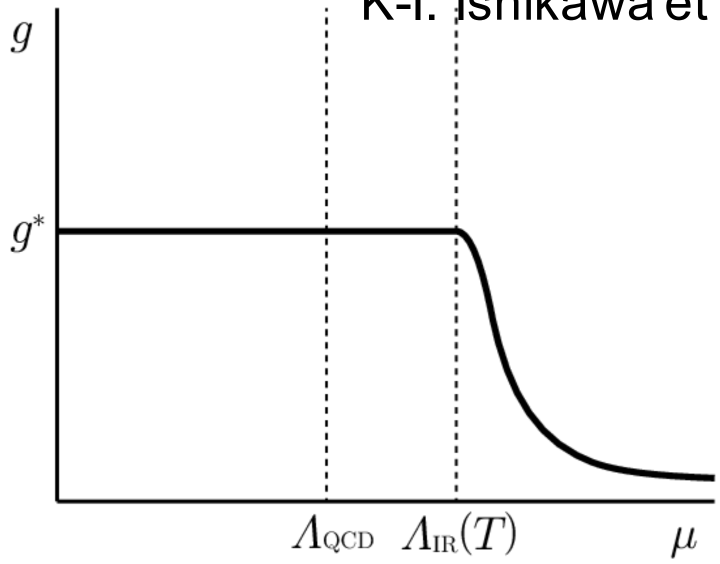
BUT THE SAME PERSON OFTEN TALKS ABOUT

Disappearance of conformal anomaly at IR fixed point:

By tuning Nf, beta function can disappear

(even if the theory itself is defined in non-conformal way) .

Figure from
K-I. Ishikawa et al. 2013



ANY DIFFERENCE?

For conformal anomaly, they examine

$$\beta(g) = \frac{g^3}{16\pi^2}\left(-\frac{11}{3}N_c + \frac{2}{3}N_f\right)$$

after gluon & quark integrals. O.K, it can be zero.
But for axial U(1) anomaly, we talk about

$$\langle \partial_\mu J_5^\mu(x) O(x') \rangle_{fermion} - \langle \delta_A O(x) \rangle_{fermion} \delta(x - x') = \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}(x) \langle O(x') \rangle_{fermion}$$

with fermion integral only (w/ classical gluon field),
which LOOKS always non-zero.

BUT THE REAL QUESTION IS

$$\langle\langle\partial_\mu J_5^\mu(x)O(x')\rangle_{fermion} - \langle\delta_A O(x)\rangle_{fermion}\delta(x-x')\rangle_{gluons}$$

$$= \left\langle \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}(x)\langle O(x')\rangle_{fermion}\right\rangle_{gluons} = 0???$$

MAIN MESSAGE OF THIS TALK

In high T QCD, whether

$$\left\langle \langle \partial_\mu J_5^\mu(x) O(x') \rangle_{fermion} - \langle \delta_A O(x) \rangle_{fermion} \delta(x - x') \right\rangle_{gluons}$$

$$= \left\langle \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}(x) \langle O(x') \rangle_{fermion} \right\rangle_{gluons} = 0???$$

or not is a non-trivial question, which can only be answered by carefully integrating over gluons (by lattice QCD).

In particular, good control of chiral symmetry (or continuum limit) is essential.

CAN U(1)ANOMALY DISAPPEAR AT FINITE T? → MANY ANSWERS.

Before 2012

- Cohen 1996, 1998 (theory)
- Bernard et al. 1996 (staggered)
- Chandrasekharan et al. 1998 (staggered)
- HotQCD 2011 (staggered)
- Ohno et al. 2011 (staggered)
- and many others

Red: YES
Blue: NO
Green: Not (directly) answered but related

After 2012

- HotQCD 2012 (Domain-wall)
- Aoki-F-Taniguchi 2012 (theory)
- Ishikawa et al2013, 2014,2017. (Wilson)
- JLQCD 2013, 2016 (overlap)
- TWQCD 2013 (optimal DW)
- LLNL/RBC 2013 (Domain-wall) [may be at higher T]
- Pelisseto and Vicari 2013(theory)
- BNakayama-Ohtsuki 2015, 2016(CFT)
- Sato-Yamada 2015(theory),
- Kanazawa & Yamamoto 2015, 2016 (theory)
- Dick et al. 2015 (OV in HISQ sea)
- Sharma et al. 2015, 2016 (OV in DW sea)
- Glozman 2015, 2016 (theory)
- Borasnyi et al. 2015 (staggered & OV)
- Brandt et al. 2016 (Wilson)
- Ejiri et al. 2016 (Wilson)
- Azcoiti 2016,2017(theory)
- Gomez-Nicola & Ruiz de Elvira 2017 (theory)
- Rorhofer et al. 2017 ……

CONTENTS

1. Is $U(1)_A$ anomaly theoretically possible to disappear?
2. Lattice QCD at high T with chiral fermions
3. Result 1: $U(1)_A$ anomaly
4. Result 2: topological susceptibility
5. Summary

SU(2) SSB AND U(1) ANOMALY LINKED BY DIRAC ZERO MODES

$U(1)_A$ breaking/restoration

Atiyah-Singer index theorem 1963

(near) zero mode spectrum of Dirac operator

Banks-Casher relation 1980

$SU(2)_L \times SU(2)_R$ breaking/restoration

* In the following, we consider Nf=2.

ATIYAH-SINGER INDEX THEOREM [1963]

(integral of) U(1) anomaly

⇆ Dirac zero-modes

$$n_+ - n_- = \frac{1}{32\pi^2}\int d^4x \mathrm{Tr} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$(n_\pm : \#$ of chirality $\pm$ zero modes$)$

(Note : RHS vanishes in T→∞ limit, since the topology is trivial in 3D theory.

BANKS-CASHER RELATION [1980]

SU(2) SSB ⇋ zero-modes of D

$$\pi\rho(\lambda=0)=\langle\bar{q}q\rangle\equiv\Sigma.$$

$$\left(\rho(\lambda)\equiv\lim_{V\to\infty}\sum_{\lambda_i\geq 0}\left\langle\frac{\delta(\lambda_i-\lambda)}{V}\right\rangle.\right)$$

λ : Dirac eigenvalue

For finite λ,

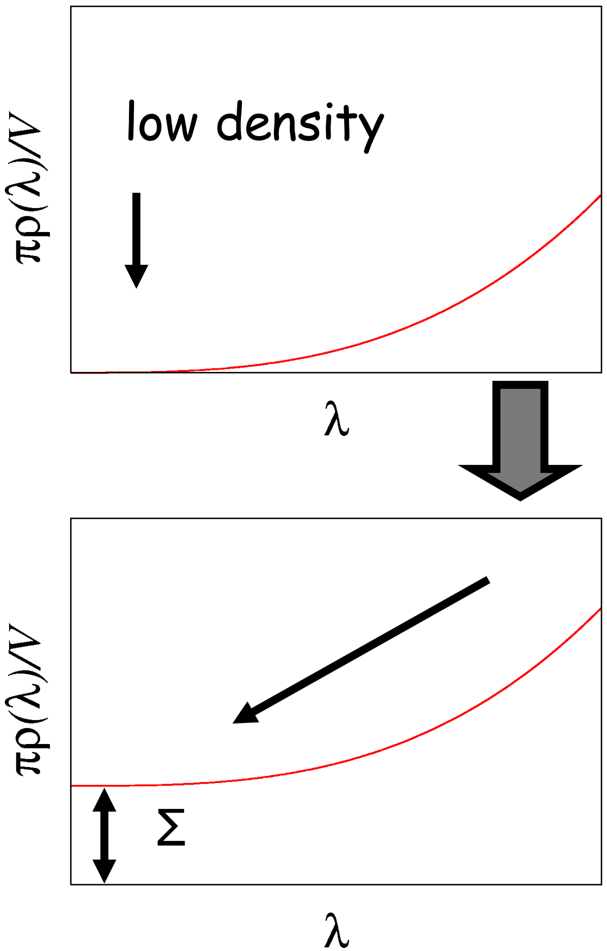
$$\rho(\lambda)=\frac{\mathrm{Re}\langle\bar{q}q\rangle(m_v=i\lambda)}{\pi}$$

BANKS-CASHER RELATION [DETAILS]

$$
\begin{aligned}
\rho(\lambda) &= \int_0^\infty d\lambda' \delta(\lambda-\lambda')\rho(\lambda') \\
&= \lim_{\epsilon\to 0}\frac{1}{\pi}\int_0^\infty d\lambda' \frac{2\epsilon}{(\lambda-\lambda')^2+\epsilon^2}\rho(\lambda') \\
&= \lim_{\epsilon\to 0}\frac{1}{\pi}\int_0^\infty d\lambda' \left[\frac{1}{i(\lambda-\lambda')+\epsilon}-\frac{1}{i(\lambda-\lambda')-\epsilon}\right]\rho(\lambda') \\
&= \lim_{\epsilon\to 0}\frac{1}{\pi}\int_0^\infty d\lambda' \left[\frac{1}{i(\lambda-\lambda')+\epsilon}-\frac{1}{i(\lambda-\lambda')-\epsilon}\right]\left\langle\sum_{\lambda_i}\frac{\delta(\lambda'-\lambda_i)}{V}\right\rangle \\
&= \lim_{\epsilon\to 0}\frac{1}{\pi V}\left\langle \mathrm{Tr}\frac{1}{D+i\lambda+\epsilon}-\mathrm{Tr}\frac{1}{D+i\lambda-\epsilon}\right\rangle \\
&= \frac{1}{\pi}\lim_{\epsilon\to 0}\left(\langle\bar{q}q\rangle_{m_v=i\lambda+\epsilon}-\langle\bar{q}q\rangle_{m_v=i\lambda-\epsilon}\right) \\
&= \frac{1}{\pi}\mathrm{Re}\langle\bar{q}q\rangle_{m_v=i\lambda},
\end{aligned}
$$

BANKS-CASHER RELATION [1980]

Why SU(2) chiral symmetry broken at T=0 ?



$$i\not{D} = \not{p} + g\not{A}$$

Free fermon When $g = 0, \quad \lambda = \pm p$

$$Area(S^3) = 2\pi^2 R^3$$

$$\begin{aligned} \rho(\lambda) &= \frac{2\pi^2\lambda^3}{V} \times \left(\frac{L}{2\pi}\right)^4 \times 3 \times 4 \times \frac{1}{2} \\ &= \frac{3}{4\pi^2}\lambda^3 \end{aligned}$$

Strong coupling

WHAT BANKS-CAHSER RELATION TELLS US

$$\langle \bar{q}q \rangle = \lim_{m \to 0} \int d\lambda \, \rho(\lambda) \frac{2m}{\lambda^2 + m^2} = \pi \rho(0)$$

[Banks-Casher 1980]

If $\rho(0) \neq 0,$ **$SU(2)_L x SU(2)_R$ broken,**
$U(1)_A$ broken.

