Mesons with charm and strangeness in nuclear matter

Philipp Gubler, JAEA

P. Gubler and K. Ohtani, Phys. Rev. D **90**, 094002 (2014).

P. Gubler and W. Weise, Phys. Lett. B **751**, 396 (2015).

P. Gubler and W. Weise, Nucl. Phys. A **954**, 125 (2016).

A. Park, P. Gubler, M. Harada, S.H. Lee, C. Nonaka and W. Park, Phys. Rev. D **93**, 054035 (2016).

K. Suzuki, P. Gubler and M. Oka, Phys. Rev. C **93**, 045209 (2016).

Talk at "New Frontiers in QCD 2018" (NFQCD2018)

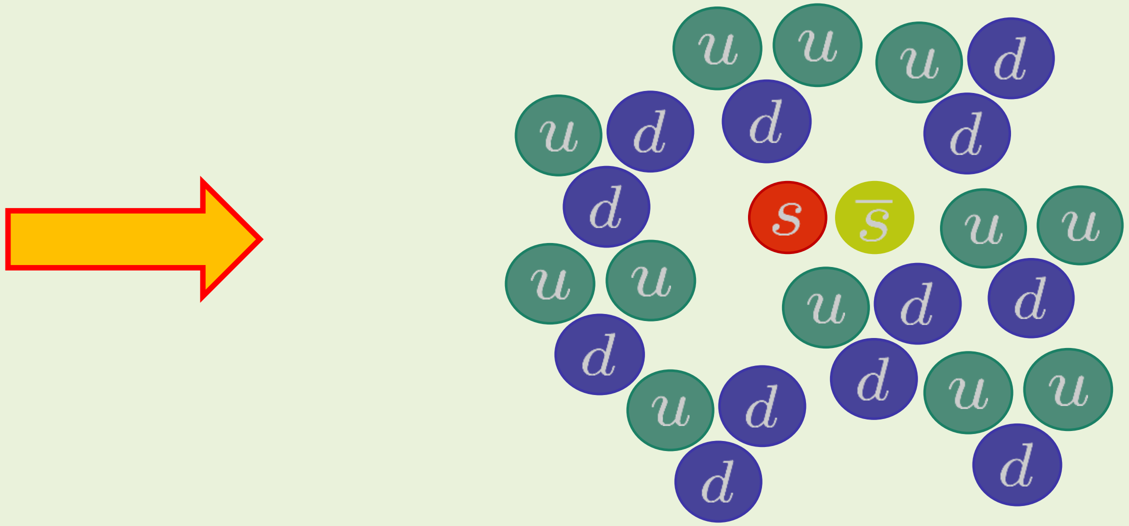
YITP, Kyoto, Japan

June 15, 2018

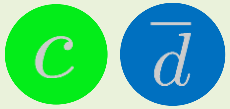
Introduction

ϕ meson

$m_\phi = 1019$ MeV

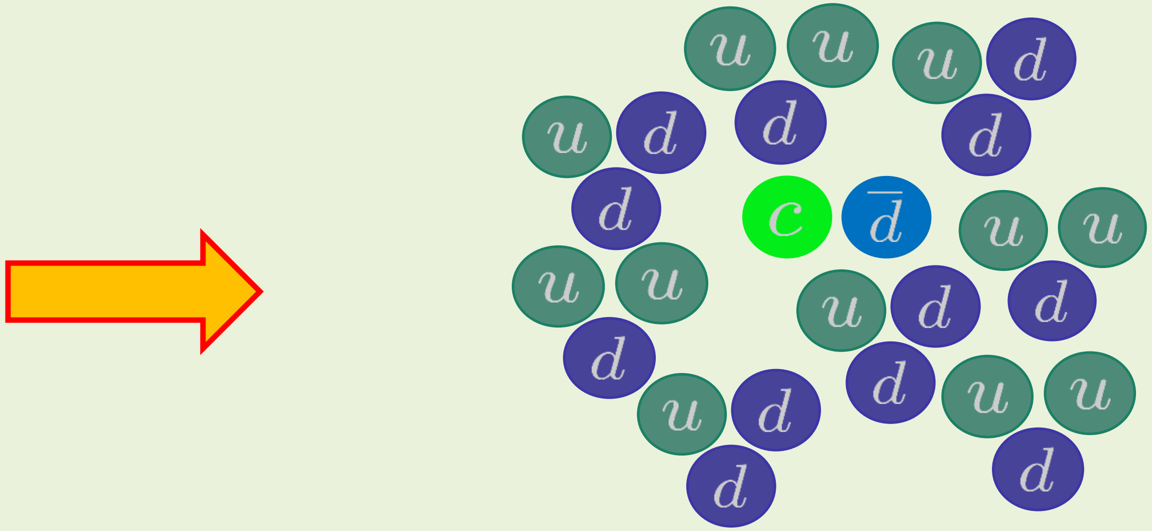


D mesons

D^+: 

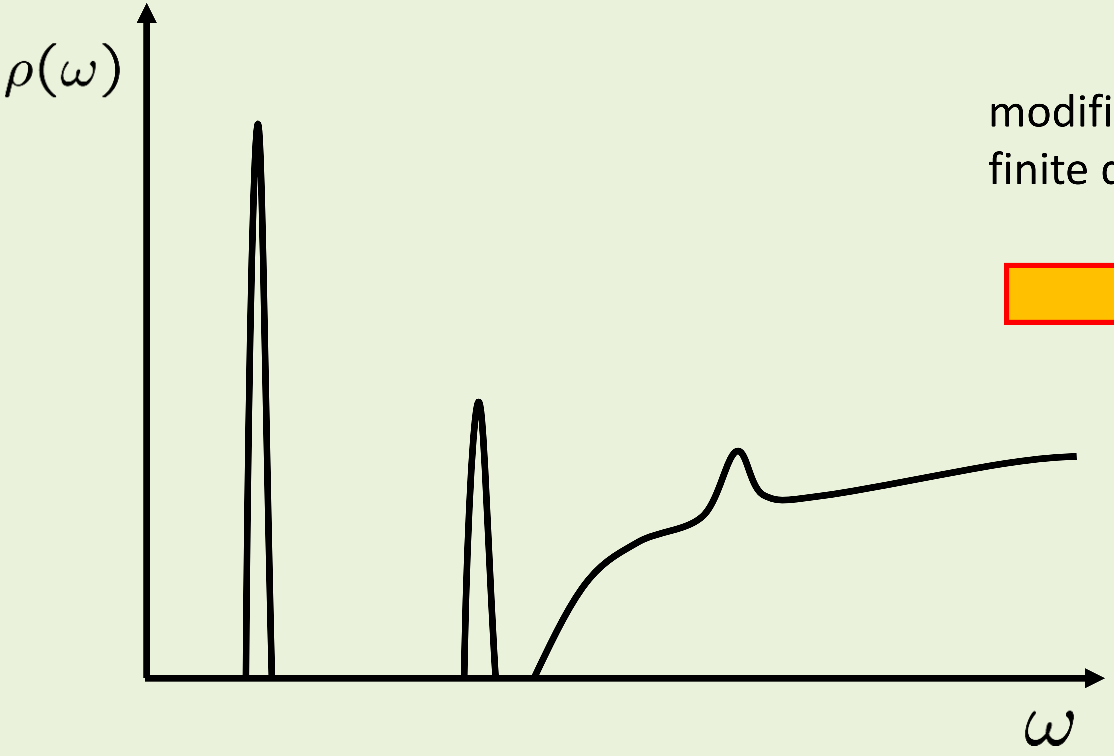
D^-: 

$m_{D^\pm} = 1870$ MeV



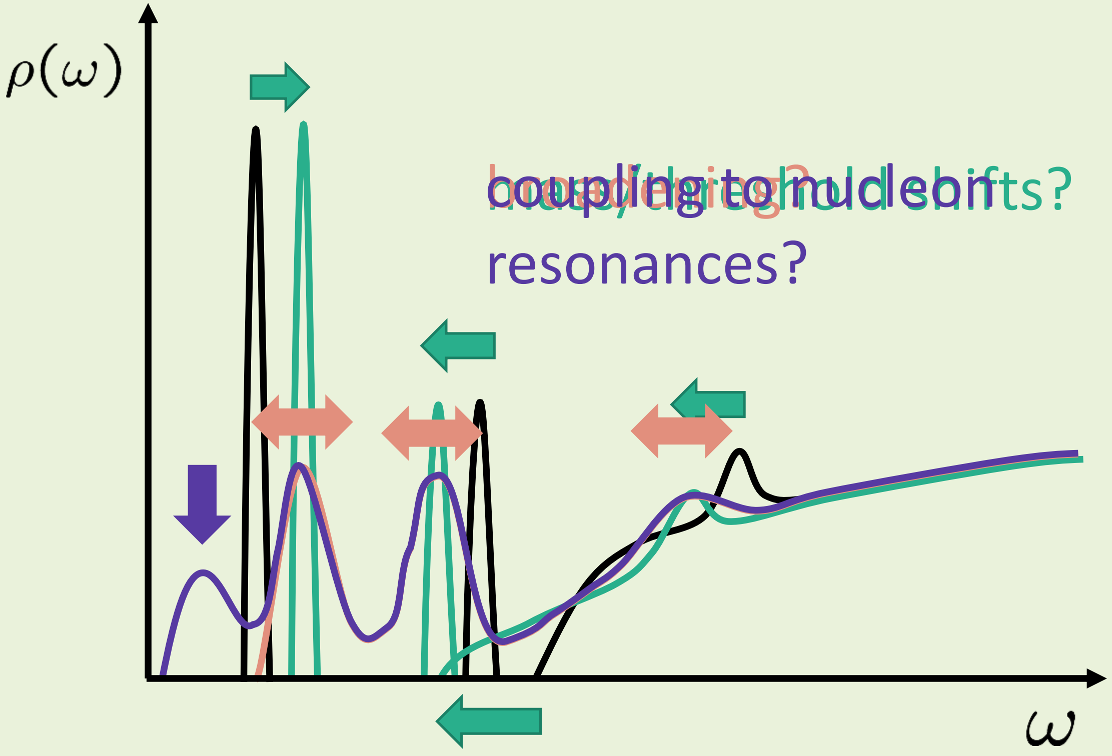
Introduction

Spectral functions at finite density



modification at finite density

How is this complicated behavior related to the behavior of QCD condensates?



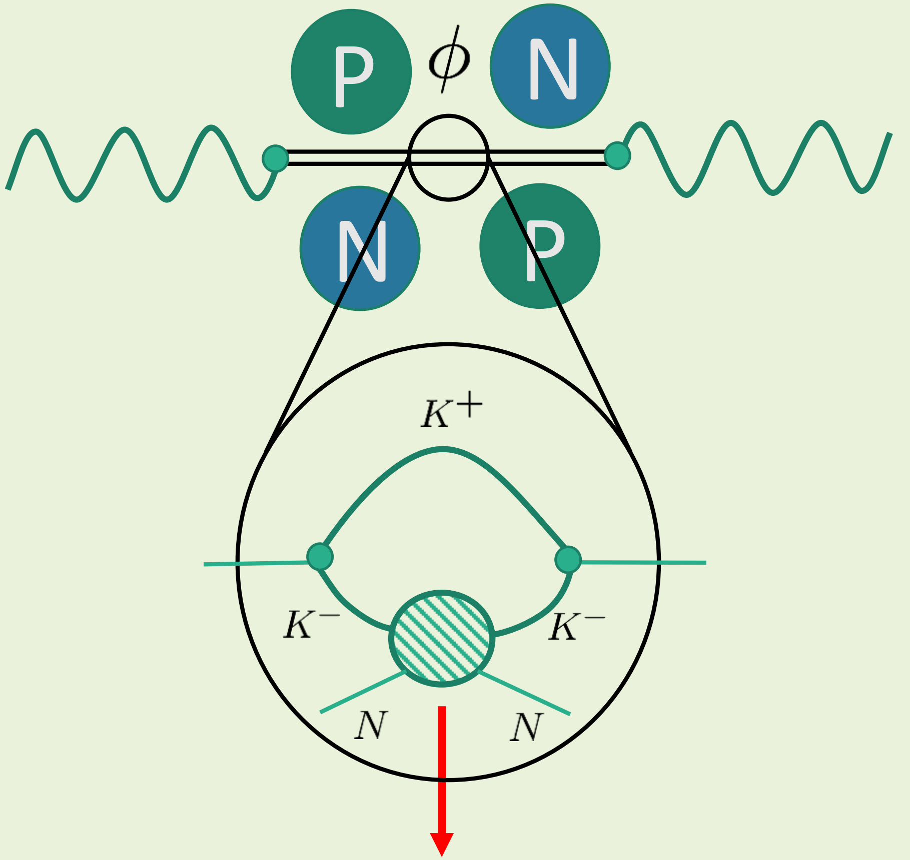
mass/threshold shifts?

broadening?

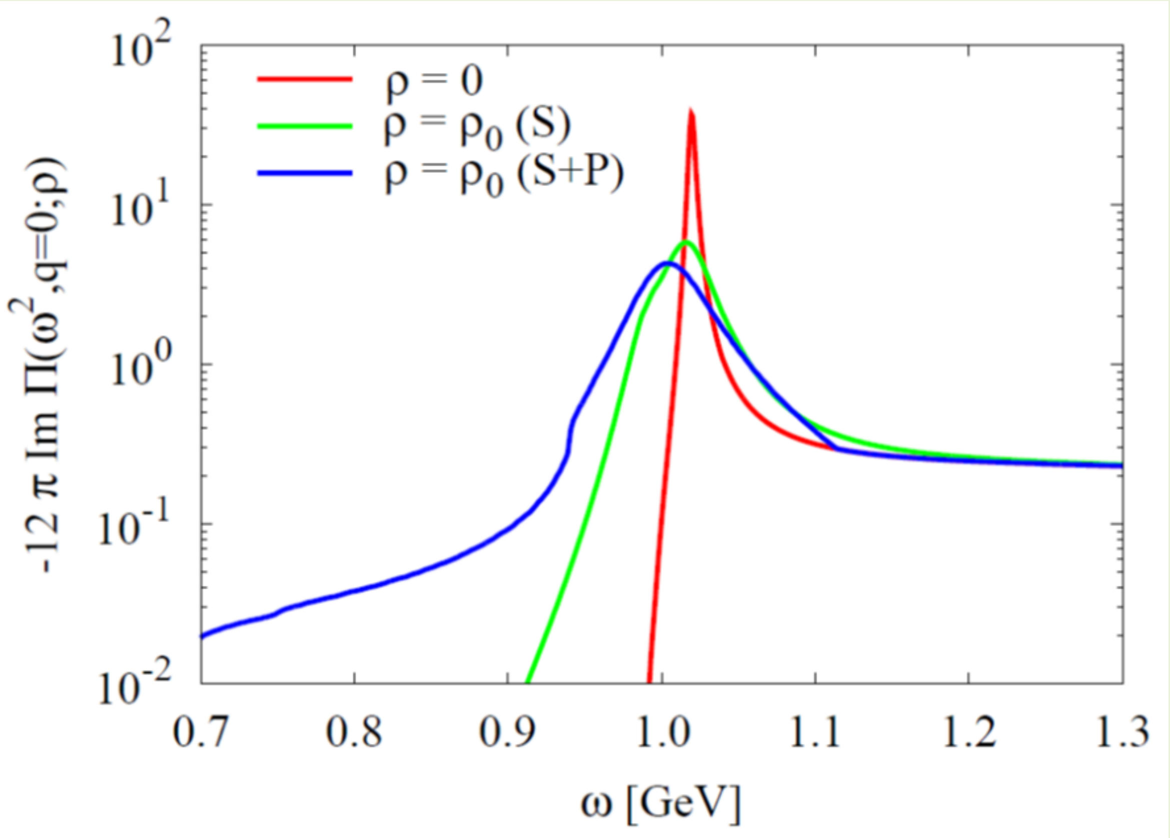
coupling to nucleon resonances?

Recent theoretical works about the ϕ

based on hadronic models



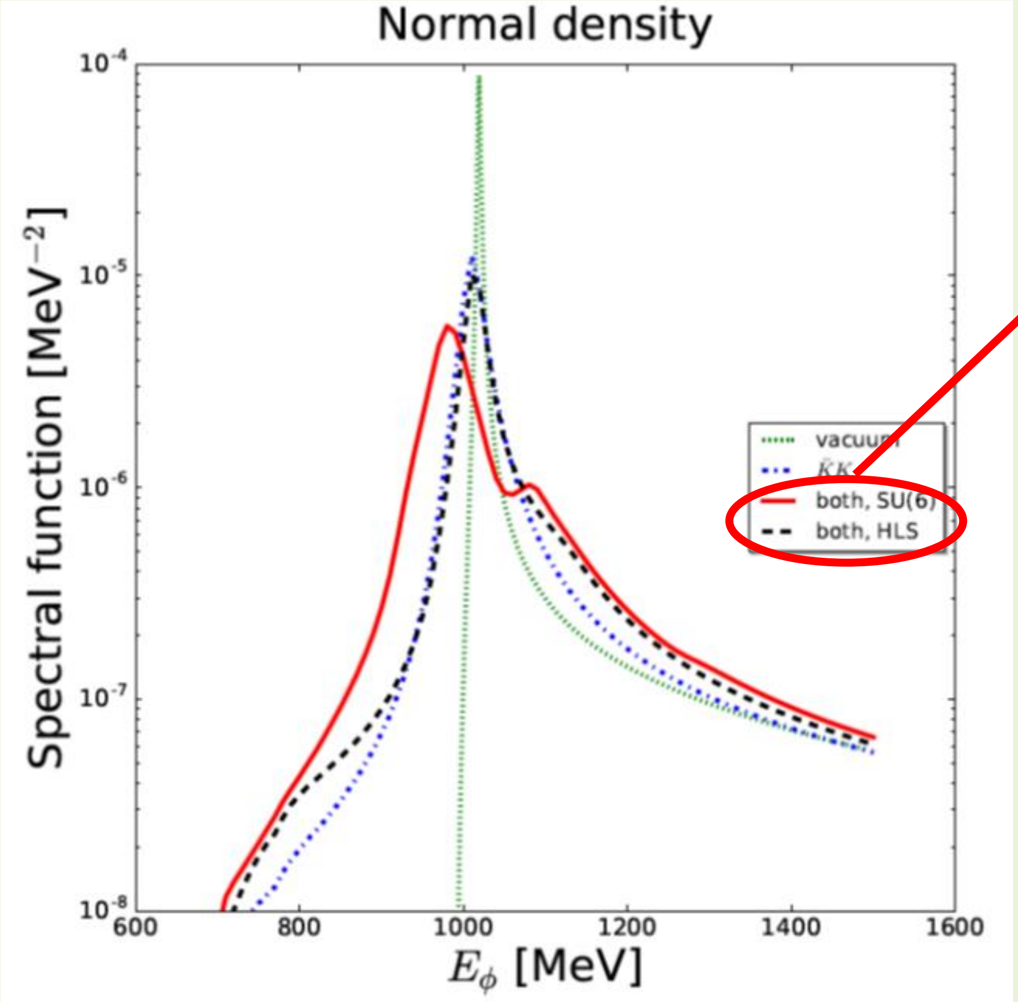
Forward $\bar{K}$N (or KN)
scattering amplitude



P. Gubler and W. Weise, Phys. Lett. B **751**, 396 (2015).
P. Gubler and W. Weise, Nucl. Phys. A **954**, 125 (2016).

Recent theoretical works about the ϕ

based on hadronic models



D. Cabrera, A.N. Hiller Blin and M.J. Vicente Vacas,
Phys. Rev. C **95**, 015201 (2017).

large dependence on details of the model incorporating Baryon - Vector meson interaction

SU(6): Spin-Flavor Symmetry extension of standard flavor SU(3)

HLS: Hidden Local Symmetry

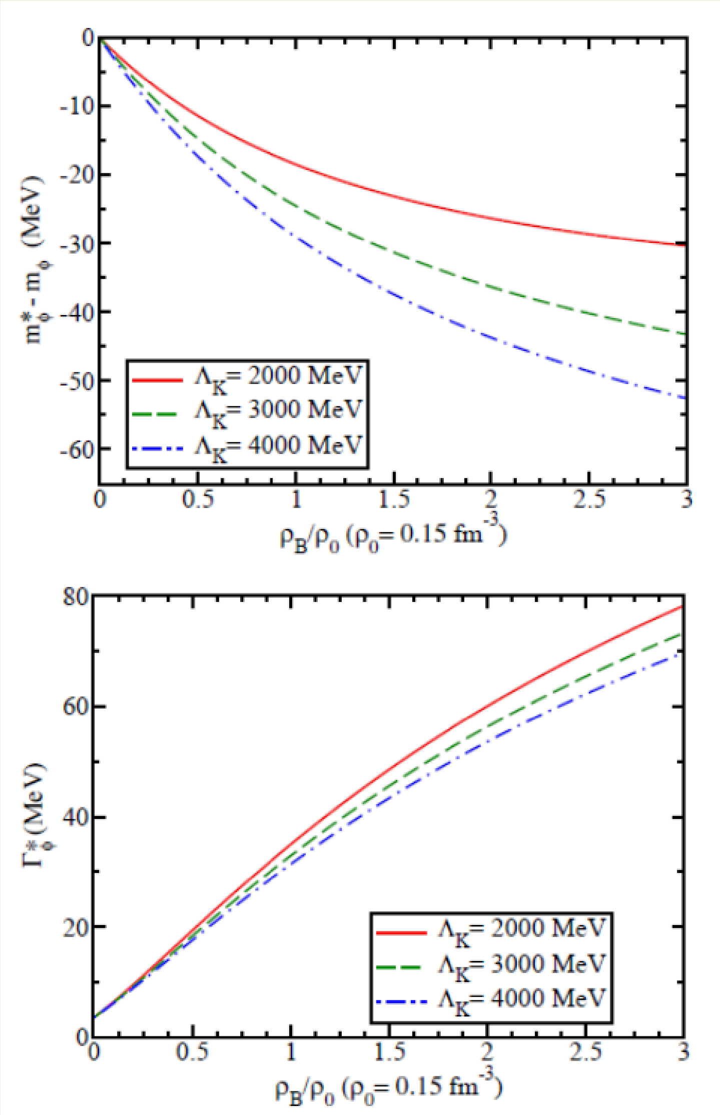
Common features:

strong broadening, small negative mass shift

See also:
D. Cabrera, A.N. Hiller Blin and M.J. Vicente Vacas,
Phys. Rev. C **96**, 034618 (2017).

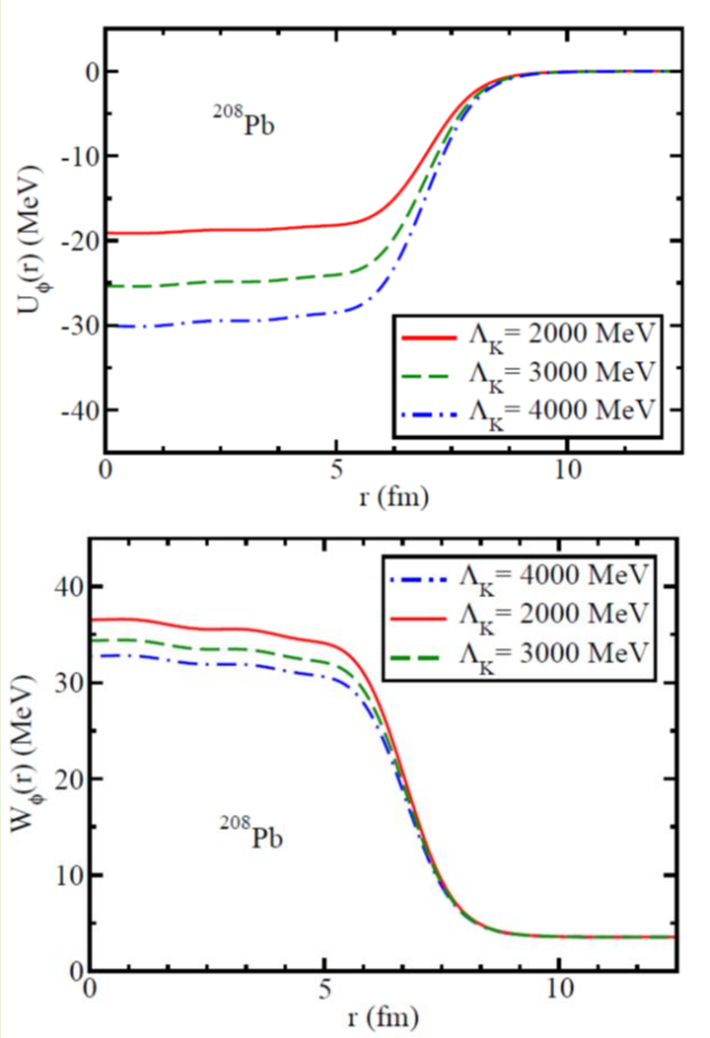
Recent theoretical works about the ϕ

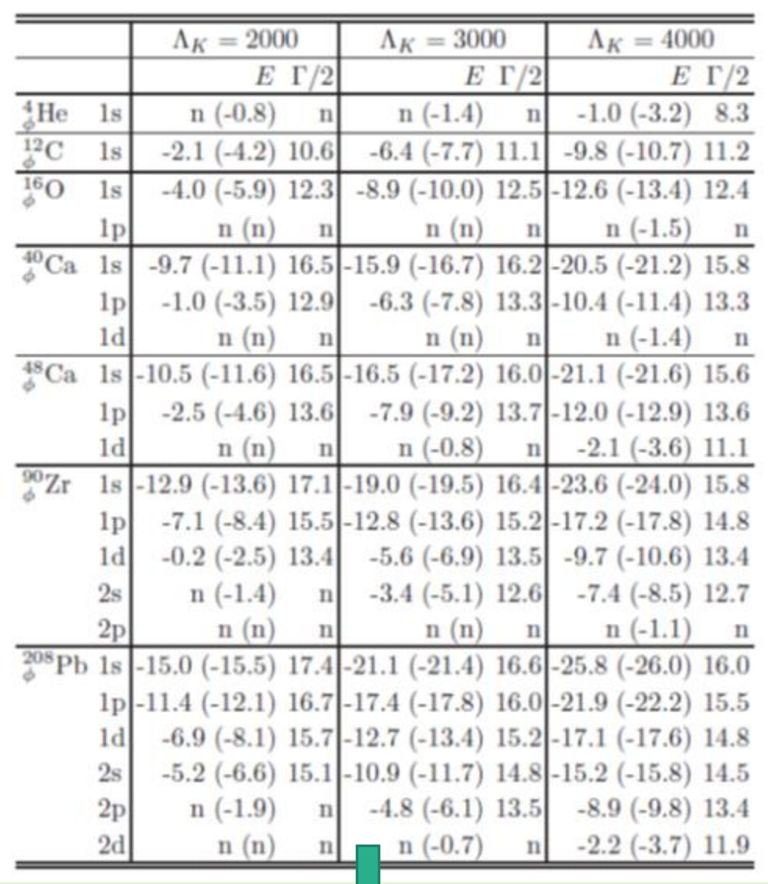
based on the quark-meson coupling model



J.J. Cobos-Martinez, K. Tsushima, G. Krein and A.W. Thomas, Phys. Lett. B **771**, 113 (2017).
J.J. Cobos-Martinez, K. Tsushima, G. Krein and A.W. Thomas, Phys. Rev. C **96**, 035201 (2017).

$$V_{\phi A}(r) = U_\phi(r) - \frac{i}{2} W_\phi(r)$$





		$\Lambda_K = 2000$		$\Lambda_K = 3000$		$\Lambda_K = 4000$	
		E	$\Gamma/2$	E	$\Gamma/2$	E	$\Gamma/2$
$^4_\phi$He	1s	n (-0.8)	n	n (-1.4)	n	-1.0 (-3.2)	8.3
$^{12}_\phi$C	1s	-2.1 (-4.2)	10.6	-6.4 (-7.7)	11.1	-9.8 (-10.7)	11.2
$^{16}_\phi$O	1s	-4.0 (-5.9)	12.3	-8.9 (-10.0)	12.5	-12.6 (-13.4)	12.4
	1p	n (n)	n	n (n)	n	n (-1.5)	n
$^{40}_\phi$Ca	1s	-9.7 (-11.1)	16.5	-15.9 (-16.7)	16.2	-20.5 (-21.2)	15.8
	1p	-1.0 (-3.5)	12.9	-6.3 (-7.8)	13.3	-10.4 (-11.4)	13.3
	1d	n (n)	n	n (n)	n	n (-1.4)	n
$^{48}_\phi$Ca	1s	-10.5 (-11.6)	16.5	-16.5 (-17.2)	16.0	-21.1 (-21.6)	15.6
	1p	-2.5 (-4.6)	13.6	-7.9 (-9.2)	13.7	-12.0 (-12.9)	13.6
	1d	n (n)	n	n (-0.8)	n	-2.1 (-3.6)	11.1
$^{90}_\phi$Zr	1s	-12.9 (-13.6)	17.1	-19.0 (-19.5)	16.4	-23.6 (-24.0)	15.8
	1p	-7.1 (-8.4)	15.5	-12.8 (-13.6)	15.2	-17.2 (-17.8)	14.8
	1d	-0.2 (-2.5)	13.4	-5.6 (-6.9)	13.5	-9.7 (-10.6)	13.4
	2s	n (-1.4)	n	-3.4 (-5.1)	12.6	-7.4 (-8.5)	12.7
	2p	n (n)	n	n (n)	n	n (-1.1)	n
$^{208}_\phi$Pb	1s	-15.0 (-15.5)	17.4	-21.1 (-21.4)	16.6	-25.8 (-26.0)	16.0
	1p	-11.4 (-12.1)	16.7	-17.4 (-17.8)	16.0	-21.9 (-22.2)	15.5
	1d	-6.9 (-8.1)	15.7	-12.7 (-13.4)	15.2	-17.1 (-17.6)	14.8
	2s	-5.2 (-6.6)	15.1	-10.9 (-11.7)	14.8	-15.2 (-15.8)	14.5
	2p	n (-1.9)	n	-4.8 (-6.1)	13.5	-8.9 (-9.8)	13.4
	2d	n (n)	n	n (-0.7)	n	-2.2 (-3.7)	11.9

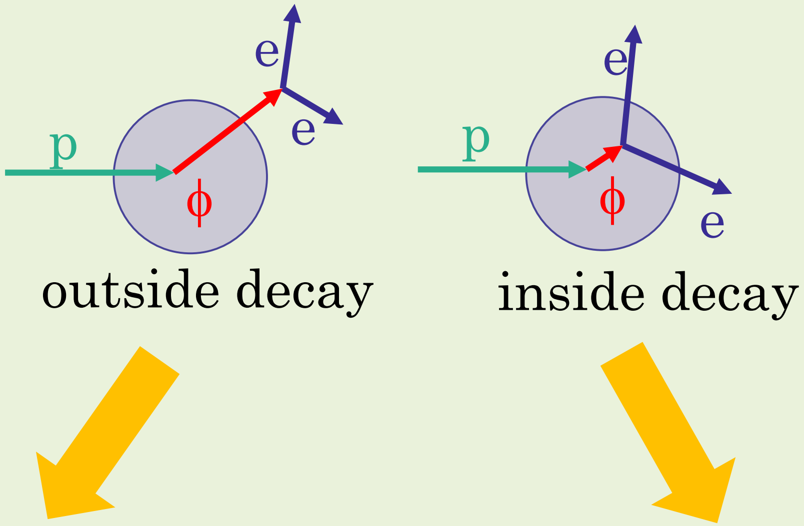
Some ϕA bound states might exist, but they have a large width

→ difficult to observe experimentally ?