If $\rho(0) = 0,$ **$SU(2)_L x SU(2)_R$ symmetric,**
$U(1)_A$ symmetric.

for quark bi-linears.

$U(1)_A$ AND $SU(2)_L \times SU(2)_R$ SHARE DIM<=3 ORDER PARAMETER(S).

Among quark bi-linears $\langle \bar{q}\Gamma q(x)\rangle$

only $\langle \bar{q}q(x)\rangle$ can have a VEV :

No dim.<=3 operator breaks $U(1)_A$ without breaking $SU(2)_L x SU(2)_R$.

How about higher dim. operators ?

-> our work [Aoki, F, Taniguchi 2012]

DIRAC SPECTRUM AND SYMMETRIES [Aoki-F-Taniguchi 2012]

$$\langle \bar{q}q \rangle = \lim_{m\to 0} \int d\lambda\, \rho(\lambda) \frac{2m}{\lambda^2 + m^2} = \pi\rho(0)$$

[Banks-Casher 1980]

Our idea = generalization of BC relation

to higher dim operators (dim=6 operators were done by T.Cohen 1996) :

$$\delta_{SU(2)} \left\langle \frac{1}{V^{N'}} \prod_i^N \left(\int dV \bar{q}\Gamma_i q \right) \right\rangle = 0$$

→ Constraints on

$$\lim_{m\to 0} \rho(\lambda)$$

$$\to \quad \delta_{U(1)_A} \left\langle \frac{1}{V^{N'}} \prod_i^N \left(\int dV \bar{q}\Gamma_i q \right) \right\rangle = 0 \; ???$$

[Aoki-F-Taniguchi 2012]

OUR RESULT 1 : MANY ORDER PARAMETERS ARE SHARED.

(under some "reasonable" assumptions)

Constraint we find

$$\lim_{m\to 0}\langle\rho(\lambda)\rangle = c|\lambda|^{\gamma}(1+O(\lambda)),\quad \gamma > 2$$

is strong enough to show

$$\delta_{U(1)_A}\left\langle \frac{1}{V^{N'}}\prod_i^N\left(\int dV\bar{q}\Gamma_i q\right)\right\rangle = 0 \text{ for } \Gamma_i{=}\tau^a \text{ and } \gamma_5\tau^a$$

for any N (up to 1/V corrections):

these order parameters are shared by $SU(2)_L xSU(2)_R$ and $U(1)_A$.

[Aoki-F-Taniguchi 2012]

OUR RESULT 2 : STRONG SUPPRESSION OF TOPOLOGICAL SUSCEPTIBILITY

We also find (in the thermodynamical limit)

$$\left(\frac{\partial}{\partial m}\right)^N \frac{\langle Q^2\rangle}{V} = 0 \quad \text{for any } N,$$

$$Q = \frac{1}{32\pi^2}\int d^4x\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}[F^{\mu\nu}F^{\rho\sigma}]$$

which implies

$$\frac{\langle Q^2\rangle}{V} = 0 \quad \text{for } m <^{\exists} m_{cr}$$

Suggests 1st order chiral transition ?

(There's no symmetry enhancement at finite quark mass.)

NOT A "SYMMETRY RESTORATION"

We allow

$$\langle \text{any } U(1)_A \text{ breaking} \rangle = \frac{1}{V^\alpha}, \quad \alpha > 0$$

Cf. conformal "symmetry" at the IR fixed point.

CONTENTS

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   Our answer = YES. $SU(2)_L \times SU(2)_R$ and $U(1)_A$ are connected through Dirac spectrum.

2. **Lattice QCD at high T with chiral fermions**

3. **Result 1: $U(1)_A$ anomaly**

4. **Result 2: topological susceptibility**

5. **Summary**

JLQCD COLLABORATION

Machines at KEK

HITACHI SR16000

shut down
last year…

IBM BG/Q

and U. of Tsukuba

Oakforest-PACS

Simulation codes :

Irolro++ (https://github.com/coppolachan/Irolro)

Grid (https://github.com/paboyle/Grid)

JLQCD FINITE T PROJECT

Members:

S. Aoki (YITP),
Y. Aoki (KEK,RBRC),
G. Cossu (Edinburgh),
HF(Osaka),
S. Hashimoto (KEK),
T. Kaneko(KEK),
K. Suzuki(KEK),
A.Tomiya(CCNU)

JLQCD FINITE T PROJECT (2014-)

[JLQCD (Cossu et al.) 2015, JLQCD(Tomiya et al.) 2016]

We simulate 2-flavor QCD.

1. **good chirality :**

   Mobius domain-wall & overlap fermion w/ OV/DW reweighting (frequent topology tunnelings)

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3. **different lattice spacings : 0.07-0.1 fm.**

Other comments

T= 190-330MeV (Tc~180MeV) with Lt=8,10,12.

3-10 different quark masses (w/ reweighting).

long MD time 20000-30000 for reweighting.

OVERLAP VS DOMAIN-WALL

Overlap Dirac operator has exact chiral symmetry

$$D_{\rm ov}(m) = \left[\frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \,{\rm sgn}(H_M)\right]$$

Measure for how much chiral sym. is violated

$$m_{\rm res} = 0.$$

(Monius) domain-wall operator is an approximation of overlap.

$$D^{4D}_{\rm DW}(m) = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \frac{1-(T(H_M))^{L_s}}{1+(T(H_M))^{L_s}}$$

$$m_{\rm res} \sim 1{\rm MeV}$$

$$H_M = \gamma_5 \frac{2D_W}{2+D_W}$$

We thought domain-wall fermion was good enough. But…

VIOLATION OF CHIRAL SYMMETRY ENHANCED AT FINITE TEMPERATURE

[JLQCD (Cossu et al.) 2015, JLQCD(Tomiya et al.) 2016]

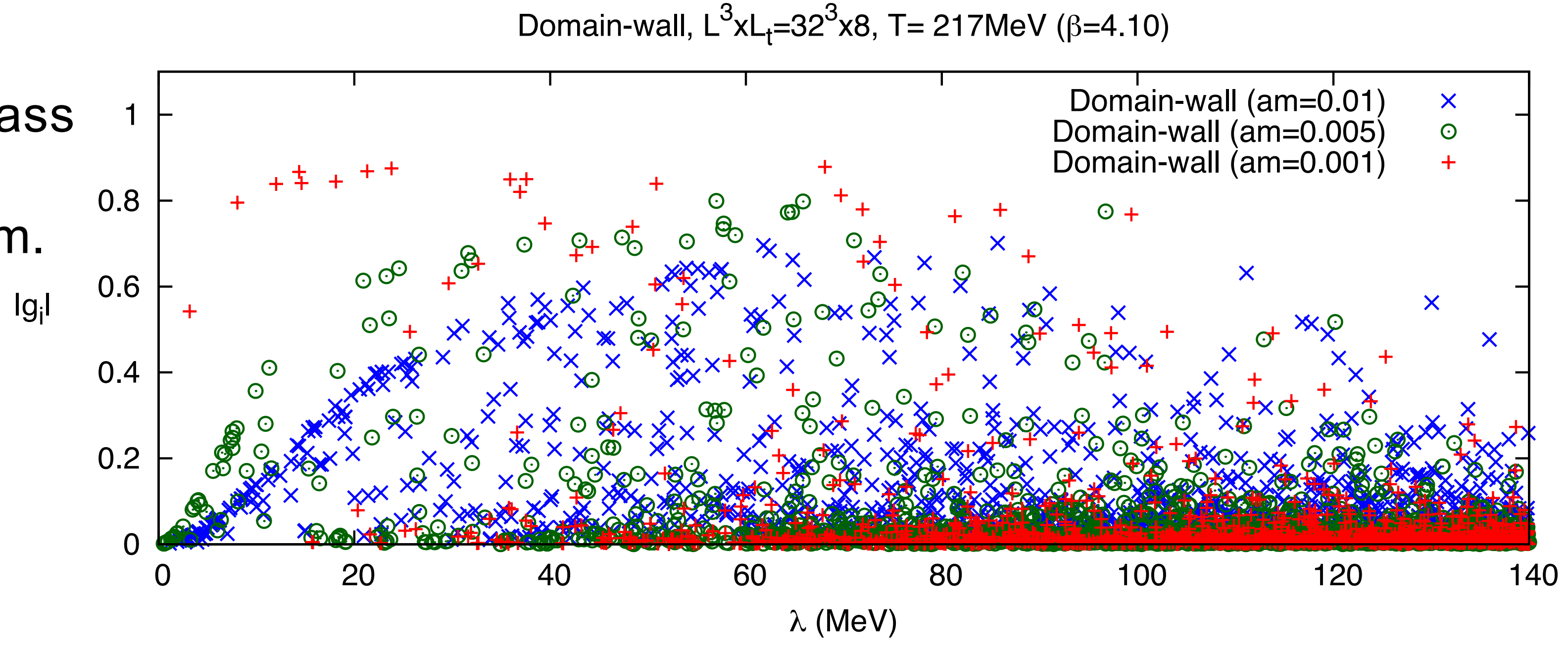
Chiral symmetry for each eigen-mode of Mobius domain-wall Dirac operator:

$$g_i = \left(v_i^\dagger, \frac{D\gamma_5 + \gamma_5 D - aRD\gamma_5 D}{\lambda_i} v_i \right)$$

→ **very bad modes appear above Tc (∼180MeV).**

Cf.) residual mass is (weighted) average of them.

For T=0, gi are consistent with residual mass.



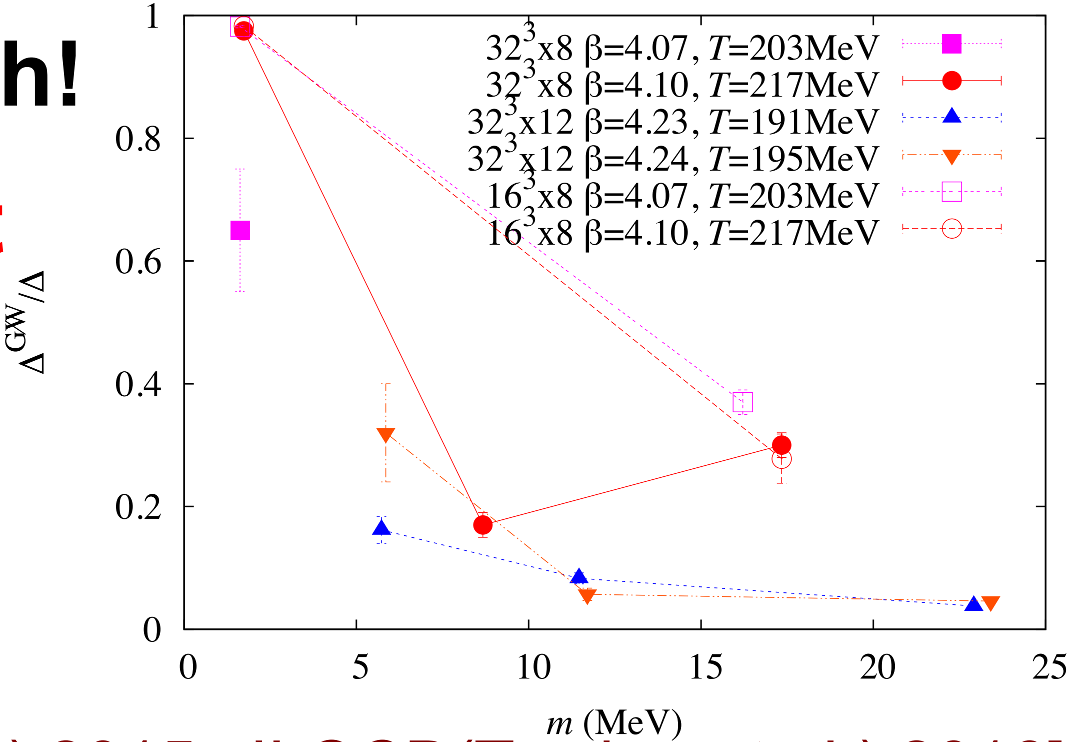
U(1)$_A$ ANOMALY IS SENSITIVE TO THE BAD MODES.

At a>0.08fm, Mobius domain-wall fermion is not good enough!

GW violation effect is 20%-100% .

(10 times of m_{res})

GW violation part in U(1)A susceptibility (definition will be given later.)



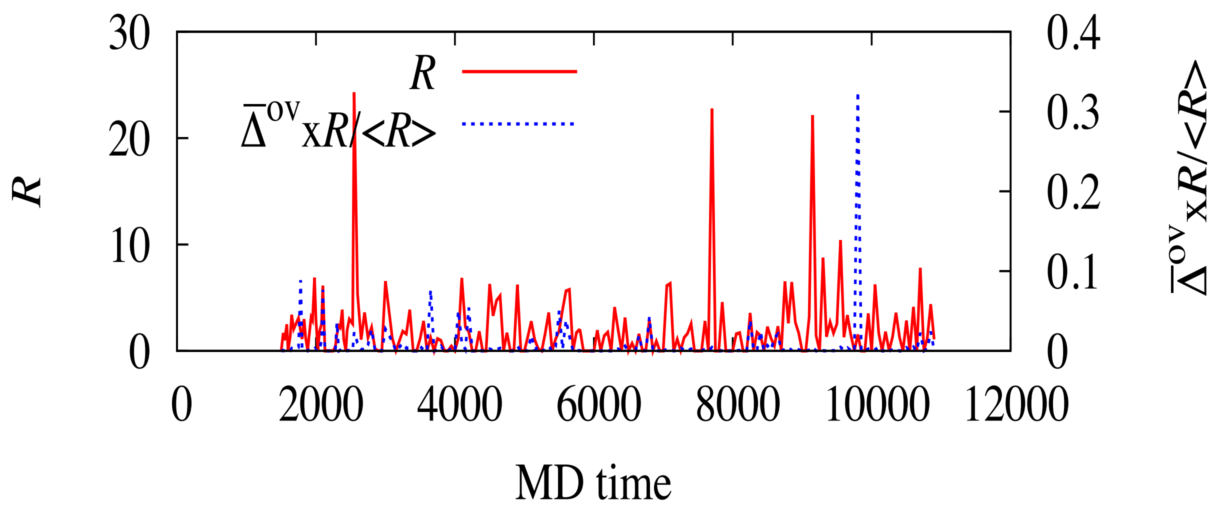
[JLQCD (Cossu et al.) 2015, JLQCD(Tomiya et al.) 2016]

OVERLAP/DOMAIN-WALL REWEIGHTING

(fermion action can be changed AFTER simulations)

$$R \equiv \frac{\det[D_{\rm ov}(m)]^2}{\det[D^{\rm 4D}_{\rm DW}(m)]^2}$$

a~0.08fm, T=1.4Tc m=6MeV



$$\langle O \rangle_{overlap} = \frac{\int dAO[\det D_{\rm ov}(m)]^2 e^{-S_G}}{\int dA[\det D_{\rm ov}(m)]^2 e^{-S_G}}$$

$$= \frac{\int dAO{\color{red}R}[\det D^{\rm 4D}_{\rm DW}(m)]^2 e^{-S_G}}{\int dA{\color{red}R}[\det D^{\rm 4D}_{\rm DW}(m)]^2 e^{-S_G}}$$

$$= \frac{\langle OR \rangle_{domain-wall}}{\langle R \rangle_{domain-wall}}$$

- R is stochastically estimated.

"EFFICIENCY" OF OV/DW REWEIGHTING

$$\frac{N_{eff}}{N} = \frac{\langle R \rangle}{N max(R)}$$

On our 2-4 fm lattices at T=1.1-1.8Tc (Tc~180MeV),

a ~ 0.1 fm : O.K. for L=2 fm, $N_{eff}/N \sim 1/20$

but does not work for 4 fm. $N_{eff}/N < 1/1000.$

($\rightarrow$ we approximate it by O(10) low-modes.)

a ~ 0.08 fm : works well (3 fm). $N_{eff}/N \sim 1/10$

a ~ 0.07 fm : domain-wall & overlap are consistent (2.4, 3.6 fm). $N_{eff}/N > 1/10$

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Our focus in this talk

VALENCE OVERLAP IN DOMAIN-WALL SEA IS MORE DANGEROUS

Dirac spectrum

DW on DW confs

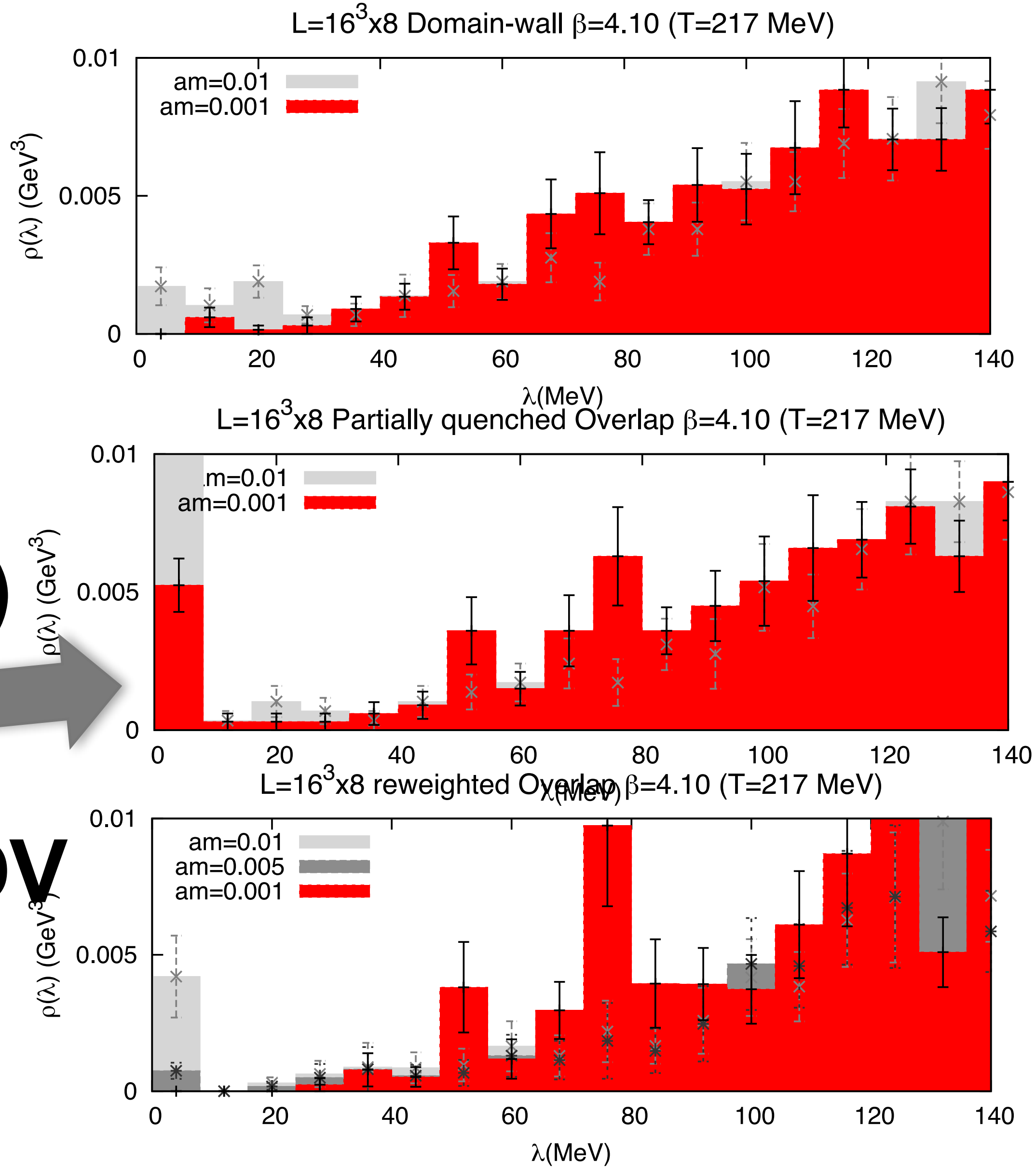
OV on DW confs

(partially quenched)