Experimental developments

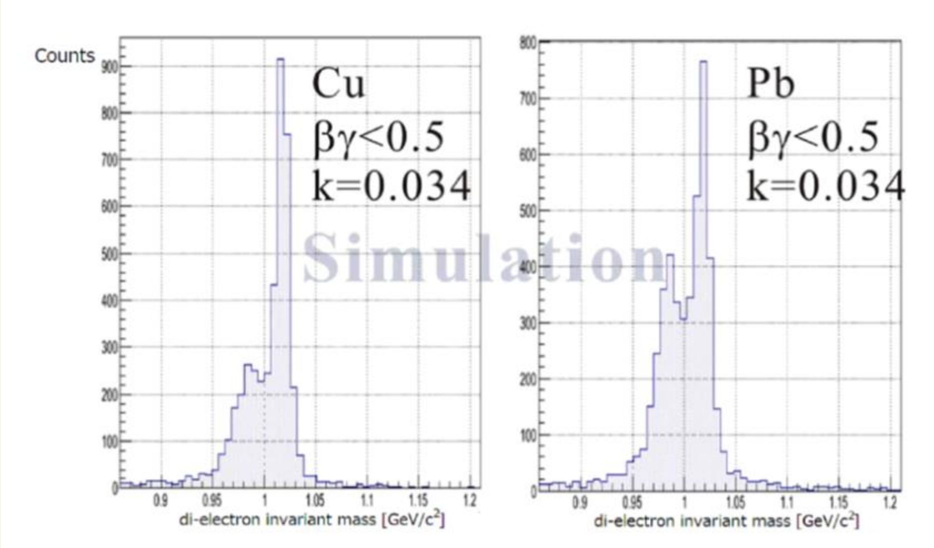
The E325 Experiment (KEK)

Slowly moving ϕ mesons are produced in 12 GeV *p*+*A* reactions
and are measured through di-leptons.



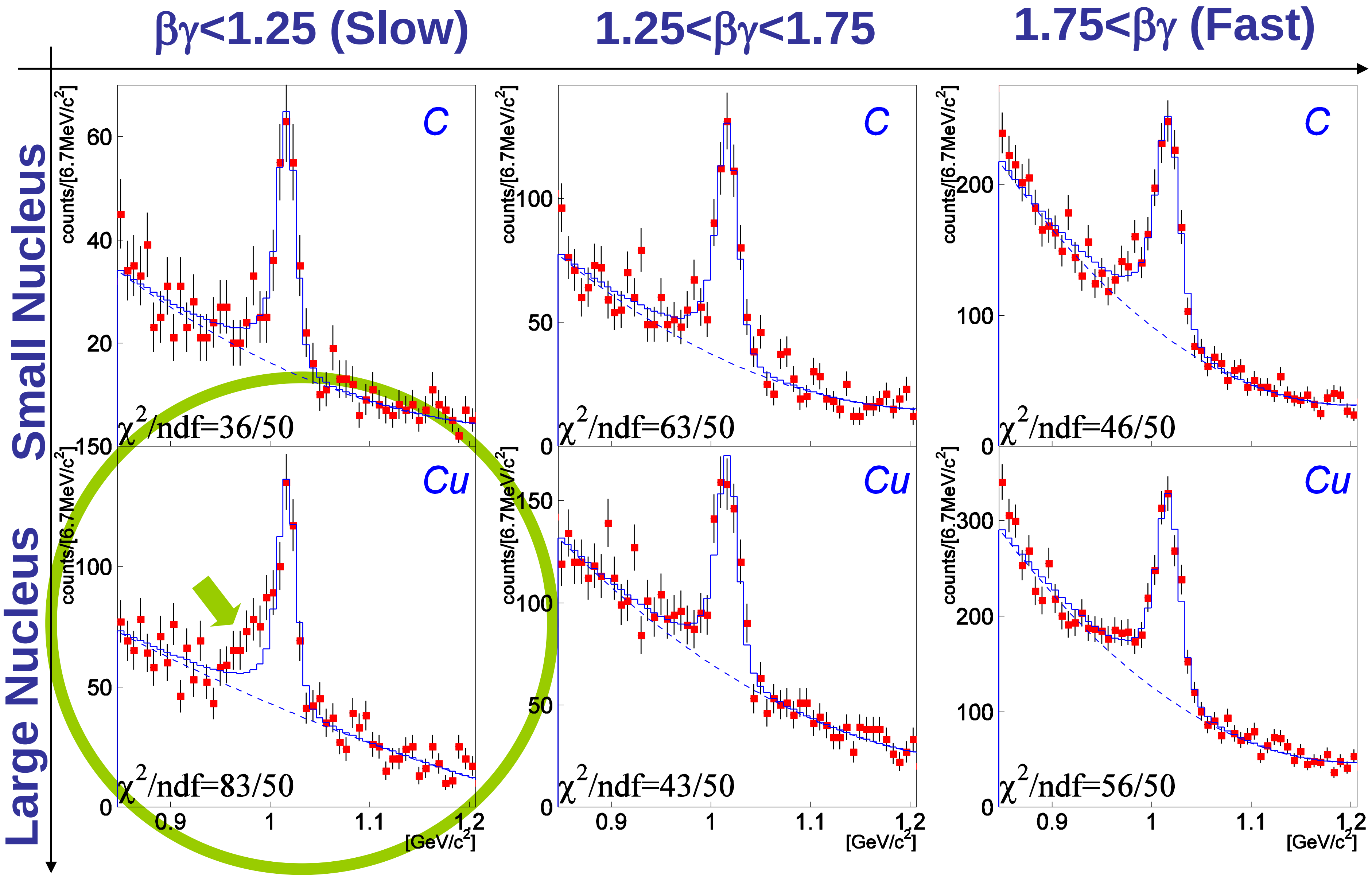
No effect
(only vacuum)

Di-lepton spectrum
reflects the modified
ϕ-meson



Y. Morino et. al. (J-PARC E16 Collaboration),
JPS Conf. Proc. 8, 022009 (2015).

Fitting Results



Experimental Conclusions

R. Muto et al, Phys. Rev. Lett. **98**, 042501 (2007).

Pole mass:

$$\frac{m_\phi(\rho)}{m_\phi(0)} = 1 - \underset{\substack{\nwarrow \\ 0.034 \pm 0.007}}{k_1}\frac{\rho}{\rho_0}$$

➡ 35 MeV negative mass shift at normal nuclear matter density

Pole width:

$$\frac{\Gamma_\phi(\rho)}{\Gamma_\phi(0)} = 1 + \underset{\substack{\nwarrow \\ 2.6 \pm 1.5}}{k_2}\frac{\rho}{\rho_0}$$

➡ Increased width to 15 MeV at normal nuclear matter density

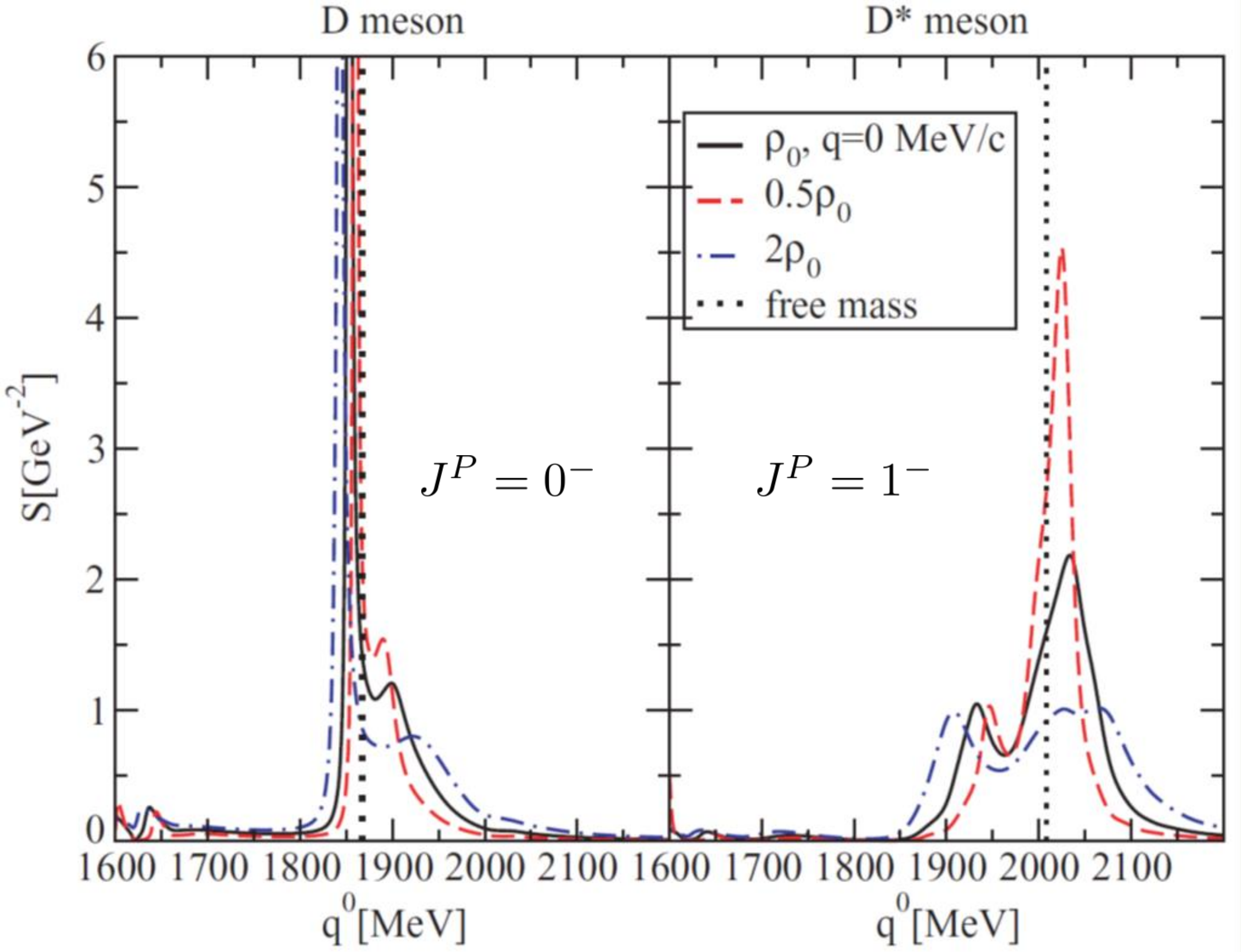
Caution!

Fit to experimental data is performed with a simple Breit-Wigner parametrization

Too simple??

Recent theoretical works about the D

based on hadronic models



Self-consistent coupled-channel unitary approach

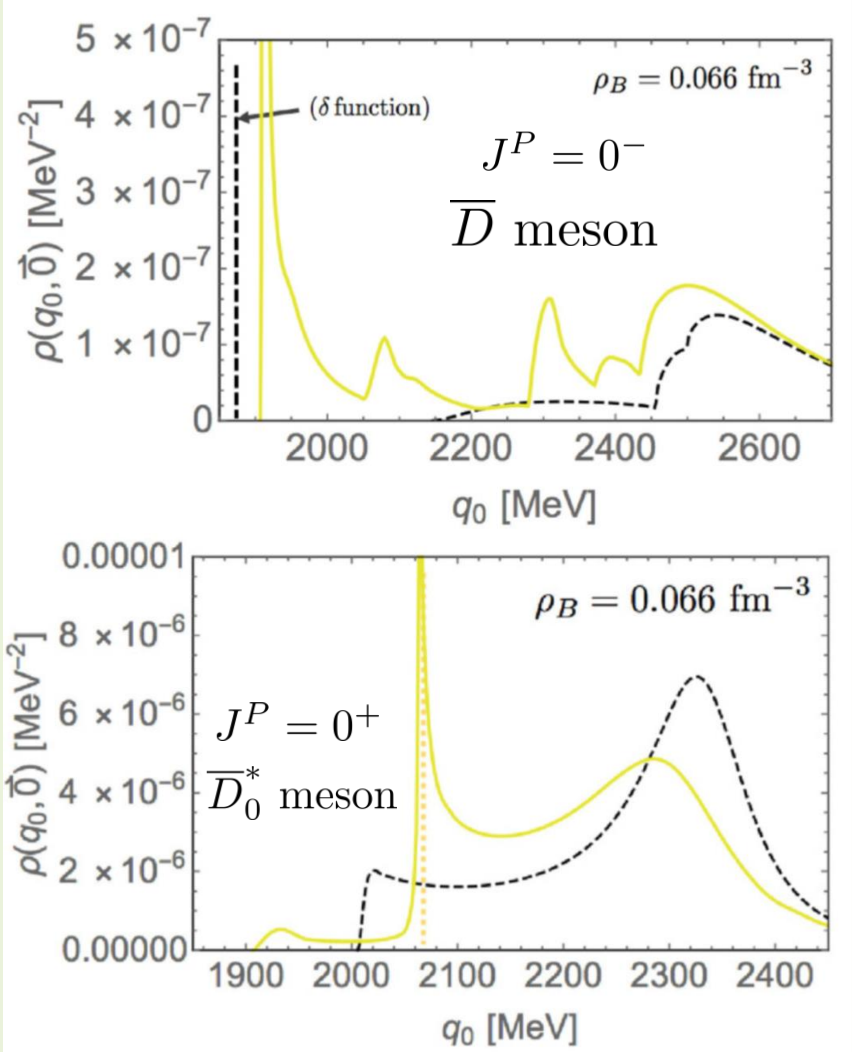
D: attractive
D*: repulsive

But:
strong broadening and additional structures appearing in the spectral function

L. Tolos, C. Garcia-Recio and J. Nieves, Phys. Rev. C **80**, 065202 (2009).

Recent theoretical works about the D

based on hadronic models



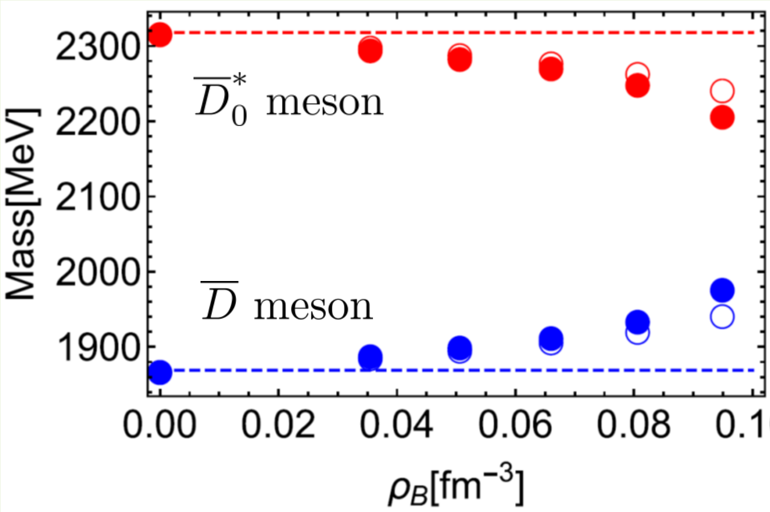
Analysis based on the linear sigma model