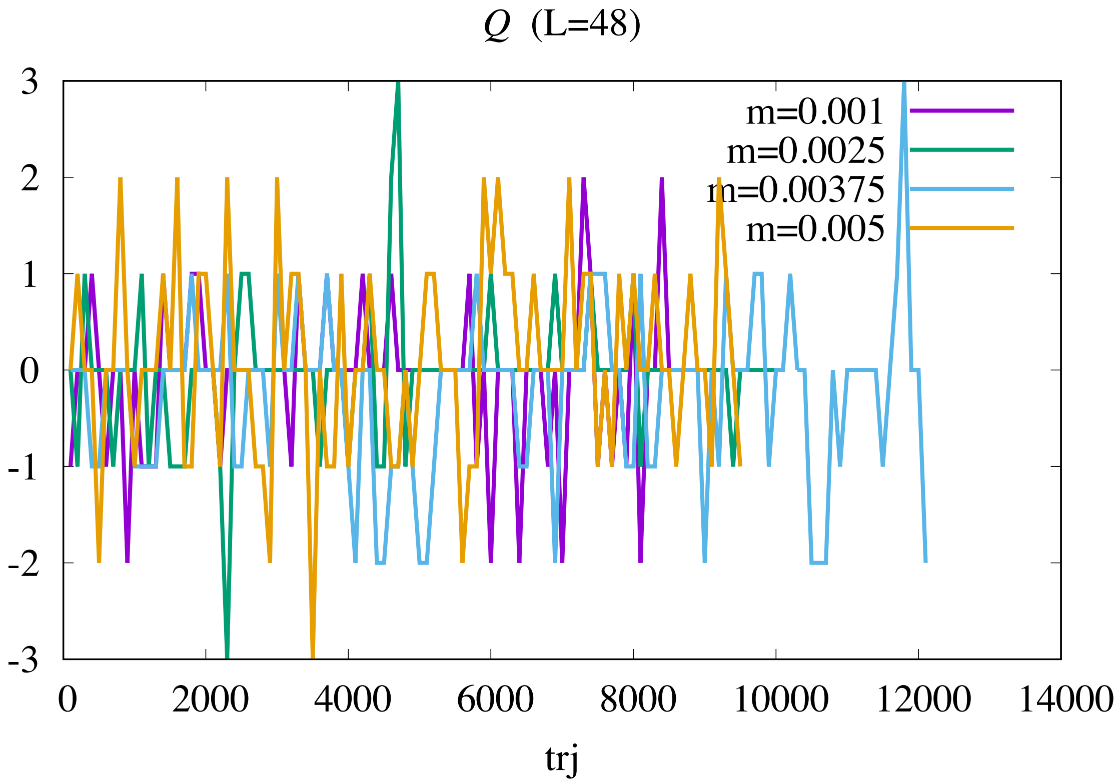
Fake zero modes

(partially quenched artifact)

OV on reweighted OV



OVERLAP/DOMAIN-WALL REWEIGHTING ALLOWS TOPOLOGY TUNNELINGS



L=48 (3.6fm)
a = 0.07fm
T = 220 MeV

Auto-correlation time of topology is O(100), small enough compared to our long trajectory length, 20000-30000 MD time.

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  Our answer = YES. $SU(2)_L x SU(2)_R$ and $U(1)_A$ are connected through Dirac spectrum.

- ✔ 2. **Lattice QCD at high T with chiral fermions**

  $U(1)_A$ at high T is sensitive to lattice artifact.
  We need good chiral sym (or careful cont. limit.).

3. **Result 1: $U(1)_A$ anomaly**

4. **Result 2: topological susceptibility**

5. **Summary**

WHAT WE OBSERVE

Axial U(1) susceptibility

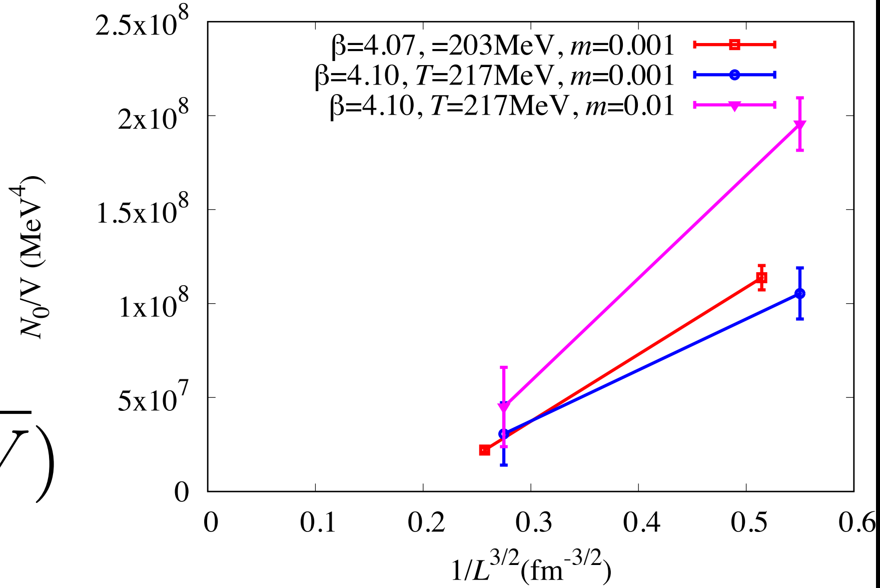
$$\Delta_{\pi-\delta} = \int d^4x \, [\langle \pi^a(x)\pi^a(0)\rangle - \langle \delta^a(x)\delta^a(0)\rangle] \, ,$$

$$\left(= \int_0^\infty d\lambda \, \rho(\lambda) \frac{2m^2}{(\lambda^2 + m^2)^2} \right)$$

We compute

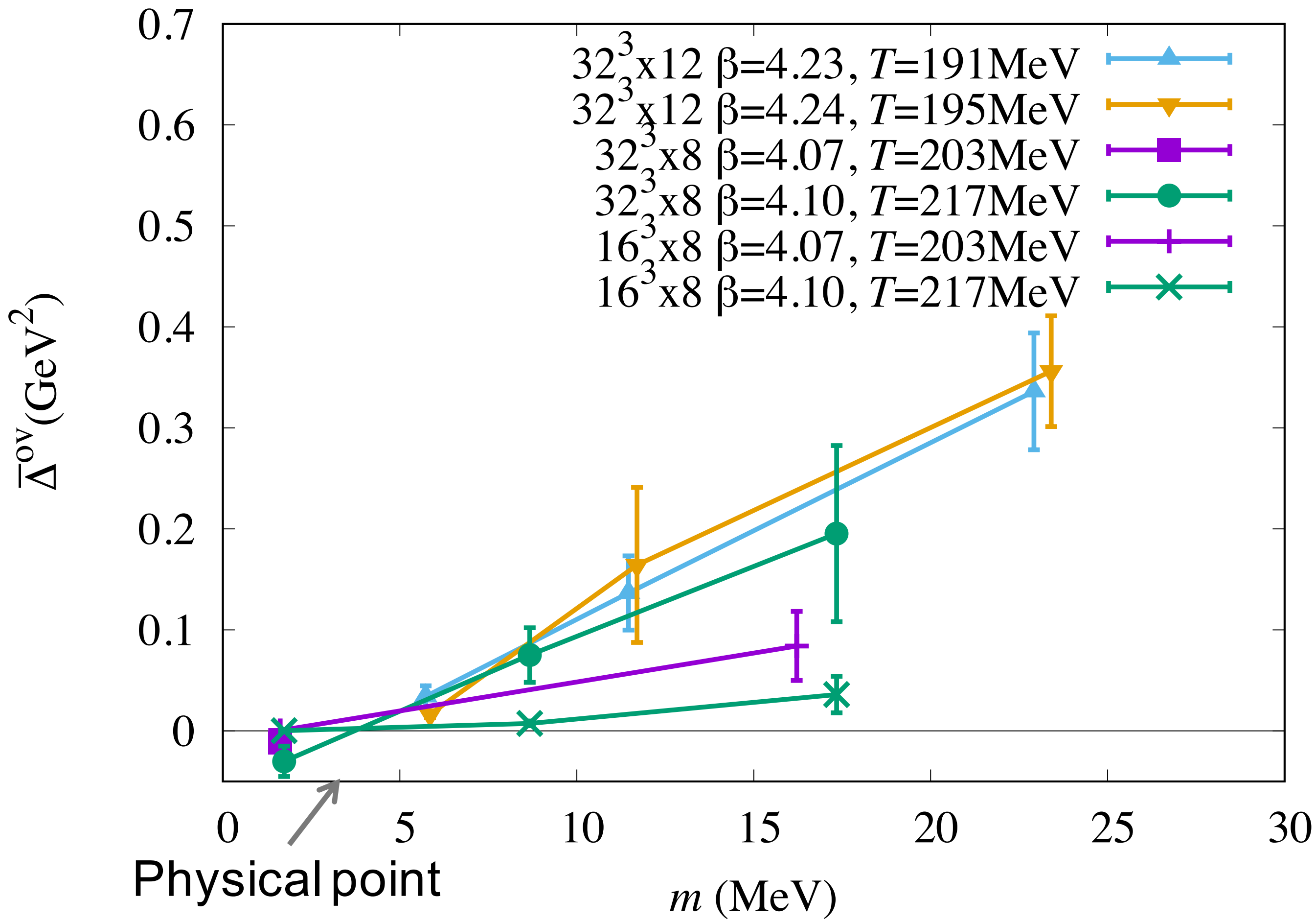
$$\bar{\Delta}^{\rm ov}_{\pi-\delta} \equiv \Delta^{\rm ov}_{\pi-\delta} - \frac{2N_0}{Vm^2} .$$

N_0 : # of zero modes $(\sim 1/\sqrt{V})$



U(1)$_A$ ANOMALY VANISHES IN THE CHIRAL LIMIT

Coarse (a>0.08fm) lattice [JLQCD(Tomiya et al.) 2016]



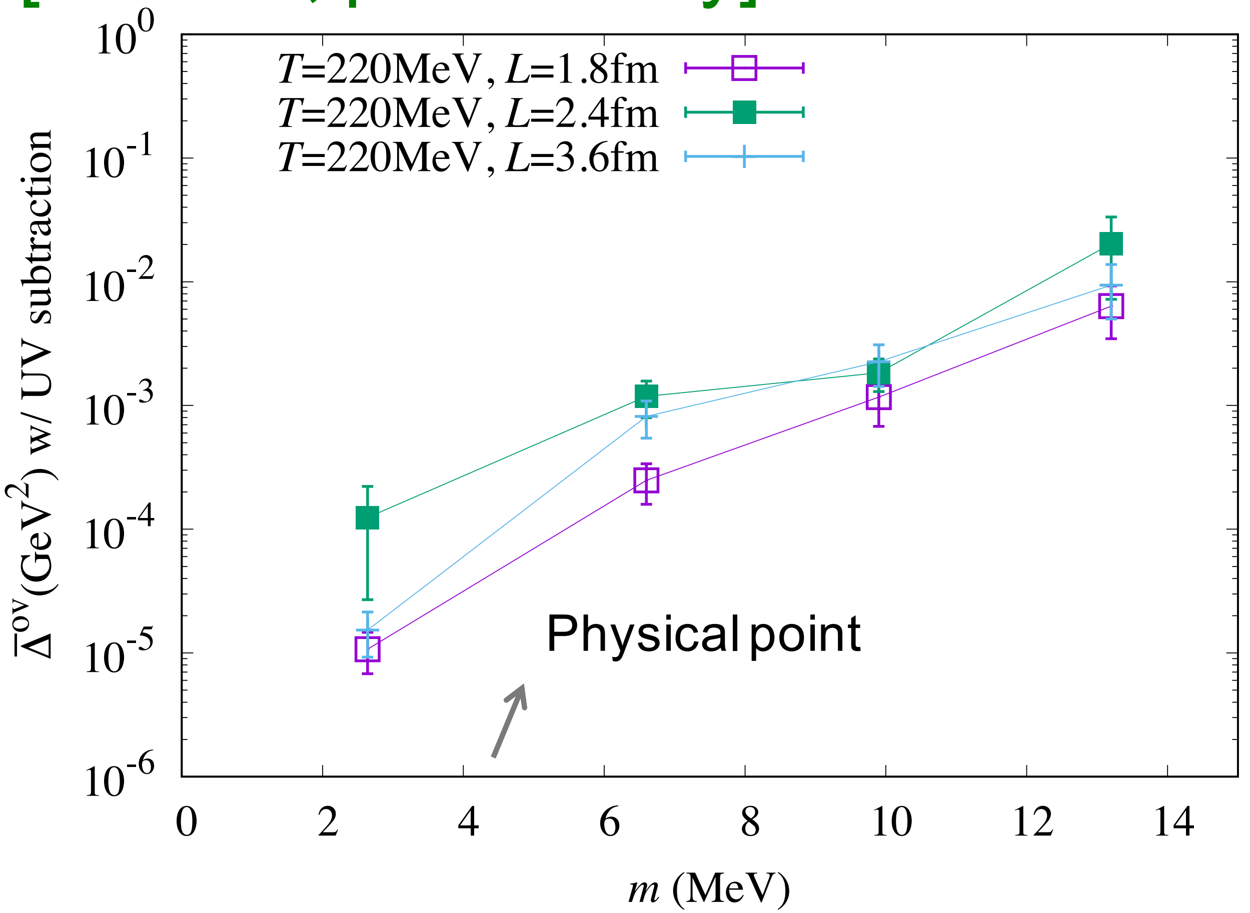
T = 1.1 – 1.2 Tc

L=16 (1.8fm) and 32 (3.6fm) results are consistent.

$$(M_{screen} L > 5.)$$

U(1) ANOMALY VANISHING ON FINE LATTICE (a~0.07fm)

[JLQCD, preliminary]



After subtraction of m^2 term (1/a divergence), the suppression looks exponential.

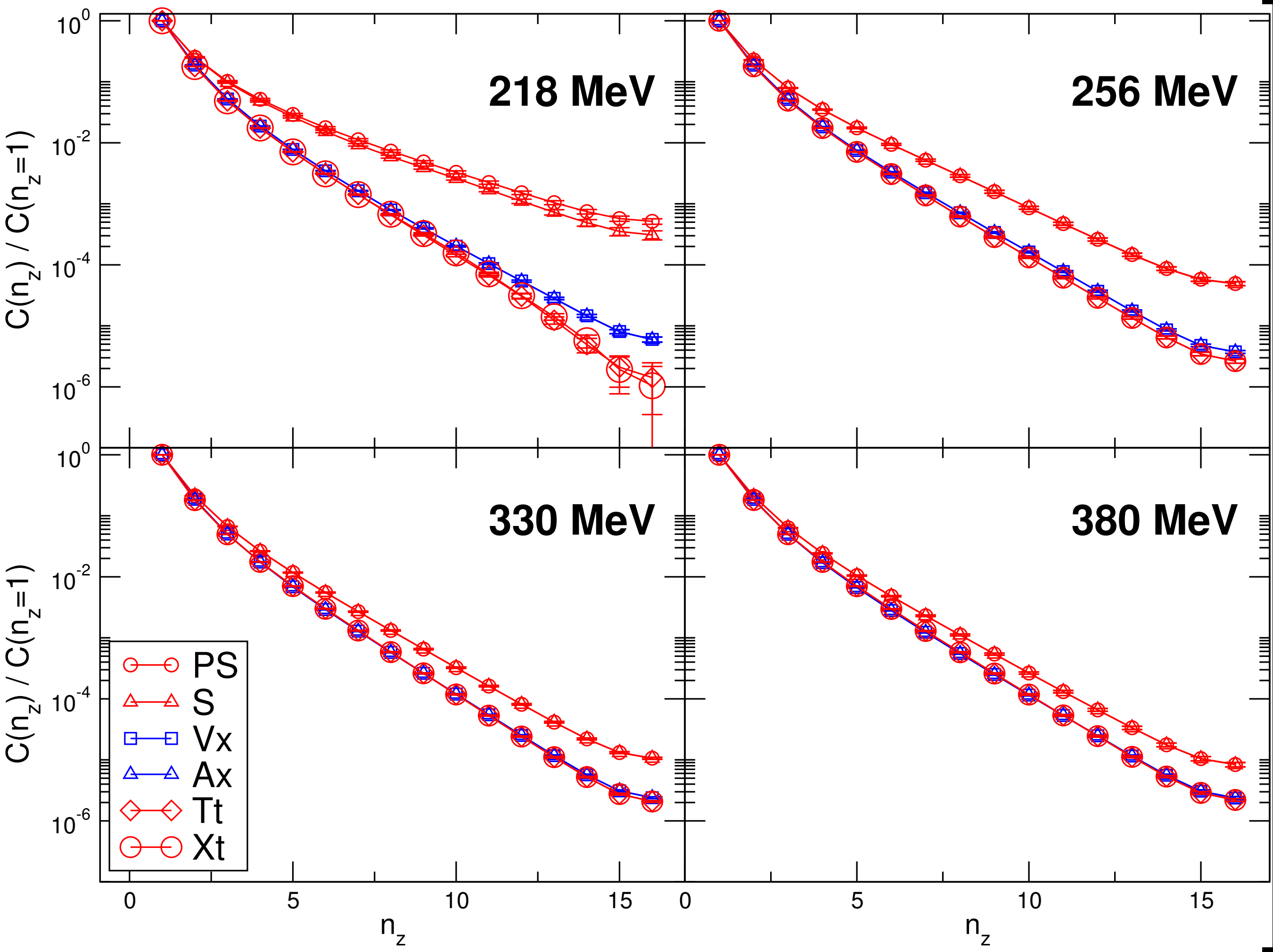
MESON CORRELATOR ITSELF SHOWS U(1) ANOMALY VANISHING

[C. Rohrhofer et al. 2017]

SU(2)xSU(2) [blue] and $U(1)_A$ (red) partners are degenerate.

[similar results reported by Brandt et al. 2016]

Further enhancement to SU(4) ? [Glozman 2015, Lang 2018]



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✔ 2. **Lattice QCD at high T with chiral fermions**

$U(1)_A$ at high T is sensitive to lattice artifact.
We need good chiral sym (or careful cont. limit.).

✔ 3. **Result 1: $U(1)_A$ anomaly**

$U(1)_A$ anomaly at T~1.1-1.4Tc (Tc~180MeV) in the chiral limit is consistent with zero.

4. **Result 2: topological susceptibility**

5. **Summary**

TOPOLOGICAL SUSCEPTIBILITY

$$\chi_t = \frac{\langle Q^2 \rangle}{V}$$

$$Q = n_+ - n_- = \frac{1}{32\pi^2} \int d^4x \mathrm{Tr} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