$\overline{\mathrm{D}}$: repulsive

chiral partners

should become degenerate for restored chiral symmetry

$\overline{\mathrm{D}}_0^*$: attractive



D. Suenaga, S. Yasui and M. Harada, Phys. Rev. C **97**, 015204 (2017).

Recent theoretical works about the D

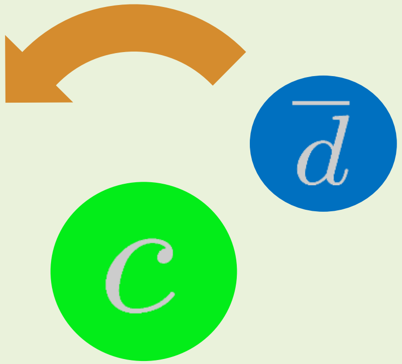
based on a non-relativistic quark model

$$E = m_c + m_q + \frac{p^2}{2m_q} + \sigma r + C$$

replace r by 1/p
and minimize E

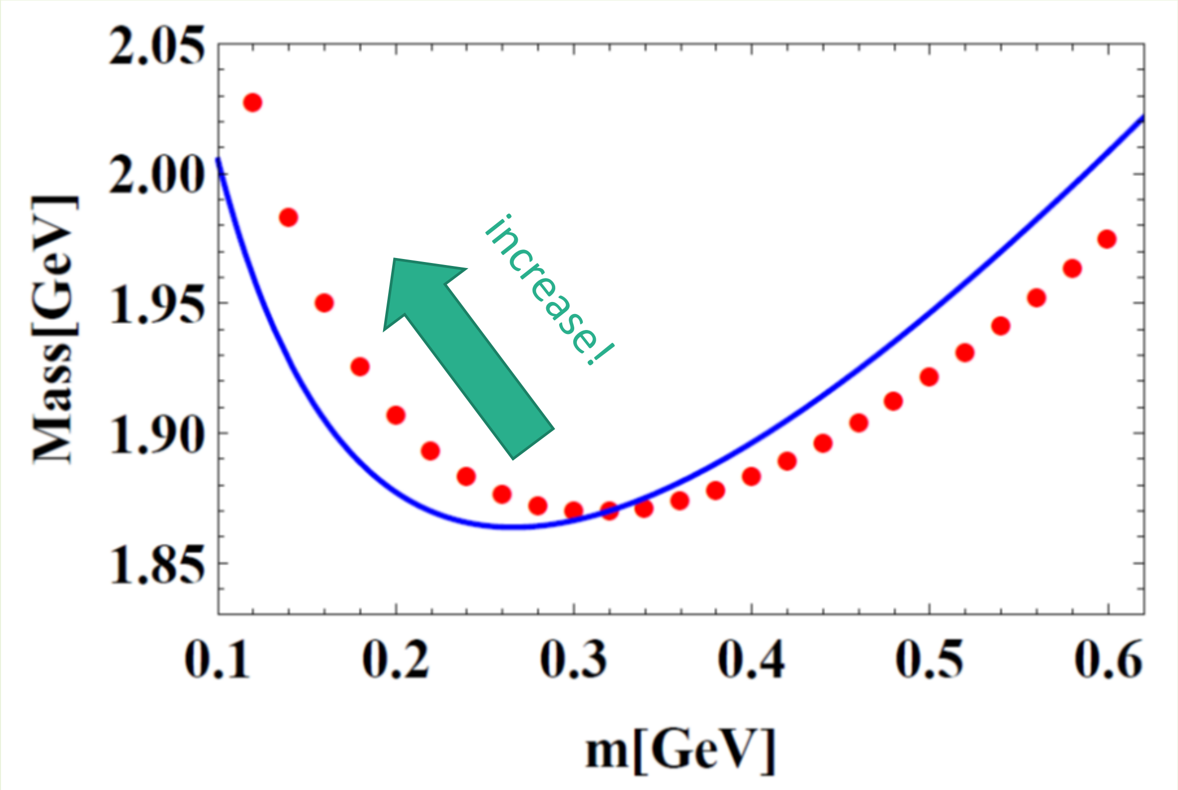
$$E_{\text{min}} = m_c + m_q + \frac{3}{2}\left(\frac{\sigma^2}{m_q}\right)^{1/3} + C$$

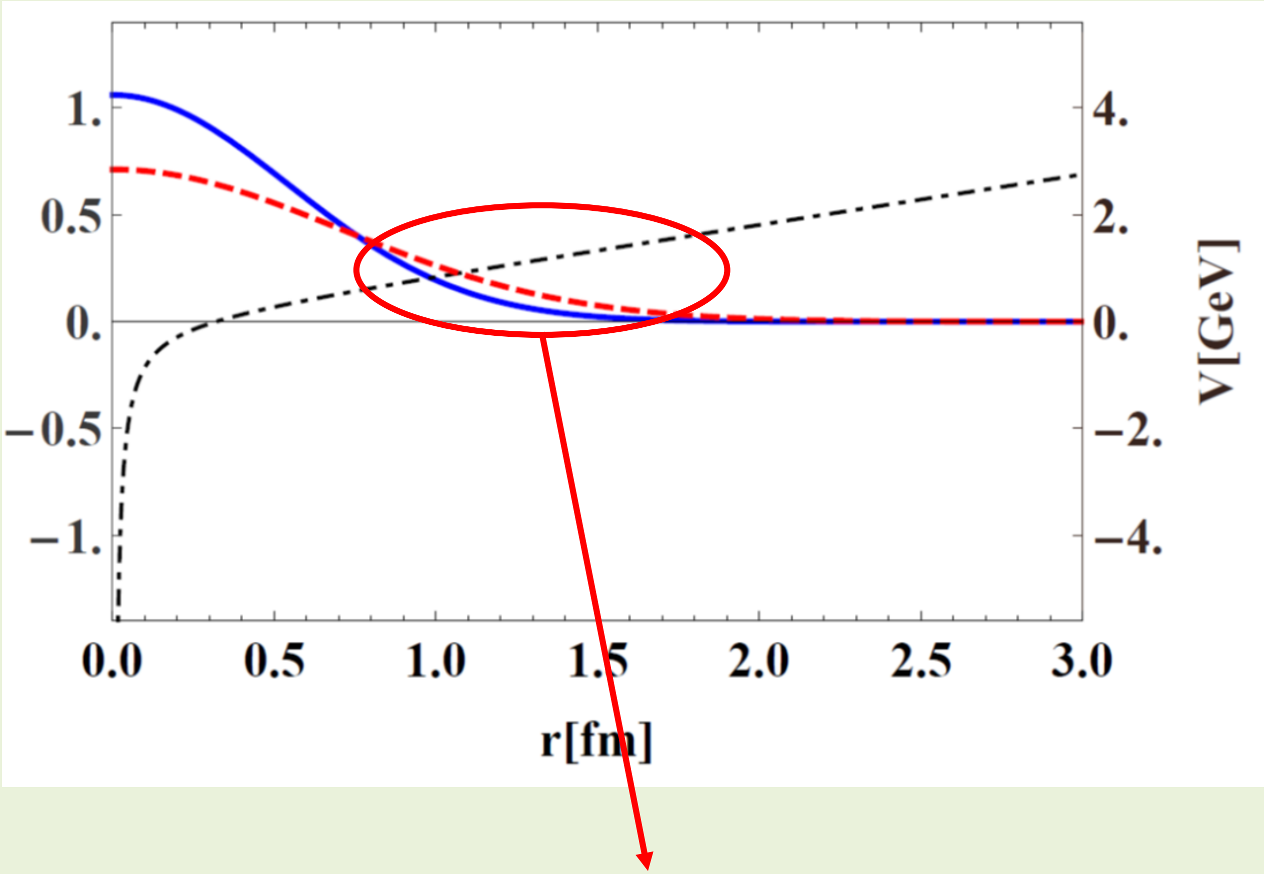
Increasing mass for sufficiently
small constituent quark masses!



A. Park, P. Gubler, M. Harada, S.H. Lee,
C. Nonaka and W. Park,
Phys. Rev. D **93**, 054035 (2016).

Comparison with more accurate quark model calculation:





Combined effect of chiral restauration and confinement!

A. Park, P. Gubler, M. Harada, S.H. Lee,
C. Nonaka and W. Park,
Phys. Rev. D **93**, 054035 (2016).

M.A. Shifman, A.I. Vainshtein and V.I. Zakharov,
Nucl. Phys. B147, 385 (1979); B147, 448 (1979).

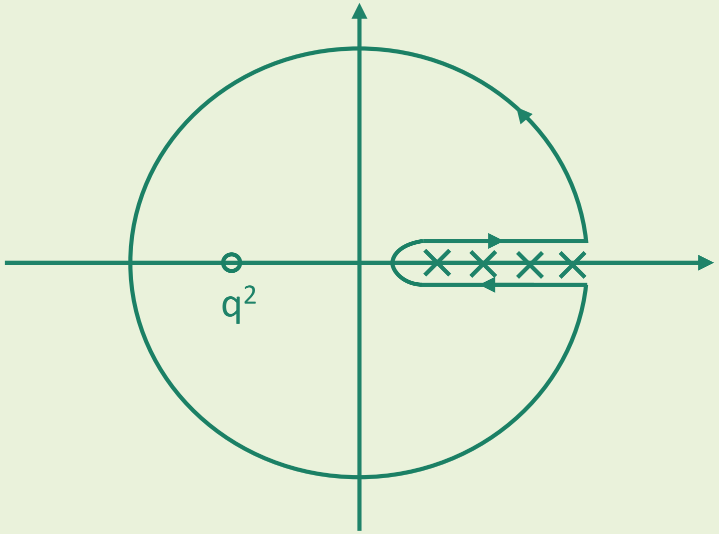
QCD sum rules

Makes use of the analytic properties of the correlation function:

$$\Pi(q^2) = i\int d^4x e^{iqx}\langle T[\chi(x)\bar{\chi}(0)]\rangle$$

$$\chi(x) = \bar{s}(x)\gamma_\mu s(x)$$

$$\chi(x) = \bar{c}(x)\gamma_5 d(x)$$



$$\to \Pi(q^2) = \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{s - q^2 - i\epsilon}$$

is calculated “perturbatively”, using OPE

spectral function of the operator χ

After the Borel transformation:

$$G_{OPE}(M^2) = \frac{1}{\pi}\int_0^\infty ds \frac{1}{M^2} e^{-\frac{s}{M^2}} \mathrm{Im}\Pi(s)$$

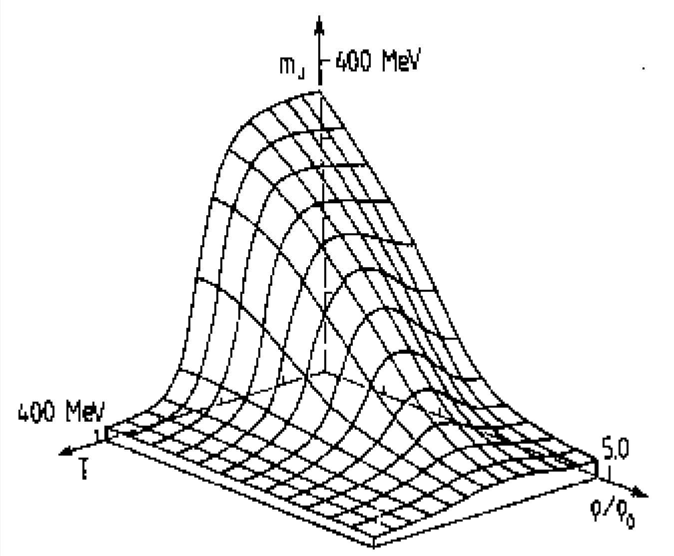
More on the operator product expansion (OPE)

perturbative Wilson coefficients

non-perturbative condensates

$$i\int d^4x e^{iqx}\langle 0|T\{\chi(x)\overline{\chi}(0)\}|0\rangle = C_I(q^2)I+\sum_n C_n(q^2)\langle 0|O_n|0\rangle$$

$$\begin{aligned}\langle 0|O_n|0\rangle = \underbrace{\begin{array}{l}\langle 0|\overline{q}q|0\rangle,\\ \langle 0|G^a_{\mu\nu}G^{a\mu\nu}|0\rangle,\\ \langle 0|\overline{q}\sigma_{\mu\nu}\frac{\lambda^a}{2}G^{a\mu\nu}q|0\rangle,\\ \langle 0|\overline{q}q\overline{q}q|0\rangle,\ldots\end{array}}_{\text{Change in hot or dense matter!}}\end{aligned}$$



Structure of QCD sum rules for the phi meson

$$\frac{1}{M^2}\int_0^\infty ds e^{-\frac{s}{M^2}}\rho(s) = c_0(\rho) + \frac{c_2(\rho)}{M^2} + \frac{c_4(\rho)}{M^4} + \frac{c_6(\rho)}{M^6} + \dots$$

In Vacuum

Dim. 0: $c_0(0) = 1 + \frac{\alpha_s}{\pi}$

Dim. 2: $c_2(0) = -6m_s^2$

Dim. 4: $c_4(0) = \frac{\pi^2}{3}\langle\frac{\alpha_s}{\pi}G^2\rangle + 8\pi^2 m_s\langle\bar{s}s\rangle$

Dim. 6: $c_6(0) = -\frac{448}{81}\kappa\pi^3\alpha_s\langle\bar{s}s\rangle^2$

Structure of QCD sum rules for the phi meson

$$\frac{1}{M^2}\int_0^\infty dse^{-\frac{s}{M^2}}\rho(s) = c_0(\rho) + \frac{c_2(\rho)}{M^2} + \frac{c_4(\rho)}{M^4} + \frac{c_6(\rho)}{M^6} + \dots$$

In Nuclear Matter

Dim. 0: $c_0(\rho) = c_0(0)$

$$\boxed{\langle\bar{s}s\rangle_\rho = \langle\bar{s}s\rangle_0 + \langle N|\bar{s}s|N\rangle\rho + \dots}$$

Dim. 2: $c_2(\rho) = c_2(0)$

Dim. 4:

$$c_4(\rho) = c_4(0) + \rho[-\frac{2}{27}M_N + \frac{56}{27}m_s\langle N|\bar{s}s|N\rangle + \frac{4}{27}m_q\langle N|\bar{q}q|N\rangle + A_2^s M_N - \frac{7}{12}\frac{\alpha_s}{\pi}A_2^g M_N]$$

Dim. 6:

$$c_6(\rho) = c_6(0) + \rho[-\frac{896}{81}\kappa_N\pi^3\alpha_s\langle\bar{s}s\rangle\langle N|\bar{s}s|N\rangle - \frac{5}{6}A_4^s M_N^3]$$

Recent results from lattice QCD

$$\sigma_{sN} = m_s \langle N | \bar{s}s | N \rangle$$

- ★ S. Durr et al. (BMW Collaboration), Phys. Rev. Lett. **116**, 172001 (2016).

  $$\sigma_{sN} = 105(41)(37)\,\mathrm{MeV}$$ (Feynman-Hellmann)

- ★ Y.-B. Yang et al. (χQCD Collaboration), Phys. Rev. D **94**, 054503 (2016).

  $$\sigma_{sN} = 40.2(11.7)(3.5)\,\mathrm{MeV}$$ (Direct)

- ★ A. Abdel-Rehim et al. (ETM Collaboration), Phys. Rev. Lett. **116**, 252001 (2016).

  $$\sigma_{sN} = 41.05(8.2)\left({}^{+1.09}_{-0.69}\right)\mathrm{MeV}$$ (Direct)

- ★ G.S. Bali et al. (RQCD Collaboration), Phys. Rev. D **93**, 094504 (2016).

  $$\sigma_{sN} = 35(12)\,\mathrm{MeV}$$ (Direct)

- ★ N. Yamanaka et al. (JLQCD Collaboration), arXiv:1805.10507 [hep-lat].