Overlap
Dirac index

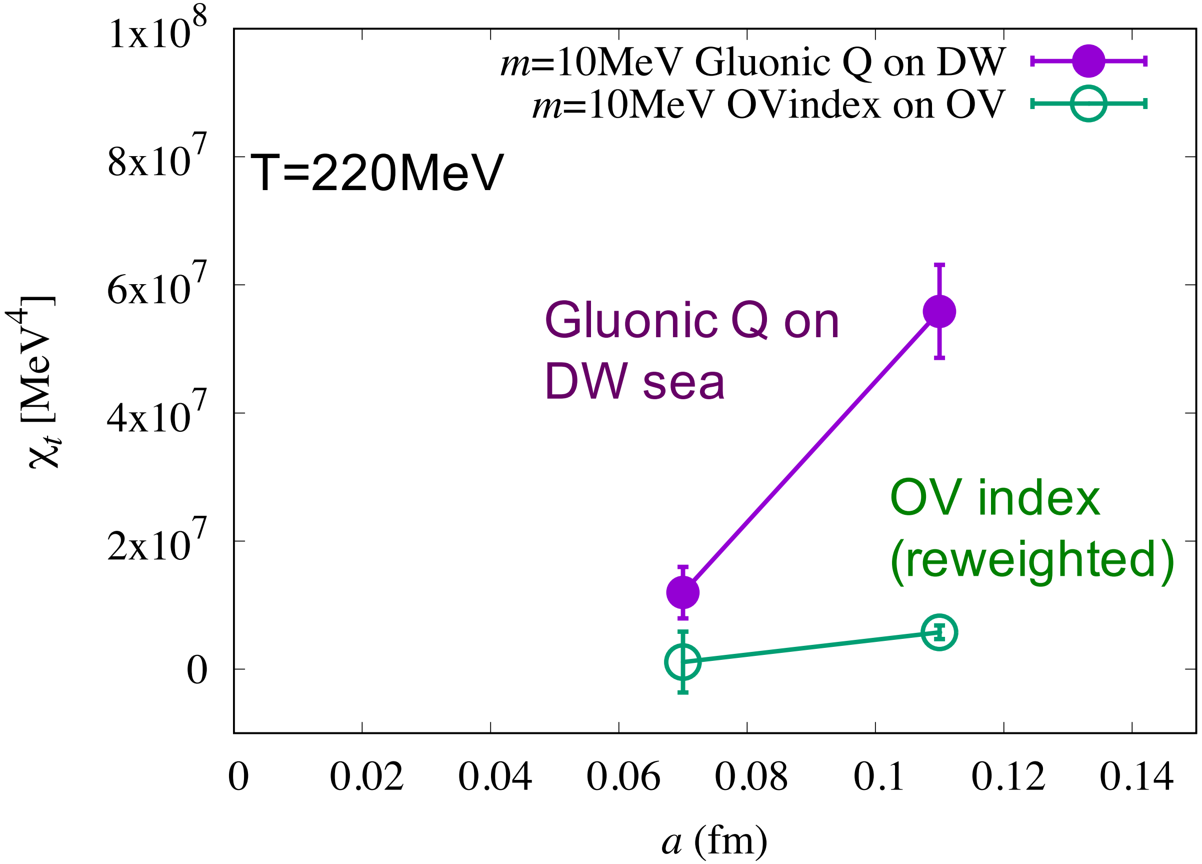
Gluonic
definition

another direct probe for $U(1)_A$ anomaly.

TOPOLOGICAL SUSCEPTIBILITY

Above Tc, it is sensitive to lattice artifact.

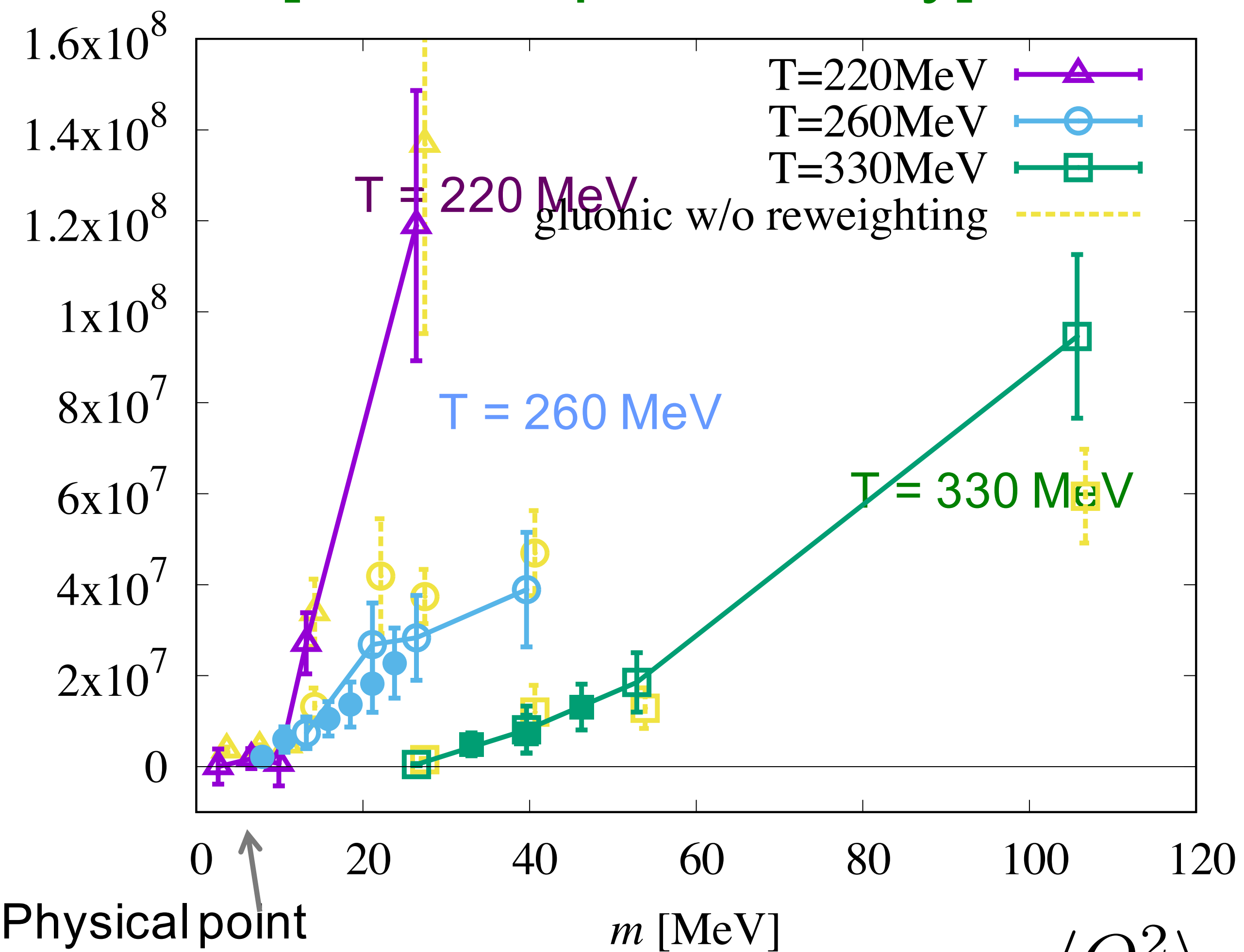
We need (reweighted) overlap fermion for a>0.08fm.



index of overlap Dirac operator is stable against lattice cut-off.

TOPOLOGICAL SUSCEPTIBILITY VANISHES *BEFORE* THE CHIRAL LIMIT

[JLQCD preliminary]



L=48 (3.6fm) & L=32 (2.4fm) results are consistent.

On our fine lattices (a~0.07fm) OV index and gluonic def. after Wilson flow ($\sqrt{8t} \sim 0.47$ fm) are also consistent.

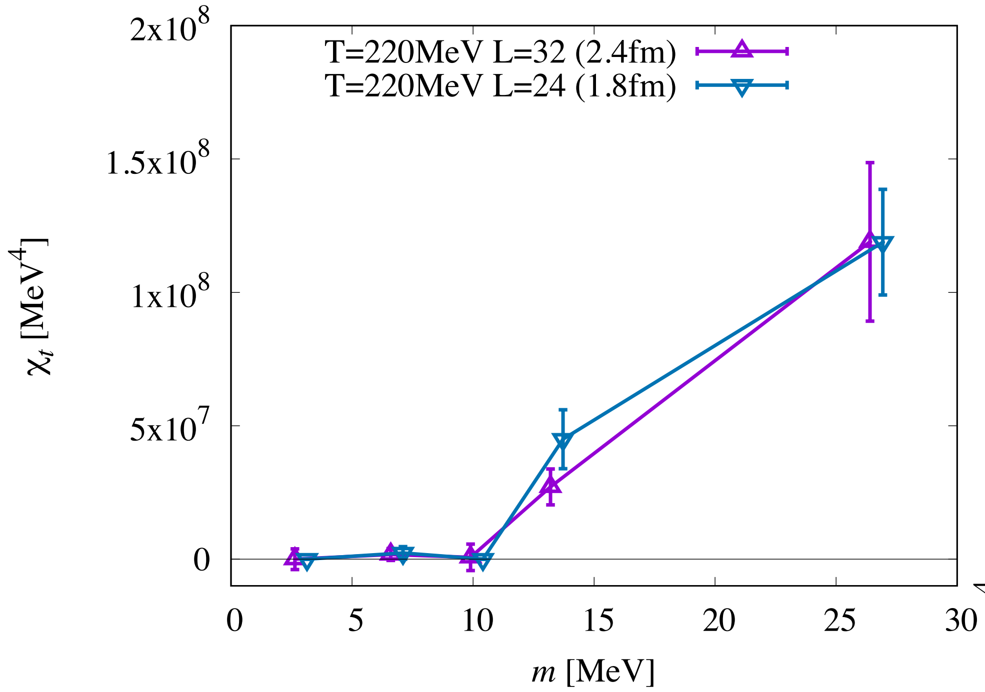
agrees with our prediction

$$\frac{\langle Q^2 \rangle}{V} = 0 \quad \text{for } m <^{\exists} m_{cr}$$

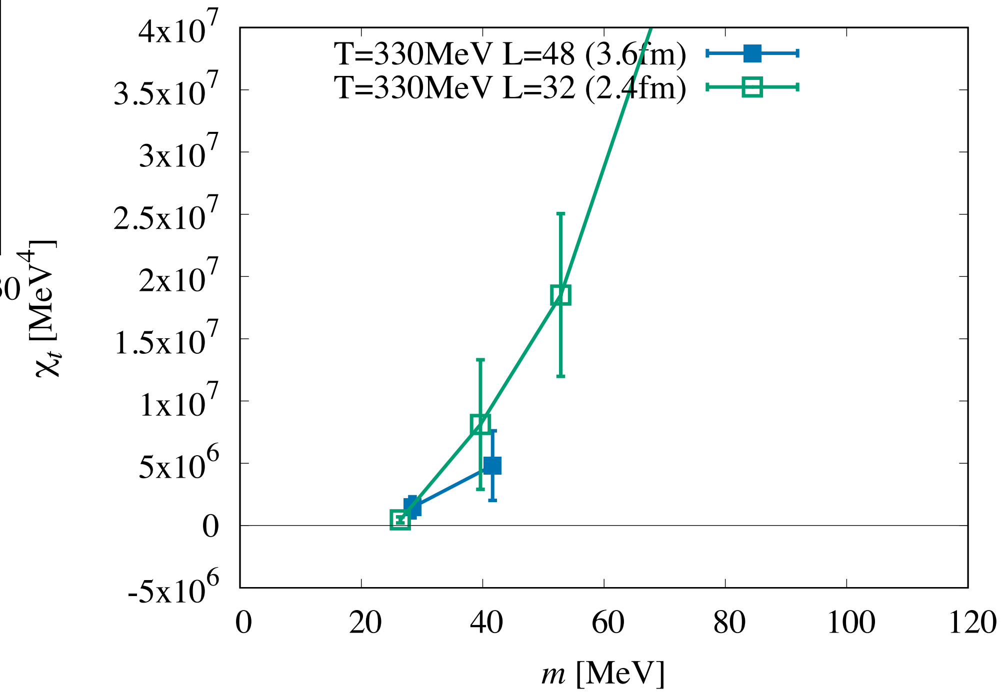
[Aoki, F, Taniguchi 2012]