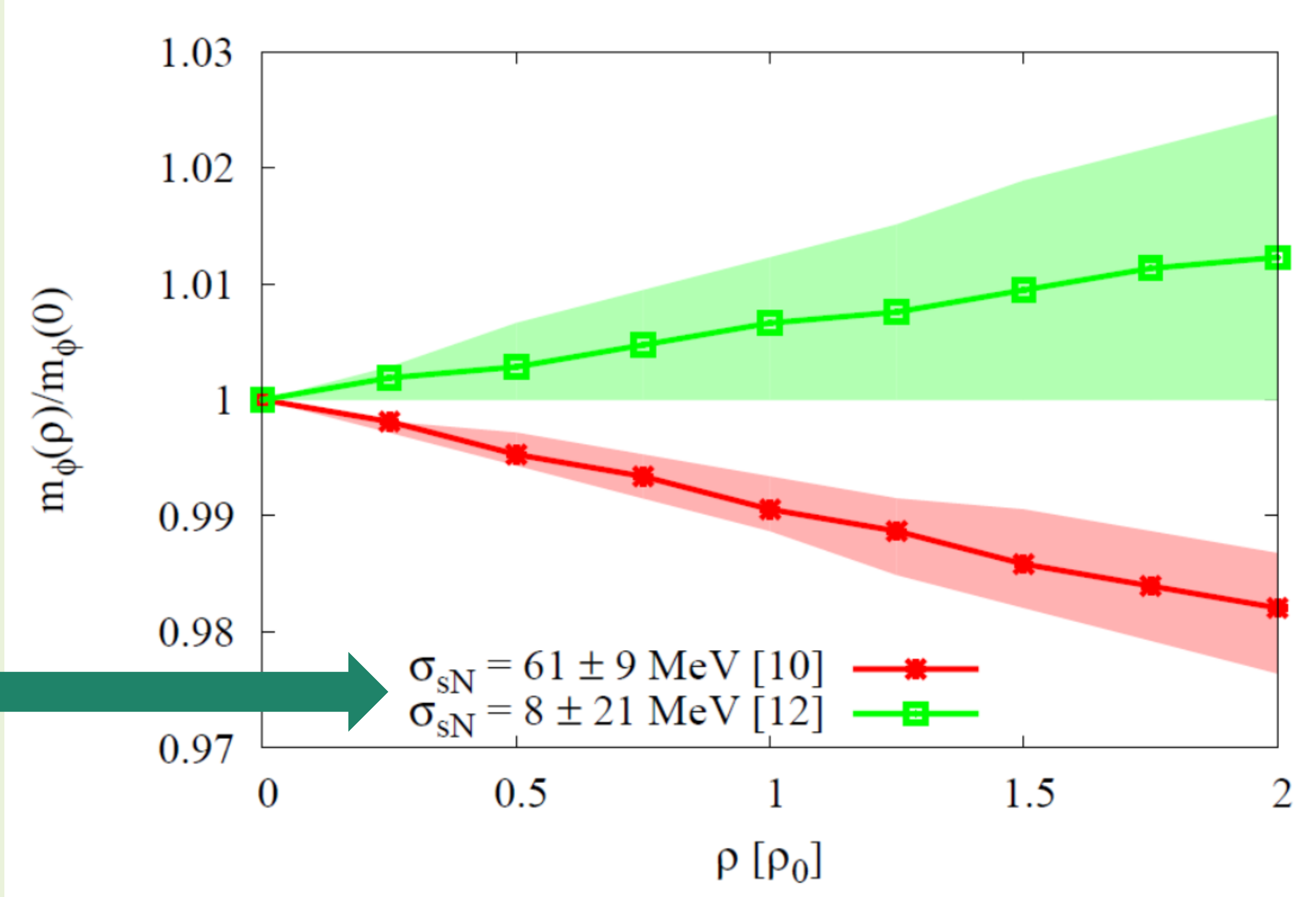
  $$\sigma_{sN} = 17(18)(9)\,\mathrm{MeV}$$ (Direct)

Results for the ϕ meson mass

Most important parameter, that determines the behavior of the ϕ meson mass at finite density:

Strangeness content of the nucleon

$$\sigma_{sN} = m_s \langle N | \bar{s}s | N \rangle$$



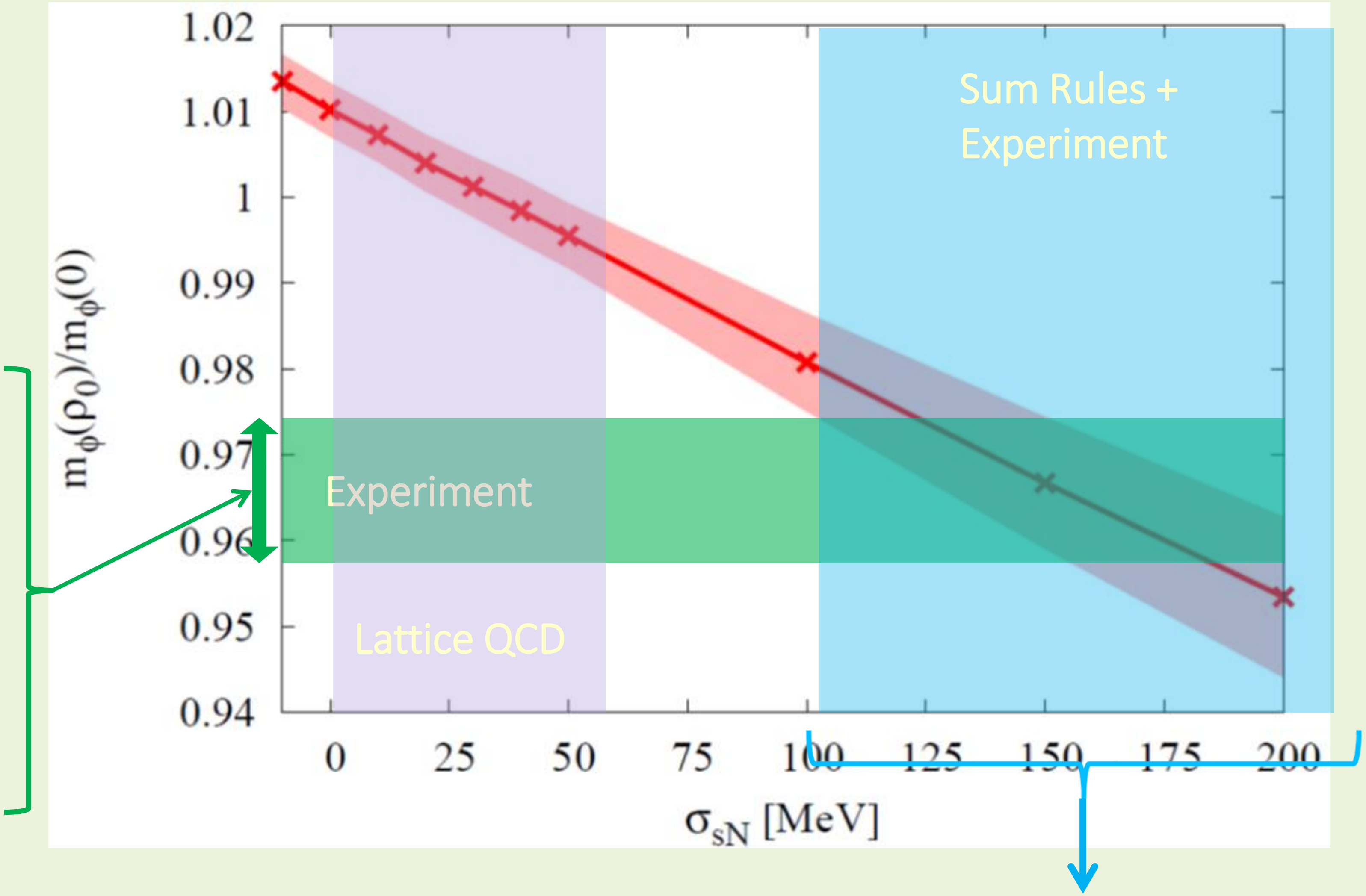
P. Gubler and K. Ohtani, Phys. Rev. D **90**, 094002 (2014).

Compare Theory with Experiment

Not consistent?

$$\frac{m_\phi(\rho)}{m_\phi(0)} = 0.966 \pm 0.007$$

Will soon be measured again with better statistics at the E16 experiment at J-PARC!



$$\sigma_{sN} \sim 160 \pm 50 \text{ MeV}$$

What about the chiral condensate?

At finite density:

$$\begin{aligned}\langle\rho|\bar{q}q|\rho\rangle &= \langle 0|\bar{q}q|0\rangle + \langle N|\bar{q}q|N\rangle\rho + \dots \\ &= \langle 0|\bar{q}q|0\rangle + \frac{1}{2m_q}\sigma_{\pi N}\rho + \dots\end{aligned}$$

πN-σ term
(value still not well known)

Newest fit to πN scattering data

M. Hoferichter, J. Ruiz de Elvira, B. Kubis and U.-G. Meißner,
Phys. Rev. Lett. **115**, 092301 (2015).

$$\sigma_{\pi N} = 59.1 \pm 3.6\,\text{MeV}$$

Recent lattice QCD determination at physical quark masses

★ S. Durr *et al.*, (BMW Collaboration), Phys. Rev. Lett. **116**, 172001 (2016).

$$\sigma_{\pi N} = 38(3)(3)\,\text{MeV}$$

★ N. Yamanaka et al. (JLQCD Collaboration), arXiv:1805.10507 [hep-lat].

$$\sigma_{\pi N} = 26(3)(^{+8}_{-5})(2)\,\text{MeV}$$

D-meson in nuclear matter

The sum rules we use:

$D^{\pm}$

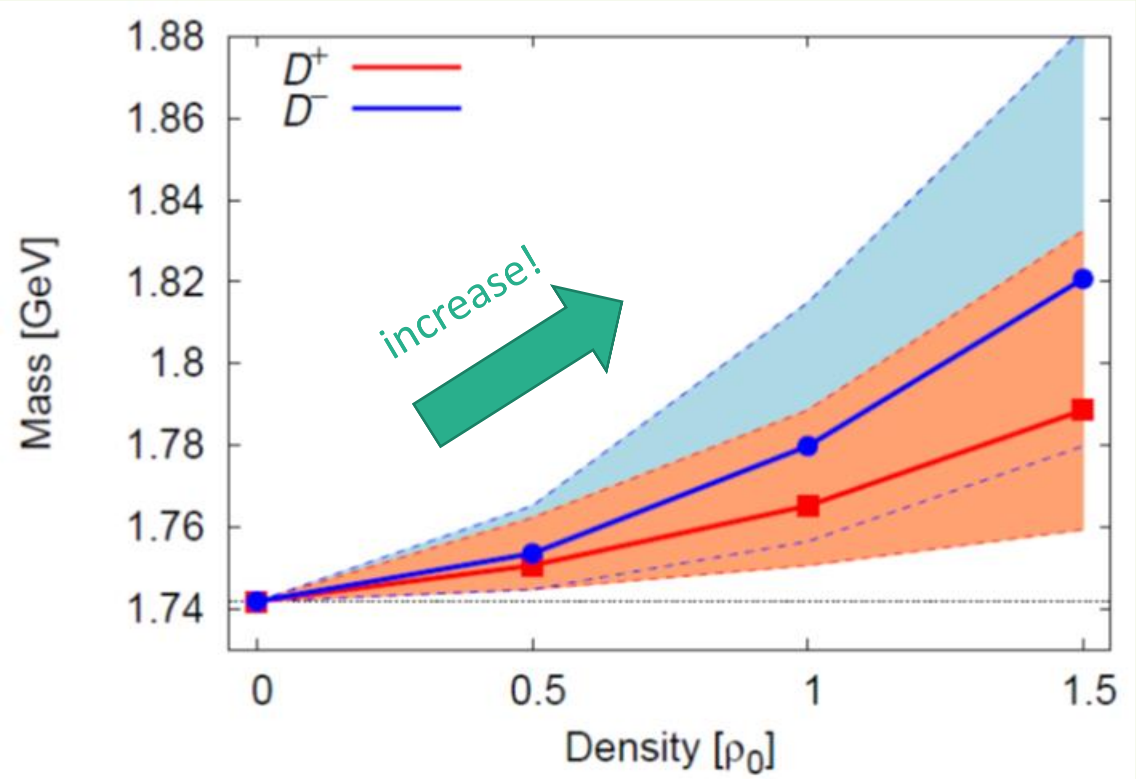
$W(\omega, \hat{s}, \tau) = \frac{\omega}{\sqrt{4\pi\tau}} e^{-(\omega^2-\hat{s})^2/4\tau}$

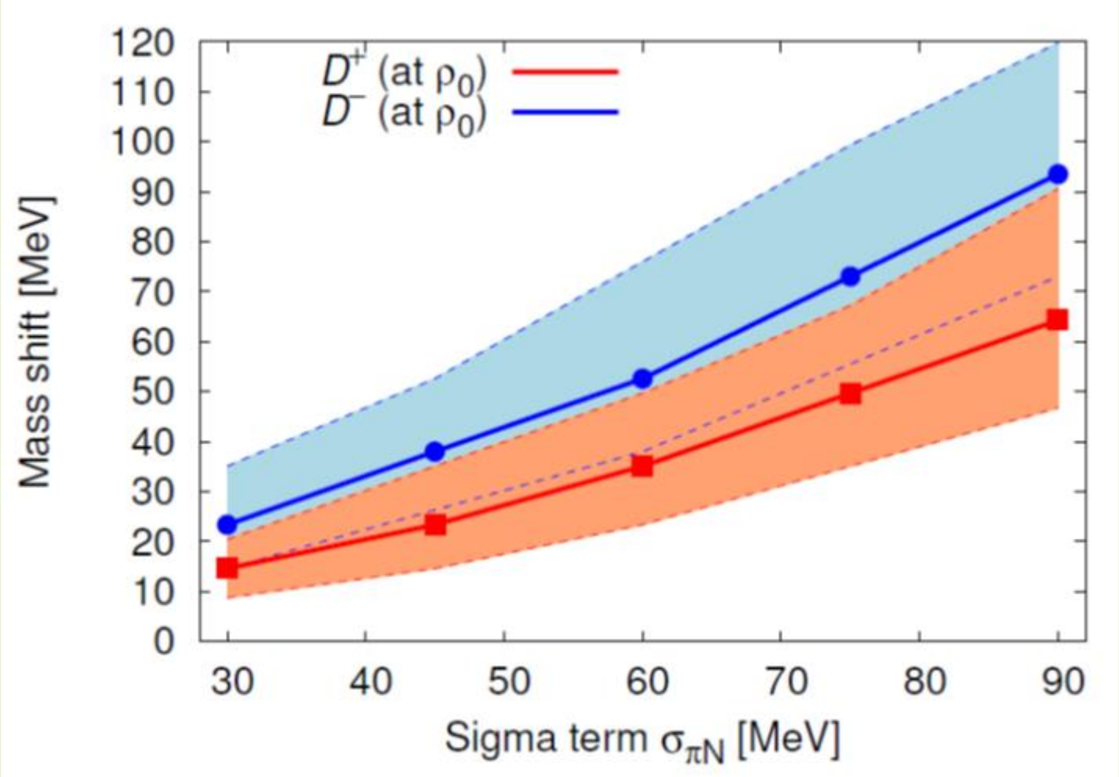
$$\int_0^\infty d\omega\, \rho^{\pm}(\omega)\, W(\omega, \hat{s}, \tau) = \tilde{G}^{\text{even}}(\hat{s}, \tau) \pm \tilde{G}^{\text{odd}}(\hat{s}, \tau)$$

$$\begin{aligned}
\tilde{G}^{\text{even}}(\hat{s}, \tau) = {} & \frac{1}{2\sqrt{4\pi\tau}} \frac{1}{\pi} \int_{m_h^2}^\infty \frac{ds}{2} e^{-\frac{(s-\hat{s})^2}{4\tau}} \text{Im}\Pi^{\text{pert}}(s) \\
& + \frac{1}{2\sqrt{4\pi\tau}} e^{-\frac{(m_h^2-\hat{s})^2}{4\tau}} \Bigl[\boxed{-m_h \langle \bar{q}q \rangle} + \frac{1}{12} \langle \frac{\alpha}{\pi} G^2 \rangle - \frac{1}{2} \left(\frac{3m_h^2 - 2\hat{s}}{4\tau} - \frac{2(m_h^2-\hat{s})^2 m_h^2}{(4\tau)^2} \right) m_h \langle \bar{q} g \sigma G q \rangle \\
& \quad + \left\{ \frac{1}{9} - \frac{5m_h^2}{36\tau}(m_h^2 - \hat{s}) + \left(-\frac{1}{3} + \frac{m_h^2(m_h^2-\hat{s})}{6\tau} \right) \ln \frac{\mu^2}{4m_h^2} \right\} \langle \frac{\alpha_s}{\pi} \left(\frac{(vG)^2}{v^2} - \frac{G^2}{4} \right) \rangle \\
& \quad - 2 \left(1 - \frac{(m_h^2-\hat{s}) m_h^2}{2\tau} \right) \langle q^\dagger i \overrightarrow{D}_0 q \rangle \\
& \quad - 4 \left(\frac{3m_h^2 - 2\hat{s}}{4\tau} - \frac{2(m_h^2-\hat{s})^2 m_h^2}{(4\tau)^2} \right) m_h \left[\langle \bar{q} \overrightarrow{D}_0^2 q \rangle - \langle \frac{1}{8} \bar{q} g \sigma G q \rangle \right] \Bigr] \\
& + \frac{1}{2\sqrt{4\pi\tau}} \int_0^\infty dy e^{-\frac{[m_h^2(1+y)^2-\hat{s}]^2}{4\tau}} \Bigl\{ -\frac{1}{3} \frac{(1+y)^2}{(2+y)^2} - \frac{\ln y}{3\tau^2} \bigl[m_h^8(1+y)^7 - 2m_h^6 \hat{s}(1+y)^5 + m_h^4(1+y)^3(\hat{s}^2 - (6+y)\tau) \\
& \quad + m_h^2 \hat{s}(4 + 5y + y^2)\tau + \tau^2 \bigr] \Bigr\} \times \langle \frac{\alpha_s}{\pi} \left(\frac{(vG)^2}{v^2} - \frac{G^2}{4} \right) \rangle,
\end{aligned}$$

$$\begin{aligned}
\tilde{G}^{\text{odd}}(\hat{s}, \tau) = {} & \frac{1}{2\sqrt{4\pi\tau}} e^{-\frac{(m_h^2-\hat{s})^2}{4\tau}} \Bigl[\boxed{m_h \langle q^\dagger q \rangle} + 4 \left(-\frac{3}{8m_h} + \frac{(4m_h^2 - 3\hat{s}) m_h}{4\tau} - \frac{2(m_h^2-\hat{s})^2 m_h^3}{(4\tau)^2} \right) \langle q^\dagger \overrightarrow{D}_0^2 q \rangle \\
& - \left(-\frac{1}{2m_h} + \frac{(m_h^2-\hat{s}) m_h}{2\tau} \right) \langle q^\dagger g \sigma G q \rangle \Bigr].
\end{aligned}$$

Results





$$\sigma_{\pi N} = 45 \pm 15\,\text{MeV}$$

To be measured at the CBM (Compressed Baryon Matter) experiment at FAIR, GSI?

And/or at J-PARC??

K. Suzuki, P. Gubler and M. Oka, Phys. Rev. C **93**, 045209 (2016).

Summary and Conclusions

★ In hadronic models, meson spectra are typically modified in a complicated manner: broadening, mass shifts, additional peaks

★ The ϕ-meson mass shift in nuclear matter constrains the strangeness content of the nucleon:

$\sigma_{sN} < 35$ MeV ↗ Increasing ϕ-meson mass in nuclear matter

$\sigma_{sN} > 35$ MeV ↘ Decreasing ϕ-meson mass in nuclear matter

★ QCD sum rule calculations suggest that the D meson mass increases with increasing density:

m_D ↗ most important parameter: $\sigma_{\pi N}$

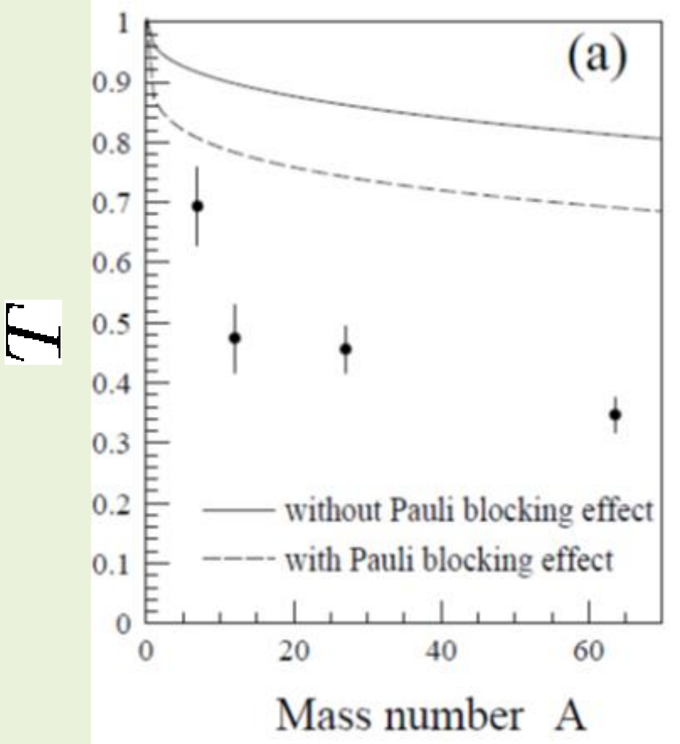
Backup slides

Other experimental results

There are some more experimental results on the ϕ-meson width in nuclear matter, based on the measurement of the transparency ratio T:

$$T = \frac{\sigma_{\gamma A\to\phi X}}{A\sigma_{\gamma N\to\phi X}}$$

Measured at SPring-8 (LEPS)



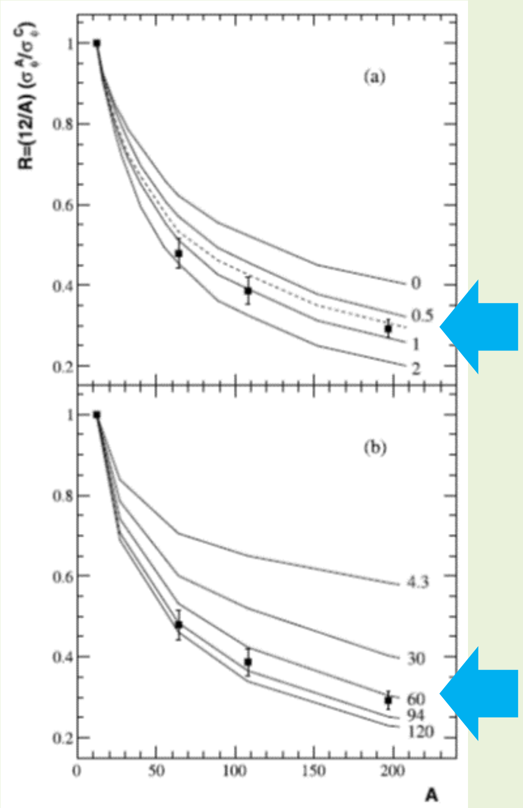
$\Gamma_\phi(\rho_0) \simeq 30\,\mathrm{MeV}$

Theoretical calculation:

D. Cabrera, L. Roca, E. Oset , H. Toki and M.J. Vicente Vacas, Nucl. Phys. **A733**, 130 (2004).

T. Ishikawa et al, Phys. Lett. B **608**, 215 (2005).

Measured at COSY-ANKE



Theoretical calculation:

V.K. Magas, L. Roca and E. Oset, Phys. Rev. C **71**, 065202 (2005).

$\Gamma_\phi(\rho_0) \simeq 27\,\mathrm{MeV}$

Theoretical calculation:

E. Ya. Paryev, J. Phys. G **36**, 015103 (2009).

$\Gamma_\phi(\rho_0) \simeq 73\,\mathrm{MeV}$

A. Polyanskiy et al, Phys. Lett. B **695**, 74 (2011).

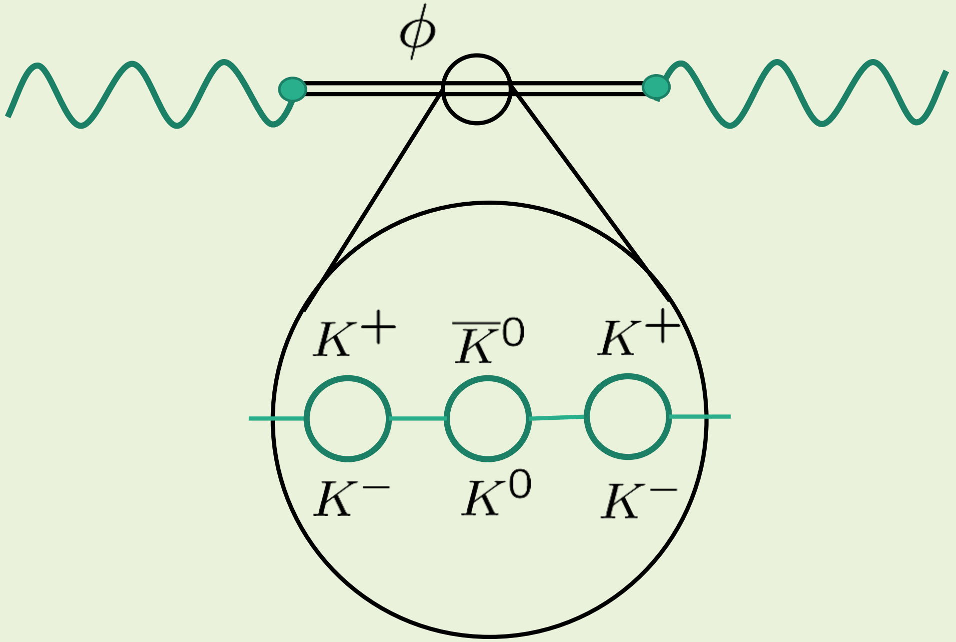
Starting point

$$j_\mu(x) = \frac{1}{3}\overline{s}(x)\gamma_\mu s(x)$$

$$\Pi_{\mu\nu}(q) = i\int d^4x e^{iqx}\langle T[j_\mu(x)j_\nu(0)]\rangle_\rho$$

$$\Pi(q^2) = \frac{1}{3q^2}\Pi^\mu_\mu(q)$$

Rewrite using hadronic degrees of freedom
(vector dominance model)



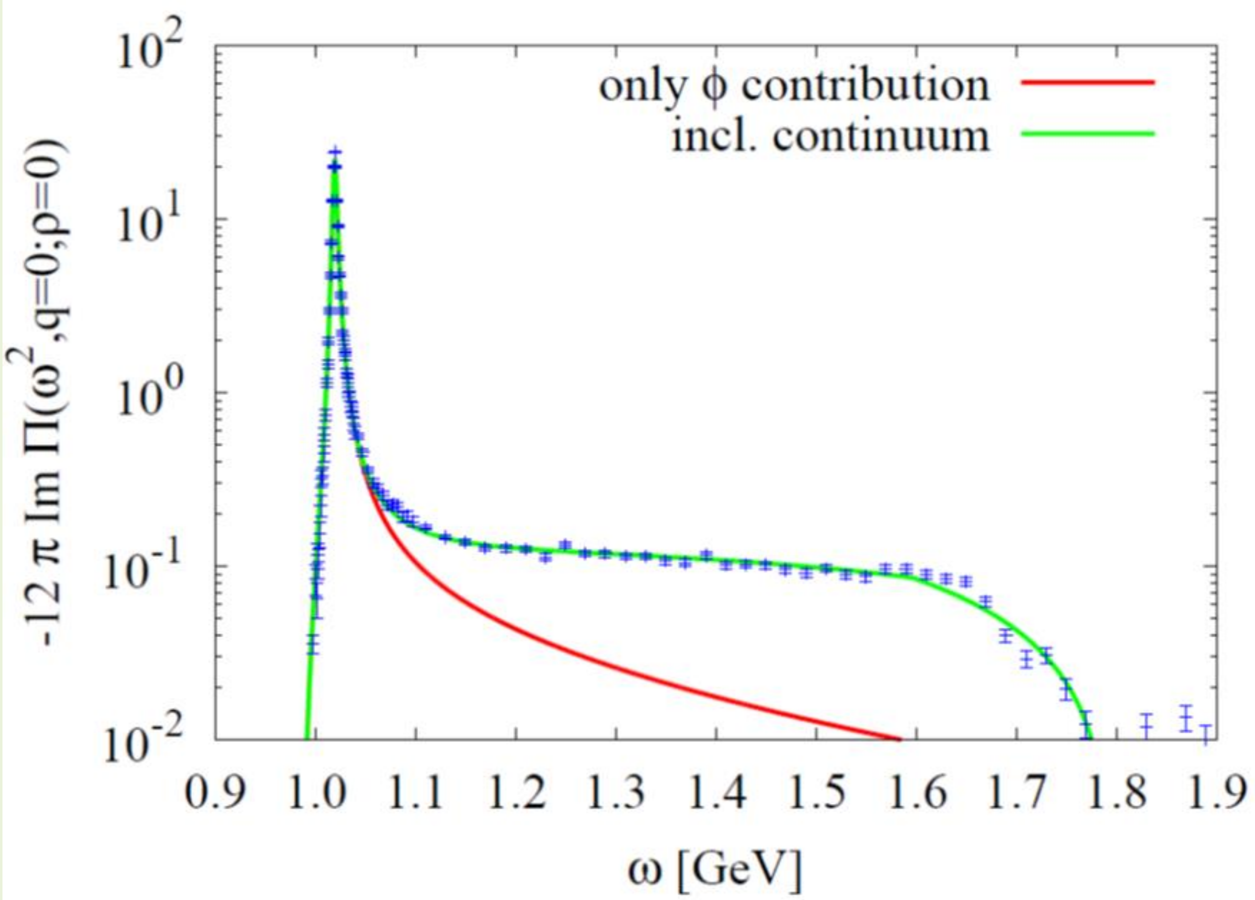
$$\mathrm{Im}\Pi(q^2) = \frac{\mathrm{Im}\Pi_\phi(q^2)}{q^2 g_\phi^2}\left|\frac{(1-a_\phi)q^2-\mathring{m}_\phi^2}{q^2-\mathring{m}_\phi^2-\Pi_\phi(q^2)}\right|^2$$

Kaon loops

Vacuum spectrum

$$\frac{\sigma(e^+e^-\to K^+K^-)}{\sigma(e^+e^-\to\mu^+\mu^-)}$$

(Vacuum)



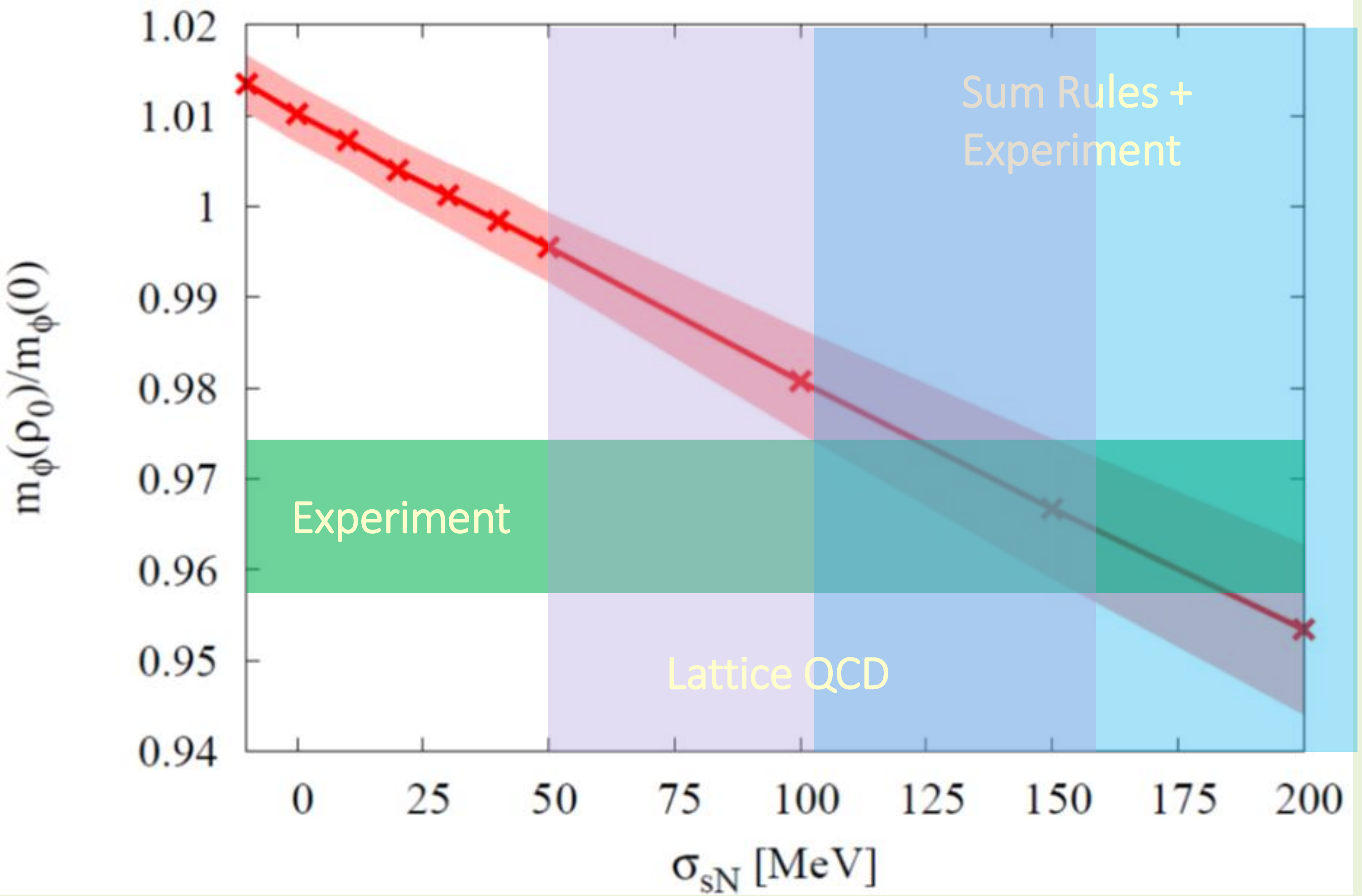
P. Gubler and W. Weise, Phys. Lett. B **751**, 396 (2015).