FINITE VOLUME DEPENDENCE

[JLQCD preliminary]

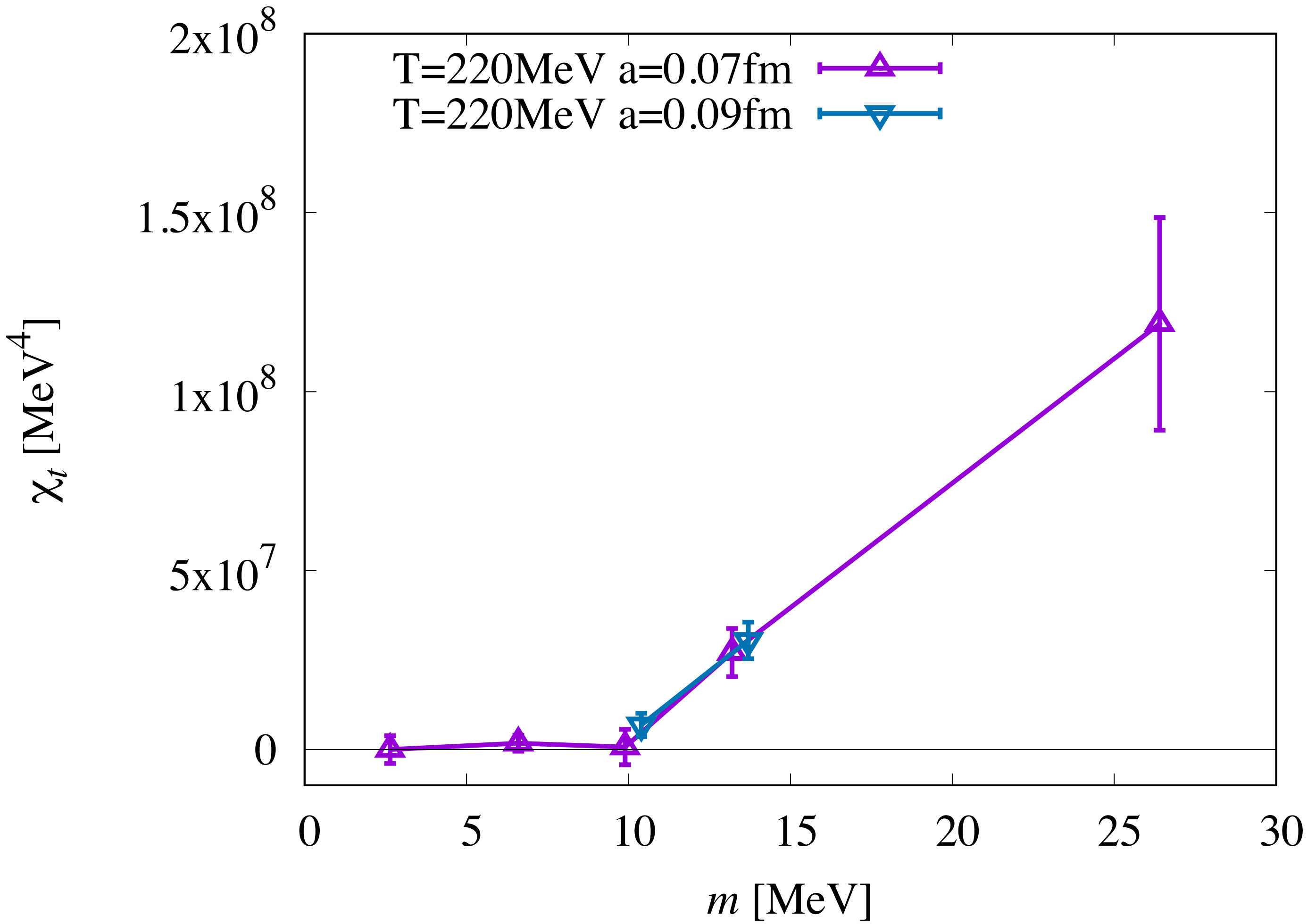


Larger lattice simulations are on-going.



FINITE LATTICE SPACING DEPENDENCE

[JLQCD preliminary]



STRONG SUPPRESSION OF TOPOLOGICAL SUSCEPTIBILITY

If our data indicates

$$\frac{\langle Q^2 \rangle}{V} = 0 \quad \text{for } m <^{\exists} m_{cr}$$

Chiral phase transition is likely to be

1st order.

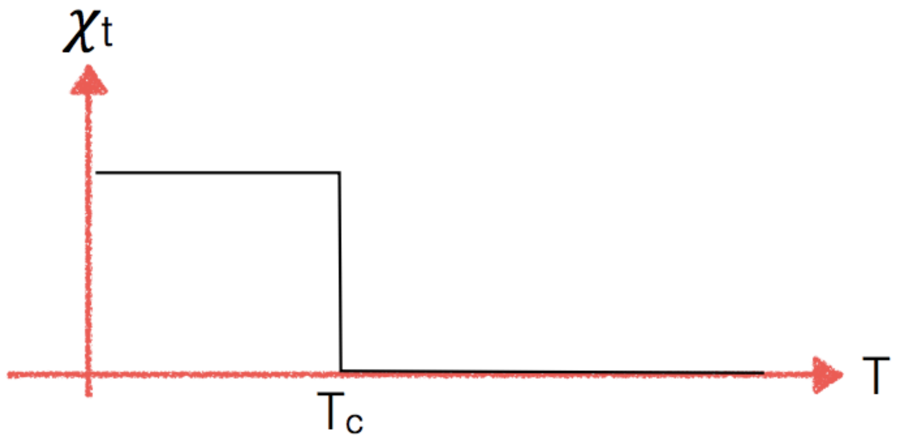
(There's no symmetry enhancement at finite quark mass.)

If $m_u, m_d < m_{cr}$, **there may be gravitational waves from QCD bubble collision in the early universe.**

CAN AXION BE A DARK MATTER?

If our result really indicates $\chi_t = 0$

and 1st order phase transition,



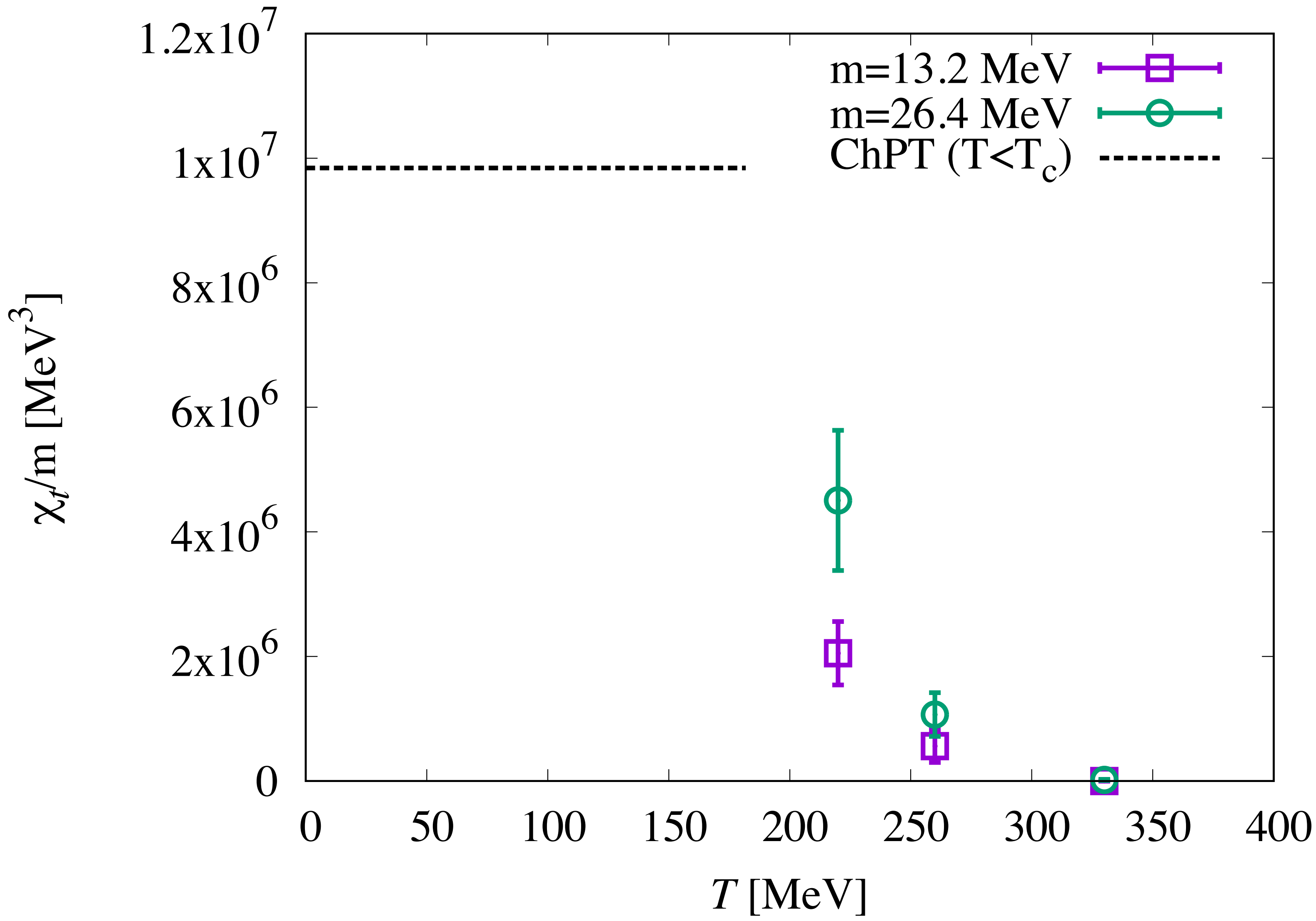
$\Omega_a \sim 2 \times 10^5 \theta_{\rm ini}^2$ independent of m_a

Figure from Kitano's talk (2015)

Axion cannot be a dark matter since too much DM created (to expand our universe).