How is this spectrum modified in nuclear matter?

Is the (modified) spectral function consistent with QCD sum rules?

Data from
J.P. Lees et al. (BABAR Collaboration), Phys. Rev. D **88**, 032013 (2013).

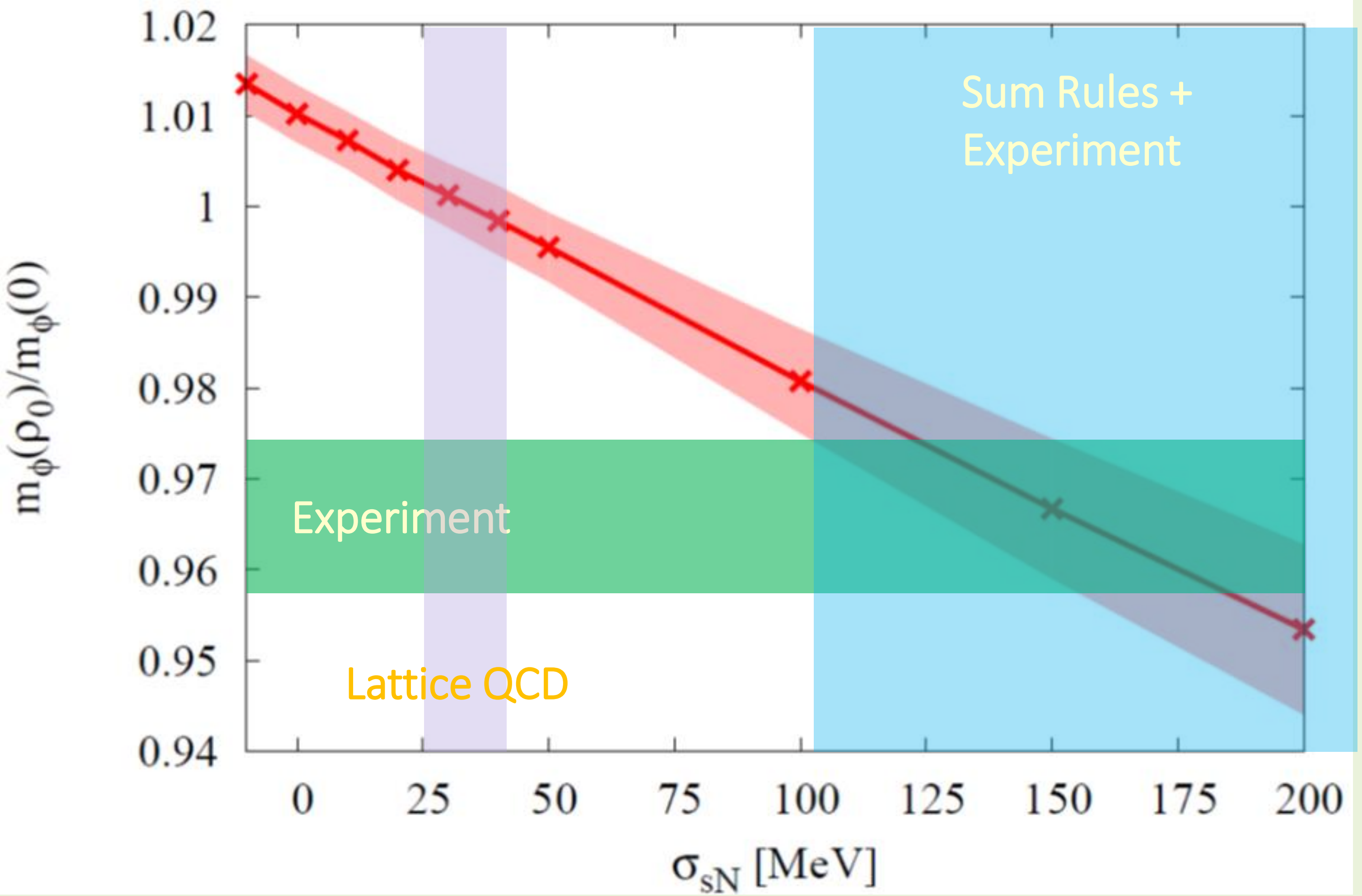
BMW version:



1.02
1.01
1
0.99
0.98
0.97
0.96
0.95
0.94
$m_\phi(\rho_0)/m_\phi(0)$
0
25
50
75
100
125
150
175
200
σ_{sN} [MeV]
Sum Rules +
Experiment
Experiment
Lattice QCD

?

χQCD version:



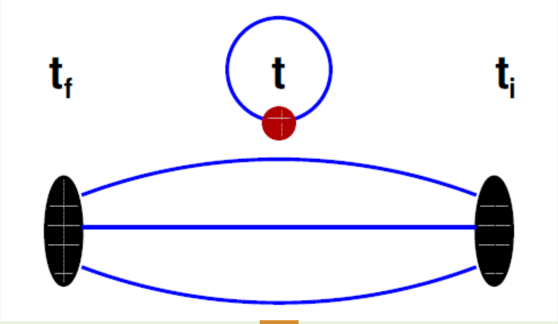
??

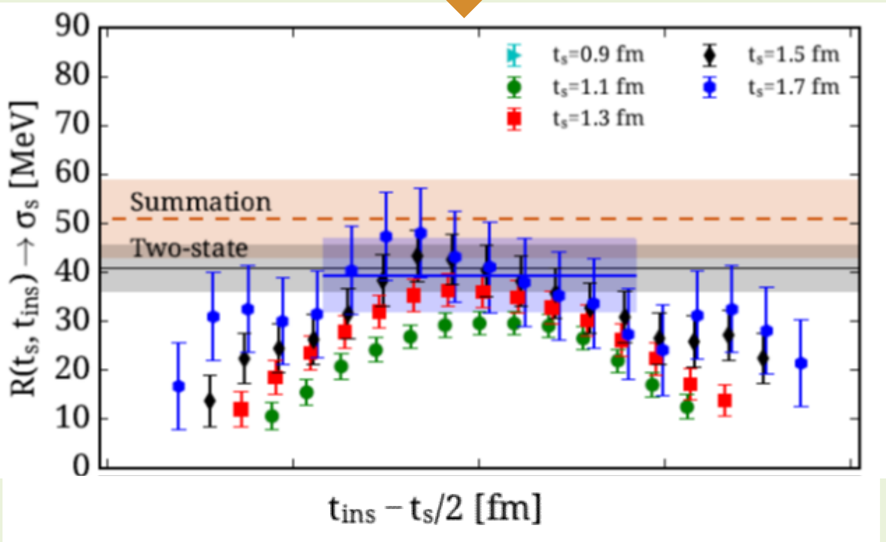
The strangeness content of the nucleon: results from lattice QCD

$$\sigma_{sN} = m_s \langle N | \bar{s}s | N \rangle$$

Two methods

Direct measurement

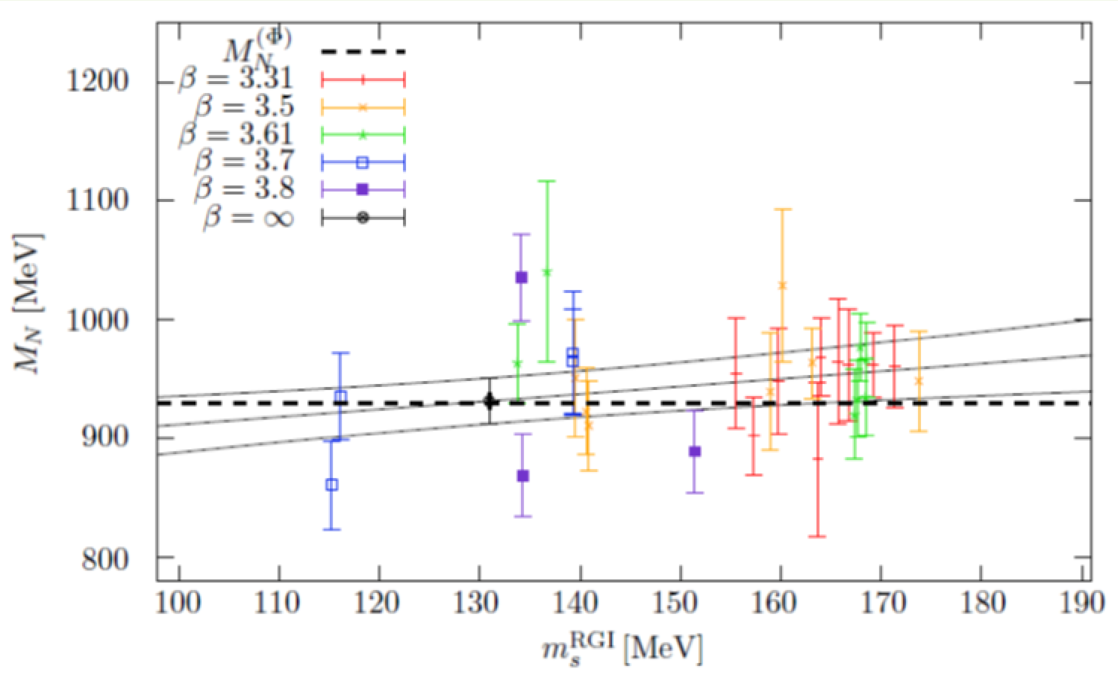




A. Abdel-Rehim et al. (ETM Collaboration), Phys. Rev. Lett. **116**, 252001 (2016).

Feynman-Hellmann theorem

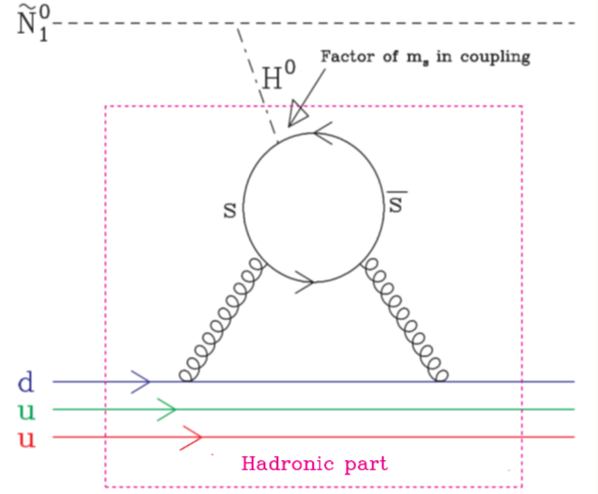
$$\sigma_{sN} = m_s \frac{\partial m_N}{\partial m_s}$$



S. Durr et al. (BMW Collaboration), Phys. Rev. Lett. **116**, 172001 (2016).

The strangeness content of the nucleon: $\sigma_{sN} = m_s \langle N | \bar{s}s | N \rangle$

Important parameter for dark-matter searches:



Neutralino:
Linear superposition of the Super-partners of the Higgs, the photon and the Z-boson

Adapted from:
W. Freeman and D. Toussaint
(MILC Collaboration),
Phys. Rev. D **88**, 054503 (2013).

$$\sigma^{(\mathrm{nucleon})}_{\mathrm{scalar}} = \frac{8G_F^2}{\pi} M_Z^2 m_{\mathrm{red}}^2 \left[\frac{F_h I_h}{m_h^2} + \frac{F_H I_H}{m_H^2} + \frac{M_Z}{2} \sum_q < N|\bar{q}q|N > \sum_i P_{\tilde{q}_i} (A_{\tilde{q}_i}^2 - B_{\tilde{q}_i}^2) \right]^2$$

most important contribution

$$I_{h,H} = k^{h,H}_{u-\mathrm{type}} g_u + k^{h,H}_{d-\mathrm{type}} g_d$$

dominates

$$g_d = \frac{2}{27} \left(m_N + \frac{23}{4} \sigma_{\pi N} + \frac{25}{2} \sigma_{sN} \right)$$

A. Bottino, F. Donato, N. Fornengo and S. Scopel, Asropart. Phys. **18**, 205 (2002).

Problem at finite ρ: sign problem!

$$Z = \int DA \, \mathrm{det}[\not{D} + m - \mu\gamma_0/2] e^{S_{\mathrm{YM}}}$$

Dirac operator   mass matrix   chemical potential

$$(\mathrm{det}[\not{D} + m - \mu\gamma_0/2])^* = \mathrm{det}[\not{D} + m + \mu^*\gamma_0/2]$$

The determinant is complex

$$\mathrm{det}[\not{D} + m - \mu\gamma_0/2] = |\,\mathrm{det}[\not{D} + m - \mu\gamma_0/2]|e^{i\theta}$$

Standard Monte-Carlo integration is essentially impossible

The basic problem to be solved

$$G_{OPE}(M) = \frac{1}{M^2}\int_0^\infty ds e^{-\frac{s}{M^2}}\rho(s)$$

given
(but only incomplete and
with error)

“Kernel”

?

This is an ill-posed problem.