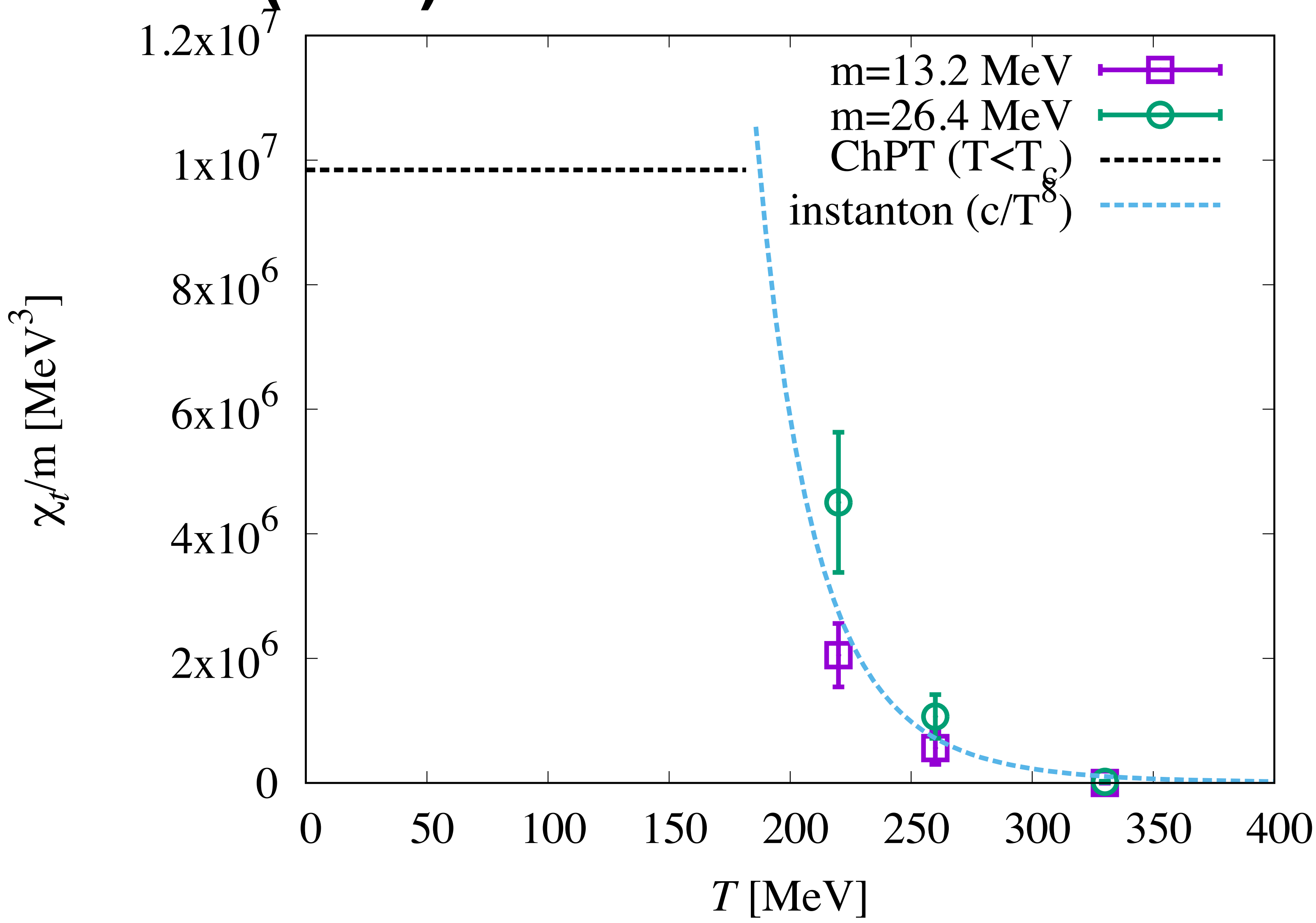
TEMPERATURE DEPENDENCE

Shows a sharp drop!

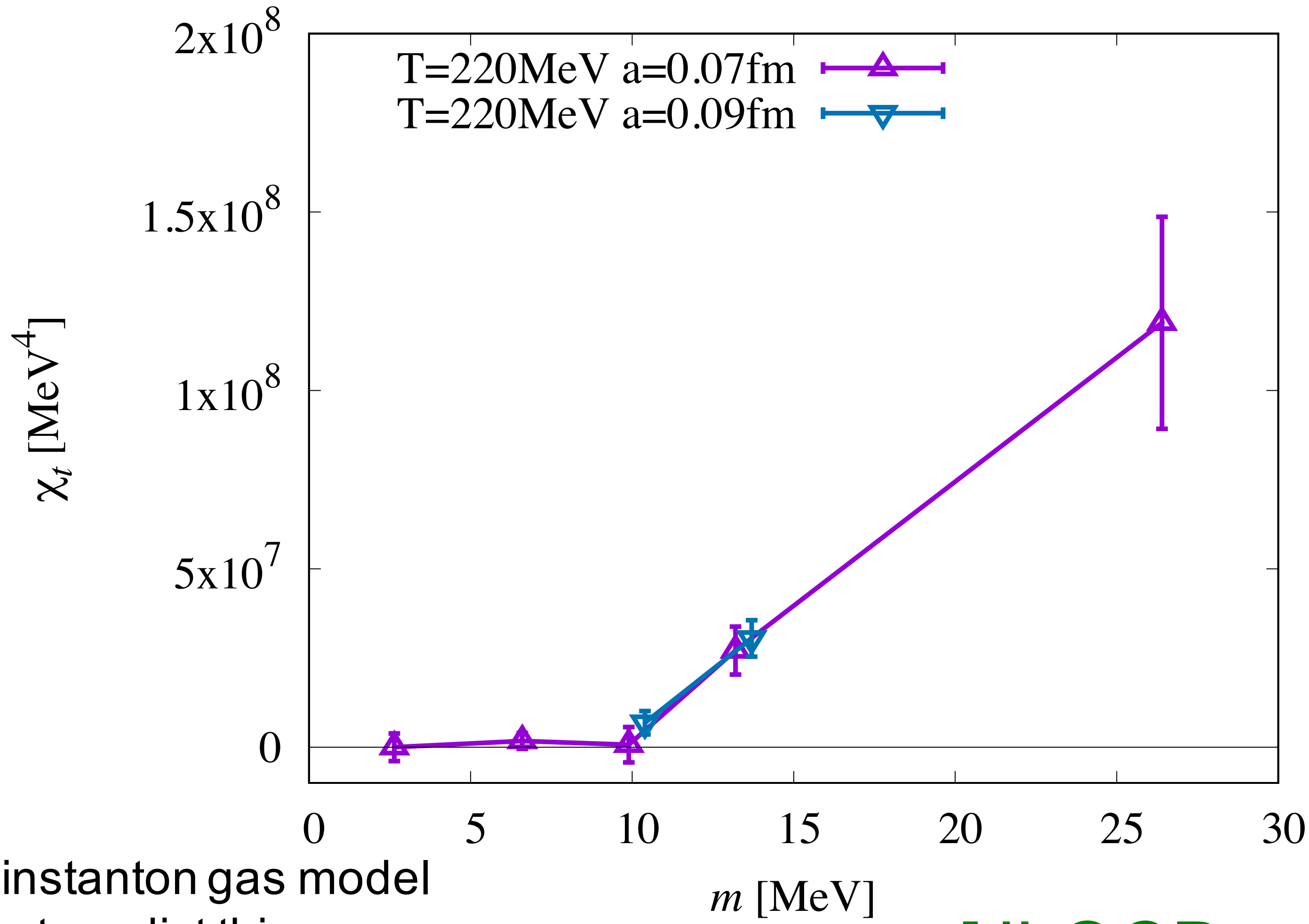


TEMPERATURE DEPENDENCE

But so does (U(1)A breaking) instanton model ($1/T^8$).



THE DROP IS STILL IMPRESSIVE.



Dilute instanton gas model does not predict this "discontinuity".

[JLQCD preliminary]

CONTENTS

✔ 1. **Is $U(1)_A$ symmetry theoretically possible ?**

Our answer = YES. $SU(2)_L x SU(2)_R$ and $U(1)_A$ are connected through Dirac spectrum.

✔ 2. **Lattice QCD at high T with chiral fermions**

$U(1)_A$ at high T is sensitive to lattice artifact.
We need good chiral sym (or careful cont. limit.).

✔ 3. **Result 1: $U(1)_A$ anomaly**

$U(1)_A$ anomaly at T~1.1-1.4Tc (Tc~180MeV) in the chiral limit is consistent with zero.

✔ 4. **Result 2: topological susceptibility**

Topological susceptibility drops *before* the chiral limit.

5. **Summary**

SUMMARY

1. $U(1)_A$ anomaly at high T is a non-trivial problem.
2. $U(1)_A$ and $SU(2)_L xSU(2)_R$ order prms. connected.
3. $U(1)_A$ is sensitive to lattice artifact at high T
   -> We need good chiral symmetry (or careful continuum limit).
4. In our simulation with chiral fermions at 3 volumes and 3-10 quark masses at T=1.1-1.8Tc (Tc~180MeV), $U(1)_A$ anomaly disappears
   [before the chiral limit]
   (suggesting 1st order transition ?).

MAIN MESSAGE OF THIS TALK

In high T QCD, whether

$$\langle\langle\partial_\mu J_5^\mu(x)O(x')\rangle_{fermion} - \langle\delta_A O(x)\rangle_{fermion}\delta(x-x')\rangle_{gluons}$$

$$= \left\langle \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}(x)\langle O(x')\rangle_{fermion}\right\rangle_{gluons} = 0???$$

or not is a non-trivial question, which can only be answered by carefully integrating over gluons (by lattice QCD).

In particular, good control of chiral symmetry (or continuum limit) is essential.

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