But, one may have additional information on ρ(ω),
which can help to constrain the problem:

- Positivity: $\rho(\omega) \geq 0$
- Asymptotic values: $\rho(\omega = 0), \rho(\omega = \infty)$

The Maximum Entropy Method

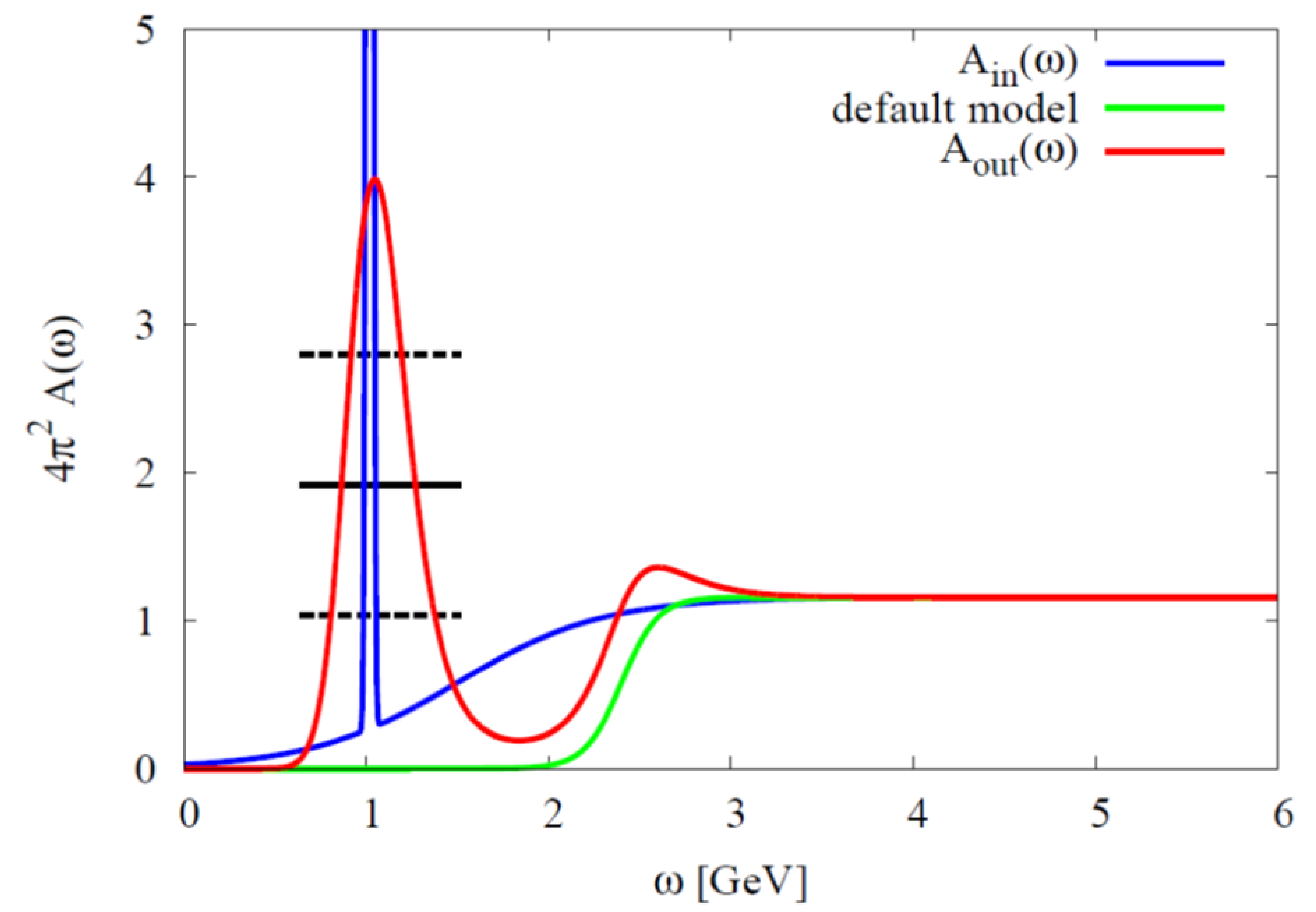
How can one include this additional information and find the most probable image of ρ(ω)?

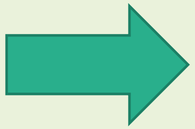
→ Bayes’ Theorem

$$P[\rho|G,I] = \frac{\underbrace{P[G|\rho,I]}_{\text{likelihood function}}\underbrace{P[\rho|I]}_{\text{prior probability}}}{P[G|I]}$$

$$\rightarrow \quad \frac{\delta P[\rho|G,I]}{\delta\rho} = 0$$

Results of test-analysis (using MEM)





Peak position can be extracted, but not the width!

P. Gubler and K. Ohtani, Phys. Rev. D **90**, 094002 (2014).

$$P[\rho|G,I] = \frac{P[G|\rho,I]P[\rho|I]}{P[G|I]}$$

likelihood function

$$P[G|\rho,I] = \frac{1}{Z_L}e^{-L[\rho]}$$

$$L[\rho] = \frac{1}{2(M_{\text{max}}-M_{\text{min}})}\int_{M_{\text{min}}}^{M_{\text{max}}} dM \frac{\left[G_{OPE}(M) - G_\rho(M)\right]^2}{\sigma^2(M)}$$

Corresponds to
ordinary χ^2-fitting.

prior probability

$$P[\rho|I] = \frac{1}{Z_s}e^{\alpha S[\rho]}$$

$$S[\rho] = \int_0^\infty d\omega\left[\rho(\omega) - m(\omega) - \rho(\omega)\log\left(\frac{\rho(\omega)}{m(\omega)}\right)\right]$$

(Shannon-Jaynes entropy)

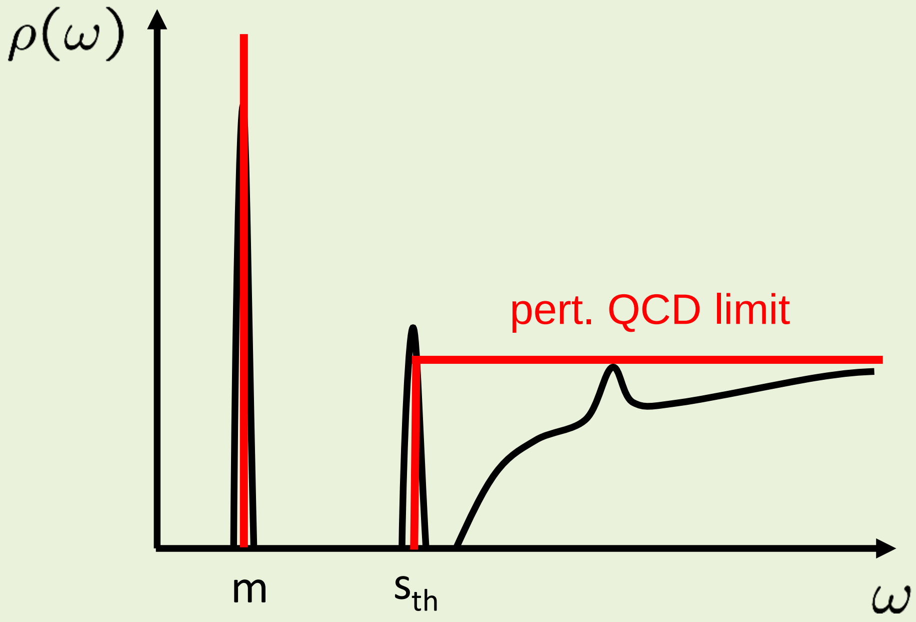
“default model”

M. Jarrel and J.E. Gubernatis, Phys. Rep. 269, 133 (1996).

M.Asakawa, T.Hatsuda and Y.Nakahara, Prog. Part. Nucl. Phys. 46, 459 (2001).

The traditional analysis method

The spectral function is approximated by a “pole + continuum” ansatz:



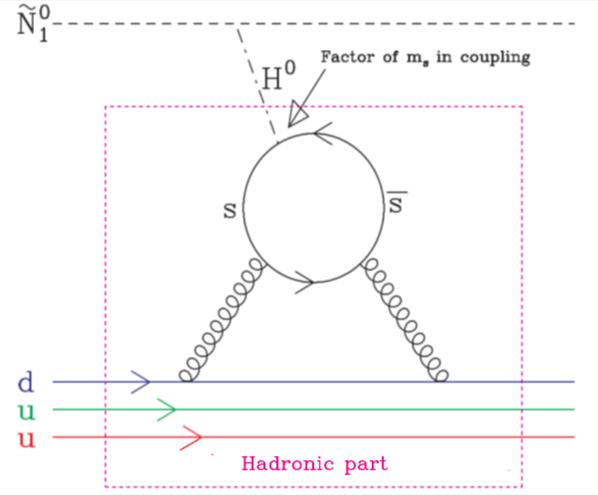
$$\rho(s) = \lambda^2\delta(s-m^2) + \theta(s-s_{\text{th}})\frac{1}{\pi}\text{Im}\Pi^{OPE}(s)$$

Even though this ansatz is very crude, it works quite well in cases for which it is phenomenologically known to be close to reality.

e.g.   -charmonium (J/ψ)

The strangeness content of the nucleon: $\langle N|\bar{s}s|N\rangle$

Important parameter for dark-matter searches:



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$$I_{h,H} = k_{u-{\rm type}}^{h,H} g_u + k_{d-{\rm type}}^{h,H} g_d$$

dominates

$$g_d = \frac{2}{27}\left(m_N + \frac{23}{4}\sigma_{\pi N} + \frac{25}{2}\sigma_{sN}\right)$$

A. Bottino, F. Donato, N. Fornengo and S. Scopel, Asropart. Phys. **18**, 205 (2002).

In-nucleus decay fractions for E325 kinematics

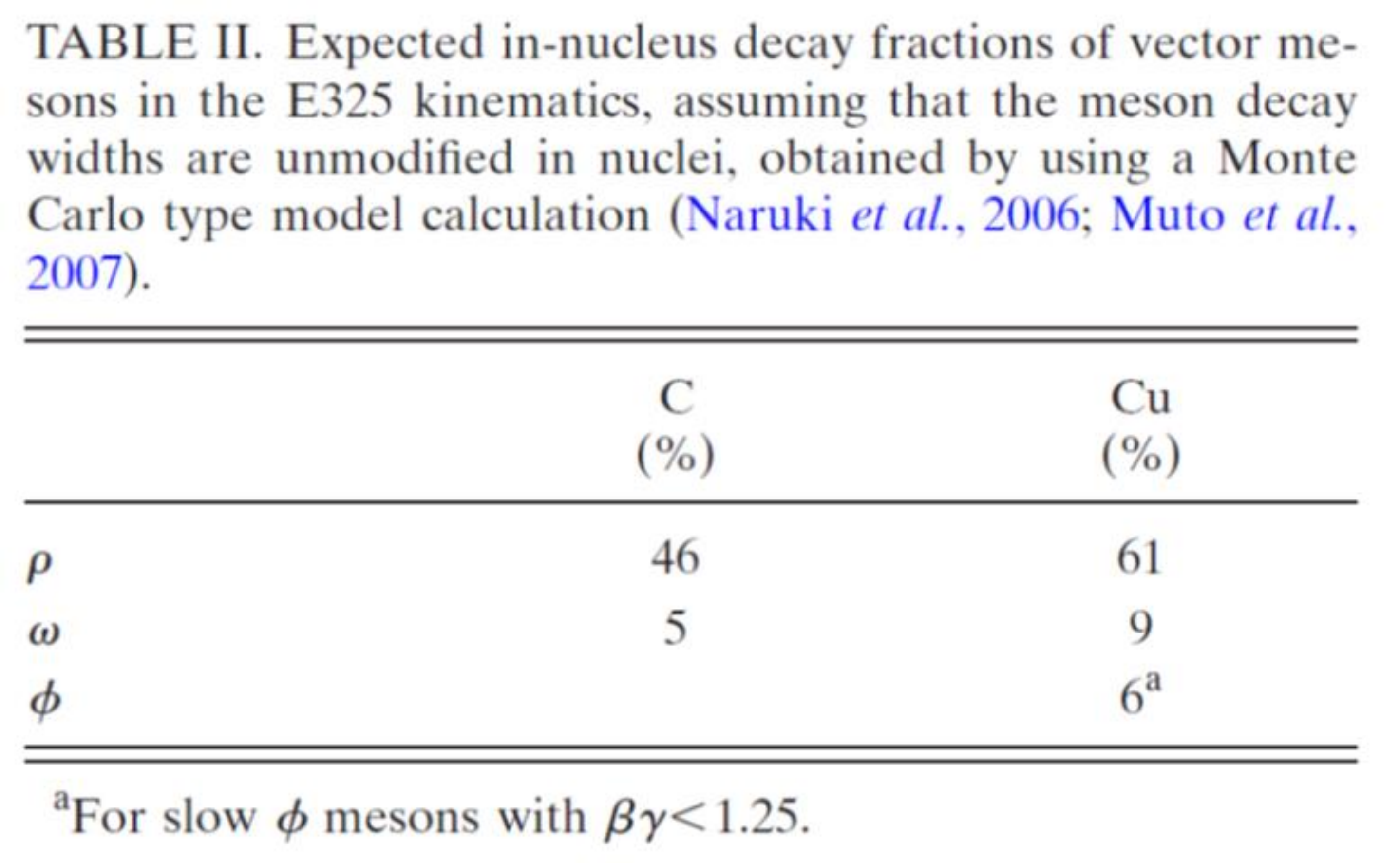


TABLE II. Expected in-nucleus decay fractions of vector mesons in the E325 kinematics, assuming that the meson decay widths are unmodified in nuclei, obtained by using a Monte Carlo type model calculation (Naruki *et al.*, 2006; Muto *et al.*, 2007).

	C (%)	Cu (%)
ρ	46	61
ω	5	9
ϕ		6[a]

[a] For slow ϕ mesons with $\beta\gamma < 1.25$.

Taken from: R.S. Hayano and T. Hatsuda, Rev. Mod. Phys. **82**, 2949 (2010).

How can this result be understood?

Let us examine the OPE at finite density more closely:

$$c_2(\rho) = c_2(0) + \rho\Big[\underset{\substack{\Updownarrow \\ -83\ \text{MeV}}}{-\frac{2}{27}M_N^0} + \underset{\substack{\Updownarrow \\ 2.2\sigma_{sN}}}{2m_s\langle N|\bar{s}s|N\rangle} + \underset{\substack{\Updownarrow \\ 38\ \text{MeV}}}{A_1^s M_N} \underset{\substack{\Updownarrow \\ -31\ \text{MeV}}}{-\frac{7}{12}\frac{\alpha_s}{\pi}A_2^g M_N}\Big]$$

↳ $\sim 2.2\rho\Big[\big(\frac{\sigma_{sN}}{1\text{MeV}}\big) - 33\Big]\text{MeV}$

↳ Dimension 4 terms governs
the behavior of the ϕ meson

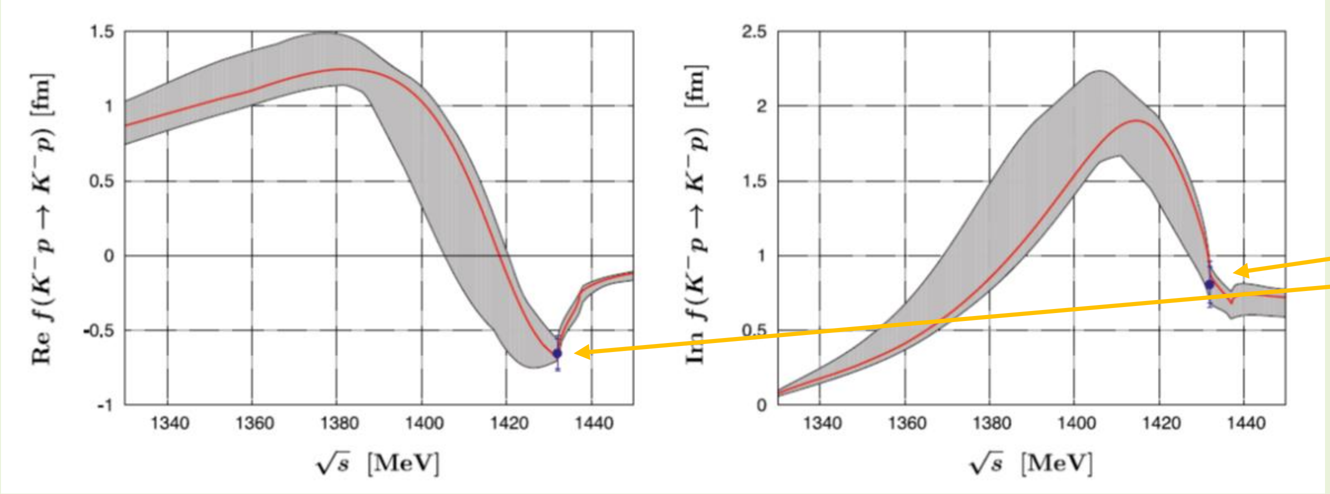
More on the free KN and $\bar{K}$N scattering amplitudes

For KN: Approximate by a real constant (↔ repulsion)

T. Waas, N. Kaiser and W. Weise, Phys. Lett. B **379**, 34 (1996).

For $\bar{K}$N: Use the latest fit based on SU(3) chiral effective field theory, coupled channels and recent experimental results (↔ attraction)

Y. Ikeda, T. Hyodo and W. Weise, Nucl. Phys. A **881**, 98 (2012).



K^-p scattering length obtained from kaonic hydrogen (SIDDHARTA Collaboration)